

The Strength and Stiffness of Geocell Support Packs

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Die Sterkte en Styfheid van Geosel Bestuttingspakke

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Summary

The strength and stiffness of geocell support packs

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In the last couple of decades, geocell reinforced soil systems have been used in challenging new applications. Although the widely different application of cellular confinement systems demand a better understanding of the fundamental behaviour of the functioning of the cellular reinforced soil systems, surprisingly little research on the fundamental behaviour of the structures and the interaction of the components has been done.

A research project has been initiated at the University of Pretoria and this thesis constitutes the first step in achieving an understanding in the functioning of geocell reinforced soil systems. This thesis is focused specifically on the geocell support pack

configuration. However, the research output is not limited to this configuration and may find wider application.

The support packs were studied at a width to height ratio of 0.5. The fill material used in this study is classified gold tailings from the Witwatersrand Complex and the geocell membranes were manufactured from a thin (nominal thickness of 0.2 mm) High Density Polyethylene (HDPE) sheet.

This study provides an understanding of the functioning of the geocell support pack by studying the constitutive behaviour of the fill and membrane material and their interaction, as well as the influence of multiple cells on the composite structures.

The behaviour of the classified tailings material is interpreted in terms of Rowe's stress-dilatancy theory and a simple robust constitutive model for the material behaviour is developed. The stress-strain behaviour of the HDPE membranes is strain-rate-dependent and two simple mathematical models for the strain-rate-dependent stress-strain behaviour of the membranes are developed.

An analytical calculation procedure for obtaining the stress-strain behaviour of the fill confined with a single geocell is developed with which some of the shortcomings of the previously presented theories are addressed. This procedure uses the models for the fill and membrane behaviour developed as part of this study.

The interaction of adjacent cells in a multiple cell geocell structure, influences its behaviour. This thesis shows that, with exception of low axial strain levels, the efficiency of a structure consisting of multiple cells of a certain size is lower than a single cell structure with the same cell size and fill. These results are contrary to previously published opinion. A method for quantifying the efficiency of a multiple cell pack is also developed.

Key words: Geocell, classified tailings, geocell reinforced soil, stope support, Hyson Cells.

Samevatting

Die sterkte en styfheid van geosel bestuttingspakke

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Gedurende die laaste paar dekades is geosel-versterkte grondsisteme in 'n wye verskeidenheid van toepassings gebruik waarvan sommige die grense van ons begrip aangaande die fundamentele gedrag van geosel-versterkte grondsisteme toets. Verbasend min navorsing is oor die fundamentele gedrag van die geosel-grondstruktuur en die interaksie van die samestellende komponente gedoen.

By die Universiteit van Pretoria is 'n navorsingsprogram van stapel gestuur om 'n beter begrip vir die funksionering van geosel-versterkte grondstrukture te ontwikkel . Hierdie proefskrif verteenwoordig die eerste stap in die bereiking van hierdie doelwit. In hierdie studie word daar op die geosel bestuttingspak konfigurasie gefokus. Die

navorsingsuitsette is egter nie beperk tot dié konfigurasie nie en mag 'n wyer toepassing vind.

Die bestuttingspakke wat in hierdie projek bestudeer is, was beperk tot 'n slankheidsverhouding (wydte tot hoogte) van 0.5. Die vulmateriaal wat gebruik is, is geklassifiseerde goudmynslik, afkomstig van die Witwatersrandkompleks en die geoselmembrane is uit 'n Hoë Digtheid Polyetelene (HDPE) membraan, 0.2 mm dik, vervaardig.

Hierdie studie help met die ontwikkeling van 'n begrip van die funksionering van geosel bestuttingspakke deur die bestudering van die spannings-vervormingsgedrag van die vulmateriaal en die membraanmateriaal en die interaksie tussen dié twee komponente. Die invloed wat die aantal selle op die gedrag van meersellige geosel-strukture het, is ook ondersoek.

Die gedrag van die geklassifiseerde goudmynslik is geïnterpreteer in terme van Rowe se spannings-volumeveranderingsteorie en 'n eenvoudige spannings-vervormingsmodel wat onsensitief vir klein veranderinge en onsekerhede in materiaalparameters is, is ontwikkel. Die spannings-vervormingsgedrag van die HDPE membrane is afhanklik van die vervormingstempo. Twee eenvoudige wiskundige modelle vir die vervormingstempo-afhanklike spannings-vervormingsgedrag word voorgestel.

'n Analitiese berekeningsprosedure om die spannings-vervormingskurwe van sand, versterk deur 'n enkel geosel, te bereken, is ontwikkel. Hiermee word sommige van die tekortkominge van die vorige teorieë aangespreek. Hierdie berekeningsprosedure maak gebruik van die spannings-vervormingsmodelle vir die grond en membrane wat as deel van hierdie studie ontwikkel is.

Die interaksie van naburige selle in 'n meersellige geosel-struktuur beïnvloed die gedrag van die saamgestelde struktuur. Hierdie studie wys dat, behalwe by klein aksiale vervormings, is die doeltreffendheid van 'n meersellige geosel struktuur met selle van 'n bepaalde grootte, laer as 'n enkelsel-struktuur met dieselfde selgrootte en vulmateriaal. Hierdie bevinding staan in teenstelling met vorige gepubliseerde opinie. 'n Metode is ook ontwikkel om die doeltreffendheid van die meersellige pak te kwantifiseer.

Sleutelterme: Geoselle, geklassifiseerde goudmynslik, geosel-versterkte grond, afboubestutting, Hyson Cells.

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