



Longitudinal Literacy and Numeracy Study:
Transitions from Preschool to School

Fostering Understanding of Early Numeracy Development

Kate Reid & Nicola Andrews

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About the authors

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Fostering Understanding of Early Numeracy Development

Executive summary

Children understand many numeracy concepts before they start learning mathematics formally at school (Song & Ginsburg, 1987; Sophian, Harley, & Manos Martin, 1995). Prior to school, children's numeracy knowledge develops significantly and these skills influence achievement in school mathematics (Malofeeva, Day, Saco, Young, & Ciancio, 2004; Starkey, Klein, & Wakeley, 2004). Promoting educators' understanding of preschool numeracy can have significant benefits for children's numeracy development and for supporting children's transitions from informal to formal mathematics learning (Floyd, Hojnoski, & Key, 2006).

Children's informal mathematical knowledge is sometimes described as a 'number sense'. Number sense has been defined in different ways but is generally thought to include a capacity to work flexibly with numbers; to compare, recognise patterns and solve problems (Gersten & Chard, 1999). Griffin (2004) argues that specific number sense content for the typical five-year-old comprises:

- knowing that numbers indicate quantity and thus have a magnitude
- understanding and using relative terms such as 'more', 'less', 'bigger' and 'smaller'
- knowing that numbers in the counting sequence have a fixed position
- understanding the sequence of numbers, for example, that 3 comes before 4
- knowing that higher numbers reflect greater quantities, and therefore 4 is greater than 3
- knowing that each count term represents a unit increase (Griffin, 2004, p. 174).

When children enter school they have a broad range of mathematical abilities (Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & Van de Rijt, 2009), but there is significant variation between individual children in how they acquire (and how quickly they acquire) different concepts (Dowker, 1998, 2005a, 2005b, 2008; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006). If children have very different skills in mathematics, it implies that teachers will encounter a wide range of abilities in the classroom even among children beginning school.

Interactions between children and adults (such as parents and early childhood educators) prior to school can encourage significant growth in children's informal mathematics. The frequency with which parents engage in direct (e.g. teaching their child to count) and indirect (e.g. measuring while cooking) numeracy activities with their child relates positively to preschool children's mathematics achievement (LeFevre et al., 2009).

Overall, however, practices that support numeracy development are reported more infrequently than those which support literacy development (LeFevre et al., 2009). This suggests that greater understanding of how growth in mathematical thinking occurs can provide guidance on different approaches to supporting children's mathematical development prior to school.

Overview

In 2012, the Australian Council for Educational Research (ACER) began the Longitudinal Literacy and Numeracy Study: Transitions from Preschool to School (LLANS:TPS). The study is part of a program of longitudinal literacy and numeracy research at ACER that started with a seven-year longitudinal study of children's developing literacy and numeracy throughout primary school, which began in 1999 with a cohort of 1000 children from 100 schools around Australia (Meiers et al., 2006). The original Longitudinal Literacy and Numeracy Study (LLANS) developed new instruments for assessing children's literacy and numeracy understanding in the first three years of primary school and described growth in skills over the entire seven years of primary school.

Two significant findings from the LLANS research informed the current study. First, even when children start their first year at school, there is a significant range of achievement in children's literacy and numeracy skills. Second, the range of achievement between lower and higher performing students tends to increase during primary school rather than diminish.

LLANS:TPS focuses on a critical period in children's development; the transition from the year prior to school (described in this report as preschool, but also known as kindergarten) into the first and second years of primary school.

Children's experiences in the early years establish a sound foundation for learning throughout primary schooling and beyond. Children begin formal schooling with wide variation in their knowledge of and skills in literacy and numeracy. It is therefore important to focus on the understanding and experiences of children in the year prior to school, to inform understanding of the distribution of achievement among children prior to school and at school entry, and to highlight the factors involved in successful transition from preschool through to the first two years of school.

Overall, LLANS:TPS aims to:

- describe literacy and numeracy development during preschool and in the first two years of school
- increase understanding of the relationship between preschool knowledge and experiences and the development of literacy and numeracy in the first two years of school.

Purpose of the report

The purpose of this report is to describe the numeracy skills of the preschool children (at the end of the year prior to starting school) who participated in the first year of LLANS:TPS. A better understanding of the numeracy knowledge and skills of preschool children has two important outcomes for early childhood educators and early years teachers: knowing what knowledge and skills can be fostered among young children, and understanding the early numeracy foundation on which formal instruction can build.

We describe patterns of understanding of preschool children participating in LLANS:TPS in six significant areas of early numeracy:

- numbers and counting
- sharing, number comparison and ordering
- calculations
- patterns
- shapes
- measurement.

The numeracy activities segment of the report is divided into sections according to the main numeracy concept discussed. Each section describes the development and importance of each numeracy concept, provides information on the skills of LLANS:TPS children on the associated activities, and highlights some everyday activities that may help to promote children's understanding in these areas. The report also includes a further resources section that includes general resources on young children's

numeracy development, specific readings on aspects of numeracy, and practical resources for teaching and learning.

