ANALYSING UNCERTAINTY AND DELAYS IN AIRCRAFT HEAVY MAINTENANCE

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Leandro Julian Salazar Rosales

Alliance Manchester Business School

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Abstract

Analysing Uncertainty and Delays in Aircraft Heavy Maintenance

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Leandro Julian Salazar Rosales

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This study investigates the influence of unscheduled maintenance activities on delays and disruptions during the execution of aircraft heavy maintenance services by developing a simulation model based on Systems Dynamics (SD) and supported by an Evidential Reasoning (ER) rule model.

The SD model studies the complex interrelationship between scheduled and unscheduled tasks and its impact on delays during a maintenance service execution. It was found that the uncertain nature of the unscheduled maintenance tasks hinders the planning, control and allocation of resources, increasing the chances to miss deadlines and incur in cost overruns. Utilising causal loop diagrams and SD simulation the research explored the relevance that the resource allocation management, the precise estimation of the unscheduled tasks and their prompt identification have on the maintenance check duration. The influence that delays and attitudes in the decision-making process have on project performance was also investigated.

The ER rule model investigates the uncertainty present during the execution of a maintenance check by providing a belief distribution of the expected unscheduled maintenance tasks. Through a non-parametric discretisation process, it was found that the size and array of distribution intervals play a key role in the model estimation accuracy. Additionally, a sensitivity analysis allowed the examination of the significance that the weight, reliability and dependence of the different pieces of evidence have on model performance. By analysing and combining historical data, the ER rule model provides a more realistic and accurate prediction to analyse variability and ambiguity.

This research extends SD capabilities by incorporating the ER rule for analysing system uncertainty. By using the belief distributions provided by the ER model, the SD model can simulate the variability of the process given certain pieces of evidence.

This study contributes to the existing knowledge in aircraft maintenance management by analysing, from a different perspective, the impact of uncertain unscheduled maintenance activities on delays and disruptions through an integrated approach using SD and the ER rule. Despite the fact that this research focuses on studying a particular problem in the airline industry, the findings and conclusions obtained could be used to understand and address problems embodying similar characteristics. Therefore, it can be argued that, due to the close similarities between the heavy maintenance process and complex projects, these contributions can be extended to the Project Management field.

Declaration

No portion of the work referred to in this thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

Copyright Statement

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List of abbreviations

AHP	Analytical Hierarchy Process
APM	Association for Project Management
ARP	Aircraft Routing Problem
AWL	Airworthiness Limitations
BD	Belief Distribution
BS	British Standard
CAA	Civil Aviation Authority, UK
CCM	Critical Chain Method
CLD	Causal Loop Diagram
CMR	Certification Maintenance Requirements
CPM	Critical Path Method
CY	Flight Cycles per Year
DEA	Data Envelopment Analysis
DES	Discrete Event Simulation
DOT	Direct Operating Costs
EASA	European Aviation Safety Agency
ENRR	Expected Non-Routine Rate
ER	Evidential Reasoning
ERP	Enterprise Resource Planning
FAA	Federal Aviation Association
FHY	Flight Hours per Year
GA	Genetic Algorithms
IATA	International Air Transport Association
ICAO	International Civil Aviation Organization
ILP	Integer Linear Problem
MAE	Mean Absolute Error
MAI	Mean Accuracy Indicator

MAPE	Mean Absolute Percentage Error
MCDA	Multi Criteria Decision Analysis
MILP	Mixed Integer Linear Problem
MOP	Multi Objective Problem
MPD	Maintenance Planning Document
MRBR	Maintenance Review Board Report
MRO	Maintenance, Repair and Overhaul Organisation
MRP	Material Requirement Planning
MSE	Mean Square Error
MSE_{DIST}	Mean Square Error of the Distribution of Maintenance Services
MSE_{SERV}	Mean Square Error for Each Maintenance Service
NRR	Non-Routine Rate
NRT	Non-Routine Tasks
OAMP	Operator Approved Maintenance Program
PERT	Program Evaluation Review Technique
PMI	Project Management Institute
R	Reliability
RAMP	Risk Analysis and Management of Projects
RT	Routine Tasks
SD	System Dynamics
SE	Maintenance Service Type
TC	Type Certificate
US DOT	U.S. Department of Transportation
W	Weight
WBDR	Weighted Belief Distribution with Reliability
α	Alpha-Index

Chapter 1: Introduction

The airline industry plays a key role in globalisation. It promotes economic growth and social development worldwide by improving the connection between people and goods, reducing transportation times, stimulating tourism and facilitating trade. However, in the last few decades this industry has undergone a severe crisis caused by a remarkably competitive and dynamic market which is extremely sensitive to external social, economic and political factors, affecting the ability of airlines to produce revenues and increasing their operating costs. To stay in business, airlines have been forced to enhance their operative and financial conditions by implementing different business strategies. Part of this pressure for improvement has been transmitted to the aircraft maintenance division due to its significant impact on safety, service quality and profits. In particular, these efforts have focused on improving turnaround times and reducing costs.

The maintenance of an aircraft and its components is a mandatory and strictly regulated duty to ensure the safety of an aircraft and its operations. Furthermore, it represents one of the main direct operating costs for an airline and is essential for providing high service quality. Maintenance, therefore, must be carried out at the lowest possible cost, provide the highest level of service and offer competitive delivery times, but without compromising quality and safety. To accomplish these objectives, commercial aviation maintenance is organised in a systematic and well-structured maintenance programme of scheduled tasks.

This thesis is based on an exploratory case study approach, where the initial empirical assumptions are explored and supported by an extensive literature review, considering the studies and opinions of experts and researchers in the field, and by using real airline operational and maintenance records.

This study investigates a significant and recurrent problem in the airline industry: *the delays and disruptions that occur during the execution of aircraft maintenance services*. Delays and cost overruns are mainly caused by the difficulty in managing a large number of maintenance activities and the considerable amount of limited resources required to accomplish them. Moreover, during the execution of the maintenance scheduled tasks, unexpected damage and failures are commonly discovered, which must be corrected by programming additional unplanned maintenance activities. As a result, the uncertainty of these unexpected maintenance activities triggers a complex interaction between scheduled and unscheduled maintenance tasks.

Diverse and valuable approaches have been utilised for studying the most common problems in aircraft maintenance from different perspectives. Due to its direct impact on daily operations, several researchers have focused on investigating the line maintenance process, in particular workforce allocation and the problem of disruption recovery. Regarding heavy maintenance, various studies have been made to address long-term planning of maintenance services and the short-term detailed scheduling of the maintenance tasks within this service. However, little attention has been paid to analysis of the uncertainty caused by unscheduled maintenance tasks and its effects on maintenance service completion, with most studies assuming a predetermined, provisional number of unplanned maintenance tasks.

In this research, it has been shown that delays and disruptions are not limited to heavy aircraft maintenance but are also frequently found in almost every complex project. A project is considered to be complex when it is in constant change during its execution and uncertainty is present throughout the process. To be considered complex, a project must also be carried out in a very constrained time frame and involve a large number of limited resources with several and sophisticated interrelationships, shared within the process and externally.

It has been argued that conventional project management tools alone fail to properly deal with highly dynamic, unstable and uncertain projects. Mathematical optimisation models and simulation modelling have been used as alternative-supportive approaches to address these types of problem in project management and in aircraft maintenance management. Optimisation models have been utilised for minimising delays and cost overruns, maximising resource utilisation and improving resource allocation, aiming to produce more accurate and robust project plans and schedules. Simulation has been extensively utilised to represent and study the operation and evolution of a project over time, to experiment and analyse how the project responds to certain changes or unexpected events and to design and assess scenarios that improve project performance. In particular, system dynamics (SD), which is a flexible simulation approach, has demonstrated its usefulness for analysing the structure and operation of complex and dynamic projects characterised by sophisticated interactions between elements. It has been especially used at strategic level for policy design and evaluation by providing a holistic perspective on system behaviour.

Given the main characteristics and challenges of the heavy maintenance process, SD is suggested as a suitable approach for analysing the interrelationship of scheduled and unscheduled tasks and its impact on delays and disruptions during aircraft heavy maintenance checks. SD was chosen for its holistic perspective of a system, as it is believed that it would be more relevant to focus on understanding the behaviour and the dynamic feedback structure rather than exhaustively describing the system and its elements. However, SD also has significant drawbacks, particularly its limitations when dealing with randomness and uncertainty within the system, which are core features in complex projects and consequently, are also present in heavy maintenance services. Therefore, it is necessary to support SD with other methodologies to overcome this limitation.

It is proposed to utilise the evidential reasoning (ER) rule as a complementary method to handle the uncertainty of the process, principally for its ability to analyse variability and ambiguity. The ER rule is used as a conjunctive probabilistic reasoning process for combining independent pieces of evidence, taking into account their weights and reliabilities, and is capable of working under highly or completely conflicting conditions. The ER rule is applied for building an inference model to estimate the expected number of unscheduled maintenance activities considering their relationship with several operational and maintenance variables.

Figure 1-1 illustrates the integration of SD and the ER rule. Once the problem and its main features have been defined and explained, several causal loop diagrams are built aiming to determine the main factors that can cause delays and disruption during the maintenance project, and to investigate the prevalent feedback structure within the system. The causal loop diagrams help to examine the complex interaction between the scheduled and unscheduled tasks that hinders the resource allocation, which in turn, causes disruption throughout the project. These diagrams assisted in building the SD simulation models by showing the main elements and the feedback structure of the system and also guided the development of the ER model by determining the main factors involved in the occurrence of damage and failures.

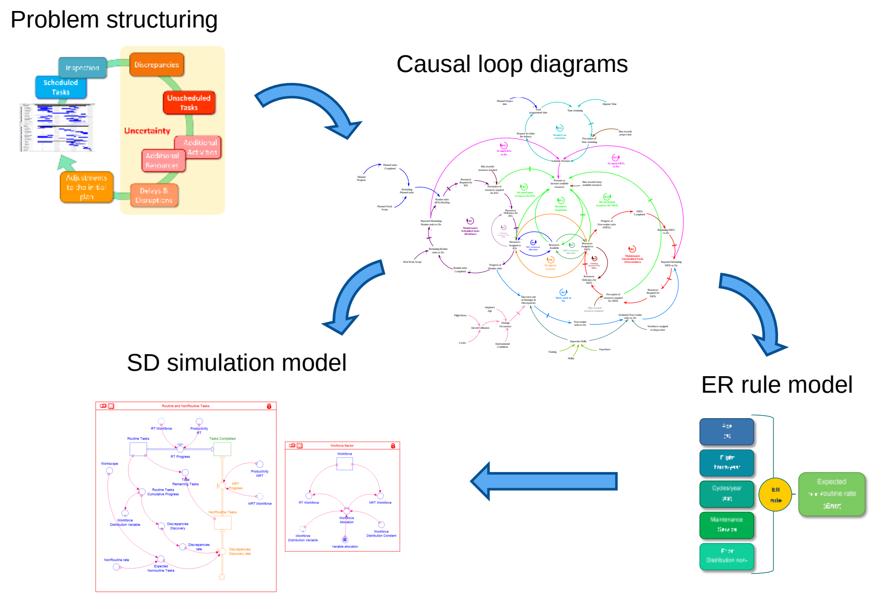


Figure 1-1 Causal loop diagrams, SD simulation model and ER rule model interaction

The SD simulation model describes and explores the impact of the occurrence and discovery of discrepancies on maintenance service duration and the effect of resource allocation on project performance. It illustrates how the management of workforce allocation becomes more difficult when unscheduled tasks begin to appear and accumulate. The model also allows the effect of different workforce allocation structures on project duration to be investigated. It shows how a large number of unscheduled tasks and the late discovery of discrepancies might cause a maintenance check to be longer than originally planned. In this way, the SD model helps to confirm the importance of defining a better estimation of the expected number of unscheduled maintenance tasks in order to improve resource allocation and accelerate the discovery of damage and failures. In addition, the SD model is used to demonstrate the influence that delays and attitudes in the decision-making process have on maintenance service duration.

The ER rule model is used to estimate the number of unscheduled maintenance tasks by combining different but complementary pieces of evidence related to the utilisation and maintenance of an airplane. The ER rule model provides a belief distribution of the expected number of unplanned maintenance tasks, given an aeroplane with a specific usage that will undergo a particular maintenance service. Instead of assuming an expected number of unscheduled activities, this belief distribution is then used in the SD model to characterise the uncertainty of the process, thus providing a more realistic perspective. The integrated SD-ER model could, therefore, be utilised as a supporting tool to experiment with and assess strategies for planning and controlling aircraft maintenance services.

This research contributes to the existing knowledge in aircraft maintenance management by examining the impact of unscheduled maintenance activities on delays and disruptions through the application of SD (utilising causal loop diagrams and SD simulation) in combination with the ER rule. SD provides a system-wide viewpoint to investigate the effect that unscheduled maintenance tasks have on maintenance check duration. The causal loop diagrams help elucidate the complex interaction between the main factors involved in delays during project execution. The SD simulation model provides a platform for exploring and analysing the impact of the occurrence and discovery of discrepancies during a maintenance check and for testing different maintenance strategies. The ER rule is used as a rigorous approach for estimating the expected number of unscheduled maintenance tasks during the execution of a maintenance check. Compared with the reviewed studies, which assume a rough estimate of the unscheduled tasks, the proposed ER model provides a more realistic prediction, producing a belief distribution of the unscheduled tasks given certain operational and maintenance conditions. It can be argued that, due to the close similarities between the heavy maintenance process and complex projects, these contributions can be extended to the Project Management field.

In addition to these contributions, it can be argued that this thesis extends capabilities of SD by incorporating the ER rule for analysing system uncertainty. By using the belief distributions provided by the ER model, the SD model can simulate the variability of the process given certain pieces of evidence. Moreover, through the several analyses carried out using the ER rule model, this research makes significant contributions to the application of the ER rule by analysing the influence that bin size, interval limits and interval arrays have on model estimation accuracy and by investigating the role that dependency, reliability and weight have on model performance.

Although this research focuses on studying a problem specific to the airline industry, its characteristics are common and occur in other complex projects across different industries. The learning and findings obtained by this research could be applied to study and explore problems with similar features, helping to understand and examine their causes and consequences. The models can be adjusted and improved to expand their applicability to the management of complex projects.

1.1 Research motivation

This research has an empirical inspiration. It was conceived during the time I worked for one of the major airlines in Mexico. Whilst I was working in the Productivity and Continuous Improvement Department in the aircraft maintenance division, I noticed that delays and disruptions during the execution of aircraft maintenance services, particularly in major maintenance checks and overhauls, were a recurrent problem with serious operational and financial implications for the airlines and maintenance shops.

Delays and cost overruns during the heavy maintenance services were not exclusive to the company I worked for. During the monitoring and analysis of maintenance checks outsourced to external companies, the problem was also observed in several airlines and maintenance shops in America, Asia and Europe, leading me to think that it was a common issue across the aviation industry.

A heavy maintenance check encompasses an exhaustive inspection and repair of the aircraft. It is generally carried out every one to two years. To perform this type of check, it is necessary to keep the aeroplane out of service for around 7 to 30 days, resulting in loss of income for the airline. During its execution, a considerable amount of maintenance tasks are performed, requiring a great number of different and limited resources. In particular, it places high demands on the workforce, which is highly skilled and costly.

Before an aircraft enters into a heavy check, a detailed plan is defined specifying all the scheduled maintenance tasks to perform and determining the required resources for accomplishing them. However, during the execution of the maintenance check, particularly during the inspection stage, damage and failures are found that need to be corrected by programming additional maintenance activities that are not considered in the initial plan. These unscheduled tasks hinder the planning and control of the whole maintenance service as they require additional resources and force adjustments to the initial plan, causing disruptions throughout the execution of the service, which in most cases results in delays at the end of the maintenance check.

A rough estimation of the expected number of unplanned tasks is incorporated in the initial plan, but most of the time this figure is based on guesses or "rules of thumb" without a valid foundation. In the best case, it is predefined by a group of experts according to their experience of the aeroplane and the maintenance check, or by considering the average behaviour of the fleet.

It was observed that despite the use of information systems for managing resources and controlling the process, once the number of unscheduled maintenance tasks starts rising beyond the point originally estimated, managing the service becomes very difficult, as one change may affect the execution of other tasks or the availability of resources for other activities. The entire maintenance process enters into a firefighting mode, focused on solving immediate issues, worsening the problem further.

A common solution that airlines and maintenance shops adopt to avoid delays is to overestimate maintenance service duration, leading to overextension of required resources. This practice causes maintenance costs to drastically increase and reduces significantly aircraft utilisation. If, however, the estimation of unscheduled maintenance tasks is very tight, this can also cause problems. It is very likely that delays and disruptions will occur during the aircraft maintenance check. This can then have a significant impact on airlines' performance by increasing inventory levels, causing a surge in overtime, increasing the incidence of errors and reworks, reducing aircraft utilisation, curtailing maintenance capacity, altering the airline itinerary and affecting service quality. This ultimately translates into increased operating costs and reduced revenues. Therefore, defining a precise number of unscheduled maintenance tasks becomes crucial in order to avoid delays and cost overruns.

Experience suggested that the problem is caused by the unpredictable nature of unscheduled tasks and the complicated planning and estimation of resources required to execute them. The problem is exacerbated during the execution of maintenance checks by the large amount of activities (scheduled and non-scheduled) to manage, the continuous changes in the plan and the large amount of resources involved in the process.

From personal experience, this seemed to be an interesting topic that was worth analysing and that it would be fruitful to explore the findings of other researchers and experts in the airline industry regarding this issue. Therefore, in chapter two the problem is contextualised and discussed further based on the evidence provided by other authors.

1.2 Research questions

Motivated by the empirical knowledge about the problem, borne out by the evidence provided by other researchers and experts, and supported by the findings in the literature review, a core exploratory research question is proposed as an alternative approach to analyse the delays and disruptions that commonly occur during aircraft heavy maintenance checks:

> *How can system dynamics in combination with the evidential reasoning rule be used to analyse the impact of uncertain unscheduled tasks on delays and disruptions during the execution of aircraft heavy maintenance services?*

The main research question is subdivided into four sub-questions that seek to investigate particular aspects of the research problem:

1) *How does the interaction between scheduled and unscheduled tasks influence resource allocation throughout the maintenance process?*
2) *How does the occurrence and discovery of damage and discrepancies affect the execution of the maintenance service?*
3) *What are the most relevant variables for estimating unscheduled maintenance tasks?*
4) *How can operational and maintenance variables be used as different pieces of evidence for estimating the expected number of unscheduled maintenance tasks?*

Attempting these questions will help to investigate the complex relationship between scheduled and unscheduled maintenance tasks, considering the large amount of resources involved during the maintenance process. It will also allow the uncertain nature of unscheduled activities to be analysed. These questions are answered by developing and integrating two different models: the system dynamics (SD) model and the evidential reasoning (ER) rule model. In general terms, the SD model analyses the complex interaction between scheduled and unscheduled tasks that hinders resource allocation during the execution of maintenance checks which might lead to delays and disruptions. The ER model aims to estimate unscheduled maintenance activities based on historical data regarding the utilisation and maintenance of an aircraft.

1.3 Research objectives

As stated in the main research question, the major objective of this study is to *investigate the influence of unscheduled activities on delays and disruptions during the execution of aircraft heavy maintenance services by developing a simulation model based on SD and supported by an ER rule model*. To achieve this core objective, a series of intermediate objectives are defined which are addressed throughout this thesis:

1. *Investigate the relevance of the proposed problem and determine if the perceived problem is real and significant.*
2. *Explain the problem and its context based on the evidence provided by other researchers and experts.*
3. *Discuss relevant theory related to the research problem.*
4. *Determine different approaches employed to analyse problems with high complexity and uncertainty.*
5. *Identify potential areas of opportunity by determining the strengths and limitations of previous works.*
6. *Formalise and justify the process that will be used to undertake this research.*
7. *Analyse the interrelationship between scheduled and unscheduled tasks and the resources required to accomplish them.*
8. *Estimate the expected number of unscheduled maintenance tasks for a particular maintenance service.*

The first two objectives are addressed in chapter 2, where the aim is to support the initial empirical assumptions about the problem, justify the practical significance of the research and describe the bases and main features of the problem. Objectives three to five are addressed by an extensive and systematic literature review, presented in chapter 3, aiming to support the theoretical significance of this study, discuss relevant works and identify potential gaps in the literature. Objective six, discussed in chapter 4, is focused on describing the methodology and research design used throughout the research. Objective seven, described in chapter 5, consists of developing a qualitative and a quantitative SD model to analyse the sophisticated interaction and dynamic complexity between the scheduled and unscheduled maintenance tasks. The last objective, presented in chapter 6, refers to the development of the ER rule model to build an inference tool to estimate the number of unscheduled maintenance tasks.

1.4 Empirical data

For building the SD and the ER rule models, both qualitative and quantitative data were used. The causal loop diagrams and the SD simulation are mainly based on qualitative information obtained through interviews and discussion sessions with experts, whereas the ER model is build using real historical operational and maintenance records of an airline.

Regarding the qualitative information, and aiming to obtain a wider point of view of the problem, nine experts in the aviation industry were consulted, all with expertise in different parts of the process, such as planning and scheduling, production control, inspection, maintenance, engineering, and supply chain. They have worked either in a strategic or tactical level, and have more than fifteen years of experience in the industry. They provided valuable comments and ideas during the interviews and the discussion meetings, which helped to develop the causal loop diagrams.

To gather the qualitative information, first, two introductory meetings were organised to approach and invite the experts to participate into the project, then a kick-off session was done to present the project and its objective. Thereafter, five interviews with some of the experts were conducted to obtain more information about the problem and its features. Later, eight feedback discussion meetings and two approval sessions were performed aiming to develop and enhance the causal loop diagrams. Finally, the SD model was evaluated through two validation sessions to compare the model behaviour with the expert’s experience about the problem.

Concerning the quantitative data, real operation and maintenance records of a commercial airline were used and a sample of ninety-one heavy maintenance services was collected, corresponding to one specific type of aeroplane from one company. The information included different key process variables, such as the aircraft age, total flight hours and total cycles, type of maintenance service, aeroplane days out of service, number of routine and non-routine tasks, and the number of man-hours required for scheduled and non-scheduled activities.

Due to the sensitivity of the information, the details of the interviews and meetings and the full values of the operational and maintenance records are not disclosed. For confidentiality reasons all sensitive data was disguised or not presented.

1.5 Thesis structure

The thesis has been organised into seven main chapters, including this introductory section, where the conception of the problem along with the motivation of the research have been explained. In this chapter, the problem and its scope were also briefly described and defined. In addition, the main objectives, the research questions and main contributions were presented and summarised.

Chapter two describes the research problem based on personal experience in the industry, supported by relevant literature. It begins by setting the background of the problem, describing the context of the airline industry, along with its relevance and main difficulties. Additionally, as the topic is highly technical, this section explains the aim, characteristics and structure of aircraft maintenance. Once the bases have been established, the problem is defined by describing the heavy maintenance process and explaining its main challenges and significant implications.

In chapter three, a literature review is performed aiming to explore relevant theory related to the topic and to discover the approaches used for studying common problems in aircraft maintenance or problems with similar characteristics of complexity and uncertainty. In this chapter, relevant topics for research are discussed, including aircraft maintenance planning and scheduling, project management, uncertainty and simulation. A consideration of potential areas of opportunity arising from gaps in the literature helps to justify and support the proposed problem and research questions.

Chapter four sets out the methodology and research design. Different ontological and epistemological paradigms are discussed, research work is positioned according to some of those paradigms and the methodological approach adopted is justified. The proposed research design is described and the two main methods utilised for analysing the problem are explained: system dynamics and the evidential reasoning rule. Based on the empirical problem and the literature, the research questions and their operationalisation are presented.

In chapter five, the SD model is developed to understand and analyse the influence that scheduled and unscheduled tasks have on maintenance service completion. The research problem is depicted in a conceptual model using several causal loop diagrams. This model is developed to describe the complex interrelationship between scheduled and unscheduled tasks that hinders the allocation of resources during the execution of a maintenance service. The conceptual model is then transformed into a quantitative SD simulation model to analyse the dynamic complexity of the system. The impact of the occurrence and discovery of damage and failures on the project completion is explored, along with different strategies of resource allocation and maintenance management attitudes. Finally, the general model behaviour is assessed based on experts' experience regarding the problem.

Chapter six comprises the development of a model based on the ER rule to estimate unscheduled maintenance activities by analysing historical data related to the usage and maintenance of an aircraft. The importance, quality and dependence of the variables used in the model are also examined. The chapter includes a sensitivity analysis to explore the influence of some features of the variables on prediction accuracy.

Finally, in chapter seven the discussions and conclusions of the research are presented. The empirical and theoretical bases of the research and its main results are summarised. The research findings and contributions are discussed and linked to the research questions. The main assumptions of the models are restated and the limitations of the research are explained. Some future research lines are suggested and closing conclusions are presented.

Chapter 2: Context and description of the problem

As was noted in the introduction chapter, considering the experience gathered during the years I worked in the airline industry, delays and disruptions were a recurrent issue during the execution of heavy maintenance checks, having a significant economic and operational effect on the airline performance. It was also observed that this problem was mainly caused by the complexity of managing the resources and the occurrence of unplanned maintenance tasks. It was also perceived that the attempted approaches were not as fruitful as expected. Hence, it is suggested to address the problem from a different and more efficient standpoint than those currently used.

Before going any further, it is necessary to determine if the observations and hypothesis are true or just based on personal perceptions, in order to justify whether the problem is real, current and relevant. For this reason, this chapter presents and describes the problem and its context from an empirical perspective and is supported by what has been discussed in literature.

In order to better understand the aircraft heavy maintenance process and to clarify the impact and the significance of this study, this chapter is structured as follows: Firstly, the problem is contextualised, describing the economic and social role of the airline industry and the challenges it has been facing since the beginning of the century. Secondly, the aim and relevance of aircraft maintenance for airlines is explained and also, due to the technical nature of the topic, some pertinent concepts are defined along with the general structure of aircraft maintenance. Finally, the heavy maintenance process and its main issues are described, presenting the research problem, its nature, relevance and implications.

2.1 Influence and challenges of the airline industry.

The aviation industry plays a key role in regional, national and international economic growth. It offers a fast, efficient and reliable method of transport for goods, workers and tourists worldwide, stimulating growth by increasing investment opportunities, enhancing economic trade by linking sellers and producers with potential markets, and providing greater alternatives for travel and tourism by reducing traveling times and improving the travel experience (International Air Transport Association (IATA), 2007; Oxford Economics, 2008).

Air transport has been a fundamental pillar of globalisation. Daily, thousands of passengers and tonnes of goods worldwide are moved by an immense and complex air communication network, positioning it as one of the most important methods of transport. The benefits of the industry, however, go far beyond transporting passengers and goods. Aviation has a close interaction with other sectors, such as commercial, industrial, tourism, energy, technology and communications, placing it as a strategic element for social development and long-term economic growth.

According to Oxford Economics (2008), in 2007 aviation generated globally US$425 billion of Gross Domestic Product (GDP), which was larger than the GDP of some G20 countries. The aviation industry contributed £52 billion towards UK GDP in 2012, 3.4% of the whole UK economy. Furthermore around 960,000 jobs, 3.3% of the UK workforce were supported by this sector, generating each employee an average of £84,000 in Gross Value Added (GVA), over 60% higher than the whole economic average across the UK (Oxford Economics, 2014). IATA (2015b) states that in 2014 over 3 billion passengers and 50 million tonnes of freight were flown worldwide across a network of almost 50,000 routes, representing a revenue of US$616 billion (passengers and cargo only, excluding ancillary). IATA (2014c) remarks on the immense economic impact of the commercial aviation, generating 58 million jobs and US$2.4 trillion on business activity.

In the same way as the aviation industry influences the global economy, air transport is also very susceptible to social, economic and political factors. Due to this sensitivity, commercial aviation worldwide has faced very difficult times, full of challenges, uncertainties and great changes. Since the beginning of the century, the industry has been severely affected by factors such as terrorism, war in the Middle East, the outbreak of contagious diseases (e.g. Severe Acute Respiratory Syndrome (SARS)), oil price fluctuations and the international economic crisis.

Despite the clear and remarkable benefits created for users and customers, the airline industry has struggled to make an adequate level of profits and therefore has failed to generate sufficient value for its equity investors. As shown in Figure 2-1, in 2008, during the global economic crisis, the revenues per passenger were almost US$220 but after expenses, taxes and debt interest, net loss per passenger was US$10.06. In 2014 airlines generated average revenues of US$220 per passenger, but net profits per passenger were only US$4.93. This figure shows the financial fragility of the industry, where a small alteration in the market or economy can erode profits (IATA (2013), figures updated based on IATA (2015b)).

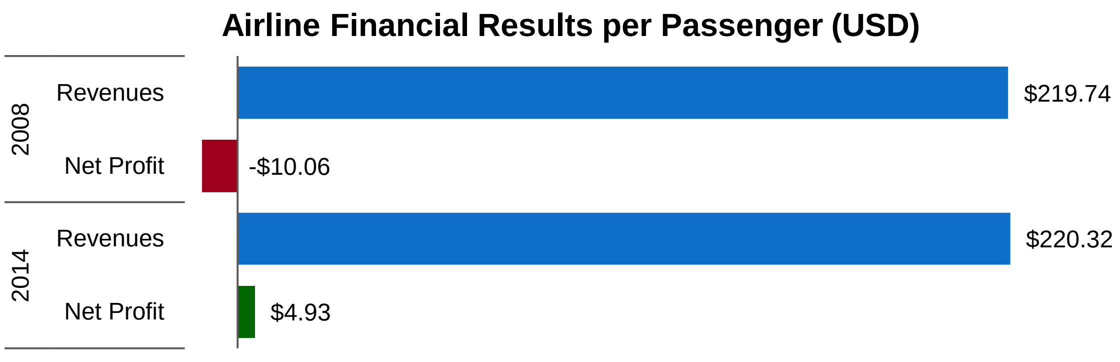


Figure 2-1 2008 vs 2014 Comparison of Average Revenues and Net Profits per Passenger (Source: IATA (2015b)).

In recent years, commercial aviation has operated in a fiercely competitive market, causing fare wars that impact upon incomes. Moreover, aviation is subject to highly variable demand, which is sensitive to economic cycles and a number of external factors. Despite having large revenue levels, the industry operates with high fixed costs causing very low profit margins, in many cases even losses. Figure 2-2 illustrates the net profit margins of the airline industry since the beginning of the 21st century, showing that poor profit margins and vulnerability to external factors characterise the industry. Doganis (2006) argues that even when the airline industry appears to be exciting, dynamic and forward-looking, it is an industry whose long-term profitability is marginal and highly cyclical. He further points out that it is an inherently unstable industry, constantly buffeted by new developments, constraints and external factors (e.g. open skies policies, global alliances, low-cost airlines, electronic commerce and privatization of state airlines).

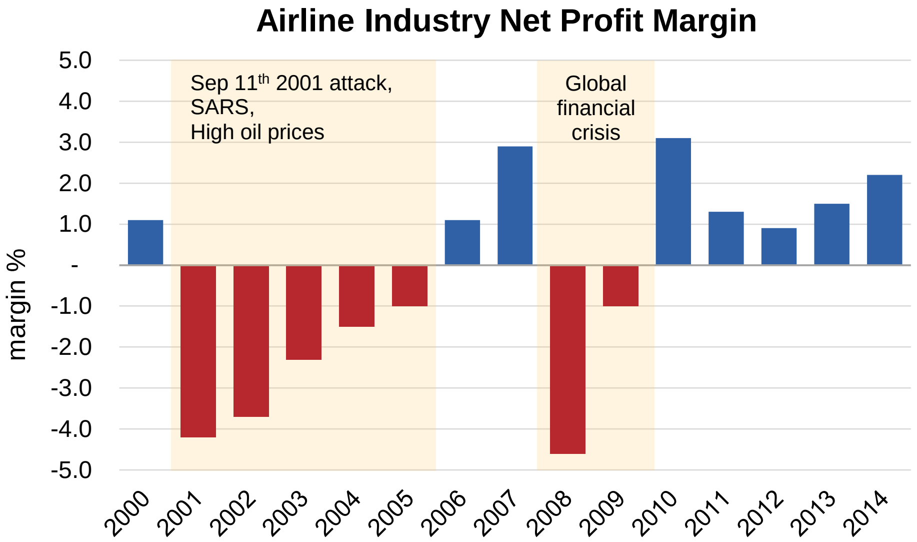


Figure 2-2 Yearly Airline Industry Net Profit Margin (Sources: IATA (2009, 2015a, 2015b)).

Developing and implementing business strategies is essential if airlines are to overcome the challenges of the industry and achieve the required efficiency and flexibility. In the hostile and uncertain environment of recent years, there has been constant pressure on airlines to reduce costs, increase revenues and improve service quality. A considerable proportion of this pressure has been transferred to the aircraft maintenance division, as this is an area of strategic importance in improving an airline's performance and has a significant financial impact. Aircraft maintenance has been studied and analysed in an attempt to find new and innovative ways to improve turnaround times and reduce costs (Thomas et al., 2008).

Aircraft maintenance is one of the cornerstones of a successful airline company. Effective, well-structured and systematic maintenance contributes to a safer, more reliable, efficient and profitable airline industry. Aircraft maintenance activities are an essential part of supplying safe and airworthy aeroplanes and are important for customer service by providing rapid turnaround times, enabling airlines to provide a continuous and reliable service that fulfils itineraries and minimises delays (Cobb, 1995; Gupta et al., 2003).

2.2 Aircraft maintenance

One of the main objectives of air transport is to provide a fast and reliable service with high safety standards. De Florio (2010) identifies man, environment and machine as the main factors involved in flight safety, describing them as three interrelated chain links, emphasising that if one of them fails, then airworthiness as a whole is compromised. Aircraft maintenance, undoubtedly, is closely linked to these three factors.

2.2.1 The aim of aircraft maintenance

Maintenance is necessary to ensure that a system or mechanism operates in good condition or to restore a failed system or faulty mechanism to an operational state (Gustavsson et al., 2014). Aubin (2004) stresses that aircraft, like almost all mechanisms invented by humans, require surveillance and continuous maintenance to ensure they can continue performing their intended function. He further points out that because aeroplanes generally operate in conditions that are inhospitable to humans and sometimes extreme, it is even more necessary to develop adequate maintenance systems that provide continuous inspection and repair.

Aircraft maintenance entails executing all the necessary activities to retain or restore the aircraft and its components to a reliable operational state in which they can perform their required design functions, with the aim of ensuring the airworthiness of the aircraft, which is its safety and operational capacity (Ayeni et al., 2011; Masmoudi and Haït, 2012). The Federal Aviation Administration (FAA) (AC-129-4A 2009) specifies that an airworthy aircraft is one that is in safe working condition and safe to fly, and fulfils its type certificate, including applicable supplemental type of certificates and airworthiness directives. International Civil Aviation Organization (ICAO) defines maintenance as: “The performance of tasks required to ensure the continuing airworthiness of an aircraft, including any one or combination of overhaul, inspection, replacement, defect rectification, and the embodiment of a modification or repair”(ICAO, 2010, pp.I–3).

2.2.2 Relevance of aircraft maintenance

Due to its crucial role in safety, aircraft maintenance is highly regulated and rigorously controlled by different aeronautical authorities (e.g. CAA, EASA, FAA, and ICAO[1]). Aircraft maintenance is, therefore, a mandatory activity for airlines since the regulations demand the aircraft operator to have a maintenance schedule programme, regardless whether it is performed internally or outsourced. This view is supported by Ward et al. (2010) who also explain that aircraft maintenance is an extremely dynamic and regulated industry characterised by complex and interdependent systems and technologies, detailed task procedures and documentation, and highly regulated management systems to ensure reliability, efficiency and safety at all times.

To guarantee airworthiness, the maintenance of an aircraft and its components must be performed under the strictest standards of quality and safety, following the requirements and procedures specified by manufacturers and in strict compliance with regulations. Furthermore, before an airline or a repair station is allowed to perform maintenance activities, they must have the relevant permits, licences and certificates for each type of maintenance service and each type

[1] CAA: Civil Aviation Authority, United Kingdom specialist aviation regulator.
EASA: European Aviation Safety Agency, European Union authority in aviation safety.
FAA: Federal Aviation Administration, United States national aviation authority.
ICAO: International Civil Aviation Organization, United Nations agency for international civil aviation.

of aircraft. (For example, the FAA Advisory Circular AC-129-4A (2009) indicates that each foreign air carrier or foreign person operating a U.S.-registered aircraft within or outside the United States must ensure that each aircraft is maintained in accordance with a program approved by the FAA). If any of the requirements mentioned above are not accomplished, the airline (or the repair station) is liable to incur large fines or to lose its licence temporarily or permanently.

In addition to its importance for the safety of the operations and its regulatory obligations, aircraft maintenance makes a substantial contribution to an airline's operating costs, representing one of the most significant Direct Operating Costs (DOC). Several authors have remarked about the influence of maintenance on airline finances, stressing that maintenance costs can fluctuate from ten to twenty percent of total operating costs depending on the type of aircraft (model, role and technology), fleet size, age and the type of operation for which the aircraft is used (short or long flights) (Friend, 1992; Kumar, 1999; Al-Garni et al., 2009; Doganis, 2009; Papakostas et al., 2010; IATA, 2014a). To highlight the relevance of maintenance costs, Figure 2-3 shows a typical airline's DOC distribution. It can be seen that maintenance plays a key role in the operating cost structure.

It is essential to emphasise that the DOC distribution can differ depending on the type of airline (traditional or low-cost) and its financial and accounting policies. Additionally, DOC can vary significantly over time, for example, the role that fuel plays in relation to costs depends closely on oil prices, thus, when oil price increases the impact of fuel on costs increases as well (IATA, 2015c; U.S. Department of Transportation (US DOT), 2015). The region where the airline operates is another factor to consider, as airport charges and taxes vary between countries. Figure 2-3 illustrates the importance and impact of maintenance for an airline's operating costs.

