Electromotive interference in a mechanically oscillating superconductor: generalized Josephson relations and self-sustained oscillations of a torsional SQUID

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We consider the superconducting phase in a moving superconductor and show that it depends on the displacement flux. Generalized constitutive relations between the phase of a superconducting interference device (SQUID) and the position of the oscillating loop are then established. In particular, we show that the Josephson current and voltage depend on both the SQUID position and velocity. The two proposed relativistic corrections to the Josephson relations come from the macroscopic displacement of a quantum condensate according to the (non-inertial) Galilean covariance of the Schrödinger equation, and the kinematic displacement of the quasi-classical interfering path. In particular, we propose an alternative demonstration for the London rotating superconductor effect (also known as the London momentum) using the covariance properties of the Schrödinger equation. As an illustration, we show how these electromotive effects can induce self-sustained oscillations of a torsional SQUID, when the entire loop oscillates due to an applied dc-current.

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The interest in nano-mechanical systems dramatically increased recently. For instance, the superposition of quantum states of a mechanical resonator has been demonstrated [1, 2], realizing an important step towards the generation of mechanically quantum dressed state at the mesoscopic level. These dressed states open wide possibilities for using mechanical resonators for quantum-limited detection and for quantum information. There are currently several routes being explored towards these applications, based on the coupling between mechanical resonators and either electrical, optical, or magnetic systems.

One of the route is to use a mechanical resonator embedded into a superconducting quantum interference device (SQUID), see *e.g.* [3–6] and references therein. The first experiment demonstrated the possibility of the detection of the resonance frequency and the quality factor of the resonator by measurements of the voltage generated across the SQUID [7]. Magnetic flux through the SQUID and the bias current served as two control parameters. Later experiments [8] investigated back-action of the SQUID exerted on the mechanical resonator and found qualitative agreement with the results of the theoretical modeling. The experiments on the new generation suspended structure, a torsional SQUID, demonstrated that the back-action may be so strong that the SQUID enters the regime of self-sustained oscillations [9]. It remained unclear, however, where such a large coupling between the superconducting and mechanical degrees of freedom comes from.

So far, the description of the experimental setup ignored possible electromotive effects of quantum nature. For instance, it is well known that a cold quantum gas exhibits vortex states under rotation, in an intuitive analogy with the Abrikosov lattice [10]. Nevertheless, the analogy is incomplete, because the Abrikosov lattice is of electromagnetic nature, whereas the circulation vortices are of mechanical origin. A situation when both these effects may compete is precisely the situation when a quantum condensate made of charged particles is mechanically displaced. Then, a superconducting condensate may magnify electromotive effects when put under displacement. This is illustrated by the striking Meissner or London momentum effects [11]. In short, the Meissner effect corresponds to the generation of a displacement current which screens an applied magnetic field, whereas the London momentum effect corresponds to the generation of a macroscopic magnetic field which screens the displacement current generated by the mechanical rotation of a superconductor. This surprising effect can only exist when electromagnetism and mechanical displacement compete together, and can be seen as the destruction of the mechanically induced vortex lattice by the generation of an electromagnetically induced lattice.

Another explanation of this effect lies in the well-known London theory of the electrodynamics of superconductors, which corresponds to the addition of an inertial term (proportional to the vector potential $\mathbf{A} \propto \mathbf{j}$) into the otherwise viscous expressions for circuit electromagnetism (*i.e.* Ohm's theory). Because of this inertial correction, the London theory is unable to take into account a change of the inertial frame, in the sense that there is no explicit need to specify in which inertial frame the superconductor is supposed to be. Nevertheless, when a normal metal is attached to a superconducting one, the normal electronic flow must be recovered at the interface. Then, London proposed to correct his theory by imposing a superconducting current to lag behind the lattice one, creating a magnetic moment by virtue of the Ampère law. To test the validity of this retarded contribution, London designed the very simple experiment of the rotating superconductor, which predicts the generation of a macroscopic magnetic field induced by the rotation of a superconducting sphere [11]. The London's prediction was soon after verified for both bulk [12] and proximity effect [13] systems, and latter for high-temperature superconductors [14] including heavy-fermion compounds [15].

