Bosonization on a lattice: The emergence of the higher harmonics

J. Ferrer*
Departamento de Física, Universidad de Oviedo, Av. Calvo Sotelo 18, 33007 Oviedo, Spain

J. González†
Instituto de Estructura de la Materia, Serrano 123, 28006 Madrid, Spain

M.-A. Martín-Delgado‡
Departamento de Física Teórica I, Universidad Complutense de Madrid, 28040 Madrid, Spain

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A general and transparent procedure to bosonize fermions placed on a lattice is presented. Harmonics higher than k_F in the one-particle Green function are shown to appear due to the compact character of real electron bands. Quantitative estimations of the role of higher harmonics are made possible by this bosonization technique.

Bosonization methods have provided us with a thorough understanding of the physics of $(1+1)$-dimensional model systems in several branches of theoretical physics such as, for example, condensed matter.[1,2] The earliest implementation of such techniques dates back to the 1960s, when Luttinger[3] proposed his model, which was subsequently solved partially by Mattis and Lieb.[4] Ten years later, Luther and Peschel[5] and Mattis[6] made the picture more concrete by combining their refined version of bosonization with known results for some models previously solved by Bethe ansatz. Soon afterward, and independently, a similar boson-fermion equivalence was also obtained by Coleman[7] in the context of the sine-Gordon model. Perhaps the most pictorial representation of a fermion in terms of bosons has been given by Mandelstam.[8] According to it, fermions should be understood in a purely bosonic theory as soliton operators interpolating between different particle vacua. Reciprocally, the generic behavior of electron liquids in $1+1$ dimensions is such that all the excitations of the Fermi sea can be classified into a set of boson operators.

The paradigm of a theory that can be solved by means of bosonization is the Luttinger model. This is a one-dimensional model in which electrons interact only through density operators of definite chirality. The total Hamiltonian can be expressed as a quadratic form of two boson fields with opposite chiralities and this fact renders the model completely integrable. There are, however, some assumptions in the Luttinger model which make it an approximate description of the physics of real electrons. The most important of them are made by considering a perfect linear dispersion relation for the original electrons and by supposing that the two branches (corresponding to the two different Fermi points) can be artificially extended *ad infinitum* in both directions [see Fig. 1(a)]. Obviously, the infinite collection of states deep inside the Fermi sea is not present at all in a real physical situation and the hypothesis that they do not modify the essential properties learned from Luttinger's model becomes crucial.

Let us explain with more detail some of the complica-

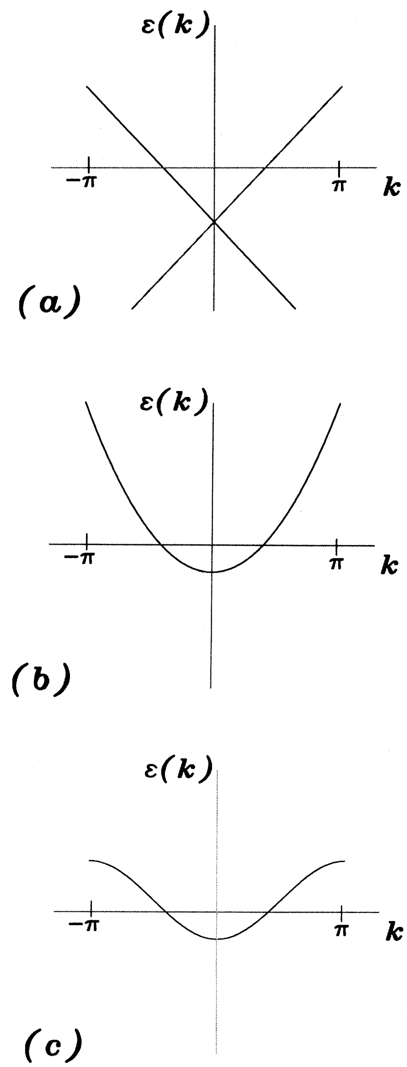

FIG. 1. Dispersion relations for (a) free electrons in the Luttinger model, (b) conventional nonrelativistic free electrons, and (c) electrons in a lattice.

