



# High Precision Prediction for $M_h$ in the MSSM

T. Hahn<sup>1</sup>, S. Heinemeyer<sup>2</sup>, W. Hollik<sup>1</sup>, H. Rzehak<sup>3</sup>, G. Weiglein<sup>4</sup>

<sup>1</sup>Max-Planck-Institut für Physik, Föhringer Ring 6, D–80805 München, Germany

<sup>2</sup>Instituto de Física de Cantabria (CSIC-UC), Santander, Spain

<sup>3</sup>Albert-Ludwigs-Universität Freiburg, Physikalisches Institut, D–79104 Freiburg, Germany

<sup>4</sup>DESY, Notkestraße 85, D–22607 Hamburg, Germany

## Abstract

The new particle discovered at the LHC can be interpreted as the lightest  $\mathcal{CP}$ -even Higgs boson of the Minimal Supersymmetric Standard Model (MSSM), where  $M_h$  denotes its mass. The experimental precision of this mass has reached the level below  $\sim 500$  MeV. In the MSSM  $M_h$  can directly be predicted from the other parameters of the model. The accuracy of this prediction should at least match the one of the experimental result. The relatively high mass value of about 125 GeV has led to many investigations where the scalar top quarks are in the multi-TeV range. We review the recent improvement in the prediction for  $M_h$  in the MSSM that has been reached by combining the existing fixed-order result, comprising the full one-loop and leading and subleading two-loop corrections, with a resummation of the leading and subleading logarithmic contributions from the scalar top sector to all orders.

**Keywords:** Higgs, Supersymmetry, mass prediction

## 1. Introduction

After the spectacular discovery of a signal in the Higgs-boson searches at the LHC by ATLAS and CMS [1, 2], now the exploration of the properties of the observed particle is in the main focus. In particular, the observation in the  $\gamma\gamma$  and the  $ZZ^{(*)} \rightarrow 4\ell$  channels made it possible to determine its mass with already a remarkable precision. Currently, the combined mass measurement from ATLAS is  $125.36 \pm 0.37 \pm 0.18$  GeV [3], and the one from CMS is  $125.03^{+0.26+0.13}_{-0.27-0.15}$  GeV [4]. This leads to the naive average of

$$M_H^{\text{LHC, today}} = 125.1 \pm 0.2 \text{ GeV}. \quad (1)$$

At the (planned) future International  $e^+e^-$  Linear Collider (ILC), using the Z-recoil method a precision of [5]

$$\delta M_H^{\text{ILC}} \lesssim 50 \text{ MeV} \quad (2)$$

is currently anticipated.

The other properties that have been determined so far, in particular the coupling strength modifiers [6, 7], as

well as spin, are compatible with the minimal realisation of the Higgs sector within the Standard Model (SM) [8]. However, the discovery can also be interpreted as a Higgs boson in a model beyond the SM, where the Minimal Supersymmetric Standard Model (MSSM) [9] is one of the leading candidates.

In the MSSM the Higgs sector consists of two scalar doublets accommodating five physical Higgs bosons. In lowest order these are the light and heavy  $\mathcal{CP}$ -even  $h$  and  $H$ , the  $\mathcal{CP}$ -odd  $A$ , and the charged Higgs bosons  $H^\pm$ . The parameters characterising the MSSM Higgs sector at lowest order are the gauge couplings, the mass of the  $\mathcal{CP}$ -odd Higgs boson,  $M_A$ , and  $\tan\beta \equiv v_2/v_1$ , the ratio of the two vacuum expectation values. Accordingly, all other masses and mixing angles can be predicted in terms of those parameters, leading to the famous tree-level upper bound for the mass of the light  $\mathcal{CP}$ -even Higgs boson,  $M_h \leq M_Z$ , determined by the mass of the  $Z$  boson,  $M_Z$ . This tree-level upper bound, which arises from the gauge sector, receives large corrections from the Yukawa sector of the theory, which

can be of  $\mathcal{O}(50\%)$  (depending on the model parameters) upon incorporating the full one-loop and the dominant two-loop contributions [10, 11, 12].

The prediction for the light  $\mathcal{CP}$ -even Higgs-boson mass in the MSSM is affected by two kinds of theoretical uncertainties. First, the parametric uncertainties induced by the experimental errors of the input parameters. Here the dominant source of parametric uncertainty is the experimental error on the top-quark mass,  $m_t$ . Very roughly, the impact of the experimental error on  $m_t$  on the prediction for  $M_h$  scales like [13]

$$\delta M_h^{\text{para}, m_t} / \delta m_t^{\text{exp}} \sim 1. \quad (3)$$

As a consequence, high-precision top-physics providing an accuracy on  $m_t$  much below the GeV-level is a crucial ingredient for precision physics in the Higgs sector [13]. The second type of uncertainties are the intrinsic theoretical uncertainties that are due to unknown higher-order corrections. An overall estimate for the lightest  $\mathcal{CP}$ -even Higgs mass of  $\delta M_h^{\text{intr}} \sim 3$  GeV had been given in Refs. [10, 12] (the more recent inclusion of the leading  $\mathcal{O}(\alpha_t \alpha_s^2)$  3-loop corrections [14], see below, has slightly reduced this estimated uncertainty by few times  $\mathcal{O}(100$  MeV)). It was pointed out that a more detailed estimate needs to take into account the dependence on the considered parameter region of the model. In particular, the uncertainty of this fixed-order prediction is somewhat larger for scalar top masses in the multi-TeV range.