Maintenance costs include routine maintenance and maintenance services performed between flights and overnight (in the industry commonly known as Line maintenance), and more extensive scheduled overhauls and major checks (known as Heavy maintenance), and the maintenance of every aircraft's components, including the engines. Aircraft maintenance costs comprise three main elements: 1) the expenses of labour and staff involved directly and indirectly in maintenance activities (which represents around 18% of the maintenance costs); 2) the expenses related to the utilization of materials and spare parts for the aircraft and its components (around 17% of the maintenance costs); and 3) the cost of subcontracting maintenance to other companies (around 65% of the maintenance expenses) (Doganis, 2009; IATA, 2014b).

Wagner & Fricke (2006) remark that within the competitive market of today's aviation industry, reducing maintenance costs is an essential strategy for airlines as well as for Maintenance, Repair and Overhaul companies (known as MROs). However, as pointed out by Samaranayake & Kiridena (2012) and Wu et al. (2004), cutting maintenance costs is not an easy task, as managing aircraft maintenance requires achieving the different, and sometimes conflicting, goals of diminishing costs and improving turnaround times without affecting quality. Furthermore, according to Cobb (1995), in the aircraft maintenance sector there is an increased demand for high quality work but with low cost service, emphasising that controlling costs is fundamental, either if the maintenance is performed in-house or outsourced.

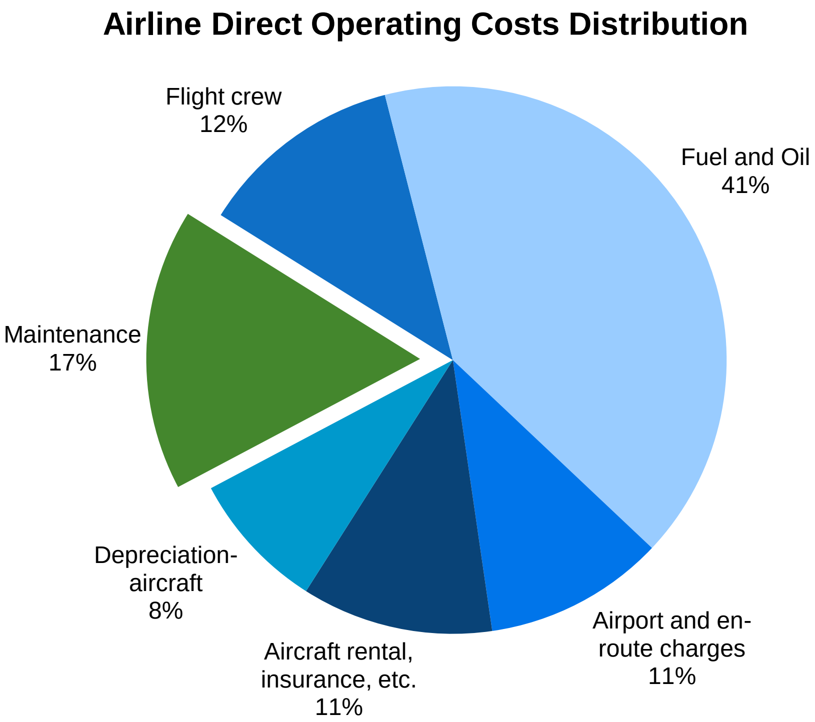


Figure 2-3 Typical Direct Operating Costs Distribution - scheduled airlines ICAO member states 2007 (Source: Doganis (2009)).

It can be summarised that aircraft maintenance is an unavoidable, compulsory and strictly regulated activity to ensure the safety of an aircraft and its operations. Moreover, it also has a significant impact on an airline's operating costs. Furthermore, due to its effect on turnaround times, it requires careful planning and coordination of all resources (tools, equipment, parts, materials, trained workforce) and the scheduling and execution of tasks in the correct sequence. For these reasons, aircraft maintenance management is a fundamental strategy for airlines and MROs, whose basic aim is to perform the maintenance at the lowest cost possible, providing the highest level of service and offering competitive delivery times but without compromising quality and safety.

2.2.3 Structure of aircraft maintenance

Aircraft maintenance needs to be planned, performed and controlled according to prescribed procedures and standards, accomplishing very specific and strict requirements, aiming to maximise system effectiveness at the lowest cost (Gupta et al., 2003; Candell et al., 2009). Radnoti (2002) explains that aircraft maintenance is structured as a systematic and scheduled programme that is jointly approved by the aeronautical authorities and manufacturers of aircraft and components. The maintenance programme clearly specifies how, and when, each individual scheduled maintenance task must be carried out. The execution of this programme prevents deterioration of, and damage to, an aircraft and its systems, preserving the standard levels of reliability and ensuring aircraft safety (FAA, 2003; Al-Garni et al., 2009).

Duffuaa & Andijani (1999) point out that a maintenance programme consists of three main elements: 1) aircraft inspections which include recurring routine inspections, minor checks and tests; 2) scheduled maintenance that comprises systems servicing, replacement of life-limited parts, periodic overhauls and special inspections; and 3) unscheduled maintenance, aiming to solve unexpected failures, generally discovered by inspections, pilot reports and failure analysis.

Figure 2-4 illustrates the process of aircraft maintenance programme development. During the certification process of a newly developed aircraft model (Type Certificate (TC)), Certification Maintenance Requirements (CMR) and Airworthiness Limitation (AWL) documents are issued to ensure that an aircraft maintains the approved design and meets the defined safety standards. The Maintenance Review Board Report (MRBR), drawn up collaboratively by manufacturers and aviation authorities and in consultation with airlines and MROs, outlines the initial minimum scheduled maintenance requirements for the airframe, engines, and systems of a specific aircraft type (Ahmadi et al., 2010; Ward et al., 2010). The maintenance requirements are designed to consider the impact of operation and aging on the occurrence of damage and on the likelihood of failures, establishing specific limits.

Subsequently, the manufacturer develops the Maintenance Planning Document (MPD) based on all TC and MRBR requirements as a framework for each operator to design and develop its individual maintenance programme (Ward et al., 2010; Masmoudi and Haït, 2012). The MPD comprises all the suggested maintenance tasks for a specific aircraft design.

Following the procedures and limits established in the MPD, the operator rearranges and customises the maintenance tasks according to its operation and maintenance policies, environmental conditions and available facilities, meeting all legal requirements. The airline also includes additional requirements requested by the aeronautical authorities (service letters, service bulletins and airworthiness directives) and activities intended to improve aircraft performance or airline service quality. This customised programme is then reviewed and approved by the corresponding aeronautical authorities and is called Operator Approved Maintenance Program (OAMP).

Once the OAMP is accepted, it is divided and organised into grouped individual maintenance tasks with similar intervals and integrated scheduled work packages or checks. These checks are programmed according to frequency, complexity and the time-on-ground required by the aeroplane. To facilitate and simplify the maintenance management and to guarantee a continuous execution of scheduled maintenance tasks, the scheduled work packages are ordered from the smallest, easiest, and most frequently performed tasks, to the most in depth tasks that are performed with less frequency, such as a major structural inspection of the aircraft (Radnoti, 2002; Sriram and Haghani, 2003; Al-Garni et al., 2009).

The scheduled maintenance tasks, and thus maintenance packages, are commonly programmed and controlled using three different periodicity measurements: 1) flight hours, 2) take-off and landings cycles, and 3) calendar time. Sriram & Haghani (2003) remark on the significance of these measurements, as they help to structure and organise the maintenance checks by specifying the frequency and maximum limits in which the checks must be performed. Al-Garni et al. (2009) explain that manufacturers generally define task limits in flight hours or cycles, but for convenience and to facilitate scheduling, operators transform them into calendar time intervals based on aircraft average daily usage.

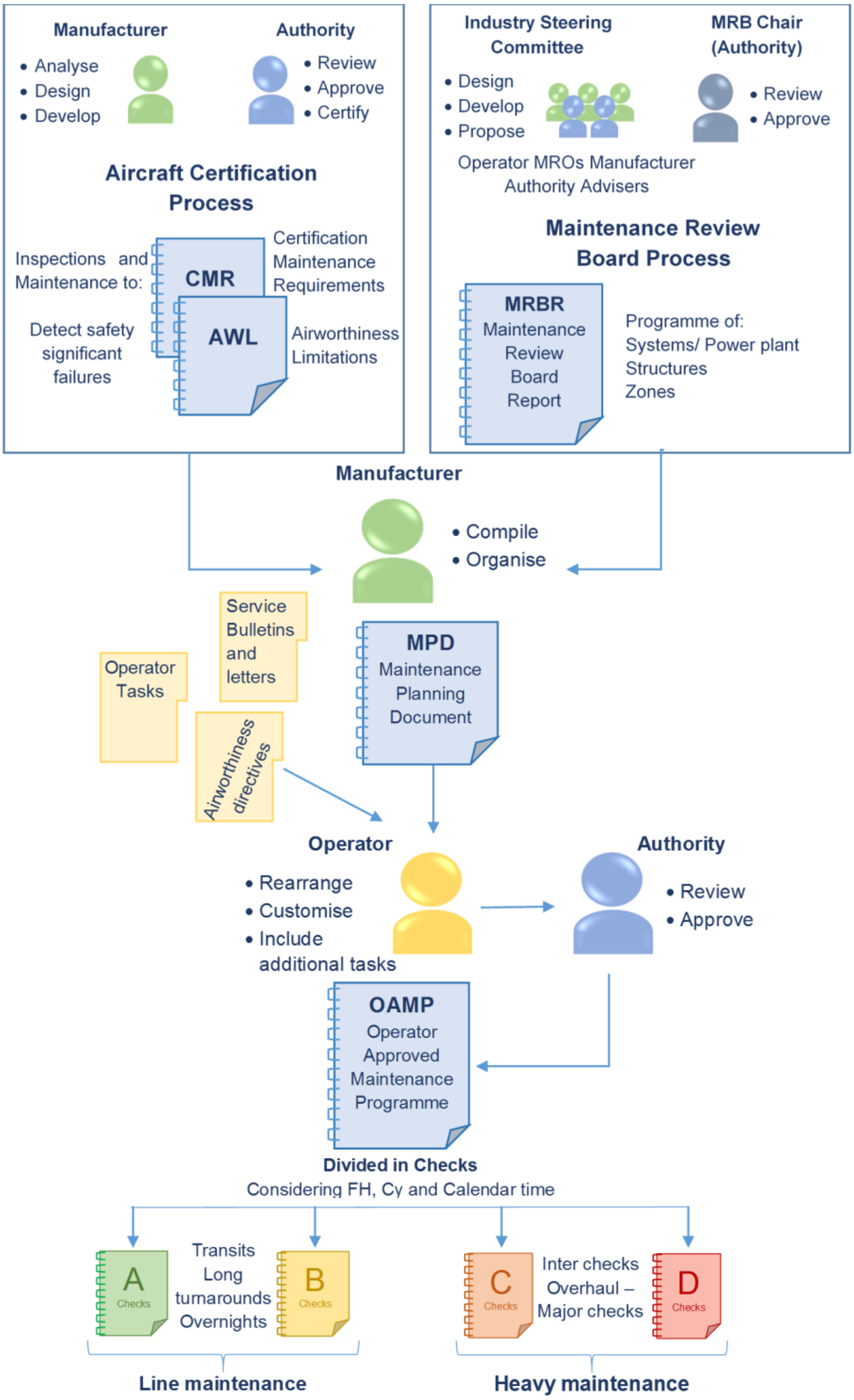


Figure 2-4 Aircraft maintenance programme development

2.2.3.1 Aircraft maintenance services

Based on the limits described above, scheduled maintenance services are generally categorized into four levels, termed A, B, C and D checks, ranging from visual inspections of essential aircraft systems continuously extending to exhaustive overhauling actions. A and B are light checks, and are commonly considered part of line maintenance; whereas C and D services are more thorough and known as heavy maintenance (FAA, 2010; Ward et al., 2010; Ayeni et al., 2011; Reiners et al., 2012).

Line maintenance involves light and frequent routine checks and inspections, troubleshooting, defect rectification, minor repairs and component replacement to ensure that the aircraft is suitable for flight. Line maintenance comprises two main groups of checks: layover or ramp checks and short or light checks. Layover checks consist of three types of job: pre-flight checks, transit checks, and daily checks. These are performed at short intervals (generally from 125 to 500 flight hours), usually at the airport gates in the transit times while the aeroplane is waiting for a flight, and can take from 30 minutes up to 6 hours. Light checks encompass A and B checks and, compared with layover checks, are executed less frequently (around 800 to 3,000 flight hours) and need more time to be completed (from 1 to 3 days), as they require the accomplishment of more scheduled tasks. During these checks, the aircraft needs to stay on the parking ramp while the maintenance is carried out, typically in long turnarounds and during overnights (Friend, 1992; FAA, 2010; Yan et al., 2004; Ayeni et al., 2011).

Heavy maintenance embraces the most sophisticated and exhaustive scheduled packages. It involves a detailed inspection and, if required, repair of the airframe, components and accessories that may entail the removal and disassembly of major components. Level C and D maintenance checks are commonly classified as heavy maintenance and are usually carried out about once every one to four years. Due to the high workload (e.g. a major check may have as many as 5,000–10,000 task cards), these types of checks require taking the aircraft out of service for large periods of time (from seven days up to 40 days), and must be performed inside a hangar using specialised equipment. They also require a considerable number of highly trained personnel. Heavy maintenance checks should be scheduled to maximise not only the use of aircraft but also maintenance facilities and resources, especially the workforce (Friend, 1992; Sriram and Haghani, 2003; Yang et al., 2003; FAA, 2010; Ward et al., 2010; Ayeni et al., 2011).

It is noteworthy that the intervals and limits of the maintenance checks described above are not fixed and should be considered only as examples, as they may vary from operator to operator depending on different factors such as aeroplane type, age and usage, fleet size, type of operations and maintenance policies. However, the concepts explained above are useful in understanding how commercial aviation maintenance is structured and organised to ensure safety, maximise the use of resources, reduce costs and improve service quality.

Based on Aubin (2004, p.11), Figure 2-5 clearly depicts the interaction between line maintenance and heavy maintenance and the key elements for an effective aircraft maintenance. It can be observed that between flights, turnarounds and overnights, visual inspections and minor routine checks are constantly carried out. However, at specific periods of time, it is necessary to keep the aeroplane-on-ground to perform thorough inspections, extensive checks and overhauls of the aircraft and its components. It is worth noting that to support the maintenance programme and to execute aircraft maintenance tasks, the appropriate facilities, equipment and tools, spare parts and materials, technical documentation and manuals and a trained workforce are all required.

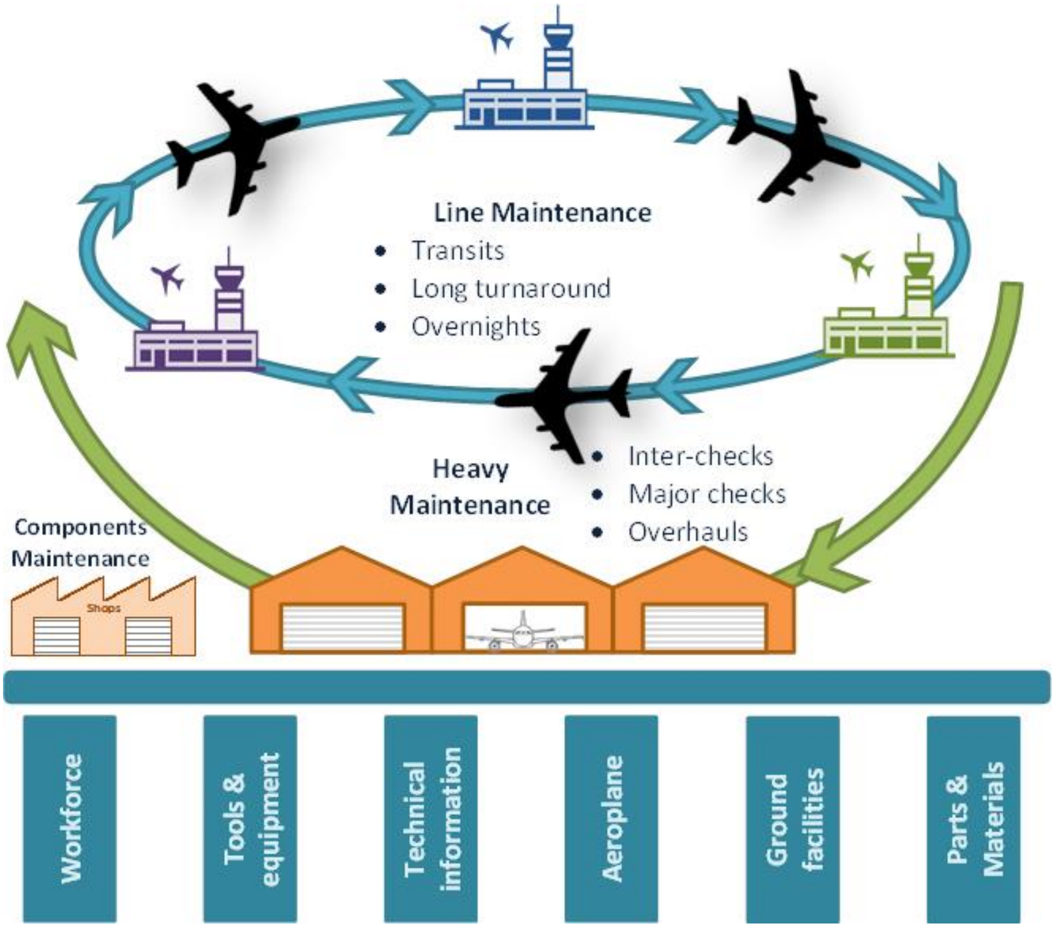


Figure 2-5 Line and heavy maintenance, and the elements required for an effective maintenance process. Based on Aubin (2004, p.11).

2.3 Aircraft heavy maintenance process

Samaranayake and Kiridena (2012) stress the importance of heavy maintenance, pointing out that these are the most challenging of maintenance tasks to undertake due to the magnitude, complexity and sophistication of an aircraft major overhaul.

For airlines and MROs, heavy maintenance is highly significant, due to its operational and financial influence (Figure 2-6). From the operational standpoint, it has a relevant impact on aircraft availability and also demands extensive use of resources. From the financial perspective, it has important repercussions for both costs and revenue generation. It can be argued that the difficulty and complexity of managing heavy maintenance services has two main elements that are closely interrelated, firstly the complexity and the huge amount of resources involved in the process; and secondly, the uncertainty of unscheduled tasks during the maintenance service.

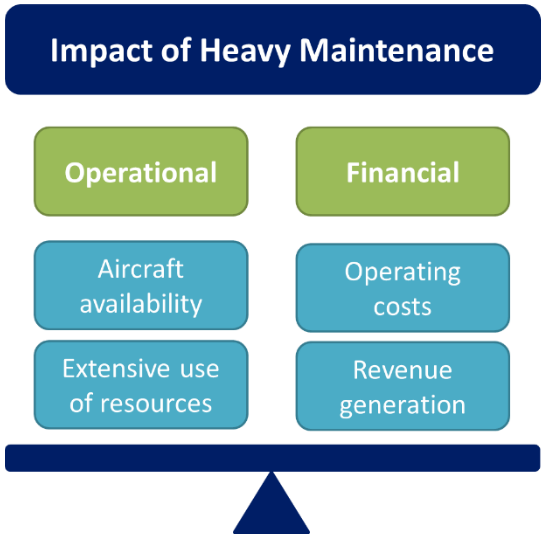


Figure 2-6 Significance of Heavy maintenance for airlines and MROs

2.3.1 Managing resources in heavy maintenance

During heavy maintenance, several resources are in constant interaction: aircraft, ground facilities, tools and equipment, parts and materials, technical information and workforce. Latorella and Drury (1992) identify operators, equipment, documentation and tasks as the main interacting elements in aircraft maintenance systems and point out that these elements interact over time with one another and with a number of external factors. Latorella and Prabhu (2000) further explain that aircraft maintenance is a complex system, where personnel execute different tasks with significant time constraints, little feedback and environmental and working conditions that are sometimes tough. Figure 2-7 depicts the relationships and interdependence between the different resources involved in major checks.

Figure 2-7 Interrelationship between heavy maintenance resources.

Heavy checks require keeping the aircraft-on-ground for large periods, generally from seven to 40 days depending on the check type, the aeroplane's age, and utilisation. During this time, the inoperative aeroplane is not producing incomes for the airline but is still generating costs. Quan et al. (2007) and Samaranayake & Kiridena (2012) remark that as a maintenance check is considered as downtime, it is imperative to follow the service plan to meet the targets and optimise aircraft turnaround time.

In addition, in heavy maintenance checks, a dedicated hangar space and all the necessary ground facilities and equipment (scaffolding, aerial and ground platforms, power generators, electrical and pneumatic supplies, etc.) are required. Moreover, the approved test equipment and tools must be available to execute the maintenance activities. The planning of all these elements must be done months or even years in advance, as their construction or supply might take a long time. Significant investment is needed to ensure the availability of these resources.

Supporting technical information for maintenance tasks (task packages, maintenance manuals, etc.) must be available and ready for consultation whenever required. Given its key role in safety and quality, it is important to have rigorous control to ensure that the correct, applicable and updated documentation is being used. In recent years, thanks to the use of technologies such as the Internet, Wi-Fi and portable electronic devices (tablets and laptops) the utilisation and management of the technical information has improved considerably.

A substantial amount of parts and materials is used in heavy maintenance, meaning that their planning, replenishment and supply have to be managed cautiously. For safety purposes, the parts and components removed, replaced and installed must be monitored, as their traceability is required by the authority. For this reason, their planning must be carried out with enough time, as the lead time for some components and parts could be high. The supply frame should ensure the availability of parts and materials without affecting the costs.

Finally, the workforce is extensively used during heavy checks due to the considerable volume of maintenance tasks to execute. For this reason, in order to reduce aircraft out of service time, it is common practice to have two or three labour shifts. Workforce management is essential during major maintenance. In the short-term, the scheduling of workforce is critical to avoid labour shortages or excesses. In the long-term it is important to plan the availability of technical labour which is licensed, highly skilled and trained, as it takes time and considerable investment to prepare such exceptionally specialised resources. Quan et al. (2007) stress that labour costs can be high, as aircraft maintenance involves intensive and highly skilled labour.

As a consequence of the massive amount of different resources involved in the execution of heavy maintenance and, more importantly, because of the complex interrelationship between them, the planning, supply and coordination of all resources must be carefully managed. A shortfall in availability of one resource can affect the management of the others and subsequently affect the whole maintenance service plan. Masmoudi and Haït (2012) remark that maintenance planning is essential to minimise aircraft downtime whilst maintaining good productivity and inventory costs, as it manages the execution of maintenance tasks, the organisation of labour and equipment and the provision of spare parts.

2.3.2 Scheduling and planning heavy maintenance checks

To successfully execute a heavy maintenance service and to guarantee safety without overrunning cost and downtime, tasks must be rigorously managed both in the long and in the short-term.

In the long-term, based on airline fleet and utilisation and considering the flight schedule, generally, a five-year service scheduling projection of the expected heavy maintenance services is created, considering the type of maintenance check to be performed, the required aircraft out of service days and the estimated man-hours (Samaranayake and Kiridena, 2012). The long-term plan is relevant because it helps to plan and supply the required resources, particularly those which are not immediately available and whose provision is significantly larger, for instance, hangar capacity and other ground facilities, the supply of special tools and parts with larger lead times and the hiring and training of technical labour.

Once the long-term plan has been drawn up, it is necessary to design a detailed short-term plan for each major maintenance service, where all the scheduled tasks (also known as routine tasks), resources and man-hours required in the service are defined and estimated to meet the time and costs established in the long-term plan. Generally, a heavy maintenance check is performed following several stages that include opening access, cleaning, inspection, programmed tasks, non-routine tasks, tests and closing access.

2.3.2.1 Scheduled maintenance tasks

Scheduled maintenance tasks can be planned precisely because they, and their requirements, are clearly described in the maintenance programme. Thus, in the definition of the short-term plan it is possible to forecast and program the resources involved and each of the activities that must be executed. In this regard, Friend (1992) explains that scheduled work is predictable and regular, similar to the production industry, and therefore can be programmed with a fair degree of accuracy. However, he further points out that even when it seems easy to plan routine activities, their planning may require assuming and estimating aircraft usage and mandatory maintenance intervals far ahead of the maintenance check that commonly leads to a sub-utilisation of the aircraft and the other resources.

2.3.2.2 Unscheduled maintenance tasks

Even within a rigorously scheduled maintenance system, unscheduled and unplanned activities, commonly known as non-routine tasks, arise during the operation of aircraft. As Friend (1992) asserts, in aircraft, as in every machine, unexpected behaviour outside specification commonly occurs but is difficult to forecast. Supporting this argument, Resto (2005) stresses that even when it is commonly believed that aircraft maintenance occurs on a regular and scheduled basis, in reality around forty to sixty per cent of all maintenance activities are unplanned and unscheduled events, generally stemming from failures or breakages where it was unknown that they would occur. In the same vein, Samaranayake and Kiridena (2012) remark that one half of the overall maintenance workload within heavy maintenance originates from unplanned maintenance activities arising out of inspections carried out during an aircraft lay-up.

Compared with routine tasks, non-routine activities can be more complex to plan, as they originate from stochastic events, hence their requirements are not easy to foresee and, therefore, difficult to forecast and program. Although the possible occurrence of unscheduled activities is considered and roughly estimated in the initial plan, generally the necessary resources to accomplish them are not immediately available. Thus, trying to prepare for all possible but uncertain non-routine tasks would be inefficient and unaffordable.

Once an undetected failure or damage occurs, it usually increases over time and spreads, producing more noticeable failure signs, leading to its detection and correction by programing a non-routine task (Fard and Melachrinoudis, 1991). Non-routine activities usually originate in four different ways. The first arises during the execution of scheduled inspections and checks after maintenance personnel find damage and discrepancies that must be corrected. The second source is aircraft logbooks, where pilots and cabin crew report abnormalities and malfunctions during the flight. The third is failure analysis, where the non-routines are generated after finding evidence of deterioration or damage during continuous monitoring and analysis of aircraft systems. Finally, non-routine tasks are required to repair damage to aeroplanes caused by external events such as hail damage, lightning strikes, bird strikes, hard landings, damage from ground equipment handlers, etc. (Duffuaa and Andijani, 1999; Resto, 2005).

As illustrated in Figure 2-8, during heavy maintenance execution, mainly during the inspection stage, unexpected damage, failures and discrepancies are discovered and need to be corrected by programming additional unscheduled tasks. These non-routine tasks might require programming supplementary activities and allocating additional resources, forcing adjustments and changes to the initial maintenance plan, causing delays and disruptions within the whole process.

Unscheduled maintenance is the most undesired event for any aircraft operator, as it tends to be more costly than scheduled maintenance. Additionally, it has significant repercussions for the spare parts supply chain and inventory management, resource planning and allocation, and execution and control of maintenance. Furthermore, these unplanned activities can lead to delays and cancellations and can have a significant impact on an aircraft's return time, cost and customer service. Therefore, for airline operators and MROs, managing these activities becomes a crucial aspect of the heavy maintenance process (Kumar, 1999; Resto, 2005; Al-Garni et al., 2009; Papakostas et al., 2010; Samaranayake and Kiridena, 2012).

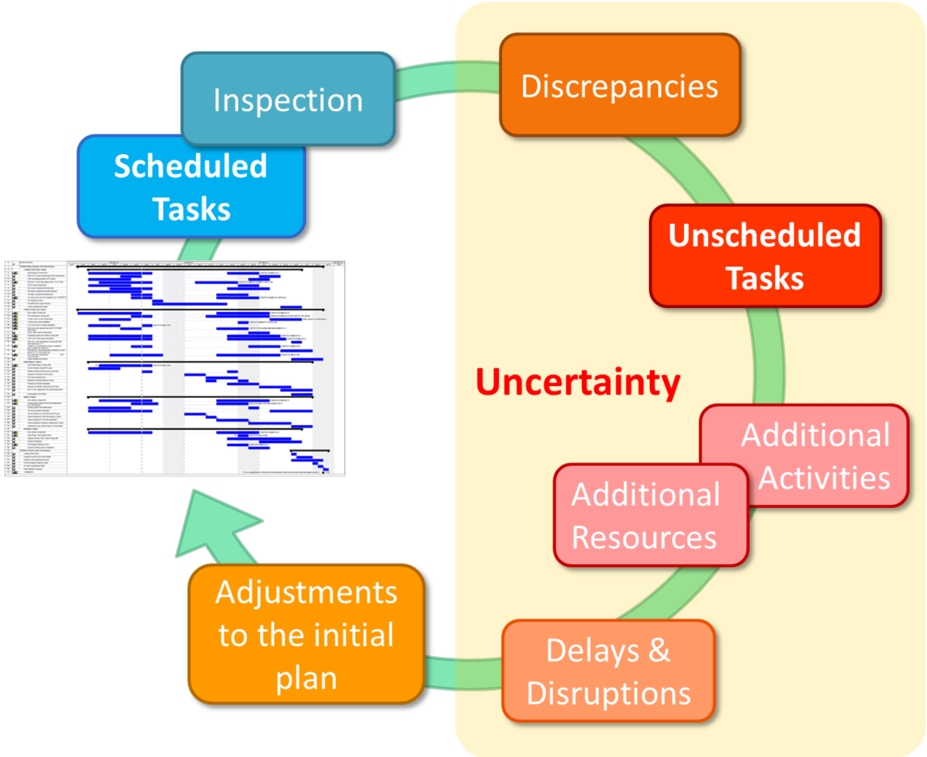


Figure 2-8 Delays and disruptions in the heavy maintenance process

Even when heavy maintenance services can be planned and scheduled by using experience and statistical data and by taking advantage of informatics tools, there is a huge uncertainty caused by the stochastic nature of unscheduled tasks. Nowadays, it is still common to see delays and disruptions during major checks, so that operators and MROs are still searching for better ways to predict non-scheduled tasks and reduce their negative effects over the scheduled plan. Papakostas et al. (2010) and Samaranayake and Kiridena (2012) support this argument, explaining that despite the progress accomplished in recent years through more sophisticated Enterprise Resource Planning (ERP) software solutions and maintenance information and decision support systems, these tools still have limitations and do not provide the necessary support for scheduling and managing uncertain and complex unscheduled maintenance tasks.

2.3.3 Heavy maintenance challenges

Srinivasan et al. (2007) identify five main challenges that are commonly faced during the heavy maintenance process. The first stems from the inherent complexity of aircraft major repair and overhaul which arises from the large number of maintenance tasks to perform, especially those due to unexpected and unpredictable damage that might require additional activities and resources. The second challenge lies in the limited amount of resources (i.e. workforce, facilities, tools and parts) available to perform the maintenance tasks, which may have a cascading effect that occurs when problems in one service start affecting other aircraft checks that share resources, similar to the problems described by Yaghootkar and Gil (2012). The third challenge is the complexity of synchronising and managing the several resources required for each maintenance task once the available resources are allocated. The fourth is the coordination and agreement of objectives and priorities between departments in an uncertain and competitive environment and the fifth arises from the execution, coordination and control of the myriad tasks that must be executed according to the plan.

The complexity of heavy maintenance might cause delays and disruptions, which may lead to serious operational, technical and economic consequences. Operationally, if the delay at the end of the service is considerable, i.e. the duration of the heavy maintenance check is longer than planned, it will alter flying schedules, delaying or, even worse, cancelling flights. Additionally, as heavy maintenance services are programmed one after another, delays can affect subsequent maintenance checks. If the problem is recurrent, it will cause a domino effect, affecting both short and long-term maintenance plans. Delays in the heavy maintenance process may have an economic impact for an airline by increasing costs or by reducing revenue. With regard to costs, for example, the pressure to reduce delays forces management to increase available workforce (hiring, boosting overtime or transferring personnel from other areas), and to purchase parts and tools urgently (paying higher prices than regular ones). Regarding revenue, if an aircraft stays on ground for longer than expected, it will affect the commercial itinerary and hence lower the number of available seats, which ultimately leads to a reduction of income-earning capacity.

The main features and challenges of heavy maintenance in the aircraft industry are shared with large and complex projects in other industries, such as construction. Both operate with high levels of uncertainty and overrunning costs and lead times occur commonly. Both require a large workforce and supplying and managing parts and materials is complicated. Masmoudi and Haït (2012) argue that heavy maintenance can be understood as a multi-project with numerous uncertainties, involving various but limited resources that must be shared either within the process or externally (i.e. a project with high variability and high dependency). Williams (2003) describes a complex project as a process in constant change during its execution, where several interdependent internal and external factors interact, many different resources are involved and uncertainty is present throughout the project.

2.4 Summary

This chapter aimed to support the empirical conception of the research by explaining and describing the problem and its context, taking into account both personal experience in the industry and the evidence provided by authors and experts in the field. The following paragraphs recap the main ideas explained in this section.

The airline industry contributes significantly to globalisation. It enhances the connection of people and goods, reduces transportation times, promotes tourism and facilitates trade and has a key role in economic growth and social development worldwide. However, it is an industry operating within a very competitive and dynamic market and is highly sensitive to external factors. It is characterised by large revenues but low-profit margin levels. For this reason, airlines have been forced to reduce costs and increase revenues without compromising service quality. Part of this pressure has been transferred to the aircraft maintenance division due to its importance for airlines.

The maintenance of an aircraft and its components is one of the main direct operating costs for an airline. Moreover, it is a mandatory duty to ensure the safety of air operations. Maintenance, therefore, must be carried out at the lowest possible cost but to the highest quality standards and in compliance with a specified schedule. To accomplish these objectives, commercial aviation maintenance is organised in a systematic and well-structured programme of scheduled tasks.

Notwithstanding the rigorous maintenance programme, during the execution of the maintenance scheduled tasks unexpected damage and failures are commonly discovered. These must be corrected by programming supplementary maintenance activities and allocating additional resources to those originally planned. The changes in the original plan hinder the management of the maintenance check, leading to interruptions during the process, delaying checks and overrunning costs.

Aircraft heavy maintenance is a critical process, which requires the aircraft to be out of service for a large period of time ranging from 7 to 30 or more days. Moreover, during its execution, a great number of maintenance tasks are performed that require a considerable amount of resources in constant interaction. Additionally, due to uncertainty related to unscheduled tasks, managing heavy maintenance is complex and most of the time inexact.

Based on the evidence presented in this chapter, the problem can be summarised as follows:

a) There is a complex interaction between scheduled and unscheduled tasks and the large amount of resources required to accomplish them.
b) Due to the stochastic nature of non-routine tasks, planning unscheduled activities and forecasting their required resources represents an important challenge for maintenance managers.
c) As a result of the previous points, adjusting an initial maintenance service plan and managing a whole process can be tortuous and complicated and might impact upon lead times, costs and even the quality of maintenance service.

In this chapter, the heavy maintenance process has been explained along with its relevance and complexity, giving rise to the problem definition. Based on real facts and supported by different studies, this chapter presented evidence to indicate that the described problem is real and relevant in the airline industry and justifies the necessity of further research. Therefore, one of the aims of the following chapter will be to perform a literature review to determine if the described problem has been studied before and how this or similar problems have been approached and analysed by other researchers.

Chapter 3: Literature review

Aircraft heavy maintenance has been shown to be a significant topic, not only due to the operational, safety and financial impact for airlines and MROs, but because of the complexity and uncertainty surrounding it. The heavy maintenance problem has been contextualised, explained and justified in the previous chapter. However, it is necessary to explore how the stated or similar problems have been addressed. Therefore, this chapter presents a comprehensive literature review, firstly, to support this research by discussing the relevant theory related to the topic and, secondly, to discover the different approaches used to study problems with high complexity and uncertainty. This critical analysis attempts to determine the strengths and limitations of previous studies and identify potential gaps where this study can contribute.

The chapter is structured as follows: firstly a review of the most commonly faced problems within the aircraft maintenance field and the methodologies used to address them, secondly, a description of the basic concepts and relevant works in project management due to the great resemblance between heavy maintenance and complex projects and thirdly a revision of uncertainty, one of the most important features of the problem, and some of the theories to explain it. Finally, as an approach to understand complex systems, a review of simulation methodologies was performed focusing on system dynamics and discrete event simulation.

3.1 Aircraft maintenance planning, scheduling and management

The main objective of this section is to review the literature of the aviation industry, mainly focused on aircraft maintenance, serving two different purposes: firstly, to identify if the proposed research problem has been studied or analysed before, and if so, how it was tackled, which methodologies were used for addressing the issue, and what were the results obtained, and secondly to determine different perspectives applied to study and tackle the most common problems in aviation maintenance, which could be strongly related to the stated problem. As a result of this examination, relevant and original studies were selected and analysed. In the following paragraphs the studies are briefly described and organised into four categories: 1) researchers interested in the planning and scheduling of maintenance, both in the long- and short-term; 2) studies focused to tackle workforce problems; 3) works that analyse the management and supply of materials and parts; and 4) those who propose alternatives to improve the maintenance process.

3.1.1 Maintenance planning and scheduling

The airline scheduling process comprises four main parts: flight schedule preparation, fleet assignment, aircraft routing, and disruption recovery. The flight schedule preparation considers the list of flight legs along with departure and arrival times. The fleet assignment specifies which aircraft types are going to operate a specific flight leg. In the aircraft routing, each aeroplane of the fleet is assigned a specific route to operate. The disruption recovery reacts to all operational disruptions by adjusting, updating and reassigning aircraft to new routes in order to minimise the affectations (Stojković et al., 2002; Sarac et al., 2006; Papakostas et al., 2010; Reiners et al., 2012).

The long-term maintenance plans are developed considering the first three categories and the time intervals and durations defined in the aircraft maintenance programme. Short-term maintenance scheduling considers the disruption recovery to adjust the maintenance activities according to the daily operations, with strict compliance with the regulations (Papakostas et al., 2010; Reiners et al., 2012).