tions that appear in real condensed matter systems and blur the sharp picture brought about by the bosonization of the Luttinger model. First, the dispersion relation for one species of free nonrelativistic fermions is simply a parabola $\epsilon(p) = p^2/2m$, which is bounded from below, but not from above [Fig. 1(b)]. Second, when these formerly free electrons are placed in a lattice and interact with the periodic substrate potential of the atoms, the dispersion relation becomes bounded from above and peaks, in the simplest case, at the Bragg point so that we are left with a compact band of width D [see Fig. 1(c)]. The aforementioned phenomena bear two related effects. One is the appearance, due to the compact character of the band, of chirality breaking processes, which mix both branches. This is tantamount to saying that one cannot divide the physical electron field operator into right and left moving pieces unambiguously. The other effect is the curvature of the band. We argue that in most cases the first effect is more relevant because it is the source of the appearance of higher harmonics in electron correlation functions, while the second one gives rise to harmless renormalizations of the parameters. In the case of the electron Green function, for instance, together with the naive frequencies $\pm k_F$ one expects higher modulations at $\pm 3k_F$, $\pm 5k_F$, etc.:

$$\langle \Psi(x)\Psi^\dagger(y)\rangle = \sum_{n=0}^{\infty} c_n \frac{i}{(x-y)^{\alpha_n}} e^{i(2n+1)k_F(x-y)} + \text{H.c.} \tag{1}$$

An underlying assumption of the bosonization technique is that only long wavelength fluctuations of the density of particles affect the physics of the problem. One is therefore allowed to average all magnitudes over distances much larger than the average distance among particles $r_s \simeq 1/k_F$, in particular the commutation relations and expectation values of density operators. We proceed now to describe qualitatively the effects of a finite density of electrons on the accuracy of the mapping of electrons to bosons.

(i) For intermediate densities, when k_F is placed approximately in the middle of the band, its curvature is very small and we can linearize it around the Fermi points with a high degree of accuracy. On the other hand, chirality-breaking processes have high energy and can be disregarded as a first approximation.

(ii) At low densities, k_F is close to the bottom of the band. In this case, the discrete character of the particles is important because the length scale set by r_s is large. One should add that the processes that break the chirality have low energies and need to be taken into account. They show up in the form of higher harmonics in the electron correlation functions.

(iii) At high densities the Fermi wave vector is close to the top of the band and, due to particle-hole symmetry, the qualitative discussion in (ii) applies.

The issue of the emergence of higher harmonics is not new and has been considered before mainly in connection with the effects of the curvature of the dispersion relation. A quite original point of view, due to Haldane,[9] rephrases the problem as the incorporation of the discrete nature of the particles into the bosonization program. In either case, it becomes clear that the boson-fermion transcription, valid within the Luttinger model, should be corrected to take into account more realistic dispersion relations. In particular, if we consider a *compact* dispersion relation and give up the perfect division between left and right movers of the Luttinger model, hybridization effects between the two chiral fields will appear, giving rise to higher harmonic modulations in the electron correlation functions. Up to date, though, there has been no attempt to understand what is technically the source of such hybridization and thereafter to propose a systematic way of correcting the original boson expression of the fermion operator.

The purpose of the present paper is to incorporate the compact character of the band, i.e., the chirality-breaking processes, into the bosonization technique in a nonperturbative, and essentially exact, way. Our goal is to set up a scheme that permits us to study *quantitatively* the role of higher harmonics. These might be relevant for the physical behavior of some experimental devices, e.g., quantum wires.[10,11] It is our belief that our procedure places bosonization in the doorway of quantitative computations of response functions of one-dimensional systems in condensed matter.

