The Universe in Perspective

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Abstract

Gödel's rotating-universe solution of Einstein's gravitational field equations is interpreted in real projective space. The compass of inertia is shown to coincide with the involuted geodesic of the closed manifold and predicts a cosmological redshift for distant events. Other cosmological criteria of antimatter imbalance, the microwave background and nucleogenesis are satisfied in a natural way. The role of black holes in steady-state circulation, universal self-similar symmetry and the need to reinterpret astronomical data in hyperspace are discussed.

1 Introduction

The basis of what is commonly claimed to be modern scientific cosmology is the theory of relativity. The cosmologically important result of the special theory is that space has a minimum of four dimensions, which is not the same as three-dimensional space and a universal time coordinate. All expanding-universe cosmologies however, assume such a system through the Robertson-Walker metric, with a universal Gaussian time coordinate and a scaled three-dimensional space:

$$\begin{aligned} ds^2 &= (\mathrm{d}x_0)^2 - R^2(t)\left[\mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2\right] \\ &= c^2\mathrm{d}t - R^2(t)\left[\frac{(\mathrm{d}r)^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)\right] \end{aligned} \quad (1)$$

It is noted at the outset [1] (p.339):

> ″that the use of a distinguished time-coordinate marks the abandonment of a completely covariant treatment of the cosmological

problem. This is the price one has to pay to simplify the cosmological models and to describe physical reality in convenient mathematical terms″.

This cautionary note is routinely ignored in cosmological discourse, which presents an expanding three-dimensional homogeneous subspace as a rigorous result of general relativity.

The only recognized mathematical system with possible relevance to large-scale cosmology is the gravitational field equations of Einstein that occur in general relativity:

$$G_{ik} \equiv R_{ik} - \frac{1}{2}(R - \Lambda)g_{ik} = -\frac{8\pi\kappa}{c^2}T_{ik} \tag{2}$$

with indices $(i, k = 0 \to 3)$. Both equations (1) and (2) are textbook material [1], to be consulted as the meaning of different terms and symbols may be in doubt.

The most important feature of the system is the implied reciprocity between the curvature tensor G_{ik} and the energy-momentum (stress) tensor T_{ik}. As both of these tensors vanish in empty Euclidean (including Minkowski) space, it is implied that matter is generated by curved space and that an accumulation of matter modifies the curvature of space.

To obtain a mathematical model of the cosmos it is necessary to find the correct metric tensor, g_{ik}, which solves the field equations (2). In fact, several different mathematically acceptable formulations are known and cosmological criteria are needed to differentiate between them. Any cosmological model should account for all, or most, of the following:

(i) galactic readshifts;

(ii) the cosmic microwave background;

(iii) matter – antimatter imbalance;

(iv) nucleogenesis and nuclide abundance;

(v) the physics of singular points;

(vi) universal self-similarity

Standard cosmology is formulated specifically to address the first two criteria, assumes special scenarios to circumvent (iii) and (iv), looks to interpret (v) as a boundary condition, and ignores (vi). As more astronomical and other physical evidence accumulates the criteria are honed more sharply and more claims of model failure emerge. Continued re-assessment of all available cosmological solutions against the physical criteria is therefore advisable.

2 The Cosmological Solutions

Solution of the field equations relies on the specification of the metric tensor as a function of non-Euclidean space-time topology. Four different solutions have been obtained:

(i) The Einstein static-universe solution;

(ii) Schwarzschild's solution for a central gravitational field;

(iii) Friedmann solutions for an expanding universe;

(iv) Gödel's solution for a rotating universe

Einstein's solution fails to predict cosmological redshifts and is no longer considered a viable model. Schwarzschild's solution leads to an infinitely imploding system and, despite a singularity at the centre, it defines the preferred model for black holes. Friedmann models are based on the Robertson-Walker metric, which, by definition, describes an infinite Euclidean universe that expands along a universal Gaussian time coordinate.

