ANALYTICAL MODELLING OF VISCO-ELASTIC BEHAVIOUR OF HOT-MIX ASPHALT

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ABSTRACT

As part of the revision of the South African Pavement Design Method (SAPDM), laboratory testing was conducted to obtain the dynamic (Complex) modulus |*E**| of hot-mix asphalt (HMA) samples. Dynamic modulus gives an indication of linear visco-elastic (LVE) behaviour of HMA materials at different temperatures and loading frequencies; and is required for computation of stresses, strains and displacements in flexible pavement analysis and design. Laboratory tests to obtain dynamic modulus are normally conducted at limited range of temperatures and loading frequencies. In order to characterize HMA mixes for pavement analysis, sigmoidal function master curves are constructed at different temperatures and loading frequencies using a time-temperature superposition principle.

Instead of using the sigmoidal function, this paper presents an alternative approach for characterising the LVE behaviour of HMA materials. This approach is based on the use of three rheological models, namely, Burger's, Huet-Sayegh and the generalised 2S2P1D. The model parameters for all three rheological models were successfully determined. The master curves were developed for all HMA mixes studied. The Cole-Cole and the Black diagrams were determined. Based on the results presented in this paper, the Huet-Sayegh and the Generalised 2S2P1D models appear to predict the LVE behaviour of HMA mixes more effectively than the Burger's model.

1 INTRODUCTION

The revision of the South African Pavement Design Method (SAPDM) requires advanced characterisation of material properties of typical South African hot-mix asphalt (HMA) mixes. As part of the process, laboratory testing was conducted at the CSIR pavement material testing laboratory to obtain dynamic (complex) modulus |*E**| of HMA samples. The dynamic modulus is a parameter that gives indication of the linear visco-elastic (LVE) behaviour of HMA materials. The dynamic modulus is required in order to compute stress, strains and displacements in flexible pavements.

The SAPDM dynamic modulus input data should be measured by means of compressive uniaxial testing using 150 mm high cylindrical specimens according to the test protocols proposed by Maina and Anochie-Boateng, (2010). The laboratory dynamic modulus testing is conducted within a selected range of temperatures and frequencies. However, the characterisation of LVE behaviour of HMA materials and prediction of pavement response requires dynamic modulus values over a wider range of temperatures and frequencies. In order to characterise LVE behaviour of HMA materials over a wider range of temperatures and loading frequencies including those not covered by laboratory testing, different models

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have been proposed. Such models include the commonly used sigmoidal function master curves using the time-temperature superposition principle (NCHRP 1-37A, 2004, Chailleux et al, 2006).

The LVE behaviour of HMA materials can also be characterised by using rheological models. The rheological models have the advantage of utilising physical elements to describe LVE behaviour of HMA materials (Xu and Solaimanian, 2009). Examples of such models include Burger's, Huet-Sayegh and the Generalised 2S2P1D (Olard and Di Benedetto, 2003; Xu and Solaimanian, 2009). The Huet-Sayegh and the generalised 2S2P1D models have been shown to be suitable for characterising the LVE of HMA materials over a wider range of temperatures and loading frequencies, whereas the Burger's model is suitable within a limited range of temperatures and frequencies (Nilsson et al, 2002; Olard and Di Benedetto, 2003; Xu and Solaimanian, 2009). Recently, Maina et al (2011) used Burger's and Huet-Sayegh models to characterise LVE behaviour of one standard South African HMA mix i.e. continuously graded asphalt mix with 60/70 penetration grade binder.

The objective of this paper is to present the results of the application of rheological models to characterise LVE behaviour of six typical South African HMA mixes. The study used dynamic modulus of the asphalt samples data from laboratory tests conducted at the CSIR pavement material laboratory. The dynamic modulus data were used to model the LVE behaviour of the HMA mixes studied using three rheological models namely, Burger's, Huet-Sayegh and the generalised 2S2P1D.

2 LINEAR VISCO-ELASTIC (LVE) BEHAVIOUR

2.1 Dynamic modulus

The dynamic modulus gives indication of LVE behaviour of HMA materials at different temperatures and loading frequencies. Several researchers have shown that the behaviour of HMA materials can be considered to be linear at small strain domain (Witczak et al, 2002; Olard and Di Benedetto, 2003; and Dougan et al, 2003). Therefore, the behaviour can be modelled using the linear viscoelastic theory. The stress-strain behaviour of LVE materials under a continuous sinusoidal loading is defined by the dynamic modulus (ASTM D 3497, 2003; NCHRP 1-37A, 2004; AASHTO TP 62, 2009). The dynamic modulus has real and imaginary parts that define the elastic and viscous behaviour of LVE materials. The absolute value of the dynamic modulus is defined as the material's dynamic modulus $|E^*|$.

