Boson-fermion pairing in a boson-fermion environment

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Propagation of a Boson-Fermion (BF) pair in a BF environment is considered. The possibility of formation of stable strongly correlated BF pairs, embedded in the continuum, is pointed out. The Fermi gas of correlated BF pairs shows a strongly modified Fermi surface. The interaction between like particles is neglected in this exploratory study. Various physical situations where our pairing mechanism could be of importance are invoked.

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The physics of ultracold atomic gases is making progress at a rapid pace, which has led to a realization of Boson-fermion mixtures of atomic gases [1–4]. Boson-fermion (BF) mixtures may exhibit the richest variety of phenomena of all. They may show very different behavior from pure fermion or pure Bose gases [5,6]. Especially interesting is a possible instability of the mixture when there is an attraction between bosons and fermions [5,7–9], as a recent experiment in fact suggests a collapse of the mixture [10].

In the present work we propose and study quite a different scenario for an attractively interacting boson-fermion mixture. To simplify the problem in a first survey we shall consider the situation where there is no interaction between atoms of the same kind. As we will discuss at the end of the paper, this is not a severe approximation to cases where the interaction between like atoms is repulsive. More precisely we want to address the question of what happens to a mixture of free fermions and bosons when a (tunable) attraction is switched on between fermions and bosons. We imagine that correlated BF pairs will be created. These BF pairs are composite fermions and as such these BF pairs should form a Fermi gas of composites. Besides in ultracold atomic gases such a situation can exist in other branches of physics. For example, in nuclear systems (e.g., neutron stars) of high-density K^- mesons and nucleons may form a gas of Λ's and the Λ's may then form a Fermi gas of their own [11]. Or in a quark-gluon plasma additional quarks may bind to preformed diquarks or color Cooper pairs (the "bosons") [12] to form a gas of nucleons in the so-called hadronization transition. Further examples may be added to this list.

For a numerical example, we take a mixture of ^{40}K (fermion) and ^{41}K (boson) atoms throughout the paper. They are known as candidates for a realization of this kind of quantum systems. While their scattering lengths are not well fixed at present, and different values have been reported experimentally [13], it is not crucial at the moment because our study will be mostly academic, elaborating on the basic phenomenon. Applications to realistic systems will be left for the future.

Let us consider a single BF pair propagating in the background of a homogeneous gas of free one-component fermions and spinless bosons. We will formulate our approach for a situation at finite temperature T, though later on in our application we will concentrate on the $T=0$ case. We have in mind an analogous study Cooper performed a long time ago [14] for the propagation of two fermions (spin up or down) in the background of a homogeneous gas of two-component free fermions. In other words we consider a situation where in the original Cooper problem one fermion type (let us say spin down) is replaced by spinless bosons. The BF propagator at finite temperature T and finite center-of-mass momentum $\mathbf{P}$ of the pair that is added to the system with momenta $\mathbf{P}/2+\mathbf{p}$ (fermion) and $\mathbf{P}/2-\mathbf{p}$ (boson) is

$$G^{t-t'}_{\mathbf{p},\mathbf{p}'}(\mathbf{P}) = -i\theta(t-t')\langle\{(b_{\mathbf{P}/2-\mathbf{p}}c_{\mathbf{P}/2+\mathbf{p}})^t,(c^\dagger_{\mathbf{P}/2+\mathbf{p}'}b^\dagger_{\mathbf{P}/2-\mathbf{p}'})^{t'}\}\rangle$$

where $\{,\}$ is the anticommutator and $c^\dagger$ and $b^\dagger$ are fermion and boson creation operators, respectively. In the ladder approximation the integral equation for $G_{\mathbf{p},\mathbf{p}'}(\mathbf{P},E)$ reads [15]

$$G_{\mathbf{p},\mathbf{p}'}(\mathbf{P},E) = G^0_{\mathbf{p}}(\mathbf{P},E)\delta(\mathbf{p}-\mathbf{p}') + \int \frac{d\mathbf{p}_1}{(2\pi)^3} G^0_{\mathbf{p}}(\mathbf{P},E)V(\mathbf{p},\mathbf{p}_1)G_{\mathbf{p}_1,\mathbf{p}'}(\mathbf{P},E). \tag{1}$$

In graphical form this equation is represented in Fig. 1.

In Eq. (1) $V(\mathbf{p},\mathbf{p}_1)$ is the BF interaction and $G^0_{\mathbf{p}}(\mathbf{P},E)$ is the free retarded BF propagator in the BF background:

FIG. 1. Graphical representation of Eq. (1). Dashed line stands for the boson, straight line for the fermion. Dotted vertical line is the interaction.

$$G^0_{\mathbf{p}}(\mathbf{P},E) = \frac{1 - f(\mathbf{P}/2 + \mathbf{p}) + g(\mathbf{P}/2 - \mathbf{p})}{E - e_f(\mathbf{P}/2 + \mathbf{p}) - e_b(\mathbf{P}/2 - \mathbf{p}) + i\eta} + \frac{(2\pi)^3 n_0}{E - P^2/2m + i\eta}\delta\left(\frac{\mathbf{P}}{2} - \mathbf{p}\right). \quad (2)$$