Our report provides a description of the LLANS:TPS research for parents, early childhood educators and primary school teachers, and is also designed as a practical document for educators in early learning contexts and in primary schools.

Who participated in LLANS:TPS?

In 2012, we approached a sample of Australian primary schools with an invitation to participate in the LLANS:TPS research. Schools that were approached to participate were part of a sampling frame of 10 South Australian schools, 34 Victorian schools, and 16 Western Australian schools. Up to two replacement schools were approached if the first primary school did not wish to participate in the research. Government and Catholic education providers and independent schools were represented in the sampling frame. A total of 18 schools agreed to participate in the research: nine schools from Victoria, three from South Australia, and five from Western Australia. One school in Queensland accepted an independent invitation to participate in the study.

Some schools with on-site early learning centres participated in the research from 2012. We obtained information from other schools on the early learning centres that provided the majority of enrolments into the first year of school. The directors of these early learning centres were approached independently and invited to participate in the research. Permission to conduct the research was obtained from relevant education authorities in each state prior to approaching schools and early learning centres.

In total, permission to participate in the research in 2012 was received from the parents of 235 children from 23 early learning centres. Children were interviewed at their preschools during September and October 2012 by researchers from ACER. On occasion, children were absent from preschool during the time of the research so researchers were unable to complete literacy interviews or numeracy interviews or both. As a result, complete numeracy data were available for 219 preschool children (118 females; 101 males). The average age of those children at the time of the interviews was four years and eleven months, although there was substantial variation in the ages of children in the study. The youngest child was aged four years and three months and the oldest was six years old. School commencement for all children in the study was planned for 2013.

Designing and administering the activities

The numeracy activities for children in the year prior to primary school incorporated a selection of important concepts in preschoolers' early numeracy development. A review of the academic literature in psychology and education provided guidance on significant concepts in numeracy for children in the year prior to school. We also reviewed the Australian Curriculum in Foundation year to identify significant concepts for children at the end of the first year at school, with particular emphasis on the Number and Algebra, and Measurement and Geometry content strands (ACARA, 2015).

The numeracy activities we developed for preschool children incorporated many hands-on materials (counters, straws, animal figurines), with instructions and questions that were written in simple, easy-to-understand language for preschool children. The numeracy activity booklet included complete instructions for researchers administering the tasks. Each question included a range of possible responses for researchers to choose from as the best description of each child's response. Figure 1 provides an example of questions from the numeracy activity booklet and the associated responses for three measurement activities from the numeracy interview.

Researchers brought the numeracy materials to the early learning centre, interviewed each child

individually in a quiet area of the centre and recorded each child's responses in the activity booklet. All researchers were teachers who were highly skilled at working with young children. In total, there were 58 short numeracy activities that each child completed in approximately 20 to 30 minutes. Some of the activities required the child to provide an extended response (e.g. to create a pattern with counters); many other activities required single word or yes/no responses (e.g. 'What is this number?'). Many children completed the numeracy interview in one session, but it was possible to separate the activities into several shorter sessions as required.

Numeracy activities: Numbers and counting

Background

As children learn to count, they develop skills that enable them to solve many different number problems. Counting was once thought of as a rote activity with little importance in children's mathematical development. Now, however, the importance of children's counting to their overall numeracy development is well recognised. Children's counting is often considered to be guided by five counting principles; three of which indicate how-to-count (stable order, one-to-one correspondence and cardinality) and two of which indicate what-to-count

MEASUREMENT								
Materials		Instructions and questions	Child's responses	1	2	3	4	5
Set of coloured straws	47	Place the six straws randomly in front of the child. Pick up the longest straw. You can move the straws if you want to. After the child makes a selection replace the straw in the random pile.	chooses the blue straw without any observable checking					
			chooses the blue straw by checking length against other straws or aligning all straws					
			chooses a straw other than blue, after checking length against other straws or aligning all straws					
			chooses a straw other than the blue, without any observable checking					
			says 'I don't know'					
			no attempt					
Set of coloured straws	48	Pick up the shortest straw. You can move the straws if you want to. After the child chooses a straw replace the straw in the random pile.	chooses the pink straw by checking length against other straws or aligning all straws					
			chooses the pink straw without any observable checking					
			chooses a straw other than the pink, after checking length against other straws or aligning all straws					
			chooses a straw other than the pink, without any observable checking					
			says 'I don't know'					
			no attempt					
Set of coloured straws	49	Put the straws in order from shortest to longest, starting with the shortest here. Point to a spot in front of the child. Also accept longest to shortest as a correct response.	orders straws from shortest to longest (R Y, G, W, R, B)					
			puts one or more straws out of order					
			says 'I don't know'					
			no attempt					

Figure 1 Examples of questions about measurement and child response categories from the preschool numeracy interview.

