Competition between Anderson localization and leakage of surface-plasmon polaritons on randomly rough periodic metal surfaces

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The competition between strong localization and radiative damping is studied for surface-plasmon polaritons propagating along a finite, randomly rough metallic surface that is periodic on average, by means of both numerical simulations and perturbation-theoretic calculations. Our results show that localization is the predominant contribution to the exponential decay of the transmission. In addition, it is found that the localization length is larger for a frequency in an allowed band of the surface-plasmon polariton dispersion relation for the underlying periodic structure than it is for a frequency in the vicinity of a gap. No significant leakage inhibition is observed near the band edge. [S0163-1829(97)03728-4]

In recent years, the study of the strong localization of electromagnetic waves in one-dimensional random systems has attracted a great deal of attention.[1–4] In particular, several works have been devoted to the characterization of weak and strong localization of surface plasmon polaritons (SPP's) on randomly rough metal surfaces.[5–14] It is well known that any amount of disorder leads to strong localization in one-dimensional systems.[3] However, in calculations of the Anderson localization length of a surface wave propagating along a randomly rough surface it is important to estimate its attenuation length caused by leakage. If this length is long compared with the decay length of the transmissivity, one can be confident that the latter is an accurate estimate of the localization length of the surface wave. In this connection, to our knowledge, none of the theoretical works reported thus far on the SPP localization[5,14] has addressed in a rigorous quantitative manner the question of SPP leakage. It is our purpose to study in this paper the interplay between SPP localization and leakage; we will describe the frequency-dependent length scales of the localization and radiative damping processes by $l(\omega)$ and $l_{rad}(\omega)$, respectively. Furthermore, in addition to the random nature of the corrugation that produces the constructive interference of multiply scattered SPP leading to localization, a strong periodic component is superimposed. This allows us to study the prediction by John[1] according to which localization should be eased or strengthened for frequencies near a band edge of the underlying periodic structure, owing to Bragg scattering.[5,14,15] Moreover, the behavior of the radiative damping at the band edge can be analyzed in light of the work predicting inhibition of leakage when the frequency lies at a band edge.[4]

The physical system we consider consists of a vacuum in the region $x_3>\zeta(x_1)$ and a metal in the region $x_3<\zeta(x_1)$. The metal is characterized by the free-electron dielectric function $\epsilon(\omega)=1-\omega_p^2/\omega^2$, where ω_p is the plasma frequency of the conduction electrons. Thus no Ohmic losses occur in the system. The electromagnetic field in it is p polarized, with the plane of incidence the x_1-x_3 plane. The surface profile function $\zeta(x_1)$, which is assumed to be nonzero only in the interval $-L/2<x_1<L/2$, is a single-valued function of x_1 that is random but periodic on average. In the present work we model this rough surface by the use of a local random impedance boundary condition[16] (IBC) on the plane $x_3=0$,

$$\frac{\partial}{\partial x_3}H_2^{>}(x_1,x_3|\omega)|_{x_3=0} = -\frac{\omega}{c}[-\epsilon(\omega)]^{-1/2}[1+s(x_1)]H_2^{>}(x_1,x_3|\omega)|_{x_3=0}, \quad (1)$$

where $H_2^{>}(x_1,x_3|\omega)$ is the single nonzero component of the total magnetic field in the vacuum region $x_3>0$. The impedance function $s(x_1)$ is the sum of a finite periodic function $s_P(x_1)$ and a zero-mean stationary Gaussian random process $s_R(x_1)$,

$$s(x_1)=[s_P(x_1)+s_R(x_1)]\mathrm{thf}(x_1,L), \quad (2)$$

where the function $\mathrm{thf}(x_1,L)=2\cosh^2(\beta L/4)/[\cosh(\beta L/2)+\cosh\beta(x_1)]$ with $\beta=100/L$ cuts off $s(x_1)$ for $|x_1|>L/2$ in a smooth fashion. For the function $s_P(x_1)$ we choose $s_P(x_1)=s_0\cos(2\pi x_1/d)$, while $s_R(x_1)$ is characterized by $\langle s_R(x_1)s_R(x_1')\rangle=\delta^2\exp(-|x_1-x_1'|^2/a^2)$, where $\delta^2=\langle s_R^2(x_1)\rangle$. The connection between the impedance function $s(x_1)$ and the surface profile function $\zeta(x_1)$ has been established recently.[17]