Here $f(\mathbf{p})$ and $g(\mathbf{p})$ are the Fermi-Dirac and Bose-Einstein distributions with chemical potentials μ_f and μ_b, respectively, and the term with the condensate fraction n_0 of bosons only appears for $T < T_{cr}$ where T_{cr} is the critical temperature for Bose condensation. We further have $e_f(p) = e_b(p) = p^2/2m$ which are the kinetic energies of fermions and bosons which we suppose of equal mass: $m_b = m_f = m$. For simplicity we disregard mass shifts from self-energy corrections which may drive the masses of fermions and bosons apart, even if in free space they are equal. Had we considered fermion-fermion (FF) propagation in a two-component Fermi gas (spin up or down), as Cooper did in his original work, then in Eq. (2) the bosonic distribution $+g(\mathbf{P}/2 - \mathbf{p})$ would have to be replaced by $-f(\mathbf{P}/2 - \mathbf{p})$ with, of course, $n_0 = 0$. As in Cooper's work, Eq. (1) only treats the propagation of one pair and neglects the influence of the other pairs on the pair under consideration. We therefore only can study situations with a very low density of BF pairs.

For the BF case we will make the schematic ansatz of separability of the force:

$$V(\mathbf{p},\mathbf{p}') = -\lambda v(p)v(p'), \quad \lambda > 0, \quad (3)$$

with a Yukawa type of form factor

$$v(p) = \frac{1}{\sqrt{m}(p^2 + \beta^2)}$$

where, in principle, the two parameters λ and β may be related to the scattering length and the effective range parameters of the low-energy BF scattering in free space [16]. However, in this exploratory study we will consider λ and β as free parameters especially in view of the fact that the interaction strength can be shifted using the Feshbach resonance phenomenon, whose application to K atoms has been discussed in [13,17]. The integral equation can then easily be solved with only a quadrature to be done numerically. The result is

$$G_{\mathbf{p},\mathbf{p}'}(\mathbf{P},E) = G^0_{\mathbf{p}}(\mathbf{P},E)\delta(\mathbf{p} - \mathbf{p}') - \frac{1}{(2\pi)^3}\frac{\lambda G^0_{\mathbf{p}}(\mathbf{P},E)v(p)v(p')G^0_{\mathbf{p}'}(\mathbf{P},E)}{1 + \lambda J_0(E,\mathbf{P})} \quad (4)$$

where

$$J_0(E,\mathbf{P}) = \int \frac{d\mathbf{p}}{(2\pi)^3} G^0_{\mathbf{p}}(\mathbf{P},E)v^2(p). \quad (5)$$

Without loss of generality we can consider the simpler propagator integrated over relative momentum

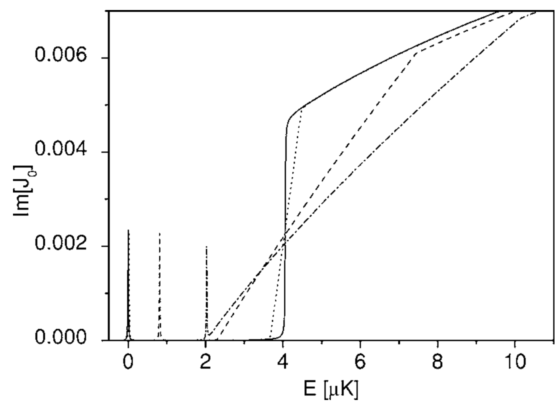

FIG. 2. Im J_0 as a function of energy E for different values of the total momenta. Solid line corresponds to $P^2/2m = 0$; dotted line to $P^2/2m = 0.01\mu_f$; dashed line to $P^2/2m = 0.4\mu_f$; dash-dotted line to $P^2/2m = \mu_f$.

$$G(\mathbf{P},E) = \int \frac{d\mathbf{p}'}{(2\pi)^3}\int d\mathbf{p}\, v(p)v(p')G_{\mathbf{p},\mathbf{p}'}(\mathbf{P},E) = \frac{J_0(E,\mathbf{P})}{1 + \lambda J_0(E,\mathbf{P})}. \quad (6)$$

We will be interested in the T matrix [18]

$$T^{\mathbf{P}}_E(\mathbf{q},\mathbf{q}') = \frac{-\lambda v(q)v(q')}{1 + \lambda J_0(E,\mathbf{P})} \quad (7)$$

and want to study the pole structure of this function, first at $T = 0$, as a function of $\mathbf{P}$. To this purpose we show in Fig. 2 the imaginary part of J_0 as a function of E for different values of the center-of-mass momentum $\mathbf{P}$. Calculations have been done for a ^{40}K—^{41}K system with equal masses: $m = m_B = m_F = 0.649 \times 10^{-22}$ g. We choose a situation where the number of bosons is much less than that of fermions: $n_F = 10^{14}$ cm^{-3}, $n_B = 0.004 n_F$. In this way the BF pair density will be very low and the perturbation of the bosons on the fermions can be kept relatively small, justifying the single pair ansatz (1). For the BF forces (3) we used $\beta = -7.89 \times 10^{-3}/a_B$ and $\lambda = \lambda' \times 10^{-9}/(2\pi a_B)^3$ where a_B is the Bohr radius and λ' is the intensity of the force which will be varied in certain limits.