(order irrelevance and abstraction) (Gelman & Gallistel, 1978). Children's knowledge of counting and their use of counting to solve number problems changes substantially in the preschool years. For instance, reciting the count numbers is evident even among relatively young preschoolers (two-to-three years) and by four-and-a-half years most preschoolers can count accurately to 20 and beyond, although they may continue to make mistakes in the teens (Sarama & Clements, 2009).

Procedural counting (reciting the numbers correctly and in order for a given set of objects) for children nearing the end of preschool is very accurate, even for quite large sets (see for instance LeFevre et al., 2006); however, although children may count well, they may not yet understand that counting is used to determine cardinality (LeFevre et al., 2006). Cardinality is an extremely important concept in young children's numeracy development, and refers to the understanding that the last number counted is the number of objects in a collection.

Conceptual knowledge of counting tends to develop more slowly than procedural knowledge and the relationships between these aspects of mathematical knowledge are not always straightforward (LeFevre et al., 2006). More stringent tests of preschool children's ability to use counting to solve problems include asking them to provide a given number of objects, or observing how they use counting to solve simple problems (such as counting to compare sets) (Sophian, 1987). Most of the four-and-a-half year olds in Sophian's (1995) study could choose the same number of objects as a group of objects they could see; they made more errors for objects placed further away, particularly when the groups were large. In these types of activities, half of all children never used counting to determine the size of the group they had to make. Counting becomes increasingly complex across the first two years of school as a strategy for solving number problems. For instance, instead of counting all of the objects in a group, older children can 'count on' from a known number. They also begin to understand counting within groups (Bobis, Clarke, Clarke, Thomas, & Wright, 2005; Sarama & Clements, 2009).

For preschool children, counting makes most sense when it is applied to concrete materials in a familiar context; problems in symbolic form tend to be more difficult. Some preschool children may begin to recognise and write common numerals; however, this skill does not necessarily reflect understanding of the quantities they represent (Sarama & Clements, 2009). Children's understanding of the meaning of numbers might be explored by asking children to read a number and create a group of objects the same as the number they have read.

What do preschool children know about numbers and counting?

Children participating in LLANS:TPS completed simple number and counting activities using small (less than 10) and large numbers (between 10 and 20). These activities showed that most children had good understanding of small whole numbers and had mastered simple counting skills. Counting tasks more complex than simply reciting the counting words (such as using counting to solve number problems) were challenging and fewer children were successful on these tasks.

Table 1 shows children's accuracy in completing different numeracy activities in the counting, reading and writing numbers section of the numeracy interview. The activities are ordered from tasks on which children were most successful to those on which they were least successful.

Table 1 Children's accuracy in numbers and counting

Activity involving numbers and counting	Percentage correct (%)
Reads number 1	95
Reads number 4	95
Shows their age on their fingers	91
Reads number 2	91
Reads number 3	91
Reads number 5	90
Counts 5 wombats	90
Counts from 1 to 10	88
Says how old they will be on their next birthday	86
Reads number 6	83
Reads number 7	83
Points to first wombat in a line	83
Reads number 8	82
Repeats the number 5 to indicate how many	75
Knows the number just after 7	72
Reads a number and puts that number of wombats (8) in a box	71
Counts backwards from 5 to 1	67
Counts from 1 to 20	61
Counts backwards from 10 to 1	58
Reads a number and puts that number of wombats (11) in a box	58
Counts 14 wombats	56
Reads number 9	52
Repeats the number 14 to indicate how many	49
Writes legible 5	44
Knows the number just before 10	42
Starts counting from 6 to at least 15	42
Points to third wombat in a line	40
Understands that moving objects does not change their number	29
Writes legible 14	10

The activities on which children were most successful were those using small whole numbers. Approximately 90 to 95 per cent of children could identify small whole numbers (1–5), could count from 1 to 10 and could count a small group of objects (five objects). Fewer children successfully completed the activities with larger numbers or when they were required to use counting in more flexible ways. Nevertheless, many children succeeded even when the activities were more demanding. For instance, 58 per cent of preschool children in the study could read the number 11 and then put 11 wombats in a box, and 42 per cent could start counting from a given number (6) and reach at least 15. Almost half of all children (44%) wrote a legible number 5.

Many children showed evidence of partial understanding even on activities on which few children were successful. They demonstrated a range of different strategies for solving problems, which, while not strictly correct, showed evidence of reasoning about the problem in mathematical ways. For example, although only 42 per cent of children counted to at least 15 starting from the number 6, a further 17 per cent of children knew how to start counting from 6 but made a mistake in their counting before reaching 15. A further 11 per cent of children chose a different number in the counting sequence from which to start counting. Children were more successful at writing the number 5 compared with the number 14. While very few children legibly wrote the number 14 (11%), another 35 per cent made reasonable attempts including writing the number 41 and other legible numerals.