The MSSM parameter space with scalar top masses in the multi-TeV range has received considerable attention recently, partly because of the relatively high value of  $M_h \approx 125$  GeV, which generically requires either large stop masses or large mixing in the scalar top sector, and partly because of the limits from searches for supersymmetric (SUSY) particles at the LHC. While within the general MSSM the lighter scalar superpartner of the top quark is allowed to be relatively light (down to values even as low as  $m_t$ ), both with respect to the direct searches and with respect to the prediction for  $M_h$  (see e.g. Ref. [15]), the situation is different in more constrained models. For instance, global fits in the Constrained MSSM (CMSSM) prefer scalar top masses in the multi-TeV range [16, 17, 18, 19].

Here we review the significantly improved prediction for  $M_h$  in the MSSM from resumming large logarithms with large scalar top masses [20], which has an important impact on the phenomenology in the region of heavy squarks and on its confrontation with the experimental results. We briefly review the relevant sectors and the new, improved prediction for  $M_h$ . The numerical analysis focuses on the effects of heavy scalar

top masses in the general MSSM and in the CMSSM. The feasibility of reaching the anticipated ILC precision will be briefly discussed.

## 2. The Higgs and scalar top sectors of the MSSM

In the MSSM with real parameters (we restrict to this case for simplicity; for the treatment of complex parameters see Refs. [21, 22, 23] and references therein), using the Feynman diagrammatic (FD) approach, the higher-order corrected  $\mathcal{CP}$ -even Higgs-boson masses are derived by finding the poles of the  $(h, H)$ -propagator matrix. The inverse of this matrix is given by

$$-i \begin{pmatrix} p^2 - m_{h, \text{tree}}^2 + \hat{\Sigma}_{hh}(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m_{H, \text{tree}}^2 + \hat{\Sigma}_{HH}(p^2) \end{pmatrix}, \quad (4)$$

where  $m_{h, H, \text{tree}}$  denote the tree-level masses, and  $\hat{\Sigma}_{hh, HH, hH}(p^2)$  are the renormalized Higgs boson self-energies evaluated at the squared external momentum  $p^2$ . For the computation of the leading contributions to those self-energies it is convenient to use the basis of the fields  $\phi_1, \phi_2$ , which are related to  $h, H$  via the (tree-level) mixing angle  $\alpha$ :

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} -\sin \alpha & \cos \alpha \\ \cos \alpha & \sin \alpha \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}. \quad (5)$$

The new higher-order corrections reviewed here originate in the top/stop sector of the MSSM. The bilinear part of the top-squark Lagrangian,

$$\mathcal{L}_{\tilde{t}, \text{mass}} = -(\tilde{t}_L^\dagger, \tilde{t}_R^\dagger) \mathbf{M}_{\tilde{t}} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}, \quad (6)$$

contains the stop mass matrix,  $\mathbf{M}_{\tilde{t}}$ , given by

$$\mathbf{M}_{\tilde{t}} = \begin{pmatrix} M_{\tilde{t}_L}^2 + m_t^2 + D_1 & m_t X_t \\ m_t X_t & M_{\tilde{t}_R}^2 + m_t^2 + D_2 \end{pmatrix}, \quad (7)$$

with

$$X_t = A_t - \mu \cot \beta. \quad (8)$$

$D_{1,2} \sim M_Z^2$  denote numerically small  $D$ -terms,  $A_t$  is the trilinear coupling between the Higgs bosons and the scalar top quarks, and  $\mu$  is the Higgs mixing parameter. The mass matrix can be diagonalized with the help of a unitary transformation  $\mathbf{U}_{\tilde{t}}$ , yielding the two stop mass eigenvalues,  $m_{\tilde{t}_1}$  and  $m_{\tilde{t}_2}$ .

For the MSSM with real parameters the status of higher-order corrections to the masses and mixing angles in the neutral Higgs sector is quite advanced.

The complete one-loop result within the MSSM is known [24, 25, 26, 27]. The by far dominant one-loop contribution is the  $\mathcal{O}(\alpha_t)$  term due to top and stop loops ( $\alpha_t \equiv h_t^2/(4\pi)$ ,  $h_t$  being the top-quark Yukawa coupling). The computation of the two-loop corrections is already quite advanced and has meanwhile reached a stage where all the presumably dominant contributions are available [23, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43]. In particular, the  $\mathcal{O}(\alpha_t\alpha_s)$ ,  $\mathcal{O}(\alpha_t^2)$  contributions to the self-energies – evaluated in the Feynman-diagrammatic (FD) and in the effective potential (EP) method – as well as the  $\mathcal{O}(\alpha_b\alpha_s)$ ,  $\mathcal{O}(\alpha_t\alpha_b)$  and  $\mathcal{O}(\alpha_b^2)$  contributions – evaluated in the EP approach – are known. (For latest corrections to the charged Higgs boson mass, see Ref. [44].)