The mathematical structure of the Friedmann models is generally accepted to define the standard model of cosmology. The insurmountable physical difficulties, associated with the singularity on one hand and infinity on the other, are shelved for later analysis. The broken symmetry between matter and antimatter is ascribed rather unconvincingly to some chance fluctuation that occured in the early universe. Recently observed [2] periodic trends in the cosmic abundance of nuclides are in serious conflict with standard guidelines. The advanced version of the standard model, known as the inflationary model, introduces several unsubstantiated assumptions such as a cosmological constant that fluctuates over hundred orders of magnitude and suggests that barionic matter represents less than 1% of the total universal mass.

Many physical scientists are sceptical about this final model as a serious picture of the universe. A growing volume of astronomical data appears to be equally incompatible with the standard model [3] and the quest for a more convincing solution gathers momentum. The one remaining unexplored alternative is examined next.

3 Gödel's Solution

It was shown by Gödel [4] that elimination of the absolute time coordinate leads to the rotation of matter relative to the compass of inertia. He proposed a metric to reflect nine special properties of the assumed four-dimensional space S, which it defines:

1. S is homogeneous;
2. (Neighbouring) world lines of matter are equidistant;
3. S has rotational symmetry;
4. A positive direction of time can be introduced in the whole system;
5. The direction of the time flow is not uniquely defined for each space-time point;
6. Every world line of matter is infinitely long and never returns to the same point;
7. There are no space-like three-spaces;
8. There is no absolute time;
9. Matter everywhere rotates relative to the compass of inertia

The Gödel metric [4]:

$$\begin{aligned} \mathrm{d}s^2 &= \alpha^2 \left[\mathrm{d}x_0^2 - \mathrm{d}x_1^2 + \left(\frac{1}{2}e^{2x_1}\right) \mathrm{d}x_2^2 - \mathrm{d}x_3^2 + 2e^{x_1}\mathrm{d}x_0\mathrm{d}x_2 \right] && (3) \\ &= \alpha^2 \left[(\mathrm{d}x_0 + e^{x_1}x_2)^2 - \mathrm{d}x_1 - \frac{1}{2}e^{2x_1}\mathrm{d}x_2^2 - \mathrm{d}x_3^2 \right] && (4) \\ &= (\mathrm{d}x_0 + e^{\alpha x_1}\mathrm{d}x_2)^2 - (\mathrm{d}x_1)^2 - \frac{1}{2}e^{2\alpha x_1}(\mathrm{d}x_2)^2 - (\mathrm{d}x_3)^2 && (5) \end{aligned}$$

contains the Schwarzschild,

$$\begin{aligned} \mathrm{d}s^2 &= c^2\mathrm{d}t^2 - \left(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2\right) && (6) \\ &= A\mathrm{d}x_0^2 - B\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right) && (7) \end{aligned}$$

and Robertson-Walker (1) metrics as special cases, with expansion factor $R(t) = 1$.

The cross term in the time interval of (3) is reminiscent of the metric of cylindrical space, rotating with constant angular frequency and can be shown [1] to be equivalent to that.

The stress tensor, as was done by Einstein, is obtained from the kinetic theory of galactic clusters, assumed to behave like dust in a fluid of average density ρ, internal pressure p and *rms* velocity $\overline{v^2}$,

$$p = \tfrac{1}{3}\overline{v^2}\rho$$

On lowering indices with the metric tensor, the contracted Riemann tensor follows as:

$$R_{ij} = -\frac{\alpha^2}{\rho} T_{ij}$$

showing that the Gödel metric is a solution of (2) if

$$\Lambda = -\frac{\alpha^2}{2} \qquad \frac{\alpha^2}{\rho} = \frac{8\pi\kappa}{c^2}$$

For $\alpha = 0$, both Λ and ρ must be zero, which implies flat space. The parameter α therefore measures the curvature of space. Although the solution will be shown to define an aesthetically more pleasing cosmology, it has been ignored for many years because it fails to predict a Doppler redshift and to give a clear definition of the compass of inertia, proposed as rotation axis.

Scrutiny of the nine criteria shows that both objections are eliminated on modification of assumption 6 by the addition of a point at infinity, which turns infinite space into projective space [5]. The metric remains the same under this transformation and some of Gödel's perceived problems, which he addressed in footnote material, are also resolved. In particular footnote 11:

> ″There exist stationary homogeneous solutions in which the world lines of matter are not equidistant. They lead, however,into difficulties in consequence of the inner friction which would arise in the ′gas′ whose molecules are the galaxies, unless the irregular motion of the galaxies is zero, and stays so″.

