'n Nuwe ontwikkeling tot die bivariate betaveld

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A new development in the bivariate beta field
In this paper, the bivariate Kummer-beta type IV distribution, which extends the Jones' bivariate beta distribution, is discussed. The probability density functions of the product and ratio of the components of this distribution are derived. Also, a shape analysis is done to investigate the effect of the new parameter.

Kummer-tipe verdelings vorm 'n prominente deel van statistiese verdelingsteorie. In die literatuur is daar verskeie Kummer-tipe verdelings voorgestel, soos die werk van Armero en Bayarri (1997), Ng en Kotz (1995), Gupta *et al.* (2001), Nagar en Gupta (2002) en Nagar en Cardeño (2001). Aandag is gegee aan eenveranderlike, meerveranderlike en matriks-Kummerverdelings.

In hierdie artikel ondersoek ons die bivariate Kummer-beta-tipe IV-verdeling wat 'n uitbreiding is van Jones se bivariate betaverdeling, waar die waarskynlikheidsdigtheidfunksie (wdf) van laasgenoemde gegee word deur:

$$f(x,y) = \frac{1}{B(a,b,c)} \frac{x^{a-1}\, y^{b-1}\, (1-x)^{b+c-1}\, (1-y)^{a+c+1}}{(1-xy)^{a+b+c}}, \quad 0 \le x, y \le 1 \qquad \text{[Verg. 1]}$$

Hierdie verdeling staan ook in die literatuur bekend as Jones se bivariate betaverdeling (Balakrishnan & Lai 2009:379) en is onafhanklik deur Jones (2001) en Olkin en Liu (2003) voorgestel. Die wdf van die Kummer-beta-tipe IV-verdeling word gegee deur

$$f(x,y) = K \frac{x^{a-1}\, y^{b-1}\, (1-x)^{b+c-1}\, (1-y)^{a+c+1}\, e^{-\psi(x+y)}}{(1-xy)^{a+b+c}}, \quad 0 \le x, y \le 1 \qquad \text{[Verg. 2]}$$

met

$$K^{-1} = \sum_{k=0}^{\infty} \frac{(a+b+c)k}{k!} \frac{{}_1F_1(a+k, a+b+c+k; -\psi)\; {}_1F_1(a+k, a+b+c+k; -\psi)}{\left(\mathrm{B}(a+c, b+k)\, \mathrm{B}(b+c, a+k)\right)^{-1}}$$

Hierdie verdeling is ontwikkel deur 'n Laplace transformasie van die bivariate beta-tipe IV. Die Laplace transformasie word gebruik om die normaliseringskonstante te bereken en bevat die Kummerfunksie, ${}_1F_1(\cdot)$ (Gradshteyn 2007, Afdeling 9.2). Die randdigtheidsfunksies van Vergelyking 2 word afgelei deur gebruik te maak van magreeksuitbreidings en die konfluente hipergeometriese reeks van twee veranderlikes, $\Phi_1(\cdot)$ (Gradshteyn 2007, Vgl 3.385).

Die verdelings van die produk en kwosiënt van onafhanklike en afhanklike stogastiese veranderlikes het verskeie toepassings (sien bv. Nagar *et al.* (2009), Gupta en Nadarajah (2008), Joarder (2009), Pham-Gia en Turkkan (2002) en Pham-Gia (2000). Eers ná 2002 geniet die produk en kwosiënt van Kummer-tipe verdelings aandag in die literatuur. Die produk en kwosiënt van *onafhanklike* Kummer-beta-veranderlikes is deur Nagar en Zarrazola (2005) ondersoek en dié van *onafhanklike* Kummer-gamma-veranderlikes deur Morán-Vásquez en Nagar (2009). Navorsing oor die produk en kwosiënt van Kummer-tipe verdelings is dus 'n huidige veld van navorsing. Ons betree 'n nuwe veld en kyk na die verdelings van die produk en kwosiënt van die *afhanklike* komponente van die bivariate Kummer-beta-tipe IV-verdeling. Om hierdie digtheidsfunksies af te lei, word gebruik gemaak van die Mellin transformasie asook die inverse Mellin transformasie (Mathai 1993, Definisie 1.8). Die uitdrukkings word verkry in terme van Meijer se G-funksie wat deesdae 'n bekende funksie in sagteware pakkette is.

Deur verskillende positiewe sowel as negatiewe waardes vir die addisionele parameter ψ aan te neem, is dit uit die ondersoek duidelik dat ψ 'n noemenswaardige effek toon op die vorms van die nuwe verdelings asook op die korrelasie van die afhanklike komponente.

Ter opsomming is dit uit die studie duidelik dat die bivariate Kummer-beta-tipe IV-verdeling meer variasie in die vorm inbring teenoor dié bekende bivariate beta-tipe IV-verdeling. Beta- en gammaverdelings word dikwels in 'n Bayes-opset gebruik, onder andere waar die betaverdeling as a priori-verdeling gebruik word en die gammaverdeling in toustaanmodelle 'n groot rol speel. Die moontlike toepassing van hierdie 'buigbare' bivariate Kummer-beta-tipe IV-verdeling in hierdie Bayes konteks moet ondersoek word. Die produk en kwosiënt word dikwels in betroubaarheidstoepassings gebruik en ook hier moet die rol van die Kummer-beta-tipe IV ondersoek word.

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