More recently, the London expression for the inertial current was considered in the framework of an effective and classical theory of elastic superconductors, with prediction of a precise acoustic sensor using Josephson systems, elasto-magnetic coupling between the motion of the superconductor and the internal magnetic moment it produces, in addition to some interesting effects in type-II superconductors, see *e.g.* [16] and references therein. It also continues to attract some fundamental interests, being at the cornerstone between mechanics and electromagnetism [17–21].

In this paper, we consider possible electromotive effects of quantum mechanical origin in a moving superconductor. The results of this study are twofold. In the first part, we will show that the London momentum naturally appears in the context of Galilean covariance of the Schrödinger equation. Generalizing this demonstration to the non-inertial case, we establish some generalized Josephson relations in a moving superconducting circuit. In the second part, we will derive constitutive relations linking the superconductor motion to the current and voltage. Subsequently, we apply the arguments to the setup of a suspended SQUID. As a simple illustration of the theory, we will describe the regime when a static current can induce self-sustained oscillations in a torsional SQUID, as shown in [9].

We start with the correspondence of the London momentum and the Galilean covariance of the Schrödinger equation, leading to the generalization of the Josephson relations when electromotive effects are taken into account. We have in mind a superconducting circuit interrupted by some Josephson junctions and when a region of the circuit is mechanically oscillating. Then a part of the circuit is in the laboratory frame, whereas the oscillating part is in a moving frame. In order to obtain the electromotive contributions in a general situation, we constraint the phase of the superconducting, macroscopic wave function to be continuous all along the circuit. When the circuit forms a loop, and when this loop is pierced by a magnetic flux $\Phi = \oint \mathbf{A} \cdot d\mathbf{l}$, the phase continuity implies that the so-called gauge covariant phase $\gamma = \varphi_0 - 2\pi\Phi/\Phi_0$ enters in the expressions, where φ_0 is the initial condition phase [22].

In any situation when part of the circuitry is moving, this gauge covariant phase fails to describe the passage from the circuit at rest to the moving part, possibly destroying the phase continuity all along the circuit. From general properties of quantum mechanics, the displacement of a particle generates specific phase factors. For a massive particle, there are two possible phase factors which can be added to the displaced wave-function. They correspond to the energy correction $e^{\mathbf{i}Et/\hbar} \sim e^{\mathbf{i}mv^2t/2\hbar}$ due to the kinetic energy the particle acquires under displacement and responsible for time dependent interference effects ; and the displacement operator of the wave function $e^{\mathbf{i}px/\hbar} \sim e^{\mathbf{i}mvx/\hbar}$ responsible for position dependent interferences.

Let us quantify this idea. The non-inertial transformation is defined as the change from the unprimed laboratory-frame to the primed moving-frame defined by $\mathbf{x}' = \mathbf{x} - \mathbf{r}(t)$ and $t' = t$. Then, the differential operators appearing in the Schrödinger equation transform according to $\partial/\partial t' = \partial/\partial t + \mathbf{v}(t) \cdot \nabla$ and $\nabla' = \nabla$ where the velocity $\mathbf{v}(t) = \dot{\mathbf{r}}$ is time dependent. Moreover, one can show that the Schrödinger equation for particle of mass m and charge q in the laboratory frame,

$$\mathbf{i}\hbar\frac{\partial\Psi}{\partial t} = \left[U + qV - \frac{\hbar^2}{2m}\left(\nabla - \mathbf{i}\frac{q}{\hbar}\mathbf{A}\right)^2\right]\Psi(\mathbf{x},t) \ , \quad (1)$$

is formally equivalent to the one with electromagnetic potentials V and $\mathbf{A}$, potential U and wave function Ψ expressed in the displaced (primed) system with the correspondence laws (full details of the calculations can be found in *e.g.* [23, 24], see also note [29])