We see that for $P = 0$, i.e., the BF pair being at rest, the imaginary part is zero below $2\mu_f = 4.06\ \mu$K. Exactly at $E = 2\mu_f$ the imaginary part jumps to a finite value, increasing afterward by the free-gas law. For finite $\mathbf{P}$ the imaginary part invades the region below $2\mu_f$ and starts with a finite slope. The threshold E_{thr} follows the law

$$2mE_{thr} = 2k_F^2 + P^2 - 2Pk_F$$

with $k_F = \sqrt{(2m/\hbar^2)\mu_f}$ the Fermi momentum. We also see a sharp peak at $E = P^2/2m$ (the finite width is numerical). This peak corresponds to the motion of the free fermions when the bosons are at rest in the condensate. We can call this peak that of the free BF pairs. The corresponding free BF propagator is

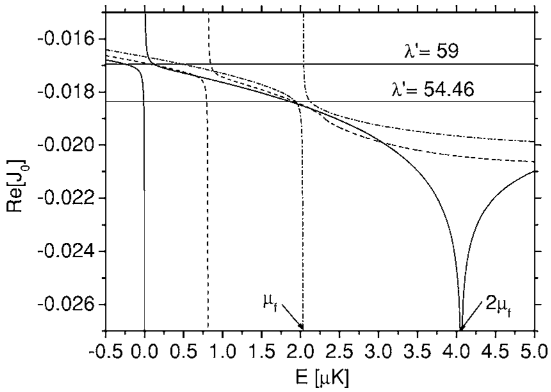

FIG. 3. $\mathrm{Re}\, J_0$ as a function of energy E for different values of the total momenta. Solid line corresponds to $P^2/2m=0$; dashed line to $P^2/2m=0.4\mu_f$; dash-dotted line to $P^2/2m=\mu_f$. Intersection with horizontal lines $-1/\lambda'$ is also shown.

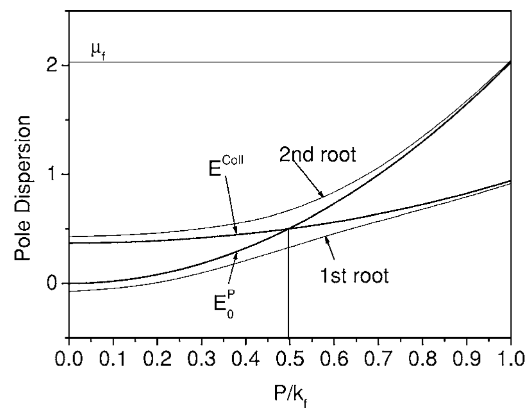

FIG. 4. Dispersion of the two true poles (thin lines) at $\lambda'=58$ in comparision with dispersion of pure collective pole E^{coll} and the free BF pair E_0^P (thick lines).

$$G^0_{n_0}(P,E)=\frac{n_0}{E-P^2/2m+i\eta}=\mathcal{P}\frac{n_0}{E-P^2/2m}-i\pi n_0\delta(E-P^2/2m). \quad (8)$$

We see that this part of the propagator is equal to the pure single-fermion propagator multiplied by n_0, which is the free-boson propagator at $T=0$. The peak of Eq. (8) and the threshold of the continuum part of $\mathrm{Im}\, J_0$ approach one another for increasing $\mathbf{P}$ and meet exactly at $E=k_F^2/2m$ when the free BF pair moves with $P=k_F$.

In Fig. 3 we show the real part of J_0 at $T=0$ for different values of $\mathbf{P}$. The poles of the T matrix (7) are determined by $\mathrm{Re}\, J_0=-\lambda^{-1}$, i.e., by the intersection of $\mathrm{Re}\, J_0$ with the horizontal line $-\lambda^{-1}$. We can see that attractive potentials always lead to two stable solutions for each $\mathbf{P}$, as long as λ does not become too large or too small. Each of these solutions is defined by contributions of two parts of J_0 One of them, which is given by the integral part of J_0, we call the collective contribution; the other one, which comes from the $n_0/(E-P^2/2m)$ part of J_0, we call the ordinary or free one. As long as both roots are well separated in energy, they can be determined by the two separate dispersion equations

$$\mathcal{P}\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{1-f(\mathbf{P}/2+\mathbf{p})+g(\mathbf{P}/2-\mathbf{p})}{E-e_f(\mathbf{P}/2+\mathbf{p})-e_b(\mathbf{P}/2-\mathbf{p})}v^2(p)=-\frac{1}{\lambda}, \quad (9)$$

$$\frac{1}{E-P^2/2m}=-\frac{1}{\lambda n_0 v^2(P/2)}. \quad (10)$$

In this work we will restrict attention to $T=0$ when the phase-space factors in Eq. (9) reduce to $g(\mathbf{p})=0$ and $f(\mathbf{p})=\theta(k_f^2-p^2)$.