We begin with a reminder of the main lines of the simple bosonization program, where the assumption of an infinite linear dispersion relation of the two-electron branches becomes essential. In the Luttinger model the electronic spectrum of the "free" Hamiltonian is that represented in Fig. 1(a). There are two types of fermion modes, say, $a_k, a_k^\dagger$ and $b_k, b_k^\dagger$, for the respective right and left branches of the spectrum. It is well known that the only excitations supported by the Fermi sea of Fig. 1(a) are density fluctuations of the form

$$\rho_{kR} = \sum_q a_{q+k}^\dagger a_q \tag{2}$$

for the right branch and

$$\rho_{kL} = \sum_q b_{q+k}^\dagger b_q \tag{3}$$

for the left branch. There are obviously other fluctuation processes in which electrons are transferred from one branch to the other, but in the Luttinger model they amount to the introduction of a conserved quantum number J. The important point is that the above currents satisfy the commutation relations

$$\begin{aligned} \left[\rho_{-\tilde{k}R}, \rho_{kR}\right] &= \delta_{k\tilde{k}}\, k\, \frac{L}{2\pi}\,, \\ \left[\rho_{-\tilde{k}L}, \rho_{kL}\right] &= -\delta_{k\tilde{k}}\, k\, \frac{L}{2\pi}\,, \end{aligned} \tag{4}$$

where L is the length of the dimension in which the electrons are confined. The linear dependence of the commutators (4) can be rigorously proved under the hypothesis of an infinite linear dispersion relation as shown in Fig. 1(a). It allows us to define boson creation and annihilation operators

$$B_k^\dagger = \begin{cases} \sqrt{\frac{2\pi}{L|k|}}\,\rho_{kR}, k>0 \\ -\sqrt{\frac{2\pi}{L|k|}}\,\rho_{kL}, k<0, \end{cases}$$

$$B_k = \begin{cases} \sqrt{\frac{2\pi}{L|k|}}\,\rho_{-kR}, k>0 \\ -\sqrt{\frac{2\pi}{L|k|}}\,\rho_{-kL}, k<0, \end{cases} \tag{5}$$

which satisfy perfect canonical commutation relations

$$\left[B_k, B_{\tilde{k}}^\dagger\right] = \delta_{k\tilde{k}}. \tag{6}$$

These oscillators can in turn be assembled into two chiral boson fields

$$\Phi_R(x) = \frac{2\pi}{L}\left(x\,N_R + i\sum_{k\neq 0}\frac{e^{-ikx}}{k}\rho_{kR}\right),$$

$$\Phi_L(x) = \frac{2\pi}{L}\left(x\,N_L + i\sum_{k\neq 0}\frac{e^{-ikx}}{k}\rho_{kL}\right). \tag{7}$$

Here N_R and N_L are the normal ordered charges for the respective channels. This boson codification of the electron excitations is only half of the boson-fermion equivalence. It can also be shown that the fermion field may be expressed in terms of the above boson fields. In particular, a correct representation for the two fermion chiralities is

$$\Psi_R(x) = :e^{i\Phi_R(x)}:,$$
$$\Psi_L(x) = :e^{-i\Phi_L(x)}: . \tag{8}$$

These are the expressions for the soliton (fermion) annihilation operators found by Mandelstam. They have the virtue of satisfying the equal-time canonical anticommutation relations of fermion operators. Finally and more important, the representation (8) reproduces the form of the fermion correlators

$$\langle \Psi_R(x)\Psi_R^\dagger(x')\rangle = \frac{i}{x-x'}, \tag{9}$$

$$\langle \Psi_L(x)\Psi_L^\dagger(x')\rangle = \frac{-i}{x-x'}. \tag{10}$$

At this point we undertake the analysis of how this program has to be modified when a more realistic, compact dispersion relation is considered in the description of the electronic system. Electrons usually experience the background periodic potential of the atomic lattice. This substrate potential changes their parabolic dispersion relation into a band [Figs. 1(b) and 1(c)]. Despite the fact that in such case no natural distinction between right and left modes can be made, we want to keep the separation into two different branches for computational purposes. In fact, even in the case of a compact spectrum of the kind shown in Fig. 1(c) the static electron correlator still shows two different modulations corresponding to the left and right branches. Suppose, for instance, that we write the mode expansion for the fermion field

$$\Psi(x) = \frac{2\pi}{L}\sum_{k=-\pi}^{0} e^{ikx}b_k + \frac{2\pi}{L}\sum_{k=0}^{\pi} e^{ikx}a_k . \tag{11}$$

Then, a straightforward computation gives the result (in the limit $L\to\infty$)

$$\langle \Psi(x)\Psi^\dagger(y)\rangle = \int_{-\pi}^{-k_F} dk\, e^{ik(x-y)} + \int_{k_F}^{\pi} dk\, e^{ik(x-y)}$$
$$= \frac{i}{x-y}e^{ik_F(x-y)} + \frac{-i}{x-y}e^{-ik_F(x-y)} . \tag{12}$$

This is exactly the same expression that one obtains for the static correlator in the Luttinger model. However, as we are going to see, the bounded character of the spectrum of boson excitations requires appropriate modifications in the intermediate steps, which lead to (12) within the bosonization approach.

