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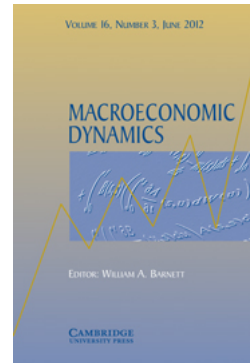
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THE USE OF A MARSHALLIAN MACROECONOMIC MODEL FOR POLICY EVALUATION: CASE OF SOUTH AFRICA

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Using a disaggregated Marshallian macroeconomic model, this paper investigates how the adoption of a set of “free market reforms” may affect the economic growth rate of South Africa. Our findings suggest that the institution of the proposed policy reforms would yield substantial growth in aggregate annual real GDP. The resulting annual GDP growth rate could range from 5.3% to 9.8%, depending on which variant of the reform policies was implemented.

Keywords: Marshallian Macroeconomic Model, Disaggregation, Transfer Functions, Macroeconomic Policy Analysis

1. INTRODUCTION

The role played by certain economic policy reforms, which we refer to as “free market reforms,” in inducing changes in economic growth has been widely discussed in the literature. Countries such as Great Britain, India, China, Estonia, Georgia, and others have experienced substantial increases in their growth rates after instituting various “free market reforms.” In Great Britain, e.g., reforms instituted by Thatcher

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that involved freeing up product and labor markets from undue restrictions, as well as tax and monetary reforms, led to considerable improvement in economic growth.

Post-apartheid South Africa has seen a succession of interesting reforms, but still the growth rate remains low (below 5%) and unemployment extremely high (23.1%).¹ The South African economy has several problems that none of the previous reforms have fully addressed. Labor unions are overwhelmingly powerful and exert a negative influence on workers' freedom to seek employment. Also, a high proportion of the labor force suffers from a lack of education, skills, and good health, which are needed to obtain employment. Further, many wishing to set up new firms find it difficult to do so under current regulations. To establish a new paradigm for the current South African economy, this paper suggests a set of policy changes similar to those that Thatcher implemented successfully in the United Kingdom during her tenure as British Prime Minister. Although controversial, her reform policies, which involved an emphasis on free enterprise and competition, produced remarkable growth in the British economy. Promoting free enterprise and competition involved adopting policies that increased the ability of firms to enter industries freely by substantially lowering the cost of entry. To assess the possible effects of lowering the cost of firm entry, we use our disaggregated Marshallian macroeconomic model (MMM-DA), which includes a cost of firm entry for each of the industrial sectors of the South African economy. As our estimates show, there is a significant and negative relationship between our proxy for the entry price and the growth of an industrial sector's real sales. Also, with higher entry prices, the model predicts a higher rate of general inflation. Last, the model predicts that lowering firm entry cost, *ceteris paribus*, generally leads to increases in output growth rate. Moreover, our model embodies a measure of labor effectiveness that represents the role played by social ingredients (health and education) in economic sectors' growth.

In addition, the Thatcher reforms involved (1) reduction of trade unions' influence, rendering labor markets much less rigid, (2) improved management of monetary policy and the money supply, and (3) a tax cut for high income groups and later the institution of a traditional "poll tax." Although much has been and could be said about these reforms, in this paper we shall just feed certain combinations of these reform measures into our MMM-DA and predict the resulting effects on important variables of the model.

An overview of the paper is as follows. In Section 2, we provide a description of (1) our model (MMM-DA) and variants of it, (2) our estimation techniques, and (3) our data. In Section 3, we discuss the fit and the predictive performance of the MMM-DA as compared to those of a benchmark autoregressive leading indicator model. Section 4 is devoted to an evaluation of the free market reforms' effects on the growth rate of the South African economy using our MMM-DA. Finally, in our concluding section, we summarize our results and indicate the direction of future research.

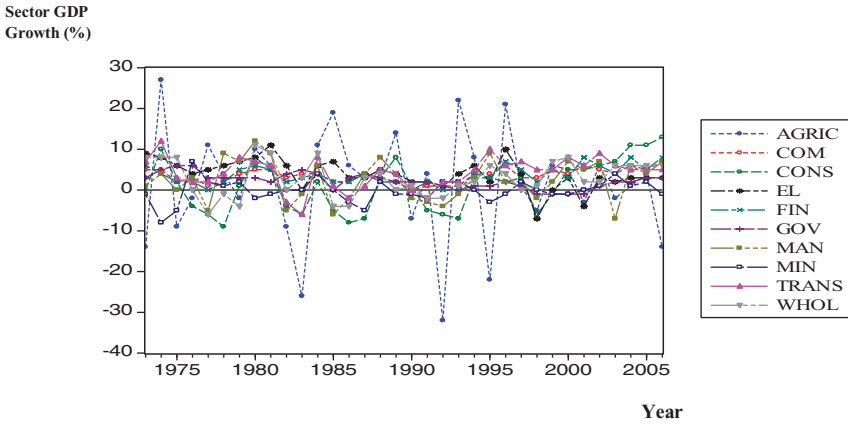


FIGURE 1. Annual growth rate of GDP per sector: data plots, 1973–2006. The growth rates of GDP at RGVA of the ten South African economic sectors are annual series in constant 2000 prices and seasonally adjusted obtained from the SARB database (<http://www.reservebank.co.za/>).

2. MODEL SPECIFICATION, ESTIMATION TECHNIQUES, AND DATA

2.1. Aggregation, Disaggregation, and Model Specification

Aggregation and disaggregation. In this paper we make use of a model disaggregated by economic sectors² of the South African economy. Good disaggregated models and data can lead to a better understanding of sectors' very different behavior; see Figure 1 for plots of the sectoral growth rates of real GDP at value-added growth.³ From Figures 1 and 2, we see that the sectors' growth rates present disparate behavior to such an extent that using aggregate data entails loss of much useful information. Moreover, users of aggregate models are unable to analyze how policy changes affect specific sectors. Also, use of aggregate data can lead to inaccurate policy recommendations. Last, use of aggregate data and relations can lead to a loss in forecast accuracy as shown, e.g., in de Alba and Zellner (1991), Zellner and Tobias (2000), and Zellner and Israilevich (2005).

Although some sectors have relatively stable shares of aggregate GDP, we see that other sectors' shares exhibit upward movement, namely Financial Services (Fin), Transport & Communication (Trans), and Community Services (Com). Electricity (El) followed an upward trend until early 1997 and then dropped (see Figure 3). The largest decline is observed for Mining. Also, there is a slight fall in Agriculture's share in recent years. The drastic changes that occurred in the political history of South Africa, together with shocks from the global economy, have greatly influenced this mapping. For example, Financial Services, as well as Transport & Communication, experienced larger increases in its share after the abolishment of the apartheid regime.

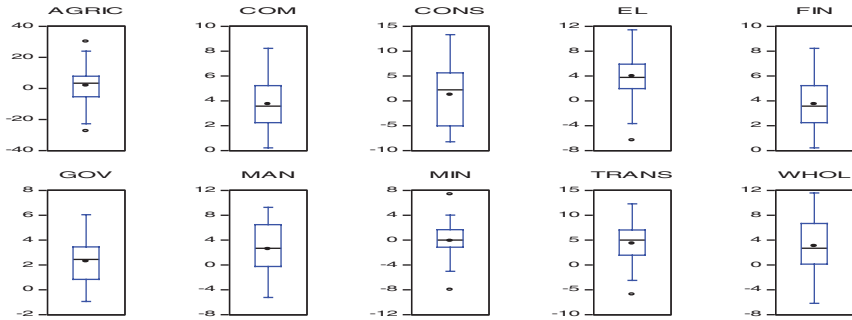


FIGURE 2. GDP annual growth (%) rates of ten South African economic sectors. We make use of well-known boxplots to provide information about the distributions of annual sector growth rates, 1972–2006. Our boxplots include the following elements: (1) the mean (point in bold), (2) a median (middle line in the box), (3) the interquartile range of the growth rates (the length of the box), (4) the outliers (extreme limits), and (5) the whiskers (vertical lines joining the outliers and the box). The GDP growth rates by sector are exactly the same series as used in Figure 1.

Development of the Marshallian model. Our current MMM-DA involves disaggregating the South African economy into the ten industrial sectors that we referred to earlier. For each sector, we, along with Alfred Marshall and others, have introduced a product market involving demand and supply equations derived from assumed optimizing behavior of firms and consumers. On aggregating over

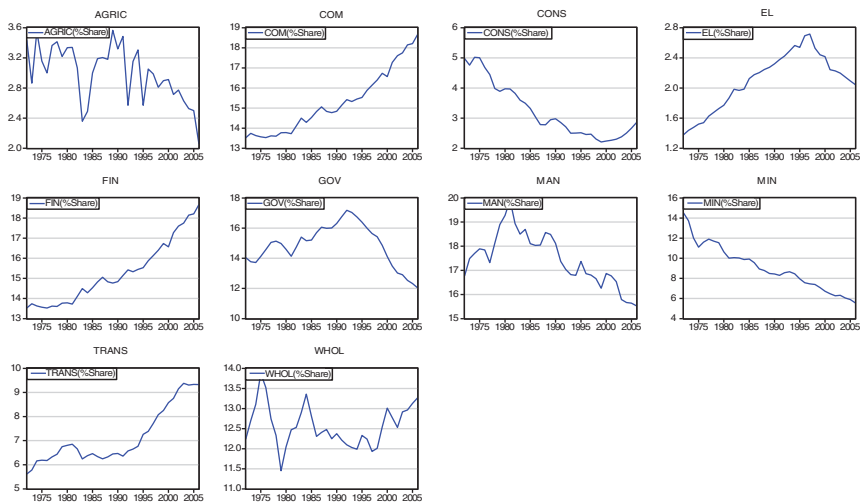


FIGURE 3. Sectors’ annual shares (% of total GDP), 1972–2006. The annual shares represent each sector’s contribution to the country’s GDP obtained from the SARB database (<http://www.reservebank.co.za/>).