$$\begin{cases} U'(\mathbf{x}',t') = U(\mathbf{x},t) + m\dot{\mathbf{v}}(t) \\ V'(\mathbf{x}',t') = V(\mathbf{x},t) - \mathbf{v}\cdot\mathbf{A}(\mathbf{x},t) \\ \mathbf{A}'(\mathbf{x}',t') = \mathbf{A}(\mathbf{x},t) \end{cases} \quad (2)$$

for displaced potentials and

$$\Psi'(\mathbf{x}',t') = \exp\left[\frac{\mathbf{i}}{\hbar}\left(\int_{t_0}^{t}\frac{mv^2}{2}dt - m\mathbf{v}\cdot\mathbf{x}\right)\right]\Psi(\mathbf{x},t) \quad (3)$$

for the wave function (t_0 being the initial time when $\mathbf{v}(t_0) = \mathbf{0}$). The simplest case of the Galilean transformation [23] with $\dot{\mathbf{v}} = \mathbf{0}$ is the obvious limit $\Psi' = \exp\left[\mathbf{i}(Et - \mathbf{x}\cdot\mathbf{p})/\hbar\right]\Psi \sim \exp\left[\mathbf{i}(m/\hbar)\left(v^2t/2 - \mathbf{x}\cdot\mathbf{v}\right)\right]\Psi$ of the transformation (2)-(3). In addition the case of a rotating frame can be discussed with a time-dependent velocity $\mathbf{v}(t)$ as well (more precisely a time-dependent angular-momentum, see [20, 24] for more details), and leads to similar covariance laws (2)-(3).

From the transformation law (3), one can show that the electromagnetic sources $\rho = q|\Psi|^2$ and $\mathbf{j} = q\hbar$ Im$\{\Psi^*\nabla\Psi\}/m$ transform as (cf. [23])

$$\begin{cases} \rho'(\mathbf{x}',t') = \rho(\mathbf{x},t) \\ \mathbf{j}'(\mathbf{x}',t') = \mathbf{j}(\mathbf{x},t) - \rho(\mathbf{x},t)\mathbf{v}(t) \end{cases} . \quad (4)$$

Here, the transformation law for the current is nothing else than the London current substitution when a superconductor is displaced [11]. In addition to its justification using conservation of current [11], thermodynamic properties [18] or equivalence principle [17, 19], we have thus shown that the London substitution for the current can be substantiated using the covariance of the Schrödinger equation at the simple Galilean relativity level. We even found that the London current is still a correct expression for non-inertial displacements. We remark that our argument is just the quantum version of the London's original one, because phase continuity and current conservation are related to each other.

Note that the full transformations (2), (3) and (4) are compatible with the usual electromagnetic gauge transformations, and that the space-time transformation laws for the electromagnetic potentials (and subsequently for the electromagnetic fields) correspond to the *magnetic-Galilean-limit* when the electric displacement current does not exist, see [23, 25] and note [30].

In addition to the generation of the London momentum in (4), the covariance laws (2)-(3) induce interference effects in coherent circuits. For instance, in order for the phase to be continuous all along a displaced superconducting path, one must include the kinematic terms (3) into the gauge-covariant phase definition. The generalized gauge-covariant phase is thus

$$\varphi = \varphi_0 - \frac{2\pi}{\Phi_0} \oint \mathbf{A} \cdot d\mathbf{l} + \frac{2\pi}{\Phi_0} \frac{m_e}{e} \oint \mathbf{v} \cdot d\mathbf{l} \tag{5}$$

where $2m_e$ is the Cooper pair mass, and the phase (difference) is defined as $\varphi = \int \nabla\varphi \cdot d\mathbf{l}$. Obviously, the velocity integral in Eq.(5) disappears when the superconducting condensate remains at rest along the path, and thus it connects the displacement velocity to the phase difference in the Josephson system and eventually to the Josephson current. This extra contribution would also disappear if there were not two distinct frames moving relative to each other with the velocity $\mathbf{v}$. For instance, the two frames of reference can be a superconductor (as the moving frame) and a measuring apparatus (say a magnetometer playing the role of the laboratory frame), like for the rotating superconducting effect [11].