With regard to heavy maintenance, two main elements comprise its programming: service scheduling, and planning and scheduling of maintenance activities. Service scheduling considers the overall scheduling of a fleet of aircraft at specific hangars taking into account the maintenance services to accomplish, the itinerary, and the available facilities; it is also known as heavy maintenance master plan. Meanwhile, planning and scheduling of maintenance activities refers to the detailed plan of a specific maintenance service for a particular aeroplane, considering all the maintenance tasks to execute, and the required materials, resources and workforce. In other words, service scheduling refers to the long-term maintenance planning, while planning and scheduling of maintenance activities is a short-term service based on the master plan (Samaranayake and Kiridena, 2012).

In order to offer a quality service, the reliability and punctuality of flights are one of the main goals of an airline, and maintenance must be aligned to this purpose. It is essential to minimize the possibility of delays without affecting the safety and by keeping the costs in control. Therefore, planning and scheduling of the maintenance activities, before and during the service is a widely addressed research field.

3.1.1.1 Long-term planning

Long-term maintenance planning is used to determine and pre-allocate the resources required to perform the maintenance activities. Long-term maintenance planning is done in conjunction with aircraft routes and crew schedules, in order to consider operational, financial and regulatory aspects. The analysed studies have paid attention to aircraft and components planning, particularly components that are essential or critical.

Regarding aircraft planning, Warrington et al. (2002) designed an aircraft reliability and maintenance model based on Discrete Event Simulation (DES), aiming to analyse the periods of time in which the aircraft shall operate without failure or without the need of maintenance, also known as the maintenance free operating period. The model helps to identify and differentiate the essential, desirable and un-desirable maintenance activities. Another relevant study was presented by Hahn and Newman (2008), which introduced a deployment and maintenance schedule for a helicopter fleet, using mixed integer linear programming, as a proposal to reduce scheduling times and avoid inconsistencies in the programme.

For critical aircraft components and parts limited by time, Fard and Melachrinoudis (1991) developed a mathematical model to determine the best inspection schedule for aircraft critical parts to minimize total maintenance costs. Their proposal is a stochastic optimization model with a non-linear objective function and a single set of bounded constraints on integer decision variables. It considers the relationship between different costs involved in the maintenance: cost of inspections, cost of repairs and cost of catastrophic failures to determine the best balance between them. Similarly, to determine which maintenance activities to perform and when, Gustavsson et al. (2014) developed an integer linear programming model to schedule preventive maintenance of the components of a system over a finite and discretized time horizon. The model is dynamic to incorporate unexpected events, which are corrected by programming corrective maintenance activities.

Particularly for engines and power plants, Bowman and Schmee (2001) proposed a model to mitigate and manage financial risk for a MRO organisation. The model, based on DES, evaluates the costs of long-term aircraft engine contracts to determine the fairest conditions for both parties (airline and MRO). The model helps the organisation to take better decisions by analysing the impact on costs and financial risk of operational and maintenance issues. Correspondingly, Weckman et al. (2006) developed a decision support approach for modelling jet engine life and estimating maintenance requirement for engine restorations. The model is based on real statistical information, a hazard model and DES, aiming to predict the jet engine removal rate, reducing additional spare engines and hence inventory costs.

3.1.1.2 Short-term scheduling

Sachon and Paté-Cornell (2000) suggest that for maintenance managers it is relevant to know the impact of certain maintenance strategies both in aircraft security and flight delays to perform an accurate decision. Therefore, they proposed a probabilistic risk analysis model, using influence diagrams, to evaluate the effects of an airline's maintenance policy on delays, cancellations and in-fight safety.

Some studies focus on the daily operation and maintenance of the aeroplane. For instance, Sarac et al. (2006) explain that due to stochastic events, long-term plans are difficult to address and hence they are often ignored by personnel in airline operations who are forced on a daily basis to develop quick, ad hoc methods to address these eventualities. As an alternative, they proposed an aircraft maintenance routing problem formulation considering maintenance resource availability constraints. The proposal uses the branch-and-price algorithm to offer a short-term planning solution (daily aircraft maintenance routing), incorporating resource availability constraints into the model. In the same vein, Papakostas et al. (2010) explain that often during line maintenance turn-around time (TAT) decisions regarding “go” or “no go” of the aeroplane for the next flight must be taken, and that the time available to take that decision is limited and the information available to analyse is considerable. They remark that currently that decision is taken reactively resulting in high operational costs and low operational reliability. Therefore, they argue that airlines need to create, simulate and evaluate different strategies of maintenance, reducing unscheduled maintenance occurrences and some of their consequences. They introduced a short-term planning model of aircraft line maintenance tasks during turn-around time at airports stations, based on multi-criteria decision-making, allowing for immediate decision-making to reduce or avoid reactive decisions. Similarly, Reiners et al. (2012) developed an algorithm extending the traditional aircraft routing problem (ARP) by including profit and robustness, aiming to mitigate the impact of unexpected events that hinder the management of the original flying schedule. They used a network flow problem formulation and greedy start heuristics.

Another approach to analyse the daily operations is proposed by Tiassou et al. (2011) and Tiassou et al. (2013). They developed a model to assess aircraft operational reliability with regard to its missions in order to be able to cope with failures. They used a stochastic dependability modelling approach to evaluate the capacity of the aircraft to keep operating for a specific time or location without reaching an adverse status. If an unexpected event occurs while the aircraft is in operation, the model assesses the ability to succeed in continuing on the remaining part of the mission.

To cope with disruptions caused by unpredictable events, flight schedules need to be updated on a daily basis, considering reassignment of aircraft, crew and passengers to reduce the effects as much as possible. However, rescheduling issues can have a significant negative impact on costs and service quality. Several works have analysed the recovery-rescheduling-rerouting problem of flight operations.

Le et al. (2011) presented a literature review of current methodologies of airline recovery optimization and airline disruption management of resources. They stress that in reality, the itinerary and flights schedule does not operate as planned, as it is commonly disrupted by different unexpected factors, such as maintenance problems or environmental conditions. They remark, therefore, that it is necessary to find a minimal cost reassignment of aircraft and crews that complies with all required safety rules, ensures the lowest impact on passengers and minimizes operational difficulties for the airline. To tackle this problem, Stojković et al. (2002) developed an optimization model to define new flight schedules based on planned crew transfers, rest periods, passenger connections and maintenance, when minor perturbations occur. Their model is a dual network based on PERT and CPM allowing both start times and activity durations as variables. Similarly, Sriram and Haghani (2003) designed an optimization model for maintenance scheduling and flight re-assignment to minimise costs. The model is formulated as a min-cost multi-commodity network with integer constraints, using origin-destination schedule as input variables and considering a heuristic approach. Meanwhile, Eggenberg et al. (2010) proposed a flexible model to determine a recovery scheme, for each plane, crew member and passenger within a certain makespan.

Another area of interest is the detailed programming of maintenance services. In this regard, Kalton and Richards (2008) emphasise that the duration of a resource constrained project can be two times or more larger than needed. They further argue that the approach used by most commercial project planning software for handling resources, is a relatively inefficient methodology for scheduling resource constrained projects. They performed a literature review about the resource-constrained project scheduling problem, and compared the performance of different scheduling engines, showing that engines can greatly impact upon the scheduling programme results. Masmoudi and Haït (2012) designed a model for project scheduling and planning problems, for modelling the periodic workload on a tactical level and the continuous workload on the operational level, particularly for heavy maintenance services. They integrated uncertainties into tactical and operational multi-resource, multi-project planning by using a fuzzy and possibilistic approach instead of a traditional stochastic approach since limited data was available. In their research, they identified different uncertainties during the aircraft heavy maintenance process and classified them into different sources. By using the model, different workload scenarios according to different time durations were proposed and assessed.

3.1.2 Workforce planning and scheduling

Workforce scheduling is a difficult task that involves personnel preferences, coverage constraints, legal restrictions and many other constraints (Van den Bergh et al., 2013). Since the workforce in aviation maintenance is highly skilled, specialized and costly, several studies have focused their attention on addressing, analysing and attempting to solve workforce issues, mainly related to the planning and scheduling of the maintenance labour. In this regard, different approaches have been utilised to study manpower problems.

Latorella and Prabhu (2000) focused on human error in aircraft maintenance and inspection, performing a literature review regarding relevant concepts and approaches for identifying, reporting, and managing human error. They point out that aviation maintenance and inspection tasks are commonly performed in an environment with time pressures, sparse feedback and sometimes difficult ambient conditions, which, in combination with human erring tendencies, result in varied forms of error which can impact upon safety, costs and quality.

Optimisation models have been commonly used for workforce scheduling. For example, Alfares (1999) proposed an Integer Programming Model for scheduling workforce of an aircraft line maintenance unit, motivated by high workload levels and by excessive overtime rates. The objective of the model is to fulfil work requirements at minimum cost by determining the optimum maintenance manpower schedule. Alfares' model addresses an actual problem in aircraft maintenance, showing real economic benefits from its implementation. However the model makes some strong assumptions that are important to mention: for instance, the non-scheduled activities were roughly estimated from a percentage derived from the routine task, even when this could be inaccurate and unrealistic. Additionally, some human and organisational factors, such as unions or performance decrease caused by fatigue, were not properly factored into the model. A similar approach was used by Beliën et al. (2013). They proposed a model to minimize workforce costs of aircraft line maintenance. The model uses an enumerative algorithm in a mixed integer linear problem (MILP) to determine the number of personnel necessary to meet the maintenance demand and to define the scheduling program that translates the availability of the staff into an adequate shift structure. It is important to note that the model deals with scheduling and staffing problems simultaneously and does not consider the possible effects of unscheduled maintenance tasks. Meanwhile, Yang et al. (2003) and Yan et al. (2004) proposed a mathematical programming model to offer a workforce supply plan, taking into account the technicians' maintenance certificates and other related operational constraints. The model considers three different flexible strategies: flexible shifts, flexible squad size and flexible working hours by applying mixed integer programs. An important point to note in this research is that for the manpower planning, relevant attributes of the technical personnel, such as their maintenance certificates and their technical skills, were considered.

Another optimisation approach is proposed by Cheung et al. (2005). They suggest that it is complicated to select and assign the most suitable person to perform skilled maintenance activities. Moreover, they remark that scheduling and allocating personnel to ensure they can meet the maintenance workload is a primarily a subjective process. Therefore, they developed a decision support expert system for manpower allocation in aircraft maintenance services. Analytical Hierarchy Process (AHP) was used for developing the model as it deals with quantitative and qualitative criteria and sub-criteria, and combined with fuzzy logic to minimize the weakness of AHP facing uncertainty. One of the strengths of the system is that it reduces subjectivity on manpower and workload allocation by using quantitative and qualitative criteria. Their results revealed that the system provides better workforce management and higher productivity for the aircraft maintenance.

Quan et al. (2007) explain that aircraft maintenance managers face the dilemma of either curtailing manpower to reduce idle time, or increasing it to improve maintenance productivity. To tackle this dilemma, they developed a multiple objective problem (MOP) to optimise the heavy maintenance process while reducing manpower costs. They applied utility theory to discover Pareto optimal solutions and evolutionary algorithms to solve the multi-objective problem. This is an original approach to optimise manpower, as it considers two different and opposite objectives. However, some important variables in workforce management, as skills and certifications, are overlooked.

Wagner and Fricke (2006) use a different method. They argue that workforce planning for both scheduled and unscheduled maintenance is fundamental for avoiding expensive overcapacities. They introduced a statistical model that represents the occurrence of failures as a stochastic process to estimate the man-hour demand necessary to handle unscheduled maintenance events on a certain fleet during line maintenance.

Another approach widely applied is the use of simulation models. Bazargan and McGrath (2003) designed a model based on DES to analyse and improve aircraft availability and maintainability. The research was motivated by poor maintenance planning causing higher rates of fleet unavailability. Through building and comparing different what-if scenarios, they proposed a workforce timetable re-schedule and the redistribution of technicians across different shifts, reducing aircraft waiting time and increasing workforce utilisation. In a similar way, to tackle low labour utilisation in line maintenance process, Gupta et al. (2003) developed a stochastic simulation model for aircraft line maintenance to improve workforce allocation and increase labour utilisation. Van den Bergh et al. (2013) also used simulation, combining DES and data envelopment analysis (DEA) to design a model for managing aircraft line maintenance workforce scheduling. The model first calculates different feasible low-cost rosters which are then evaluated through the DES model. Finally, the most efficient alternatives are determined using DEA.

3.1.3 Management and supply of parts

In aircraft maintenance, large amounts of parts and materials are required. Additionally, they must fulfil strict regulations to be approved and certified, hence becoming significantly expensive and requiring large periods for supplying. For these reasons, the management and supply of parts has been an area of great interest for several researchers.

For example, Ghobbar and Friend (2004) stress that implementing material requirement planning (MRP) in the aviation industry could be complicated, as the aircraft maintenance process is complex and full of uncertainties, thus the need for spare parts is unpredictable. They performed a review in the industry using interviews to determine the most common inventory procedures and policies used by airline operators and MROs, and to analyse the use of MRP for controlling inventory, reducing stock and therefore costs. In the same vein, Cohen and Wille (2006) investigated the effect that sharing part consumption data from heavy maintenance checks from different MRO and airline operators has on the planning and procurement of parts. They found that adopting a coordinated data-driven approach for managing parts might reduce the service parts inventory and improve parts availability.

Kilpi and Vepsäläinen (2004) developed a statistical model to assess the effect of reducing the inventory level through the implementation of an inventory pooling strategy, focusing on the spare units needed to cover unscheduled removals. They concluded that inventory pooling of spare components between airlines can significantly reduce inventory costs. Similarly, but focused on aircraft engines, Joo and Min (2011) explain that engine modular design makes field maintenance simple and agile, although it increases considerably the cost of the spare modules and inventory. They remark that rigorous inventory control of spare modules is essential for cost-efficient maintenance, and introduced a dynamic model for scheduling preventive maintenance of engine modules. Their algorithm seeks to minimise the difference between due time for preventive maintenance and actual time for preventive maintenance, considering that the spare modules to support this type of maintenance are limited.

3.1.4 Process improvement

Aircraft maintenance is a complex process which requires a large amount of resources and involves several activities. The inherent uncertainty that characterises this process makes it challenging to manage. Moreover, due to its significant impact on costs and revenues for the airline, there is constant pressure to improve the process. Different researchers have studied and proposed new approaches to achieve this goal. Their proposals range from the use of mathematical models and simulation or the application of information technologies to the adoption of a continuous improvement philosophy.

Simulation has been widely used as a supporting tool for aircraft maintenance improvement. Duffuaa and Andijani (1999) designed a framework for a simulation model that integrates maintenance activities and operations. They advocate that in the airline industry it is essential to treat maintenance and operations as one system, because of the high degree of dependency between them. Meanwhile, Cobb (1995) proposed a simulation model in aircraft line maintenance to reduce turnaround time, improve quality and reduce cost. Even though the model does not consider the uncertainties of the maintenance process, it serves to test the effects of changes in the maintenance process. The results are used to estimate operational capabilities.

Correspondingly, and focused on aircraft critical components, Adamides et al. (2004) used system dynamics to analyse the long-term performance of aircraft engine maintenance, arguing that life-cycle engine maintenance is a dynamic system influenced by engine reliability and maintenance, operational requirements, and maintenance infrastructure. The model is used as a decision making supporting tool to compare different maintenance scenarios, and also as a learning tool to understand the drivers of successful and poor maintenance and operational performance.

Another example of simulation is given by Horning et al. (2012), who developed an interesting model based on DES to simulate flight operations of an aircraft fleet and the resulting preventive and corrective maintenance of each aircraft, considering also its condition-based maintenance. Through the assessment of different maintenance policies, the model helps to forecast and evaluate the availability and flying rates for the fleet and for individual aircraft.

Mohaghegh et al. (2009) remark that despite the significant improvements to analyse and model safety and risk assessment, there is a lack of organizational safety risk frameworks. Therefore, they proposed a hybrid approach to model organizational safety risk, incorporating organizational factors. The hybrid approach integrates System Dynamics (SD), Bayesian Belief Network (BBN), Event Sequence Diagram (ESD), and Fault Tree (FT). Likewise, Ostrom and Wilhelmsen (2008) developed a risk model in aviation maintenance to improve aeroplane reliability and safety by using probabilistic risk assessment methodologies. They further suggest that structural damage increases the risk of an accident, emphasising the relevance of analysing, detecting and assessing vulnerabilities during the inspection process for risk reduction.

A different aspect worth studying is the approached by Jian and Hong-fu (2004) who stress that determining and budgeting maintenance costs are difficult tasks, due to the uncertainty and complexity that characterise the process. However, these are fundamental tasks for cost efficiency improvement and consumption reduction. To overcome this problem, they designed a maintenance cost-forecasting model based on the project-evaluating method and the cost-estimating relationships (CERs) method.

Considering the available statistical data from operational and maintenance records, Al-Garni et al. (2009) state that monitoring and analysing this information is a fundamental task for airlines and MROs, to give a timely follow-up to the performance of the aircraft and its components, and to evidence any possible failures. However, they add that this monitoring is commonly performed using sophisticated and complex statistical and other analytical tools that might be difficult to explain and understand for practitioners. Therefore, they proposed the use of graphical techniques, using cumulative and mean cumulative function plots for extracting key management information from field failures. In recent years the amount of information available has increased dramatically and with it the complexity of its mining and analysis. In this regard, Wang et al. (2009) explain that nowadays information systems are used in every aspect of maintenance support, becoming effective tools to assist the decision making and support the execution of maintenance. Thus, they proposed an evaluation framework to measure the effectiveness of the Aviation Equipment Maintenance Support (AEMS) informationalization.

Ayeni et al. (2011), through a thorough and systematic review of the literature, evaluated the perception, progress and implementation of “Lean” philosophy within the aviation maintenance industry, particularly in MRO organisations. They concluded that it is feasible to apply “lean” in the aviation industry; however, it is not sufficient to achieve all the objectives set by the organization. Besides, they found that “lean” has been mainly applied as a waste reduction methodology rather than as a process improvement philosophy. An important drawback pointed out in this research is that some basic and common practices in “lean” are difficult to implement in the aircraft maintenance sector due to the complexity and uncertainty to estimate accurately the required resources. In a similar way, with a lean and continuous improvement approach, Ward et al. (2010) proposed an integral model of the operational system to enhance efficiency and customer satisfaction through improving aircraft maintenance processes without neglecting safety and quality. The model is based on participatory action research and blocker resolution process aiming to identify and eliminate those activities that hinder maintenance performance.

An additional way to improve the maintenance process is by enhancing its supporting processes. For instance, Cheung et al. (2005) designed a model using genetic algorithms (GA) to maximize ground support vehicles utilization and to enhance the logistics of aircraft maintenance activities by providing a faster but reliable way of managing ground equipment.

Finally, focused on heavy maintenance, Srinivasan et al. (2007) remark that long lead times for repairing and overhauling aircraft is a serious and common problem for MROs. Hence, they implemented Critical Chain Method aiming to reduce turnaround time. Similarly, Samaranayake and Kiridena (2012) argue that current approaches in aircraft heavy maintenance for planning and scheduling have a limited capacity to deal with contingencies arising out of inspections. They proposed a single integrated framework considering uncertainties and supported by unified data structures.

3.1.4.1 Summary of aircraft maintenance studies

After reviewing and analysing the relevant literature concerning the common problems of the aviation industry, some important findings emerge. Regarding the methodologies utilised to address the planning, scheduling and management of aircraft maintenance, two main approaches were used: mathematical optimisation models and simulation modelling. Optimisation was principally adopted to minimise costs and turnaround time, or to maximise aircraft utilisation and availability, by scheduling or managing different resources, like materials and parts, or workforce. Meanwhile, different simulation approaches have been utilised as forecasting and scenario building tools, to support the decision making and management thinking.

It is also worth noting the different processes analysed. The three main aircraft maintenance areas, namely, line maintenance, heavy maintenance, and components maintenance, have been studied to address their particular problems. The most addressed process has been line maintenance, because of its direct impact upon and close relationship with daily operations. Many researchers have analysed it from different perspectives and using several approaches, mainly focused on short-term maintenance scheduling, particularly to analyse the recovery scheduling problem. Optimisation models have been the most common approach for addressing these types of problems.

Heavy maintenance has been studied from different angles due to the complexity and uncertainty that characterises it. Some studies have considered heavy maintenance as a complex project and have addressed it by using project management techniques such as CPM, PERT and CCM. Others have proposed the use of continuous improvement tools, for instance, lean and six-sigma. Another approach is the utilisation of MRP for supply management and production planning. The application of simulation methodologies like discrete event simulation and system dynamics has also been proposed. However, as stressed by Samaranayake and Kiridena (2012) most of the studies in heavy maintenance have focused on the service scheduling problem, i.e. on the long-term, disregarding the detailed planning and scheduling of maintenance tasks and their corresponding resources.

The studies in component maintenance have concentrated on critical components, particularly aircraft engines. The reason for this preference could be due to their significant impact on the operation, their strict regulation, and their high costs. Studies have focused on optimising the inventory, increase the "time on wing” and improving the maintenance process and supply, amongst other factors.

Workforce is one of the most, if not the most, important resources for aircraft maintenance; it is costly, highly skilled with a lengthy training and certification process. For this reason, workforce planning and scheduling have attracted significant interest in the field. Nevertheless most of the studies have focused primarily on line maintenance, aiming to reduce idle time and costs and at the same time improve workforce productivity.

As discussed in chapter two, delays and disruptions during the heavy maintenance process are relevant, real and actual problems in the aviation industry. However, few studies have addressed these problems by analysing and assessing the impact of non-scheduled activities on the delays and disruptions and how that might affect the heavy maintenance goals. Supporting this argument, Samaranayake and Kiridena (2012) suggest that in heavy maintenance there is poor integration between the planning of materials and resources, the scheduling and control of maintenance tasks and the execution of maintenance activities. They also emphasise the importance of dealing with uncertainty associated with heavy maintenance planning and scheduling. Regarding uncertainties, Papakostas et al. (2010) further assert that nowadays, the decision support process in aircraft maintenance is mainly reactive and focused on resolving unscheduled maintenance activities.

Most of the solutions offered by the studies reviewed fail to consider a systemic understanding of the problem and do not take into account the impact that each part has within the whole process and the relationship between them. Some studies did not consider uncertainties, or made very rigid assumptions about them. Due to the different fleet, a global optimisation approach is not the best way to address heavy maintenance planning in a MRO (Masmoudi and Haït, 2012). Moreover, traditional project management methodologies do not have the ability to unfold different contingent scenarios to deal with the uncertainty associated with the unscheduled activities (Samaranayake and Kiridena, 2012). Other approaches, like continuous improvement or IT developments, require a deep cultural change in the organisation as well as in the philosophy of thinking and its implementation requires a huge effort.

After describing the main characteristics of aircraft heavy maintenance and reviewing significant approaches used to address the most frequent challenges of this process, a great similarity between aircraft maintenance and complex projects management is evident. Therefore, it would be fruitful to discuss relevant concepts in project management and learn how problems with similar characteristics have been addressed by using this discipline.

3.2 Project management

Projects, from very ancient times, have been managed in very diverse forms. However, the concept of project management as a discipline is relatively recent. Currently, projects are carried out in a globalised and highly dynamic environment and in a constantly changing world, characterised by large availability of information and extremely competitive markets. Project managers face constant and continuous pressure to minimise costs and increase profits, whilst also improving quality, efficiency, productivity and customer satisfaction. Project management has evolved to support the achievement of project goals in a broad range of fields, mainly those with high levels of complexity and uncertainty. Hence, before discussing relevant approaches for analysing problems with similar characteristics to the research problem proposed, it is necessary to explain the most significant concepts in the project management field.

Project management can be defined as the application of knowledge, skills, experience, processes, methods, tools and techniques to achieve the project objectives (Project Management Institute (PMI), 2013; Association for Project Management (APM), 2015b). In addition, according to the British Standard BS 6079-2:2000, project management considers, the planning, monitoring and control of a project and the motivation of the people involved, in order to achieve the project objectives according to the specified time, cost, quality and performance (MS/2 British Standards Institution, 2000). Furthermore, it is emphasised in the BS 6079-1:2010 that project management is beneficial for an organization in different ways as it enables a more efficient use of resources and has been shown to be an effective tool for managing many types of change (MS/2 British Standards Institution, 2010).

3.2.1 Project

A project “is a temporary organization to which resources are assigned to do work to deliver beneficial change” (Turner, 2009, p.2). Similarly, the PMI (2013) defines a project as a “temporary endeavour undertaken to create a unique product, service, or result” (PMI, 2013, p.2). Projects can also be seen as engines of change (MS/2 British Standards Institution, 2010). They have distinct attributes that characterise and differentiate them from other routine work. The combination of different factors, such as context, time frame and objective, make a project *unique*. Additionally, they are *finite*, which means they have a well-defined starting and ending date. The *objective* and *expected results* of a project are known and clearly stated at its definition. In order to meet these objectives, different *resources* must be assigned. However, projects are always subjected to several *constraints*, generally measured in terms of time, cost and quality. Importantly, projects are rarely isolated as they have *constant interaction* with other projects and external entities. Lastly, projects are usually carried out under *risk* and *uncertainty* conditions (Williams, 2003; Buttrick, 2009; MS/2 British Standards Institution, 2010; PMI, 2013).

A project is not a static element: rather it is an entity in constant dynamism that evolves through different stages to accomplish its objectives. These stages, or phases, are commonly known as the project life-cycle and although different authors refer to them in different ways, in principle they share a similar meaning. The PMI points out that the project phases’ names and numbers are determined by the needs of the organisation involved in the project, the nature of the project itself and its area of application. The PMI further adds that these steps are generally sequential and time bound, with a specific start and ending point. Generally, a project is composed by the following phases: 1) Appraisal, where the viability and feasibility of the project are assessed to determine if it is possible or beneficial to carry out the project. 2) Design, where the concept of the project is formalised by defining the plan of activities and by determining the resources, budget and time required to perform the project. 3) Execution, where the activities are carried out and the resource utilised. Rigorous monitoring must be performed during execution to ensure that the project is run according to plan and to determine whether it is necessary to take corrective actions to adjust it. Finally, 4) Close-out, which refers to the stage once the project has been completed, where the project is formally finished, all relevant documentation is completed, the product or service is delivered and the resources are released. Usually, a final evaluation and feedback is performed to determine what went wrong and what went well (PMI, 2013).

It is important to highlight that in the final stage of the project life-cycle, a project is considered to be completed when its objectives have been reached, but also it can be terminated because the objectives cannot be met or if the project is no longer needed, or if the client asks to terminate it (PMI, 2013).

3.2.2 Complex project

Complex project is a concept widely used in project management literature, yet it is commonly applied as a synonym for large projects. Nevertheless, its definition is not as straightforward as it might seem, and very few studies in the field have formally defined it.

Smith (2003b) explains that uncertainty is inherent in all projects, but he emphasises that the higher the uncertainty level, the greater the complexity of the project. In the same vein, Meredith and Mantel (2012) point out that in many complex projects, as a consequence of uncertainty, not all activities are fully understood until some earlier activities have been completed. Complexity is not just a question of the number of components or their relationship, but of numerous interactions and the sophisticated connection between them. In other words, complexity represents the interrelationship of components and how they act together within the project (Eve et al., 1997, cited in Hetland, 2003).

According to Remington (2010), a complex project is one that shows a critical size, a very constrained timeframe, a high level of ambiguity and a deep interconnectedness. Furthermore, he argues that a complex project is indeed a complex adaptive system characterised by a nested structure, a sophisticated communication network, both inside and outside, negative feedback loops that induce stability, but also positive loops that encourage change, adaptiveness in response to external stimulus in the environment, and extreme dependence on initial conditions. Supporting this argument, Stacey (2011) points out the main principles of complex systems. He suggests that due to the non-linear relationship of positive and negative feedbacks, unexpected and counterintuitive results are normally produced by complex systems. Moreover, the sensitivity to change is variable in complex systems, where the links between cause and effect could be distant in time and space. A complex system can be more sensitive to some changes than to others and uncertainty makes it difficult to determine which change will have the greater impact.

Williams (2003) argues that project complexity arises from two main dimensions: structural complexity and uncertainty. The former is sub-divided into size, or number of elements, and interdependence of elements, and the latter into uncertainty in goals and uncertainty in methods. Size implies that the larger the number of elements in the project, and the larger the number of activities to perform, the higher its complexity. Interdependence of elements suggests that the large number of interrelations and the complicated interweaving of elements also increases the complexity in the project. As a result of these two dimensions, structural complexity leads to a complex system in which the whole is more than the sum of the parts. Complexity in goals arises because there is vagueness or ambiguity in the project’s objectives, or because the objectives are continually changing with the clients' requirements. Uncertainty in methods occurs when methodologies or steps to perform are imprecise, unclear or unknown, causing continuous changes in the plan and project structure. The two types of uncertainty generate perturbations in the system, which result in complex, dynamic behaviour.

Interestingly, though using different concepts, Remington (2010) adopts a similar perspective regarding complexity, suggesting that there are four types of complexity in projects, namely, *structural, technical, directional* and *temporal*. *Structural complexity* is generally found in large projects and it arises due to difficulty in managing and controlling the great number of interconnected tasks and activities. Projects developing new products or services, or applying a new technique or methodology without precedents experience *Technical Complexity*, characterised by technical or design problems caused by the interconnection between multiple interdependent solution options. *Directional Complexity* is the result of conflict or ambiguity in the interpretation of goals and objectives, and is exhibited by projects with multiple and unshared goals or divergent and unclear goal paths. Finally, *Temporal Complexity* is present in projects carried out in highly unstable and dynamic conditions generally outside the control of the project team. It arises from uncertainty regarding future constraints, unsteady environment or even concerns about the survival of the system.

In summary, as already discussed, and supported by Latorella and Prabhu (2000), Ward et al. (2010) and Masmoudi and Haït (2012), aircraft heavy maintenance is a highly dynamic and intricate process, where uncertainty is present during execution. It involves several constrained resources that interact both internally and externally and so fulfils the above definition of a complex project. It can, therefore, be studied from this perspective. Moreover, the uncertainty, complexity, and uniqueness of project activities make control more difficult and deviation from plans more probable, because plans are formulated for a set of contingencies that cannot be preconceived because they have no precedent (Sydow and Staber, 2002).

3.2.3 Risk

Risk is associated with project management and this association is particularly true for complex projects. Smith (2003b) explains that "risk" comes from the Latin root "*risicare*", which means "to dare", emphasising that from this etymological meaning, risk represents more a choice rather than fate. According to Buttrick (2009), risk represents any potential uncertainty, threat or occurrence that may affect the achievement, objectives and benefits of the project. Smith (2002), however, argues that risk can bring opportunities as well as threats. It does not, therefore, necessarily have negative consequences and can have positive ones.

Some relevant definitions of risk in the context of project management are the following:

In the BS 6079-2 2000, risk is defined as the "combination of the probability or frequency of occurrence of a defined threat or opportunity and the magnitude of the consequences of the occurrence" (MS/2 British Standards Institution, 2000, p.11).

According to the HM Treasury, risk is the "uncertainty of outcome, whether positive opportunity or negative threat, of actions and events. The risk has to be assessed in respect of the combination of the likelihood of something happening, and the impact which arises if it does actually happen" (HM Treasury, 2004, p.9).

The Project Management Institute defines risk as "an uncertain event or condition that, if it occurs, has a positive or negative effect on one or more project objectives such as scope, schedule, cost, and quality" (PMI, 2009, p.310).

From these definitions, two relevant elements can be identified: uncertainty and consequences or impact. Uncertainty refers to unexpected or unforeseen events that might happen either just before or during the project execution. Uncertainty is generally described as a "probability", "possibility" or "likelihood" of an unexpected event occurring. Sometimes, there is enough information or knowledge to estimate the uncertainty using a probability distribution, but in others it is based on perceptions and subjective judgments. If an unexpected event occurs, the consequences may be positive or negative and so can be seen either as opportunities or threats. This might influence the performance and objectives of the project.

3.2.4 Risk management

According to the Association for Project Management (2015a), risk management is the “process that allows individual risk events and overall risk to be understood and managed proactively, optimising success by minimising threats and maximising opportunities”. The Project Management Institute (PMI) (2009) explains that project management addresses the uncertainty in project estimates and assumptions by assessing and analysing the potential unexpected events and conditions that might occur before or during the project. The PMI further indicates that for successful project management, project risk management is neither an optional activity nor a substitute for other project management processes and must, therefore, be considered in all project phases.

The opinions of authors differ regarding the steps that comprise the project risk management process. For instance, in the context of construction projects, Perry and Hayes (1985), Smith (2003a) and Wood and Elis (2003) identify a three-step process for risk management, namely, *identification, analysis* and *response of risk.* The Risk Analysis and Management of Projects approach (RAMP) considers six separate stages known as opportunity identification, appraisal, investment planning, asset creation, operation and close-down (Institution of Civil Engineers and the Faculty and Institute of Actuaries, 2002). Meanwhile, the Project Management Institute (PMI) (2009, 2013) defines six main steps in the risk management process: risk management plan, risk identification, qualitative risk analysis, quantitative risk analysis, risk responses plan and risk control. (Lester, 2014) proposes 5 stages for risk management: awareness, identification, assessment, evaluation and management of risk.

All these classifications have remarkable similarities despite the different stages proposed. Considering them all, it can be suggested that project risk management should consider: 1) Clear definition of risk management objectives, process and scope. 2) Identification and categorisation of potential risks that might influence performance and objectives, including documentation of their main features. 3) A qualitative assessment to analyse the two elements of risk: uncertainty and impact, in order to prioritise the different risks. 4) A quantitative analysis that involves the application of a variety of numerical tools to assess the effect of the identified risks on project performance. 5) Action plans to take advantage of opportunities and to minimise the effect of threats. 6) The implementation of control and the adjustment of the proposed action plans to ensure their effectiveness.

Smith et al. (2006) remark that risk management does not mean predicting the future: rather, it means understanding the project to make better decisions regarding its management. Finally, it can be summarised that risk management is a process to systematically identify, assess, analyse, understand and manage uncertainties and their corresponding consequences. Therefore, it becomes an essential part of project management to ensure the success of the project.

3.2.5 Relevant approaches in project management

Different approaches have been used to analyse a wide range of problems in project management and this section summarises those considered as outstanding due to the close relation to the problem subject of this research. For instance, in risk management, Chan and Kumaraswamy (1997), Charoenngam and Yeh (1999) and Sambasivan and Soon (2007) identified and evaluated the typical risk factors that cause delays in construction projects. (Bowers, 1994) proposed a framework to determine, customise and combine uncertainty estimates based on quantitative and qualitative data to determine the project uncertainty rating. Lee (2005) developed a simulation model to determine the probability of completing a project in the specified time, considering the randomness and stochastic nature of the activities' duration. Schatteman et al. (2008) proposed a heuristic methodology for determining and evaluating major risk factors, their probability of occurrence and their impact on project duration, to define a robust project plan that can cope with disruptions.

Regarding resource management, several methods have been applied. Dynamic programming was used by Elmaghraby (1993) to minimise the project completion time considering the relation between the resources allocated and the activity duration. Hapke et al. (1994) focused on software development and developed a fuzzy project scheduling system to allocate personnel between dependent activities under uncertain activity duration. Fatemi Ghomi and Ashjari (2002) used a simulation approach to improve the utilisation of common resources during the execution of concurrent projects. Genetic algorithms have been commonly used for resource planning and allocation (Hegazy, 1999; Chang et al., 2001; Valls et al., 2009; Yannibelli and Amandi, 2011). Other optimisation techniques have been also applied to the management of resources. Leus and Herroelen (2004) used a branch-and- bound algorithm to develop a resource allocation model for projects with variable activity duration. Brucker et al. (2011) suggested that workforce scheduling problems can be formulated as integer linear programs and that the assignation of shifts to personnel and the allocation of workforce to specific activities can be solved by applying network-flow algorithms.

Several approaches to the planning and scheduling of activities in project management have been utilised to support traditional techniques and mitigate their drawbacks. Herroelen and Leus (2005) point out that the fundamental approaches for scheduling under uncertainty are reactive scheduling, stochastic project scheduling, fuzzy project scheduling, robust (proactive) scheduling and sensitivity analysis.

Fuzzy concepts have been widely used as a different way to represent and analyse uncertainty in projects execution. Leu et al. (2001) used fuzzy set theory in combination with genetic algorithms for a multi objective time-cost trade off under different risk levels. Özdamar and Alanya (2001) used fuzzy numbers to represent the uncertainties related to activity durations and the interrelation between them and applied them in conjunction with a generic heuristic algorithm to planning software development projects. Oliveros and Fayek (2005) proposed a fuzzy logic model for monitoring and controlling construction projects. Li and Chen (2007) and Long and Ohsato (2008) used fuzzy sets to support a critical chain method for estimating the buffer size. Castro-Lacouture et al. (2009) developed different fuzzy mathematical models for the multi-objective optimisation of project schedules taking into account time, cost, and unexpected materials. Zammori et al. (2009) used fuzzy logic and multi criteria decision analysis (MCDA) to support critical path method (CPM) considering the critical parameters of the project.

Optimisation models have been widely used to study planning and scheduling problems. Golenko-Ginzburg and Gonik (1997, 1998) utilised a zero-one integer programming problem for the former and a heuristic approach for minimising the project duration for the latter. Fleszar and Hindi (2004) used a variable neighbourhood search to reduce the solution search to address the resource-constrained project scheduling problem. Zhu et al. (2004) developed a recovery plan to get back on track as soon as possible at minimum cost using integer linear programming. Budai et al. (2005) used a NP-hard mathematical formulation and a heuristic approach to minimise out of service and maintenance costs for a preventive maintenance scheduling programme. Rabbani et al. (2007) proposed a heuristic algorithm by combining CCM concepts and resource-constrained project scheduling methods, aiming to minimise expected project duration and its variance.

