MARKOVIAN MODELLING

IN BUSINESS RISK ANALYSIS

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ABSTRACT

Risk analysis in a business context is largely intangible, allowing limited opportunity for the development of means by which the risk can be more securely assured, and in the long run, appropriately managed. This paper proposes that by means of Markovian analysis a basis can be established for the development of appropriate models. From the analysis, differential equations are established to solve into meaningful formulae for the system and solved via numerical methods. The use and development of this approach is further illustrated by means of an example.

OPSOMMING

Risiko-analise, in die besigheidsomgewing, is 'n relatief wasige vakgebied wat beperkte geleentheid bied vir die ontwikkeling van prosedures waarmee die risiko met groter sekerheid bepaal kan word en oor die langtermyn toepaslik bestuur kan word. In hierdie artikel word voorgestel dat Markovanalise gebruik kan word om 'n basis te verskaf vir die ontwikkeling van bruikbare modelle. Uitgaande van die analise kan differensiaal-vergelykings opgestel word waarmee die gedrag van die stelsel beskryf kan word. Hierdie differensiaalvergelyking lan met behulp van numeriese metodes opgelos word. Die gebruik en ontwikkeling van hierdie metodiek word geïllustreer deur 'n voorbeeld.

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Project risk [2,4,5]

Understanding risk inherent in a project is important to the investor as well as the other stakeholders (A business change initiative of significant stature is typically undertaken in project form). The investor must thoroughly understand the project's flexibility, while on the other hand the other stakeholders, as early as the contemplation stage, must make provision for contingencies in order that they do not find themselves in a compromised position due to an oversight on their part. The assumption of the reasonable person is made where the path chosen, is the one not necessarily the most frugal, but the most reliable. Reliability impacts on cost-effectiveness albeit indirectly.

Risk pertains to the chance that an outcome may not prove to be as planned. This definition implies that risk revolves around the concept "planned", even though the occurrence may be either predominantly unfavourable or coincidentally favourable. This concept of residing in an unplanned state or transferring to another state (or corrective state) is the precept upon which model development presented in this paper is based. Markovian analysis is well suited to this type of problem.

Markovian analysis [1,3]

The state of a project and its components can be likened to an engineering system. Assume that a system's components can each fail at a specific hazard rate (λ_i) or be repaired at a specific repair rate (μ_i). Depending on the combination of the state of failure and the state of repair of the various components, the system will acquire a profile. This profile will have cost associated with it as well as decisions regarding re-emergence to the base state of satisfactory repair. Consider the following example. A system comprises of two components providing three states, namely:

- Both components functioning correctly (state S_0).
- One component is in a state of failure (state S_1).
- Both components is in a state of failure (state S_2).

If both components are repairable, the situation may be modelled as shown in figure 1.

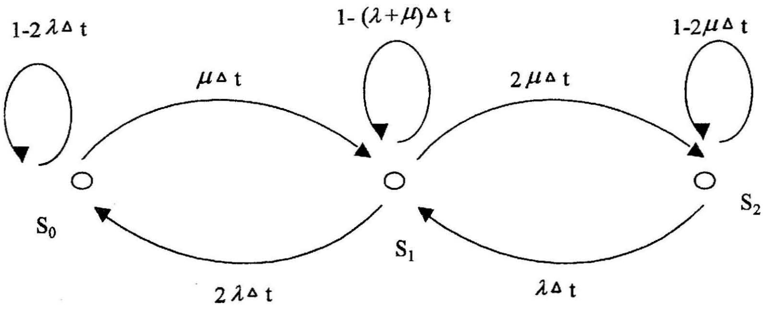

Figure 1 - Simple three state Repairable System

The differential equations for the diagram (figure 1) are:

$$\frac{\delta P_{s_0}(t)}{\delta t} = -2\lambda P_{s_0}(t) + \mu P_{s_1}(t)$$

$$\frac{\delta P_{s_1}(t)}{\delta t} = 2\lambda P_{s_0}(t) - (\mu + \lambda) P_{s_1}(t) + 2\mu P_{s_1}(t)$$

$$\frac{\delta P_{s_2}(t)}{\delta t} = \lambda P_{s_1}(t) - 2\mu P_{s_2}(t)$$

By means of Laplace transforms (or other appropriate means), these differential equations may be solved and values for $P_{s_0}(t)$, $P_{s_1}(t)$ and $P_{s_2}(t)$ obtained. The reliability of the system is:

$$R_s(t) = P_{s_0}(t) \quad \text{for a series system, and}$$

$$R_p(t) = P_{s_0}(t) + P_{s_1}(t) + P_{s_2}(t) \quad \text{for a parallel system.}$$

$R_{s/p}(t)$ implies the expected reliability of the system at a specific point in time taking into account the distribution governing the hazard or repair rate of the various components. In project terms, the hazard rate addresses the probability of a project task falling into an undesirable state, while the repair rate performs the actions (and naturally the cost incurred) taken to place the project into a position where it can be successfully continued.