The inclusion of the relativistic correction in (5) suggests that some subsequent contributions will be associated to the so-called second Josephson relation. Indeed, the second Josephson relation connects the time derivative of the superconducting phase (difference) with the energy difference across the weak-links, *i.e.* to the electromagnetic work the superconducting charges undergo when traveling across the junction. The total electromagnetic work is found according to the complete Lorentz force. Using the generic relation [26]

$$\frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{S} = \iint \left[\frac{\partial \mathbf{B}}{\partial t} - \nabla \times \mathbf{v} \times \mathbf{B} + \mathbf{v} \left(\nabla \cdot \mathbf{B} \right) \right] \cdot d\mathbf{S} \tag{6}$$

for any $\mathbf{B}$ and $\mathbf{S}$ fields, when the infinitesimal surface element $d\mathbf{S}$ is time dependent, with $\mathbf{v}(t)$ the velocity field of the contour, one obtains

$$\frac{d\varphi}{dt} = \frac{2\pi}{\Phi_0} \frac{m_e}{e} \frac{d}{dt} \oint \mathbf{v} \cdot d\mathbf{l} + \frac{2\pi}{\Phi_0} \oint \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \cdot d\mathbf{l} \tag{7}$$

for the phase-voltage relation, using Faraday's law. For $\mathbf{v} = \mathbf{0}$, we recover the (second) Josephson relation $\dot{\varphi} = 2eV/\hbar$ [22]. In addition to the first term, which generates electromagnetic fields due to the motion of accelerating charges, the last term in Eq.(7) is another electromotive one, caused by the moving surface the magnetic flux threads. Thus, even a static magnetic field can generate such a retardation effect, due to the kinematic displacement of the interfering paths in the presence of an electromagnetic field.

The two expressions (5) and (7) and their explicit derivations are our first results. We believe they are rather general, because the notation $\varphi = \int \nabla\varphi \cdot d\mathbf{l}$ above was a generic phase difference along a superconducting path, eventually a closed one. Because their derivation used the covariance properties of the Schrödinger equation under a non-inertial transformation, Eqs.(5) and (7) describe any situation when phase coherence is preserved at large distance between two sub-systems moving relatively to each other with velocity $\mathbf{v}(t)$ (after proper identification of the elementary mass and charge at works in some specific examples, which may not necessarily be the Cooper pair ones). Obviously, superconductivity exhibits such a possibility, and we will focus on superconducting systems in the following.

In order to apply relations (5) and (7) to explicit circuits, we remark that Eqs.(2) and (3) are presumably invalid for a true velocity field $\mathbf{v}(\mathbf{x}, t)$: the covariance of the Schrödinger equation would be destroyed if $\mathbf{r}$ was a function of both time and position, because of the appearance of mixing derivatives [24]. In addition, the fact that the velocity explicitly depends in time only was crucial to obtain the London expression (4) for the current (recall that $\mathbf{j}$ is defined as a gradient interference). Nevertheless, we note that only some space integrals of the velocity appear in (5) and (7), which supposedly signifies that a weak spatial dependency of $\mathbf{v}(\mathbf{x}, t)$ (say with some characteristic magnitude in space smaller than the size of the contour of the integral) would not alter the generalized Josephson relations found here (see also [19, 20] when alternative demonstrations of (5) are given for a generic $\mathbf{v}(\mathbf{x}, t)$). Due to their integral representations, we also remark that only *global* properties of the velocity field appear in the expressions for the phase difference (5) and its time derivative (7).

We now apply the generalized Josephson relations (5) and (7) to the SQUID geometry, see *e.g.* [7–9]. More precisely, we discuss the problem of an *entirely suspended* SQUID, the so-called torsional SQUID, when the entire loop is oscillating and pierced by a magnetic flux, as realized recently [9].