Simulation techniques and computer-based systems have been also used for planning and scheduling. Lu and AbouRizk (2000) and Zhang et al. (2002) applied discrete event simulation (DES) to improve CPM and PERT methodologies. Cho and Eppinger (2005) developed a simulation model based on the design structure matrix (DSM) to identify leverage points for process improvements and to evaluate alternative planning and execution strategies. Hegazy and Menesi (2010) used the main concepts of the critical path method (CPM) to develop the critical path segments (CPS) model that breaks down the duration of each activity into separate time segments to avoid complex network relationships and facilitate resource allocation. Bruni et al. (2011) designed a computer-supported system for making a robust project plan less sensitive to unexpected events by considering the available probabilities to assess the project reliability and minimise the impact of possible disruptions during project execution.

Other approaches use MCDA to support decision making by evaluating the importance and criticality of certain activities (Mota et al., 2009) or to evaluate the project performance according to different indicators considering the point of view of different stakeholders (Marques et al., 2011). Bayesian networks have also been used to estimate delays in maintenance projects (Demelo and Sanchez, 2008).

Several approaches have been proposed to support the traditional methodologies of project management, e.g. CPM, PERT and CCM, especially to deal with complex projects and high levels of uncertainty. Most of the proposals focus on mathematical optimisation models and computer-aided simulation models. Their applications range from an operative and tactical level, to define comprehensive project schedules as well as detailed resource allocation, to a strategic level as a decision support tool for project managers to monitor the project status and test different policies.

3.3 Uncertainty

Uncertainty is an important concept for this research. In aircraft maintenance, uncertainty is mainly caused by the potential occurrence of unexpected damage and failures that must be corrected by programming unscheduled maintenance activities, which might require additional resources and tasks, complicating the planning, scheduling and execution of the maintenance service.

Uncertainty is commonly used in a variety of fields, including project management, to express complexity and the unknown. However the concept of uncertainty might be misunderstood and is sometimes confused with the concept of risk. Therefore, it is important to establish a formal definition of uncertainty and discuss different arguments about it.

Chapman and Ward (2002, 2003) define uncertainty as a lack of certainty involving variability, ambiguity and deficiency or absence of data, information, knowledge and understanding. Meredith and Mantel (2012) explain that uncertainty refers to having partial or no information about the situation or outcomes, often due to ambiguity or complexity. A broader definition is offered by Klir and Wierman (1999), who specify that uncertainty represents the degree of information deficiency, indicating how incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory or deficient the information available is.

Uncertainty is inherent to many aspects of life and Klir and Wierman (1999) suggest that uncertainty cannot be avoided when dealing with real-world problems, being present at the empirical, cognitive and social level, resulting in errors, misunderstandings, vagueness, ambiguity and unavailability. Moreover, uncertainty is always present at a certain level in decision making, and although it can be understood, managed, and even reduced, it cannot be eliminated (Belton and Stewart, 2002). In the same vein, Smith (2003b) explains that in real world projects there is always a certain degree of uncertainty, as it is extremely difficult to predict and control all events relating to major projects.

Risk and uncertainty are used indistinctly as synonyms in daily life. However, it is important to point out the differences between them. According to Knight (1921), the practical difference between risk and uncertainty is that in the former the distribution of the outcome is known, while in the latter it is unknown because the situation being addressed is unique. In a similar way, Pidd (2004a) explains that risk and uncertainty are used to discriminate between options. He further highlights that a decision concerning risk is one where, although it is not known precisely what is going to happen, enough information exists to estimate a probability distribution of the outcomes. In contrast, a decision concerning uncertainty relates to unique or uncommon situations where there is no objective way to build the probability distribution and the outcome, therefore, is unknown and must be estimated by different means. Williams (2003) uses a different terminology, but also differentiates between these two concepts, arguing that there are *aleatoric* and *epistemic* uncertainties, the first resulting from stochastic events and the second from lack of knowledge. Importantly, he highlights that the latter increases the complexity of a project, hindering its management.

Chapman and Ward (2002) explain the relation between uncertainty and risk from a different perspective, stating that risk is a consequence or implication of an uncertain event or phenomenon. They further point out that these unexpected consequences may be wanted (positive) or unwanted (negative). Uncertain events and their implications must, therefore, be understood properly in order to manage them successfully. Uncertainty and risk have the same relationship in the context of project management. Smith (2003b) remarks that risk is a consequence of uncertainty; it is generally interpreted as factors which might impact upon the achievement of project objectives. According to the Project Management Institute (PMI) (2013), project risk is an uncertain event that, when it occurs, has a positive or a negative influence on a project's objectives. From these points of view, it can be argued that risk involves the uncertainty of an event, but also its consequences.

3.3.1 Classification of uncertainty

Uncertainty has been classified in different ways using different perspectives for particular contexts. This gives a better understanding for analysis and management. These different classifications, however, implicitly share some relevant concepts.

In terms of multi-criteria decision making, Belton and Stewart (2002) suggest that *internal* and *external* uncertainty can be differentiated. They explain that internal uncertainty comprises two aspects: the structure of the model and the judgmental inputs of the model. The first focuses on problem structuring and model building and mainly relates to ambiguity and ignorance. The second considers the elicitation of information and the utilisation of the model and mainly relates to imprecision. They describe external uncertainty as a lack of knowledge about the consequences of a particular choice, subdividing it in two groups: uncertainty about related decision areas and uncertainty about the environment. The former refers to the influence that some decisions can have on other interconnected decisions and the insecurity this can cause. The latter denotes the concern about problems outside the control of the decision maker, representing lack of knowledge or understanding and randomness.

In a project management context, Hetland (2003) identifies four different categories of uncertainty, based on the crossed-relation between the availability of the information (how much is known) and whether a project is closed or open. In a closed project, it is well defined what to do and how to do it, whereas in an open project this is not the case. Therefore, the first type of uncertainty, where there is full knowledge about the possible outcomes and the project is closed, is called *deterministic uncertainty*; hence it is just a question of calculating the probability of occurrence of each outcome. The second category is known as *lack of information* and also occurs in a closed project. However, in this type of uncertainty, there are missing pieces of information that might result in an incomplete or ambiguous understanding of the project and the challenge is how to estimate the outcome using the incomplete available data. The third category is called *variability* and occurs in an open project when the information is available. This approach differs from the deterministic uncertainty because in this case the number of outcomes, their values and their probabilities of occurrence are assigned subjectively based on the judgement or intuition of experts supported by historical facts. The last category is known as *undetermined uncertainty* and refers to an absence of information in an open project. It generally occurs in unique and highly specialised projects where there is not enough information available. This type of uncertainty represents the highest degree of complexity.

From the information analysis and management point of view, Klir and Wierman (1999) identify three different types of uncertainty: *fuzziness, strife or conflict* and *non-specificity*, clarifying that the last two are commonly grouped and recognised in a wider category known as ambiguity, as shown in Figure 3-1. Fuzziness refers to the uncertainty that arises from the vagueness of linguistic expressions or from a lack of clear or sharp definition. Strife is the uncertainty that results from conflict or incongruence between different pieces of evidence regarding a set of alternatives. Non-specificity is the type of uncertainty that occurs when some alternatives are left unspecified or there is imprecision in characterising the different alternatives. Therefore, ambiguity expresses the difficulty in determining or characterising a set of alternatives, either by discrepancy or imprecision.

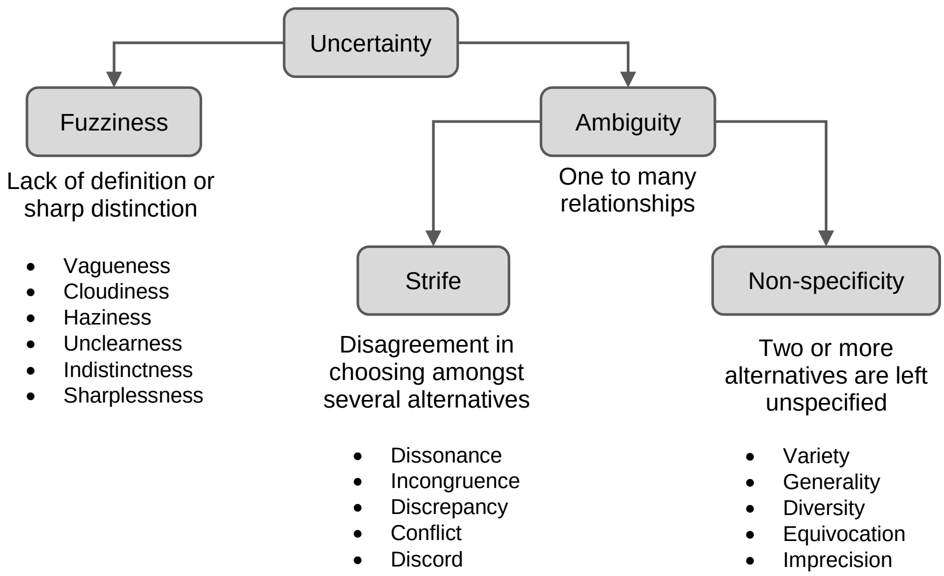


Figure 3-1 Basic uncertainty types (Klir and Wierman, 1999, p.103)

In aircraft maintenance, Masmoudi and Haït (2012) identify six different types of uncertainty and classify them at the tactical and operational level. At the tactical level, they define three types of uncertainty. Firstly, *uncertainty in the release date* where the days that the aircraft is going to stay out of service are not precisely known and must be estimated. Secondly, *uncertainty in the workload* where despite the scheduled tasks being known in advance, the programming of unscheduled tasks hinders the planning of the workload. Finally, *uncertainty in procurement delays*, where unplanned maintenance might require parts and material not included in the original supply plan and affects procurement significantly. At the operational level, they also define three types of uncertainty. Firstly, *unexpected lack of resources* (principally the absence of personnel) which might affect planned daily progress. Secondly, *task duration*: in aviation maintenance it is difficult to set a standard time for each activity, as it depends on different factors, e.g. the skill level of the personnel, the condition of the aircraft, external conditions, etc. Finally, *updates in the maintenance program*: sometimes manufacturers, authorities or operators require adjustments to the maintenance plan by including, modifying or eliminating maintenance tasks. Masmoudi and Haït (2012) further argue that the problem of tactical planning under uncertainty for aircraft maintenance had never been studied until their proposed study.

3.4 Alternative uncertainty theories

Traditionally, and for more than three hundred years, uncertainty has been described and studied using probability theory. However, uncertainty is a multidimensional concept formed from distinct types of uncertainty and, in its simplest form, probability theory focuses on only one of these dimensions (Klir and Wierman, 1999). During the second half of the 20th century, other notable theories emerged that aimed to characterise the different aspects of uncertainty, namely, fuzzy set theory, evidence theory and possibility theory (Klir and Wierman, 1999; Klir, 2003; Rogova and Nimier, 2004; Maskell, 2008). Each particular uncertainty theory describes different forms of uncertainty (predictive, prescriptive, diagnostic, etc.) in terms of a certain type of monotonic measure (or a pair of measures) by using a specific formalised language system (Klir, 2003).

3.4.1 Fuzzy set theory

Fuzzy set theory was proposed by Zadeh (1965) for modelling uncertainty associated with vagueness, imprecision and lack of information about the system, providing a “mathematical way to represent vagueness in humanistic systems” (Özdamar and Alanya, 2001, p.159). A fuzzy set “is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one” (Zadeh, 1965, p.338). A relevant characteristic of fuzzy sets is their capability to represent gradual transitions from membership to non-membership, which is important in describing the vagueness and imprecision of expressions in natural language (Klir, 2001). Fuzzy set theory is a generalisation of classical set theory, which represents a particular case when the membership of an element can be defined unambiguously, i.e. where it is clear if the element does or does not belong to the set.

3.4.2 Evidence theory

Evidence theory, also known as Dempster-Shafer theory, was proposed by Dempster (1967; 1968) and further formalised by Shafer (1976). It consists of two dual semi-continuous non-additive measures called belief and plausibility measures. (Klir and Wierman, 1999). In the Dempster-Shafer theory, an outcome of a random experiment is known as a proposition. The set of all possible, mutually exclusive and collectively exhaustive propositions is called the frame of discernment and the set of all subsets of the frame of discernment is known as the power set. A basic probability mass can then be assigned to the subsets that form the power set (Yang and Xu, 2013). Therefore, the belief measure of a particular preposition describes all basic probability masses assigned exactly to that preposition and its smaller subsets, whereas the plausibility measure of a specific preposition represents all possible basic probability masses that could be assigned to the preposition and its smaller subsets. The belief and the plausibility measures can, therefore, be interpreted as the lower and upper bounds of probability to which a certain preposition is supported. The difference between the belief and plausibility measures of a given preposition defines the degree of ignorance for that preposition (Yager, 1987; Dubois and Prade, 1991; Tonon, 2004; Yang and Xu, 2013). In contrast to classical probability theory, where probability masses are only assigned to singleton subsets, in evidence theory probability masses can be assigned to all subsets of the frame of discernment (Borotschnig et al., 1999).

The core of Dempster-Shafer theory is Dempster's rule, which constitutes a conjunctive probabilistic inference process to combine independent evidence that is fully reliable. However, Dempster's rule cannot be applied when two pieces of evidence support different propositions and this lack of definition has led to a counter-intuitive problem when the rule is used to combine evidence in high (or near complete) conflict (Zadeh, 1984; Zadeh, 1986; Haenni, 2005; Yang and Xu, 2013).

Different evidence combination rules have been developed to overcome problems associated with the non-definition and counter-intuitive problem associated with Dempster's rule (Yang and Xu, 2013). Yager's rule (Yager, 1987) suggests allocating conflicting beliefs to the frame of discernment. Dubois and Prade's rule (Dubois and Prade, 1988) proposes the allocation of conflict beliefs locally between focal propositions where the conflict occurs. Proportional Conflict Redistribution 5 (PCR5) rule (Smarandache and Dezert, 2005) also proposes allocating conflicts locally. However, in their attempt to deal with conflict, these rules have lost their probabilistic nature, as they no longer constitute a conjunctive or probabilistic reasoning process (Yang and Xu, 2013).

3.4.2.1 Evidential Reasoning (ER) rule

A new rule for evidence combination has been recently proposed by Yang and Xu (2013). This new evidential reasoning rule allows multiple pieces of independent evidence to be combined conjunctively and in any order. An important feature of the ER rule is that it considers the weight, or importance, and the reliability or quality, of each piece of evidence. It constitutes a generic conjunctive probabilistic reasoning process or a generalised Bayesian inference process (Yang and Xu, 2014). The ER rule consists of two main elements: the bounded sum of their individual support and the orthogonal sum of their collective support. Through a new reliability perturbation analysis, the ER Rule overcomes the non-definition and counterintuitive problems associated with Dempster's Rule (Yang and Xu, 2013).

Yang and Xu (2013) demonstrated that Dempster's rule is a special case of the ER rule when all pieces of evidence are fully reliable. They also prove that the ER algorithm is a special case of the ER rule, when reliability and weight are the same in all pieces of evidence and the latter is normalised.

3.4.3 Possibility theory

Possibility theory was proposed by Zadeh (1978) and further investigated by Dubois and Prade (1988, 1994) to handle certain types of uncertainty. Similarly to evidence theory, it uses two dual semi-continuous fuzzy measures to deal with incomplete information or partial ignorance: possibility, or lower semi-continuous, and necessity, or upper semi-continuous, both measures are commonly understood as functions of the power set of a given possible set of prepositions (Klir, 2003; Masmoudi and Haït, 2012; Borotschnig et al., 1999). Possibility theory is a natural mathematical framework for describing and analysing uncertainty associated with fuzzy propositions, where possibility and necessity are a special case of plausibility and belief measures respectively and whose focal elements are nested. It represents a specific branch of evidence theory (Klir and Wierman, 1999; Klir, 2003). In general terms, instead of providing a single value like probability theory, possibility theory provides a “minimum possible value” and a “maximum necessary value”.

3.5 Simulation

Since the development of computers, computer-based simulation has become a useful tool for analysing complex systems and for decision support. Swain (1993) argues that simulation can be useful for studying and analysing how a system reacts to changes in flow patterns or for obtaining statistical estimates of important performance variables. Pidd (2004a) describes three common approaches to analyse particular problems in management science: experiment directly on the real system, construct and apply a mathematical model, or simulate the system. He further adds that simulation can have significant advantages over the other two approaches. For example, compared with direct experimentation, simulation can be cheaper, faster, safer and easier than experimenting or practising in the real system. Compared with traditional mathematical modelling, simulation can cope with dynamic effects. Robinson (2004) also explains the importance of simulation, stating that it helps to deal with variability, interconnectedness and complexity, three common characteristics of operations systems.

Due to its flexible capabilities, simulation has recently gained popularity for studying relevant problems in the airline industry. Regarding aircraft maintenance, Cobb (1995) remarks that simulation can support airlines and MROs for analysing and assessing long-term strategic changes and short-term operational and tactical changes. Bazargan and McGrath (2003) argue that simulation of maintenance activities can be a superior technique compared with traditional analytical approaches. Supporting this argument, Gupta et al. (2003) explain that aircraft maintenance is plagued by planning difficulties mainly caused by unpredictable failures and they argue that stochastic simulation in particular presents relevant advantages over mathematical modelling to study situations where randomness is present. Duffuaa and Andijani (1999) point out three key features of maintenance that make simulation the most viable approach: complex interactions between maintenance and other organization functions, the close interrelationship between the elements involved in maintenance and the constant uncertainty that characterises maintenance.

Simulation can be understood as the imitation of the operation of a real-world process or system over time (commonly designing and using mathematical or logical models computer-based), to describe, explain and predict the behaviour of the real process/system (Hoover and Perry, 1989; Banks et al., 2000). Likewise, Madachy (2008) defines simulation as the numerical evaluation of a model to describe and analyse a particular system, experimenting with given inputs to see how the system performs; he further points out that simulation is used to explain system behaviour, improve existing systems, or to design new complex systems. Law and Kelton (2000) distinguish between static and dynamic simulation, where the former imitates a system at a point in time and the latter imitates a system as it changes through time. Correspondingly, Robinson (2004) describes dynamic simulation as (computer-aided) experimentation with a simplified and limited version of an operations system as it changes through time, aiming to better understand and improve the real system. Rotaru et al. (2014) state that the simulation of a system comprises building a valid model to replicate the parts of interest of that system, then using the model and experimenting within it to explore and understand the system’s functioning and performance under certain conditions and in response to certain courses of action, with the aim of providing scientific support for management thinking and decision support for process systems.

Pidd (2004a) explains that computer simulation is the utilisation of computer-aided models to analyse and experiment through processing and analysing inputs to explore the effects or outputs they might have in the real system. However, he indicates that any simulation approach is limited and influenced by certain assumptions and considerations based on the particular perception of reality. How the model (and the simulation) is developed and expressed depends on the specific “worldview”.

It can be summarised that simulation is the act of using or “playing” with the models with the aim of learning from them. Simulation allows experimentation with the model and specific input variables in a controlled and safe environment to understand and analyse how the model, and therefore the real system that it emulates, behaves. It also allows comparison between different input variables to determine which perform the best.

The close relationship between real systems (or worldview) and their models and the learning cycle are clearly explained by Sterman (2000). He remarks that through the simulation process, models are used to learn and practice more quickly, effectively and safely how the system works and behaves, just as pilots do in flight simulators. Knowledge acquired during the simulation stage can then be applied to reality with more confidence. Subsequently, users of the model and the real system can compare the behaviour of both and feed back to the designers ways to improve the model to more closely resemble reality. In other words, knowledge acquired during simulation is used in the real world and knowledge acquired in the real world is used to change and improve the model.

From simulation definition, two relevant concepts arise: system and model. It is important to explore them both in detail.

3.5.1 System

According to Bertalanffy (1971), a system can be understood as a combination of elements interacting complexly with one another and with the environment. He further adds that complexity can arise from three different factors: 1) the number of elements, 2) the "species" or types of elements and 3) the relations between them. In the same vein, Ackoff (1971) defines a system as a group of interrelated elements, where each system's element is connected to every other element, directly or indirectly, and no subset of elements is unrelated to any other subset. Forrester (1973), Coyle (1996), and Law and Kelton (2000) describe a system as an organised group of elements that work together and interact for a common purpose. Likewise, Roberts et al (1983) summarise that a system is a cluster of interacting components that function in combination for a particular objective. From these definitions, it is clear that a system is formed by a set of elements that interact with one another to create a more complex structure with the aim of accomplishing a specific objective. To achieve the collective objective, each element has its own place and mission within the system.

Checkland (1981) defines four different types of systems: 1) Natural designed systems, which are formed and maintained without human intervention (e.g. the solar system, an atom, or a cell). 2) Designed physical systems, which are created by humans and have a physical or tangible representation (e.g. an aircraft, a building, or an electricity distribution system). 3) Designed abstract systems, which are non-tangible or conceptual systems designed or conceived by humans (e.g. mathematics). 4) Human activity systems, which are systems that are created, consciously or unconsciously, by human activity (e.g. the family, a city, political or economic systems).

Madachy (2008) indicates that systems can be categorised as "open" or "closed". In an open system, outputs do not influence inputs. An open system is not aware of past performance or behaviour. In contrast, a closed system is affected by its own behaviour over time. In other words, it is a feedback system that changes through loops connecting past actions to control future actions.

Pidd (2003) identifies four main characteristics that any system should have: 1) Boundaries, which are the limits that define the environment of the system and determine the elements that are inside and outside of it. These boundaries, however, are not necessarily a physical barrier and are often difficult to define. 2) Components, which are the elements that constitute the system. A system should contain at least two elements and there should be at least a bidirectional interaction between them. 3) Internal organization, which is each element inside the system having its own and well-defined role and working well in combination with other elements to achieve the system's objectives. 4) Behaviour, which is the purpose and particular properties of the system as a whole. This is the result of complex interaction between constituent elements and not just the simple sum of the individual roles of each element.

3.5.2 Model

Traditionally and in general terms, a model is a representation of reality. Madachy (2008) explains that it can be conceived as a representation or abstraction of real or conceptual systems of reality with the aim of assisting the study, exploration and understanding of a real system. Morecroft (2007) defines a model as a tangible aid to imagination and learning, a temporary and dynamic object that helps with the comprehension and exploration of a partially understood world. Bayer et al. (2014) suggest that models are interfaces that act as representative and boundary objects. As representative elements (i.e. as a simplified and limited version of the real world), models help to represent, explore and understand reality. They also allow experimentation and simulation assuming different inputs and analysing outputs. As boundary objects models, as tools for processing and transmission of information, promote learning and understanding amongst the stakeholders. They also encourage participation, engagement and teamwork between group members. In the same vein, Schultz and Sullivan (1972) and Williams (2003) remark that models have significant benefits during the building process. They enhance cohesion, participation and commitment amongst the modelling group and stakeholders and promote continuous learning.

Pidd (2003) proposes an interesting and well-founded definition of a model as “an external and explicit representation of part of reality as seen by the people who wish to use that model to understand, to change, to manage and to control that part of reality” (Pidd, 2003, p.12). From Pidd’s definition, three main components can be identified: reality, people and objective. “Reality” is the element in which the model is based and which it tries to emulate and exemplify. However, according to Pidd, it is difficult to describe “true reality”, as much depends on the person who sees, explores or experiments with that reality. Additionally, to define the scope of the model, the reality is bounded so that it only considers the elements of interest to the person designing or utilising the model. “People” refers to who conceives, designs and utilises the model to manage or understand reality. “Objective” denotes the purpose for which the model was designed. Pidd concludes that from a management perspective, a model should help people to understand, change, manage and control reality.

Pidd further defines a model as “an external and explicit representation”. However, other authors (Forrester, 1971; Papert, 1980; Sterman, 2000; Morecroft, 2007) have pointed out the existence and usefulness of other internal and more abstract models known as “mental models”, which are the ideas and perceptions that people carry in their minds about the way something works (Morecroft, 2007). Sterman (2000), Pidd (2004b) and Morecroft (2007) describe the relation and interaction between mental and formal models. Firstly, a person or group has their own ideas and insights about how a specific system works, i.e. a mental model. Then, with the help of this mental model, they create a tangible and explicit model about the system being analysed, i.e. a formal model. After designing, building, exploring and analysing the formal model and its operation, their initial mental model changes and adapts according to the new knowledge acquired, thus creating an evolutionary cycle between the two models.

Williams (2003) summarises seven features that a good model should have: 1) An empirical basis, supported by objective and coherent data. 2) A theoretical grounding, in accordance with the relevant body of knowledge. 3) Coherency, with elements and interactions not contradicting. 4) Simplicity, considering only the most relevant points of the real system. 5) Handles the complexity of the system and mimics real behaviour. 6) Adds value by helping with exploration and understanding of the system it is representing. 7) Impacts upon decision making by facilitating it and enhancing management thinking.

Models can be classified in different ways. Madachy (2008) categorises models as discrete, continuous or a combination, depending upon how time is analysed during simulation. In discrete models, time is advanced based on specific events, i.e. model's state changes at aperiodic times from one event to the next in a discrete manner. In contrast, in continuous modelling, such as system dynamics, time is advanced in constant periods and during these periods all time-dependent variables in the model are recalculated. Madachy (2008) further explains that models can be categorised as deterministic or stochastic according to the type of variables they handle. Deterministic models do not consider any probabilistic element in their structure, whereas stochastic models take into account randomness in their components. It is possible to have a combination of the previous classifications, e.g. continuous deterministic or stochastic models, or discrete deterministic or stochastic models.

Pidd (2003) proposes a broader classification of models, arguing that from the management science perspective there are "*hard*" (or *quantitative*) and "*soft*" (or *qualitative*) models. A "*hard*" model can be understood as a mathematical, analytical or quantitative model. This type of model is the most commonly used in business process modelling for decision and control. In contrast, "soft" models are less concrete but are still formal. They are a set of tools to help understand how people conceive and interpret reality and the system being analysed. "Soft" models are considered as interpretive approaches. They are more focused on strategic management and planning, as they seek to analyse disagreements and uncertainties amongst a group of people, with the aim of achieving consensus and commitment to action.

The most common and widely used modelling approaches for studying and analysing management problems are System Dynamics (SD) and Discrete Event Simulation (DES). The former can be defined as a continuous and deterministic approach and the latter as a stochastic and discrete modelling tool. In the following sections, both modelling approaches are discussed in more detail.

3.6 System Dynamics (SD)

The concept of System Dynamics (SD) was introduced by Forrester in 1958 as the "study of the information-feedback characteristics of industrial activity to show how organizational structure, amplification (in policies), and time delays (in decisions and actions) interact to influence the success of the enterprise" (Forrester 1964, p.13). Although it was firstly known as industrial dynamics, the term evolved to system dynamics because its application area grew beyond industrial problems (Forrester, 2007a). Forrester, probably influenced by the work of Tustin

(1953), suggested the application of concepts commonly used to analyse mechanical and electrical control systems to study human-management problems.

The concept of SD has evolved to become a methodology for designing, building and applying simulation models to study and manage highly dynamic and complex systems (Ford et al., 2004). Coyle (1996) states that SD deals with the time-dependent behaviour of managed systems with the aim of describing the system and understanding, through qualitative and quantitative models, how information feedback governs its behaviour. This allows for the design of robust information feedback structures and control policies through simulation. Sterman (2000) explains that SD is a method for improving learning in complex systems using “management flight simulators” (generally computer simulation models) as a representation of reality to help understand dynamic complexity, identify the sources of policy resistance and design more effective policies. Sterman also stresses that, in order to be useful, SD simulation models must emulate the behaviour of the real system so that they respond appropriately, not only in conditions already observed, but also those not yet encountered.

According to Coyle (1977), Sterman (2000) and Forrester (2007a) SD is an interdisciplinary methodology because it connects exact and social sciences. SD is based on control theory developed in mathematics, physics and engineering, but deals with social, economic and management problems, traditionally studied in the social science field. It is highly supported by computer sciences as simulation models can be designed and built more easily using computers, resulting in larger, more robust and complex models that can handle more variables and data.

One of the main features of SD is its ability to represent the dynamics of complex systems as feedback processes. Forrester (1964, p. 14) establishes that "an information-feedback system exists whenever the environment leads to a decision that results in action which affects the environment and thereby influences future decisions". Richardson and Pugh (1981) state that system dynamics simulation models are useful for managing processes that change significantly over time and have feedback loops where the information is transmitted. Similarly, Sterman (2000) argues that all systems, no matter how complex they are, consist of networks of positive and negative feedbacks, and all dynamics arise from the interaction between these loops. He also states that the dynamics of a system are determined by its feedback processes, along with stock and flow structures, time delays and nonlinearities. Stocks depict stored quantities, show the state of the system and generate the information on which decisions are based. Flows represents the rate of increase or decrease in stocks.

Dangerfield (2014) summarises the main characteristics of system dynamics models. He argues that they: 1) model the elements of interest as aggregates or groups, not making any distinction between individual elements. 2) Regard the dynamics of a system as generally being caused by endogenous factors, such as internal feedback and interaction between variables. 3) Draw a clear distinction between resource flows and information flows, where the latter causes the former to change. 4) Adopt a continuous simulation approach, where the model is simulated at constant intervals of time. Flows are assumed to be continuous and represented by differential equations.

5) Are primarily focused on the behaviour of stocks in the system, represented by integral equations. 6) Have the ability to handle hard and soft variables, considering the latter as being relevant to model performance.

Pidd (2003) and Brailsford (2014) set out two different ways in which SD can be used: “soft SD” and “hard SD”. “Soft SD” is qualitative and focuses on designing conceptual models to represent feedback structure and complex interactions between variables. This approach is based on understanding the problem and learning from the complexity of the system. To achieve this objective, “soft SD” uses different mapping and graphical tools such as mental models, causal loop diagrams (also known as influence diagrams) and sector maps. It aims to understand the impact of policies and strategies rather than serving as an analytical tool for prediction or decision support. “Soft SD” is a tool for learning, communication and debate. “Hard SD” focuses on developing a computer-based simulation model of the system. Generally, “hard SD” uses the insights obtained by “soft SD” to build a simulation model. It aims to simulate and analyse the dynamic of the system and to act as a decision-support tool through the comparison and assessment of scenarios. “Hard SD” centres its attention on mimicking the behaviour of the system to compare the effects of new policies or strategies on the system performance. It is also used as an estimation tool of “alternative futures” by experimenting on critical variables in the model. As illustrated by Morecroft (2007), the complete quantitative model is “an inference engine to diagnose performance problems; a virtual world to experience dynamic complexity and stimulate imagination; and a laboratory to design and test new policies and strategies” (Morecroft, 2007, p.85). Both qualitative and quantitative aspects work together to make SD a methodology for analysing the dynamic complexity of a particular system. As summarised by Lane (2000), SD provides hard system models from a soft and interpretative modelling process (cited in Morecroft, 2007, p.150).

SD has been successfully applied to a wide range of areas, both in the public and private sectors, to study and analyse social and engineering problems. For example, in the public sector it has been applied to health-care, the military and defence, urban development, road maintenance, environmental and energy issues, politics, population strategies, government policies, natural disaster assessment and anti- terrorism policies. In the private sector SD has been used in the finance sector, aerospace and aviation, construction, mining and oil extraction, energy production and distribution, manufacturing and industrial engineering and information technology and systems. (Wolstenholme, 1990; Sterman, 2001; Koelling and Schwandt, 2005; Forrester, 2007b; Tako and Robinson, 2010; Thompson and Bank, 2010).

3.6.1 System dynamics in project management

SD has been widely applied to project management in different areas, such as large scale projects in construction, shipbuilding, aerospace, and energy, software development and information technologies, research and development projects, maintenance, aviation, health-care and the military and defence. The application of SD models encompasses a broad range of project management issues. Bendoly (2013) highlights the positive effects that SD and systems thinking can have for project management, especially for understanding the dynamics of systems and their features.

SD has been used to analyse cost and scheduling overrunning, one of the most common problems in project management. Reichelt and Lyneis (1999) discuss the main causes of projects overrunning. Ford et al. (2004) studied constructability reviews to reduce highway project durations. Eden et al. (2005) compare the measured mile approach with SD to analyse cost overruns. Park (2005) focuses on resource management to reduce cost and schedule overruns. Lee et al. (2006b) analyse the impact of negative iterative cycles on construction performance. Ford et al. (2007) investigate the impact of project controls on cost and schedule overruns. Lee and Peña-Mora (2007) study the causes and consequences of error and changes in the project objectives. Peña-Mora et al. (2008) propose a hybrid model SD-DES to improve project performance. Park and Peña-Mora (2003) and Fard and Pena-Mora (2007) study the effect of non-value adding activities caused by changes and errors in the project. Han et al. (2013) analyse the effects of design errors in project costs and schedule.

In the context of risk management, Lyneis et al. (2001) use SD to identify risks and assess the effects of different processes and organization changes on project performance. Rodrigues (2001) proposes the use of SD modelling within the PMBOK to provide a framework for managing project risk dynamics. Dulac et al. (2005) analyse organizational safety culture for safety and risk management in complex engineering projects. Nasirzadeh et al. (2008) apply SD and fuzzy logic to analyse and manage risk in construction projects, characterising the imprecise and uncertain nature of risks.

To analyse the impact of labour in project management, Abdel-Hamid (1989) simulate different policies for managing human resources in software projects to investigate their effect on project behaviour. Chapman (1998) studies the impact of changing key project personnel on construction project duration. Bayer and Gann (2006) examine the causes and effects of workload fluctuations in a project-based professional service organisation. An et al. (2007) propose a “workforce supply chain” that recruits, develop and deploy workforce in a timely manner to fulfil project objectives.

Another common application of SD in project management has been to the litigation process. Williams et al. (1995a) analyse delays and disruptions caused by changes and delays in the design process. Williams et al. (1995b) examine the impact of delays and in-development product enhancements in highly parallel and time-constrained projects. Ackermann et al. (1997) evaluate the cost of delays and disruptions during the channel tunnel project. Howick and Eden (2001) investigate the consequences of compressing large projects due to client pressure, delays and disruptions. Cooper et al. (2002) discuss the impact of learning lessons from project to project. Williams et al. (2003) propose a logical, transparent, auditable and sustainable approach to make litigation claims for delays and disruptions within the projects. Howick (2003) analyses SD as an approach for handling delays and disruptions for litigation purpose, pointing out its strengths and limitations.

Other applications of SD in project management include planning and scheduling (Abdel-Hamid, 1993; Coyle, 1996; Ceylan and Ford, 2002; Ford, 2002; Adamides et al., 2004; Han et al., 2014), the analysis of tipping points (Taylor and Ford, 2006; Taylor and Ford, 2008), the exploration of the impact of changes and rework during project execution (Love et al., 1999; Love et al., 2002; Cooper and Reichelt, 2004; Love et al., 2008), the analysis of concurrent projects (Ford and Sterman, 2003; Yaghootkar and Gil, 2012) and the management of resource allocation (Joglekar and Ford, 2005; Laslo and Goldberg, 2008; Lee et al., 2008).

A possible explanation of the successful adoption of SD in project management is suggested by Lyneis and Ford (2007). They argue that planning and managing projects is very challenging, mainly because project conditions and performance constantly change over time as a result of feedback responses, principally involving nonlinear relationships and the accumulation of project progress and resources. Several studies note that traditional project management tools are inadequate for dealing with the dynamism of complex systems, as these tools conceive a project statically and focus only on part of the project, (Ford and Sterman, 1998; Reichelt and Lyneis, 1999; Lyneis et al., 2001; Lee et al., 2006a; Lee and Peña-Mora, 2007). These studies also suggest that given this inefficient management, most projects miss budget and schedule objectives, requiring additional resources. Rodrigues and Bowers (1996) suggest that SD, in contrast, can be a suitable approach for analysing dynamic complexity in project management due to its holistic vision of the project rather than a sum of individual elements, the non-linear analysis of feedbacks and its flexibility for modelling the dynamic project structure.

Lyneis and Ford (2007) identify four general categories where SD has been applied in project management: 1) *Post-mortem assessments for disputes and learning*. SD has been used as a tool to analyse and assess the performance of a project once it has been finalised, as a post-project evaluation to determine what went wrong and what the causes were. These kinds of SD models are commonly used in litigation cases or to learn lessons for future projects. 2) *Project estimating and risk assessment*, where SD is applied to estimate the resources and time required to complete the project under specific circumstances. It is also used to evaluate the impact of unpredictable but possible events that could affect project performance. Here, SD is used as a pre-project analysis to explore possible outcomes. 3) *Change management, risk management, and project control*. In this case, SD is used as a tool for monitoring, assessing and controlling the performance of a project by comparing it with the project plan. If some changes or adjustments are necessary, the model helps to determine the best alternatives and to evaluate possible consequences. The model is used here as a “during the project” analysis, acting as a monitor or dashboard. 4) *Management training and education*, where SD models are used as learning tools for explaining methodology and for understanding the behaviour and complexity of the system, acting as laboratories or flight simulators for training project managers.

Summarising, it can be argued that SD is a tool for understanding and managing highly dynamic and complex systems that has been successfully applied to project management to analyse and solve numerous problems that impact upon the development and performance of a project and the achievement of its objectives.

3.7 Discrete Event Simulation (DES)

Discrete Event Simulation (DES) is founded on Monte Carlo simulation (Robinson, 2014) and is a stochastic discrete simulation methodology for modelling systems as networks of queues and activities as they evolve over time, where the stated variables change only at countable points over time. The conditions for these activities and the order in which they may occur can be extremely complex (Law and Kelton, 2000; Brailsford, 2014). Belton and Stewart (2002) explain that DES models the changes of individual entities in a system of queues, and through the utilisation of theoretical and empirical statistical distributions to represent variability, the stochastic nature of the system is represented. To simulate a DES model, a considerable amount of quantitative data is required, plus the input distributions for all sources of stochastic variation. Furthermore, as DES simulation is a sampling process, it requires a large number of simulation runs to obtain better and more accurate results that help the understanding of output distribution (Brailsford, 2014; Robinson, 2014).