In the laboratory testing of the dynamic modulus, the sinusoidal loading is applied to cylindrical asphalt specimen at a range of temperatures and frequencies. For sinusoidal loading, the applied stress and the corresponding strain response can be expressed in a complex form by equations 1 and 2 respectively (Huang, 2004).

$$\sigma^* = \sigma_0 e^{i\omega t} \quad \textbf{(1)}$$

$$\varepsilon^* = \varepsilon_0 e^{i(\omega t - \delta)} \quad \textbf{(2)}$$

Where σ is the applied stress, σ_0 is the stress amplitude; ε is the strain response, ε_0 is the strain amplitude; ω is angular frequency, which is related to frequency by $\omega=2\pi f$; $f=1/T$; t is time, and T is period; δ is the phase angle related to the time the strain lags behind the stress. Phase angle is an indicator of the viscous (or elastic) properties of the viscoelastic material. For a pure elastic material, δ=0º, and for a pure viscous material, δ=90º. Mathematically, the dynamic modulus is defined as the maximum (peak) dynamic stress divided by the recoverable maximum (peak) axial strain.

From Equations 1 and 2 the dynamic modulus, $E^*(i\omega)$, is defined as the dynamic quantity in Equation 3.

$$E^*(i\omega) = \frac{\sigma^*}{\varepsilon^*} = \frac{\sigma_0}{\varepsilon_0} e^{i\delta} = E' + iE'' \quad \textbf{(3)}$$

The real part (E') of the dynamic modulus is the storage modulus (considered as elastic part) and the imaginary part (E'') is the loss modulus (considered as viscous part). The dynamic modulus $|E^*|$ is the absolute value of the dynamic modulus, which is defined mathematically in equation 4.

$$\left| E^* \right| = \frac{\sigma_0}{\varepsilon_0} \quad \textbf{(4)}$$

2.2 Rheological models

A combination of springs (elastic element) and linear dash-pots (viscous elements) in series or parallel can be used to describe LVE behaviour of asphalt materials (Huang, 2004). Models built in this manner are known as rheological or mechanical models. Literature contains many rheological models which include the Maxwell, Kelvin, generalised Maxwell, generalised Kelvin, Burger's, Huet-Sayegh and the Generalised 2S2P1D (Olard and Di Benedetto, 2003; Xu and Solaimanian, 2009).

Nilsson et al (2002) showed that the burger's model is suitable for describing visco-elastic properties within a limited range of temperatures and frequencies, whereas the Huet-Sayegh model is suitable for characterising the behaviour of LVE over a wider range of temperatures and frequencies. In order to account for the 'pseudo' permanent deformation of asphalt materials, Olard and Di Benedetto (2003) proposed a generalised 2S2P1D model which is based on the generalised Huet-Sayegh model.

In this paper, the three rheological models, namely, Burger's, Huet-Sayegh and the generalised 2S2P1D are used to model the LVE behaviour of six South African HMA mixes. These models are briefly discussed in sections 2.2.1 to 2.2.3 below.

2.2.1 Burger's model

The Burger's model is represented by an arrangement of one Kelvin element (Delayed Elastic) and one Maxwell element (Elastic Viscous) connected in series. Figure 1 shows the representation of the Burger's model (Xu and Solaimanian, 2009). The Kelvin model is used to represent the delayed elastic response, while the elastic spring in the Maxwell model represents the instantaneous elastic response and the dash-pot represents the creep behaviour of the visco-elastic material. Using the Burger's model, the dynamic modulus is given by equation 5.

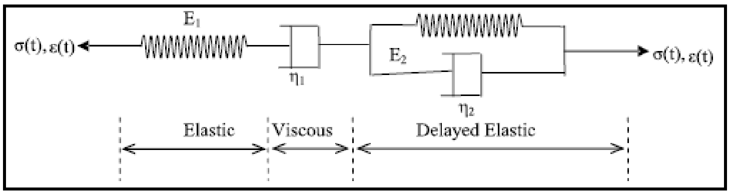


Figure 1: Representation of Burger's model (Xu and Solaimanian, 2009)

$$E^*(i\omega) = \frac{E}{1+(i\omega\frac{\eta}{E})^{-1}} + \frac{E_1}{1+(i\omega\frac{\eta_1}{E_1})^{-1}} \qquad \textbf{(5)}$$

Where, E^* is the dynamic modulus, i^2 is a complex number defined by $i^2 = -1$, ω is frequency (Hz), E , E_1, η and η_1 are constants.

2.2.2 Huet-Sayegh model

The Huet-Sayegh model consists of combinations of two parallel branches as shown in Figure 2 (Xu and Solaimanian, 2009). The first branch has two parabolic dash-pots (*h* and *k*) and a spring ($E_1 = E_\infty - E_0$) in series. E_∞ represents a purely elastic modulus whereas E_0 represents the long term behaviour of materials. The second branch consists of a single spring (E_0). The dynamic modulus using Huet-Sayegh model is given by equation 6 (Olard and Di Benedetto, 2003).

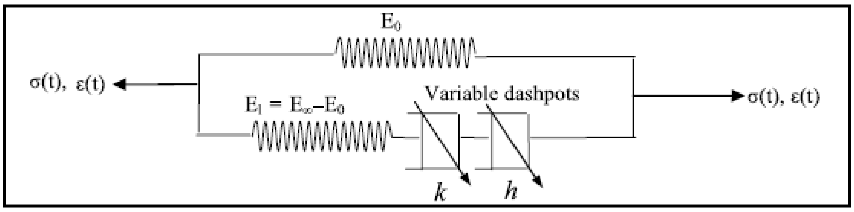


Figure 2: Representation of Huet-Sayegh model (Xu and Solaimanian, 2009)

$$E^*(i\omega\tau) = E_0 + \frac{E_\infty - E_0}{1+\delta(i\omega\tau)^{-k} + (i\omega\tau)^{-h}} \qquad \textbf{(6)}$$

Where, E^* is the dynamic modulus, i^2 is a complex number defined by $i^2 = -1$, ω is frequency (Hz), E_∞ , E_0, *δ, k, h,* and τ are constants with *0<k<h<1*. The temperature dependency of the Huet-Sayegh model is governed by parameter *τ*. The parameter is known as characteristic time and is expressed as function of temperature by using equation 7.

$$\ln(\tau) = a + bT + cT^2 \qquad \textbf{(7)}$$

Where, T is temperature and *a, b* and *c* are regression constants.