In Fig. 4 we show for $\lambda'=58$ the dispersion of the collective pole (9) and of the ordinary pole (10) which describes to very good approximation [with the parameters used here $\lambda n_0 v^2(P/2)\ll 1$] the center-of-mass motion of a noninteracting BF pair, i.e., $E_0^P=P^2/2m$ (thick lines). On the same figure we also show the true dispersion of the two roots (thin lines). We notice that at $P\simeq k_F/2$ there is a level crossing between Eqs. (9) and (10). However, in reality, due to the no-crossing rule [19] and the level-level repulsion, the two roots do *not* cross but, as is well known [19], nevertheless exchange their character around crossing. For $P\lesssim k_F/2$ the collective pole is above the ordinary root whereas for $P\gtrsim k_F/2$ it is the inverse. This interchange has dramatic consequences: all BF pairs with center-of-mass momenta $k_F/2\lesssim P<k_F$ will populate the lower branch, i.e., the collective pole. Due to its strong collectivity the upper part of the Fermi sphere becomes strongly modified, as we will see later. Of course, this interpretation is qualitative, since we only considered a single BF pair and, as in the case of ordinary Cooper pairing [20], pair-pair interaction may modify the scenario quantitatively. How much of the original free Fermi surface melts and turns into a new momentum-space shell filled with a gas of BF pairs depends, of course, on the interaction. For $\lambda'<58$ the new shell will be thinner than the one of Fig. 4 and at $\lambda'=\lambda'_{cr}$ the shell of BF pairs disappears. For the parameters used here this happens for $\lambda'_{cr}=54.46$. One may, however, also define another critical value $\lambda'=\lambda'_{tot}$ which corresponds to the conversion of practically all original bosons into BF pairs. For our case this occurs at $\lambda'_{tot}\simeq 58.75$. Increasing the interaction further, part of the BF pairs will be converted into bound BF molecules with negative binding energy. The various scenarios are depicted in Fig. 5 where we show the dispersion of the collective pole (9) in comparison with the free-gas BF dispersion $E_0^P=P^2/2m$ for four cases $\lambda'=\lambda'_{cr}=54.46$, $\lambda'=58$, $\lambda'=\lambda'_{tot}\simeq 58.75$, and $\lambda'=59$. We see that for $\lambda'=\lambda'_{tot}$ the new dispersion of the BF pairs undershoots the free-gas BF dispersion everywhere and that for $\lambda'=59$ molecules appear in the range $0\leqslant P\lesssim 0.47k_F$.

For $\lambda'<\lambda'_{cr}$ still a *stable* pole, i.e., with no imaginary part, exists down to infinitesimally small attraction where for $P=0$ the collective pole hits the value $2\mu_f$. This is due to the logarithmic divergency seen in Fig. 3, which is of the same origin as in the original Cooper problem of fermions [14], namely, the sharp Fermi function in Eq. (9) at $P=T=0$. In regions where the collective pole lies above the ordinary BF pole one would call the collective pole a BF pair vibration which for $\lambda'\lesssim\lambda'_{cr}$ can become of considerable collectivity as seen in Fig. 5. Whether as in nuclear physics [19] such pair

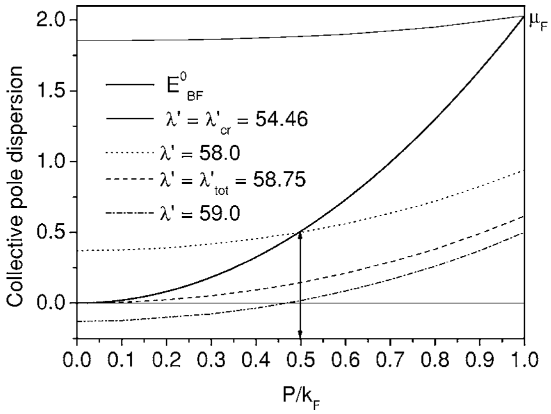

FIG. 5. Dispersion of the collective poles at different values of interaction strength, together with the free BF dispersion E_0^P.

vibrations can be detected experimentally is an open question.

To evaluate the ratio of the fermions and bosons which participate in the composite BF pair let us consider the fermion (boson) occupation numbers

$$n_{\mathbf{p}}(\mu,T) = \int \frac{dE}{2\pi} f(E) A(\mathbf{p},E) \tag{11}$$

which can be found by the help of the single-particle spectral function $A(\mathbf{p},E)$,

$$A_{b,f}(\mathbf{p},E) = \frac{-2\,\mathrm{Im}\,\Sigma^{ret}_{b,f}(E,\mathbf{p})}{[E - e^{b,f}_{\mathbf{p}} - \mathrm{Re}\,\Sigma^{ret}_{b,f}(E,\mathbf{p})]^2 + \mathrm{Im}\,\Sigma^{ret}_{b,f}(E,\mathbf{p})^2} \tag{12}$$

calculated via the retarded self-energy $\Sigma^{ret}_{b,f}(E,\mathbf{p})$. The retarded self-energy can be defined through the Matsubara self-energies $\Sigma_{b,f}(z_n,\mathbf{p})$ (again we keep the formalism general and work at finite temperature; however, at the end we set $T=0$). To find $\Sigma_{b,f}(z_n,\mathbf{p})$ we express them in terms of the T matrix calculated in the ladder approximation,

$$\Sigma_{b,f}(z_n,\mathbf{p}) = \pm T \sum_{z_{n'}} \int \frac{d\mathbf{p}'}{(2\pi)^3} T^{\mathbf{K}}_{z_n+z_{n'}}(\mathbf{q},\mathbf{q}) G^0_{f,b}(z_{n'},\mathbf{p}') \tag{13}$$

where $\mathbf{K}=\mathbf{p}+\mathbf{p}'$, $\mathbf{q}=(\mathbf{p}-\mathbf{p}')/2$, and $G^0_{b,f}(z_{n'},\mathbf{p}')$ are the free single-particle Green's functions (GFs)

$$G^0_b(z_n,\mathbf{p}) = \frac{1}{iz_n - e^b_{\mathbf{p}}} - (2\pi)^3 T^{-1} n_0 \delta(\mathbf{p}) \delta_{n,0},$$