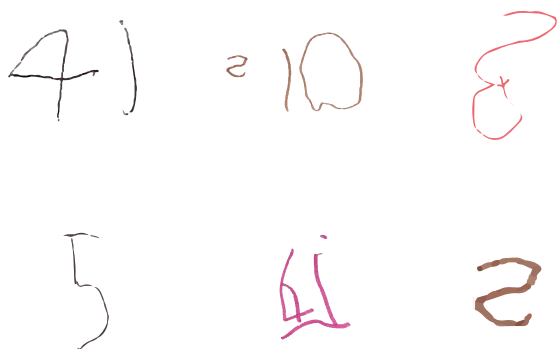


Figure 2 Children were more successful at writing the number 5 compared with the number 14

Activities to promote understanding of counting, reading and writing numbers

Many different types of activities are used in early learning contexts to develop children’s understanding of counting and to help them to recognise and produce numbers in the counting sequence from

1 to 20. Whole-number counting activities can be easily integrated into daily activities. Some examples of commonly used activities are described below.

Everyday counting

- Opportunities arise regularly to count aloud with children. For example, count the animals on the front cover of a book, the lunchboxes in a tub, or the chairs around a table. Point to each object as it is counted to model one-to-one correspondence between the object and the number word. After finishing the count, reinforce that the last number said is the same as the number of objects counted.
- Have children ‘count off’ around a circle. Each child says the number after the person next to them.

Songs and rhymes

- Children’s songs can be used to act out their numerical meaning. For example, ‘Ten in the bed’ is useful in practising counting back from 10 and for demonstrating the meaning of counting back by ones (one less).

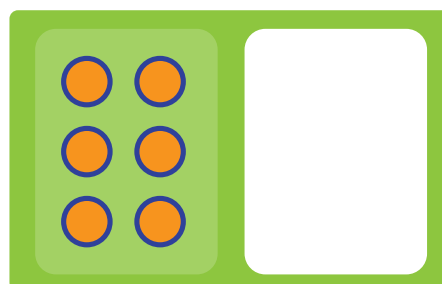
Number cards

Create a range of number cards for single- or two-digit numbers. Cards could be made in two ways:

- a numeral is written/printed on one side and the other side is left blank. The child makes a collection of objects (shells, counters, paperclips etc) to model the number.

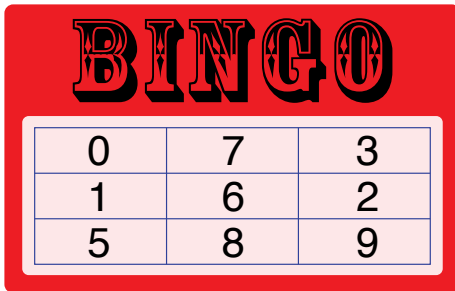


- a quantity of dots represent a number on one side of the card; the other side is blank. The child counts the dots and writes the numeral on the blank side.



Bingo

Create or purchase a class set of bingo boards like the one shown. Make each bingo board different by using different one-digit numbers in different orders. Each number should only appear once.



- From a bag of 10 number cards (digits 0–9), choose a number at random and call it out to the children.
- If a child recognises that number on their board, they may cover it with a counter.
- Children call out 'Bingo!' when three numbers in a row (horizontal, vertical or diagonal) are covered.
- The child that calls 'Bingo!' reads aloud each of the three numbers that have made the winning row.

Variations: Make boards with one- and two-digit numbers depending on the ability of the children. Or, provide the children with empty bingo boards and have them write either one- or two-digit numbers in each space.

Numeracy activities: Sharing, number comparison and ordering

Background

Developing a concept of number equivalence is particularly important for young children as they build their mathematical understanding. Building knowledge of equivalence means that children recognise that a group of three apples is numerically equivalent to a group of three oranges, because the number of objects in each group is identical (Mix, 1999).

Children's early understanding of equivalence is often investigated through using sharing and comparison activities, where children have to create two groups that have the same number, or when they have to work out whether two groups have the same number of objects in each. Children explore ideas about equal shares informally in different types of sharing activities. It has been suggested that sharing between recipients has mathematical properties (one-to-one correspondence, equivalence), that children's sharing might relate to other mathematical skills (such as

counting), but that children do not always understand the mathematics underlying sharing (Frydman & Bryant, 1988; Squire & Bryant, 2002).

Preschool children often proficiently share material between recipients (e.g. blocks between dolls) in a one-to-one manner so that equal shares result; however, they often do not understand that the sharing process creates groups of the same number. For instance, preschool children usually count to work out how many objects are in each group. Even if they know the number of objects in one group, they do not always understand that sharing one-to-one to each recipient means that each group contains the same number of objects (Frydman & Bryant, 1988; Muldoon, Lewis, & Towse, 2005).

The concept of sharing continues to be used as children enter primary school. Children's introduction to fractions often occurs in the context of equal share problems (Empson, 2001). Children as young as five years seem to be able to grasp, with a minimum of instruction, the relationship between the size of shares and the number of recipients. Sharing activities appear to be an important approach to understanding fractions (Sophian, Garyantes, & Chang, 1997).

What do preschool children know about sharing, number comparison and ordering?

Children participating in LLANS:TPS completed several activities related to sharing, number comparison and ordering. Some of these tasks were challenging and placed greater demands on children's use of counting to solve problems.