firms, we obtain the industry supply equation, which depends on the number of firms in operation, a variable that does not appear in many macroeconomic models. To determine the number of firms in operation, we introduce a firm entry–exit equation in each sector such that when positive profits exist in the industrial sector, firms enter to compete away the profits and to help the sector return to a new equilibrium, as described in many price theory texts. Further, the firms in our industrial sectors demand labor, capital, and money services in markets for these factors of production. Also, consumers demand outputs and money services and supply labor and savings and the government supplies money to the money market. Likewise, the government collects taxes and produces a range of goods and services that are demanded by firms and consumers in the model. See Zellner and Israilevich (2005) for one-sector, two-sector, and n -sector versions of our MMM and their properties, along with some results of forecasting experiments with the model [Zellner and Chen (2001)]. Further, in their paper, Zellner and Israilevich (2005) ascertained that an MMM in its discrete version is in the form of a chaotic model that has various types of oscillatory behavior. Using their two-sector version of the model, they have established that it can provide a wide variety of possible solutions, including output rates of growth with “bubbles and busts” behavior.

The complete model (see the Appendix) includes five major components. First, it includes the sectors’ supply equations, derived by aggregating individual firms’ supply functions. Second, the MMM-DA includes the sectors’ product demand equations, derived by aggregating individual consumers’ demand functions, which include traditional demand shifters such as real disposable income and real money balances. Third, sectors’ firm entry–exit relations incorporate the link between firms’ entry–exit behavior and the gap between actual and equilibrium profits. In this regard, Veloce and Zellner (1985) have discussed the effects of a failure to take account of entry and exit behavior on the analysis of industries’ behavior, using data for a Canadian manufacturing industry. Fourth, the MMM-DA incorporates firms’ factor demand functions for labor, capital, and money services and supply functions for these factors in a set of factor markets. In regard to labor in a mixed economy such as South Africa, further disaggregation of the market is much needed. For example, the labor market can be disaggregated as follows: (1) nonunionized skilled labor, (2) nonunionized unskilled labor, (3) unionized skilled labor, and (4) unionized unskilled labor. Such an extension of our model will be easy to implement provided that we have appropriate data sets. However, due to data shortfalls, this study accounts for only one labor market that includes all these features. In our model, the impact of labor unions is currently captured through the firms’ entry and exit function. Labor unions exert pressure that leads to higher wages and increases in other operating costs. The upshot of this then is simple: fewer firms are willing to join the industry.

The fifth major component of our model is the government sector, which produces goods and services, demands factors of production in the factor markets, supplies money to the money market, taxes producers and consumers, and provides regulatory policies.

Each industrial sector has a number of firms operating at time t , each with a Cobb–Douglas production function $Q_i = A_i(z_i L_i)^{\alpha_i} K_i^{\beta_i}$, where A_i is the product of (1) A_N , a neutral technological change factor, (2) A_K , a capital augmentation factor, and (3) A_L , a labor technological augmentation factor. Therefore, $z_i L_i$ represents firms’ level of effective labor input and K_i represents a firm’s input of capital services.⁴ Also, as suggested in many studies, e.g., Zellner and Israilevich (2005), money services can be introduced as an additional input in the production process: $Q_i = A_i(z_i L_i)^{\alpha_i} K_i^{\beta_i} M_i^{\gamma_i}$.

Transfer function equations. As discussed in the literature [see, e.g., Zellner and Palm (2004)], dynamic simultaneous equations models, such as our MMM-DA, have a variety of associated algebraic representations, including reduced form equations, restricted reduced form equations, final equations, and transfer function equations. For our purposes, the transfer function representation of our MMM-DA is very useful, given that we do not have data on all of our variables. Each transfer equation links current and lagged values of an endogenous variable, e.g., each sector’s output growth rate, to its own lagged values and to current and lagged values of the exogenous variables. Thus, for the sector output growth rate variables, we have a set of ten transfer equations that can be estimated and used in forecasting and policy analysis without the need for data on other endogenous variables, e.g., prices and numbers of firms in operation that we do not have.

Shown below are the transfer functions, as derived from the complete model (see the Appendix), for the rates of change of real sectoral sales of the i th sector with $i = 1, 2, \dots, 10$, where $\lambda(L)$ and $\gamma(L)$ are lag operators. Also, (1) X is a set of exogenous variables, (2) S , the GDP at value added (RGVA), (3) W , the wage rate, (4) r , the interest rate, (5) A , the technological factor productivity, and (6) EC , the entry cost, which represents a combination of all costs incurred in starting a new firm. Besides, we have (i) Y for disposable income, (ii) IY , world income, using U.S. income as proxy, because the United States is one of the largest export destinations for South African products, (iii) D , the number of demanders of sectors’ outputs, (iv) SP , the stock price index, (v) M , the money supply M2, and (vi) ε , μ , and ν for the error terms. With the exception of r , each of the lowercase letters represents the rate of change of the corresponding capital letters; e.g.,

$$\ln \left(\frac{S_{i,t}}{S_{i,t-1}} \right) = s_{i,t} \quad \text{with } i = 1, 2, \dots, 10.$$

The transfer function for $s_{i,t}$ derived from our dynamic structural model is described below and in the Appendix:

$$\begin{aligned} [\lambda(L) - \gamma(L)] \cdot s_{i,t} = & -\gamma(L)[\delta_{0,i} - \delta_{1,i}S_{i,t-1} - \kappa_{1,i}w_{i,t} - \kappa_{2,i}r_t - \kappa_{3,i}a_{i,t} \\ & - \kappa_{4,i}z_{i,t} - \kappa_{5,i}ec_{i,t} - \kappa_{6,i}sp_{t-2} - \kappa_{7,i}m2_{t-2} - \varepsilon_{t,i} - \nu_{t,i}] \\ & + \lambda(L)[\Delta_{1,i}y_t + \Delta_{2,i}iy_t + \Delta_{3,i}d_t + \mu_{i,t}]. \end{aligned} \tag{1}$$

As regards error terms' properties, we note that when white noise error terms are introduced into the structural equations, the error terms in the transfer functions will be autocorrelated. If the structural equations' error terms are autocorrelated, then the transfer functions' error terms can have a variety of possible properties, e.g., MA(1), and perhaps white noise in certain cases. Because data are not available on all the structural equations' variables, it is not possible to estimate the structural equations and determine the properties of the structural error terms. Thus, we decided to fit the transfer functions using a generalized least square (GLS) criterion and to check whether the error terms are autocorrelated. We find that they are not, according to estimates of the autocorrelation functions for each sector's error terms.

Considering that current theories on agents' expectations in macroeconomic modeling remain somehow disparate, we have determined the lag structure of our transfer functions using Box–Jenkins model identification techniques [see Box and Jenkins (1970)]. Also, use of the Akaike information criterion [AIC; see Akaike (1973)] for our transfer function–selection problem led to results similar to those that we obtained using Box–Jenkins procedures.

Due to unavailability of disaggregated data on sector prices and numbers of firms in each sector, we have only estimated the ten sectoral GDP transfer function equations shown above in (1).

2.2. The Iterative Seemingly Unrelated Regression Transfer Function Estimation Technique

To estimate the set of ten transfer functions in (1) associated with our MMM-DA model, the iterative seemingly unrelated regression (ISUR) technique has been utilized. The ISUR method provides estimates using a GLS approach: see Zellner (1962) and Judge et al. (1985) for discussions of iterative SUR GLS estimation of a set of regression equations. Also, see Zellner and Ando (2010) for Bayesian estimation techniques for the SUR model. Note that, as is well known, the use of ISUR takes account of differing variances of error terms as well as correlations of error terms in different functions. Further, it yields consistent and asymptotically efficient estimators.

In future work, it will be interesting to compute Bayesian estimates and predictions for our transfer function system and to compare results with those produced by the ISUR procedure. In this connection, we note that with a flat prior and a normal likelihood function, the ISUR estimate is equal to the modal value of the posterior distribution, which is an optimal estimate relative to a zero–one loss function, as is well known. Further, in large samples, the posterior distribution will assume a normal shape with the posterior mean equal to the maximum likelihood estimate (MLE), as shown by Jeffreys (1967) and others, which will also be equal to the ISUR estimate. Thus, in large samples, the MLE will be equal to the mean and to the modal value of the posterior and our ISUR estimates have a number of alternative justifications. In small to medium-sized samples, the ISUR estimate

will be equal to the modal value of the posterior density based on a uniform prior, as noted above, and will thus be optimal relative to a zero–one loss function, i.e., zero loss if the estimate is close to the true value and unit loss if it is not.

2.3. Data

The data used in this paper for implementing our ten-equation transfer function model were collected on a yearly basis from 1973 onward. Ten economic sectors were considered that account for the overall national sales output.⁵ The main data sources used in this paper are (i) the SARB (South African Reserve Bank) database, (ii) the International Financial Statistics (IFS) database, and (iii) the World Bank Indicators (WBI). Data on local leading indicators such as stock price index (SP)⁶ and real money supply (M2)⁷ were obtained from the IFS database, whereas data on world leading indicators (*IY*) come from the WBI.⁸ Other types of national data such as (1) real firms' sales (*S*), (2) real disposable income (*Y*), (3) real interest rate (*r*), (4) real wage rates (*W*), (5) number of households (*D*), (6) labor effectiveness (*Z*), a measure of labor productivity, and (7) real firms' entry cost (EC) were collected from the SARB database (<http://www.reservebank.co.za/>). The proxy used for firms' entry cost is "other taxes on production." It includes all costs incurred by firms, independent of the value and quantity of goods and services produced or sold, such as charges for paperwork required to set up a firm in the industry, as well as certain other transactions related to fixed assets, etc.