To represent a system, DES uses four elements: entities, queues, activities and resources. Entities "are the individual elements of the system that are being simulated and whose behaviour is being explicitly tracked" (Pidd, 2004a, p.64). Each has specific features or attributes that enable them to be distinguished from other entities and to control their own behaviour (Pidd, 2004a). Queues are where entities wait for available resources in order to be processed. Cassandras and Lafortune (2008) point out that in queuing systems three elements are involved: the entities that are waiting to be treated, the resources which are being waited for and the space where the waiting is done. Activities are the performance of any kind of work and change entities in specific ways. "The operations and procedures that are initiated at each event are known as activities and these activities are what transform the state of the entities" (Pidd 2004a, p.66). Resources are the system elements that must be available to perform activities. Like entities, they are individual elements, but their behaviour is not modelled individually. Instead, they are treated as countable items (Pidd, 2004a). In summary, in a DES model individual *entities* flow through a certain number of *activities*. During the process, the *entities* have to wait in *queues* for available *resources*.

According to Robinson (2014) there are three main features of DES: 1) modelling of individual entities, 2) time handling, and 3) randomness in the system.

In DES, the system is represented as individual entities that move through a network of queues and activities (Tako and Robinson, 2014). The entities in the system are distinct objects that are perfectly traceable, each having their own characteristics that determine what happens to them (Brailsford, 2014).

Regarding time handling in DES, unlike continuous time simulation, a system is modelled as a series of "events", that is the points in the time at which the state of the system changes (Robinson, 2004). The evolutionary state of the system depends on the occurrence of asynchronous discrete events through time (Cassandras and Lafortune, 2008). In other words, DES models are simulated in unequal time-steps only when the system state changes (Brailsford, 2014)

An event in the context of discrete-event simulation can be defined as “an instant of time at which a significant state change occurs in the system” (Pidd, 2004a, p.65). An event can be understood as occurring instantaneously and producing a transition from one state value to another (Cassandras and Lafortune, 2008). There are two different types of events that might occur: firstly, bound or booked events that are programmed to occur at a specific point in time and depend on a particular action taken and secondly conditional events that cause changes in the system when certain conditions in the model are met (Pidd, 2003; Robinson, 2004; Cassandras and Lafortune, 2008). According to Robinson (2014) interconnected randomness is the key factor in system performance and is at the heart of DES modelling.

In DES, the randomness of the system occurs over the length of time an activity takes (Robinson, 2014). Randomness is modelled by sampling activity durations for each entity from probability distributions (Brailsford, 2014). In other words, to represent the stochastic nature of the system, DES utilises probabilistic distributions to characterise the unpredictability and complexity of that system.

According to Brailsford (2014), DES is the most widely practically used modelling operational research approach. She explains that DES models are often used for comparison of scenarios, prediction or optimising specified performance criteria and are generally applied at a tactical or operational level. DES has been widely used for understanding, analysing and improving the design and operation of systems, but has also been used to help and support strategic decision making (Robinson, 2014). It has been traditionally used in the manufacturing sector, but has recently also been applied to the service sector (Tako and Robinson, 2014).

With regard to the specific context of this study, DES has also been used to study and analyse different problems in process and project management and also to address common issues in aircraft maintenance.

Fanti et al. (1997) used DES in process management to analyse complex resource sharing and deadlocks in flexible production systems. They simulated resources interactions and proposed different control policies to ban or allow specific resource allocation strategies and minimize deadlocks. Louit and Knights (2001) developed a model using DES to analyse and evaluate different strategies, aiming to improve the maintenance of mining equipment by identifying the critical root-factors that impact upon a mine maintenance system and to determine the best improvement initiatives through testing. Krause et al. (2004) proposed DES as a decision support tool for shipbuilding to test and evaluate different scenarios in investment planning, scheduling, and resource planning.

Lu and AbouRizk (2000) proposed a simplified CPM/PERT model for project management to evaluate different scenarios based on computer simulation and risk analysis. The model incorporates DES into traditional CPM/PERT for modelling entities, their particular attributes and the points of time (events) where the entities are processed. The model aims to overcome the limitations of traditional CPM/PERT approaches and to significantly reduce computation time. Zhang et al. (2002) proposed a model combining DES and the critical path method (CPM) for construction project scheduling and planning, aiming to enhance modelling capabilities under complex scenarios and to assist in planning resource allocation policies. In their model, DES is used to obtain late-time information in addition to resource utilization statistics. Cho and Eppinger (2005) applied the design structure matrix (DSM) and parallel discrete-event simulation method for analysing and managing the development of complex design projects. They argued that their proposed model can be used to increase understanding of the real behaviour of design projects, to improve project management and to assess and compare planning and execution strategies. Kouskouras and Georgiou (2007) employed DES for managing a software project. The model works as a decision support tool for controlling and monitoring the project and to determine the best planning alternative. Setamanit et al. (2007) developed a hybrid model based on SD and DES to analyse and assess different task allocation strategies to configure global software development (GSD) projects. The model also allows for the analysis of the impact that different factors have on the studied allocation strategies.

With respect to aircraft maintenance applications, and as already discussed in section 3.1, Bowman and Schmee (2001) proposed a model to mitigate and manage financial risk for a Maintenance, Repair and Overhaul organization (MRO). Warrington et al. (2002) designed an aircraft reliability and maintenance model based on DES. Bazargan and McGrath (2003) proposed a model to improve aircraft availability and maintainability. Weckman et al. (2006) developed a decision support approach for modelling jet engine life and estimating maintenance requirement for engine restorations. Allgood, Olama and Lake (2010) designed a DES model combined with human factor analysis to study, analyse and evaluate performance improvements for an aviation cargo flow and security inspection system. In their model, they compared different scenarios by changing several model parameters and then assessing various indicators of the system's performance to assess its ability to service current needs and respond to additional requests. The model helped them to design new inspection requirements and policies without affecting operational cost or incurring in shipping delays. Horning et al. (2012) developed an interesting model based on DES to simulate the flight operations of an aircraft fleet and the resulting maintenance requirements. More recently, Van den Bergh et al. (2013) combined DES and data envelopment analysis (DEA) to design a model for managing aircraft line maintenance workforce scheduling.

3.8 Comparison of system dynamics and discrete event simulation

Different authors have studied from different perspectives the differences and similarities between SD and DES (Sweetser 1999; Brailsford and Hilton 2001; Morecroft and Robinson 2005; 2014; Greasley 2009; Tako and Robinson 2010; 2014; Brailsford 2014; Han et al. 2014). Given these reviews, the main differences between SD and DES can be summarised as follows.

Modelling structure: although both approaches seek to represent the behaviour of a real system over time in a simplified manner, their modelling structure clearly differs. In SD, the system is represented as a set of interconnected stocks and flows modelled in continuous time-steps. The stocks are used to describe the accumulation of elements, while flows regulate the amount of resources that enter or leave the stocks. In DES, the system is depicted as individual entities passing through a series of queues and activities at discrete time intervals. Queues are formed by entities waiting to be processed, while activities perform changes on entities.

Time handling is one of the main points of difference between SD and DES. In SD, system state changes are continuous; however, it uses a semi-continuous simulation approach, represented by differential equations that on a computer are modelled using small constant time steps to describe the changes in the system state. In DES, system state transitions take place at discrete points of time that are modelled at asynchronous time steps, when a change in the system (known as "event") occur.

Entities perspective: in SD, the elements of a system are modelled as a conglomeration of elements. There is no distinction between elements and they have the same features, so all that is measured is their flow through the system, not the specific status of a particular element. SD models the entities in a global perspective. In contrast, DES keeps track of each element. As each entity has particular characteristics, it is possible to accurately determine its status and movement through the system. DES models the entities in an individual perspective.

Modelling approach: SD models are deterministic, where the growth-decay of stocks is modelled as a delay, usually following an exponential or s-shaped behaviour. DES models are stochastic, with activity durations represented as random variables, usually based on probability distributions.

Modelling complexity is related to the previous point. SD explains the dynamic complexity of a system by using feedback structures (deterministic complexity). DES explains it by using several interrelated random processes (stochastic complexity).

Data requirements: SD models do not use a lot of data as they are more concerned with the structure of the model, using hard and soft data to model and represent the feedback structure. In contrast, DES models require a large amount of data to represent the randomness of the system, to model each individual entity and to maintain the list of events, but can model great complexity and detail.

Validation: In general terms, SD uses an open-box validation approach, where the structure, consistency, coherence and meaningfulness of the conceptual and mathematical models must be checked. However, due to its general perspective and the use of qualitative and quantitative data, validation in SD can be very subjective, complicated and arduous. DES mainly utilises a closed-box validation approach, where due to the large amount of data involved in the simulation process, a plethora of statistical and analytical tools can be used to compare the final results and behaviour these type of models against the actual results observed in the real system.

Application: due to their holistic perspective, SD models are more commonly used at a strategic level as learning laboratories, providing a broad picture (quantitative and qualitative) of system behaviour. DES, with its analytical point of view, has been widely used at operational and tactical level as operation simulators, providing statistical estimations of system performance.

Brailsford (2014) suggests that the main philosophical difference between SD and DES lies in the way the world is perceived, and therefore how the problem is conceptualised and modelled. She summarises that while SD is a top-down approach, DES is bottom-up methodology, that is, SD conceives the world as a system of dynamically interconnected elements and takes a 'helicopter view' of the world, whereas DES, takes a "microscope view", looking at the system in detail, analysing variability between individual components.

3.9 Summary

Assessing the literature review focused on the aviation industry, it can be seen that several approaches have been applied to analyse aircraft maintenance problems, providing different and valuable proposals for solving them from different perspectives. Due to its direct impact on daily operations, most of the studies were focused on the line maintenance process, particularly in managing the workforce and for disruption recovery. Analyses of the heavy maintenance process aim mainly to address the long-term planning and short-term scheduling of the maintenance service. However, despite the relevance of the problem, only a few studies attempted to analyse the uncertainty caused by unscheduled maintenance tasks.

It can be argued that aircraft heavy maintenance services have more than one of the characteristics of a complex project: uncertainty, number of elements and interaction between them. It can be argued that each heavy maintenance process is a complex project.

Every project has a certain degree of risk and this is greater for complex projects. Risk, as discussed previously, is compounded by a latent uncertain event and the consequences, either positive or negative, for project performance. In terms of heavy maintenance, risk is understood as the occurrence of unexpected damages or failures, events, which must be corrected by programming unscheduled maintenance activities which might require additional resources and tasks hindering the planning and execution of the service. Overrunning costs, delays in the delivery time and even poor service quality are potential consequences of these unexpected events.

Uncertainty is inherent in any project and aircraft maintenance is no exception. It cannot be avoided as it is extremely difficult to predict and control all events on major projects and to possess all relevant information. Traditionally, uncertainty has been associated with the randomness of an outcome and described by using probabilities. However, the concept is more sophisticated than just variability, as it also considers ambiguity, deficiency, imprecision, conflict or absence of information. These must all be considered to achieve a deeper understanding of uncertainty. In aircraft maintenance, as in most complex projects, the probability of the occurrence of an unexpected event is unknown in advance and generally a rough assessment is made based on perceptions or subjective judgements, giving rise to vague and ambiguous estimates.

Similarly to aircraft maintenance studies, two main perspectives have been utilised to study the planning, scheduling of complex projects under uncertainty: mathematical optimisation models and computer simulation models. In general terms, the former seeks to minimize delays or budget overruns, or to maximize resource utilisation, by designing robust and reliable project plans capable of dealing with the effects of unexpected events. This is usually accomplished by extending activity duration, increasing the budget or the availability of resources, a practice commonly called buffering. However, a major drawback of the optimisation models is that they are usually static and so not able to adapt to changes in the system. Therefore, they are generally used to define an initial project plan. Simulation has been widely used to characterise, analyse and predict the operation and evolution of a system over time. Its flexibility enables it to handle dynamism, irregularity, uncertainty, interrelatedness and complexity. In project management and aircraft maintenance, simulation has been particularly applied to optimisation and estimation and as a decision supporting tool. Nevertheless, it has also some disadvantages. It normally requires a considerable amount of data and computation time and so can be time-consuming.

SD is a flexible simulation approach for studying and managing dynamic and complex systems characterised by sophisticated interrelations between elements. It has been successfully applied to project management because of its capacity for analysing projects from a holistic perspective, representing their dynamism and non-linear relationships, and for handling qualitative and quantitative data. Most of the SD studies in project management have focused on studying and analysing project resources and their influence on project performance, the assessment of project changes and their relationship with project time and budget overruns and the causes and impact of project delays and disruptions. These studies have used SD throughout the whole life-cycle of the project, i.e. during the design and planning for estimation and risk assessment, in the execution as a monitoring and controlling tool and after the close-out for evaluation and learning.

DES is one of the most widely applied simulation approaches. It is used for modelling complex systems as they change at specific points in time and has the ability to capture in great detail the condition and flow of each element that passes through the system. In DES, to epitomise the stochastic nature of the system, the random duration of activities is described by utilising probabilistic distributions. However, modelling to such a level of detail and complexity requires a large amount of data, previous knowledge about probability distributions and the performance of a considerable number of runs of the model. Due to its analytic and comprehensive perspective, DES has primarily been used at operational and tactical level for estimation and optimisation.

Both simulation methodologies have shown their usefulness for analysing complex systems and have been extensively used in project management. However, aircraft heavy maintenance is a highly dynamic process, involving a large amount of scheduled and unscheduled maintenance tasks and requiring a vast number of resources that are in constant interaction. Moreover, maintenance services tend to vary significantly between them. For these reasons, it would be more useful and clear to have a broader view of the problem, describing the whole behaviour and feedback structure of the system rather than giving a detailed perspective of the status and complex evolution of each maintenance task throughout the maintenance service.

In light of the literature review, it can be argued that little investigation has been carried out into the effect of unexpected events on aircraft heavy maintenance. Considering the two key aspects of the problem, the complex interrelationship between the variables involved in the process and the uncertainty caused by the non-scheduled tasks, it can be suggested that SD provides a suitable methodology for analysing the impact of unscheduled tasks on delays and disruptions during the heavy maintenance process.

Despite all the advantages that SD offers to analyse dynamic and complex projects, it is not exempt from limitations and does not provide a complete solution by itself. Many authors have pointed out the drawbacks of SD (Jackson, 2001; Tako and Robinson, 2010; Brailsford and Hilton, 2001; Brailsford, 2014), mainly its inability to represent and describe randomness in a system. To overcome this limitation it will be necessary to support SD by using alternative tools to strengthen and enrich the proposed solutions. Sterman (2000) remarks that SD modelling does not stand alone and the best results are achieved in combination with other tools.

In the case of aircraft heavy maintenance, it is particularly necessary to analyse the stochastic nature of unscheduled tasks and to explore their relationship with different operational and maintenance variables in order to find a suitable way to estimate them. It has been observed through experience and in the literature review that a common practice in the airline industry is to roughly estimate the expected number of non-routine tasks by using a “rule of thumb” based on expert judgements or in the best case by using average values based on the behaviour of the fleet.

It is important, therefore, to propose a realistic and rigorous way for estimating the expected number of non-routine tasks considering not only the variability of information but also other dimensions of uncertainty such as ambiguity and imprecision. It is proposed to analyse from a different perspective the uncertainty of unforeseen events that can cause delays to and disruption of complex projects by using evidential reasoning as an inference process rather than traditional probability approaches.

It is proposed to combine the system dynamics model with the evidential reasoning model to assess the impact of unscheduled tasks on the delays and disruptions within the heavy maintenance. The first model will address the complexity of the process, while the second will focus on the stochastic nature of the unscheduled tasks.

This research is focused on a particular but common problem in aviation maintenance. However, delays and disruptions caused by unscheduled tasks are not exclusive to this field and are a frequent and recurrent problem in other complex projects. Thus, the models, results and learning developed during this research can be generalised and applied to analyse problems with similar characteristics.

Chapter 4: Methodology and research design

A current problem in the aircraft maintenance industry has been presented. Its existence and relevance have been supported by other studies and, through an extensive literature review, potential opportunity areas and promising methods have been identified. The next step is to define a formal and systematic process in which this research will be carried out to address the proposed problem. This chapter aims to formalise and describe the methodology and research design adopted in this thesis and is structured as follows: firstly, the philosophical bases are discussed by positioning the ontological and epistemological assumptions considered in this research to further define the methodological approach adopted. Secondly, the proposed research design is explained along with its different steps. Thirdly, the two main methods utilised in this research for analysing the problem are discussed: system dynamics (SD) and the evidential reasoning (ER) rule. Finally, the research questions in which this research is grounded are presented and subsequently operationalised to explain how they are going to be addressed.

4.1 Ontological and epistemological basis of the research

Before going any further, it is important to establish the ontological and epistemological foundations of this research to formalize the type of methodology to utilise. The root of the dilemma between choosing quantitative or qualitative research is more philosophical than methodological, since the researcher's selection of the philosophical assumptions largely influences the choice of the methodology approach (Dobson, 2002; Krauss, 2005).

According to Burrell and Morgan (1979) social theory can be usefully analysed in terms of four broad paradigms: functionalist, interpretive, radical humanist and radical structuralist. The functionalist paradigm considers society to be concrete, real, objective and systemic. Its essential nature can, therefore, be known and measured to determine human behaviour. Functionalism is a regulative, rational and pragmatic approach focused on producing useful empirical knowledge. In contrast, the interpretative paradigm assumes that the social world does not concretely exist, as it is the result of the subjective experience of individuals. Therefore, its meaning, regularity and effect on human behaviour is highly variable. Knowledge is based on a set of subjectively determined concepts and rules. The radical humanist paradigm is focused on finding ways to link knowledge and action as a way to create change and transcend alienation. Finally, the radical structuralist paradigm is concerned with understanding social conflicts and contradictions, using knowledge and action as a way to transcend domination (Barley, 1980; Morgan, 1980).

Each of these paradigms can be classified according to their location within two meta-theoretical dimensions: the nature of social science and the nature of society. Both dimensions are characterised by opposing perspectives: objective vs. subjective in the case of the former and regulation vs. change in the case of the latter. The four paradigms group various schools of thought, with diverse approaches and standpoints, but share common fundamental assumptions about the nature of the reality that they address (Morgan, 1980). Burrell and Morgan (1979) conceptualise the subjective and objective perspectives of social sciences in terms of four philosophical assumptions related to ontology, epistemology, human nature and methodology.

Ontology refers to assumptions and convictions about the essence, nature and existence of the world or a certain phenomenon (Burrell and Morgan, 1979; Mingers and Brocklesby, 1997; Meredith, 2001; Rotaru et al., 2014). Nominalism, also known as conventionalism, conceives the social world as a cognitive and artificial creation formed exclusively by names and labels. It does not acknowledge any real structure and is defined by Burrell and Morgan (1979) as a subjectivist approach. On the other hand, realism, located in the objectivist perspective, considers that reality exists even if there is no awareness of its presence as it is created by tangible and immutable structures external to human cognition (Burrell and Morgan, 1979).

Epistemology is the theory of knowledge. It comprises the study of the nature, bases and scope of acquired knowledge, how it is used to understand the world or a particular phenomenon and how it is transmitted to others (Burrell and Morgan, 1979; Mingers and Brocklesby, 1997; Meredith, 2001; Rotaru et al., 2014). Burrell and Morgan (1979) considers anti-positivism, or interpretivism, as part of the subjectivist approach and positivism as objectivist. Anti-positivism assumes that the social world is relativistic and is only understood with reference to the individuals involved in the studied phenomena. This perspective asserts that the social sciences are not able to generate objective knowledge. Positivism, on the other hand, attempts to describe the social world based on observed causal relationships and regularities. This epistemological approach is characterised by the ability to verify hypothesis through experimentation and therefore it dominates the natural sciences.

Human nature encompasses the relationship between individuals and their environment and how the individual is perceived in a given social-scientific theory (Burrell and Morgan, 1979). Burrell and Morgan (1979) identify two opposite approaches: voluntarism, which is subjectivist and determinism, which is objectivist. Voluntarism regards individuals as creative and autonomous beings, constructing and controlling their own environment. Determinism considers individuals and their experiences and activities as products of the environment conditioned by external situations.

These three previous philosophical assumptions shape and influence the methodological approach that refers to the systematic way in which knowledge is pursued and acquired (Burrell and Morgan, 1979; Smyth and Morris, 2007). The ideographic standpoint, which can be classified as subjectivist, assumes that the social world can only be understood by considering the relationship of the individual with their environment in order to gain insight about the subject under investigation. The objectivist nomothetic approach considers that social sciences can be studied using systematic and rigorous techniques focused on analysing data and testing hypotheses (Burrell and Morgan, 1979).

The two proposed methods for this research are quantitative approaches and as such are located in the functionalist paradigm according to the (Burrell and Morgan, 1979) framework. However, they are also able to consider and handle subjective information. With this in mind and given the interpretative nature of the models (mental and formal), it may be more appropriate to place them somewhere between the interpretative and functionalist paradigms.

Considering the philosophical essence of the problem, as per the ontological dimension, this research assumes a realistic stance, regarding organizations and their problems, in this particular case airlines and MROs, as real, concrete and formal structures which can be observed and studied, where their relevant problems can be analysed and measured.

At first glance, given the conditions of the problem and considering the characteristics of the proposed methods, it appears that this research makes a positivist epistemological assumption, takes a deterministic approach towards human nature and adopts a nomothetic methodology. However, assuming this position might be extreme, inflexible and unrealistic, excluding individuals' interpretations about the problem and considering that the decision maker has no influence on the environment and thus on the problem. Project overruns, for example, would be presumed to depend exclusively on external factors, with managers' decisions and perceptions not affecting project performance.

It would be more fruitful to adopt an intermediate position between the positivist and interpretivist approaches with less radical assumptions about human nature, allowing for a more flexible methodological approach. This view is supported by Gioia and Pitre (1990), Weaver and Gioia (1994) and Remenyi et al. (1998) who agree that rather than assume a dichotomy between objective-subjective, these approaches should be considered as complementary, with their overlapping boundaries offering a transition across paradigms. In the same vein, Blatter and Haverland (2012) propose an “epistemological middle ground” that rejects fundamentalist and extreme epistemological positions. This suggests that social science research is based on thorough reflection on the relationship between empirical evidence and abstract concepts and assumes them to be commensurable.

Likewise, the structuration theory of Giddens (1984) proposes that the objective and subjective perspectives are intertwined and it is not possible to separate them. This theory allows for the articulation of a set of relationships amongst the competing perspectives, where reality arises from the constant interaction between structure and meaning, rather than one or the other (Weaver and Gioia, 1994; Mingers and Brocklesby, 1997).

Similarly, critical realism (Bhaskar, 1975, 1979, 1994) describes a coexistence of intransitive and transitive objects of knowledge, with the former describing real entities that exist independently to individual perspective and the latter resulting from individual experience, representations and descriptions used to produced knowledge. Social theories, therefore, cannot be purely descriptive or evaluative. There must be balance between them, as facts and values cannot be separated (Mingers and Brocklesby, 1997; Mingers, 2000, 2003, 2006). Critical realism provides a “position that would allow the combined use of the objectivist worldview that is based on the assumption of existence of ‘an objective reality which can be measured and described’ and a more subjectivist position that allows ‘the mind of the decision maker’ to interpret the reality based on the available, including value-laden, data” (Rotaru et al., 2014, p.98).

Taking a more flexible and inclusive philosophical position on the objective-subjective dichotomy can allow for a multi-methodological position, where “methods and techniques from the original competing paradigms may be combined without the agent having to constantly adjust their philosophical position depending upon whichever method is being used at any time” (Mingers and Brocklesby, 1997, p.498). This position can enrich research by using distinct methods and tools to overcome the individual weaknesses of particular methods. Mingers (2003) highlights the benefits of considering a multi-methodological approach. The real world is multidimensional and material, social and personal factors should all be considered. Different ways of appreciation, analysis, assessment, and action are required. Such a wider perspective may enhance the richness and reliability of results.

In the light of this, it is proposed to use an exploratory perspective based on a case study rather than a nomothetic approach. A case study seems the most suitable approach as it can help to gain insight into the problem by describing and testing the initial hypothesis via a holistic in-depth exploration of the dynamics, complexity and uniqueness of the organization and the project in question, using both quantitative and qualitative data (Eisenhardt, 1989; Yin, 2003; Simons, 2009; Farquhar, 2012). The exploratory approach is proposed as a comprehensive and systematic method for improving the understanding of a particular phenomenon, especially when little is known about it, by determining and examining significant issues and crucial variables and defining the necessary questions and hypotheses (Eisenhardt, 1989; Stebbins, 2001; Yin, 2003; Streb, 2010). In this research, the initial empirical assumptions regarding the problem were further explored and supported using a wide-ranging literature review, leading to the formulation of the research questions and supporting the use of the proposed methods.

4.2 Research design

This research has an empirical motivation based on personal experience in the airline industry and supported by the insights of various authors and experts in the field. From the evidence presented thus far, it can be argued that delays and disruptions during the execution of aircraft heavy maintenance services is a real, frequent and relevant problem with important economic and operational consequences for both airlines and MROs. Schedule and budget overruns in maintenance checks stem from the complexity of managing resources and the occurrence of unplanned maintenance tasks. The literature review has shown that these features and problems are not exclusive to the heavy maintenance process but are common in complex projects across different industries such as construction, engineering, manufacturing and software development.

Prior studies have noted the deficiency of the traditional project management methods when dealing with highly dynamic, unstable and uncertain projects. To overcome these limitations, simulation has been applied as an alternative approach for studying complex projects, due to its capability to capture the development of a project over time. System dynamics (SD) has proved to be a useful method for analysing the operation and behaviour of complex and dynamic projects, especially for conceptualising projects from a holistic perspective and for capturing the dynamism of elements and the relationships between them.

Considering the main features of the problem and the ability of SD to analyse dynamic and complex systems, it is suggested to use SD for considering the effect of unscheduled tasks on delays and disruptions within aircraft heavy maintenance checks. However, it has been shown that SD has limited capabilities for dealing with randomness in the system. To cope with this drawback, it is proposed to use the evidential reasoning rule as a complementary tool to handle uncertainty.

The SD model helps to analyse the interrelationship of scheduled and unscheduled tasks and its impact on delays and disruptions and also to explore the effect on project performance of the occurrence and discovery of damage and discrepancies during unscheduled tasks. The ER rule model complements the SD model by estimating unscheduled maintenance tasks by analysing historical data related to the usage and maintenance of an aeroplane, taking into account its importance and quality. The enhanced SD model can be used as a decision supporting tool by exploring different policies and strategies for the planning and control of the maintenance service.

Research design is the coherent and logical process of merging the research elements, aiming to successfully address the research problem. It considers the creation, collection, organisation, measurement, analysis and interpretation of data (Vaus, 2001; Perri and Bellamy, 2012). In this thesis, to accomplish the research objectives, an iterative research design process is proposed, based on the research frameworks proposed by Eisenhardt (1989), Yin (2003), Hernández Sampieri et al. (2010) and Creswell (2014), and using as a reference the modelling processes suggested by Forrester (1994), Sterman (2000) and Pidd (2004a). The research design consists of eight phases as described below:

(1) *Problem structuring*: based on experience and the literature review, the process and the problem are explained and contextualised, the relevance of the problem is supported and the suitability of the proposed methods is discussed. The aim of this phase is to define the problem and its main features, delineate the research objectives and scope, and define the research questions.
(2) *Conceptual model*: the SD model refers to the initial causal-loop diagram that depicts the main components and their relationships, and the possible causes of the problem. The ER rule model describes the basic configuration for aggregating the variables to estimate the non-routine rate. The aim of these preliminary models is to aid the gathering of data.
(3) *Data collection*: where qualitative and quantitative data is collected, organised and analysed. Continuous discussions with experts and practitioners are used to validate the dynamical hypothesis and to improve the causal loop diagrams. Maintenance and operational records are used as input variables for building the ER rule model to analyse the uncertainty of the non-routine tasks, and these records are also used to build and test the SD simulation model.

(4) *Model refining*: the improvement of the initial concepts by including additional information from the previous step and taking into account experts' modifications to describe the problem more accurately. For the SD model, considering adjustments in the variables, relationships and feedback loops to improve the structure of the model. For the ER rule model, considering the inclusion of or changes in the input variables.

(5) *Formulation and simulation:* describing the transition from the conceptual model to a detailed mathematical model. For the SD model, considering the development of the stock-flow diagram and the definition of its equations to finally complete the simulation model. For the ER rule model, referring to the application of the ER rule process to combine different pieces of evidence to estimate the non-routine rate.

(6) *Model validation:* assessing if the model complies with its intended purpose. For the SD model, a white-box validation approach is considered as this step in SD could be very difficult and subjective. The model is tested and verified throughout the modelling process, with reference to the opinions of experts regarding its meaningfulness. Verifying the structure of the model and the mathematical equations and evaluating the results. For the ER model, a black-box validation approach is carried out where the estimated non-routine rates are compared with the observed values to assess the model performance. Additionally, a sensitivity analysis is carried out to determine the effect of certain variables on model efficiency.

(7) *Scenario analysis:* the use of the models for building and comparing scenarios to allow the testing of different policies and for learning the behaviour of the system under certain circumstances, for instance, to assess the impact of different resource allocation strategies and the effect of the occurrence and discovery of discrepancies on project performance.

(8) *Findings and conclusions*: with the results of both models and the knowledge gained during the investigation process, the research questions can be answered and relevant findings can be discussed, stating also the limitations of the study and suggesting potential further research.

4.3 System dynamics modelling

System dynamics comprises the development of qualitative and quantitative models. Causal loop diagrams are used to describe the structure of the system and the relevant interrelation between its elements. Stock and flow diagrams are used to build a dynamic model and simulator depicting the evolution and performance of the system through the time.

SD models are useful for coping with dynamic complexity resulting from the sophisticated relationship between the elements of the system, where a change in the system can have an impact elsewhere in the short-, medium- or long- term. However, the cause-effect relation is not always obvious and sometimes even counterintuitive (Sterman, 2001).

Moreover, SD models are not intended to predict or forecast the future. They serve to build scenarios and explore potential futures that may unfold if the assumptions behind the scenarios turn out to be true. They are used to test the consequences of choosing certain strategies or policies, to anticipate problems, reveal surprises and hidden pitfalls (Morecroft, 2007).

4.3.1 Causal loop diagrams

A causal loop or influence diagram is a graphical representation of the main system structure, depicting the interrelation between the elements within the system. Causal loop diagrams, (CLDs) through the abstraction and representation of mental models of individuals and teams, help to change the problem perspective from a cause-effect relation to a complex structure formed by feedback loops. They are particularly useful for enhancing the understanding of a problem by capturing potential causes of dynamics and feedback, and for illustrating qualitative and quantitative information (Sterman, 2000; Morecroft, 2007; Brailsford, 2014).

CLDs are used to describe cause and effect relationships between the variables and to represent the feedback structure of the system. The cause-effect relation consists of two or more variables connected by arrows, while an interconnected array of arrows and variables depicts the feedback process. The basic elements in the CLDs are: variables, causal links, link polarity, loops, loop identifier and delays, as shown in Figure 4-1.

Variables are the elements in the system that change due to certain stimuli outside or inside. *Causal links* illustrate the direction of the causal effect between the variables and are represented using arrows. *Link polarity* describes the direction of change when independent variables change and is depicted by a positive (+) or negative (-) symbol. A positive or negative link means that when the cause increases or decreases the effect will change accordingly. Link polarities describe the structure of the system rather than the variable's behaviour as they explain what would happen if there was a change, not what actually is happening (Sterman, 2000).

A *loop* refers to a closed chain of cause and effect (Madachy, 2008) in which a cycle is completed when the variable is influenced by the change initially triggered by it. According to its effect, there are positive and negative loops also called reinforcing or balancing loops respectively. A positive or reinforcing loop acts as an amplifier or a reinforcing mechanism of growth: for example, an increase in one variable will eventually lead to a significant boost of that variable. In contrast, a negative or balancing loop acts as a controller or a mechanism of equilibrium: for instance, an increment in one variable will eventually lead to a reduction of that variable. The main reinforcing and balancing loops in the CLD are depicted by using a *loop identifier*, which must have the same direction of the loop that it is describing. It should also be numbered and include a brief textual description (Dangerfield, 2014).

Delays are usually represented as two dashed lines crossing a causal link indicating a lag in the cause-effect relation between two variables, i.e. if there is a delay, the cause will take longer to have an effect (Morecroft, 2007).

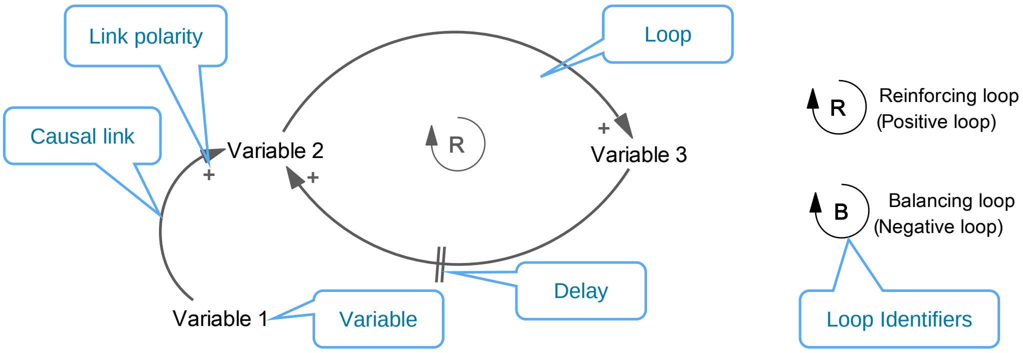


Figure 4-1 Causal loop diagrams notation

Figure 4-2 shows a simple example of a causal loop diagram representing changes in population. The diagram is formed by two loops, one reinforcing and one balancing. For instance, if average fertility increases, the birth rate will increase too leading to a growth in the population, which in turn will cause a further rise in the birth rate, causing a surge in the population, closing the reinforcing loop. If population grows, death rate will also increase. This increment in the death rate will cause a reduction in the population, balancing this loop. However, the death rate is affected by the average life time expectancy, where if the latter increases, there will be fewer deaths and if it decreases, more deaths will be expected.

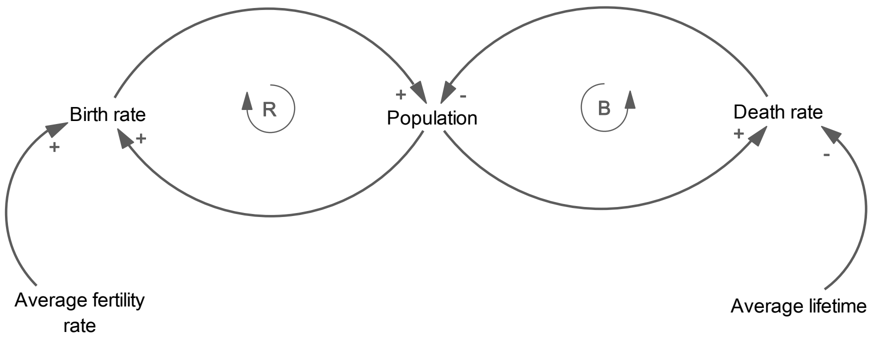


Figure 4-2 Causal loop diagram example: the dynamics of population. Based on (Sterman, 2000, p.138)

Different authors have suggested significant benefits of CLDs. Some even consider CLDs as a separate method from SD, highlighting its capacity to generate insight about the system and its feedback structure, materialise and communicate mental models, create consensus and help problem solving, even without using quantitative models and computer simulation (Wolstenholme, 1993; Ballé, 1994; Sherwood, 2011; Merrill et al., 2013).

4.3.2 Stock and flow diagrams

Though CLDs are very useful to broaden the conceptualising of the problem and to improve the understanding of the system and its feedback structure, they are not able to capture the stock and flow structure of the system and fail to show the changes and performance of the system through the time (Sterman, 2000; Morecroft, 2007). The CLDs are generally used to develop the stock-flow diagrams, which are an essential step for building a quantitative SD model (Brailsford, 2014).

Stock-flow models resemble a hydraulic system formed by connections of tanks, pipes and valves, where a homogeneous mass of elements flows through the system. A stock and flow network consists of four main components, shown in Figure 4-3: 1) Stocks, represented by rectangles, serve as containers to accumulate the elements that flow through the network. 2) Flows, represented by a thick arrow, describe the movement of elements across the network. When an arrow points towards a stock, it adds elements and is called *inflow*. When the arrow emerges from the stock, it drains elements out of it, known as *outflow*. 3) Valves or rates are located in the middle of a flow to regulate or control the movement of elements in the network. 4) Clouds represent the initial and final points of the flow. Whatever is beyond these limits is out of the scope of the model (Sterman, 2000; Morecroft, 2007; Dangerfield, 2014).

In addition to these four components, two supplementary elements are required to describe the feedback structure of the system, causal links and auxiliary variables (also known as converters). The former are represented by a light arrow, similar to the causal loop diagram, describing the influence of one variable on another. The latter is an intermediate variable intended to represent something planned or desired, a target or a management goal. Once all the components are integrated, the stock-flow diagram has the ability to represent the two fundamental concepts of SD theory: stocks and flows, and feedback (Sterman, 2000).