The public code `FeynHiggs` [45, 29, 10, 21, 20] includes all of the above corrections<sup>1</sup>, where the on-shell (OS) scheme for the renormalization of the scalar quark sector has been used (another public code, based on the Renormalization Group (RG) improved Effective Potential, is `CPsuperH` [47]). A full 2-loop effective potential calculation (supplemented by the momentum dependence for the leading pieces and the leading 3-loop corrections) has been published [48, 49]. However, no computer code is publicly available (see, however, Ref. [50]). Most recently another leading 3-loop calculation at  $\mathcal{O}(\alpha_t\alpha_s^2)$  became available (based on a  $\overline{\text{DR}}$  or a “hybrid” renormalisation scheme for the scalar top sector), where the numerical evaluation depends on the various SUSY mass hierarchies [14], resulting in the code `H3m` (which adds the 3-loop corrections to the `FeynHiggs` result).

### 3. Improved calculation of $M_h$

We review here the improved prediction for  $M_h$  where we combine the fixed-order result obtained in the OS scheme with an all-order resummation of the leading and subleading contributions from the scalar top sector. We have obtained the latter from an analysis of the RG Equations (RGEs) at the two-loop level [51]. Assuming a common mass scale  $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$  ( $M_S \gg M_Z$ ) for all relevant SUSY mass parameters, the quartic Higgs coupling  $\lambda$  can be evolved via SM RGEs from  $M_S$  to the scale  $Q$  (we choose  $Q = m_t$  in the following) where  $M_h^2$  is to be evaluated (see, for instance, Ref. [32] and references therein),

$$M_h^2 = 2\lambda(m_t)v^2. \quad (9)$$

<sup>1</sup>More recent refinements in the  $M_h$  calculation that have been included (or are currently included) into `FeynHiggs` can be found in Refs. [43, 23, 46].

Here  $v \sim 174$  GeV denotes the vacuum expectation value of the SM. Three coupled RGEs, the ones for

$$\lambda, h_t, g_s \quad (10)$$

are relevant for this evolution, with the strong coupling constant given as  $\alpha_s = g_s^2/(4\pi)$ . Since SM RGEs are used, the relevant parameters are given in the  $\overline{\text{MS}}$  scheme. We incorporate the one-loop threshold corrections to  $\lambda(M_S)$  as given in Ref. [32],

$$\lambda(M_S) = \frac{3h_t^4(M_S)}{8\pi^2} \frac{X_t^2}{M_S^2} \left[ 1 - \frac{1}{12} \frac{X_t^2}{M_S^2} \right], \quad (11)$$

where as mentioned above  $X_t$  is an  $\overline{\text{MS}}$  parameter. Furthermore, in Eq. (11) we have set the SM gauge couplings to  $g = g' = 0$ , ensuring that Eq. (9) consists of the “pure loop correction” and will be denoted  $(\Delta M_h^2)^{\text{RGE}}$  below. Using RGEs at two-loop order [51], including fermionic contributions from the top sector only, leads to a prediction for the corrections to  $M_h^2$  including leading and subleading logarithmic contributions at  $n$ -loop order,

$$L^n \text{ and } L^{(n-1)}, \quad L \equiv \ln\left(\frac{M_S}{m_t}\right), \quad (12)$$

originating from the top/stop sector of the MSSM.

We have obtained both analytic solutions of the RGEs up to the 7-loop level as well as a numerical solution incorporating the leading and subleading logarithmic contributions up to all orders. In a similar way in Ref. [49] the leading logarithms at 3- and 4-loop order have been evaluated analytically. Most recently a calculation using 3-loop SM RGEs appeared in Ref. [52].

A particular complication arises in the combination of the higher-order logarithmic contributions obtained from solving the RGEs with the fixed-order FD result implemented in `FeynHiggs` comprising corrections up to the two-loop level in the OS scheme. We have used the parametrisation of the FD result in terms of the running top-quark mass at the scale  $m_t$ ,

$$\overline{m}_t = \frac{m_t^{\text{pole}}}{1 + \frac{4}{3\pi}\alpha_s(m_t^{\text{pole}}) - \frac{1}{2\pi}\alpha_t(m_t^{\text{pole}})}, \quad (13)$$

where  $m_t^{\text{pole}}$  denotes the top-quark pole mass. Avoiding double counting of the logarithmic contributions up to the two-loop level and consistently taking into account the different schemes employed in the FD and the RGE approach, the correction  $\Delta M_h^2$  takes the form

$$\begin{aligned} \Delta M_h^2 &= (\Delta M_h^2)^{\text{RGE}}(X_t^{\overline{\text{MS}}}) - (\Delta M_h^2)^{\text{FD,LL1,LL2}}(X_t^{\text{OS}}), \\ M_h^2 &= (M_h^2)^{\text{FD}} + \Delta M_h^2. \end{aligned} \quad (14)$$

Here  $(M_h^2)^{\text{FD}}$  denotes the fixed-order FD result,  $(\Delta M_h^2)^{\text{FD,LL1,LL2}}$  are the logarithmic contributions up to the two-loop level obtained with the FD approach in the OS scheme, while  $(\Delta M_h^2)^{\text{RGE}}$  are the leading and sub-leading logarithmic contributions (either up to a certain loop order or summed to all orders) obtained in the RGE approach, as evaluated via Eq. (9). In all terms of Eq. (14) the top-quark mass is parametrised in terms of  $\overline{m}_t$ ; the relation between  $X_t^{\overline{\text{MS}}}$  and  $X_t^{\text{OS}}$  is given by

$$X_t^{\overline{\text{MS}}} = X_t^{\text{OS}} \left[ 1 + 2L \left( \frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi} \right) \right] \quad (15)$$

up to non-logarithmic terms, and there are no logarithmic contributions in the relation between  $M_S^{\overline{\text{MS}}}$  and  $M_S^{\text{OS}}$ .