3. PREDICTIVE PERFORMANCE: MMM-DA VERSUS AR(3)LI

The analyses reported below are based on the use of transfer functions for sector output growth rates derived from our MMM-DA. As mentioned earlier, our estimates have been obtained using ISUR estimation of the transfer functions in (1) for ten sectors of the South African economy, whereas the lag structures of the transfer functions have been specified using the Box–Jenkins transfer functions identification technique. As shown by the following results, not only do the sector transfer functions fit the South African data rather well (see Figure 4), but they also provide reasonable one year-ahead forecasts (see Figure 5).

With a few exceptions, mainly caused by uncontrolled structural breaks, our disaggregated model fits each of the ten sectors of the South African economy remarkably well. These results are encouraging especially when we note that our equations are not highly overparameterized.

Figure 5 shows the predictive performance of our MMM-DA for twelve-point forecasts (1995–2006), with some sectors such as Electricity, Manufacturing, Agriculture, and Mining providing dependable predictions of turning points. Weaker predictive performance for some sectors probably indicates that these sectors' equations need to be improved, perhaps by introducing additional explanatory variables and/or changing the lag structures. Also, the year 1995 was hard to

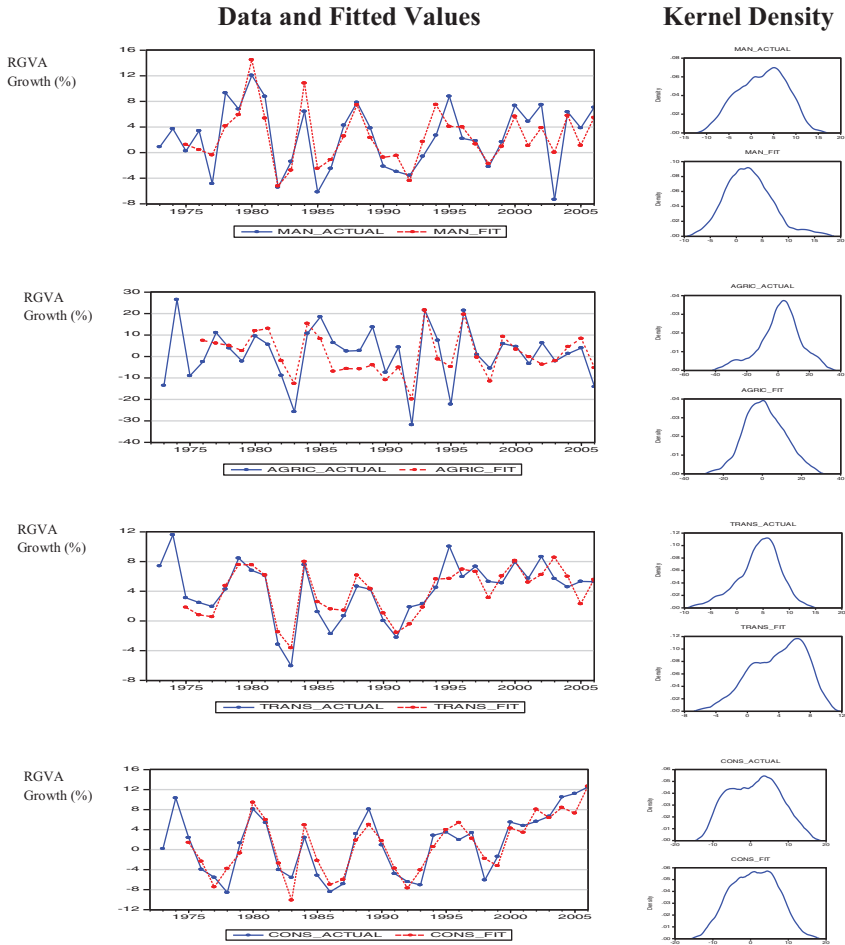


FIGURE 4. Actual series versus fitted values, 1972–2006: data and fitted values kernel density. See Figure 1 for acronyms. Actual series represent sectors’ GDP annual growth rates obtained from the SARB database (<http://www.reservebank.co.za/>). The fits are obtained from estimating our transfer equations using ISUR. The kernel density constitutes a refined version of the histogram of the growth rate of RGVA computed using an advanced algorithm, the fast Fourier transform.

predict, especially for agriculture. This was mainly due to the major political outbreak characterizing the shift from the apartheid regime to the democratic South African regime.

As mentioned earlier, the benchmark model used in this paper is an autoregressive leading indicator model of order 3, denoted by AR(3)LI, that is specified as

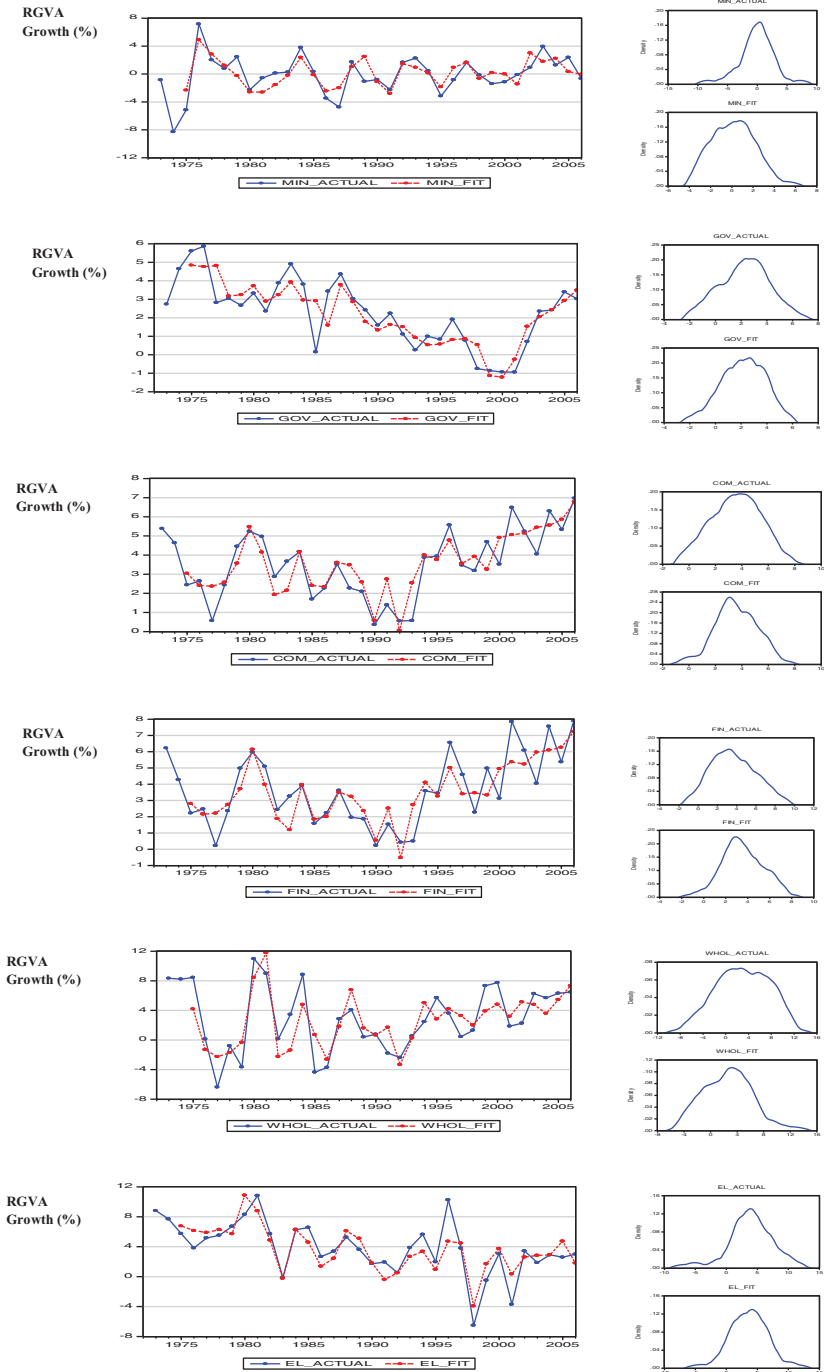


FIGURE 4. Continued.