For simplicity, we suppose the entire loop to behave as a simple harmonic oscillator of mass m, length ℓ, quality factor Q, and resonance frequency ω_0, according to

$$\ddot{x}+\frac{\omega_0}{Q}\dot{x}+\omega_0^2 x=g\,(i+j) \tag{8}$$

where $g\approx \ell B I_0/m$ is a geometric acceleration, such that $g\,(i+j)$ represents the Lorentz force acting on the oscillator when B is the external magnetic field, $(i+j)$ being the total current through the loop, see (11). The overdot refers to total time derivative.

Let us now make plausible the appearance of a non-trivial electromotive effect for torsional SQUID. If only a part of the loop was oscillating, the contribution $\oint \mathbf{v}{\cdot}d\mathbf{l}\approx 0$ in (5) would vanish, since the velocity of the oscillator would mainly be orthogonal to the loop in that case. Nevertheless, in a torsional SQUID, this relation becomes non-local since the entire loop is oscillating, and the usual relation $\oint \mathbf{v}{\cdot}d\mathbf{l}=\iint(\nabla\times\mathbf{v})\cdot d\mathbf{S}$ applies, with $\mathbf{S}$ the surface-vector of the entire loop. Due to the non-connectedness of the ring geometry, such a term does *not* vanish in elastic media [27, 28]. This involved theorem of differential geometry can be picturesquely described in the following way: the torsional SQUID exhibits torsional elastic modes, which in turn create some global vorticity $\nabla\times\mathbf{v}$ around the loop, and thus generate a non-vanishing electromotive contribution in (5). More precisely, this global vorticity generates a pseudo-angular-momentum to the elastic loop. Then one can use the correspondence between a pure time-dependent and non-inertial Galilean transformation (as used to obtain (4), (5), and (7)) and the non-inertial transformation towards a rotating frame (see *e.g.* [24]) to ensure that the contribution $\oint \mathbf{v}{\cdot}d\mathbf{l}\neq 0$ is non-trivial for a torsional SQUID. In the following, we suppose $\oint \mathbf{v}\cdot d\mathbf{l}\approx \ell\dot{x}$ for notational simplicity.

The non-mechanical SQUID is characterized by two degrees of freedom, which we take to be the sum and the difference of the phases of the junctions, respectively $\varphi_\pm$ [22]. When the SQUID is mechanically oscillating, these two phases are modified due to the electromotive effects in (5) and (7). First, the phase difference reads then

$$\varphi_-=-\varphi_e-\kappa_- x+\frac{k}{\omega_0}\dot{x}-\beta j\ , \tag{9}$$

where $\beta=4\pi L I_0/\Phi_0$ is the self-inductance of the loop, $\varphi_e=\pi\Phi/\Phi_0$, Φ the external flux across the loop and $\Phi_0=\pi\hbar/e$ the superconducting flux quantum, $\kappa_-\approx \pi B\ell/\Phi_0$ is a flux-to-phase ratio, and $k\approx(m_e\omega_0/\hbar)\,\ell$ within our approximation. Note that the extra contribution is just the usual phase factor $\exp\left[\mathbf{i}\mathbf{p}\cdot\mathbf{x}\right]$ corresponding to the extra path length due to the displacement of the mechanical oscillator, when the impulsion $\mathbf{p}\approx m\mathbf{v}/\hbar$ is related to the velocity of the mechanical loop, the length of which is ℓ. Also, k is the inverse of an effective Compton wavelength [13], related to the inertia of the electrons because of their lag behind the mechanical oscillations of the lattice.

Comparing the two mechanical contributions in (9) leads to the ratio $B/\omega_0\approx m_e/2e\approx 3\text{ng/C}$ which seems extremely small. Nevertheless, being a velocity term, the Compton contribution has to be compared with the Q-factor of the oscillator, which may eventually be large enough to make interesting relativistic effects observable at the nanoscale. Also, it is interesting to mention that the k term in (9) survives in the absence of magnetic field. Thus, the observation of this electromotive effect in the absence of external magnetic field should be a clear demonstration that a non-trivial displacement of charges generates a complete electromagnetic field at the quantum level.