It is worthwhile to remark that the particular energy values of the electron modes are irrelevant for the purpose of computing the static correlators. The only important point is that the Fermi sea comprises now a connected set of states from $k=-k_F$ to $k=k_F$. Given that we do not have an infinite dispersion relation anymore, we would like to write tentatively the set of chiral currents

$$\rho_{kR} = \sum_{0<q+k,q<\pi} a_{q+k}^\dagger\, a_q ,$$
$$\rho_{kL} = \sum_{-\pi<q+k,q<0} b_{q+k}^\dagger\, b_q. \tag{13}$$

The first exercise in order to test the bosonization procedure is to check the linear dependence of the commutator of currents with like chirality:

$$[\rho_{-kR},\rho_{kR}] = \left[\sum_{0<q-k,q<\pi} a_{q-k}^\dagger\, a_q, \sum_{0<r+k,r<\pi} a_{r+k}^\dagger\, a_r\right]$$
$$= \sum_{0<q-k,r<\pi}\delta_{q,r+k}a_{q-k}^\dagger a_r$$
$$- \sum_{0<r+k,q<\pi}\delta_{r,q-k}a_{r+k}^\dagger a_q . \tag{14}$$

It is important to realize that in these sums all the subindices run from 0 to π. For this reason, one can see that for sufficiently small values of k the first sum in (14) has $L/(2\pi)$ times k more contributions than the other. This agrees with the linear dependence in (4). However, in the case of a band less than half filled, when $k > k_F$ there are not enough excitations of the Fermi sea and the commutator remains equal to $L/(2\pi)\,k_F$, up to a value of $k=\pi-k_F$. From that value it begins to decrease linearly and reaches 0 at $k=\pi$. This picture is valid, as we have said, for values of k_F between 0 and $\pi/2$. When the band is more than half filled we get a similar form of the commutator, but with a linear increase up to $\pi-k_F$ and later a linear decrease from k_F to π.

We may pause at this point to think of the physical reasons for this deviation of the commutator from a perfect linear dependence. They can indeed be found by looking at the very essence of the computation performed above. As a matter of fact, *the value of the commutator is a measure of the number of available one-particle excitations over the Fermi sea.* In the case of a band less than half filled, for instance, it is clear that for small values of momentum transfer k there is no problem in exciting $L/(2\pi)\,k$ electrons from below the Fermi level to states above it. When $k > k_F$, though, we cannot continue pulling out right modes once we reach the bottom of the band and the number of available excitations is less than $L/(2\pi)\,k$. This argument explains also why the actual number remains constant and equal to $L/(2\pi)\,k_F$, up to a momentum transfer $\pi - k_F$.

However, this clear interpretation of the functional dependence of the commutator also shows that the present picture is physically incorrect. In fact, it is only our artificial division between right and left modes that has prevented us from considering another set of admissible one-particle excitations for $k > k_F$. These correspond to the transfer of electrons below the Fermi level in the range $[-k_F, 0]$ to states above the Fermi level in the right branch. Obviously, there is no reason for not considering these excitations on the same footing as those taken into account before within the same branch. Bearing this in mind, the following definition of the currents seems more natural

$$\rho_{kR} = \sum_{0<q+k,q<\pi} a^\dagger_{q+k}\, a_q + \sum_{-k_F<q<0} a^\dagger_{q+k}\, b_q \ ,$$
$$\rho_{kL} = \sum_{-\pi<q+k,q<0} b^\dagger_{q+k}\, b_q + \sum_{0<q<k_F} b^\dagger_{q+k}\, a_q \ . \tag{15}$$

It is clear that the correct counting of excitations leads to a situation in which for momentum transfer $k = 2k_F$ the number of them equals the maximum value $L/(2\pi)\,2k_F$, including the extreme process in which an electron slightly below the Fermi level is transferred above it at the other Fermi point. This value $L/(2\pi)2k_F$ is also the cutoff for the commutator. The correct physical picture says, then, that the commutator should be a linear function growing up to $L/(2\pi)\,2k_F$ at $k = 2k_F$, remaining constant until $k = \pi - 2k_F$, and then linearly decreasing to 0 at $k = \pi$.