$$G^0_f(z_n,\mathbf{p}) = \frac{1}{iz_n - e^f_{\mathbf{p}}}. \tag{14}$$

The sum over the Matsubara frequencies can be performed using the spectral representation of the T matrix and the single-particle GF and transforming the sum into a contour integral. The corresponding imaginary parts will be

$$\mathrm{Im}\,\Sigma^{ret}_b(E,\mathbf{p}) = \int \frac{d\mathbf{p}'}{(2\pi)^3}\,\mathrm{Im}\,T^{\mathbf{K}}_{e^f_{\mathbf{p}'}+E}(\mathbf{q},\mathbf{q}) \times [f(e^f_{\mathbf{p}'}) - f(e^f_{\mathbf{p}'}+E)], \tag{15}$$

$$\mathrm{Im}\,\Sigma^{ret}_f(E,\mathbf{p}) = n_0\,\mathrm{Im}\,T^{\mathbf{p}}_{E+i0}(\mathbf{p}/2,\mathbf{p}/2) + \int \frac{d\mathbf{p}'}{(2\pi)^3}\,\mathrm{Im}\,T^{\mathbf{K}}_{e^b_{\mathbf{p}'}+E+i0}(\mathbf{q},\mathbf{q}) \times [g(e^b_{\mathbf{p}'}) + f(e^b_{\mathbf{p}'}+E)]. \tag{16}$$

The real parts can be calculated from the imaginary ones by using the dispersion relation

$$\mathrm{Re}\,\Sigma(E,\mathbf{p}) = \Sigma^0(\mathbf{p}) + \mathcal{P}\int \frac{dE'}{\pi} \frac{\Gamma(E',\mathbf{p})}{E-E'} \tag{17}$$

where $\Gamma(E,\mathbf{p}) = i[\Sigma^{ret}(E,\mathbf{p}) - \Sigma^{adv}(E,\mathbf{p})]$.

For the energies below E^{thr} the quantity $J^I_0(E,K)$ is zero and the T matrix exhibits poles at $1+\lambda J^R_0(E,K)=0$. The corresponding value of $\mathrm{Im}\,T^{\mathbf{K}}_{e^b_{\mathbf{p}'}+E+i0}(\mathbf{q},\mathbf{q})$ in this case is the following:

$$\mathrm{Im}\,T^{\mathbf{K}}_{E+i0}(\mathbf{q},\mathbf{q}) = V(\mathbf{q},\mathbf{q}) \sum_\nu A^K_\nu \delta(E - E^K_\nu) \tag{18}$$

where

$$A^K_\nu = \left[\lambda \frac{dJ^R_0(E,K)}{dE}\right]^{-1}_{E=E^K_\nu} \tag{19}$$

and E^K_ν is the solution of the secular equation $1+\lambda J^R_0(E^K_\nu,K)=0$.

For the calculation of the occupation numbers we use the quasiparticle approximation (we checked this to be very accurate)

$$A_{b,f}(\mathbf{p},E) = \frac{2\pi\delta(E - \xi^{b,f}_p)}{|1 - (d/dE)\mathrm{Re}\,\Sigma^{ret}_{b,f}(E,\mathbf{p})|_{E=\xi^{b,f}_p}} \tag{20}$$

where $\xi^{b,f}_p$ is the solution of the following equation:

$$\xi^{b,f}_p - e^{b,f}_{\mathbf{p}} - \mathrm{Re}\,\Sigma^{ret}_{b,f}(\xi^{b,f}_p,\mathbf{p}) = 0. \tag{21}$$

The fermionic distribution function $n^f_p(\mu_f,T)$ was calculated at zero temperature in the approximation that $\mathrm{Im}\,\Sigma^{ret}_f(E,\mathbf{p})$ of Eq. (16) contains only the term with the boson condensate. We estimated that this term gives the by far largest contribution in comparison with the second term. At weak interaction the redistribution of the fermions due to their interaction with bosons is small. As an example, in Fig. 6(a) we show n^f_p calculated for $\lambda'=52.0$. We can see that the usual Fermi step function of the free fermions converts into two steps. To interpret such a behavior we show in Fig. 6(b) the dispersion of the quasifermion energies, solutions of Eq. (21). Since the mass operator (16) contains via the T matrix the two poles we have discussed in Figs. 3 and 4, Eq. (21) will have three roots which in the weak-coupling limit considered here correspond approximately to the free solution $\xi_{1p}\sim e^f_p$, a second one ξ_{2p} corresponding to Eq. (10) and

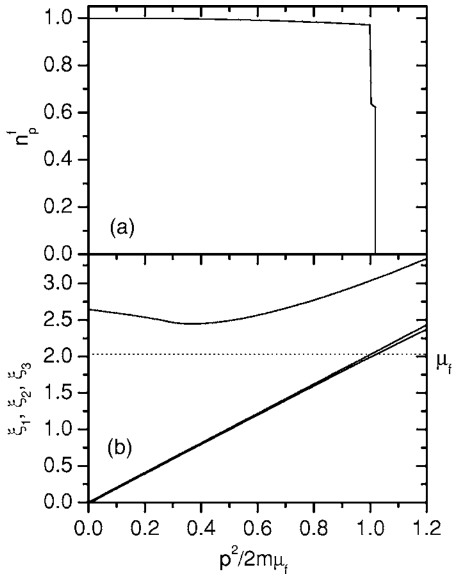

FIG. 6. Fermion occupation numbers (a) and dispersion of the quasifermion energies (b) at $\lambda'=52.0$.

almost degenerate with ξ_{1p} up to the level crossing, and a third one which corresponds to the collective BF pair (9) corresponding to the highest root in Fig. 6(b). Since our interaction strength is still sufficiently weak so that the original chemical potential is barely changed, the intersection at $p^2/2m\mu_F \simeq 1.02$ of the ξ_{2p} with μ_F gives the upper step in Fig. 6(a), whereas the solution ξ_{1p} still intersects at $p^2/2m \simeq \mu_F$ and gives rise to the intermediate step in Fig. 6(a). When the interaction further decreases, the two lowest roots become completely degenerate and we have the usual Fermi step.