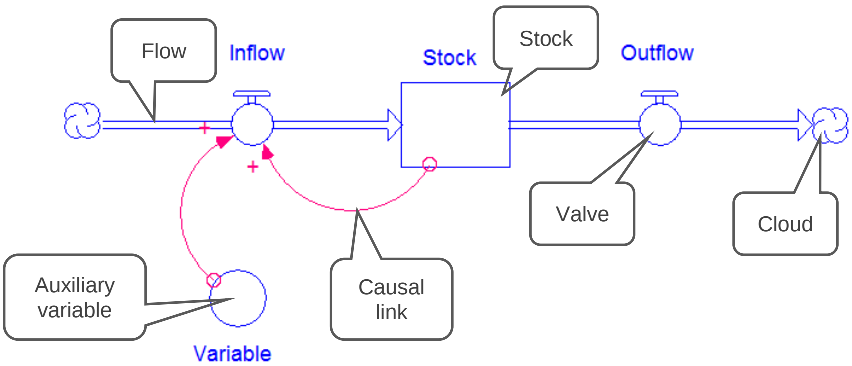


Figure 4-3 Stock and flow diagram notation

Figure 4-4 illustrates an example of a stock and flow diagram, depicting population dynamics. The population stock represents the accumulated number of individuals at a particular point in time. Total population increases through the number of births per unit of time and decreases by the number of deaths in the same period. The number of births depends on two variables: the number of population members and the fractional birth rate, which is the expected number of births per capita. In a similar way, the number of deaths results from population size and fractional death rate. The fractional birth and death rates depend on the relation between population size and carrying capacity. The latter expresses the maximum capacity of the environment to support a certain population, for instance because of availability of food or physical space.

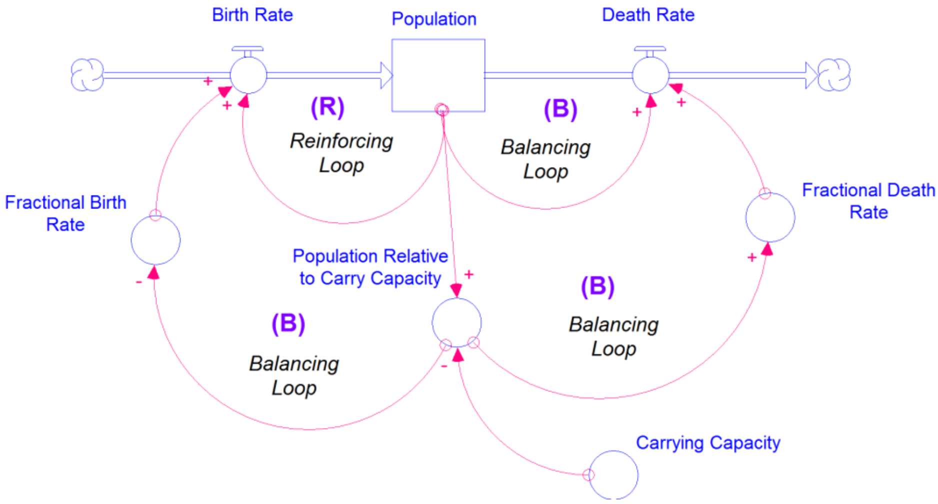


Figure 4-4 Stock-flow diagram example: the dynamics of population. Based on (Sterman, 2000, p.285)

4.3.3 System dynamics equations

In theory, System Dynamics is a constant time advance methodology, but in practice this can only be achieved using a semi-continuous simulation, based on small time steps represented by differential equations (Dangerfield, 2014). Modelling in finely sliced time steps is a technical and computational strategy to establish the time units in which the system evolves, in other words, the stocks are updated according to their related inflows and outflows to yield a new system state at every time step (Morecroft, 2007). The smaller the time step (*dt*) the more precise the model will be. However, it will also increase computation load and the risk of round-off and truncation error, making it important to define an appropriate *dt* according to the model features. Morecroft (2007) suggests using a *dt* for no longer than a quarter of the smallest time unit in the model.

The final step to complete the quantitative model is to transform the stock-flow diagrams into equations. Stocks and flows have a precise and unambiguous mathematical meaning. Stocks accumulate or integrate their flows, while a flow depicts the rate of increase or decrease of a stock. The status of a stock is represented by an integral equation from an initial time t_0 to the current time t, considering the stock level at t_0 and the difference of the inflow and outflow between t_0 and t, as shown in equation (4-1). Correspondingly, the net rate of change in a stock is given by a differential equation describing the difference between the inflow and outflow at a particular point in time, presented in equation (4-2). The flows are equations in terms of the stock and other state variables and parameters (Sterman, 2000).

$$Stock(t) = \int_{t_0}^{t} [Inflow(t) - Outflow(t)]dt + Stock(t_0) \tag{4-1}$$

$$\frac{dStock(t)}{dt} = Inflow(t) - Outflow(t) \tag{4-2}$$

Every component in a stock-flow diagram is defined by an equation, a parameter or a graphical function. It is worth mentioning that each component in the model is associated with its respective units to help the representation and description of the model. Therefore, it is very important to conduct a dimensional analysis throughout the modelling process to ensure the consistency of units and measures.

Transforming a qualitative causal loop diagram into a complete and robust system dynamics model and simulator that describes as closely as possible the system and its main troubles requires experience and takes a lot of time and effort, but the result is much more than a diagram or a static analytical solution. It is an inference engine to diagnose performance problems, a virtual world to explore dynamic complexity and stimulate learning and a laboratory to design and experiment new policies and strategies (Morecroft, 2007).

4.3.4 Validation

As described in the literature review, a model is a representation of a real or conceptual system that aims to analyse and understand that system. In management science, models are used to understand, improve and manage real systems and as a decision support tool for testing strategies and policies in the system. A model's definition has two main elements: the real system and the model, where the main objective of the model is to represent in the best way the real system. However, important questions arise. How to assess the quality and accuracy of the model? How to be sure that model is representing the real system? How to measure if the model is accomplishing with its objective and if it is really useful?

Pidd (2003) points out that validation is not as straightforward as it seems, and highlights the importance of the epistemological perspectives during the model assessment. Citing Déry et al. (1993) he presents three different standpoints. First, the philosophical, which describes that the model should be based on objective research, rigorously tested, expressed mathematically, shown to be useful and should pass crucial tests designed to show its adequacy. Second, the historical perspective, which considers that validation changes in time and location and refers to the acceptability of the model to the expert community who are operating with a dominant paradigm. Finally, the sociological perspective, strongly related to the previous, assesses the validity of the model according to its acceptance, understanding and usefulness for the intended community.

Three terms are commonly used during the validation process: validity, credibility and acceptability. Validity expresses the accuracy of the model to represent the real system. Credibility describes the willingness of the users to take decisions based on the information obtained from the model. Acceptability includes the model and the whole modelling process, but also the relation between the modeller and the user and can be understood as the result of the validity and credibility (Robinson and Pidd, 1995).

Validation is the process that ensures that a model is suitable and adequate for the purpose for which it was built (Pidd, 2003; 2004b). Validity and testing is the process of confidence building amongst those who will use the model (Forrester and Senge, 1980; Sterman, 2000). However a thorough and complete validation that ensures the model is fully correct and exactly represents the real system is impossible. It is, therefore, important to bear in mind that the validation process has certain limitations (Pidd, 2004a; Sterman, 2000).

It would be impractical to carry out an assessment once the model has been completed. As the previous definitions stated, validation is a process not just a single step and therefore should be performed at different stages during modelling. In this regard, Pidd (2003) emphasises that assessment and validation are activities that should be performed at every stage of the simulation project. Morecroft (2007) adds that validation includes different tests that need to be performed throughout the modelling process with the aim of assessing the quality of both the model and the model building process. He proposes three categories of tests that have been proved particularly useful in practice: tests of model structure, tests of model behaviour and tests for learning.

Model structure tests help to evaluate if the feedback structure and equation formulations of the model are consistent with the available facts and descriptive knowledge of the real system. The model behaviour tests compare and assess the results of simulation with real system behaviour. Tests for learning are intended to evaluate if the model users have learnt and gained insight about the structure and behaviour of the system (Morecroft, 2007). Recalling the idea that a full validation is impossible, Morecroft (2007) remarks that these tests do not prove whether a model is valid in the sense of being the perfect depiction of the real system. They can, however, when used in combination during the modelling process, demonstrate to modellers and clients that the model is of acceptable quality to meet the intended purpose, providing a reliable basis for understanding and analysis.

Taking into consideration the previous definitions and philosophical standpoints, it is worth noting that during the validation process at least two points must be ensured: firstly, that the model should represent and depict the real system as closely as possible and secondly, it has to be meaningful, relevant and useful for the user. In order to achieve these purposes, the validation must be performed in different steps throughout the modelling process.

Considering the validation tests suggested by Morecroft (2007), it is necessary for this particular project to firstly verify the structure and design of conceptual models with reference to the experts in the field, i.e. through feedback sessions to revise, discuss and improve the conceptual models in order to capture and include the relevant missing points of the problem and ensure that the structure and representation of the model is consistent with the knowledge and perception of the system. Later, during the design and building of the stock and flow model and also in the development of the ER rule model, the mathematical equations must be reviewed and verified to ensure that they are coherent in characterising the problem and dimensionally have a proper meaning compared with what they represent in the real world. Additionally, it is necessary to run more tests under extreme conditions, to see if the results of the equations make sense when their inputs take extreme values. The next set of tests consists of evaluating overall system performance. Here, it is necessary to use real data, i.e. maintenance and operational records, to compare with estimated results obtained throughout the model. Finally, it is necessary to review the results with reference to expert opinion, check if they match overall system behaviour and, through building scenarios, evaluate the usefulness, learning and benefits to users obtained by using the model.

4.4 Evidential Reasoning rule

The evidential reasoning (ER) rule is a general conjunctive probabilistic reasoning process for combining independent pieces of evidence, taking into account their weights and reliabilities. The ER rule is established as a rational and rigorous reasoning method rather than a model to explain human judgements. The ER rule explains that the combined degree of belief in which two pieces of independent evidence jointly support a proposition is formed by two parts: the bounded sum of their individual support and the orthogonal sum of their collective support (Yang and Xu, 2013).

It has been demonstrated by Yang and Xu (2013) that the ER algorithm (Yang, 2001; Yang and Xu, 2002; Yang et al., 2006) and Dempster's rule in the theory of evidence (Dempster, 1967; Dempster, 1968; Shafer, 1976) are special cases of the ER rule. In the case of the ER algorithm, the reliability is equal to the weight and the weights are normalised for all pieces of evidence. In Dempster's rule, each piece of evidence is considered fully reliable.

One of the advantages of the ER rule is that it can be used to combine different pieces of evidence regardless of the order of aggregation and without affecting the final results. This feature is due to its being based on the orthogonal sum operation and therefore it acquires the basic properties of being associative and commutative (Yang and Xu, 2013; Yang and Xu, 2014). Furthermore, through the use of a novel reliability perturbation analysis, the ER rule can be applied to combine multiple pieces of evidence that are fully reliable but highly or completely conflicting, overcoming in this way the non-definition and counter-intuitive drawbacks of Dempster's rule (Zadeh, 1986; Murphy, 2000; Haenni, 2005; Huynh et al., 2006).

In the ER rule, three elements are necessary when combining two pieces of evidence: their *belief distributions*, *weight* and *reliability*.

Firstly, a piece of evidence is represented by a belief distribution (*BD*) that is defined on the power set of the frame of discernment, which is a set of hypotheses that are mutually exclusive and collectively exhaustive, thus allowing the assignment of basic probabilities not only to singleton hypotheses but also to any of their subsets (Yang and Xu, 2013). A belief distribution is considered as the most natural and flexible generalisation of conventional probability distribution as it resembles the human rational process, allowing inexact reasoning at whatever level of abstraction (Gordon and Shortliffe, 1985). Particularly, a BD is equivalent to a conventional probability distribution when basic probabilities are only assigned to singleton hypotheses (Yang and Xu, 2013).

Weight represents the importance of a piece of evidence based on the decision maker's preferences over the evidence, whereas reliability measures the quality of a piece of evidence to provide correct assessment of or a solution to a given problem. While weight is subjective, reliability aims to be an objective measurement. The former relies on who makes the judgement, whilst the latter assesses the evidence independently of who may use it (Smarandache et al., 2010). Yang and Xu (2014) explain that weight and reliability are the same if all the pieces of evidence are measured in the same joint space. However, when the different pieces of evidence are gathered from different sources and measured in different ways, the weight and reliability are dissimilar. Additionally, Yang and Xu stress the importance of the part that the weight and reliability of all pieces of evidence play in the inference process. Therefore, they need to be estimated with care and rigour.

In the ER rule, these three elements are used to define what is known as a weighted belief distribution with reliability ($WBDR$), where m_j represents the $WBDR$ of a particular piece of evidence e_j, and j is the j^{th} number of independent pieces of evidence, as expressed in equation (4-3) (Yang and Xu, 2013):

$$m_j = \left\{ \left(\theta, \tilde{m}_{\theta,j}\right), \forall\theta \subseteq \Theta; \left(P(\Theta), \tilde{m}_{P(\Theta),j}\right) \right\} \tag{4-3}$$

Considering that Θ is a set of mutually exclusive and collectively exhaustive propositions referred as a frame of discernment, and that $P(\Theta)$ represents the power set of Θ consisting of 2^{Θ} subsets of Θ; then $\tilde{m}_{\theta,j}$ measures the degree of support for a proposition θ from evidence e_j with both the weight and reliability of e_j taken into account, as defined in equation (4-4) (Yang and Xu, 2013; Yang and Xu, 2014):

$$\tilde{m}_{\theta,j} = \begin{cases} 0 & \theta = \emptyset \\ c_{rw,j} m_{\theta,j} & \theta \subseteq \Theta, \theta \neq \emptyset \\ c_{rw,j}\left(1 - r_j\right) & \theta = P(\Theta) \end{cases} \tag{4-4}$$

where $m_{\theta,j} = w_j p_{\theta,j}$ is referred to as a basic probability mass, $c_{rw,j} = 1/(1 + w_j - r_j)$ is a normalisation factor, which is uniquely determined to satisfy $\sum_{\theta \subseteq \Theta} \tilde{m}_{\theta,j} + \tilde{m}_{P(\Theta),j} = 1$ given that $\sum_{\theta \subseteq \Theta} p_{\theta,j} = 1$, and $1 - r_j$ represents the unreliability of evidence e_j.

Finally, the combined degree of belief in which two independent pieces of evidence e_1 and e_2 jointly support θ, denoted by $p_{\theta,e(2)}$, is generated by the orthogonal sum of their weighted belief distributions with reliability and represented by $\hat{m}_{\theta,e(2)}$, given as follows (4-5) (Yang and Xu, 2013; Yang and Xu, 2014):

$$\begin{gathered} p_{\theta,e(2)} = \begin{cases} 0 & \theta = \emptyset \\ \dfrac{\hat{m}_{\theta,e(2)}}{\sum_{D \subseteq \Theta} \hat{m}_{D,e(2)}} & \theta \subseteq \Theta, \theta \neq \emptyset \end{cases} \\ \hat{m}_{\theta,e(2)} = \left[(1 - r_2) m_{\theta,1} + (1 - r_1) m_{\theta,2}\right] + \sum_{B \cap C = \theta} m_{B,1} m_{C,2} \quad \forall\theta \subseteq \Theta \\ \hat{m}_{P(\Theta),e(2)} = (1 - r_2)(1 - r_1) \end{gathered} \tag{4-5}$$

From equation (4-5) it can be seen that the combined degree of belief of the two pieces of evidence is formed by two parts: the bounded sum of their individual support (the first term of the equation in square brackets) and the orthogonal sum of their collective support (the last term of the equation). The first term refers to the sum of the degree of individual support of the basic probability mass (or *weighted belief distribution*) $m_{\theta,1}$ from the first piece of evidence e_1 restricted by the unreliability $(1-r_2)$ from the second piece of evidence e_2, with its counterpart, i.e. the degree of individual support $m_{\theta,2}$ from e_2 limited by $(1-r_1)$. In other words, the first term of the ER rule indicates that the higher the reliability of a piece of evidence, the less opportunity it leaves for other evidence to provide its information. The second term of the ER rule denotes the orthogonal sum of the collective support of the evidences, which measures the degree of intersected or heterogeneous support for the proposition θ from both pieces of evidence e_1 and e_2 (Yang and Xu, 2013).

Similarly, the recursive ER rule to combine in any order L pieces of independent evidence e_i $(i = 1,2,\ldots,L)$ with weight w_i and reliability r_i is given in eq. (4-6) (Yang and Xu, 2013):

$$p_{\theta,e(L)} = \begin{cases} 0 & \theta = \emptyset \\ \dfrac{\widehat{m}_{\theta,e(L)}}{\sum_{D\subseteq\Theta}\widehat{m}_{D,e(L)}} & \theta \subseteq \Theta, \theta \neq \emptyset \end{cases}$$

$$\widehat{m}_{\theta,e(i)} = \left[(1-r_i)m_{\theta,e(i-1)} + m_{P(\Theta),e(i-1)}m_{\theta,i}\right] + \sum_{B\cap C=\theta} m_{B,e(i-1)}m_{C,i} \quad \forall\theta \subseteq \Theta \tag{4-6}$$

$$\widehat{m}_{P(\Theta),e(i)} = (1-r_i)m_{P(\Theta),e(i-1)}$$

Yang and Xu (2013) demonstrate that when the weight of a piece of evidence is zero, the evidence will not play any role in the aggregation process, i.e. evidence with a null importance can be regarded as a neutral element. Nevertheless, this property is not equivalent for a piece of evidence with zero reliability.

Yang and Xu (2014) propose the ER rule as a generalised Bayesian inference process, explaining that whilst traditional Bayesian inference requires accurate probabilities and likelihoods to obtain reliable results, the ER rule can be used even when evidence is ambiguous and inaccurate.

4.4.1 Dependency between the pieces of evidence

According to Yang and Xu (2013, 2014), the ER rule is appropriate only when pieces of evidence are independent, in other words, if the information provided by a piece of evidence does not depend on information carried by other pieces of evidence. They further suggest that other aggregation rules should be used to aggregate multiple dependent pieces of evidence. However, recently Yang and Xu (2015) have proposed a novel approach to deal with the dependency of pieces of evidence by including a new variable in the ER rule (referred to in this study as alpha-index). The alpha-index($\propto$) is introduced in the second term of the ER rule, as shown in Equations (4-7) and (4-8), i.e. in the orthogonal sum of the collective support of the evidences, as shown in eq. (4-7) for the first two pieces of evidence and in eq. (4-8) for the recursive ER rule. The aim of alpha-index is to measure the degree of dependency between the pieces of evidence and to regulate their joint support towards a specific proposition, thus taking into account the overlapping of the information provided by the pieces of evidence. Alpha can take non-negative values, where one means that the pieces of evidence are independent from each other. However, as the value of alpha decreases, the dependency amongst the variables gets higher, and vice versa.

$$\hat{m}_{\theta,e(2)} = \left[(1-r_2)m_{\theta,1} + (1-r_1)m_{\theta,2}\right] + \sum_{B\cap C=\theta} \propto m_{B,1}m_{C,2} \quad \forall\theta \subseteq \Theta \tag{4-7}$$

$$\hat{m}_{\theta,e(i)} = \left[(1-r_i)m_{\theta,e(i-1)} + m_{P(\Theta),e(i-1)}m_{\theta,i}\right] + \sum_{B\cap C=\theta} \propto m_{B,e(i-1)}m_{C,i} \quad \forall\theta \subseteq \Theta \tag{4-8}$$

To determine the dependency between the different pieces of evidence, the alpha-index value is obtained based on the ideas and results proposed by Yang and Xu (2013, 2014). They prove the equivalence between the ER rule and Bayes' rule under certain conditions, explaining that if all the pieces of evidence are independent and fully reliable, and if the probability is assigned only to a singleton hypothesis, then Bayes' rule can be considered as a special case of the ER rule. They showed that under these special conditions the results obtained using the ER rule were the same as those when applying Bayes' rule.

When a combined belief distribution obtained using the ER rule is not equal to the observed joint probability distribution and when it is assumed that all pieces of evidence are independent and fully reliable, and only singleton hypotheses are considered, it is shown that this difference is incurred because the pieces of evidence are not independent and there is a certain degree of duplication of support provided by the different pieces of evidence (Yang and Xu, 2015). Therefore, alpha-index is introduced and can be calculated by building an optimization model with the aim of minimising the difference between the combined belief distribution obtained by the ER rule and the observed joint distribution by adjusting the values of $\propto$.

4.5 Operationalisation of the research questions

Driven by an empirical motivation (chapter 1) and reinforced by the evidence provided by other researchers (chapter 2) and the findings in the literature review (chapter 3), a core exploratory research question is proposed as a different approach to analyse the delays and disruptions that commonly occur during aircraft heavy maintenance checks:

> *How can system dynamics in combination with the evidential reasoning rule be used to analyse the impact of uncertain unscheduled tasks on delays and disruptions during the execution of aircraft heavy maintenance services?*

This core question is then subdivided into four sub-questions that address particular aspects of the research problem aiming to understand and analyse its different dimensions. Their operationalisation is briefly described in the following paragraphs:

1) *How does the interaction between scheduled and unscheduled tasks influence resource allocation throughout the maintenance process?*

This question aims to investigate the complex interaction between scheduled and unscheduled maintenance tasks and describe how it may complicate the management of resources, leading to pressures upon the programme during the maintenance check. This question is addressed by the development of the causal-loop diagram and is further analysed with the SD simulation model.

2) *How do the occurrence and discovery of damage and discrepancies affect the execution of the maintenance service?*

Despite the fact that the relation between the number of non-routines and the duration of the maintenance service appears to be straightforward, the impact of the stage of its discovery remains unknown. This question explores the role of both variables on project performance by comparing different scenarios based on the SD simulation model.

3) *What are the most relevant variables for estimating unscheduled maintenance tasks?*

This question is addressed by identifying the key variables that contribute most to the estimation of unexpected maintenance activities. The quality and relevance of each of the variables is assessed, determining also their role in the process of estimation.

4) *How can operational and maintenance variables be used as different pieces evidence for estimating the expected number of unscheduled maintenance tasks?*

To answer this question, the evidential reasoning rule is proposed as a Bayesian inference process to aggregate the key variables identified in the previous question taking into account their weight and reliability aiming to determine the expected number of unscheduled maintenance tasks.

Answering these questions will help understand and analyse the complex interaction of routine and non-routine maintenance tasks with the large amount of resources required during the process and the uncertain nature of unscheduled activities. This is mainly achieved by developing and integrating two different models: the system dynamics model described in chapter 5 and the evidential reasoning rule model explained in chapter 6.

4.6 Summary

In this chapter, the methodology and research design were discussed. This research adopts a realism ontological assumption but assumes a more relaxed epistemological position, considering an intermediate point between the subjective-objective paradigms. Based on the main findings and knowledge acquired in the three previous chapters, an exploratory case study has been chosen as the most appropriate approach for addressing the problem discussed in this thesis.

An iterative research design process is suggested for achieving the research objectives and overall is as follows: first, based on the experience and supported by the literature review, the problem is described, delimited and understood and the research questions are defined. Initial conceptual models are then developed which are further improved and refined by collecting real data and considering expert opinions. The conceptual models are transformed into detailed mathematical and simulation models. Subsequently, they must be tested and validated to then be used for scenario analysis. Finally, the results are used for answering the research questions and drawing conclusions.

Two different methods have been proposed to analyse this problem: Systems Dynamics as a tool for understanding and analysing highly complex and dynamic systems, and the Evidential Reasoning Rule as a generalised Bayesian inference process for combining independent pieces of evidence.

One main research question is proposed for analysing the delays that commonly occur during aircraft maintenance checks. This question is split into four sub-questions to deal with specific aspects of the problem: the complex resource allocation when scheduled and unscheduled tasks have to be managed, the effects on project performance of the occurrence and discovery of unexpected events, the uncertain nature of the unscheduled tasks and the variables related to their occurrence.

The following chapters seek to answer these research questions by developing firstly a SD model (chapter 5) to understand and analyse the interrelationship of routine and non-routine tasks and its impact on maintenance service duration, and then an ER rule model (chapter 6) to estimate the non-routine tasks for a specific maintenance check by analysing historical data related to the usage and maintenance of an aeroplane.

Chapter 5: Analysing delays and disruptions in aircraft heavy maintenance using System Dynamics

As already mentioned, managing and planning aircraft heavy maintenance services represents a real challenge. It requires keeping an aeroplane out of service for a long period of time, ranging from around 7 to 40 days. Additionally, during its execution, large amounts of limited, specialised and costly resources are utilised and there is a constant and intricate interaction between them. Heavy maintenance services are also likely to bring about eventualities with high levels of uncertainty mainly caused by the stochastic nature of the unscheduled maintenance tasks. If these contingencies occur they could cause delays in and disruptions to the process, which may lead to serious operational, technical and economic consequences for airlines and MROs.

Several authors have discussed the significance of heavy maintenance, stressing its magnitude, complexity and sophistication. For instance, Srinivasan et al. (2007) explain that the complexity of heavy maintenance is due to five different but interrelated factors: 1) The large number of maintenance activities to execute, especially the additional and unplanned tasks to solve unexpected damage and failures. 2) The limited amount of resources available to perform the maintenance tasks (both scheduled and unscheduled) which also commonly need to be shared with other simultaneous services. 3) Managing and synchronising the two previous factors, i.e. allocating the required resources for each maintenance task in a timely manner. 4) Coordination of objectives and priorities between departments in an uncertain and competitive environment. 5) Implementation of the whole service according to the plan.

From the characteristics mentioned above, it is evident that aircraft heavy maintenance shares many aspects with complex projects. For instance: they have to accomplish well-defined targets and are subject to specific constraints, such as deadlines, restricted budgets, limited resources, strict regulations, etc. Complex projects have critical size comprised of a large number of activities that need to be performed in a very constrained timeframe. They are highly dynamic, where numerous interdependent internal and external elements interact, involve various but limited resources that must be shared either within the process or externally, and uncertainty and ambiguity are present throughout their execution (Williams, 2003; Remington and Pollack, 2010; Masmoudi and Haït, 2012).

Aircraft heavy maintenance also encompasses several of the main features of complex systems. According to Sterman (1992, 2000), Remington (2010) and Stacey (2011) a dynamic and complex system is characterised by being highly dynamic, tightly coupled, governed by positive and negative feedback processes, nonlinear, history-dependent (some actions are irreversible), self-organizing, adaptive, counterintuitive, and involves "hard" and "soft" data.

Computer simulation has been shown to be a useful approach to analyse aircraft maintenance due to its ability to characterise the system's response to different changes capturing the uncertainty, complexities, and interactions present throughout the maintenance process (Cobb, 1995; Duffuaa and Andijani, 1999; Gupta et al., 2003).

System Dynamics (SD) is a tool for understanding and managing highly dynamic and complex systems, which has also been successfully applied to project management due to its capacity for analysing a whole project as a system and representing dynamic and complex behaviour. In this chapter it is proposed to use SD as a methodology to analyse the problem of delays and disruptions within the heavy maintenance process and their relationship with scheduled and unscheduled tasks.

In this chapter, both components of SD, “soft” and “hard”, have been used for analysing the problem. Firstly, a conceptual model is developed by building several causal loop diagrams. Through their description, the problem and the relationships between the variables become clearer. Secondly, two quantitative models are built based on stock and flow notation and their structure and the main results are discussed.

5.1 Model articulation

Prior to describing the qualitative and quantitative models, some premises and configurations must be considered. The scope of the model, the software utilised for the simulation and the general process of the data collection will be explained in the following paragraphs.

5.1.1 Model boundaries

Bearing in mind Morecroft's (2007) suggestion of not modelling the system itself but rather the dynamic complexity of it, these models are focused on describing the main problems faced during the execution of heavy maintenance services, which have been described previously. Neither the long-term planning nor the initial plan scheduling processes are described in the models, as they are beyond the scope of this research. Nor was the provision of resources considered: the procurement and supply of parts, the selection, recruitment and training of labour, and the provision and management of tools and parts were excluded. However, the models describe the execution of an aircraft heavy maintenance service, emphasising the arising and discovery of non-routine tasks, the resulting sophisticated management of both, scheduled and unscheduled activities and the constant battle for resources between these two types of tasks.

5.1.2 Simulation software

Different SD software tools were analysed, evaluating and comparing their features, advantages and disadvantages. Variables such as popularity, interface, cost and robustness were considered. After a trial period, and taking into account the particular problem characteristics, two different software packages were selected. Vensim 6.3D was chosen to develop the causal loop diagrams for its easy and friendly interface, while for the stock and flow diagrams and mathematical models, iThink 10.0.2 emerged as the best option for its robustness and large selection of built-in functions.

5.1.3 Data collection

Qualitative and quantitative information was used for developing the causal loop diagrams and the stock and flow models. Concerning the qualitative information, and aiming to obtain a holistic perspective of the problem, nine experts in the field were consulted, all with a similar background but with expertise in different parts of the process as summarised in Table 5-1. In average, they have more than twenty years of experience in the industry, either as managers or tactical personnel, having worked for different airlines in Mexico and with experience in heavy maintenance services around the world. Some of them are currently working in academia, studying the management of aircraft maintenance. Together, they made valuable contributions during several feedback sessions to enhance the structure of the conceptual model.

Table 5-1 Experts profile

Level	**Expert**	**Academia**	**Engineering**	**Inspection**	**Maintenance**	**Planning and scheduling**	**Production control**	**Supply chain**	**Years of experience (approx.)**
Strategic	A								25
	B								30
	C								25
	D								30
	E								40
Tactic	F								15
	G								10
	H								15
	I								10

The gathering of the qualitative information was undertaken following the process illustrated in Figure 5-1. Firstly, two introductory meetings were carried out as an initial approach to the experts in the industry to present the project. Then, after the project was accepted, there was a kick-off meeting with all the experts involved aiming to describe the objective, research project and familiarise them with the generalities of the SD methodology. Afterwards, five specific open interviews were conducted to gain more insight about the problem and its main features. The experts to interview were selected based on their experience and time availability.

The following step was the development of the causal loop diagrams, which comprised eight feedback sessions and two approval meetings. Based on personal experience and considering the main comments from the interviews an initial causal loop diagram was built. Then, throughout the feedback meetings it was discussed and improved to finally, when most of the comments were addressed, the causal loop diagrams were agreed in the approval sessions.

As the final stage, the SD simulation model was built grounded in the causal loop diagrams. In this step, due to the difficulty in obtaining sensitive information regarding the performance of maintenance services, it was necessary to evaluate the overall model performance by comparing its general behaviour and results with the experts' experience about the problem. For this reason, two different experimentation and validation sessions were conducted to play with the model and assess its results.

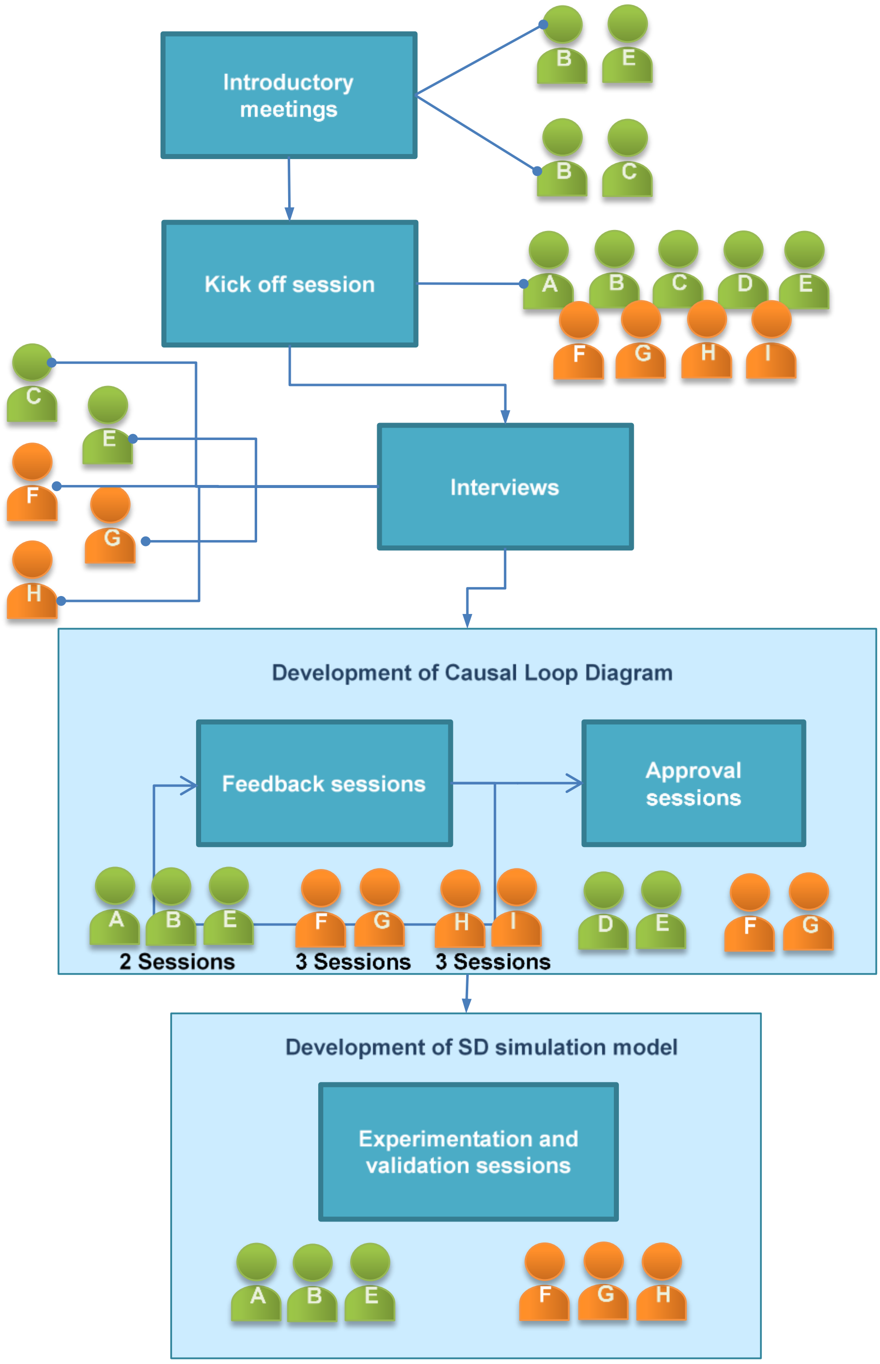


Figure 5-1 Qualitative data collection

Regarding the quantitative data, real maintenance records were used, corresponding to one specific type of aeroplane from a commercial airline. The information included different key process variables, such as the type of maintenance service, aeroplane days on ground, number of routine and non-routine tasks, number of man-hours required for scheduled and non-scheduled activities, and available headcount, amongst others.

For confidentiality reasons, the details of the meetings and maintenance records are not disclosed. Some of the comments and issues discussed, as well as the particularities regarding the data, may be very sensitive and compromising. Moreover, it can be argued that the improvements to the model obtained throughout these meetings are of more interest to this study than the details themselves.

5.2 Conceptual model development

One of the challenges in understanding a complex system is to represent the complicated interrelationship between variables and their constant dynamism. One of the tools for system thinking that can help overcome this challenge is causal loop diagrams. Sterman (2000) states that causal loop diagrams are helpful for capturing hypotheses about the causes of dynamics, eliciting mental models of individuals and teams, and for representing and communicating possible feedbacks responsible for a problem. Morecroft (2007) adds that causal loop diagrams can help to change the perspective of problems from simple cause-effect to a more complex structure with feedback loops. Building a causal loop diagram that represents a problem, its dynamism and the interrelationship between variables therefore becomes a necessary and useful step in the modelling process.

In order to do so, an initial conceptual model was created using personal experience. Then, after several feedback meetings where the model was iteratively explained and discussed, the model was refined and enhanced based on experts' knowledge of the problem. The final conceptual blueprint is discussed below.

5.2.1 Understanding delays and disruptions in aircraft heavy maintenance process

The causal loop diagram presented in Figure 5-2 is formed by fourteen feedback loops that depict delays and disruptions in aircraft heavy maintenance process, which are mainly caused by unscheduled tasks. It illustrates the interaction between resources and their impact on the completion of a maintenance check. The diagram also includes perceptions, attitudes and delays in reacting during the project, which might increase variability and hinder coordination of maintenance services. To facilitate the explanation and understanding, it is separated into different sections shown from Figure 5-3 to Figure 5-7.[2]

[2] A larger version of these figures has been included in Appendix A. Please refer to this section for a detailed view of the causal loop diagrams.