Table 2 shows children's accuracy in completing the activities in the sharing section of the numeracy interview. The activities are ordered from tasks on which children were most successful to those on which they were least successful.

Table 2 Children's accuracy in sharing, number comparison and ordering activities

Activity involving sharing, number comparison and ordering	Percentage correct (%)
Chooses the smallest number (3)	83
Shares 8 between 2 recipients	78
Chooses the largest number (9)	70
Orders the numbers 4, 6 and 7	60
Shares 13 between 2 recipients	52
Decides whether two groups have the same (8 compared with 8) using counting	39
Decides which group has more (8 compared with 9) using counting	30

Children were relatively successful at sharing an even number of counters equally between two recipients (teddies, see Figure 2). Children could successfully complete this task by distributing the counters in a one-to-one manner without understanding that this strategy produced groups of the same size. This was evident when children were asked to share an odd number of counters (13) equally between two recipients. Approximately half of the preschool children in the study allocated six counters to each teddy and left one counter aside, or they allocated six counters to one teddy and seven counters to the other teddy and asked for one more counter so as to make the groups equal. The remaining children shared the counters but the number given to each recipient was not equal.

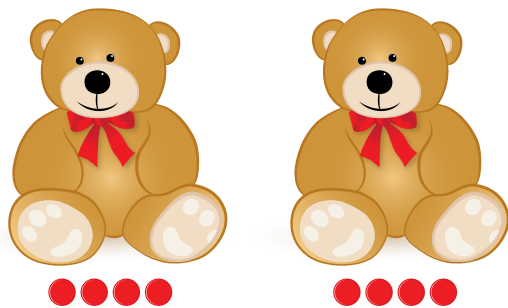


Figure 3 Sharing eight counters between two recipients

Fewer children were successful at using counting to compare the number in two groups. For instance, children were asked to determine whether the trees in Figure 3 had the same number of cockatoos. Some children's counting was not accurate and others misapplied counting by trying to count both groups as one set of objects. Many children had not developed an understanding of how to use counting to solve these kinds of problems and answered these questions with a non-numerical response; for instance, they indicated that objects in one of the groups looked bigger.

Children compared the sizes of different groups in order to select the largest and smallest number and to order groups of different sizes. Most children identified the smallest group (83%) and the majority identified the largest group (70%), but fewer children were able to order groups containing four, six and seven kangaroos in ascending order (60%).

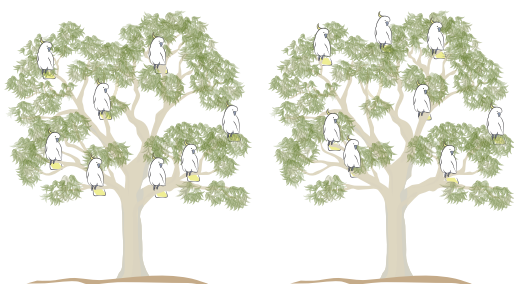


Figure 4 Comparing the number of cockatoos in two trees

Activities to promote understanding of sharing, number comparison and ordering

The following activities provide children with opportunities to share and compare equivalence in both number and measurement contexts.

Equal legs

- Have children each draw three monsters or aliens without legs.
- Provide each child with 12, 15 or 18 matches, straws or other concrete material to use as legs.
- Have the children divide the legs equally among the monsters/aliens.

Equal pouring

- Fill a jug with water and give children two or three cups.
- Have the children take turns pouring the water into the cups so there is an equal amount in each.

'More than' wins!

- Provide pairs of children with a collection of cards that have up to 10 dots on them.
- Each child gets half of the cards.
- Each child turns over a card at the same time.
- The child that turns over the card that has more dots on it collects both cards.

Numeracy activities: Calculations

Background

Children acquire many skills prior to school that establish a foundation for developing formal calculation skills such as addition and subtraction (Levine, Jordan, & Huttenlocher, 1992). The development of skills such as counting to determine the number of objects in a group, comparing the size of two numbers to work out which is larger, and understanding that moving a group of objects does not change the number of objects are important skills that contribute to understanding addition and subtraction (Levine et al., 1992). Quite young preschool children show that they understand that counting can be used to solve addition and subtraction tasks (Zur & Gelman, 2004). For instance, toddlers aged less than two years old will reach into a box the correct number of times to retrieve objects added together by a researcher.

There is evidence that preschool children can undertake very simple calculation problems with concrete materials, with contextualised calculation and symbolic problems developing later (Levine et al., 1992). There is also evidence of reasoning about addition and subtraction among younger preschoolers who are yet to develop counting skills. For instance, children aged three can reason approximately about the results of subtraction involving large quantities (Slaughter, Kamppi, & Paynter, 2006).

What do preschool children know about simple calculations?

A small number of calculation problems were included among the LLANS:TPS numeracy activities. These activities were presented in supported contexts (e.g. addition and subtraction in the context of story problems) rather than in symbolic form. These activities were expected to be particularly difficult for children of this age; yet, we also expected that a small number of children would have begun to develop skills in effective calculation using small numbers.