Since our higher-order contributions beyond 2-loop have been derived under the assumption  $M_A \gg M_Z$ , to a good approximation these corrections can be incorporated as a shift in the prediction for the  $\phi_2\phi_2$  self-energy (where  $\Delta M_h^2$  enters with a coefficient  $1/\sin^2\beta$ ). In this way the new higher-order contributions enter not only the prediction for  $M_h$ , but also consistently all other Higgs sector observables that are evaluated in FeynHiggs, such as the effective mixing angle  $\alpha_{\text{eff}}$  or the finite field renormalization constant matrix  $\mathbf{Z}_h$  [21].

The latest version of the code, FeynHiggs 2.10.0, which is available at [feynhiggs.de](http://feynhiggs.de), contains those improved predictions as well as a refined estimate of the theoretical uncertainties from unknown higher-order corrections. Taking into account the leading and sub-leading logarithmic contributions in higher orders reduces the uncertainty of the remaining unknown higher-order corrections. Accordingly, the estimate of the uncertainties arising from corrections beyond two-loop order in the top/stop sector is adjusted such that the impact of replacing the running top-quark mass by the pole mass (see Ref. [10]) is evaluated only for the non-logarithmic corrections rather than for the full two-loop contributions implemented in FeynHiggs. First investigations using this new uncertainty estimate can be found in Refs. [17, 53].

Further refinements of the RGE resummed result are possible, in particular

- (i) extending the result to the case of a large splitting between the left- and right-handed soft SUSY-breaking terms in the scalar top sector [38],
- (ii) extending the result to the case of small  $M_A$  or  $\mu$  (close to  $M_Z$ ),
- (iii) including the corresponding contributions from the (s)bottom sector.

Some details in these directions can be found in Ref. [52]. We leave those refinements for future work.

#### 4. Numerical analysis

In this section we review the analysis of the phenomenological implications of the improved  $M_h$  prediction for large stop mass scales, as evaluated with FeynHiggs 2.10.0. In Ref. [20] it was shown that  $M_h(X_t/M_S)$  has local maxima for  $X_t/M_S \approx \pm 2$ , where  $X_t$  denotes  $X_t^{\text{OS}}$  (for  $M_S$  the difference in the two schemes is negligible). The  $M_h$  values at  $X_t/M_S \approx +2$  are slightly larger than the ones at  $X_t/M_S \approx -2$ . Local minima are reached around  $X_t/M_S \approx 0$ .

These findings motivate the choice of parameters used in Fig. 1. Here we show  $M_h$  as a function of  $M_S$  for  $X_t = 0$  and  $X_t/M_S = +2$ , corresponding to the local minimum and the maximum value of  $M_h$  as a function of  $X_t/M_S$ , respectively. The other parameters are  $M_A = M_2 = \mu = 1000$  GeV,  $m_{\tilde{g}} = 1600$  GeV (where  $M_2$  is the SU(2) gaugino mass term,  $\mu$  the Higgsino mass parameter and  $m_{\tilde{g}}$  the gluino mass) and  $\tan\beta = 10$ . The plot shows for the two values of  $X_t/M_S$  the fixed-order FD result containing corrections up to the two-loop level (labelled as “FH295”, which refers to the previous version of the code FeynHiggs) as well as the latter result supplemented with the analytic solution of the RGEs up to the 3-loop, ... 7-loop level (labelled as “3-loop” ... “7-loop”). The curve labelled as “LL+NLL” represents our full result, where the FD contribution is supplemented by the leading and next-to-leading logarithms summed to all orders. One can see that the impact of the higher-order logarithmic contributions is relatively small for  $M_S = \mathcal{O}(1 \text{ TeV})$ , while large differences between the fixed-order result and the improved results occur for large values of  $M_S$ . The 3-loop logarithmic contribution is found to have the largest impact in this context, but for  $M_S \gtrsim 2500(6000)$  GeV for  $X_t/M_S = 2(0)$  also contributions beyond 3-loop are important. A convergence of the higher-order logarithmic contributions towards the full resummed result is clearly visible. At  $M_S = 20$  TeV the difference between the 7-loop result and the full resummed result is around 900(200) MeV for  $X_t/M_S = 2(0)$ . The corresponding deviations stay below 100 MeV for  $M_S \lesssim 10$  TeV. The plot furthermore shows that for  $M_S \approx 7$  TeV (and the value of  $\tan\beta = 10$  chosen here) a predicted value of  $M_h$  of about 125 GeV is obtained even for the case of vanishing mixing in the scalar top sector ( $X_t = 0$ ). Since

the predicted value of  $M_h$  grows further with increasing  $M_S$  it becomes apparent that the measured mass of the observed signal, when interpreted as  $M_h$ , can be used (within the current experimental and theoretical uncertainties) to derive an *upper bound* on the mass scale  $M_S$  in the scalar top sector, see also Ref. [54]. However, more robust statements in this direction will require a careful analysis of still present intrinsic as well as the parametric uncertainties.