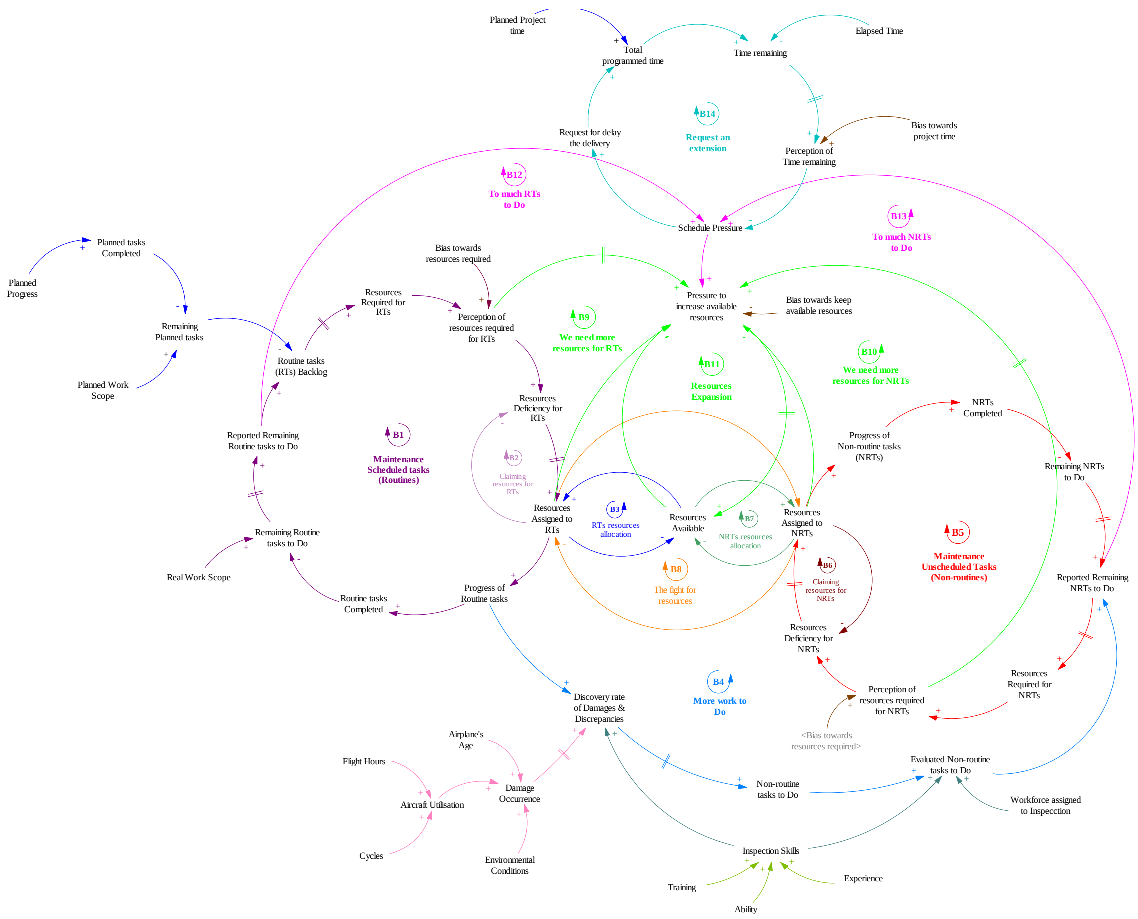


Figure 5-2 Delays and disruptions in aircraft heavy maintenance process

5.2.1.1 *Managing maintenance scheduled tasks*

Figure 5-3 portrays one of the basic and common problems in project management: *the delays within the project*. In the upper left-hand side, the project's plan is depicted. The planned progress leads to an increase in completed tasks, which in turn causes reduction in remaining tasks according to the plan. Meanwhile, a rise in work-scope results in an increment in the planned remaining tasks.

The plan is then compared against the *real* status of the project. For this, the remaining tasks according to the plan are compared with the *reported* remaining routine tasks (RTs). If the latter is bigger than the former there is a backlog of tasks to do. In order to reduce this backlog more resources are required, though there is a delay in noticing this need. Additionally, there might be a difference between the real resources needed and the perception of this additional requirement, which is influenced by the attitude of management to the backlog of tasks.

Subsequently, the perception of required resources is compared with resources already assigned to routine tasks. If the resources assigned are lower than needed, there is deficiency (balancing loop B2). Therefore, it is necessary to allocate available resources to execute routine tasks, aiming to reduce resource deficiency. However, this allocation of additional resources depletes the availability of resources for additional tasks (balancing loop B3).

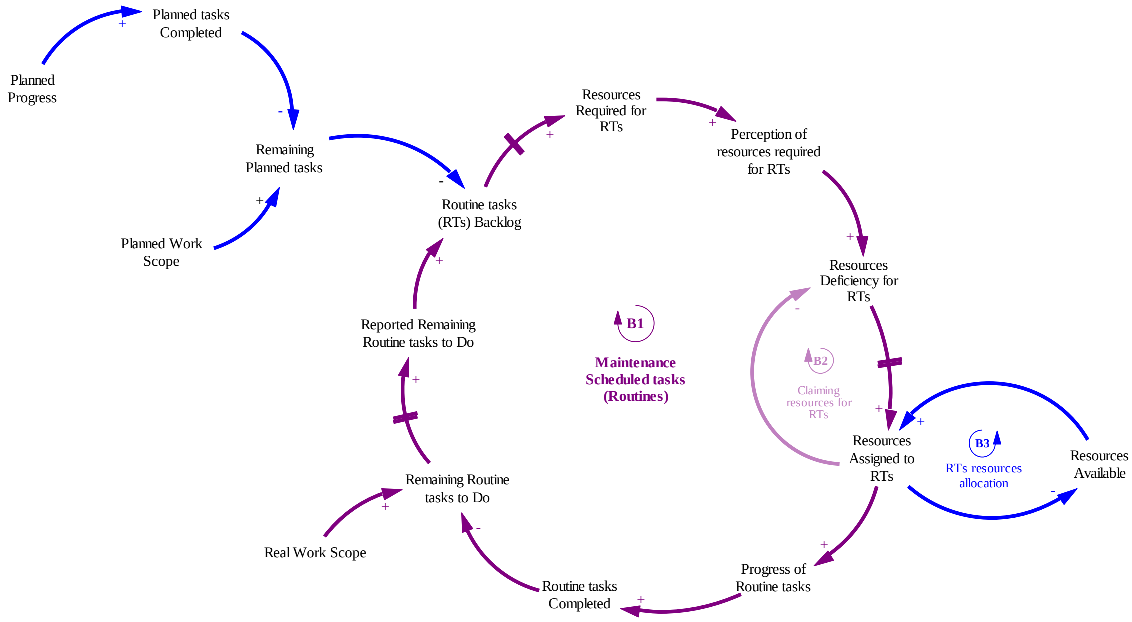


Figure 5-3 Scheduled tasks and resources allocation

If more resources are assigned to execute routine tasks, an increase in the progress of these tasks is to be expected that will result in more tasks being completed. However, whilst a rise in routine progress leads to a reduction in remaining tasks, an inclusion of additional activities into the work-scope augments the number of remaining tasks to do. There is also a delay in reporting the remaining tasks to perform. Finally, if the remaining tasks drop, routine backlog reduces too, closing in this way the balancing loop B1 of maintenance scheduled tasks.

Summarising, loop B1 “Maintenance scheduled tasks” compares the plan against the real progress of scheduled tasks. If there is a backlog of remaining tasks, more resources are allocated in order to increase progress and the number of tasks completed, reducing remaining tasks and also backlog. Once backlog is reduced, pressure to increase resources is eased.

5.2.1.2 The occurrence of damage and discrepancies

Figure 5-4 adds the loop B4 called “*More work to do*” that represents uncertainty in the problem. During the execution of routine tasks, unexpected damage and discrepancies can be found that must be corrected by programming non-routine tasks. Non-routines must be evaluated, their severity assessed and the necessary resources to execute them estimated. In the diagram, it can be seen that the discovery of additional discrepancies or failures leads to more non-routines, which need to be evaluated. Once assessed, these will increase the number of remaining non-routine tasks.

The origin and cause of uncertainty is a relevant question. The occurrence of damage and discrepancies is due to several external factors such as aircraft age, utilisation or environmental conditions. The main issue is that it is difficult to determine precisely the number of discrepancies, their type, their severity or when they may occur. Therefore, in the industry, damage and failures are commonly estimated by experts considering the external factors mentioned above. If the number of discrepancies is unknown and uncertain, the discrepancy discovery rate and therefore the number of unplanned tasks will also be uncertain, hindering the planning and coordination of activities and resources.

Moreover, the discrepancy discovery rate and the evaluation of non-routine tasks are affected by the skills of the workforce, particularly inspectors. Employees with a higher level of skills can detect damage more easily and also evaluate it more accurately. Workforce skills depend basically on the training, experience and ability of personnel.

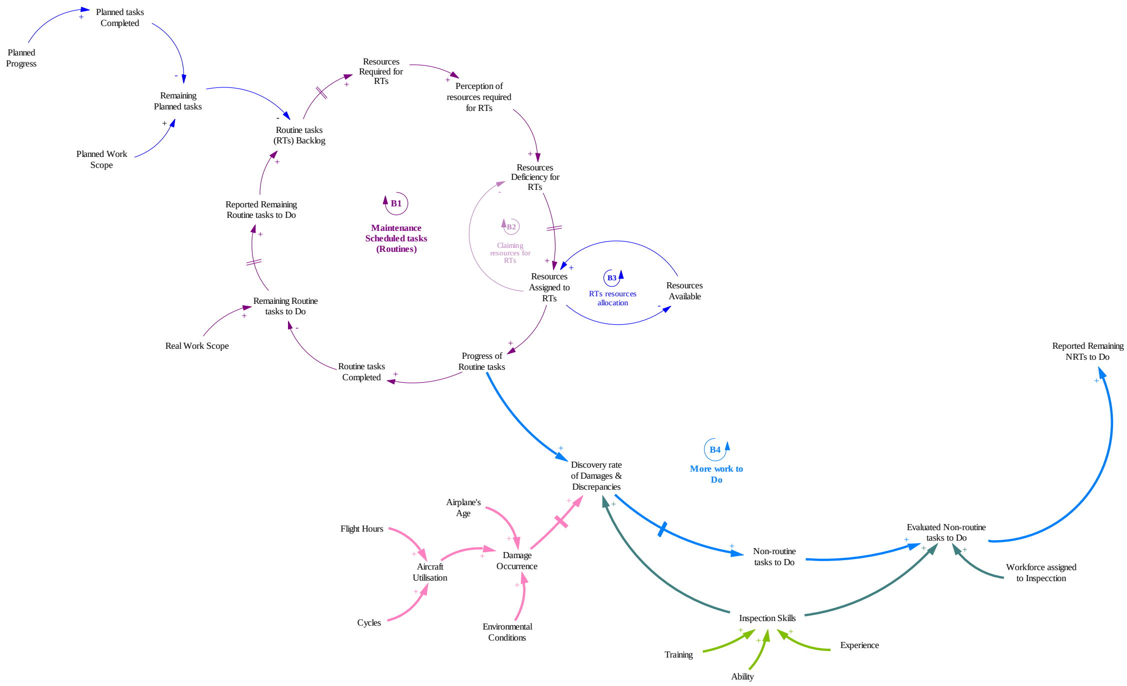


Figure 5-4 Occurrence and discovery of discrepancies

5.2.1.3 Managing maintenance unscheduled tasks and the fight for resources

The main loops in Figure 5-5 are B5 "*Maintenance unscheduled tasks*" and B8 "*The fight for resources*". The principle of loop B5 is very similar to loop B1 as it shows that the greater the number of remaining non-routine tasks, the more resources are required to increase progress and complete more unscheduled activities, balancing the loop by reducing the number of remaining non-routines.

Loop B8 illustrates a constant fight for resources between scheduled and unscheduled tasks. If more resources are assigned to execute routine tasks, we could expect an increment in progress and, therefore, faster completion. However, this causes a decrease in available resources and this in turn affects the allocation of resources to perform unscheduled activities. This leads to a negative effect on the progress of non-routines, delaying their completion. In contrast, when more resources are allocated to execute unscheduled activities, it will have a positive effect on their progress, but a negative impact on the completion of scheduled activities. For this reason, finding the best resource allocation policy is fundamental in order to optimise the use of resources and to reduce maintenance check duration.

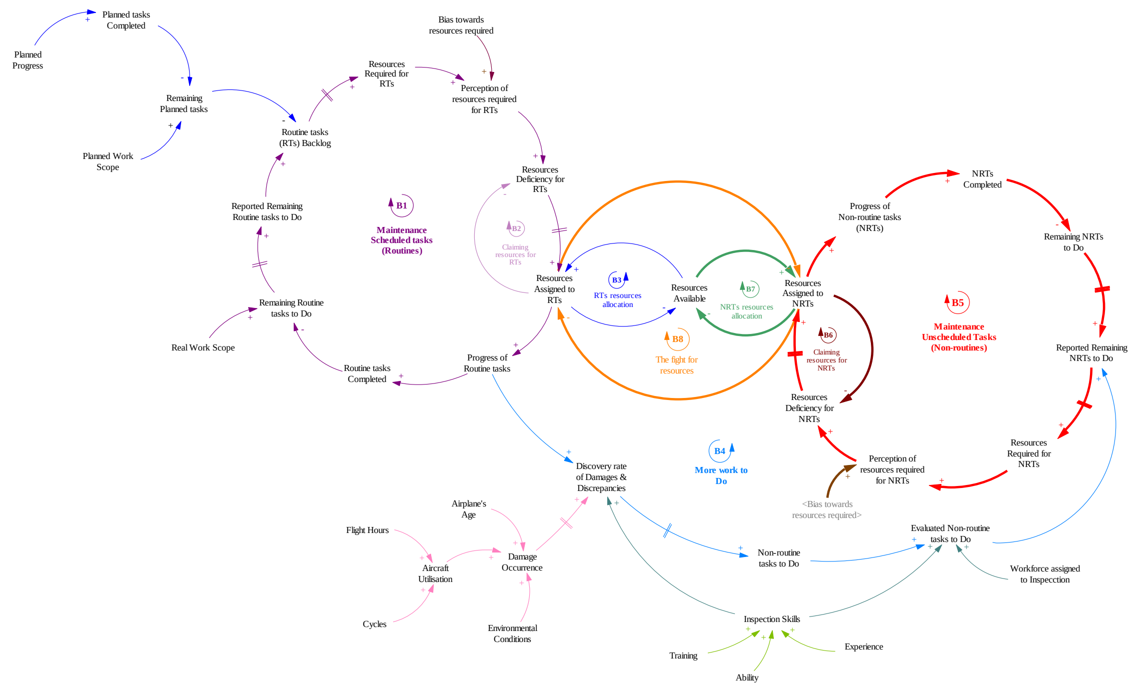


Figure 5-5 Unscheduled tasks and the fight for resources

5.2.1.4 *Increasing resources*

Loops B9, B10 and B11, illustrated in Figure 5-6, describe the process for increasing available resources. When the resources required to perform scheduled and unscheduled tasks are greater than the sum of available and assigned resources, the pressure to enlarge the available resources increases. This leads to an increase in the resources available, consequently closing the balancing loops and relaxing the pressure to expand the resources.

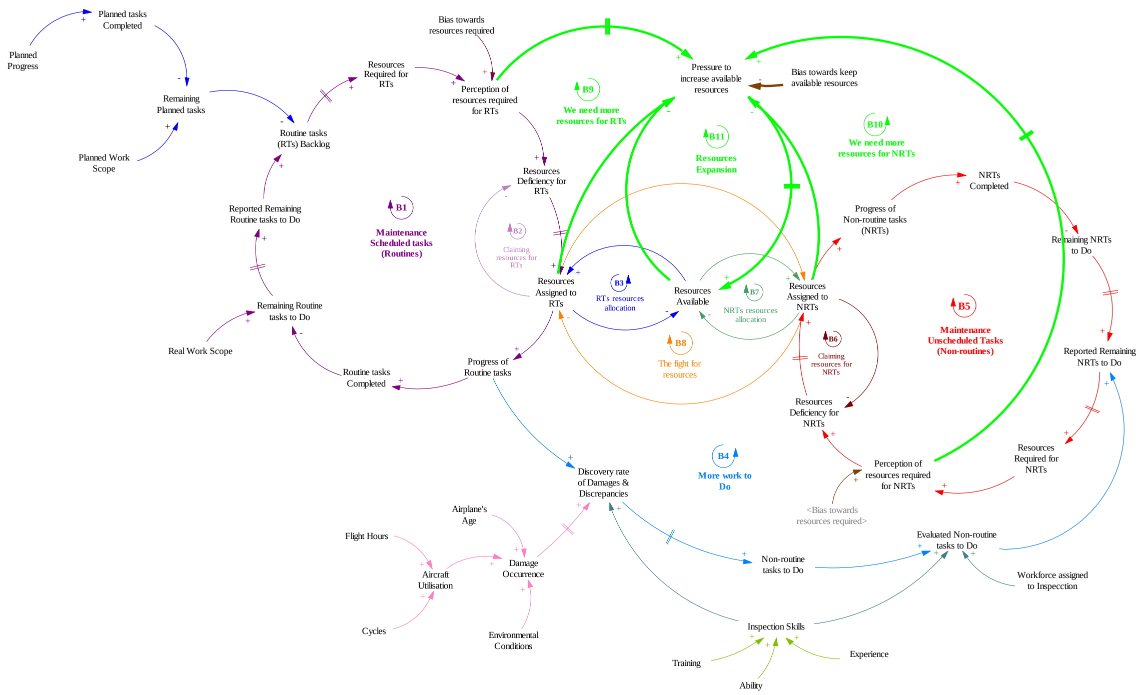


Figure 5-6 Increase of available resources

It is worth mentioning the delays and perceptions present in these loops. Firstly, there is a delay, or lack of acceptance, in recognising that the resources available are not enough to overcome the need for allocating more resources to the process. Secondly, increasing the number of resources available requires considerable time to obtain or hire them, particularly for the specialised resources utilised in the aviation industry. It is also important to mention the attitude of management towards increasing available resources. In this regard, the experts pointed out that at the beginning of the maintenance service, there is a significant reluctance from the management to increase resources, whereas at the end of the project, if the backlog of tasks and the accumulated delays are extensive, the tendency is to increase them even more than needed to overcome the problem as soon as possible.

5.2.1.5 Requesting an extension

Figure 5-7 in loops B12, B13 and B14 illustrates the alternative of asking for a maintenance service extension, thus reducing the pressure to finish the project on time. If the remaining routine and non-routine tasks increase, the pressure to finish the maintenance service on time starts rising. Additionally, if the time remaining is running out, the pressure to finish on time grows. Therefore, if all other alternatives, as explained in the previous loops, fail to complete the maintenance check on time, the last option is to ask for an extension to deliver an aeroplane after the expected date. Once the extension is approved, the total project time increases and hence the time remaining also increases, also diminishing time pressure.

Once all loops are reviewed and explained, the causal loop diagram presented in Figure 5-2 looks less complicated and is easier to understand. It is important to mention that, for practical purposes and to facilitate visualization of the main problem, this loop diagram considers the term *resources* from a general perspective. However, in real life it is not that simple. In heavy maintenance, different types of resources are generally involved, the most relevant being workforce, parts and materials, and tools and equipment.

The *workforce* plays a key role in heavy maintenance, as labour is extensively used when a large number of maintenance tasks need to be executed. In the short-term, the scheduling of workforce is critical to avoid shortages or excesses of personnel. In the long-term, it is also important to plan technical labour, since a considerable investment in time and money is required to hire, prepare and train highly specialised personnel. A large quantity of *parts and materials* is used in heavy maintenance and their lead time can be significant. The planning, replenishment and supply of components and parts must be managed carefully to ensure the availability of parts and materials without increasing costs unnecessarily. Approved test *equipment and specialised tools* must be available to execute maintenance activities. The planning and procurement of these resources should be made months in advance, as it may take a long time for them to be available. Considerable investment may also be required to obtain these resources.

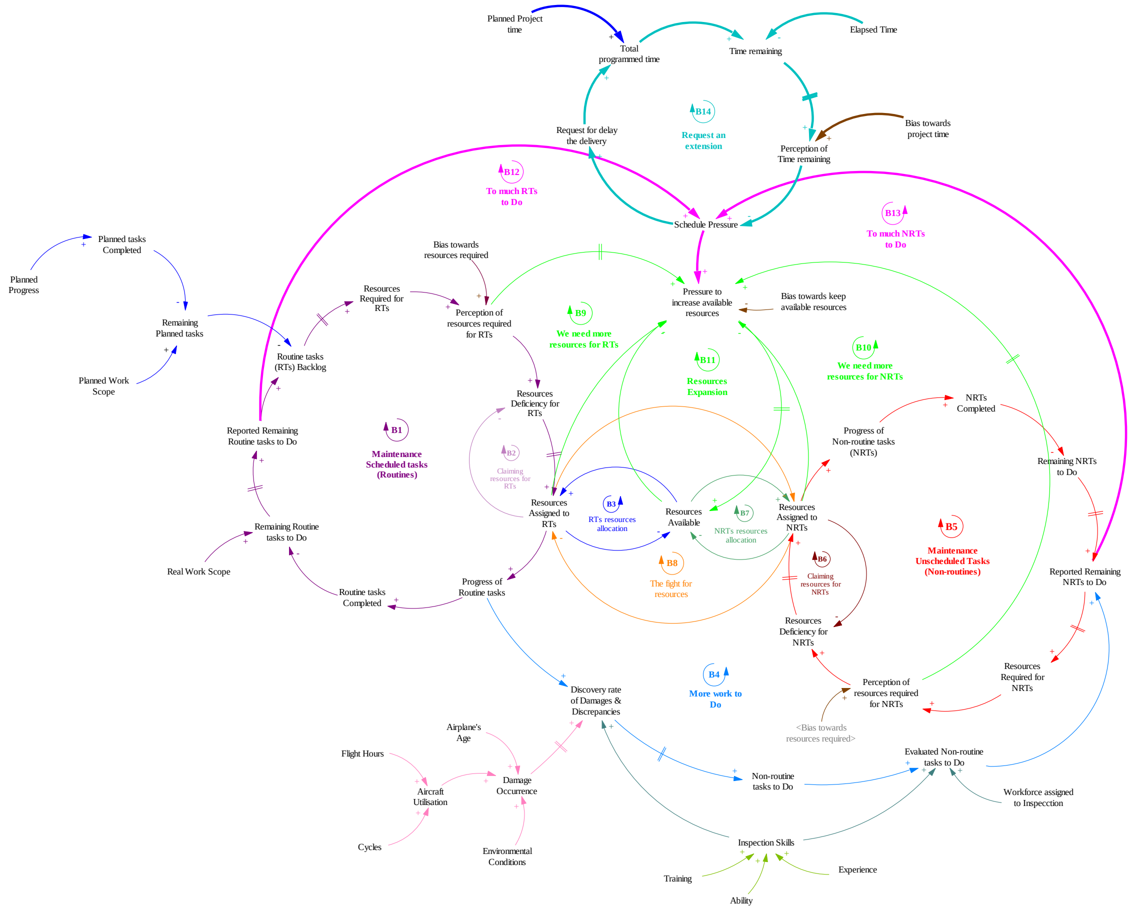


Figure 5-7 Extend the project deadline

5.2.2 Integrating workforce, parts and materials, and tools and equipment

Therefore, based on the generic model of Figure 5-2, three models are proposed that depict workforce, parts and materials, and tools and equipment. These three models are interrelated as they are immersed in the same system. Figure 5-8 illustrates this idea, showing each model as a different layer but interacting all together in one main model that relies on the three different types of resources.

Notwithstanding the close resemblance between the three models, each layer has particular characteristics due to the different nature of each type of resource. Each also has qualities that can either positively or negatively influence the progress of maintenance checks and the performance of the project as a whole.

Generally, workforce has been considered as the most important resource during the management of a project. Particularly in aircraft maintenance, workforce has been deemed to require special attention due to its peculiarities during the process. The following sections discuss different alternatives to increasing the availability of workforce and the negative effects triggered by implementing this strategy.

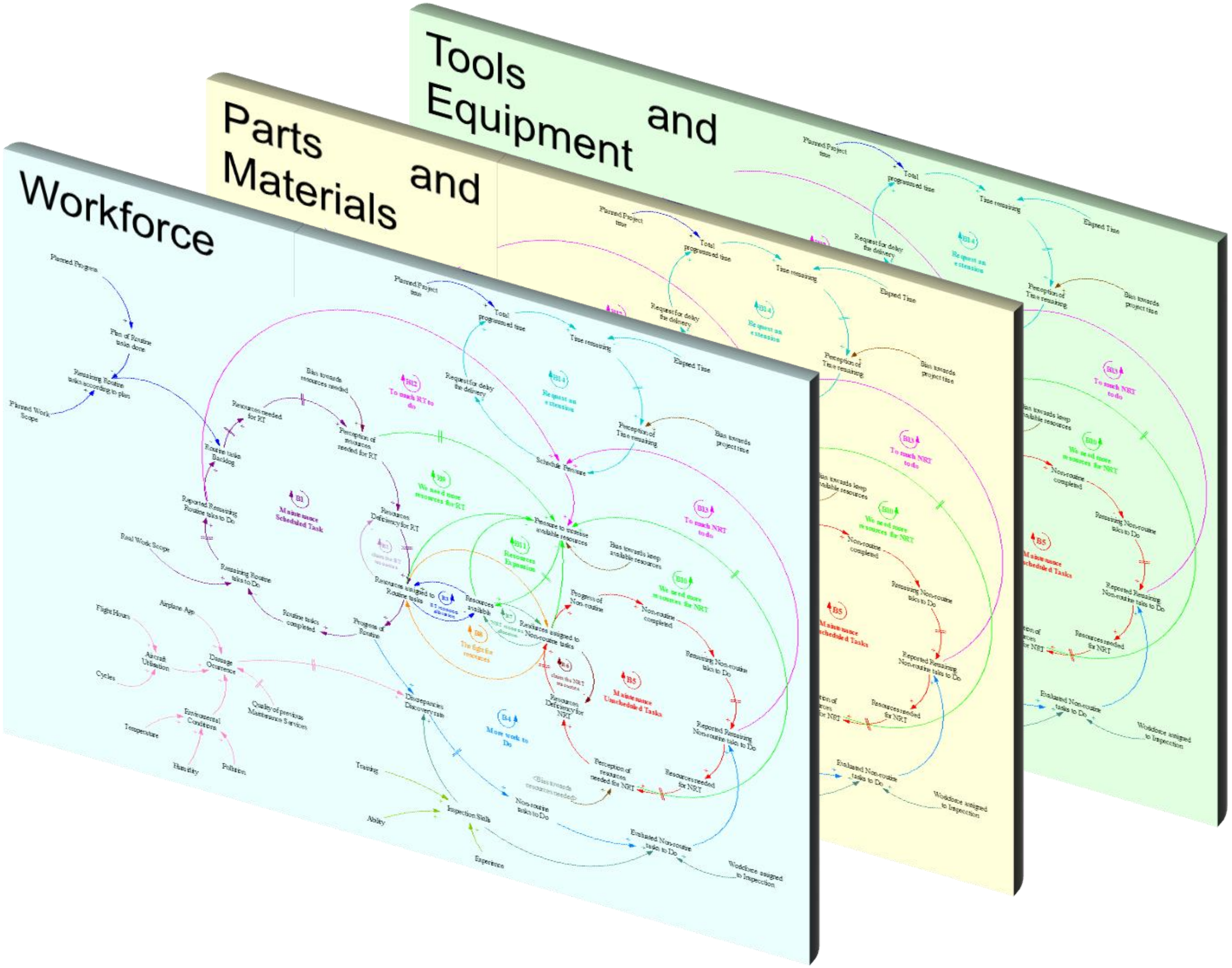


Figure 5-8 Interrelationship between workforce, parts and materials, and tools and equipment

5.2.2.1 Increasing workforce availability

Figure 5-9 depicts three different ways of extending the available workforce: increasing work intensity, increasing overtime and hiring more people. Loop Ba describes how increasing work intensity leads to an improvement in productivity and boosts progress. Loop Bb depicts the effect of incrementing overtime, enlarging the amount of man-hours available. Finally, loop Bc illustrates that extending the headcount by hiring more people also results in more workforce being available that can be allocated to perform scheduled and unscheduled tasks.

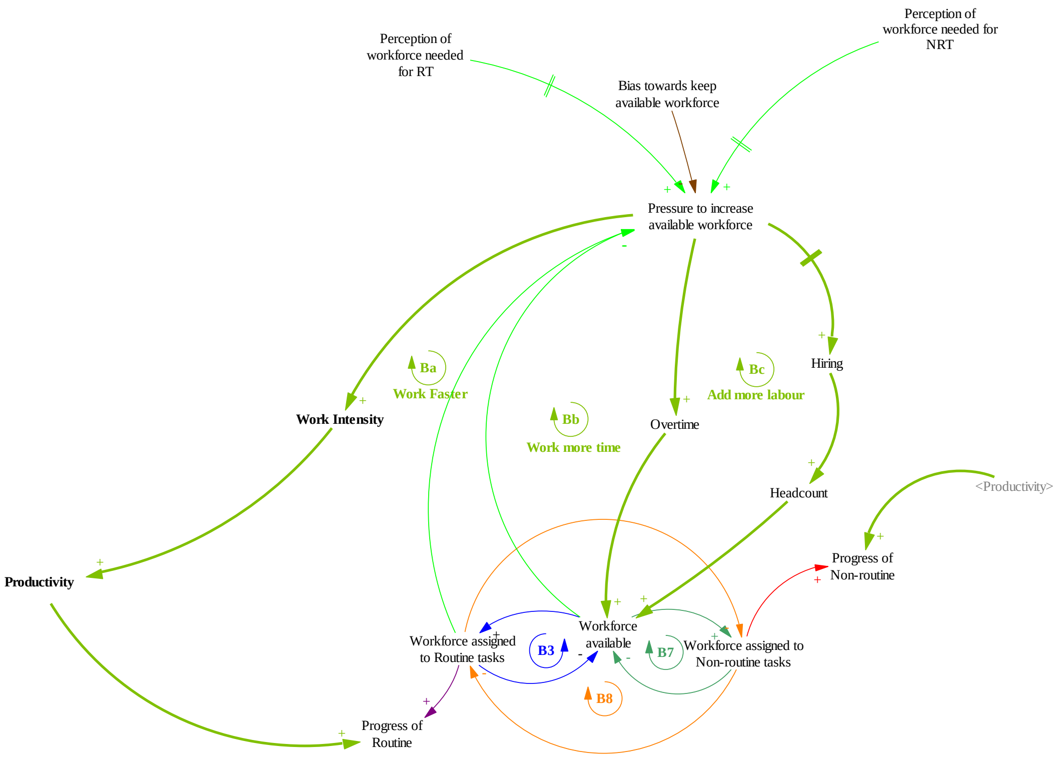


Figure 5-9 Ways to increase workforce

5.2.2.2 Ripple and knock-on effects of increasing workforce availability

However, expanding the workforce as described above also leads to negative effects. Figure 5-10 shows the ripple and knock-on effects that attempting to increment the availability of workforce might have on process performance. Firstly, increasing work intensity increases the possibility of errors, damage or accidents, affecting negatively the progress of the tasks (reinforcing loop Rd). Secondly, increasing overtime causes more fatigue in the medium and long-term, reducing productivity and also raising the error fraction (loop Rb). Finally, increasing the headcount intensifies congestion and hinders workforce management, affecting productivity (loop Rc). Newly hired labour is also normally less skilled and may be more prone to error, impacting negatively on productivity and affecting progress (loop Ra).

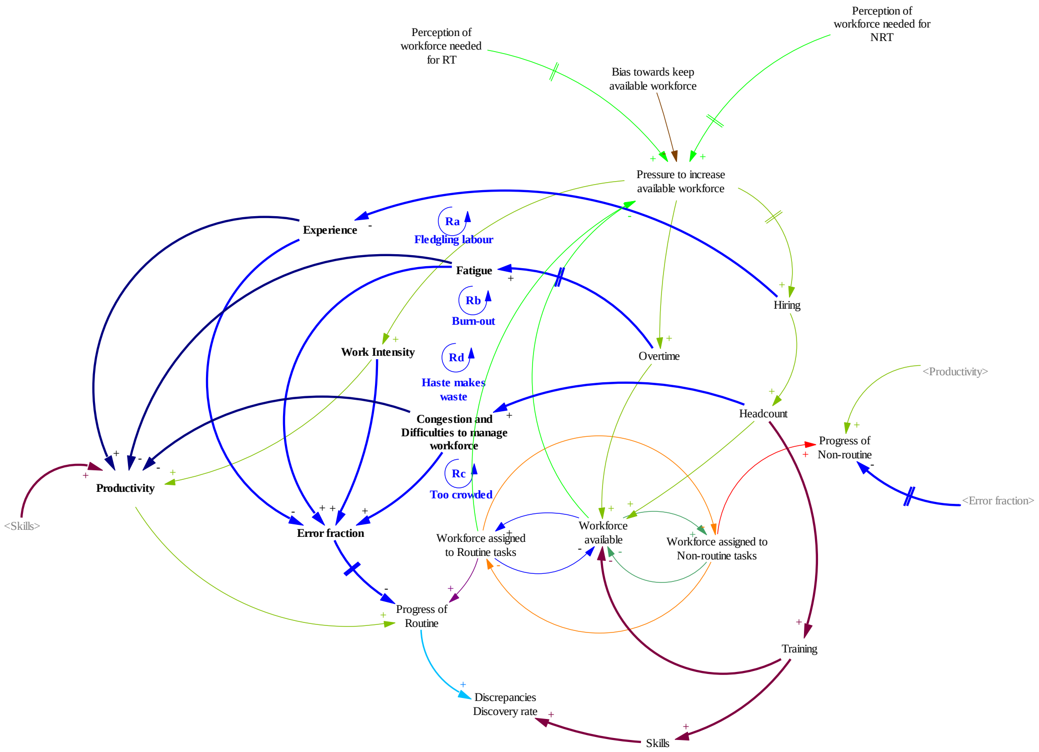


Figure 5-10 Ripple and knock-on effects of increasing workforce

5.3 Quantitative models

Although the conceptual model helped to enhance the awareness of the problem and to improve the understanding of the complex feedback structure of the system, it is not capable of accurately representing the dynamic behaviour of the system through time. Therefore, based on the acquired knowledge from the conceptual model, two SD simulation models were developed to characterise and analyse the dynamism of the aircraft maintenance services to then evaluate different maintenance strategies.

Two different simulation models were developed to analyse specific aspects of the problem. The first focuses on analysing and describing the effect of the occurrence and discovery of damage and failures on project duration and the impact of different resource allocation policies on the performance of the maintenance service. The second examines and explains the influence of perceptions, attitudes and delays in decision-making on maintenance service performance. Both models are presented in the following sections.

5.3.1 Analysing scheduled and unscheduled maintenance tasks

This section describes a simulation model that analyses the interaction between routine and non-routine tasks and their effect on the duration of an aircraft heavy maintenance service. The model studies two major aspects of the problem: 1) the arising of non-routine tasks, which depends basically on two factors: the occurrence of discrepancies and the discovery of discrepancies, and 2) the impact of workforce allocation between scheduled and non-scheduled tasks on project performance, as stressed in the causal loop diagram.

It is worth noting at this stage that some important assumptions have been made. These assumptions aim to simplify the model and the simulation, but they also restrict the scope of the model. Productivity is assumed to be constant and the task progress is assumed to follow a linear pattern. In reality, these variables may have a more complex behaviour.

Figure 5-11 and Figure 5-12 illustrate the two sectors of the stock and flow model. The first describes the interaction between routine and non-routine tasks and the occurrence and discovery of discrepancies, while the second represents workforce allocation.

Figure 5-11 describes the process in which the routine and non-routine tasks are carried out. The number of routine tasks to be performed decreases according to the average progress, increasing in turn the stock of tasks completed. The average routine tasks executed per day are determined by the number of technicians assigned to these activities and the average productivity per worker, represented by equations (5-1) to (5-5).

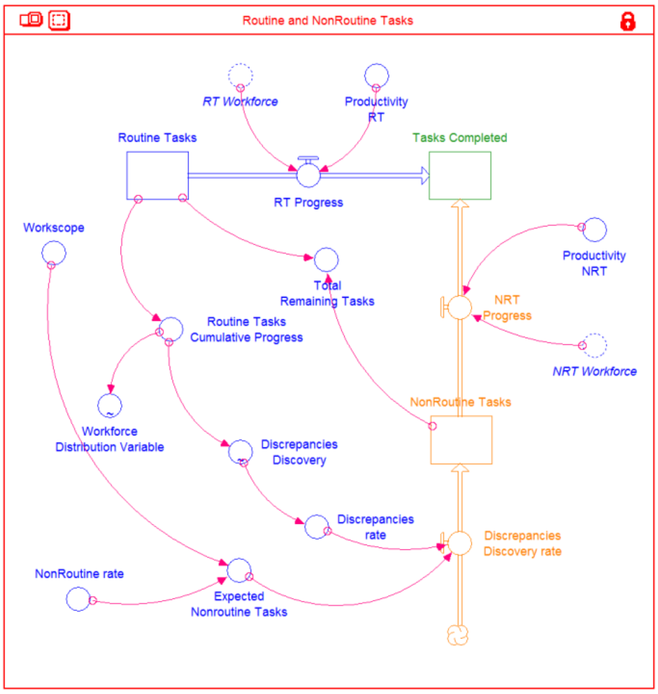


Figure 5-11 Occurrence and discovery of discrepancies

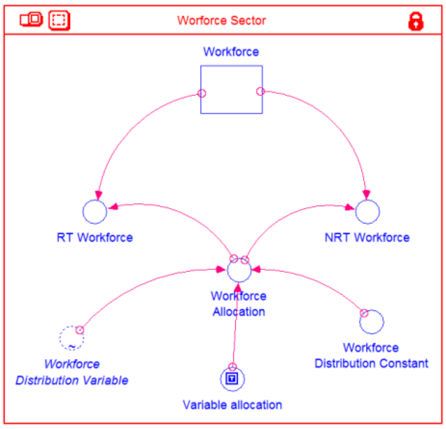


Figure 5-12 Workforce allocation

Variable	Productivity RT $$Productivity\ RT(t) = 2 \quad \{constant\} \qquad (5\text{-}1)$$
Type	Converter
Units	Tasks/Worker/Day
Description	Daily average productivity per Worker assigned to the routine tasks. The productivity may vary depending on the skills and training of the workforce. Its value can be assigned based on experts' judgements or using an average value of the maintenance records. It can be used to explore the impact of productivity on project duration.

Variable	RT Workforce
	$$RT\ Workforce(t) = Workforce(t) * Workforce\ Allocation(t) \tag{5-2}$$
Type	Converter
Units	Workers
Description	Number of Workers assigned to execute routine tasks.

Variable	RT Progress
	$$RT\ Progress(t) = RT\ Workforce(t) * Productivity\ RT(t) \tag{5-3}$$
Type	Rate
Units	Tasks/Day
Description	Average number of routine tasks performed per day.

Variable	Routine tasks
	$$Routine\ Tasks(t_0) = Workscope(t_0) \quad \{constant\}$$ $$Routine\ Tasks(t) = Routine\ Tasks(t-dt) - \int_{t-dt}^{t} RT\ Progress(t)dt \tag{5-4}$$
Type	Stock
Units	Tasks
Description	Number of routine tasks to be executed.