Application [2]

Case background

Consider a South African based textile business producing yarn and socks[1]. With the previous dispensation, it was reasonably lucrative to establish a labour intensive business in one of the homelands like Bophuthatswana. The reason for this was the incentives that the administrations put forward to encourage economic development and employment. This form of aid was further assisted by the banning of union activity, even in the light of minimal wages and poor related measures of remuneration.

The breaking up of the previous regions and the reunification of the country has resulted in the elimination of these business "privileges". The business is now subject to a uniform labour law with higher minimal wage and remuneration levels. This can indeed now be enforced by either law or legalised collective labour action.

The textile industry in South Africa is under strain, facing competitive pressures particularly from the so-called Asian tigers. They are able to produce at huge volumes with cost structures lower than locally available. For example the Asian labour cost is generally lower than the local total cost attributable to labour.

The company has already been struggling to maintain market share, but with the two new business drivers, i.e. the higher labour cost pressures and the increased Asian competition, it faces no alternative but to make a fundamental change to the way that business is done. The company brought in a business consultancy, which assessed the current operations in addition to the alternatives available. Two clear options emerged, namely:

- Close down the sock operations in the old Bophuthatswana region, open up a material weaving operation in the Western Cape and move the yarn facility down to this site as well.

- Reduce the current labour complement by 60% while maintaining near minimal remuneration structures, terminate the non-profitable sock lines, eliminate preferential internal stock sales and outsource the distribution business.

These two options were put to the board for evaluation. The first option was terminated due mainly to the start-up effort required, i.e. new staff, new site, relocation costs and a relatively new business. The second option was therefore selected, not due to its attractiveness, but because it was the only other feasible option available.

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[1] While this application is based on realistic events, individuals and organisations, it in no way refers to any particular individual or organisation.

Management involved labour where the future was laid out. The deal was that if they achieved a particular target of improvement (35%) on a sustainable basis, then after two years, it would be possible, not only to improve the remuneration base, but also pay out meaningful bonuses. Negotiations with the shop stewards was very difficult for two reasons, namely (1) management still had a bias for the pre-unionised paradigm and (2) labour was immature in union business politics. Nevertheless, the targets were set amidst a feeling from labour that they were on the wrong end of a win-lose situation.

Application modelling

In order to analyse the dynamics of the possible business states at the macro level, Markovian models were employed [3]. The state diagram is shown in figure 2.

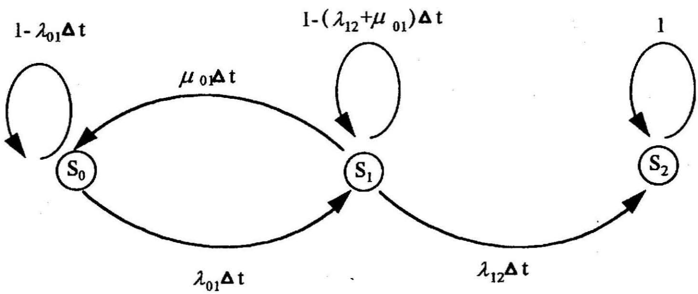

Figure 2 - Textile Business State Change Diagram

- State S_0 refers to the starting state of the business before any change takes place.
- State S_1 refers to the state of the company in the event of major labour problems, strikes etc. which will cause significant damage to the business.
- State S_2 refers to the business being liquidated due to irreparable damage from the labour problems. This is an absorbing state as indicated.
- μ_{01} refers to the "repair rate" between states S_0 and S_1.
- λ_{01} refers to the "hazard rate" between states S_0 and S_1.
- λ_{12} refers to the "hazard rate" between states S_1 and S_2.

The following equations are applicable :

$$\frac{\delta P_{s_0}(t)}{\delta t} = -\lambda_{01} P_{s_0}(t) + \mu_{01} P_{s_1}(t)$$

$$\frac{\delta P_{s_1}(t)}{\delta t} = \lambda_{01} P_{s_0}(t) - (\mu_{01} + \lambda_{12}) P_{s_1}(t)$$

$$\frac{\delta P_{s_2}(t)}{\delta t} = \lambda_{12} P_{s_1}(t)$$

This can be solved by means of Laplace transformations, as follows :

$$sP_{S_0}(s) - P_{S_0}(0) = -\lambda_{01} P_{S_0}(s) + \mu_{01} P_{S_1}(s)$$

The business change starts at state

S_0, $\quad P_{S0}(0) = 1$, $\quad$ and $P_{S1}(0) = 0$

therefore:

$$P_{S_0} = \frac{1 + \mu_{01} P_{S_1}(s)}{s + \lambda_{01}}$$

Similarly,

$$P_{S_1} = \frac{\lambda_{01} P_{S0}(s)}{s + (\mu_{01} + \lambda_{12})}$$

and therefore:

$$P_{S_0}(s) = \frac{s + \mu_{01} + \lambda_{12}}{s^2 + s(\mu_{01} + \lambda_{12} + \lambda_{01}) + \lambda_{01}\lambda_{12}}$$