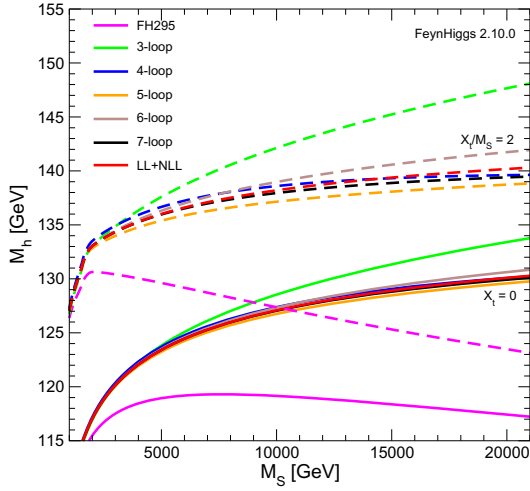


Figure 1:  $M_h$  as a function of  $M_S$  for  $X_t = 0$  (solid) and  $X_t/M_S = 2$  (dashed). The full result (“LL+NLL”) is compared with results containing the logarithmic contributions up to the 3-loop, ... 7-loop level and with the fixed-order FD result (“FH295”).

In Fig. 2 we compare our result with the prediction obtained from the code H3m [14]. The comparison was performed in the CMSSM with the parameters set to  $m_0 = m_{1/2} = 200 \text{ GeV} \dots 15000 \text{ GeV}$ ,  $A_0 = 0$ ,  $\tan\beta = 10$  and  $\mu > 0$ . The spectra were generated with SoftSusy 3.3.10 [55]. The H3m result shown as blue line, containing the terms in  $O(\alpha_t \alpha_s^2) \times O(L^3, L^2, L^1, L^0)$ , can be compared with the FeynHiggs 3-loop result,  $O(L^3, L^2)$ , but restricted to  $O(\alpha_t \alpha_s^2)$  (green dashed). We find that the latter result agrees rather well with H3m, with maximal deviation of  $O(1 \text{ GeV})$  for  $M_S \lesssim 10 \text{ TeV}$ . The observed deviations can be attributed to the terms of  $O(L^1, L^0)$  included in H3m, to the SUSY mass hierarchies taken into account in H3m, and to the use of different scalar top renormalization schemes employed in the two codes (where the latter effect is already expected to be at the GeV-level). Further investigations will be needed to explore the source of the main differences. However, adding also the 3-loop  $O(\alpha_t^2 \alpha_s, \alpha_t^3) \times O(L^3, L^2)$  terms (solid green), as included in the FeynHiggs result, leads to a strong reduction of  $M_h$  by  $\sim 5 \text{ GeV}$

for  $M_S = 10 \text{ TeV}$  (see also Ref. [49]). Going to the full resummed FeynHiggs result (red) exhibits a further, but smaller reduction of  $M_h$  of about 2 GeV for  $M_S = 10 \text{ TeV}$ , even larger changes are found for  $M_S > 10 \text{ TeV}$ . Consequently, 3-loop corrections at  $O(\alpha_t^2 \alpha_s, \alpha_t^3)$  as well as corrections beyond 3-loop are clearly important for a precise  $M_h$  prediction.

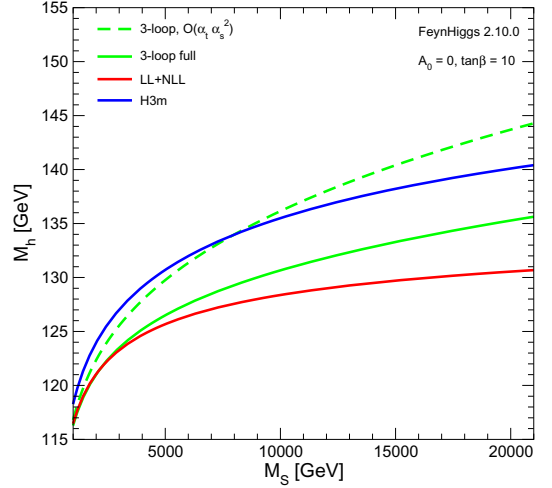


Figure 2:  $M_h$  as a function of  $M_S$ . Comparison of FeynHiggs (red) with H3m (blue). In green we show the FeynHiggs 3-loop result at  $O(\alpha_t \alpha_s^2)$  ( $O(\alpha_t \alpha_s^2, \alpha_t^2 \alpha_s, \alpha_t^3)$ ) as dashed (solid) line.

Finally, in Fig. 3 we analyze the effects of the new  $M_h$  evaluation in the CMSSM [53]. In the upper (lower) plot we show the  $m_{1/2}-m_0$  plane for  $\tan\beta = 10(30)$ ,  $A_0 = 2.5 m_0$  and  $\mu > 0$ . Regions where the lightest SUSY particle (LSP) is charged are shaded brown, those where there is no consistent electroweak vacuum are shaded mauve, regions excluded by  $\text{BR}(b \rightarrow s\gamma)$  measurements at the  $2\sigma$  level are shaded green, those favored by the SUSY interpretation of  $(g-2)_\mu$  are shaded pink (with lines indicating the  $1\sigma$  (dashed) and  $2\sigma$  ranges (solid)), and strips with an LSP density appropriate to make up all the cold dark matter are shaded dark blue. The 95% CL limit from the ATLAS MET search is shown as a continuous purple contour, and the 68 and 95% CL limits from the CMS and LHCb measurements of  $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$  are shown as continuous green contours, see Ref. [53] for details. Finally, the labelled continuous black lines are contours of  $M_h$  calculated with FeynHiggs 2.10.0, and the dash-dotted red lines are contours of  $M_h$  calculated with FeynHiggs 2.8.6. It can be seen that the improved  $M_h$  calculation leads to an upward shift, such that  $M_h \sim 125 \text{ GeV}$  can be reached for lower values of  $m_{1/2}$  and  $m_0$ .