With the growth of λ the low-lying BF roots become more and more collective and when λ' approaches the value equal to ~ 54.0 the BF roots corresponding to total momenta $K \sim k_F$ become collective enough to change the fermion distribution strongly. In Fig. 7(a) we thus show n_p^f for $\lambda' = 54.0$ and the corresponding dispersion is displayed in Fig. 7(b). After the level crossing, as already discussed above, Eqs. (9) and (10) exchange their properties and therefore for $p^2/2m\mu_F \gtrsim 1$ the lowest root becomes the collective BF pair and the highest one becomes degenerate with the free solution.

At the interaction $\lambda' \gtrsim 54.0$ level crossing takes place below the Fermi momentum. Such a case is displayed in Figs. 8(a) and 8(b) for $\lambda'=56.0$. We can see one small step at the momenta slightly below k_F, which will disappear with an increase of the interaction, and a rather long tail in the Fermi distribution which corresponds to the strong collective BF pairs. With further increase of the interaction this tail goes to infinity whereas the energy of the collective BF pair goes to zero. After that the BF pair converts into the molecular state and it is necessary to apply another kind of theory.

We prefer not to increase λ' further because the redistribution of n_p^f strongly varies with λ' and for strong λ' the one pair approximation becomes invalid. On the other hand the stronger values of λ' employed in Figs. 4 and 5 have just been chosen for illustration purposes and for a qualitative

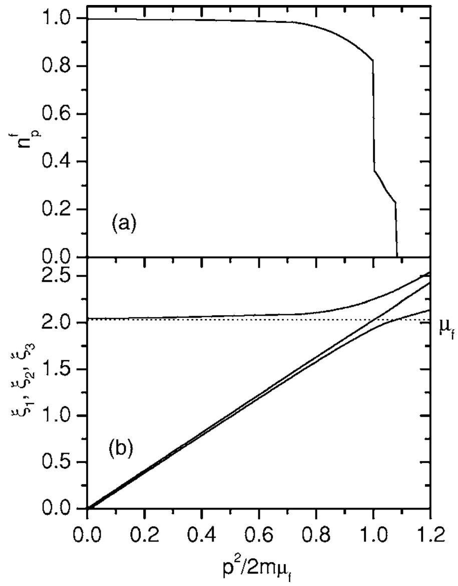

FIG. 7. Same as Fig. 6 but for $\lambda'=54.0$.

discussion of the roots of $\mathrm{Re}\, J_0 = -\lambda^{-1}$ this seems quite appropriate.

We therefore very nicely see that with the parameters chosen we have around μ_F a mixture of the gas of the old free fermions and the new composite ones formed out of a boson and a fermion. The interaction was chosen sufficiently weak so that the one-pair description is approximately valid and yet sufficiently strong that the coexistence of the two Fermi gases can clearly be seen.

In principle with Eqs. (1), (12), and (15) one can also calculate the new boson occupation numbers, i.e., of those bosons which, due to the BF correlations, are scattered out of the condensate. However, with our choice of parameters, the influence of the bosons on the fermions remains modest and the one-pair approximation is justified. On the other hand,

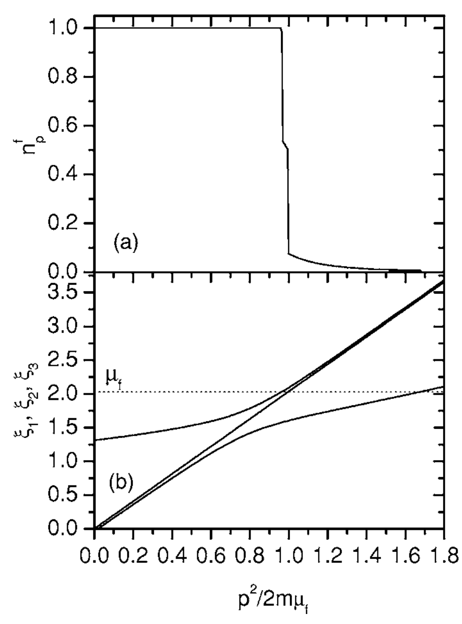

FIG. 8. Same as Fig. 6 but for $\lambda'=56.0$.

the action of the fermions on the bosons even for relatively small interactions is enormous because of the much greater number of fermions than bosons invalidating the one-pair approximation. One therefore would have to iterate the calculation for the occupation numbers, that is, we insert the new occupation numbers into Eq. (1), solve it, calculate new occupation numbers, and so on up to convergence. At the same time we will have to readjust the chemical potentials for bosons and fermions to preserve the number of particles. This will be done in future work and we do not show the boson occupation here.