The phase difference is not affected by the contribution (7), and the $\mathbf{v}\times\mathbf{B}$ term affects only the phase sum φ_+. Eq.(7) implies that the voltage in a torsional SQUID is defined as

$$V=\frac{\Phi_0}{2\pi}\frac{d}{dt}\left[\varphi_+ +\kappa_+ x-\frac{k}{\omega_0}\dot{x}\right] \tag{10}$$

with $\kappa_+\approx\kappa_-\approx \pi B\ell/\Phi_0$ in our approximation. Expression (10) appears natural: because the position of the oscillating loop is equivalent to a flux for the SQUID, its velocity $\dot{x}$ generates a voltage. The Compton k-term plays the role of a usual Bremsstrahlung in (7).

With the help of (9) and (10), the equations of motion for the SQUID can be easily written in the quasi-classical approximation (also called RCSJ-model [22]),

$$\begin{cases} i=\sin\varphi_+\cos\varphi_- +\dfrac{\dot{\varphi}_+ +\kappa_+\dot{x}-k\ddot{x}/\omega_0}{\omega_c} \\ \qquad +\dfrac{RC}{\omega_c}\left(\ddot{\varphi}_+ +\kappa_+\ddot{x}-\dfrac{k}{\omega_0}\dddot{x}\right) \\ j=\sin\varphi_-\cos\varphi_+ +\dfrac{\dot{\varphi}_-}{\omega_c}+\dfrac{RC}{\omega_c}\ddot{\varphi}_- \end{cases} \tag{11}$$

with $\omega_c=2\pi R I_0/\Phi_0$ the characteristic frequency of the SQUID, I_0 the critical current of each Josephson junction, R and C the resistance and capacitance of the shunted Josephson junctions, $i=I/2I_0$ and $j=J/2I_0$, where I and J are the bias and self-circulating current, respectively.

Eqs.(8)-(11) represent the set of non-linear coupled differential equations of a torsional SQUID. The limit $\kappa_\pm\to 0$ and $k\to 0$ in (11) corresponds to the usual SQUID description without any electromotive effect [22]. The equations (8)-(11) are extremely hard to solve. Nevertheless, one can easily show that they lead to self-sustained oscillations of the loop induced by a static current.

Indeed, in the over-damped limit and without self inductance (*i.e.* $RC/\omega_c\to 0$ and $\beta\to 0$), the macroscopic time averaged voltage can be approximated as [22]

$$\frac{\langle V\rangle}{RI_0}=\sqrt{\left(\frac{I}{2I_0}\right)^2-\cos^2\left(\varphi_e+\kappa_- x-\frac{k}{\omega_0}\dot{x}\right)} \tag{12}$$

in the limit $\omega_0/\omega_c \ll 1$. This limit is realized in actual experiments [7–9], when $\omega_0/\omega_c \lesssim (10^{-3} - 10^{-4})$. Then, in the harmonic equation of motion (8), one has to realize that the Lorentz force is present only in the resistive branch of the SQUID response, and Eq.(8) can be approximated as

$$\ddot{x} + \frac{\omega_0}{Q}\dot{x} + \omega_0^2 x \approx g\frac{\langle V \rangle}{RI_0} \tag{13}$$

which exhibits self-sustained oscillations when $gk/\omega_0^2 \gtrsim Q^{-1}$ or in our approximation $\ell^2 B I_0 Q/m\omega_0 \gtrsim \hbar/m_e$, the quantum of circulation. Thus, to apply a static current to a torsional SQUID can generate self-sustained mechanical oscillations, as already demonstrated in [9]. Here, we present an alternative explanation based on electromotive interferences in the mechanical loop, instead of the capacitive coupling already discussed in [9]. Note that the equations of motion in both cases are similar in the limit $k/\omega_0 \ll 1$ in (12), which make the origin of the self-sustained oscillations demonstrated in [9] a bit unclear. Nevertheless, for the experiment [9], the capacitive coupling seems unable to generate the self-sustained oscillation by a few orders of magnitude, whereas the condition $gk/\omega_0^2 \gtrsim Q^{-1}$ is verified.