2.2.3 2S2P1D model

Olard and Di Benedetto (2003) extended the Huet-Sayegh model by adding a linear dash-spot in series with the two parabolic elements in order to account for the permanent deformation of bitumen binders and mixes. The generalized model is a combination of two springs, two parabolic creep elements and one dash-spot, and is abbreviated as 2S2P1D. Figure 8 shows the representation of the 2S2P1D model. The dynamic modulus using the generalised 2S2P1D model is given by equation 8 (Olard and Di Benedetto, 2003).

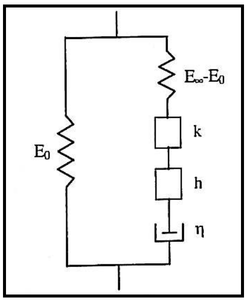


Figure 3: Representation of 2S2P1D model (Olard and Di Benedetto, 2003)

$$E^*(i\omega\tau) = E_0 + \frac{E_\infty - E_0}{1 + \delta(i\omega\tau)^{-k} + (i\omega\tau)^{-h} + (i\omega\beta\tau)^{-1}} \quad \textbf{(8)}$$

Where, E^* is the dynamic modulus, i^2 is a complex number defined by $i^2 = -1$, ω is frequency (Hz), E_∞, E_0, δ, k, h, β and τ are constants with $0<k<h<1$. Similar to the Huet-Sayegh model, the temperature dependency of the 2S2P1D model is governed by characteristic time (τ) which is given by equation 3. All the model parameters in equations 5 to 8 can be obtained by using non-linear regression analysis process using the dynamic modulus test results.

3 MATERIALS AND LABORATORY TESTING

3.1 Materials

Six types of South African HMA mixes were used in this study, namely:

- Bitumen-Treated Base (BTB) mix with a 40/50 penetration grade binder.
- Coarse continuously graded mix with A-E2 modified binder.
- Medium continuously graded mix with A-E2 modified binder.
- High Modulus Asphalt (HiMA) mix with a 20/30 penetration grade binder
- Medium continuously graded mix with a 60/70 penetration grade binder, and
- Bitumen-rubber asphalt semi-open graded mix (BRASO).

The HMA mixes were designed by Much Asphalt and prepared at the CSIR pavement material testing laboratory.

3.2 Dynamic modulus testing

A Universal Testing Machine (UTM-25) device was used for conducting the dynamic modulus testing of HMA samples using the test protocols developed by Anochie-Boateng et al. (2010), and presented by Maina and Anochie-Boateng (2010). The UTM-25 system was used to apply a continuous sinusoidal compressive load pulse on the gyratory compacted specimens (100 mm in diameter x 150 mm high) at five test temperatures (-5, 5, 20, 40, 55°C) and six loading frequencies (25, 10, 5, 1, 0.5, 0.1 Hz). Figure 4 show the UTM-25 setup and dynamic modulus test samples.

(a) (b)

Figure 4: (a) UTM-25 set up at the CSIR, (b) dynamic modulus test samples

4 DATA ANALYSIS, RESULTS AND DISCUSSIONS

4.1 Determination of rheological model parameters

The parameters of the rheological models (Burger's, Huet-Sayegh and the generalised 2S2P1D) were determined for each of the HMA mixes studied. The model parameters were determined by using non-linear least square regression of the set of laboratory measured dynamic modulus and corresponding phase angles at different frequencies and temperatures. Five test temperatures (-5, 5, 20, 40 and 55°C) and six loading frequencies (25, 10, 5, 1, 0.5 and 0.1 Hz) were used. Table 1 gives model parameters for each of the HMA mixes studied. The reference temperature used for the determination of the model parameters was 20°C, this temperature is the same as the one adopted for the SAPDM HMA materials test protocol (Maina and Anochie-Boateng, 2010). The plots of the laboratory measured dynamic modulus versus the predicted dynamic modulus using rheological models (equations 5, 6 and 8) and parameters presented in Table 1 are shown in Figures 5 to 10. It can be seen that the Huet-Sayegh and the generalised 2S2P1D give an excellent prediction of the measured dynamic modulus. On the hand, the Burger's model gives a poor prediction of the measured dynamic modulus.

Figure 11 shows plots of Cole-Cole and Black diagrams for BTB 40/50 mix. Again, the Burger's model gave poor prediction of real and the imaginary part of the dynamic modulus as compared to the Huet-Sayegh and the generalised 2S2P1D models. The plots for other mixes follow the same trend; however, due to space constraints the Cole-Cole and Black diagrams for other mixes could not be included in the paper.