The solution remains the same[1], only the geometric meaning changes, as already recognized by Gödel himself:

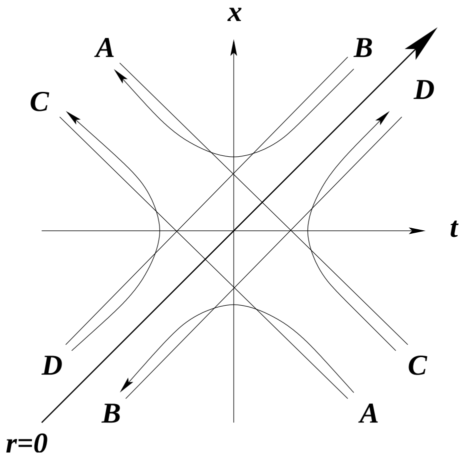
> ″The space S is the direct product of a straight line and the three-space S_0, defined by $x_0 = 0$, and obtained from a space R of constant curvature. This definition of S_0 leads to an elegant presentation of its group of transformations. To this end we map the points of R on the hyperbolic quaternions $u_0+u_1j_1+u_2j_2+u_3j_3$ by means of projective coordinates $u_0u_1u_2u_3$.

In terms of this simple alternative geometry, geodesic transplantation fixes points on the line element to occur along the double cover of a narrow Möbius

[1]In view of modern observations Gödel's concerns appear less serious. Galaxies are now known to wander and even collide. There is hardly any reason why not to explore the alternative possibility.

band to define the compass of inertia. It is a known property of a Möbius band that a point, which moves along the double cover, close to one edge, rotates around the central line without intersecting it.

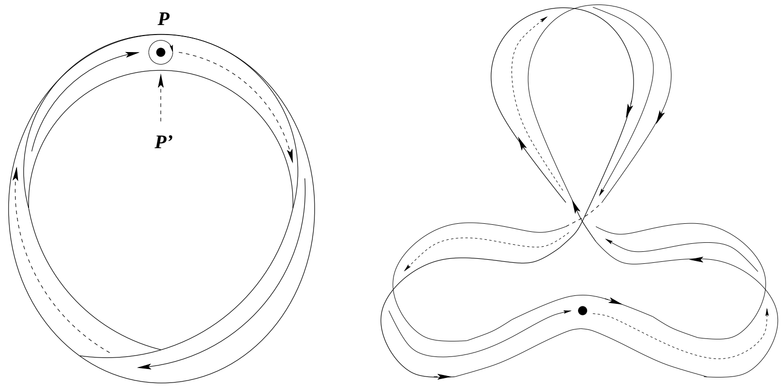
To further elucidate this description it will now be shown that a Möbius strip represents a section through a closed projective plane embedded in four-dimensional space. As an example consider four-dimensional Minkowski space. A pseudocircle in this space,



$$x^2 - (ct)^2 = \pm r^2$$

consists of two hyperbolas [5]. By following the asymptotes of the hyperbolas as an involuted closed curve, two Möbius strips occur as sections through the projective plane. Any geodesic in projective Gödel space therefore rotates around an inertial compass. The constant separation between symmetry-related points determines the number of turns before returning to the starting point. To define a non-orientable plane this number must be odd. All essential properties are recovered, without loss of generality, by assuming a single Möbius twist, shown below, alongside a one-sided ribbon with five twists.

Although the Gödel metric solves the field equations, large-scale astronomical observations in the system can only be understood as features of the topological space [6]. The real projective plane is topologically closed, simply-connected, one-sided and non-orientable, with Euler characteristic $\chi = 1$. Moving a normal to the surface at P (*e.g.* time axis) along the continuous path to P', so that the foot of the normal remains in contact with the surface, demonstrates the one-sided non-orientable property. As the real projective plane and the disc are homeomorphic they obey the same fixed



point theorem: for any continuous transformation of a disc to itself, there is at least one point which is mapped to itself. The Gödel rotation defines such a transformation. For $\varphi \neq 2\pi n$ there is only one fixed point, namely the centre. Since all coordinate points are equivalent the fixed point can be placed anywhere.