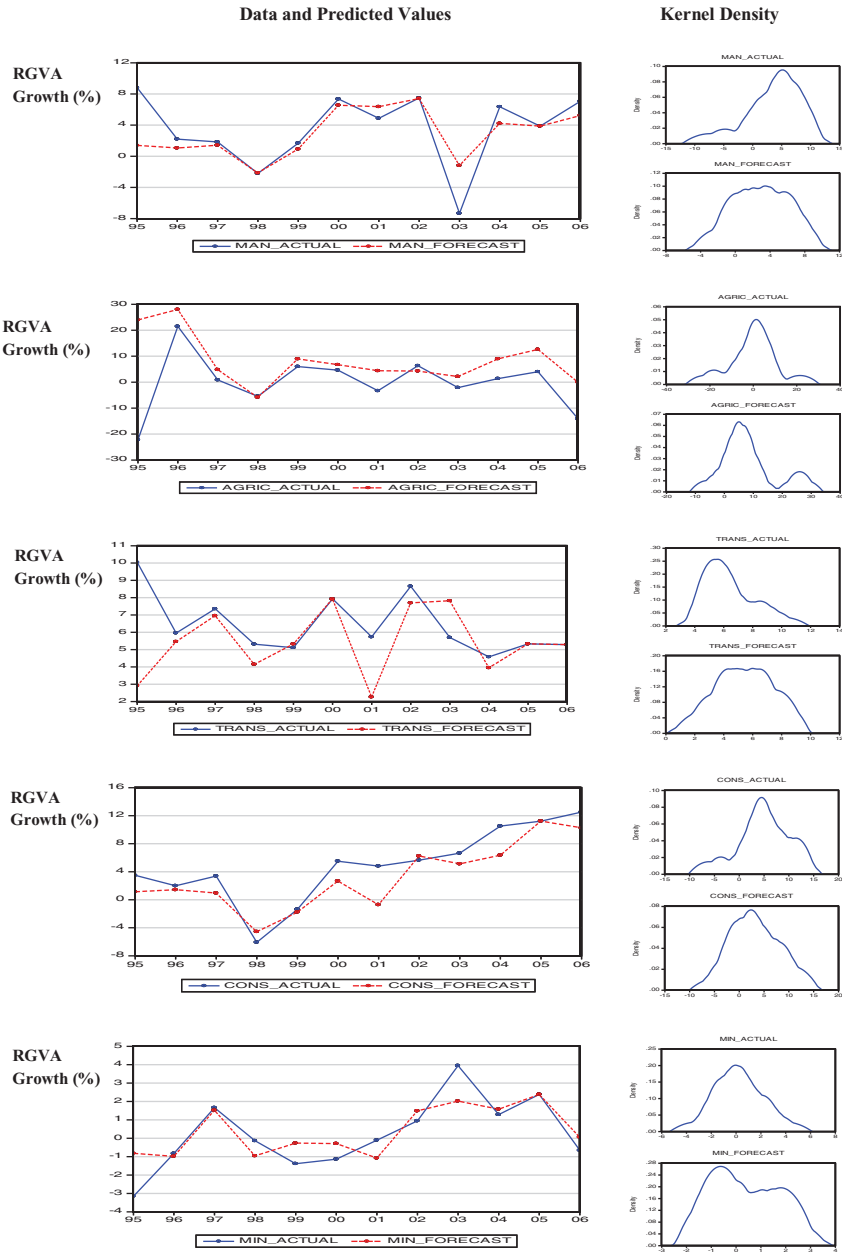


FIGURE 5. Actual versus one year-ahead predictions, 1995–2006: data and predicted values kernel density. See Figure 1 for acronyms. Results (forecasts) are obtained by computing one year-ahead forecasts (predictions) of individual sectors’ RGVA growth rates. The exogenous variables in the prediction period are assumed to have known values equal to their observed values.

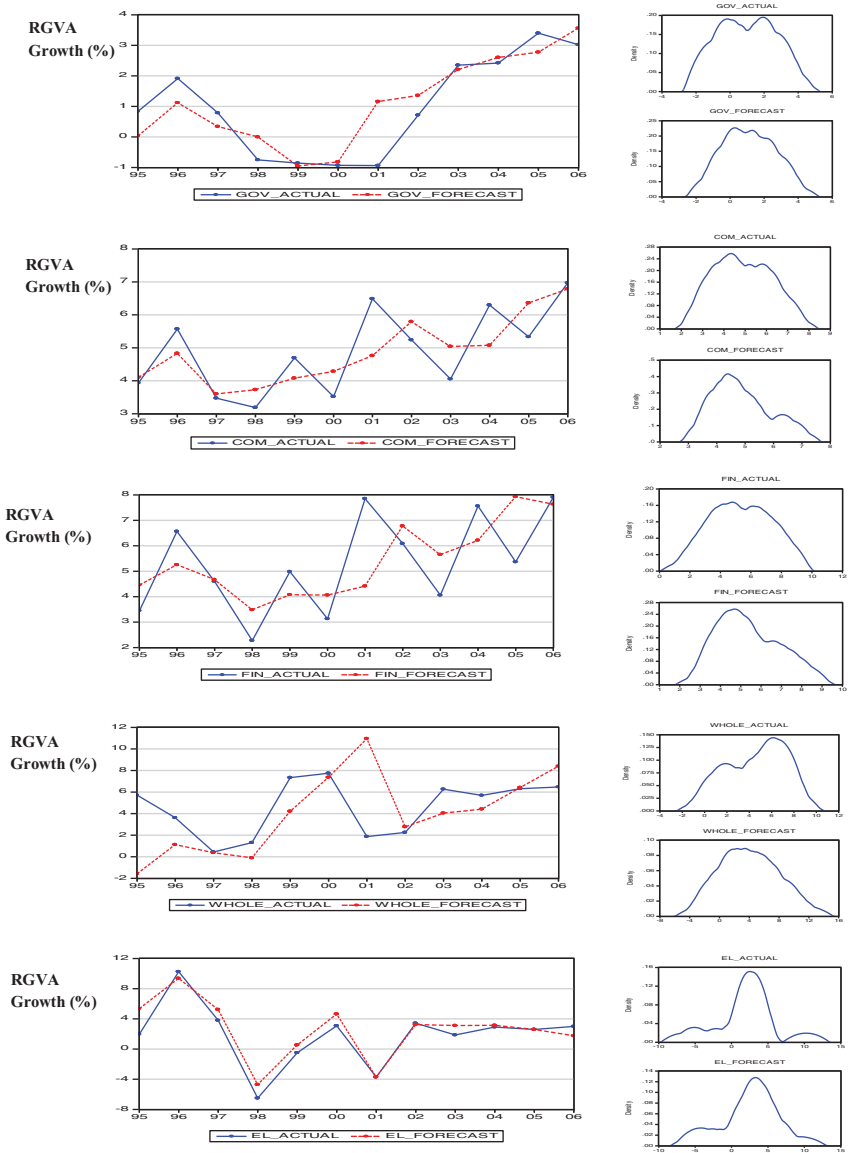


FIGURE 5. Continued.

follows:

$$s_{i,t} = \theta_0 + \theta_1 s_{i,t-1} + \theta_2 s_{i,t-2} + \theta_3 s_{i,t-3} + \theta_4 \ln \left(\frac{SP_{Q(t-3)}}{SP_{Q(t-4)}} \right) + \theta_5 \ln \left(\frac{M2_{(t-1)}}{M2_{(t-2)}} \right) + \varepsilon_{St}. \quad (2)$$

TABLE 1. RMSE and MAE (%) of one year-ahead predictions, 1997–2006

Sectors	AR(3)LI		MMM-DA (with observed values of exogenous variables)		MMM-DA (with predicted values of exogenous variables)	
	MAE	RMSE	MAE	RMSE	MAE	RMSE
Community	1.83	1.22	0.65	0.82	0.76	1.68
Electricity	2.44	3.29	0.73	1.00	1.21	1.77
Finance services	3.70	1.80	1.08	1.48	2.46	2.80
Wholesale	2.41	2.57	1.68	2.96	3.30	3.45
Transport & commun.	1.66	1.89	0.75	1.27	1.01	1.39
Mining	1.42	1.57	0.61	0.83	1.40	1.83
Manufacturing	3.53	4.07	1.14	2.02	2.27	2.64
Government	0.92	0.99	0.47	0.73	1.20	1.47
Construction	4.22	4.93	1.77	2.44	2.34	2.43
Agriculture	2.86	4.90	4.48	6.06	6.78	6.97

Note: Results are obtained from computing sectors' root mean squared errors (RMSEs) and mean average errors (MAEs) based on observed and predicted values of exogenous variables.

Autoregressive (AR) models in general have been extensively used in the forecasting literature. As opposed to autoregressive models of order 1, AR(1), the AR(3)LI model allows both real and complex roots. Also, the use of rates of change of real stock prices (SP) and of real money (M2) as leading indicators in the AR(3) models has produced substantial improvement in the predictive ability of this class of models [see Zellner and Tobias (2000)]. Therefore, choosing the forecasting performance of an AR(3)LI as a benchmark for comparison with the forecasting performance of our MMM-DA provides a rather good test of the latter's predictive ability.

In Table 1, we compare the predictive ability of our MMM-DA and that of the benchmark AR(3)LI model using known and unknown future values of exogenous variables. Future exogenous variables' values are predicted using estimated ARIMA models. As shown in Table 1, forecasting using predicted exogenous variables leads to larger mean average errors (MAEs) and root mean squared errors (RMSEs). The errors in predicting the exogenous variables tend to drive up the MAEs or the RMSEs of the MMM-DA predictions, as expected.

From the information in Table 1 and in Figure 5, it is evident that MMM-DA predicts reasonably well, and much better than the benchmark AR(3)LI model. However, it is important to note that the MAEs and RMSEs results for the agriculture sector are quite large. When we look at the actual series, we find that the growth rate of agriculture jumped from -20% to $+20\%$ in the period 1995–1996 after facing a major decline in 1994. This was due to a major structural break linked to the end of apartheid. Land in the country was owned entirely by white farmers, who had growing concerns about their future after the 1993 national

elections. There was heavy pressure for instituting land redistribution starting at that time, and farmers had the fear that it could all turn into chaos, as happened in other African states, and that fear affected tremendously the sector's output growth rates and our ability to forecast them.