The magnetic field $H_2^{>}(x_1,x_3|\omega)$ consists of the sum of the field of an incoming SPP and a scattered field

$$H_2^{>}(x_1,x_3|\omega)=\exp[ik(\omega)x_1-\beta_0(\omega)x_3] + \int_{-\infty}^{\infty}\frac{dq}{2\pi}R(q,\omega)\exp[iqx_1+i\alpha_0(q,\omega)x_3], \quad (3)$$

where $k(\omega)=(\omega/c)[1-\epsilon^{-1}(\omega)]^{1/2}$, $\beta_0(\omega)=(\omega/c)[-\epsilon(\omega)]^{-1/2}$, and $\alpha_0(q,\omega)=[(\omega^2/c^2)-q^2]^{1/2}$;

$\mathrm{Re}[\alpha_0(q,\omega)]>0$ and $\mathrm{Im}[\alpha_0(q,\omega)]>0$. When Eq. (3) is substituted into the boundary condition (1), we obtain for the integral equation satisfied by the scattering amplitude $R(q,\omega)$

$$R(q,\omega)=\beta_0(\omega)G_0(q,\omega)\hat{s}[q-k(\omega)]+\beta_0(\omega)G_0(q,\omega)\times\int_{-\infty}^{\infty}\frac{dp}{2\pi}\hat{s}(q-p)R(p,\omega), \tag{4}$$

where

$$G_0(q,\omega)=i\epsilon(\omega)/\{\epsilon(\omega)\alpha_0(q,\omega)+i(\omega/c)[-\epsilon(\omega)]^{1/2}\}$$

is the Green's function for a SPP on the unperturbed surface $[s(x_1)\equiv 0]$ and $\hat{s}(Q)$ the Fourier transform of $s(x_1)$. The integral equation (4) was solved numerically by converting it into a matrix equation by the use of a quadrature scheme.

For $x_1 \gg L/2$, the magnetic field in the vacuum region $x_3>0$, evaluated on the surface, is found to have the form

$$H_2^{>}(x_1,0|\omega)=\exp[ik(\omega)x_1]+r(\omega)\times\exp[-ik(\omega)x_1], \quad x_1\ll-\frac{L}{2} \tag{5a}$$

$$=t(\omega)\exp[ik(\omega)x_1], \quad x_1\gg\frac{L}{2}, \tag{5b}$$

where the SPP reflection and transmission amplitudes $r(\omega)$ and $t(\omega)$ are given by

$$r(\omega)=iG_0^{-1}(-k(\omega),\omega)R(-k(\omega),\omega)C(-k(\omega),\omega), \tag{6a}$$

$$t(\omega)=1+iG_0^{-1}(k(\omega),\omega)R(k(\omega),\omega)C(k(\omega),\omega). \tag{6b}$$

In Eqs. (6) $C(\pm k(\omega),\omega)=\omega/c[-\epsilon(\omega)]^{1/2}k(\omega)$, as obtained from

$$C(q,\omega)=\{\epsilon(\omega)\alpha_0(q,\omega)-i(\omega/c)[-\epsilon(\omega)]^{1/2}\}/2i\epsilon(\omega)k(\omega).$$

The corresponding SPP reflection and transmission coefficients are $R(\omega)=|r(\omega)|^2$ and $T(\omega)=|t(\omega)|^2$, respectively. Part of the energy in the incident SPP is converted into volume electromagnetic waves in the vacuum, propagating away from the surface. The total scattered power, normalized by the power in the incident SPP, is given by

$$S(\omega)=\int_{-\pi/2}^{\pi/2} d\theta_s\frac{\omega^2}{2\pi c^2}\frac{\beta_0(\omega)}{2k(\omega)}\cos^2\theta_s\left|R\left(\frac{\omega}{c}\sin\theta_s,\omega\right)\right|^2. \tag{7}$$

The integrand represents the fraction of the energy of the incident SPP that is scattered into an angular region of width $d\theta_s$ about the scattering angle θ_s.