Cosmic rotation, which appeared to be in geocentric mode, has been observed in radio astronomy [7] and excited the comment that:

> ″This would have drastic cosmological consequences, since it would violate Mach's principle and the widely held assumption of large-scale isotropy″.

Topologically there is no mystery. Any large-scale rotation must appear to be centred at the observer and indeed, the identity of the Einstein and Gödel stress tensors shows that the field equations have two basically different solutions for the same T_{ij}, one rotating and the other static. The Gödel solution therefore is consistent with the general theory of relativity, but not with Mach's principle [1], p.377.

The fact that all conics are equivalent in projective space [9] could have dramatic effects on the interpretation of astronomical observations. The observation that some galaxies seem to rotate in rigid mode may therefore simply be an artefact that occurs in perspective and creates the illusion of a dark-matter halo, known as Zwicky's paradox.

4 The Cosmological Model

The Gödel solution of the relativistic gravitational field equations (2) is compatible with an incoherent matter distribution, in uniform rotation about a compass of inertia that coincides with the involuted geodesic of high-dimensional projective geometry. In one-dimensional analogue the geodesic traverses a Möbius band, which is the interface between symmetry-related segments of the non-orientable surface that constitutes a double cover. It is natural to identify the interface with the physical three-dimensional vacuum which is interpenetrated by a pair of chiral subspaces that define the material universe. Curvature of the surface occurs with respect to the local time coordinate of the non-Euclidean manifold, and is everywhere perpendicular to the surface. The interpenetrating dual spaces are seperated in time, which prevents mutual annihilation.

This model can hardly be more unlike a universe that expands in three-dimensional Euclidean space. The special-relativistic requirement of four-dimensional space, with the curvature of general relativity superimposed, demands that space-time has a minimum of five dimensions. Locally perceived three-dimensional space therefore is an illusion and extrapolation of local structure, beyond the Galactic borders, a gross distortion. The well-known Kaluza-Klein theory, which is aimed specifically at the electromagnetic field, is by no means the only model for a five-dimensional hyperspace. The Thierrin model [8] with its four equivalent space coordinates accounts for relativistic effects by simple trigonometry, readily interpreted in projective space. (Time flow is perceived to be directed by the fourth space dimension.) While the dimensionality of space-time remains largely speculative, the argument will not be pursued any further, beyond pointing out that the naïve interpretation of spectroscopic redshift as a three-dimensional Doppler effect is a futile attempt to visualize four-dimensional events. By a remarkable coincidence Segal [10] has shown that a chronometric redshift and a Planckian microwave background occur in the same projective space as the modified Gödel solution and that it satisfies all the main cosmological criteria.

4.0.1 Galactic redshifts

According to general relativity the cosmos is a four-dimensional manifold, $\overline{M}$. At each point there is given a convex cone of infinitesimal future directions, in the tangent space to the manifold. Space and time coordinates are entangled in $\overline{M} = T \times S$, but in the vicinity of an observer space-time events have a linear temporal order T^1 and a three-dimensional space S^3. An observer is said to collapse the event into a stationary state by splitting space-time into *space* and *time* components. The non-Euclidean space-time model, which may be globally acausal, becomes locally Minkowskian.

The universal space-time geometry itself is not necessarily directly observed; no apparent departures from a Euclidean model have been found by classical measurements. It might be argued that curved space-time coordinates are split into flat space and time coordinates. There is no direct observational basis for asserting that the Cosmos is Minkowskian at large distances and times.

Locally, Minkowski and universal space are identical as causal manifolds, but the universal time is not equivalent to the time registered in the local Minkowski frame. Quantum mechanically temporal evolution and energy are defined by conjugate operators. The operator $-i(\partial/\partial t)$, which defines the energy (or frequency) depends on the geometry of the stationary state. The difference between standard time and Minkowski time therefore becomes measurable in the form of alterations produced in the apparent frequency of a freely propagated photon. The relative shiftcan be calculated as the frequency difference as measured by two observers. one of whom is at rest relative to the other.