Table 1: Model parameters

Burger's model						
Parameter	BTB 40/50	Coarse AE2	Medium AE2	HiMA	Medium 60/70	BRASO
$E(MPa)$	2590	574	3983	8517	1316	1638
$E_1(MPa)$	21532	16204	14439	15508	22931	10397
$\eta(MPa.s)$	100000	100000	100000	133926	100000	100000
$\eta_1(MPa.s)$	500	500	16	92	500	27
Huet-Sayegh model						
$E_0(MPa)$	90	100	80	95	45	50
$E_\infty(MPa)$	44000	44143	44143	44143	44143	44143
δ	2.09930	2.01465	2.84880	1.48776	1.59336	3.14119
k	0.14901	0.10398	0.13867	0.09319	0.15751	0.11029

h	0.65687	0.51212	0.47061	0.33497	0.53542	0.46230
τ	0.04454	0.00369	0.00414	0.03852	0.01393	0.00067
Generalised 2S2P1D model						
E_0*(MPa)*	90	95	90	95	90	80
E_∞*(MPa)*	45000	44143	44143	44143	45000	45000
δ	2.10921	2.01190	2.81089	1.48848	1.53368	3.16781
k	0.14397	0.10387	0.13787	0.09322	0.14903	0.10845
h	0.64730	0.51144	0.46818	0.33503	0.51966	0.45902
β	300	3136526	3136526	3136526	300	2000
τ	0.04100	0.00368	0.00393	0.03858	0.01164	0.00060

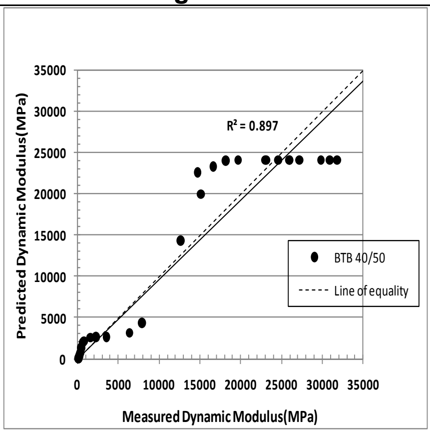
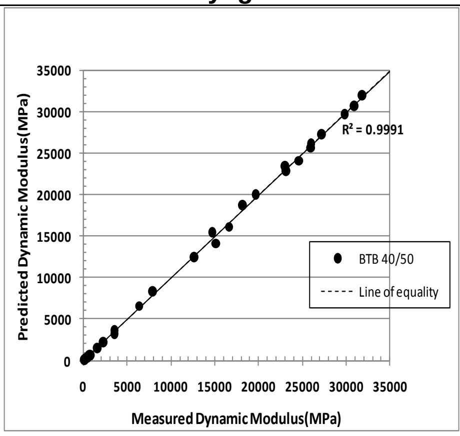
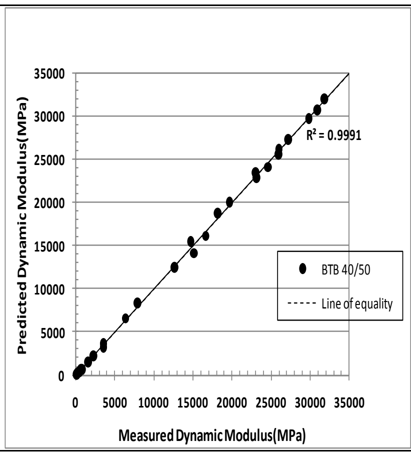
Burger's model	Huet-Sayegh model	Generalised 2S2P1D model
		

Figure 5: Measured versus predicted dynamic modulus for BTB 40/50 mix

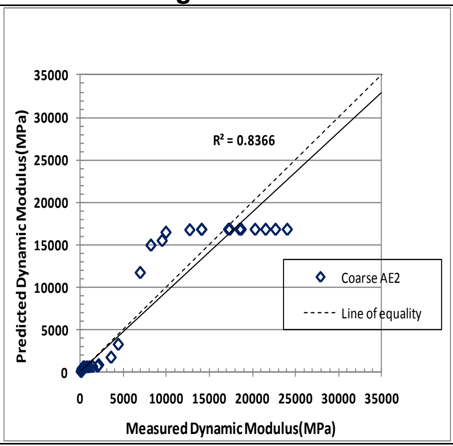
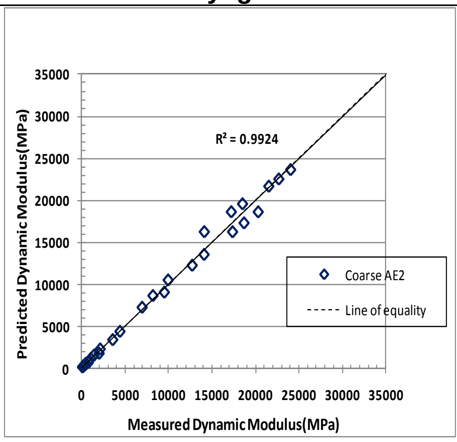
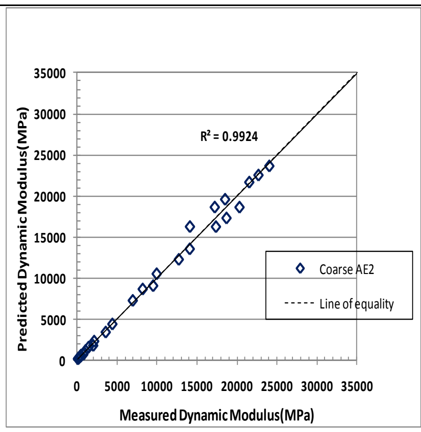
Burger's model	Huet-Sayegh model	Generalised 2S2P1D model
		