Since we are interested in studying the localization and radiative damping of SPP on a random surface impedance that is periodic on average, with special emphasis on the importance of the underlying periodic structure, it is necessary to determine the SPP dispersion relation for an infinite sinusoidal grating. This is done by numerically looking for the zeros of the determinant arising in the homogeneous problem with the use of the IBC. For a period $d=392.5$ nm, $s_0=3$, and a plasma wavelength $\lambda_p=157.1$ nm, our results reveal that a gap opens up for $0.45<\omega/cG<0.4945$, where $G=2\pi/d$; we will be concerned in what follows with the influence of the frequency when approaching the edge of the lower band at $\omega_L/cG=0.45$. To establish the accuracy and efficiency of the numerical calculations, we have studied the SPP scattering from a finite sinusoidal grating consisting of an integer number N_d of periods d. Our results, not shown here, satisfy the energy conservation condition $R+T+S=1$ within a 0.01% error and agree with those obtained in Refs. 14 and 16 for similar geometries.

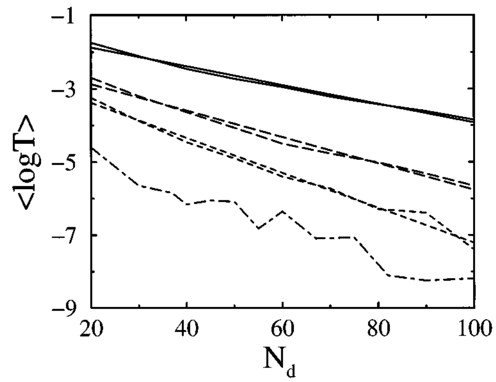

FIG. 1. Average of the logarithm of the transmission coefficient as a function of the number of periods $N_d=L/d$ for a finite, periodic on average, random metal grating with $d=392.5$ nm, $s_0=3$, $a=d/2$, $\delta=0.2$, and $\lambda_p=157.1$ nm. Solid curve, $\omega/cG=0.3$; long-dashed curve, $\omega/cG=0.4$; dashed curve, $\omega/cG=0.46$. The dash-dotted curve represents $\ln T$ for a pure sinusoidal grating ($\delta=0$) for a frequency in the gap $\omega/cG=0.46$.

Let us now turn to the random surface impedance that is periodic on average [cf. Eq. (2)]. For each realization of $s(x_1)$, whose random component is generated numerically,[9] R, T, and S are calculated in the manner described above. The mean values of the quantities of physical interest are then obtained by averaging them over the ensemble of realizations of $s(x_1)$. The quantity of interest in the present calculations is the self-averaging quantity $\langle \ln T\rangle$, regarded as a function of the length L of the random segment of the metal surface and of the frequency ω of the incident SPP. This is because in a one-dimensional random system this quantity is expected to decay exponentially with increasing L, with a decay length that is identified as the localization length of the corresponding wave in the system. In Fig. 1 the length dependence of $\langle \ln T\rangle$ is shown for three frequencies $\omega/cG=0.3$, 0.4, and 0.46. In all three cases, a linear dependence with negative slope is found, manifesting an exponentially decaying behavior of the transmission channel. Note that for the frequency $\omega/cG=0.46$, in the gap of the SPP dispersion relation for the infinite sinusoidal grating, the transmission is larger in the presence of roughness than it is for a pure sinusoidal grating (included in Fig. 1 for the sake of comparison), in agreement with Ref. 19. It should also be pointed out that the randomness, despite its being small compared to the periodicity strength ($s_0/\delta=15$), results in the transmission being exponentially decaying even for frequencies lying in the band of the pure periodic grating. But there is yet another possible mechanism besides localization responsible for such behavior: radiative damping. By assuming that both lead to a frequency-dependent exponential decay of the SPP transmission coefficient scaled by the localization length $l(\omega)$ and the radiative damping length $l_{rad}(\omega)$, respectively, one can write the Lyapunov exponent $l_T(\omega)$ as[3]

$$\langle \ln T\rangle \propto -\frac{L}{l_T(\omega)} \quad \text{with} \quad l_T^{-1}(\omega)=l^{-1}(\omega)+l_{rad}^{-1}(\omega). \quad (8)$$

Since the results shown in Fig. 1 provide information about $l_T(\omega)$, one needs to know either $l(\omega)$ or $l_{rad}(\omega)$ in order to determine which mechanism is the dominant one in this particular situation.