Minkowski space, M, is assumed embedded in a more general universal closed (compact) space $\overline{M}$, the so-called conformal space. This is the projective space proposed [11] as a model of the universe by Oswald Veblen in 1933. Roughly speaking, $\overline{M}$ is obtained from M by adding a light cone at infinity. More precisely, it is the double cover of the space so generated. Segal [10] refers to $\overline{M}$ as *unispace* and to the natural time τ in this space as *unitime.*

Temporal generators (*e.g.* energies) are basically different between unispace and local Minkowski space. The unispace generator is strictly greater than the Minkowski energy. An observation consists of the local decomposition of either the curved unispace or flat Minkowski space into space and time components. Physically only one of these analyses can be globally valid. Therefore, if global conservation of energy is valid in one analysis, it cannot be valid in the other, for the respective energy operators do not commute.

Conventionally flat frequency is represented by the operator $-i(\partial/\partial t)$, but in terms of the unitime it becomes $-i(\partial/\partial\tau)$. The experimental observation of redshifts indicates conclusively that local measurements correspond to flat dynamic variables, but that the universe operates on curved dynamics. In the process of measurement the frequency shifts from the actual $\nu'(\tau)$ to the stationary value νt. By symmetry

$$\nu' = \frac{1+\cos\tau}{2}\nu$$

which predicts a so-called *chronometric* redshift:

$$z = \frac{\nu-\nu'}{\nu'} = \frac{1-\cos\tau}{1+\cos\tau} = \tan^2\left(\frac{\tau}{2}\right)$$

The excess energy appears with redshifting, and is then diffused in space in a fashion which causes no observable local particle production.

The apparent motion between the cosmos and the stationary Minkowski frame is entirely virtual. To establish a cosmic distance scale, stationary states at two different points, r apart, are compared, choosing unit radius for unispace; leading to

$$z = \tan^2(\rho/2) = \tan^2(r/2R)$$

where R is a Minkowskian radius of the universe.

4.0.2 Microwave Background

As $\rho \to \pi$, at the antipodal point in the projective space, the redshift approaches totality as $z \to \infty$. In the stationary Minkowski frame the antipode is infinitely distant at an infinite time coordinate. The conditions leading to the derivation of the redshift formula are not met for radiation with a propagation interval close to a half-circuit in space, and such photons will appear entirely delocalized and severely redshifted, consituting the isotropic microwave background. Segal ascribes the Planckian distribution to the conservation of energy, which is tantamount to the fact that any closed space must eventually impose a Planckian spectrum on stray radiation.

4.0.3 Chiral Matter

A predicted gradual change in chirality with transplantation along the geodesic follows the gauge transformation of electric charge, such that [12]:

$$\Psi' = \Psi \exp\left[-(2\pi i\hbar)\int e\phi_i \mathrm{d}x_i\right]$$

as $x_i \to -x_i$, and an involution of π turns matter into conjugate antimatter. There are no separate matter and antimatter domains as chiral forms transform smoothly into each other, preserving an element of PCT symmetry in the interface.

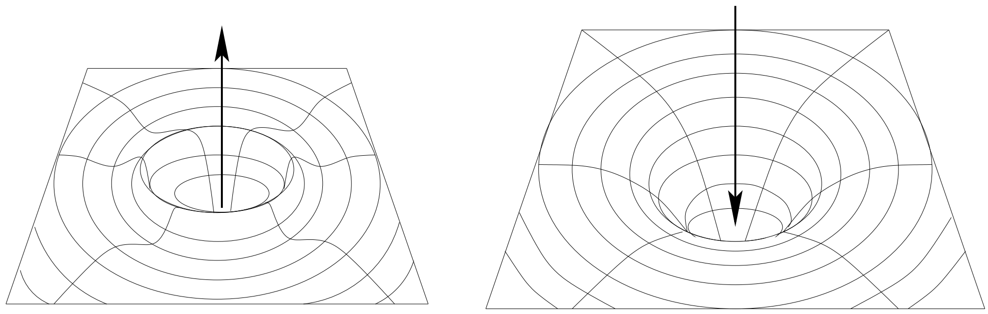
The secondary criteria are functions of a steady state that reflects the exchange of matter and energy across the vacuum interface.

4.1 The Steady State

Mathematical singularities in physical theory appear for only one reason – a wrong model.