If we let the roots of the equation be x and y respectively:

$$P_{S_0}(s) = \frac{s + \mu_{01} + \lambda_{12}}{(s-x)-(s-y)} = \frac{A}{s-x} + \frac{B}{s-y}$$

By substitution, solving for A and B respectively, provides the following results:

$$B = \frac{y + \mu_{01} + \lambda_{12}}{y-x}$$

$$A = \frac{x + \mu_{01} + \lambda_{12}}{x-y}$$

Applying the inverse Laplace transform:

$$P_{S_0}(t) = Ae^{xt} + Be^{yt}$$

and solving for x and y results in the following:

$$s^2 + s(\mu_{01} + \lambda_{12} + \lambda_{01}) + (\lambda_{01}\lambda_{12}) = 0$$

$$s = \frac{-(\mu_{01} + \lambda_{12} + \lambda_{01}) \pm \sqrt{\mu_{01}^2 + 2\mu_{01}\lambda_{12} + 2\mu_{01}\lambda_{01} - 2\lambda_{01}\lambda_{12} + \lambda_{01}^2 + \lambda_{12}^2}}{2}$$

It is now possible to derive the value of $P_{S0}(t)$ for various values of t, hazard rates and repair rates. Analysing the sensitivity of the hazard and repair rates provides insight into those types of risk management actions that can be put into place as illustrated in table 1 and figure 3.

Table 1 – Scenarios Using Various Repair and Hazard Rates

	$P_{S0}(1)$	$P_{S0}(2)$	$P_{S0}(3)$
λ_{01}	0.05	0.025	0.05
λ_{12}	0.05	0.05	0.05
μ_{01}	0.5	0.5	0.75
Ps(1 day[2])	0.96	0.98	0.97
Ps(1 week)	0.90	0.95	0.93
Ps(1 month)	0.81	0.90	0.86
Ps(6 months)	0.43	0.65	0.56
Ps(1 year)	0.20	0.43	0.32
Ps(2 years)	0.04	0.20	0.11
Ps(3 years)	0.01	0.09	0.04
Ps(5 years)	0.00	0.02	0.00
Ps(10 years)	0.00	0.00	0.00

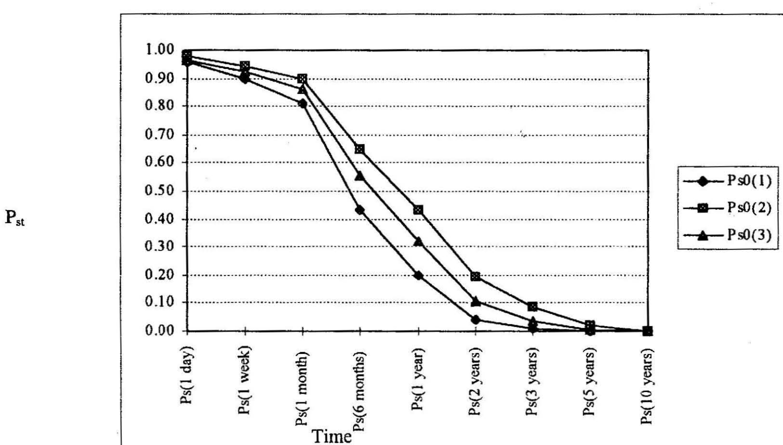

Figure 3 - Scenarios Using Various Repair and Hazard Rates

[2] 1 day = 1 time unit.

P_{S0}(1) refers to a situation where the two "hazard rates" are equal and the "repair rate" is 0.5. This is also used as the control values in the sensitivity analysis. In order to determine the requirements of risk management, both the "hazard rate" (λ_{01}) [**P_{S0}(2)**] and the "repair rate" (μ_{01}) [**P_{S0}(3)**]is improved by 50% (or the values of 0.025 and 0.25). From figure 3, it is clear that more leverage can be gained from addressing the "hazard rate", than by addressing the "repair rate". This would imply that appropriate pro-active risk management would be a better strategy for lengthening the life of a business intervention solution.

Conclusion

Markovian modelling provides useful means for the analysis of business related risks. It not only provides the ability to understand the probabilities attributable to the risks of various business states, but also gives insight into their progression over time. The disadvantage however is the mathematical computational effort required which increases directly in relation with the system complexity under consideration.

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