Additionally, the MMM-DA's predictive ability is well demonstrated by observing the number of turning points that are well forecasted across different sectors (see Figure 5). In general, the model seems to do well in forecasting turning points correctly in a number of cases. However, the performance in certain sectors is not entirely satisfactory. This may be explained by the fact that the model specification is more appropriate for some sectors than others and/or that some sectors have higher data quality.

4. IMPLEMENTING AND EVALUATING THE FREE MARKET REFORMS

As mentioned earlier, this paper focuses on three types of reforms, namely, (1) freeing firms' entry by lowering their cost of entry, (2) a tax cut on the incomes of workers and employers, and (3) an improvement of labor effectiveness through improved education and health. The paper makes use of "other taxes on production" as a proxy for entry cost. Other taxes on production consist of all costs incurred by firms, other than production costs. Such taxes or charges cover paperwork required to set up a firm in the industry as well as certain other transactions related to fixed assets, etc. Because it comprises costs that are independent of the value or quantity of goods and services produced, we consider it to be the most appropriate proxy for entry cost available in the SARB database.

The flat-tax rate cut is simply captured through an overall increase in national disposable income. We do not suggest any specific type of tax cut. We rather allow the policy makers to choose any combination that helps increase national disposable income. As regards labor effectiveness (z), the details of its linkages with health and education are well described in a related study that was conducted using South African data; see Ngoie et al. (2009). Therefore, when we introduce an increase in z , it is the result of an increase in investment in health and education programs that is translated into a more effective labor force. Needless to say, reforms that result in increased labor effectiveness must be well designed and well implemented. In this paper, we simply utilize the outcome of such reforms without necessarily providing the design and implementation techniques that produced them. Generally, freeing up markets induces more competitiveness in different sectors, and therefore we may observe not only an increase in the number of firms in existence, but also more firms seeking to make their employees more productive. Also, having more competitive firms in the sectors helps to increase tax revenues. With more money available, the government can invest more in good health and education programs.

The three sets of free market reforms that we have implemented using this model are interlinked. Freeing up the market by introducing lower entry costs increases the number of firms operating in the sectors. Having more firms translates into more

employees and therefore higher disposable income. Also, having firms become more competitive provides incentives for them to dispense appropriate training for their employees that makes them more cost-effective.

Considering the various objections that may arise to such reforms, e.g., an increase in the budget deficit and potential distortions in public sector activities, we respond as follows. It is important to consider how much government revenue is actually lost due to restrictive firm entry requirements, power abuse from labor unions, and other economic distortions. With increased competition from other emerging economies, South Africa faces the risk of capital and skilled labor emigrating to lower tax nations that have fewer firm entry requirements and less pressure from labor unions. In addition, post-apartheid South Africa is known for its fiscal discipline. For the two fiscal years 2007 and 2008, the country recorded budget surpluses. A deficit that was announced for the fiscal year 2009 due to the global recession had major repercussions on the country's economy. The country can afford reforms while staying within acceptable budget deficit boundaries and use the growth outcomes of the reforms to experience budget surpluses.

When policy shocks are introduced into a macroeconomic model, a simple but highly recommended requirement is to check that the model behaves reasonably when the shocks are pushed to extremes. Impulse responses have been of widespread use for this purpose, although the transfer function is simply the Laplace transform of the impulse function. The impulse response function indicates how a shock is transmitted from the policy variables to the endogenous variables, taking account of the dynamic properties of the model. In this paper, in order to obtain the response standard errors, we use Monte Carlo simulations (50,000 iterations) and apply the Cholesky decomposition with adjusted degrees of freedom (see Figure A.1).

As shown in Table 2, when all the reforms are implemented simultaneously with a 1% shock (level at which the concerned variable is increased) on each of the variables (EC , Y , z), our model predicts that the country's real GDP growth will gain 0.8 percentage points in a year. That produces a growth rate of real GDP of 5.5% compared to 4.7%, initially recorded for the year 2006. Supposing that the reforms are much stronger, e.g., a five-percentage point increase in the growth rate of the same variables, a gain of 4.1 percentage points in real GDP is produced. That will increase the country's growth rate to 8.8%. The five-percentage point policy shock can be considered as an extremely large program implementing the reforms, whereas the one-percentage point shock represents a more modest program. Also, because the implementation of education and health reforms as a way to raise labor efficiency produces long-term effects, we have decided to reassess the growth outcomes by (1) allowing the reforms to raise z gradually (over five years) rather than instantaneously and (2) without the health and education reforms.

We then assume a sustained increase in the growth of labor effectiveness as a result of rigorous reforms in health and education, meaning five years of a

TABLE 2. Country's aggregate growth rate (RGVA growth) resulting from the implemented reforms

Reform types	Allocation	Reform size (percentage points)	GDP growth rate (%)	
			After 1 year	After 5 years
Tax cut and entry cost	Equal	1	5.3 (0.98)	7.9 (1.01)
		5	8.0 (1.00)	—
Tax cut, entry cost, and labor effectiveness	Equal	1	5.5 (0.99)	9.6 (1.02)
		5	8.8 (1.01)	—
Tax cut, entry cost, and labor effectiveness	Optimal	1	5.7 (0.99)	9.8 (1.02)
		5	9.8 (1.01)	—

Notes: Table 2 presents elasticities of the policy variables used for the reform. Estimates have been obtained using the transfer functions. The values in parentheses represent the predictive standard errors corresponding to each shock. The predictive standard errors constitute summarized measure of the estimated variance of the equation's residual. In this case, we first obtained the predictive standard error for each sector and then computed the aggregate predictive standard errors using a median growth equation obtained by converting the sectors' growth equations into levels and multiplying each level by the corresponding weight. After obtaining the median growth rate, we compute the standard error of regression.

consistent one–percentage point increase in z . Alongside, we assume a consistent one–percentage point increase in the growth of disposable income and a consistent one–percentage point decrease in the growth of entry cost. The resulting GDP growth rate will be 9.6%.

Furthermore, for a one–percentage point increase in the growth rate of disposable income (Y) concomitant with a one–percentage point decrease in the growth rate of entry cost (EC) with change in z , the country's GDP growth rate rises by 0.6 percentage point, from 4.7% to 5.3%. Assuming a five–percentage point shock, the resulting GDP growth is 8.0%. From such findings, we may conclude that reforms on education and health do not have instantaneous effects and therefore require more time before producing substantial effects. Also, the use of a disaggregated model has permitted us to determine how different sectors react to the reforms.

In Table 2, we report results of what we call a more “optimal” implementation of these three types of reforms; see, e.g., Tinbergen (1956). The reforms are calibrated according to the sectors' level of responsiveness. For example, instead of reducing entry cost in sectors that are naturally regulated monopolies, this money can be used to reduce entry costs of other, more open sectors. As we mentioned earlier, sectors may have different types of reactions when it comes to improving labor effectiveness. Capital-intensive sectors may react differently than labor-intensive sectors. It is therefore relevant to reallocate funds by shifting them from sectors where they are less productive to sectors with higher returns. Indeed, given a social welfare function of the type used by Tinbergen and others, it may be possible to determine an optimal allocation of a given amount of funds for different reform programs in different sectors.

In another calculation, we have reallocated funds for financing reforms so that sectors having larger recorded growth gains from increased labor effectiveness as compared to lower entry cost will receive more funds to promote labor effectiveness and vice versa. Such a reallocation of funds for reforms provides a much larger GDP (at value added) growth gain, 5.2% instead of 4.1%. That will raise the predicted annual GDP growth rate to (1) 5.7% for a one-percentage point shock and (2) 9.8% for a five-percentage point shock.

5. CONCLUSION AND STUDY LIMITATIONS

The present research considers the impact of free market reforms on the performance of the South African economy. The paper makes use of a disaggregated Marshallian macroeconometric model that was shown to fit the data and to predict well to evaluate the economic effects of the reforms. As regards free market reforms, similar to Thatcher-like reforms, the results in this paper indicate that such reforms are likely to produce a remarkable improvement in the South African growth rate. When carefully implemented, institution of all the three sets of free market reforms (five-percentage point shocks) is predicted to raise the South African annual real GDP growth rate to 8.8% with a uniform allocation of the reforms over the ten economic sectors and to 9.8% using a more reasonable allocation of reforms over sectors. These results are indeed encouraging, and will be studied further in the future with other variants of our MMM-DA, with data not only for South Africa but also for other countries. Also, this paper provides a clear response to potential objections against the implementation of such reforms. When we consider how much revenue is lost due to overbearing entry charges, high tax rates difficult to bear for middle- and low-income groups, and the heavy-handed labor unions that exist in the country, it is obvious that such reforms are needed.

In addition, the current research provides evidence that a disaggregated Marshallian macroeconometric model is a useful tool for understanding and predicting a country's overall economic behavior and the behavior of important industrial sectors. In the present study, lack of data on important sector variables led to the use of the model's implied transfer function equations for the sectors' real sales growth rates. With additional data on sector prices, number of firms in operation, etc., the full MMM-DA model can be estimated and used to explain and predict a wider range of variables, probably with added precision given that use of more data involves an increase in information available for estimation and prediction purposes. Also, disaggregation helps to avoid aggregation biases emphasized by Theil (1979) and many others; see, e.g., de Alba and Zellner (1991), Zellner and Tobias (2000), Zellner and Chen (2001), Zellner and Israilevich (2005), and Kim (2007) for some effects of disaggregation on the quality of models' forecasts. Moreover, in the present paper, the Marshallian modeling process has been broadened by (1) further analysis and implementation of entry costs, (2) more explicit allowance for a human capital component in the production process (labor effectiveness), and (3) addition of a foreign sector.

It is anticipated that further use and development of the MMM-DA will yield additional explanatory, predictive, and policy-making results that will be useful to many.