Table 3 shows children's accuracy in completing the activities in the calculation section of the numeracy interview. The activities are ordered from tasks on which children were most successful to those on which they were least successful.

Table 3 Children's accuracy in calculation activities

Activity involving calculation	Percentage correct (%)
Solves addition story problem ($2 + 3$)	37
Solves addition with objects ($5 + 3$)	31
Solves subtraction story problem ($7 - 5$)	29
Divides 6 into 3 equal groups	10

Approximately one third of preschool children in the study were able to solve the following simple addition and subtraction problems presented within a story context.

Imagine there are two wombats under a tree. Three more wombats come along.
How many wombats are under the tree altogether?'

Imagine there are seven wombats under a tree. Five of the wombats leave.
How many wombats are still under the tree?'

A small number of children used advanced strategies to solve the addition story problem. Fourteen children already knew the number fact $2 + 3 = 5$ and three children used a strategy of counting on from the number 2. By far the most common strategy for

children who answered this problem correctly was for them to count on their fingers or to represent the problem using the wombats and then use counting to work out the total number (55 children).

Most of the strategies used on the subtraction story problem involved children counting on their fingers or representing the problem using the wombats and then using counting to work out the total number (45 children). Only two children knew the number fact $7 - 5 = 2$, and two children counted backwards from 7 to 2 to work out the answer.

Children who were successful at working out the answer to $5 + 3$ used similar strategies. A very small number of children knew the number fact $5 + 3 = 8$ (six children) and others counted on from 5 to 8 (six children). The remaining children mostly counted to solve the problem.

Dividing a group of six wombats into three equal groups was particularly difficult. Only 10 per cent of children created three groups with two wombats in each group. Many of the remaining children (64%) created two groups with three in each group.



Figure 5 Dividing wombats into three equal groups

Activities to promote understanding of calculation

A range of simple games can be used to develop understanding of calculation. Two examples are described.

Bean bag totals

- Children work in pairs. Each pair has a hoop and each child in the pair has a selection of different coloured bean bags.
- Children each throw their bean bags at the hoop.
- Ask the children to describe what happened. 'I threw in four blue bean bags. Kate threw in three yellow bean bags. Altogether we have seven bean bags.'

Skittles

- Arrange 10 skittles and have a child roll a ball to knock as many down as possible.
- Ask the children to describe what has happened. 'We had 10 skittles. John knocked three down. There are seven left.'

Numeracy activities: Patterns

Background

An understanding of pattern and structure is essential to developing mathematical thinking. A pattern in mathematics involves some form of repeated, predictable relationship (Mulligan & Mitchelmore, 2009). Young children's ability to recognise, replicate and create patterns is also thought to be related to their mathematics achievement in school (McGarvey, 2012; Warren, Miller, & Cooper, 2012). The development of many different mathematical skills, such as counting, multiplicative reasoning, proportional reasoning, measurement and reasoning about data are reliant on an understanding of pattern and structure (Papic, 2007; Papic & Mulligan, 2005).

Working with pattern and structure is evident in all strands of mathematics from preschool through to the early years of school. In number, pattern may be explored in the context of grouping a set of 20 objects in different ways (as five repeated groups of four, or four repeated groups of five) (Mulligan & Mitchelmore, 2009).

Children's ability to identify simple patterns of the form ABABAB develops early and often occurs through noticing regularities in the everyday world (such as patterns on clothing) (Mulligan & Mitchelmore, 2009; Papic, Mulligan, & Bobis, 2009; Warren et al., 2012). These simple repeating patterns are often initially encountered in repeated patterns using geometric shapes and alternating colours and later in symbolic contexts that entail understanding repeated patterns such as 3, 6, 9 ... (Mulligan & Mitchelmore, 2009).

What do preschool children know about simple patterns?

The three problems in the patterns section of the in LLANS:TPS numeracy interview required children to copy a pattern of counters that varied in both colour and number (Figure 5), to extend the pattern for at least two repetitions, and to make a completely new pattern using the coloured counters.

Copying the pattern was relatively easy. Children could solve this problem by simply matching counters

one-to-one. Seventy-one per cent of children copied the pattern of coloured counters. Extending the pattern successfully was more difficult: 48 per cent of children extended the pattern for at least two repetitions.



Figure 6 Pattern of counters for children to copy and extend

Children found it more difficult to generate their own pattern using the counters. Nineteen per cent of children made a pattern that varied in colour and the number of counters (such as the example in Figure 7), and 21 per cent made a pattern that varied only in colour. Some children had partial success at creating their own pattern, with 11 per cent making a recognisable pattern that contained errors. The remaining children found it difficult to create their own pattern, with no obvious pattern evident in the creations of 48 per cent of children.



Figure 7 Pattern of counters varying in number and colour

Activities to promote understanding of patterns

Children's knowledge of pattern and structure can be encouraged in early learning contexts through activities and games that encourage children to develop patterns and to recognise and label patterns in their everyday environment.