The most important effect of the lattice is therefore to replace the commutators in (4) by a *bounded* function in the interval $[0, \pi]$, which we will call $L/(2\pi)\,f(k)$, i.e.,

$$[\rho_{-kR}, \rho_{kR}] = \frac{L}{2\pi}\, f(k). \tag{16}$$

$f(k)$ is depicted in Fig. 2. This in turn modifies the properties of the bosons that one can build from the currents (15). The correct definition of boson creation and annihilation operators should be now

$$B^\dagger_k = \begin{cases} \sqrt{\frac{2\pi}{Lf(|k|)}}\,\rho_{kR}, k>0 \\ -\sqrt{\frac{2\pi}{Lf(|k|)}}\,\rho_{kL}, k<0, \end{cases}$$
$$B_k = \begin{cases} \sqrt{\frac{2\pi}{Lf(|k|)}}\,\rho_{-kR}, k>0 \\ -\sqrt{\frac{2\pi}{Lf(|k|)}}\,\rho_{-kL}, k<0, \end{cases} \tag{17}$$

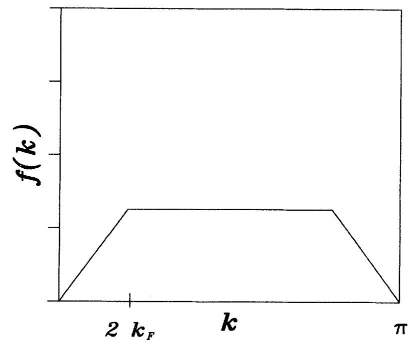

FIG. 2. Function $f(k)$ defined in the text.

in order to preserve the canonical commutation relations (6). We want to maintain at this point the relation that exists in the Luttinger model between the fields $\rho_L(x), \rho_R(x)$ and $\Phi_L(x), \Phi_R(x)$

$$\rho_L(x) = \frac{L}{2\pi}\,\nabla\Phi_L(x), \;\; \rho_R(x) = \frac{L}{2\pi}\,\nabla\Phi_R(x) \tag{18}$$

with the only difference that now ∇ is the lattice derivative. The two boson fields

$$\Phi_L(x) = i\sum_{k<0}\sqrt{\frac{2\pi}{L}}\frac{\sqrt{f(|k|)}}{2\sin(k/2)}(e^{-ikx}B^\dagger_k - e^{ikx}B_k)\ ,$$
$$\Phi_R(x) = i\sum_{k>0}\sqrt{\frac{2\pi}{L}}\frac{\sqrt{f(|k|)}}{2\sin(k/2)}(e^{-ikx}B^\dagger_k - e^{ikx}B_k) \tag{19}$$

are chiral in the sense that $\Phi_R(x)$, for instance, creates excitations in the forward direction and destroys them in the backward direction. Their properties, though, are nonstandard, since they account in their structure for the finiteness of the number of modes of the lattice. Over very large distances we should expect from them the same behavior found in the Luttinger model. This is guaranteed by the fact that the function $f(k)$ is linear in k for small values of the argument. Over smaller distances, though, we start to feel the effects of the discreteness of the number of particles.