Figure 6: Measured versus predicted dynamic modulus for Coarse AE2 mix

Burger's model	Huet-Sayegh model	Generalised 2S2P1D model

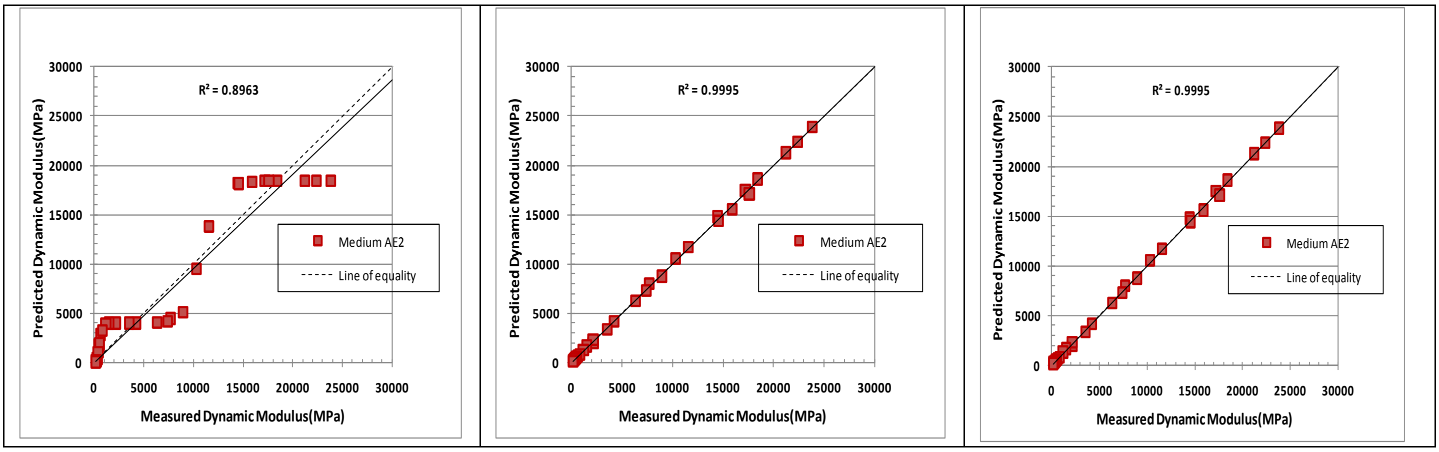


Figure 7: Measured versus predicted dynamic modulus for Medium AE2 mix

Burger's model	Huet-Sayegh model	Generalised 2S2P1D model

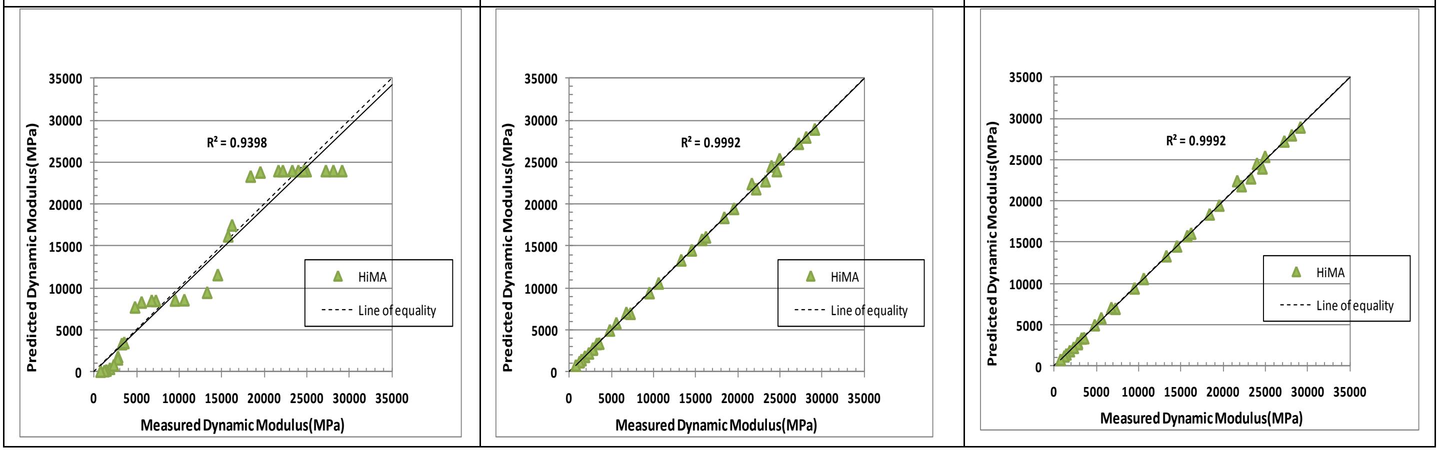


Figure 8: Measured versus predicted dynamic modulus for HiMA mix

Burger's model	Huet-Sayegh model	Generalised 2S2P1D model

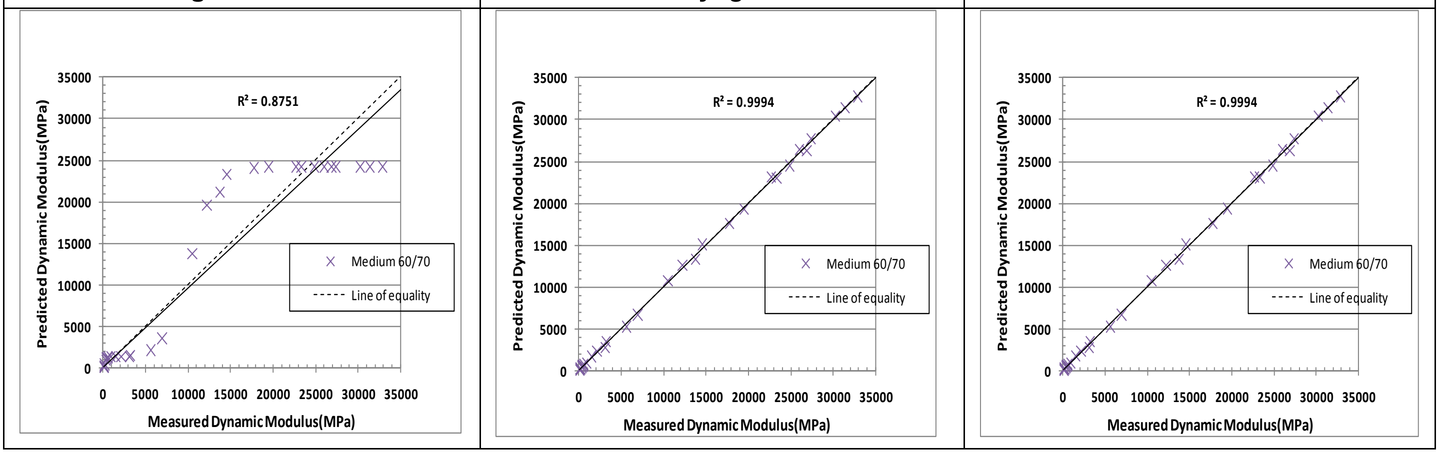


Figure 9: Measured versus predicted dynamic modulus for Medium 60/70 mix

Burger's model	Huet-Sayegh model	Generalised 2S2P1D model

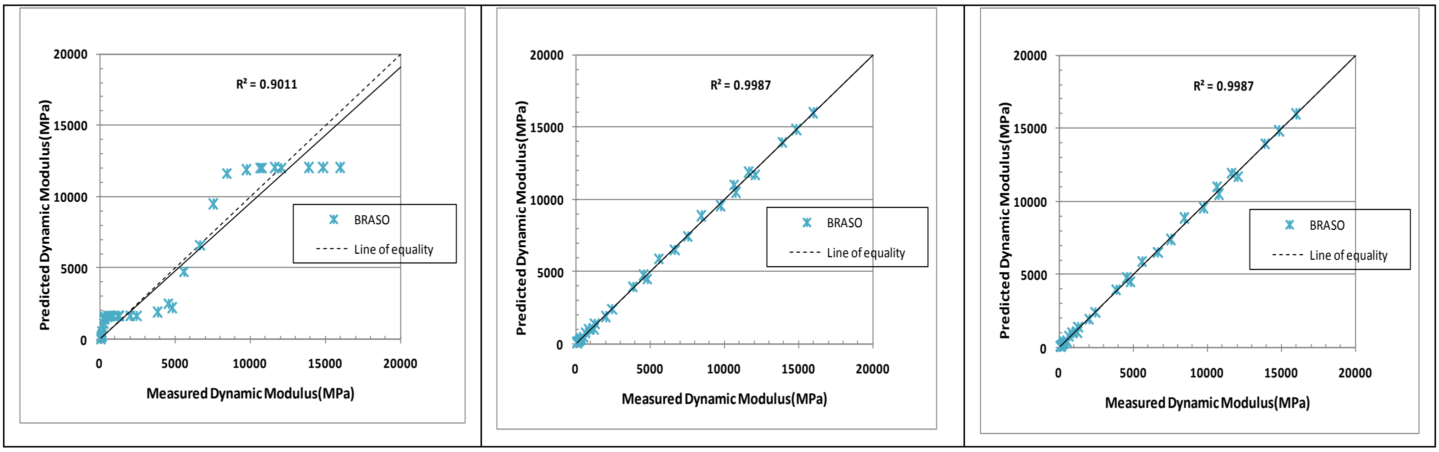


Figure 10: Measured versus predicted dynamic modulus for BRASO mix

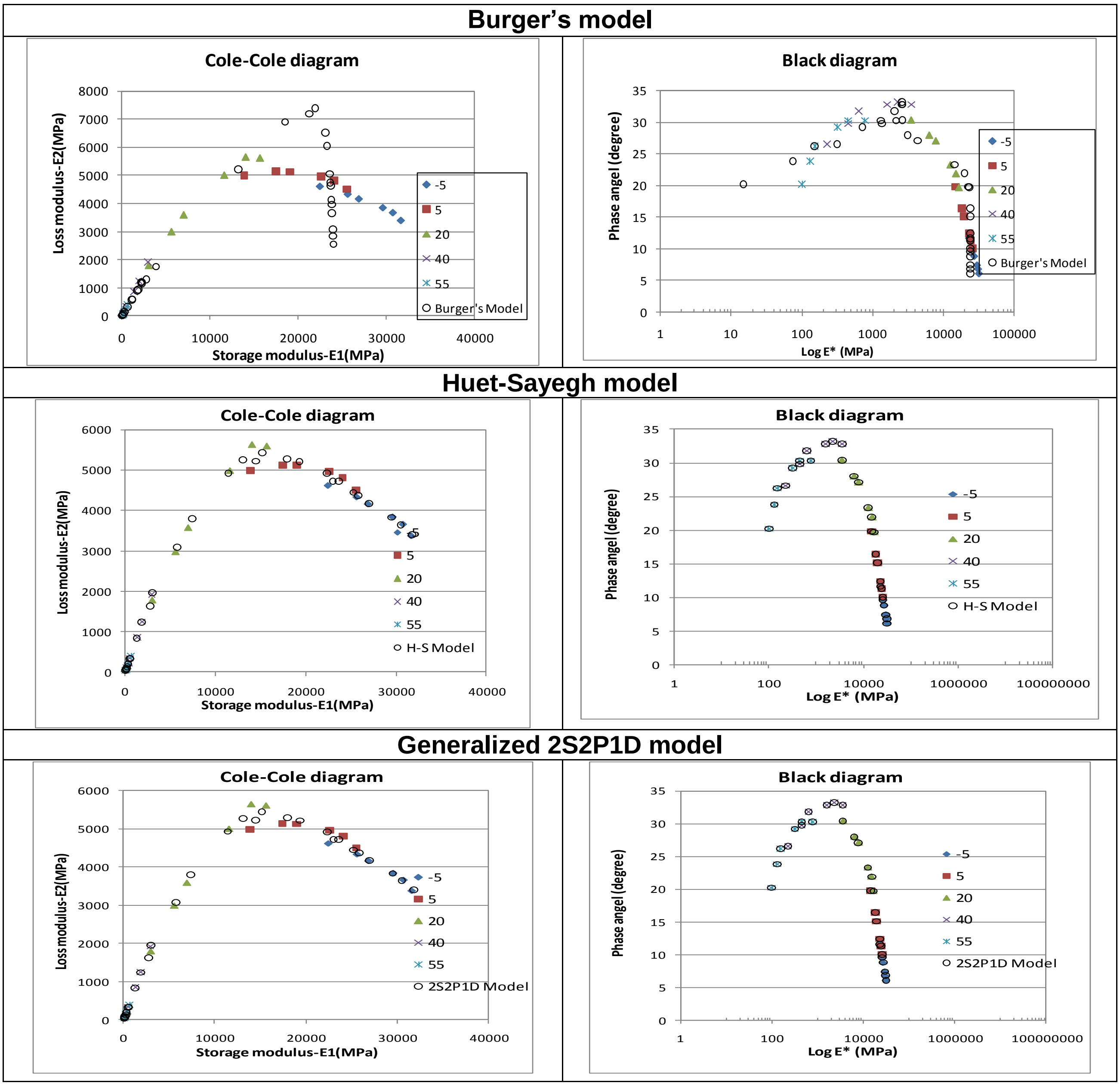


Figure 11: Cole-Cole and Black diagrams for BTB 40/50 mix

4.2 Development of master curves

Master curves are used to characterise the time-temperature dependency of dynamic modulus of asphalt materials. The time-temperature superposition principle allows the