By means of energy conservation arguments,[20] $l_{rad}(\omega)$ could be inferred from the numerical results for the total scattered power $\langle S\rangle$ as a function of the surface length since $\langle S\rangle = L/l_{rad}$. Unfortunately, our numerical results for the mean scattered intensity, not shown here, are strongly affected by the large radiation losses owing to edge-diffraction effects (up to 90% of the total incoming SPP power). Thus, even though the relative error remains indeed low, the fact that this strong contribution has to be subtracted from $\langle S\rangle$ makes it unfeasible to discern the real behavior of $\langle S\rangle$ as a function of the grating length.

To elude these edge effects, we have developed an analytical calculation based on the two-potential formalism through a perturbative treatment of the random component, the periodic component being treated exactly:[18] This provides an independent calculation of $\langle S\rangle$ through terms of second order in δ and leads to the result

$$l_{rad}^{-1}(\omega)=\frac{2\beta_0(\omega)\delta^2}{k(\omega)}\sum_m \sum_n \sum_{m'} \sum_{n'} \delta_{m-n,m'-n'} \times \int_{-\omega/c}^{\omega/c} \frac{dq}{2\pi}\alpha_0(q,\omega)F_{mn}(q,\omega)F^*_{m'n'}(q,\omega) \times g(|q-k(\omega)+(m-n)G|), \quad (9)$$

where

$$F_{mn}(q,\omega)=\beta_0(\omega)G_{0,m}(q,\omega)\{\delta_{n0}+\tfrac{1}{2}s_0\beta_0(\omega) \times[G_{n,1}(k(\omega),\omega)+G_{n,-1}(k(\omega),\omega)]\}$$

[$G_{m,n}(q,\omega)$ is the reduced Green's function for SPP on a periodically corrugated surface[21]]. The function $g(|Q|)=\sqrt{\pi}a\exp(-Q^2a^2/4)$ is the power spectrum of the random surface roughness. The radiative damping lengths thus obtained are $l_{rad}(\omega/cG=0.3)=712d$, $l_{rad}(\omega/cG=0.4)=606d$, and $l_{rad}(\omega/cG=0.46)=974d$. These values are one order of magnitude larger than those of $l_T(\omega)$ calculated by fitting the $\langle \ln T\rangle$ curves in Fig. 1 to straight lines. This manifests that radiative damping is negligible within the length scale we are dealing with in this work, so that strong localization of SPP is the physical mechanism giving rise to the linear decay of $\langle \ln T\rangle$ shown in Fig. 1. The values of $l(\omega)$ for the three frequencies studied here are presented in Table I. It is evident that the localization length becomes shorter as the frequency of the incoming SPP approaches the band edge of the SPP dispersion relation for the underlying periodic structure, in agreement with similar behaviors predicted and observed in different physical situations.[1,15] In addition, the results of Ref. 14 for a slightly different metal surface also point in that direction, although the absence of a quantitative estimate of $l_{rad}(\omega)$ makes the interpretation therein more complicated.

TABLE I. Results for $l_T(\omega)$ (from numerical simulations) and $l_{rad}(\omega)$ (from perturbation-theoretic calculations), along with the localization length $l(\omega)$ obtained from them, for different frequencies. The parameters are $d=392.5$ nm, $s_0=3$, $a=d/2$, $\delta=0.2$, and $\lambda_p=157.1$ nm.