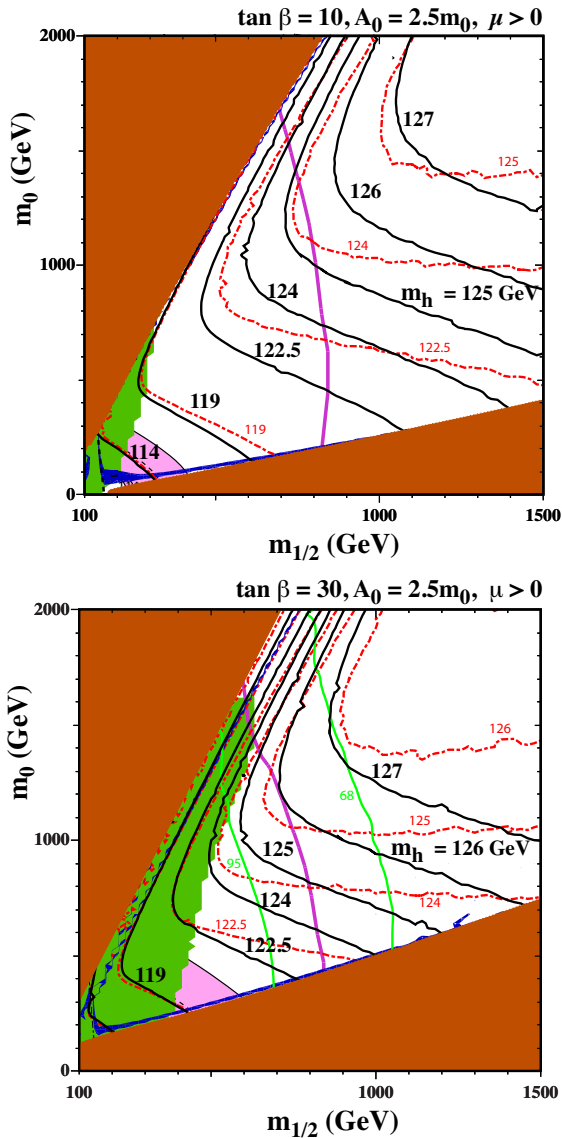


Figure 3: The  $m_{1/2}$ - $m_0$  plane in the CMSSM for  $\tan\beta = 10(30)$ ,  $A_0 = 2.5m_0$  and  $\mu > 0$  in the upper (lower) plot [53]. The black (red) lines correspond to  $M_h$  evaluated with FeynHiggs 2.10.0 (2.8.6). For the various shadings see text.

In view of the anticipated future accuracy at the ILC, as given in Eq. (2), the remaining theory uncertainties in the current status of the  $M_h$  evaluations will have to be re-analyzed carefully. It can be expected, see also Ref. [53], that for scalar top mass scales below the few-TeV level the intrinsic uncertainty is now, i.e. including the resummed contributions, at or below the level of  $\sim 2$  GeV. However, still substantial further refinements will be needed to reach the sub-GeV level. On the other hand, no investigation of the size of the intrinsic uncer-

tainties has been performed for scalar top masses in the multi-TeV range, as explored here. Consequently, the prospects of reaching the sub-GeV level in  $\delta M_h^{\text{intr}}$  are so far unclear.

## 5. Conclusions

We have reviewed the improved prediction for the light  $C\mathcal{P}$ -even Higgs-boson mass in the MSSM, obtained by combining the FD result at the one- and two-loop level with an all-order resummation of the leading and subleading logarithmic contributions from the top/stop sector obtained from solving the two-loop RGEs. Particular care has been taken to consistently match these two different types of corrections. The result, providing the most precise prediction for  $M_h$  in the presence of large masses of the scalar partners of the top quark, has been implemented into the public code FeynHiggs and can be obtained at `feynhiggs.de`. We have found a sizable effect of the higher-order logarithmic contributions to  $M_h$  in the MSSM and in particular in the CMSSM, where  $M_h \sim 125$  GeV can be reached for lower values of  $m_{1/2}$  and  $m_0$ .

## Acknowledgements

We thank O. Buchmueller, M. Dolan, J. Ells, J. Marrouche, K. Olive and K. de Vries with whom parts of the results presented here have been obtained. S.H. thanks the ICHEP14 organizers for a very stimulating and exciting atmosphere.