construction of master curves using dynamic modulus data obtained over a limited range of temperatures and frequencies. This enables characterisation of LVE behaviour of HMA materials over a wider range of temperatures and frequencies. The validity of time-temperature superposition principle for asphalt mixes containing modified bitumen binders is being questioned in literature (Lesueur, 2009; Di Bennedetto et al., 2010). However, Di Benedetto et al. (2010) showed that asphalt mixes containing modified bitumen binders tend to obey partial time-temperature superposition principle; hence, the master curves can still be constructed. It is recommended that future research should investigate whether this conclusion is valid for South African HMA mixes.

The temperature dependency of dynamic modulus |E*| is incorporated by the reduced frequency f_r given by equation 9. The reduced frequency is defined as the actual loading frequency multiplied by the time-temperature shift factor, $a(T)$.

$$\begin{aligned} f_r &= a(T) \times f \\ \log f_r &= \log f + \log a(T) \end{aligned} \qquad \textbf{(9)}$$

where:

f_r = frequency, Hz
$a(T)$ = shift factor as a function of temperature
T = temperature

The procedures for determination of shift factor in equation 9 are contained in CSIR HMA materials test protocol (Maina and Anochie-Boateng, 2010). The reduced frequencies obtained by using equation 9 were used together with the parameters of Burger's, Huet-Sayegh and the generalised 2S2P1D models to develop master curves of the six HMA mixes studied. The master curves are shown in Figure 12. For comparison purposes the master curves developed by using sigmoidal function are included in the Figure (Maina and Anochie-Boateng, 2010). From Figure 12, it can be seen that the Huet-Sayegh and the generalised models yielded very good results, whereas the Burger's model could not result in smooth continuous master curves. It is interesting to note that all the models ranked the dynamic modulus of the six mixes in the same way. The HiMA mix is stiffer followed by the BTB 40/50, Medium 60/70, Coarse AE2, Medium AE2 and BRASO.