NOTES

1. These and other data cited below have been obtained from Statistics South Africa, Q2 2008.
2. The economic sectors considered are (1) Agriculture (AGRIC), (2) Mining (MIN), (3) Manufacturing (MAN), (4) Financial Services (FIN), (5) Wholesale (WHOL), (6) Transport and Communication (TRANS), (7) Construction and Building (CONS), (8) Government (GOV), (9) Community Services (COM), and (10) Electricity (EL).
3. In this study we make use of real GDP measured at Value Added, which is called Real Gross Value Added.
4. There is an extensive literature on the relationship between the labor augmentation factor (z) and health, education, and other social components. Ngoie et al. (2009) constitutes our closest reference using South African data. In that paper, the authors have estimated the parameters that link education and health to labor augmentation using sectoral data.
5. The sectors considered are the following: (1) Manufacturing; (2) Agriculture, Fishing and Forestry; (3) Construction and Buildings; (4) Mining; (5) Government; (6) Community Services; (7) Transport and Telecommunication; (8) Financial services; (9) Wholesales, Retail, Catering and Accommodation; and (10) Electricity, Gas and Water.
6. SP is published in the International Financial Statistics (IFS) under the code IFS; 19962 MB.ZF.
7. The series for M2 is published in the IFS under the code IFS; 19959 MB.ZF.
8. The series for IY is published under the code WDI; A111, NYGNPMKTPCD.

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APPENDIX: MODEL SPECIFICATION

Under the assumptions that firms and consumers are optimizers and firms face entry costs, the MMM-DA can be derived as follows.

A.1. PRODUCT MARKET

The sales supply equation (in real terms) has been derived from the firms’ profit-maximizing supply function Q by multiplying both sides by P to obtain PQ :

$$S_{Sit} = A_{it}^{\frac{1}{1-\alpha_i-\beta_i}} \cdot \alpha_i^{\frac{\alpha_i}{1-\alpha_i-\beta_i}} \cdot \beta_i^{\frac{\beta_i}{1-\alpha_i-\beta_i}} \cdot N_{it} \cdot w_{it}^{-\frac{\alpha_i}{1-\alpha_i-\beta_i}} \cdot r_{it}^{-\frac{\beta_i}{1-\alpha_i-\beta_i}} \cdot P_{it}^{1+\alpha_i\varphi_{Li}+\beta_i} \cdot (P_{Qit}^e)^{-\alpha_i\varphi_{Li}-\beta_i\varphi_{Ki}+\frac{\alpha_i+\beta_i}{1-\alpha_i-\beta_i}} \cdot z_{it}^{-\alpha_i}. \tag{A.1}$$

We have assumed Cobb–Douglas production functions for firms in the sectors.

As regards the sales demand equation, it is just the product price P multiplied by the usual demand function:

$$S_{Dit} = P_{it} \cdot \left[C_{Si} (P_{Qit}^e)^{\lambda_{1i}} \cdot (Y_{dt})^{\lambda_{2i}} \cdot (D_{it})^{\lambda_{3i}} \prod_{j=1}^m X_{jt}^{\lambda_{ji}} \cdot \left(\frac{P_{Qit}^e}{P_{Qit}} \right)^{\Delta_i} \right]. \tag{A.2}$$

In (A.3), the market equilibrium profit within a given sector is represented by $\bar{\pi}_{it}$:

$$\frac{\dot{N}_{it}}{N_{it}} = C'_{Ei} (\pi_{it}^a - \bar{\pi}_{it}). \tag{A.3}$$

Assuming that a firm’s actual profit π_i^a constitutes a proportion ℓ of its sales supply S_{Si} and $\pi_{it}^a = \ell S_{Sit}$, we can transform (A.3) as follows:

$$\frac{\dot{N}_{it}}{N_{it}} = C_{Ei} (S_{Sit} - \pi_{it}^e). \tag{A.4}$$

In this regard, we assume that (1) $\pi_i^e = \bar{\pi}_i/\ell$, (2) $C_{Ei} = aEC_i^x = C'_{Ei}\ell$, and (3) Γ is the firms’ entry cost per sector, which have a negative impact on firms’ entry.

Additionally, we have considered two output prices, the expected price (P_Q^e) and the current price (P_Q). At the beginning of period t , firms base all their production activities on the expected price. However, should the actual price be set, firms follow an adjustment process that is captured through the parameter ϕ in our optimizing equations.

As we have done for all the equations below, the above demand and supply equations can be expressed in growth terms (discrete time denoting variables’ rates of change) by logging

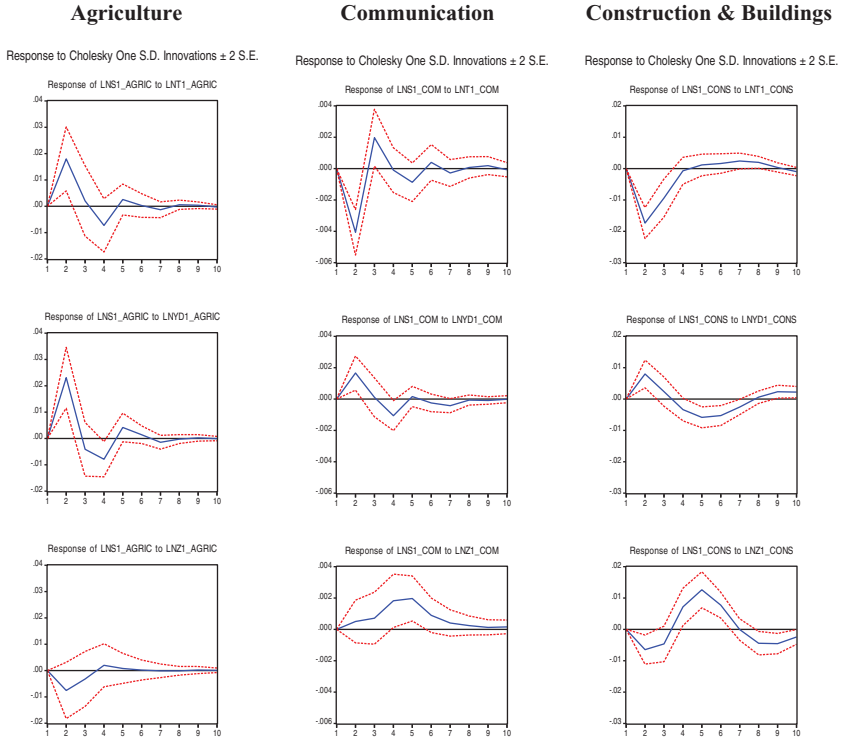


FIGURE A.1. Impulse responses using Cholesky decomposition. LNS1 represents the GDP (at value added) growth rate of the given sector, whereas LNT1 represents the entry cost (in growth terms) and LNZ1 the level of labor effectiveness (in growth terms). Impulse response functions describe the response of our endogenous variable (in this case it will be the sector’s GDP growth rate) as a reaction to a one-time impulse [Hamilton (1994)].

both sides and differentiating with respect to time. The new equations system includes (1) a sales supply function,

$$\frac{\dot{S}_{Sit}}{S_{Sit}} = \theta_{1i} \frac{\dot{A}_{it}}{A_{it}} + \frac{\dot{N}_t}{N_t} + \theta_{2i} \frac{\dot{P}_{it}}{P_{it}} + \theta_{3i} \frac{\dot{w}_{it}}{w_{it}} + \theta_{4i} \frac{\dot{r}_{it}}{r_{it}} + \sum_{l=1}^T \sigma_{li} \frac{\dot{P}_{lit}}{P_{lit}} + \theta_{5i} \frac{\dot{z}_{it}}{z_{it}}, \quad (\text{A.5})$$

(2) a sales demand function (A.6),

$$\frac{\dot{S}_{Dit}}{S_{Dit}} = (1 - \Delta_i) \frac{\dot{P}_{Qit}}{P_{Qit}} + (\lambda_{1i} + \Delta_i) \frac{\dot{P}_{Qit}^e}{P_{Qit}} + \lambda_{2i} \frac{\dot{Y}_{dit}}{Y_{dit}} + \lambda_{3i} \frac{\dot{D}_{it}}{D_{it}} + \chi_{j1i} \frac{\dot{IY}_{it}}{IY_{it}}, \quad (\text{A.6})$$

and (3) an entry/exit function for each sector (A.4),

$$\frac{\dot{N}_{it}}{N_{it}} = C_{Ei} (S_{Sit} - \pi_{it}^e). \quad (\text{A.4})$$

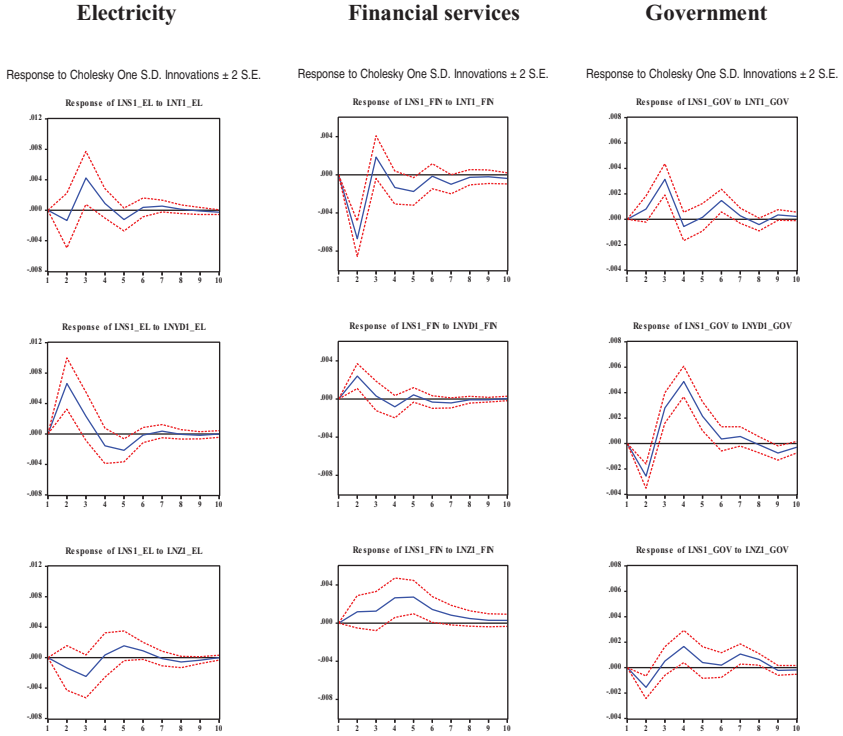


FIGURE A.1. Continued.