## References

- [1] G. Aad et al. [ATLAS Collaboration], *Phys. Lett. B* **716** (2012) 1 [arXiv:1207.7214 [hep-ex]].
- [2] S. Chatrchyan et al. [CMS Collaboration], *Phys. Lett. B* **716** (2012) 30 [arXiv:1207.7235 [hep-ex]].
- [3] G. Aad et al. [ATLAS Collaboration], *Phys. Rev. D* **90** (2014) 052004 [arXiv:1406.3827 [hep-ex]].
- [4] CMS Collaboration [CMS Collaboration], CMS-PAS-HIG-14-009.
- [5] H. Baer et al., arXiv:1306.6352 [hep-ph].
- [6] S. Heinemeyer et al. [LHC Higgs Cross Section Working Group Collaboration], arXiv:1307.1347 [hep-ph].
- [7] A. David et al. [LHC Higgs Cross Section Working Group Collaboration], arXiv:1209.0040 [hep-ph].
- [8] E. Gross, talk given at “Rencontres de Moriond EW 2014”, see <https://indico.in2p3.fr/materialDisplay.py?contribId=218&sessionId=8&materialId=slides&confId=9116>, E. Musella, talk given at “Rencontres de Moriond EW 2014”, <https://indico.in2p3.fr/materialDisplay.py?contribId=219&sessionId=8&materialId=slides&confId=9116>.