Although the Gödel solution is free of singularities the need to accommodate black holes in the cosmic model requires an interpretation of the

Schwarzschild singularity which occurs with infinite curvature of space-time. An interpretation is rather obvious. Such a high degree of curvature must clearly rupture the interface between adjacent sides of the postulated cosmic double cover. Rather than disappear into a singularity, the matter, swal-



lowed up by a black hole, therefore reappears, with inverted chirality, on the opposite side of the interface. Such a connection is known as an Einstein–Rosen bridge, never precisely localized before. A schematic drawing of the gravitational gradients suggests sufficient asymmetry to interpret the bridge as operating between a black hole and a cosmic volcanic source that injects matter into space.

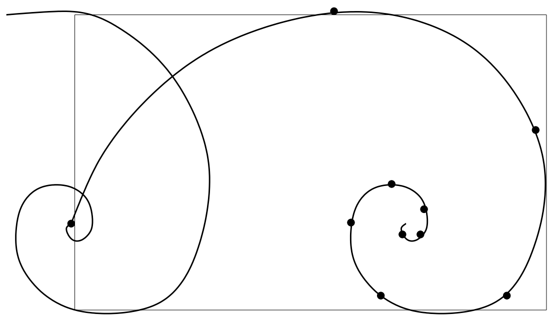
Many sources with prominent emission spectra, such as Seyfert galaxies and quasars, which could be of this type, have been observed. According to this interpretation cosmic matter is neither dispersed nor created in time, but recycled. The constant two-way flow across the interface has reached a steady state which gives the universe the appearance of being static.

The observed periodicity in the cosmic abundance of atomic nuclides [2] finds a ready explanation in this steady-state recycling. Under simulated high pressure all matter collapses into elementary forms with inversion of chirality. On emergence, through the throat of the black hole, complex nuclides are reconstituted by the fusion of α-particles in an equilibrium process involving all compositions with the proton:neutron ratio of unity. On release into free space only those isotopes with appropriately adjusted composition survive radioactive transformation, giving rise to a numerically precise range of nuclides. The nucleogenesis of standard cosmology cannot reproduce the observed mix

5 Universal Self Similarity

It has recently been shown [13] that the Bode–Titius law, which hints at some harmonious regular organization of planetary motion in the solar system, is dictated by a more general self-similar symmetry that applies from sub-atomic systems to galactic spirals. The common parameter is the golden ratio, $\tau = 0.61803\ldots$. Any such cosmic symmetry should be dictated by a successful cosmological model.

The golden ratio is superimposed on a logarithmic spiral, $r = \mu^\theta$, by setting $\mu = \tau^{2/\pi}$ to produce the golden spiral, $r = \tau^{2\theta/\pi}$ that leads to the Bode–Titius law. The orbital radii of planets and moons in the solar system are characterized correctly by divergence angles of $n\pi/5$ ($\simeq n\tau$) on the spiral that fits into a golden rectangle, as shown diagrammatically for $n = 2$.



It is now conjectured that the Gödel compass of inertia has the same structure with five involutions, which divide the universe into ten segments with alternating chirality. The topological structure remains the same as for the single involution considered before, but the five-fold self-similarity is now imprinted on all space.

The consequences of rotating space with periodically alternating chirality have never been contemplated before. If it has some dynamo effect it could explain the appearance of Einstein's cosmological constant Λ, which stabilizes Gödel's solution.

6 Conclusion

The idea of a rotating closed chiral universe is an irreducible concept which is only defined in hyperspace. Unlike the concept of an expanding universe, which is routinely projected into infinite three-dimensional Euclidean space, there is no incentive to perform such a visualization here. The possibility of observing multiple images of astronomical objects are obvious and real. With a regular scattering of black holes and their conjugates, concepts such as gravitational lensing and astronomical measures of distance and time need serious reconsideration. Rescaling by a million orders of magnitude may not be unreasonable. The Hubble telescope is like an observer, tricked into reporting from a hall of mirrors.

The volume of astronomical data has grown too large for disentanglement until a well-defined geometrical framework is in place. Further speculation about great attractors, megawalls, quantized redshifts and dark matter are doomed to remain unproductive with a quasi-three dimensional model of the universe in place. It is not even clear which extra-Galactic objects have real existence in geocentric projection.

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