The above considerations are exemplified by the computation of the correlator

$$\langle e^{i\Phi_R(x)} e^{-i\Phi_R(0)}\rangle\ , \tag{20}$$

which in the Luttinger model equals the fermion propagator (9). A straightforward calculation leads, in the limit $L \to \infty$, to

$$\langle e^{i\Phi_R(x)} e^{-i\Phi_R(0)}\rangle = \exp\left\{-\int_0^\pi dk\frac{f(k)}{4\sin^2(k/2)}\left(1 - e^{ikx}\right)\right\} = e^{-I}\ . \tag{21}$$

We are mainly interested in the behavior of the correlator at large values of x. The evaluation of I in this regime still appears to be unfeasible, but in the limit of small k_F (compared to π) we may consider the effects of the integration over large values of k as irrelevant. We can then approximate I by

$$\begin{aligned} I &\approx \int_0^\Lambda dk \frac{f(k)}{k^2} \left(1 - e^{ikx}\right) \\ &= \int_0^{2k_F} dk \frac{1}{k} \left(1 - e^{ikx}\right) + 2 k_F \int_{2k_F}^{\Lambda} dk \frac{1}{k^2} \left(1 - e^{ikx}\right) \\ &\approx \ln(2 k_F x) + \gamma_E + 1 - i\frac{\pi}{2} - \frac{e^{i2k_F x}}{(2k_F x)^2} \\ &\quad + F(x,\Lambda) + \cdots . \end{aligned} \tag{22}$$

Λ plays here the role of an upper cutoff and the function $F(x,\Lambda)$ is

$$F(x,\Lambda) = -2\,\frac{k_F}{\Lambda} + 2\,i\,\frac{k_F}{\Lambda}\,\frac{e^{i\Lambda x}}{\Lambda x} . \tag{23}$$

At small values of k_F/Λ the influence of the cutoff can be disregarded and we get the asymptotic expansion for the correlator

$$\langle e^{i\Phi_R(x)} e^{-i\Phi_R(0)} \rangle = C\,\frac{i}{2k_F x} + C\,e^{i2k_F x}\,\frac{i}{(2k_F x)^3} + \cdots . \tag{24}$$

The first term corresponds to the right-handed piece of the electron propagator (12), while the rest are contributions that arise from the structure of the boson field operators over distances corresponding to the mean separation among particles.

The main conclusion that follows from the evaluation of (20) is that the boson representation (8) of the two fermion chiralities cannot be correct since it produces spurious contributions to the electron propagator as shown in (24). We stress again the fact that the electron propagator is given in any event by the expression (12). It suggests that the left-right chiral decomposition is still at work in the free theory of Fig. 1(c), showing no other harmonics than those at k_F and $-k_F$. The structure of the higher-order contributions in (24), in particular the modulation at $2k_F$, shows that the boson representation (8) can be conveniently corrected in order to cancel out spurious terms in the fermion propagator. Actually, it is not a coincidence that the subdominant order in (24) is just *the opposite* of the dominant contribution from

$$\langle e^{i[\Phi_L(x)+\Phi_R(x)]} e^{i\Phi_R(x)} e^{-i[\Phi_L(0)+\Phi_R(0)]} e^{-i\Phi_R(0)} \rangle . \tag{25}$$

Thus the correct boson representation of the chiral fermion operators is

$$\Psi_R(x) = e^{i\Phi_R(x)} + c_1 e^{i2k_F x} e^{i[\Phi_L(x)+\Phi_R(x)]}\, e^{i\Phi_R(x)} + \cdots , \tag{26}$$

$$\Psi_L(x) = e^{-i\Phi_L(x)} + c_1 e^{-i2k_F x} e^{-i[\Phi_R(x)+\Phi_L(x)]} e^{-i\Phi_L(x)} + \cdots \tag{27}$$

By keeping the decomposition

$$\Psi(x) = e^{-ik_F x}\Psi_L(x) + e^{ik_F x}\Psi_R(x) \tag{28}$$

it is not difficult to see that the use of (26) and (27) reproduces the correct expression of the electron propagator (12), provided that $c_1 = C^{-1}$. The form of the corrections in (26) and (27) coincides with what has been advocated by other authors.[9] Here we have accomplished a quantitative derivation of them, precise enough to determine the coefficients of the series within a given model. Our argumentation also clarifies conceptually that it is the bosonization method that introduces the higher harmonic contributions, as in (26) and (27), though in some instances, such as that of the free-electron system, the only modulations in the electron propagator are at k_F and $-k_F$.