- [9] H. Nilles, *Phys. Rept.* **110** (1984) 1;  
H. Haber and G. Kane, *Phys. Rept.* **117** (1985) 75;  
R. Barbieri, *Riv. Nuovo Cim.* **11** (1988) 1.
- [10] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich and G. Weiglein, *Eur. Phys. J. C* **28** (2003) 133 [arXiv:hep-ph/0212020].
- [11] A. Djouadi, *Phys. Rept.* **459** (2008) 1 [arXiv:hep-ph/0503173];  
S. Heinemeyer, *Int. J. Mod. Phys. A* **21** (2006) 2659 [arXiv:hep-ph/0407244].
- [12] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Rept.* **425** (2006) 265 [arXiv:hep-ph/0412214].
- [13] S. Heinemeyer, S. Kraml, W. Porod and G. Weiglein, *JHEP* **0309** (2003) 075 [arXiv:hep-ph/0306181].
- [14] R. Harlander, P. Kant, L. Mihaila and M. Steinhauser, *Phys. Rev. Lett.* **100** (2008) 191602 [*Phys. Rev. Lett.* **101** (2008) 039901] [arXiv:0803.0672 [hep-ph]]; *JHEP* **1008** (2010) 104 [arXiv:1005.5709 [hep-ph]].
- [15] S. Heinemeyer, O. Stål and G. Weiglein, *Phys. Lett. B* **710** (2012) 201 [arXiv:1112.3026 [hep-ph]];  
L. Hall, D. Pinner and J. Ruderman, *JHEP* **1204** (2012) 131 [arXiv:1112.2703 [hep-ph]];  
H. Baer, V. Barger and A. Mustafayev, *Phys. Rev. D* **85** (2012) 075010 [arXiv:1112.3017 [hep-ph]];  
A. Arbey, M. Battaglia, A. Djouadi, F. Mahmoudi and J. Quevillon, *Phys. Lett. B* **708** (2012) 162 [arXiv:1112.3028 [hep-ph]];  
P. Draper, P. Meade, M. Reece and D. Shih, *Phys. Rev. D* **85** (2012) 095007 [arXiv:1112.3068 [hep-ph]].
- [16] O. Buchmueller et al., *Eur. Phys. J. C* **72** (2012) 2243 [arXiv:1207.7315 [hep-ph]].
- [17] O. Buchmueller et al., *Eur. Phys. J. C* **74** (2014) 2922 [arXiv:1312.5250 [hep-ph]].
- [18] O. Buchmueller et al., arXiv:1408.4060 [hep-ph].
- [19] P. Bechtle et al., arXiv:1310.3045 [hep-ph].
- [20] T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *Phys. Rev. Lett.* **112** (2014) 141801 [arXiv:1312.4937 [hep-ph]].
- [21] M. Frank, T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *JHEP* **0702** (2007) 047 [arXiv:hep-ph/0611326].
- [22] S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *Phys. Lett. B* **652** (2007) 300 [arXiv:0705.0746 [hep-ph]].
- [23] W. Hollik and S. Paßehr, *Phys. Lett. B* **733** (2014) 144 [arXiv:1401.8275 [hep-ph]]; arXiv:1409.1687 [hep-ph].
- [24] J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett. B* **257** (1991) 83;  
Y. Okada, M. Yamaguchi and T. Yanagida, *Prog. Theor. Phys.* **85** (1991) 1;  
H. Haber and R. Hempfling, *Phys. Rev. Lett.* **66** (1991) 1815.
- [25] A. Brignole, *Phys. Lett. B* **281** (1992) 284.
- [26] P. Chankowski, S. Pokorski and J. Rosiek, *Phys. Lett. B* **286** (1992) 307; *Nucl. Phys. B* **423** (1994) 437 [arXiv:hep-ph/9303309].
- [27] A. Dabelstein, *Nucl. Phys. B* **456** (1995) 25 [arXiv:hep-ph/9503443]; *Z. Phys. C* **67** (1995) 495 [arXiv:hep-ph/9409375].
- [28] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Rev. D* **58** (1998) 091701 [arXiv:hep-ph/9803277]; *Phys. Lett. B* **440** (1998) 296 [arXiv:hep-ph/9807423].
- [29] S. Heinemeyer, W. Hollik and G. Weiglein, *Eur. Phys. J. C* **9** (1999) 343 [arXiv:hep-ph/9812472].
- [30] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Lett. B* **455** (1999) 179 [arXiv:hep-ph/9903404].
- [31] S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *Eur. Phys. J. C* **39** (2005) 465 [arXiv:hep-ph/0411114].
- [32] M. Carena, H. Haber, S. Heinemeyer, W. Hollik, C. Wagner, and G. Weiglein, *Nucl. Phys. B* **580** (2000) 29 [arXiv:hep-ph/0001002].
- [33] R. Zhang, *Phys. Lett. B* **447** (1999) 89 [arXiv:hep-ph/9808299];  
J. Espinosa and R. Zhang, *JHEP* **0003** (2000) 026 [arXiv:hep-ph/9912236].
- [34] G. Degrassi, P. Slavich and F. Zwirner, *Nucl. Phys. B* **611** (2001) 403 [arXiv:hep-ph/0105096].
- [35] R. Hempfling and A. Hoang, *Phys. Lett. B* **331** (1994) 99 [arXiv:hep-ph/9401219].
- [36] A. Brignole, G. Degrassi, P. Slavich and F. Zwirner, *Nucl. Phys. B* **631** (2002) 195 [arXiv:hep-ph/0112177].
- [37] J. Espinosa and R. Zhang, *Nucl. Phys. B* **586** (2000) 3 [arXiv:hep-ph/0003246].
- [38] J. Espinosa and I. Navarro, *Nucl. Phys. B* **615** (2001) 82 [arXiv:hep-ph/0104047].
- [39] A. Brignole, G. Degrassi, P. Slavich and F. Zwirner, *Nucl. Phys. B* **643** (2002) 79 [arXiv:hep-ph/0206101].
- [40] G. Degrassi, A. Dedes and P. Slavich, *Nucl. Phys. B* **672** (2003) 144 [arXiv:hep-ph/0305127].
- [41] M. Carena, J. Espinosa, M. Quirós and C. Wagner, *Phys. Lett. B* **355** (1995) 209 [arXiv:hep-ph/9504316];  
M. Carena, M. Quirós and C. Wagner, *Nucl. Phys. B* **461** (1996) 407 [arXiv:hep-ph/9508343].
- [42] J. Casas, J. Espinosa, M. Quirós and A. Riotto, *Nucl. Phys. B* **436** (1995) 3, [Erratum-ibid. **B 439** (1995) 466] [arXiv:hep-ph/9407389].
- [43] S. Borowka, T. Hahn, S. Heinemeyer, G. Heinrich and W. Hollik, *Eur. Phys. J. C* **74** (2014) 2994 [arXiv:1404.7074 [hep-ph]].
- [44] M. Frank et al., *Phys. Rev. D* **88** (2013) 055013 [arXiv:1306.1156 [hep-ph]].
- [45] S. Heinemeyer, W. Hollik and G. Weiglein, *Comput. Phys. Commun.* **124** (2000) 76 [arXiv:hep-ph/9812320];  
T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *Comput. Phys. Commun.* **180** (2009) 1426; see: [www.feynhiggs.de](http://www.feynhiggs.de).
- [46] M. Gómez, T. Hahn, S. Heinemeyer and M. Rehman, to appear in *Phys. Rev. D*, arXiv:1408.0663 [hep-ph].
- [47] J. Lee et al., *Comput. Phys. Commun.* **156** (2004) 283; J. Lee et al., *Comput. Phys. Commun.* **180** (2009) 312; *Comput. Phys. Commun.* **184** (2013) 1220.
- [48] S. Martin, *Phys. Rev. D* **65** (2002) 116003 [arXiv:hep-ph/0111209]; *Phys. Rev. D* **66** (2002) 096001 [arXiv:hep-ph/0206136]; *Phys. Rev. D* **67** (2003) 095012 [arXiv:hep-ph/0211366]; *Phys. Rev. D* **68** (2003) 075002 [arXiv:hep-ph/0307101]; *Phys. Rev. D* **70** (2004) 016005 [arXiv:hep-ph/0312092]; *Phys. Rev. D* **71** (2005) 116004 [arXiv:hep-ph/0502168]; S. Martin and D. Robertson, *Comput. Phys. Commun.* **174** (2006) 133 [arXiv:hep-ph/0501132].
- [49] S. Martin, *Phys. Rev. D* **75** (2007) 055005 [arXiv:hep-ph/0701051].
- [50] S. Martin and D. Robertson, arXiv:1407.4336 [hep-ph].
- [51] J. Espinosa and M. Quiros, *Phys. Lett. B* **266** (1991) 389; H. Arason et al., *Phys. Rev. D* **46** (1992) 3945.
- [52] P. Draper, G. Lee and C. Wagner, *Phys. Rev. D* **89** (2014) 055023 [arXiv:1312.5743 [hep-ph]].
- [53] O. Buchmueller et al., *Eur. Phys. J. C* **74** (2014) 2809 [arXiv:1312.5233 [hep-ph]].
- [54] M. Cabrera, J. Casas and A. Delgado, *Phys. Rev. Lett.* **108** (2012) 021802 [arXiv:1108.3867 [hep-ph]];  
G. Degrassi, S. Di Vita, J. Elias-Miro, J. Espinosa, G. Giudice, G. Isidori and A. Strumia, *JHEP* **1208** (2012) 098 [arXiv:1205.6497 [hep-ph]].
- [55] B. Allanach, *Comput. Phys. Commun.* **143** (2002) 305. [arXiv:hep-ph/0104145].