The reader may have noticed that our approach is equivalent to a particle-particle random-phase approximation (RPA) formulation [19] in the BF channel. Since our BF pairs are discrete states an eigenvalue variant of the present approach can give useful additional information. We briefly present this in the Appendix.

In conclusion we considered boson-fermion propagation in a BF environment and found that the original free gas converts for sufficiently strong attractive interaction into a completely different state of matter as a Fermi gas of BF Cooper pairs with a strongly modified Fermi surface. The most interesting feature concerns the fact that, due to the Pauli exclusion principle, this transition can occur for interaction strength *insufficient* to form bound BF molecules in free space. In other words the BF pairs are at *positive* energies much analogous to Cooper pairs in a pure Fermi gas. On the other hand the collective BF pairs are still fermions building a Fermi gas of composites and a Fermi surface. Whether this transition has anything to do with the recently discovered collapse of a BF mixture [10] remains to be seen.

The only system parameter we varied in our work is the strength of the interaction. Of course, a variety of other parameters could be changed: the densities of bosons and fermions can be varied in strong proportions, their masses could be strongly different, we worked strictly at zero temperature only, we consider a homogeneous system and not the geometry of traps, etc. Such investigations will be performed in the future. It also should again be mentioned that in this pioneering work we considered only a very idealized situation, disregarding any interaction between like particles, i.e., between bosons or between fermions. We suspect that as long as the interactions between like particles are repulsive nothing qualitative will change: the Fermi surface will become slightly rounded and some depletion of the condensed bosons will occur. The constellation of moderate repulsion between particles of the same kind and attraction between different kinds is not unrealistic [21,22].

In this respect we also mention that an attractive BF interaction can induce, via, e.g., second-order processes, an effective attraction between fermions [23]. In this paper we consider a weak-coupling scenario where the interaction is weaker than the one needed to form a BF molecule. Therefore we suppose that induced FF attraction is weak and in any case weaker than the direct FF repulsion which we implicitly can assume here. In any case, in this work we are only treating a one-component Fermi gas where *s*-wave scattering is suppressed, unless the force is finite range. Then only dipole or higher odd multipole interactions could lead to FF attraction. On the other hand, if one considered fermions with spin together with bosons, FF attraction is possible more easily and in that case our scenario may still change strongly. Indeed, it is conceivable that in this case standard purely fermionic Cooper pairs form new Cooper pairs of triples in pairing up with bosons. A strong enhancement of our present effect could occur since now bosons (the FF Cooper pairs) pair with bosons (atoms). What exactly will happen under these conditions is unknown at this point. Of course with attraction among fermions, it is also conceivable that two BF Cooper pairs form a quartet. Those quartets would be different from the purely fermionic quartets which may be possible when four different species of fermions are trapped in a pure Fermi gas, as recently discussed in the nuclear physics context (α particles [24,25]). One sees that a great variety of quantum condensation phenomena may still be explored with ultracold atomic gases consisting of bosons and fermions.

Note added. Recently we learned about the related work in [26] where, however, the BF pairs are treated in the molecular state and not as collective BF pairs embedded as sharp states in the continuum as in this work.

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APPENDIX

A further more formal but eventually useful aspect of our theory is to relate it to the language of a particle-particle RPA [19] in the BF channel. We only consider this here at $T=0$ and will present the extension to $T\neq 0$ in a future publication. With this purpose we write the following RPA excitation operator:

$$Q_l^\dagger = \sum_{\mathbf{pq}} \frac{X_{\mathbf{pq}}^l c_{\mathbf{p}}^\dagger b_{\mathbf{q}}^\dagger}{\sqrt{1+\delta_{\mathbf{q},0} n_0}} + \sum_{\mathbf{h}} \frac{Y_{\mathbf{h}}^l c_{\mathbf{h}}^\dagger b_0^\dagger}{\sqrt{n_0}} \qquad \text{(A1)}$$

where $p(h)$ is the fermion momentum above (below) the Fermi sea [a particle (hole) state] and q is a boson momentum which can take on all values. With the definition of an excited state of the $(N+2)$-particle system (the addition mode)

$$|l\rangle = Q_l^\dagger|\,\rangle \qquad \text{(A2)}$$

we arrive with the usual condition

$$Q_l|\rangle = 0 \qquad \text{(A3)}$$

at the following secular equation [19]:

$$\langle\,|\{\delta Q,[H,Q_l^\dagger]\}|\rangle = E_l\langle|\{\delta Q,Q_l^\dagger\}|\rangle \qquad \text{(A4)}$$

where δQ is a variation with respect to either X or Y. The usual linearization of Eq. (A4) consists in evaluating the expectation values with the uncorrelated ground state that is a product of a Slater determinant and an ideal Bose condensate. Defining the Hamiltonian as

$$H=\sum_{\mathbf{q}}(\varepsilon^b_{\mathbf{q}}b^\dagger_{\mathbf{q}}b_{\mathbf{q}}+\varepsilon^f_{\mathbf{q}}c^\dagger_{\mathbf{q}}c_{\mathbf{q}})+\sum_{q_1q_2q_3q_4}V_{\mathbf{q}_1\mathbf{q}_2,\mathbf{q}_3\mathbf{q}_4}b^\dagger_{\mathbf{q}_1}c^\dagger_{\mathbf{q}_2}c_{\mathbf{q}_3}b_{\mathbf{q}_4} \tag{A5}$$