In conclusion, we established and discussed electromotive phenomena in the Josephson physics, based on the gauge-covariance of the Schrödinger equation under a non-inertial Galilean transformation. It leads to both an alternative demonstration of the London momentum (4), and to generalized Josephson relations taking into account the displacement of the superconductor, see (5) and (7). We then apply these generic expressions to the case of a torsional SQUID. When only a part of the SQUID is oscillating, there is in general no electromotive effect associated with the displacement. In contrary, when the entire loop is oscillating as in the torsional SQUID, we established some generalized differential equations describing the coupled evolution of the mechanical and charge degrees of freedom, see (7)-(11). In addition, the electromotive effects can lead to self-sustained oscillations of the torsional SQUID. The electromotive effects we discussed may be relevant for the description of the measurement of the motion of a quasi-classic elastic loop, and/or for the description of the back-actions a SQUID exerts onto itself when the entire loop is elastic. More precisely, the Compton-like k-term seems to be important for back-action effect, because it generates magnetic flux ; whereas the kinetic interference κ_+-term transforms existing magnetic field into voltage, and might be important for detection purpose when the voltage is monitored.

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[28] K. Hackl and U. Zastrow, Journal of Elasticity **19**, 3 (1988).
[29] Actually Brown and Holland discuss the *inertial* Galilean covariance of the Schrödinger equation when an electromagnetic field is taken into account [23], whereas Takagi discusses the non-inertial covariance (*i.e.* when the velocity is time-dependent in the Galilean transformation $\mathbf{x}' = \mathbf{x} - \mathbf{r}(t)$) of the Schrödinger equation *without* external fields [24]. To fuse these two results is straightforward, resulting in the covariance rules (2)-(3) when a non-inertial Galilean transformation is applied to the Schrödinger equation in the presence of electromagnetic fields.
[30] In the so-called magnetic limit, the Galilean-covariant equations of electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ are the usual ones $\partial\mathbf{B}/\partial t + \nabla\times\mathbf{E} = \mathbf{0}$ and $\nabla\cdot\mathbf{B} = 0$, whereas the $\mathbf{H}$ and $\mathbf{D}$ fields fail to verify the charge conservation: $\nabla\times\mathbf{H} = \mathbf{j}$ and $\nabla\cdot\mathbf{D} = \rho$. Nevertheless, one still verifies $\mathbf{B} = \nabla\times\mathbf{A}$ and $\mathbf{E} = -\nabla V - \partial\mathbf{A}/\partial t$, which guaranty gauge invariance of the fields. Injecting $\tilde{\mathbf{A}} = \mathbf{A} - \nabla\xi$ and $\tilde{V} = V + \partial\xi/\partial t$ for any field $\xi(\mathbf{x}, t)$ in (2) will not modify the covariance of the potentials transformation laws: then the covariance laws (2) and the gauge invariance of the fields are compatible. Along [23], we would like to reinforce the surprising result that the covariance laws of the Schrödinger equations follow the *magnetic* limit (2) of the so-called Galilean electromagnetism for the potentials whereas the sources transform alongside the *electric* limit of Galilean electromagnetism, these two limits being incompatible with each other in the Galilean relativity. So in short the London momentum effect (given by the transformation (4)) follows from the covariance rules of quantum mechanics, and would be difficult (if not impossible) to demonstrate using the Galilean covariance of the classical dynamics. The price to pay for our demonstration here is to stick with a pure time-dependent velocity $\mathbf{v}(t)$ (see after (7) for the reason). The reader is referred to the pedagogical articles [23, 25] for more details about the Galilean electromagnetism and its covariant transformations in the electric and magnetic limits.