A.2. FACTOR MARKETS

We assume that the sectoral labor supply function is given by

$$z_{Lit} = C_{Li} \left(\frac{w_i}{P_{Qit}} \right)^{\psi_1} \left(\frac{Y_{it}}{P_{Qit}} \right)^{\psi_2} \left(\frac{P_{Qit}}{P_{Qit}^e} \right)^{\psi_3} (D_{Lit})^{\psi_4} \left(\prod_{j=1}^l v_{jt}^{\phi_j} \right), \tag{A.7}$$

where D_L is the total number of labor providers within the sector and the v variables are labor supply shifters:

$$\begin{aligned} \frac{\dot{z}_{Lit}}{z_{Lit}} &= \psi_1 \left(\frac{\dot{w}_i}{w_i} - \frac{\dot{P}_{Qit}}{P_{Qit}} \right) + \psi_2 \left(\frac{\dot{Y}_{it}}{Y_{it}} - \frac{\dot{P}_{Qit}}{P_{Qit}} \right) + \psi_3 \left(\frac{\dot{P}_{Qit}}{P_{Qit}} - \frac{\dot{P}_{Qit}^e}{P_{Qit}^e} \right) + \psi_4 \left(\frac{\dot{D}_{Lit}}{D_{Lit}} \right) \\ &+ \sum_{j=1}^l \varphi_j \left(\frac{\dot{v}_{jt}}{v_{jt}} \right). \end{aligned} \tag{A.8}$$

The demand for labor, derived from profit maximization on the part of firms, is given by

$$z_{Lit} = \alpha_i \cdot N_{it} \cdot \left(\frac{S_{Sit}}{w_i} \right) \cdot \left(\frac{P_{Qit}^e}{P_{Qit}} \right)^{1+\beta_i \varphi_{Ki} + (\alpha_i - 1) \varphi_{Li}}, \tag{A.9}$$

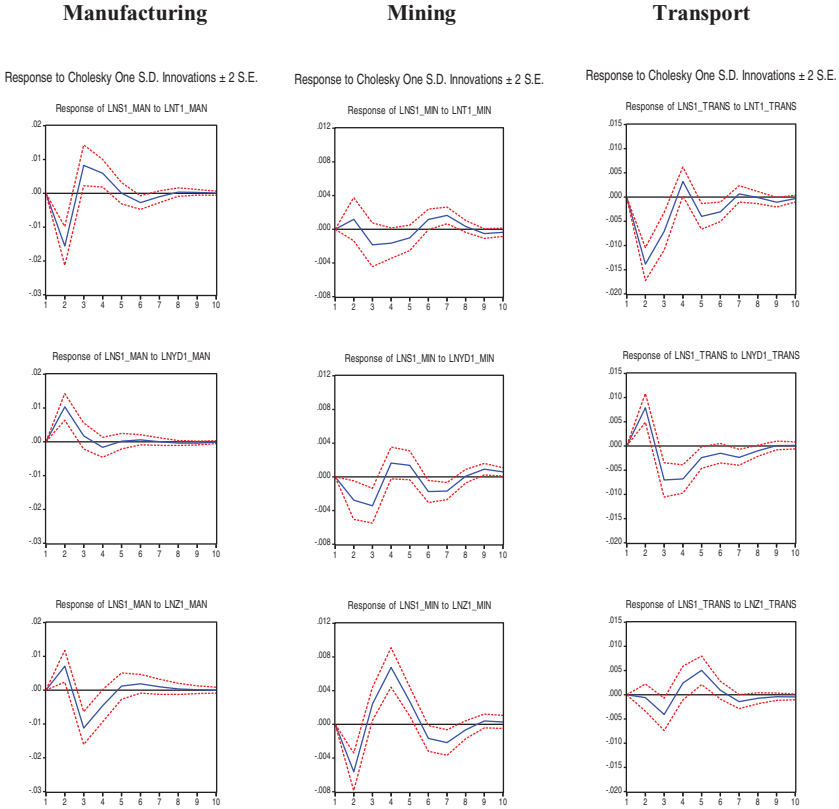


FIGURE A.1. Continued.

$$\frac{\dot{z}L_{it}}{zL_{it}} = \frac{\dot{N}_{it}}{N_{it}} + \frac{\dot{S}_{Sit}}{S_{Sit}} - \frac{\dot{w}_{it}}{w_{it}} - (1 + \beta_i\varphi_{Ki} + (\alpha_i - 1)\varphi_{Li}) \frac{\dot{P}_{Qit}}{P_{Qit}} + (1 + \beta_i\varphi_{Ki} + (\alpha_i - 1)\varphi_{Li}) \frac{\dot{P}_{Qit}^e}{P_{Qit}^e}, \tag{A.10}$$

$$\frac{\dot{z}L_{it}}{zL_{it}} = \frac{\dot{N}_{it}}{N_{it}} + \frac{\dot{S}_{Sit}}{S_{Sit}} - \frac{\dot{w}_{it}}{w_{it}} + (1 + \beta_i\varphi_{Ki} + (\alpha_i - 1)\varphi_{Li}) \left[\frac{\dot{P}_{Qit}^e}{P_{Qit}^e} - \frac{\dot{P}_{Qit}}{P_{Qit}} \right]. \tag{A.11}$$

As for labor, capital equations are obtained from firms' profit maximization. Capital supply is

$$K_{it} = C_{Ki}(r_t)^{\gamma_1} \left(\frac{Y}{P_{Qit}} \right)^{\gamma_2} \left(\frac{P_{Qit}}{P_{Qit}^e} \right)^{\gamma_3} (D_{Kit})^{\gamma_4} \left(\prod_{j=1}^n u_j^{\delta_j} \right), \tag{A.12}$$

Wholesale

Response to Cholesky One S.D. Innovations ± 2 S.E.

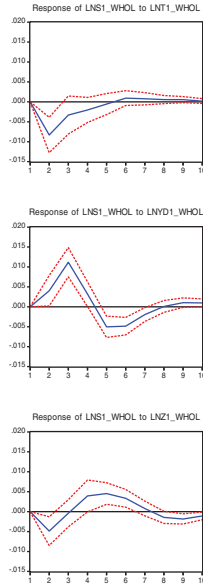


FIGURE A.1. Continued.

$$\begin{aligned} \frac{\dot{K}_{it}}{K_{it}} &= \gamma_1 \left(\frac{\dot{r}_t}{r_t} \right) + \gamma_2 \left(\frac{\dot{Y}_t}{\dot{Y}_t} - \frac{\dot{P}_{Qit}}{P_{Qit}} \right) + \gamma_3 \left(\frac{\dot{P}_{Qit}}{P_{Qit}} - \frac{\dot{P}_{Qit}^e}{P_{Qit}^e} \right) + \gamma_4 \left(\frac{\dot{D}_{Kit}}{D_{Kit}} \right) \\ &+ \sum_{j=1}^n \delta_j \left(\frac{\dot{u}_{jt}}{u_{jt}} \right), \end{aligned} \tag{A.13}$$

where D_K represents the total number of capital providers, which includes (1) government, (2) domestic providers, and (3) foreign providers; u represents the capital supply shifters; and r represents the real interest rate.