Body patterns

- Use your body and movements to demonstrate a pattern. For example, tap your head, shoulders, knees, head, shoulders, knees etc.
- Have the children copy the pattern with their own body.
- Ask a child to make their own body pattern to show the class.
- Have the children copy that pattern.

Art patterns

- Use different textures, materials, paints, glitter and stickers to create patterns.

Pattern walk

- Go for a walk around the local environment and have children look for patterns. These might be visible in tiling or paving, fences etc. Take photographs and have the children identify the patterns they see.

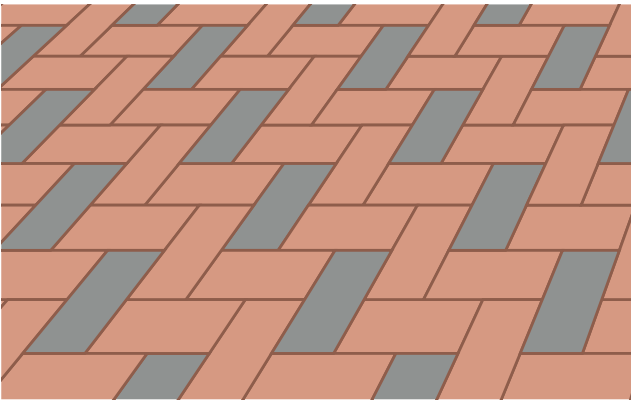
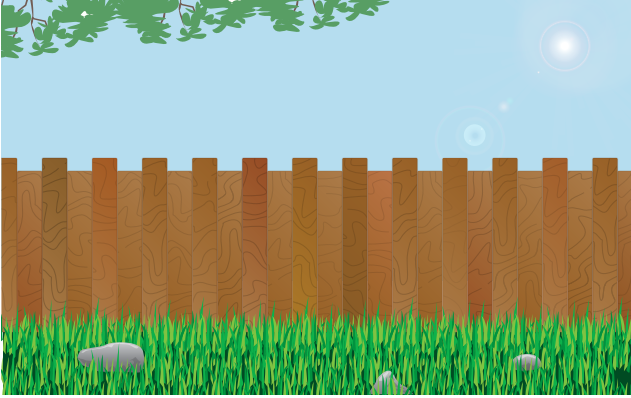


Figure 8 Examples of patterns found in an outdoor environment

Numeracy activities: Shapes

Background

Most research on children’s early numeracy has focused on whole number knowledge, with much less evidence for the development of geometric and spatial thinking (Clements, Sarama, & DiBiase, 2004). Nonetheless, there is evidence of development in spatial and geometric thinking among young children prior to school. Mental rotation (or the ability to imagine objects in different orientations) and perspective taking (or imaging movements with respect to objects) are two components of spatial thinking that researchers have investigated with preschool children (Newcombe & Frick, 2010). Children also begin to learn the relational terms that allow them to describe position (e.g. in, on, under, next to, up, down) (Clements, Sarama, & DiBiase, 2009).

Typical activities undertaken in preschool to foster spatial thinking include learning of the names of shapes (including dissimilar instances such as odd-shaped triangles) and introducing maps into the curriculum. Children tend to be fairly accurate by age four in identifying circles and squares, with rectangles and triangles more difficult (Clements, Sarama, & DiBiase, 2009; Clements, Swaminathan, Hannibal, & Sarama, 1999). However, in classifying objects on the basis of shape, children tend to be misled by aspects of shapes that are mathematically irrelevant (e.g. colour) (Clements et al., 2004). Young children also begin to develop ideas about congruence (whether two shapes are the same). In this respect, preschoolers may recognise similarity between shapes but find it difficult to explain the characteristics that define the category of shape (Clements et al., 1999).

What do preschool children know about shapes?

Almost all children in LLANS:TPS could name the shapes circle, triangle and square. Preschool children also found it relatively easy to select a different example of the shape they had named. Using the picture card in Figure 9, the children named shapes that the researcher pointed to and selected another example of the same shape.



Figure 9 Naming and identifying examples of different shapes

Table 4 shows children’s accuracy in completing the activities in the shapes section of the numeracy interview. The activities are ordered from tasks on which children were most successful to those on which they were least successful.

Table 4 Children's accuracy in shapes activities

Activity involving shapes	Percentage correct (%)
Names a triangle	99
Names a circle	97
Identifies another square	95
Identifies another triangle	93
Names a square	90
Identifies another circle	87

Activities to promote understanding of shapes

The shape hunt provides a simple activity to encourage children to become familiar with common shapes that appear in their environment.

Shape hunt

- Take children outside for a 'shape hunt'.
- Get children to look around the outdoor area for particular shapes.
- Have children draw the shapes they find.
- Take photos of some key shapes.
- When back inside, show the photographs taken and discuss what the shape in each is.