ω/cG	$l_T(\omega)/d$	$l_{rad}(\omega)/d$	$l(\omega)/d$
0.3	39 ± 2	712	41 ± 2
0.4	28 ± 1	606	29 ± 1
0.46	21 ± 1	974	21 ± 1

With respect to the behavior of $l_{rad}(\omega)$, it has been argued that leakage should either vanish or substantially decrease at the band edge frequency.[4] This is not observed in our numerical simulation calculations for $\langle S\rangle$, nor is it observed in Ref. 14. Nevertheless, as pointed out above, there exist strong radiation losses produced at the edges of the finite gratings, which might exhibit a very weak frequency dependence, thus hindering the observation of such leakage inhibition. Our perturbative calculations, which do not account for those edge effects, do not reveal any significant decrease in the leakage for frequencies near the band edge.

It is also of interest to study the surface magnetic field for given realizations of $s(x_1)$. This is shown in Fig. 2 for $N_d=50$ and two frequencies: $\omega/cG=0.4$ (solid curve) and $\omega/cG=0.46$ (dashed curve). We observe for both frequencies that the surface field, leaving aside some strong enhancements close to the left edge of the grating, decays very rapidly with increasing length upon entering in the grating, as expected. However, there exists for the lower frequency in

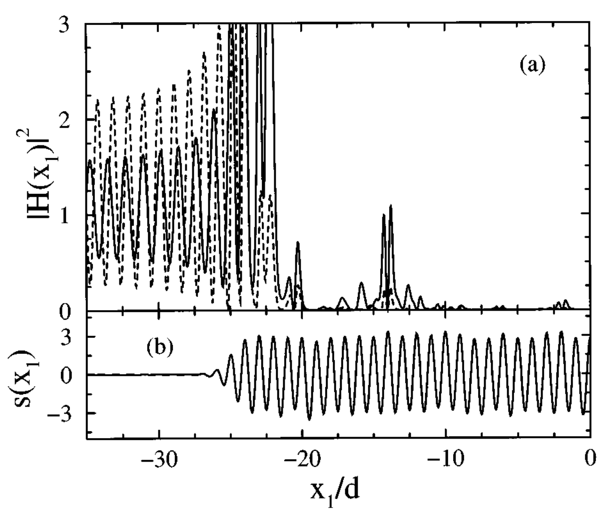

FIG. 2. (a) Square modulus of the total surface magnetic field for a single realization of a random, periodic on average, metal grating with $d=392.5$ nm, $s_0=3$, $N_d=50$, $a=d/2$, $\delta=0.2$, and $\lambda_p=157.1$ nm. Solid curve, $\omega/cG=0.4$; dashed curve, $\omega/cG=0.46$. (b) The surface impedance profile is also included.

Fig. 2 a small region inside the random grating, sufficiently far from the edges, where the field appears to be enhanced. This suggests that a localized state has been excited by the incident SPP. The fact that there is no correlation between the surface impedance profile and the shape of the square-modulus of the magnetic field within that region supports this suggestion. In addition, the disappearance of such localized state upon changing the frequency, which in turn modifies the phase coherence conditions responsible for it, agrees with our previous argument. Similar surface field localized states have been encountered for other realizations of the random surface impedance. Furthermore, these localized states seem to be similar in nature to those obtained through incident light excitation in Refs. 5 and 13.

In summary, by means of a theoretical formulation that describes in a rigorous manner the scattering of a surface plasmon polariton propagating along a planar vacuum-metal interface by a one-dimensional obstacle modeled through an impedance boundary condition, the angular spectrum of the scattered electromagnetic field in the vacuum half-space above the metal surface can be calculated. By calculating the leakage length $l_{rad}(\omega)$ (through perturbation theory) and the total decay length $l_T(\omega)$ (through numerical simulation calculations), the localization length $l(\omega)$ has been obtained. In all the cases studied here, it has been shown that localization predominates over leakage. The localization length is larger for frequencies in the band than it is for a frequency in the vicinity of the gap. We have found no evidence of leakage inhibition at the band edges, contrary to the prediction in Ref. 4. It should be emphasized that, to our knowledge, these are the first rigorous calculations up to now that explicitly and quantitatively address the issue of leakage of surface electromagnetic waves, thereby allowing us to jaccurately obtain the localization length. Finally, our results for the surface magnetic field for given realizations of the random surface impedance suggest the existence of localized states.

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