Burger's model	**Huet-Sayegh model**

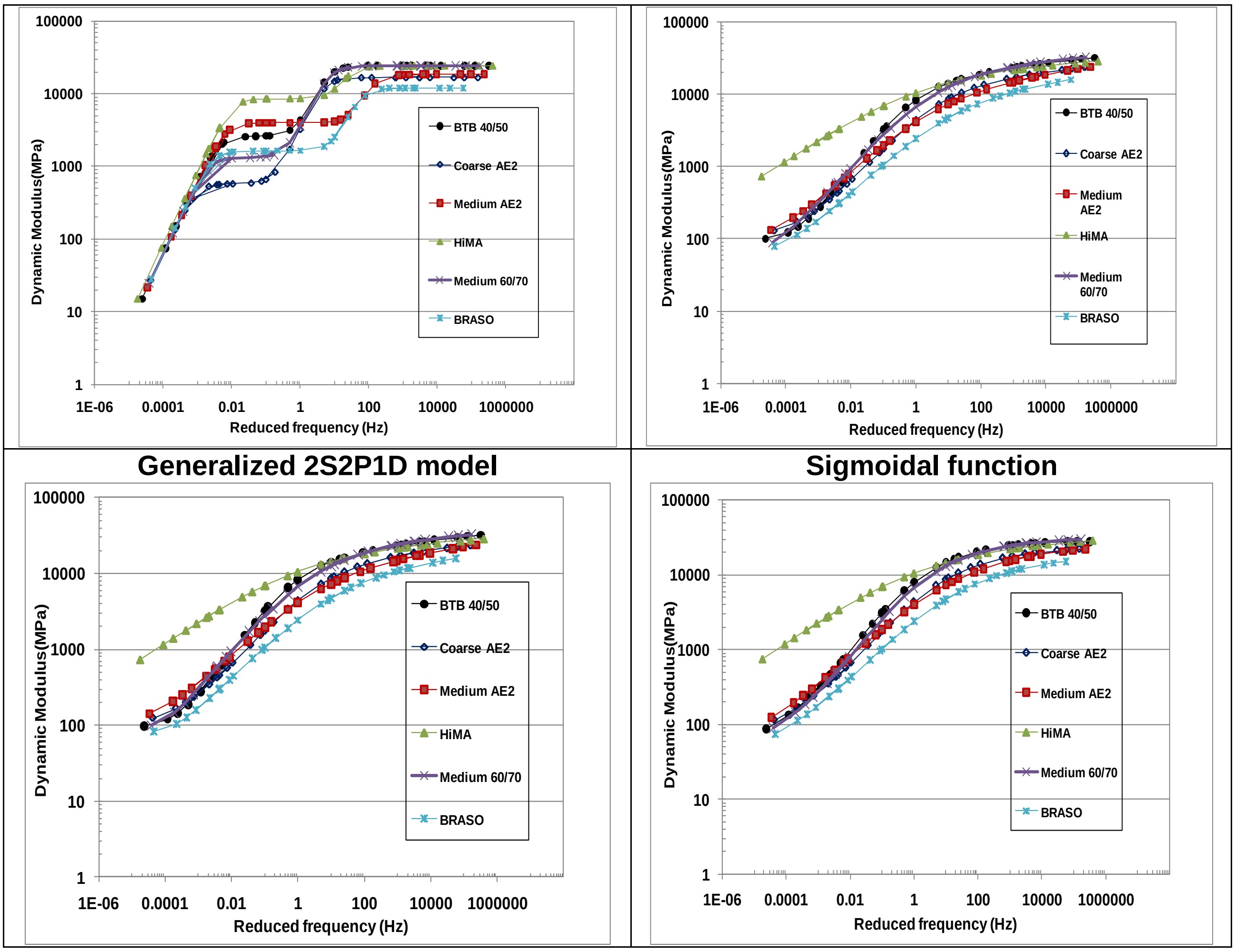


Figure 12: Master Curves of the six HMA mixes studied

5 CONCLUSIONS AND RECOMMENDATIONS

The paper presented the results of using rheological models to characterise LVE behaviour of six South African HMA mixes. Based on the results presented in this paper, the following conclusions and recommendations are made.

- The model parameters for Burger's, Huet-Sayegh and the generalised 2S2P1D models were successfully determined for the six HMA mixes studied.
- The master curves were successfully developed, and the Cole-Cole and Black diagrams were presented for all the mixes studied.
- It appears that Huet-Sayegh and the generalised 2S2P1D models describe properties of the HMA mixes studied better than the Burger's model. The Huet-Sayegh and the generalised 2S2P1D models predicted the dynamic modulus of the HMA materials accurately. Furthermore, smooth continuous master curves, Cole-Cole and Black diagrams were obtained using the two models.
- It is recommended that for the future updates of the SAPDM, the Huet-Sayegh and generalised 2S2P1D models should be considered.

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