Numeracy activities: Measurement

Background

During preschool, children also show that they are reasoning about measurement concepts and will demonstrate understanding of length or area by using strategies such as overlaying or aligning objects to determine whether they have the same length or area (Bart & Yuzawa, 2007). Development in children's concepts of measurement has also been investigated in the context of sharing tasks. Initially, when preschool children are asked to divide continuous material equally (length or area) they may do so randomly or focus on the numerical equivalence of the number of pieces. Only later do preschool children begin to exhibit strategies that allow them to approximately measure the equality of shares (Miller, 1984).

Children's understanding of length and area conservation tends to develop more slowly than number conservation. Conservation is the ability

to understand which transformations affect overall quantity (e.g. spreading a number of objects apart does not affect the total number). Three-year-olds generally believe that all types of transformations affect amount; five-year-olds have better understanding of length and area conservation, but still understand number conservation better (Miller, 1989).

What do preschool children know about measurement?

Preschool children in LLANS:TPS undertook activities that relied on understanding simple measurement concepts related to length, area, volume and mass. In general, these activities were challenging for the majority of children. At the same time, many children of this age exhibited very sophisticated understanding of strategies to solve non-numerical problems (e.g., folding a straw in half to work out where to divide it to make two equal shares).

Table 5 shows children's accuracy in completing the activities in the measurement section of the numeracy interview. The activities are ordered from tasks on which children were most successful to those on which they were least successful.

Table 5 Children's accuracy in measurement activities

Activity involving measurement	Percentage correct (%)
Identifies heaviest container	81
Identifies lightest container	77
Chooses the longest straw	62
Chooses the shortest straw	46
Folds a straw in half to identify the midpoint	44
Chooses a straw the same length as a given object	41
Orders straws from shortest to longest	38
Indicates a box will fit more small objects than large objects	33
Works out how many small squares will cover a large square	31
Overlaps squares to decide which is larger	1

In many cases, children in the study were not familiar with the strategies that would help them to solve a particular problem. For example, children often appeared to choose a straw at random when they were asked to select a straw of a particular length. Length comparison strategies were understood by many children, whereas almost no children used the strategy of overlapping to work out an area

comparison problem. Another common strategy occurred when children used non-quantitative explanations for quantitative problems. For instance, 50 per cent of children explained whether more of a smaller or a larger object would fit in a box by referring to an irrelevant dimension (e.g. the object's colour). Many children gave a similar type of explanation when they compared two sets of objects and often referred to the object's size rather than the number of objects in each group.



Figure 10 Ordering straws from shortest to longest

In contrast, the majority of children understood the concepts of 'heaviest' and 'lightest' and could accurately determine which of three containers met these criteria.

Activities to promote understanding of measurement

Opportunities to measure different quantities occur in a range of everyday activities. Children can be encouraged to compare, order and divide different materials to build understanding of measurement strategies.

Caterpillars

- Have each child use playdough to make one caterpillar.
- Ask children to find someone else with a caterpillar that is the same length as, longer than or shorter than their caterpillar.

Pouring and filling

- Facilitate open-learning experiences for children by providing water tables with different sized containers, spoons etc.
- Allow children to experiment with which containers hold more or less; how many of this container will fill that container etc.

Conclusions

This report describes how preschool children develop understanding of numeracy concepts prior to school. It used the findings from the numeracy activities in the first year of the LLANS:TPS to illustrate understanding of numeracy concepts. It also provides some practical activities for promoting understanding of children's early numeracy. The overarching aim of this report is to highlight the importance of early numeracy and to enhance understanding of the range of numeracy skills that children are developing prior to school commencement.

Research such as the LLANS:TPS demonstrates that preschool children have a wide variety of numeracy skills in the year prior to school. This research, in conjunction with other research on early numeracy, highlights the numeracy concepts that are more and less difficult for children of this age. Moreover, it is clear that even in the year prior to starting school, there is significant variation in children's numeracy skills. The development of informal numeracy or number sense provides a sound foundation for learning mathematics at school. Where such knowledge and skills are lacking, children's mathematics development at school may be affected. We know that much can be done in the years prior to school, both at home and in early learning centres, to encourage the development of children's numeracy. Two important approaches to building strong foundations for school mathematics learning are to enhance the profile of early numeracy and to provide encouragement to early childhood educators in order to actively foster growth in early numeracy.

The Longitudinal Literacy and Numeracy Study: Transitions from Preschool to School has followed the children in the study into their first and second years of primary schooling. Information gathered from these numeracy interviews will allow us to describe how children's numeracy develops as they transition from early childhood contexts to primary school and to determine the relationship between preschool children's knowledge and skills and their later mathematical development.

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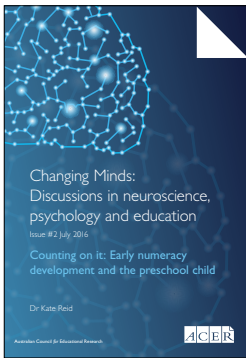
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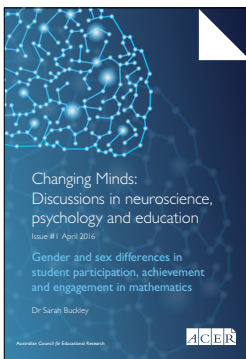
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