Capital demand is as follows:

$$K_{it} = \beta_i \cdot N_{it} \cdot \left(\frac{S_{Sit}}{r_t} \right) \cdot \left(\frac{P_{Qit}^e}{P_{Qit}} \right)^{1+\alpha_i\varphi_{Li}+(\beta_i-1)\varphi_{Ki}}, \tag{A.14}$$

$$\begin{aligned} \frac{\dot{K}_{it}}{K_{it}} &= \frac{\dot{N}_{it}}{N_{it}} + \frac{\dot{S}_{Sit}}{S_{Sit}} - \frac{\dot{r}_t}{r_t} - [1 + \alpha_i\varphi_{Li} + (\beta_i - 1)\varphi_{Ki}] \left(\frac{\dot{P}_{Qit}}{P_{Qit}} \right) \\ &+ [1 + \alpha_i\varphi_{Li} + (\beta_i - 1)\varphi_{Ki}] \left(\frac{\dot{P}_{Qit}^e}{P_{Qit}^e} \right), \end{aligned} \tag{A.15}$$

$$\frac{\dot{K}_{it}}{K_{it}} = \frac{\dot{N}_{it}}{N_{it}} + \frac{\dot{S}_{Sit}}{S_{Sit}} - \frac{\dot{r}_t}{r_t} + [1 + \alpha_t \varphi_{L_t} + (\beta_i - 1) \varphi_{K_t}] \left(\frac{P_{Qit}^e}{P_{Qit}} - \frac{P_{Qit}^e}{P_{Qit}} \right). \tag{A.16}$$

Real money balances as a factor of production are demanded by firms and the government; households also require the services of real money balances. Money supply is as follows:

$$M_{Sit} = C_{M_{Si}} \cdot P_{Qit}^{\pi_1} \cdot r_t^{\pi_2}, \tag{A.17}$$

$$\frac{\dot{M}_{Sit}}{M_{Sit}} = \pi_1 \left(\frac{P_{Qit}^e}{P_{Qit}} \right) + \pi_2 \left(\frac{r_t}{r_t} \right). \tag{A.18}$$

Money demand is as follows:

$$M_{it}^d = C_{M_{it}^d} \cdot (D_{it})^{\nabla_1} \cdot (N_{it})^{\nabla_2} \cdot \left(\frac{r_t}{P_{Qit}^e} \right)^{\nabla_3} \cdot \left(\frac{S_{Sit}}{P_{Qit}^e} \right)^{\nabla_4} \cdot \left(\frac{P_{Qit}}{P_{Qit}^e} \right)^{\nabla_5}, \tag{A.19}$$

$$\begin{aligned} \frac{\dot{M}_{it}^d}{M_{it}^d} &= \nabla_1 \left(\frac{\dot{D}_{it}}{D_{it}} \right) + \nabla_2 \left(\frac{\dot{N}_{it}}{N_{it}} \right) + \nabla_3 \left(\frac{\dot{r}_t}{r_t} - \frac{P_{Qit}^e}{P_{Qit}^e} \right) + \nabla_4 \left(\frac{\dot{S}_{Sit}}{S_{Sit}} - \frac{P_{Qit}^e}{P_{Qit}^e} \right) \\ &+ \nabla_5 \left(\frac{P_{Qit}}{P_{Qit}^e} - \frac{P_{Qit}^e}{P_{Qit}^e} \right). \end{aligned} \tag{A.20}$$

As shown in Zellner and Palm (2004), transfer functions can be derived mathematically from dynamic linear structural equation models. In his seminal work, Quenouille (1957) indicated a specific way to represent linear multiple time series processes [see Zellner and Palm (2004)],

$$H(L) z_t = F(L) \varepsilon_t, \tag{A.21}$$

$\begin{matrix} mx1 & & mxm & mx1 \end{matrix}$

where (1) $z_t' = (z_{1t}, z_{2t}, \dots, z_{mt})$ is a vector of random variables and (2) $\varepsilon_t' = (\varepsilon_{1t}, \varepsilon_{2t}, \dots, \varepsilon_{mt})$ is the random error vector. $H(L)$ and $F(L)$ are full rank matrices containing polynomial lag operators as elements. Quenouille’s model was extended by Zellner and Palm (2004) to structural econometric models by allowing $z_t' = (y_t', x_t')$, where y_t represents a vector of the endogenous variables and x_t a vector of the exogenous variables. Then (A.21) becomes

$$\begin{bmatrix} H_{11}(L) & H_{12}(L) \\ H_{21}(L) & H_{22}(L) \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} F_{11}(L) & F_{12}(L) \\ F_{21}(L) & F_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}.$$

This system can be written as follows, given that the assumption of x_t being exogenous

implies $H_{21}(L) = 0$, $F_{12}(L) = 0$, and $F_{21}(L) = 0$ y_t :

$$H_{11}(L)y_t + H_{12}(L)x_t = F_{11}(L)\varepsilon_{1t}, \tag{A.22}$$

$$H_{22}(L)x_t = F_{22}(L)\varepsilon_{2t}. \tag{A.23}$$

From the system above, we can derive the transfer functions by multiplying both sides of (A.22) by H_{11}^{-1} to obtain

$$y_t = -H_{11}^{-1}H_{12}(L)x_t + H_{11}^{-1}F_{11}\varepsilon_{1t}. \tag{A.24}$$

From $H_{11}^{-1} = H_{11}^{\text{adj}}/|H_{11}|$, (A.24) can be expressed as

$$|H_{11}|y_t = -H_{11}^{\text{adj}}H_{12}(L)x_t + H_{11}^{\text{adj}}F_{11}(L)\varepsilon_{1t}.$$

Transfer functions for the endogenous variables in our MMM-DA are obtained from (A.24).

Our specific product market model for the i th sector expressed in matrix lag operator form is

$$\begin{aligned} \begin{bmatrix} 1 & -\lambda(L) & -1 \\ 1 & -\gamma(L) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_{i,t} \\ p_{i,t} \\ n_{i,t} \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \\ \delta_{0,i} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \delta_{1,i} \end{bmatrix} S_{i,t-1} + \begin{bmatrix} \kappa_{1,i} \\ 0 \\ 0 \end{bmatrix} w_{i,t} + \begin{bmatrix} \kappa_{2,i} \\ 0 \\ 0 \end{bmatrix} r_t \\ &+ \begin{bmatrix} \kappa_{3,i} \\ 0 \\ 0 \end{bmatrix} a_{i,t} + \begin{bmatrix} \kappa_{4,i} \\ 0 \\ 0 \end{bmatrix} z_{i,t} + \begin{bmatrix} \kappa_{5,i} \\ 0 \\ 0 \end{bmatrix} ec_{i,t} + \begin{bmatrix} \kappa_{6,i} \\ 0 \\ 0 \end{bmatrix} sp_{i,t-2} + \begin{bmatrix} \kappa_{7,i} \\ 0 \\ 0 \end{bmatrix} m_{2,t-2} \\ &+ \begin{bmatrix} 0 \\ \Delta_{1,i} \\ 0 \end{bmatrix} y_t + \begin{bmatrix} 0 \\ \Delta_{2,i} \\ 0 \end{bmatrix} iy_t + \begin{bmatrix} 0 \\ \Delta_{3,i} \\ 0 \end{bmatrix} d_t + \begin{bmatrix} \varepsilon_{i,t} \\ \mu_{i,t} \\ v_{i,t} \end{bmatrix}. \end{aligned} \tag{A.25}$$

To obtain the transfer equations, we multiply both sides of (A.25) by the adjoint matrix A^* ($A^* = \det A \cdot A^{-1}$), with

$$A = \begin{bmatrix} 1 & -\lambda(L) & -1 \\ 1 & -\gamma(L) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad A^{-1} = \frac{1}{\det A} \begin{bmatrix} -\gamma(L) & \lambda(L) & -\gamma(L) \\ -1 & 1 & -1 \\ 0 & 0 & \lambda(L) - \gamma(L) \end{bmatrix}.$$

Therefore

$$A^* = \begin{bmatrix} -\gamma(L) & \lambda(L) & -\gamma(L) \\ -1 & 1 & -1 \\ 0 & 0 & \lambda(L) - \gamma(L) \end{bmatrix}.$$

After both sides of (A.25) are multiplied by A^* ,

$$\begin{aligned}
 [\lambda(L) - \gamma(L)] \cdot \begin{bmatrix} s_{i,t} \\ p_{i,t} \\ n_{i,t} \end{bmatrix} &= \begin{bmatrix} -\gamma(L)\delta_{0,i} \\ -\delta_{0,i} \\ \delta_{0,i} [\lambda(L) - \gamma(L)] \end{bmatrix} + \begin{bmatrix} -\gamma(L)\delta_{1,i} \\ -\delta_{1,i} \\ \delta_{1,i} [\lambda(L) - \gamma(L)] \end{bmatrix} s_{i,t-1} \\
 &+ \begin{bmatrix} -\gamma(L)\kappa_{1,i} \\ -\kappa_{1,i} \\ 0 \end{bmatrix} w_{i,t} + \begin{bmatrix} -\gamma(L)\kappa_{2,i} \\ -\kappa_{2,i} \\ 0 \end{bmatrix} r_t + \begin{bmatrix} -\gamma(L)\kappa_{3,i} \\ -\kappa_{3,i} \\ 0 \end{bmatrix} a_{i,t} + \begin{bmatrix} -\gamma(L)\kappa_{4,i} \\ -\kappa_{4,i} \\ 0 \end{bmatrix} z_{i,t} \\
 &+ \begin{bmatrix} -\gamma(L)\kappa_{5,i} \\ -\kappa_{5,i} \\ 0 \end{bmatrix} ec_{i,t} + \begin{bmatrix} -\gamma(L)\kappa_{6,i} \\ -\kappa_{6,i} \\ 0 \end{bmatrix} sp_{i,t-2} + \begin{bmatrix} -\gamma(L)\kappa_{7,i} \\ -\kappa_{7,i} \\ 0 \end{bmatrix} m_{2i,t-2} + \begin{bmatrix} \lambda(L)\Delta_{1,i} \\ \Delta_{1,i} \\ 0 \end{bmatrix} y_t \\
 &+ \begin{bmatrix} \lambda(L)\Delta_{2,i} \\ \Delta_{2,i} \\ 0 \end{bmatrix} iy_t + \begin{bmatrix} \lambda(L)\Delta_{3,i} \\ \Delta_{3,i} \\ 0 \end{bmatrix} d_t + \begin{bmatrix} -\gamma(L)\varepsilon_{i,t} + \lambda(L)\mu_{i,t} - \gamma(L)v_{i,t} \\ -\varepsilon_{i,t} + \mu_{i,t} - v_{i,t} \\ [\lambda(L) - \gamma(L)]v_{i,t} \end{bmatrix}.
 \end{aligned}
 \tag{A.26}$$

(A.26) is the transfer equations system for the variables s_{it} , p_{it} , and n_{it} .