We follow at this point the standard bosonization procedure by which the boson representation of the fermion operators remains unchanged after switching the interaction. Thus we are in the position to make explicit statements regarding the structure of the electron propagator in the interacting theory. In general there are couplings in the interacting Hamiltonian which mix explicitly the two chiral parts of the electron field. This mixing has to be considered along with the underlying chiral mixing already present in the boson representation. The interplay between them gives rise, in addition to the standard k_F modulation, to $3k_F$ and higher-order modulation terms in the electron propagator, as we are going to see in what follows. It is worth mentioning that the signal of the $3k_F$ modulation has been observed numerically by Ogata and Shiba[12] in the strong coupling limit of the one-dimensional Hubbard model.

Let us take, for the sake of simplicity, a simple g-ology model consisting of forward scattering terms of g_2 and g_4 type[1]

$$\begin{aligned} H = &\int dx\; i v_F (\Psi_R^\dagger \partial_x \Psi_R - \Psi_L^\dagger \partial_x \Psi_L) \\ &+ \int dx \Big\{ g_2 \Psi_R^\dagger \Psi_R \Psi_L^\dagger \Psi_L \\ &+ \frac{g_4}{2} \left[(\Psi_R^\dagger \Psi_R)^2 + (\Psi_L^\dagger \Psi_L)^2 \right] \Big\} . \end{aligned} \tag{29}$$

As is well known, in the boson representation this Hamiltonian is diagonalized by the canonical transformation

$$\begin{pmatrix} \Phi_L \\ \Phi_R \end{pmatrix} = \begin{pmatrix} \cosh\lambda & -\sinh\lambda \\ -\sinh\lambda & \cosh\lambda \end{pmatrix} \begin{pmatrix} \tilde\Phi_L \\ \tilde\Phi_R \end{pmatrix} \tag{30}$$

with

$$\tanh 2\lambda = \frac{g_2}{2\pi v_F + g_4} . \tag{31}$$

By implementing this transformation to free boson fields in the computation of the fermion propagator we get

$$\langle \Psi_R(x)\Psi_R^\dagger(0)\rangle = \langle e^{i\Phi_R(x)}e^{-i\Phi_R(0)}\rangle + |c_1|^2 e^{i2k_Fx}\langle e^{i[\Phi_L(x)+\Phi_R(x)]}e^{i\Phi_R(x)}e^{-i[\Phi_L(0)+\Phi_R(0)]}e^{-i\Phi_R(0)}\rangle + \cdots$$
$$= \frac{d_1}{(2k_Fx)^{1+2(\sinh\lambda)^2}} + e^{i2k_Fx}\left[\frac{d_2}{(2k_Fx)^{3+2(\sinh\lambda)^2}} + \frac{d_3}{(2k_Fx)^{1+2(\cosh\lambda-\sinh\lambda)^2+2(\sinh\lambda)^2}}\right] + \cdots . \quad (32)$$

d_1, d_2, and d_3 are known constants whose explicit value is not relevant for the purpose of the current discussion. The important issue here is that the $2k_F$ oscillation, which translates into a $3k_F$ oscillation of the electron propagator, does not cancel out anymore. Only when $\sinh\lambda = 0$ (free case) the cancellation takes place. As we advanced previously, the higher harmonic oscillations show up explicitly in the interacting fermion propagator. In the long distance limit and for $\lambda > 0$, of the two terms in the last line of (32) the second one is actually more relevant as its exponent turns out to be smaller than that of the first.

To summarize, we have found in this paper that the natural way to understand the emergence of higher harmonics in nonstandard bosonization formulas is the consideration of a compact dispersion relation for the fermions in one dimension. As a paradigm of this situation we have taken an electron system on the lattice. In our approach, we have obtained the higher harmonics within a purely kinematical framework. In this fashion we have followed the standard bosonization procedure where the boson representation is first proposed for the free theory and subsequently the interacting theory is solved without changing the bosonization prescription.

As is apparent from (26) and (27), our bosonization formulas are quantitative in the sense that we obtain explicit values for the amplitudes associated with the higher harmonic terms. The extension of the present work to more complicated interacting fermion systems and to fermions with spin is currently under study.

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* Electronic address: jav@fmc.uam.es
† Electronic address: imtjg64@cc.csic.es
‡ Electronic address: mardel@fis.ucm.es

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