where

$$\varepsilon^{f,b}_{\mathbf{k}}=e^{f,b}_{\mathbf{k}}-\mu_{f,b},$$

Eq. (A4) then reads as

$$\begin{pmatrix} A_{\mathbf{p}'\mathbf{q}',\mathbf{pq}} & D_{\mathbf{p}'\mathbf{q}',\mathbf{h}0} \\ B_{\mathbf{h}'0,\mathbf{pq}} & C_{\mathbf{h}'0,\mathbf{h}0}\end{pmatrix}\begin{pmatrix} X^l_{\mathbf{pq}} \\ Y^l_{\mathbf{h}}\end{pmatrix}=E_l\begin{pmatrix} X^l_{\mathbf{p}'\mathbf{q}'} \\ Y^l_{\mathbf{h}'}\end{pmatrix} \tag{A6}$$

where

$$A_{\mathbf{p}'\mathbf{q}',\mathbf{pq}}=\delta_{\mathbf{pp}'}\delta_{\mathbf{qq}'}(\varepsilon^f_{\mathbf{p}}+\varepsilon^b_{\mathbf{q}})+\sqrt{1+\delta_{\mathbf{q}',0}n_0}V_{\mathbf{p}'\mathbf{q}',\mathbf{pq}}\sqrt{1+\delta_{\mathbf{q},0}n_0},$$

$$B_{\mathbf{h}'0,\mathbf{pq}}=V_{\mathbf{h}'0,\mathbf{pq}}\sqrt{n_0}\sqrt{1+\delta_{\mathbf{q},0}n_0},$$

$$D_{\mathbf{h}'0,\mathbf{pq}}=V_{\mathbf{p}'\mathbf{q}',0\mathbf{h}}\sqrt{n_0}\sqrt{1+\delta_{\mathbf{q}',0}n_0},$$

$$C_{\mathbf{h}'0,\mathbf{h}0}=\delta_{\mathbf{hh}'}(\varepsilon^f_{\mathbf{h}}+\varepsilon^b_0)+V_{\mathbf{h}'0,\mathbf{h}0}n_0. \tag{A7}$$

We immediately see that with $V_{\mathbf{p}'\mathbf{q}',\mathbf{pq}}=-\lambda v(\mathbf{p}'-\mathbf{q}')v(\mathbf{p}-\mathbf{q})\delta_{\mathbf{p}+\mathbf{q},\mathbf{p}'+\mathbf{q}'}$ the eigenvalues are given as before by $\mathrm{Re}\,J_0=-\lambda^{-1}$ and therefore the RPA description is completely equivalent to the Green's function approach we used at the beginning.

The interesting aspect of this formulation is that Eq. (A6) contains a quasifermion approximation, i.e., the BF pairs in Eq. (A1) are treated as ideal fermions,

$$F^\dagger_{\mathbf{pq}}=\frac{c^\dagger_{\mathbf{p}}b^\dagger_{\mathbf{q}}}{\sqrt{1+\delta_{\mathbf{q},0}n_0}}$$

$$F^\dagger_{\mathbf{h}}=\frac{c^\dagger_{\mathbf{h}}b^\dagger_0}{\sqrt{n_0}} \tag{A8}$$

with

$$\{F_{\mathbf{p}'\mathbf{q}'},F^\dagger_{\mathbf{pq}}\}=\delta_{\mathbf{pp}'}\delta_{\mathbf{qq}'},$$

$$\{F_{\mathbf{h}'},F^\dagger_{\mathbf{h}}\}=\delta_{\mathbf{hh}'}.$$

This is quite an analogy to the standard RPA for a pure Fermi system where a fermion pair $c^\dagger_p c^\dagger_{p'}$ is treated as a quasiboson [19]. This quasifermion approximation contained in Eq. (A6) allows to write down the approximate ground state. It is given by a Slater determinant of the new BF pairs,

$$|\,\rangle\sim\exp\left[\sum_{\mathbf{p}'\mathbf{q}'\mathbf{h}'}Z_{\mathbf{p}'\mathbf{q}'\mathbf{h}'}F^\dagger_{\mathbf{p}'\mathbf{q}'}F_{\mathbf{h}'}\right]|uc\rangle \tag{A9}$$

where $|\mathrm{uc}\rangle$ is the uncorrelated vacuum. From the condition (A3) we also can find the system of equations which defines the coefficients $Z_{p'q'h'}$:

$$Y^l_{\mathbf{h}}+\sum_{\mathbf{pq}}X^l_{\mathbf{pq}}Z_{\mathbf{pqh}'}=0. \tag{A10}$$

The initial Hamiltonian (A4) in the basis of the quasifermions (A1) will be the following:

$$H=\sum_{l\gtrless 0}E_lQ^\dagger_lQ_l \tag{A11}$$

where $l<0$ corresponds to the negative roots of Eq. (A6) or (which is the same) to the roots of the RPA for the $N-2$ system (the removal mode). And finally we can define the following correlation energy:

$$E_{corr}=\langle H\rangle-\langle uc|H|uc\rangle=\sum_{l<0}E_l-\mathrm{Tr}C, \tag{A12}$$

which may be evaluated for realistic systems in the future. This correlation energy calculated in BF pp-RPA is exactly the analog to the correalation energy calculated for an electron gas in ph-RPA via the summation of ring diagrams [18]. We therefore see that our BF calculation also leads to an improved equation of state.

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