Variable	Tasks Completed
	$$Tasks\ Completed(t_0) = 0 \quad \{constant\}$$ $$\begin{aligned} Tasks\ Completed(t) \\ &= Tasks\ Completed(t-dt) \\ &+ \int_{t-dt}^{t} (RT\ Progress(t)dt + NRT\ Progress(t)dt) \end{aligned} \tag{5-5}$$
Type	Stock
Units	Tasks
Description	Number of maintenance tasks completed (scheduled and unscheduled).

The expected number of non-routine tasks during a service is calculated based on the total number of programmed tasks and by the estimation of unscheduled maintenance tasks per each scheduled task. The cumulative percentage of discrepancies found during the service is directly proportional to the cumulative progress of routine tasks performed. Correspondingly, the discrepancies rate represents the percentage of discrepancies discovered per day. Equations (5-6) to (5-8) present the details of these variables.

Variable	Work-scope
	$$Workscope(t) = 5000 \quad \{constant\} \tag{5-6}$$
Type	Converter
Units	Tasks
Description	Number of scheduled tasks defined in the original maintenance plan. The value depends on the type of maintenance check to execute.

Variable	NonRoutine rate
	$$NonRoutine\ rate\ (t) = 0.5 \quad \{Constant\} \tag{5-7}$$
Type	Converter
Units	Proportion
Description	Proportion of expected unscheduled tasks per scheduled task. It is usually estimated based on experience or as an average of historical data. This constant is used to analyse the impact of the expected number of non-routine tasks on the service workload and thus the project duration.

Variable	Expected non-routine tasks
	$$Expected\ Nonroutine\ Tasks(t) = Workscope(t) * NonRoutine\ rate(t) \tag{5-8}$$
Type	Converter
Units	Tasks
Description	Expected number of unscheduled tasks to be found during the service.

Variable	Routine tasks cumulative progress
	$$Routine\ Tasks\ Cumulative\ Progress(t) = (1 - (Routine\ Tasks(t))/(RoutineTasks(t_0))) * 100 \tag{5-9}$$
Type	Converter
Units	Percentage
Description	Percentage of performed scheduled tasks.

Variable	Discrepancies discovery
	$$Discrepancies\ Discovery(t) = f(Routine\ Tasks\ Cumulative\ Progress(t)) \tag{5-10}$$
Type	Converter
Units	Percentage
Description	Percentage of discrepancies found by routine tasks progress. This is obtained by a graphical function that relates the progress of the routine tasks at a particular point in time with the expected proportion of non-routine activities discovered during the execution of the maintenance service.

Variable	Discrepancies rate
	$$Discrepancies\ rate(t) = DERIVN(Discrepancies\ Discovery(t), 1) \tag{5-11}$$
Type	Converter
Units	Percentage
Description	Marginal percentage of discrepancies found by routine tasks progress. This equation calculates the specific proportion of expected non-routine tasks at a particular point in time given the cumulative value provided by the Discrepancies discovery variable. The equation uses the DERIVN function that calculates the nth-order time derivative of a given input.

The expected number of non-routine tasks and the proportion discovered daily determines the average of non-routine tasks to be carried out, which will be processed by the progress in the non-routine. This average progress is influenced by the number of workers assigned to execute these activities and their daily average productivity. Finally, the total remaining tasks is used as an indicator to measure the work left to be done, considering both scheduled and unscheduled tasks. The information regarding these variables is included in equations (5-12) to (5-17).

Variable	Discrepancies discovery rate
	$$Discrepancies\ Discovery\ rate(t) = \frac{Discrepancies\ rate(t)}{100} * Expected\ Nonroutine\ Task(t) \quad (5\text{-}12)$$
Type	Rate
Units	Tasks/Day
Description	Number of expected non-routine tasks found per day.

Variable	Non-routine tasks
	$$NonRoutine\ Tasks\ (t_0) = 0 \quad \{constant\}$$ $$NonRoutine\ Tasks\ (t) = NonRoutine\ Tasks(t-dt) + \int_{t-dt}^{t} (Discrepancy\ Discovery\ rate(t)dt - NRT\ Progress(t)dt) \quad (5\text{-}13)$$
Type	Stock
Units	Tasks
Description	Number of non-routine tasks to be executed.

Variable	NRT Progress
	$$NRT\ Progress(t) = NRT\ Workforce(t) * Productivity\ NRT(t) \quad (5\text{-}14)$$
Type	Rate
Units	Tasks/Day
Description	Average number of non-routine tasks performed per day.

Variable	Productivity NRT
	$$Productivity\ NRT(t) = 2 \quad \{constant\} \quad (5\text{-}15)$$
Type	Converter
Units	Tasks/Worker/Day
Description	Daily average productivity per worker assigned to the non-routine tasks. The productivity may vary depending on the skills and training of the workforce. Its value can be assigned based on experts' judgements or using an average value of the maintenance records. It can be used to explore the impact of productivity on project duration.

Variable	NRT Worforce
	$$NRT\ Workforce(t) = Workforce(t) * (1 - Workforce\ Allocation(t)) \quad (5\text{-}16)$$
Type	Converter
Units	Workers
Description	Number of workers assigned to execute non-routine tasks.

Variable	Total remaining tasks
	$$Total\ Remaining\ Tasks(t) = Routine\ Tasks(t) + NonRoutine\ Tasks(t) \quad (5\text{-}17)$$
Type	Converter
Units	Tasks
Description	Total number of tasks remaining to be performed.

Figure 5-12 describes the workforce allocation to execute scheduled and unscheduled activities. The workforce stock represents the headcount available. Two different strategies can be evaluated for allocating the workforce, one considering a constant distribution of the labour throughout the whole service and the other assuming a variable allocation during the execution of the project. Depending on the chosen strategy, the available workforce is distributed to perform the scheduled and unscheduled maintenance tasks. Equations (5-18) to (5-22) present the details of these variables.

Variable	Workforce
	$$Workforce\ (t) = 200 \quad \{constant\} \tag{5-18}$$
Type	Stock
Units	Workers
Description	Headcount available to perform maintenance tasks. The workforce level can be used to test strategies about headcount.

Variable	Workforce allocation
	$$IF \quad Variable\ Allocation\ =\ 1\ , \quad THEN \quad Workforce\ Distribution\ Variable(t) \quad ELSE \quad Workforce\ Distribution\ Constant(t) \tag{5-19}$$
Type	Converter
Units	Percentage
Description	Proportion of headcount allocated to the routine, either variable or constant.

Variable	Variable allocation
	$$When{:} \quad Variable\ allocation = 1, \qquad Workforce\ distribution\ is\ variable \quad When{:} \quad Variable\ allocation = 0, \qquad Workforce\ distribution\ is\ constant \tag{5-20}$$
Type	Converter
Units	Binary
Description	Strategy of workforce allocation, either variable or constant.

Variable	Workforce Distribution constant
	$$Workforce\ Distribution\ constant(t) = 0.5 \quad \{constant\} \tag{5-21}$$
Type	Converter
Units	Proportion
Description	Constant proportion of headcount allocated to the routine. Using different allocation values can be used to assess the influence of workforce allocation in project duration.

Variable	Workforce distribution variable $$Workforce\ Distribution\ Variable(t) = f(Routine\ Tasks\ Cumulative\ Progress(t)) \quad (5\text{-}22)$$
Type	Converter
Units	Percentage
Description	Variable proportion of headcount allocated to the routine. It is obtained by a graphical function that relates the progress of the routine tasks at a particular point in time with the proportion of workforce assigned to perform routine activities. This function is used to test different strategies of workforce allocation.

Several experiments were carried out to explore the influence that the occurrence of non-routine tasks and the workforce allocation have on the duration of the maintenance check. In the following sections the findings of these experiments are discussed.

5.3.1.1 Occurrence of non-routine activities

The occurrence of non-routine tasks stems from the occurrence and discovery of unexpected damage and failures, referred to as discrepancies. The occurrence of discrepancies is expressed by the non-routine rate that indicates the estimated number of unscheduled activities per scheduled task (for instance, a non-routine rate of 0.8 expresses that for each 10 routine tasks, 8 non-routine tasks might occur). Generally, this value is an estimate assigned considering different variables mainly related to the operation and usage of an aeroplane. However, as damages and discrepancies depend on several factors, it is difficult to accurately predict their number and severity, and this uncertainty about discrepancies and damages complicates the forecasting and planning of unscheduled activities.

During the simulation process, it was proved that the higher the non-routine rate, the greater the expected number of unscheduled tasks to be carried out, resulting in a significant increment of the project workload, increasing the chances of missing the maintenance check deadline. Underestimating the number of non-routine tasks might lead to having fewer resources than required to ensure project completion, extending the days out of service of the aeroplane. Overestimating the expected number of unscheduled activities results in a sub-utilisation of the resources and does not guarantee early completion as the project milestones and deadlines might slip. Therefore, defining a precise non-routine rate becomes essential and it is necessary to explore more rigorous methods to determine it more accurately.

The discovery of discrepancies is also important, as it represents the rate with which damage and discrepancies are found during the execution of scheduled activities. Usually, according to experts in the industry, around 75% to 80% of non-routines should be found in the earliest 15% to 20% of the service duration in order to have enough time to carry out the non-routine tasks. Figure 5-13 illustrates a typical distribution of the discovery of non-routine tasks with respect to the routine progress. As can be seen, most of the discrepancies are found in the first quarter of the service.

Figure 5-13 Discrepancies discovery distribution 1

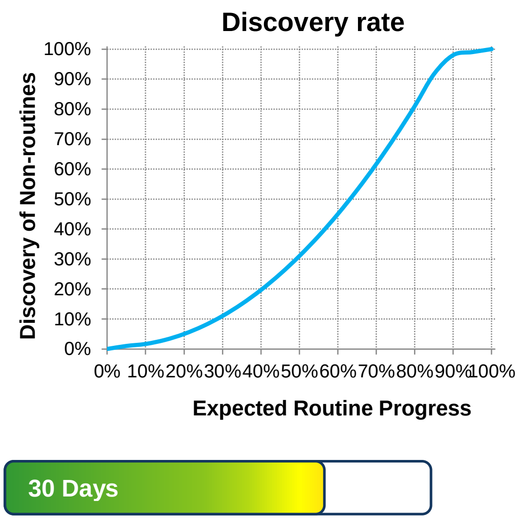


Figure 5-14 Discrepancies discovery distribution 2

It is to be expected that the later the discovery of discrepancies, the greater the project duration, as demonstrated by comparing the discovery rates shown in Figure 5-13 and Figure 5-14. In the first distribution, where approximately 95% of the discrepancies are discovered in the first half of the service, the maintenance check is completed in approximately 25 days. In the second distribution, where only 30% of the discrepancies have been found by the middle of the service, the maintenance check requires approximately 30 days to be completed.

5.3.1.2 Workforce allocation

Another relevant finding discovered through the simulation was the significance of workforce allocation for completion of a maintenance service. In order to execute routine and non-routine tasks, workforce must be assigned to both activities. However, the difficulty lies in defining the number of people that should be allocated to complete scheduled and non-scheduled tasks. Figure 5-15, illustrates that if the majority of personnel is assigned to execute routine tasks (line 1), the routine activities will be finished rapidly and also most of the discrepancies (line 2) will be discovered early. However the progress of non-routine tasks (line 3) will be very slow, which impacts upon the overall performance of the process and will delay the completion of the maintenance service (line 4). In contrast, as shown in Figure 5-16, if the majority of workforce is allocated to non-routines, the progress of routine tasks will be very slow and the discovery of discrepancies will also be delayed, resulting in the late execution of non-routine tasks. The consequence will be, as in the previous case, delay in completion of a project.

Figure 5-17 compares different workforce allocation values and their impact on the completion time of a maintenance service. The first line represents the allocation of 90% of the workforce to the routine, which leads to completing the project in more than 40 days. The second line depicts a distribution of 80% with a completion time of 30 days. Significantly, allocating 60% of the workforce to the routine (line 4) means completing the project in approximately 21 days. However, allocating the workforce equally (line 5) increases again the duration of the project to 25 days. All of these alternatives consider a constant allocation of workforce. Nevertheless, it is possible to have a variable distribution of labour during the maintenance service.

Figure 5-18 shows a variable workforce allocation, where initially most of the labour is assigned to the routine, but is gradually reduced until only 55% of the resources are left to the scheduled tasks. As a result of this strategy, line 6 in Figure 5-17 shows a considerable reduction in project duration, completing the maintenance service in less than 20 days. These results depict the impact of workforce allocation on the performance of the maintenance service and emphasise the relevance of finding the best distribution of labour to reduce project time.

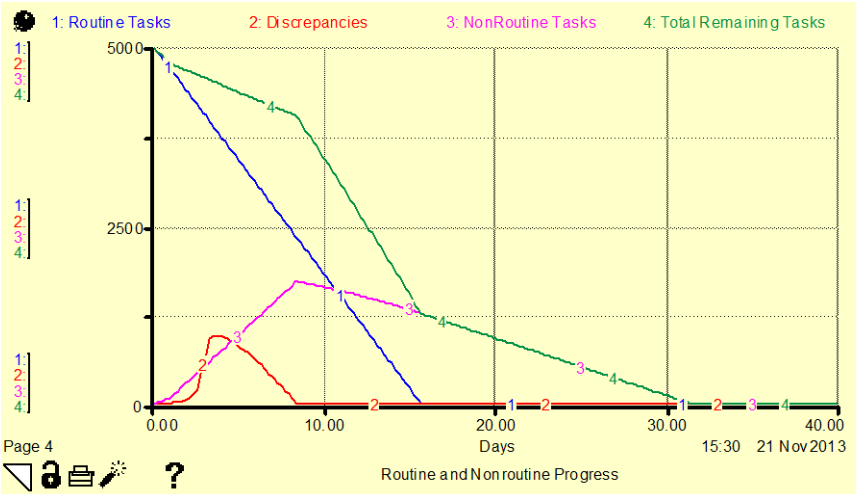


Figure 5-15 Routine and non-routine tasks (80% workforce allocated to routine)

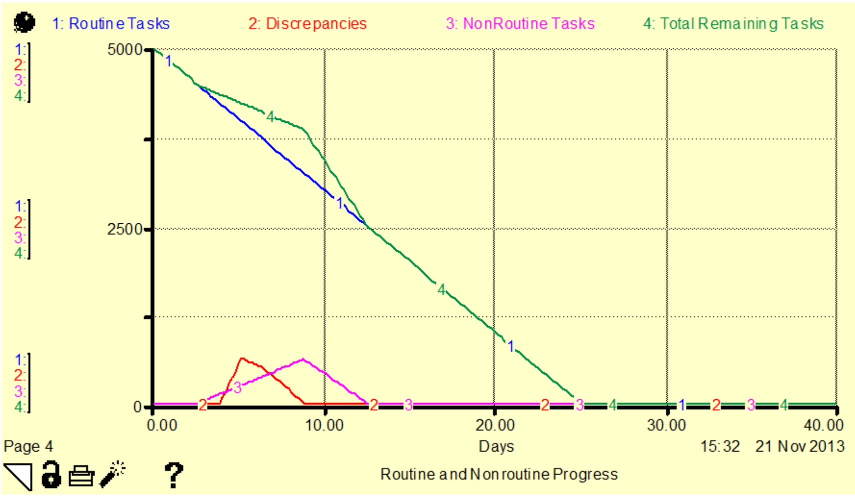


Figure 5-16 Routine and non-routine tasks (50% workforce allocated to routine)

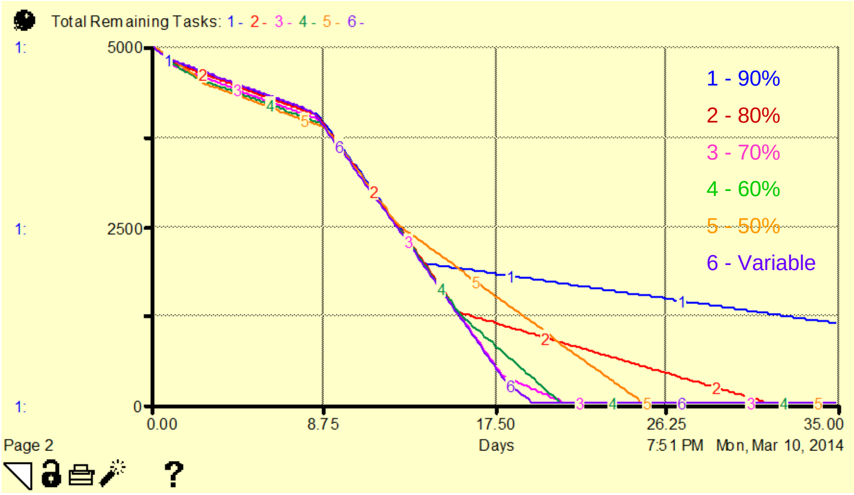


Figure 5-17 Remaining tasks and project duration (different workforce allocation)

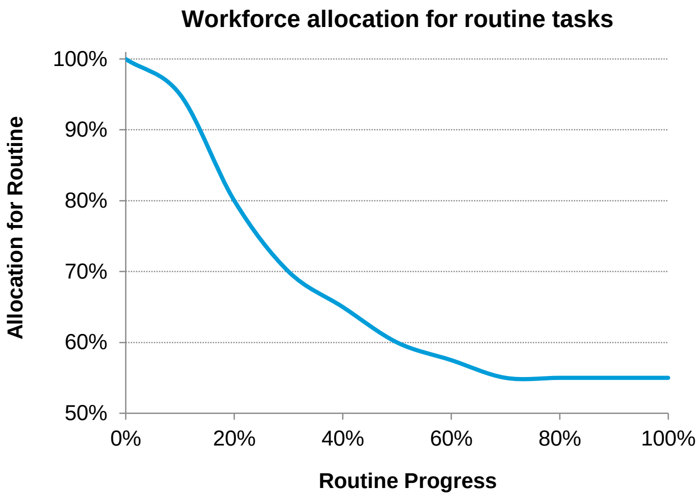


Figure 5-18 Variable workforce allocation for routine tasks

5.3.2 Managing maintenance scheduled tasks

The model discussed in this section concerns the causal loops B1, B2 and B3 and is presented in section 5.2.1.1. These loops describe the allocation of workforce to execute maintenance tasks according to the plan. If there is a backlog of tasks more resources are allocated in order to increase progress and reduce the remaining tasks. Once the backlog is reduced, the pressure to increase resources is relaxed.

In this model, only the scheduled maintenance tasks are studied in order to analyse the influence that perceptions, attitudes and delays in decision making have on project duration. It shows that even without modelling the uncertainty derived from unscheduled tasks it is still complex and challenging to manage a heavy maintenance service. As in the previous model, several assumptions were made to facilitate modelling and simulation. Productivity is considered to be constant throughout the simulation and the progress of tasks is assumed to be linear.

The model comprises four main sections, analysing planned progress, real progress of the maintenance scheduled tasks, project duration and management of the required workforce to execute the activities, as shown in Figure 5-19.

The routine tasks plan sector portrays the execution of scheduled activities according to the maintenance plan. The number of maintenance tasks to perform is defined in the work-scope of the plan, and the average estimated progress per day is obtained considering this work-scope and the estimated project time. The planned tasks are gradually performed based on this average rate of execution. The variables involved in this sector are described in equations (5-23) to (5-28).

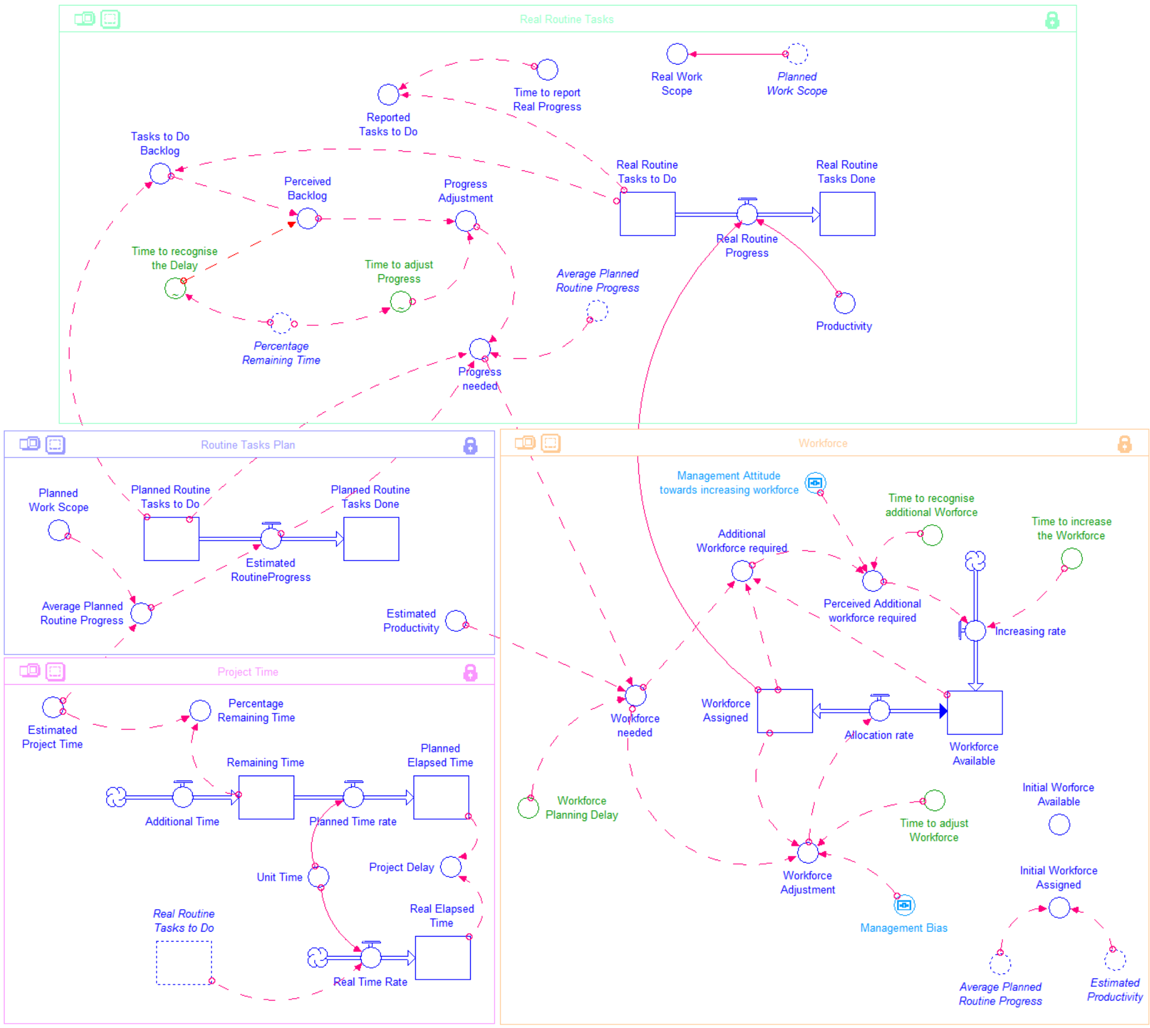


Figure 5-19 Managing maintenance scheduled tasks

Variable	Average planned routine progress	
	$$Average\ Planned\ Routine\ Progress(t) = Planned\ Work\ Scope(t)/Estimated\ Project\ Time(t)$$	(5-23)
Type	Converter	
Units	Tasks/Day	
Description	Number of tasks to do per day according to the plan.	

Variable	Estimated productivity
	$$Estimated\ Productivity(t) = 2 + \mathrm{step}(1{,}10) \quad \{constant\} \tag{5-24}$$
Type	Converter
Units	Tasks/Workers/Day
Description	Planned or perceived number of tasks performed by technician per day. Estimated productivity is assumed to have a value of 2. On day 10 of the project the value is increased to 1 unit, i.e. a total value of 3. This increment aims to experiment with the effect of false perception on productivity.

Variable	Planned work-scope
	$$Planned\ WorkScope(t) = 5000 \quad \{constant\} \tag{5-25}$$
Type	Converter
Units	Tasks
Description	Number of maintenance tasks to execute according to the plan.

Variable	Planned routine tasks to do
	$$Planned\ Routine\ Tasks\ to\ Do(t_0) = Planned\ WorkScope(t_0) \quad \{constant\}$$ $$Planned\ Routine\ Tasks\ to\ Do(t) = Planned\ Routine\ Tasks\ to\ Do(t-dt) - \int_{t-dt}^{t} Estimated\ RoutineProgress(t)dt \tag{5-26}$$
Type	Stock
Units	Tasks
Description	Number of maintenance tasks to perform according to the plan.

Variable	Planned routine tasks done
	$$Planned\ Routine\ Tasks\ Done(t_0) = 0 \quad \{constant\}$$ $$Planned\ Routine\ Tasks\ Done(t) = Planned\ Routine\ Tasks\ Done(t-dt) + \int_{t-dt}^{t} Estimated\ RoutineProgress(t)dt \tag{5-27}$$
Type	Stock
Units	Tasks
Description	Number of maintenance tasks that should have been done according to the plan.

Variable	Estimated routine progress
	$$Estimated\ Routine\ Progress(t) = Average\ Planned\ Routine\ Progress\ (t) \tag{5-28}$$
Type	Rate
Units	Tasks/Day
Description	Daily work rate defined in the maintenance plan.

The project time sector describes the duration of the maintenance service. It compares the planned duration of the project with the real time required to accomplish all the maintenance tasks. The remaining time stock measures the number of days left for completing the project according to the plan. By comparing the planned and real elapsed days, the total delay of the project can be determined. Equations (5-29) to (5-36) present the characteristics of the variables of this sector.

Variable	Estimated project time
	$$Estimated\ Project\ Time = 50 \quad \{constant\} \tag{5-29}$$
Type	Converter
Units	Day (constant)
Description	Total project time frame according to the maintenance plan.

Variable	Percentage remaining time
	$$Percentage\ Remaining\ Time(t) = Remaining\_Time(t)/Estimated\_Project\_Time(t) * 100 \tag{5-30}$$
Type	Converter
Units	Percentage
Description	Remaining project time expressed as a percentage.

Variable	Project delay
	$$Project\ delay(t) = Real\_\_Elapsed\_Time(t) - Planned\_Elapsed\_Time(t) \tag{5-31}$$
Type	Converter
Units	Days
Description	Project delay compared with the plan expressed in day.

Variable	Planned elapsed time
	$$Planned\ elapsed\ time(t_0) = 0 \quad \{constant\}$$ $$Planned\ elapsed\ time(t) = Planned\ elapsed\ time(t-dt) + \int_{t-dt}^{t} Planned\ time\ rate(t)dt \tag{5-32}$$
Type	Stock
Units	Days
Description	Project elapsed days according to the plan.

Variable	Real elapsed time
	$$Real\ elapsed\ time(t_0) = 0 \quad \{constant\}$$ $$Real\ elapsed\ time(t) = Real\ elapsed\ time(t-dt) + \int_{t-dt}^{t} Real\ time\ rate(t)dt \tag{5-33}$$
Type	Stock
Units	Days
Description	Real elapsed days of the project.

Variable	Remaining time
	$$Remaining\ time(t_0) = Estimated\_Project\_Time(t) \quad \{constant\}$$ $$Remaining\ time(t) = Remaining\ time(t-dt) - \int_{t-dt}^{t} Planned\ time\ rate(t)dt \quad (5\text{-}34)$$
Type	Stock
Units	Days
Description	Days left to complete the project according to the plan.

Variable	Planned time rate
	$$Planned\ time\ rate(t) = Unit\ Time(t) = 1 \quad \{Constant\} \quad (5\text{-}35)$$
Type	Rate
Units	Days/Day
Description	Counter of days.

Variable	Real time rate
	$$IF\ Real\_Routine\_Tasks\_to\_Do > 0$$ $$THEN\ Unit\ Time(t) = 1 \quad \{Constant\} \quad (5\text{-}36)$$ $$ELSE\ 0 \quad \{Constant\}$$
Type	Rate
Units	Days/Day
Description	Counter of days.

The real routine tasks sector depicts the execution of the maintenance project and the adjustments made to meet the planned times, detailed in equations (5-37) to (5-47). The number of scheduled tasks is defined in the real work-scope of the service. However, this figure is often higher than the one defined in the plan, generating a backlog of activities since the beginning of service that need to be corrected. The rate at which the routine tasks are performed is calculated from the number of workers assigned to the process and their average productivity. During the process, it is common to have a divergence between the real and reported number of tasks to be done. This difference stems from the delay in reporting daily progress. A false perception of delay may lead to assigning more resources than required. The backlog of tasks is determined by comparing the planned against the real tasks to perform. If there is a backlog in the process, it is frequently perceived belatedly. After the backlog of tasks has been recognised, there is also a delay in adjusting the progress needed to meet the planned targets. The delays in recognising the backlog of tasks and the adjustment in the pace of work vary depending on the stage of the project. Typically, at the beginning of the project when there is no rush for completing on time, it takes more time to recognise that adjustments in the process are required, but as the project approaches its deadline and the pressure to finish on time increases, any required adjustment in the process will be quickly identified. If the pace of work needs to be increased, more workers are required, which leads the model to the workforce sector.

Variable	Perceived backlog $Perceived\ backlog(t)$ $= SMTH1(Tasks\_to\_Do\_Backlog(t), Time\_to\_\_recognise\_\_the\_Delay(t))$	(5-37)
Type	Converter	
Units	Tasks	
Description	Management perception regarding the project backlog of tasks. It uses a smoothing function (SMTH1) to model the delay in the variable.	

Variable	Productivity $Productivity(t) = 2 \quad \{constant\}$	(5-38)
Type	Converter	
Units	Tasks/Workers/Day	
Description	Current average number of tasks performed by technician per day.	

Variable	Progress adjustment $Progress\ adjustment(t) = Perceived\_\_Backlog(t)/Time\_to\_\_adjust\_Progress(t)$	(5-39)
Type	Converter	
Units	Tasks/Day	
Description	Adjusted daily work rate required to meet the planned requirements.	

Variable	Progress needed $Progress\ needed(t) =$ $IF, \quad Planned\_Routine\_Tasks\_to\_Do(t) > 0$ $THEN, \quad Estimated\_\_RoutineProgress(t) + Progress\_\_Adjustment(t)$ $ELSE, \quad Average\_Planned\_\_Routine\_Progress(t) + Progress\_\_Adjustment(t)$	(5-40)
Type	Converter	
Units	Tasks/Day	
Description	Tasks per day that need to be done to complete the project according to the plan.	

Variable	Real work-scope $Real\ Work - Scope(t) = 5000 \quad \{constant\}$	(5-41)
Type	Converter	
Units	Tasks	
Description	Real number of maintenance tasks to execute during the maintenance check.	

Variable	Reported tasks to do $Reported\ tasks\ to\ do(t)$ $= Delay(Real\_Routine\_\_Tasks\_to\_Do(t), Time\_to\_report\_\_Real\_Progress(t))$	(5-42)
Type	Converter	
Units	Tasks	
Description	The registered number of tasks to do.	

Variable	Tasks to do backlog $$Tasks\ to\ do\ backlog(t) = Real\_Routine\_Tasks\ to\ Do(t) - Planned\_Routine\_Tasks\_to\_Do(t)$$	(5-43)
Type	Converter	
Units	Tasks	
Description	Accumulation of uncompleted tasks compared with that planned tasks that should have been done.	

Variable	Time to report real progress $$Time\ to\ report\ real\ progress(t) = 1 \quad \{constant\}$$	(5-44)
Type	Converter	
Units	Days	
Description	Delay in registering and reporting the real progress.	

Variable	Time to adjust progress $$Time\ to\ adjust\ progress(t) = f(Percentage\ Remaining\ Time(t))$$	(5-45)
Type	Converter	
Units	Days	
Description	Delay to acknowledge the real progress. Represents a variable delay according to the project progress and is defined by a graphical function that relates the delay with the remaining time of the project.	

Variable	Time to recognise the delay $$Time\ to\ recognise\ the\ delay\ (t) = f(Percentage\ Remaining\ Time(t))$$	(5-46)
Type	Converter	
Units	Days	
Description	Delay to acknowledge the backlog of tasks. Represents a variable delay according to the project progress and is defined by a graphical function that relates the delay with the remaining time of the project.	

Variable	Real routine progress $$Real\ routine\ progress(t) = Workforce\_\_Assigned(t) * Productivity(t)$$	(5-47)
Type	Rate	
Units	Tasks/Day	
Description	Number of tasks executed by day.	

Variable	Real routine tasks to do $$Real\ routine\ tasks\ to\ do(t_0) = Real\ WorkScope(t) \quad \{constant\}$$ $$Real\ routine\ tasks\ to\ do(t) = Real\ routine\ tasks\ to\ do(t-dt) - \int_{t-dt}^{t} Real\ Routine\ Progress(t)dt$$	(5-48)
Type	Stock	
Units	Tasks	
Description	Number of maintenance tasks to perform.	

Variable	Real routine tasks done
	$Real\ routine\ tasks\ done(t_0) = Real\ WorkScope(t) \quad \{constant\}$
	$$Real\ routine\ tasks\ done(t) = Real\ routine\ tasks\ done(t-dt) + \int_{t-dt}^{t} Real\ Routine\ Progress(t)dt \quad (5\text{-}49)$$
Type	Stock
Units	Tasks
Description	Number of maintenance tasks already executed.

The workforce sector describes the allocation of labour to execute maintenance tasks. Once the necessity of increasing the progress rate has been recognised, the requirement is translated into the need to increase workforce assigned to the execution of the maintenance tasks. However, the additional workforce requirement is inaccurately informed by perceived rather than real process productivity and there is also a delay in recognising the adjustment in the workforce. Allocating available technicians to the process requires time and is subject to the attitude of the management towards increasing the workforce assigned to the process. This attitude tends to vary depending on the status of the maintenance check. At the early stages of the project or if the perception of delay is weak, there is a reluctance to assign more resources and a lower percentage is allocated than required, but at the end of the service when the pressure to finish on time increases, management is willing to assign even more resources than originally requested. When more technicians are allocated to perform maintenance tasks, the progress rate increases and the balancing loop is closed.

If the available headcount is insufficient to cope with the demand to allocate more technicians to the process, the pressure to expand the roster increases. However, this requirement is also affected by the attitudes and perceptions of the management. Generally, the administration is slow to accept that it is necessary to adjust the number of available workers and is resistant to the idea of augmenting the headcount. Once the decision to increase the available workforce has been made either by withdrawing personnel from other processes or by hiring new ones, it will take time to take effect due to the specialised nature of these resources. A detailed description of the variables considered within this sector is included in equations (5-51) to (5-65).

Variable	Initial workforce available
	$Initial\ workforce\ available(t) = 50 \quad \{constant\}$ (5-50)
Type	Converter
Units	Workers
Description	Headcount available at the beginning of the project.

Variable	Additional workforce required

$$Additional\ workforce\ required(t) = IF\ Workforce_{needed}(t) > \left(Workforce_{Available}(t) + Workforce_{Assigned}(t)\right) THEN\ Workforce_{needed}(t) - \left(Workforce_{Available}(t) + Workforce_{Assigned}(t)\right), ELSE\ 0 \quad (5\text{-}51)$$

Type	Converter
Units	Workers
Description	Additional workers needed to cope with the demand of work.

Variable	Initial workforce assigned

$$Initial\ workforce\ assigned(t) = Average\_Planned\_\_Routine\_Progress(t) / Estimated\_\_Productivity(t) \quad (5\text{-}52)$$

Type	Converter
Units	Workers
Description	Workers assigned to perform routines tasks at the beginning of the service.

Variable	Management attitude towards increasing workforce

$$Management\ attitude\ towards\ increasing\ workforce(t): Values\ from\ 0.5\ to\ 1.5\ \{range\} \quad (5\text{-}53)$$

Type	Converter
Units	Proportion
Description	Stance towards augmenting the headcount.

Variable	Management bias

$$Management\ bias: Values\ from\ 0.5\ to\ 1.5\ \{range\} \quad (5\text{-}54)$$

Type	Converter
Units	Proportion
Description	Stance towards allocating more labour to execute routine tasks.

Variable	Perceived additional workforce required

$$Perceived\ additional\ workforce\ required(t) = SMTH1(Additional\_\_Workforce\_required(t) * Mgmt\_Attitude\_towards\_increasing\_workforce(t), Time\_toRecognise\_add\_workforce(t) \quad (5\text{-}55)$$

Type	Converter
Units	Workers
Description	Perception regarding the additional workers needed to cope with the demand of work.

Variable	Time to increase the workforce

$$Time\ to\ increase\ the\ workforce(t) = 5 \quad \{constant\} \quad (5\text{-}56)$$

Type	Converter
Units	Days
Description	Delay in increasing the available workforce.

Variable	Time to recognise additional workforce
	$$Time\ to\ recognise\ additional\ workforce(t) = 2\ \{constant\} \quad (5\text{-}57)$$
Type	Converter
Units	Days
Description	Delay in accepting that more resources are required.

Variable	Time to adjust workforce
	$$Time\ to\ adjust\ workforce(t) \quad \{constant\} \quad (5\text{-}58)$$
Type	Converter
Units	Days
Description	Delay in acknowledging that more technicians need to be assigned to routines.

Variable	Workforce adjustment
	$$Workforce\ adjustment(t) = ((Workforce\_needed(t) - Workforce\_Assigned(t))/Time\_to\_adjust\_Workforce(t)) * Management\_Bias(t) \quad (5\text{-}59)$$
Type	Converter
Units	Workers/Day
Description	Adapts the resource allocation according to the project status.

Variable	Workforce needed
	$$Workforce\ needed(t) = SMTH1(Progress\_needed(t) / Estimated\_Productivity(t), Workforce\_Planning\_Delay(t)) \quad (5\text{-}60)$$
Type	Converter
Units	Workers
Description	The number of workers required to meet the project goals.

Variable	Workforce planning delay
	$$Workforce\ planning\ delay(t) \quad \{constant\} \quad (5\text{-}61)$$
Type	Converter
Units	Days
Description	Delay in recognising the requirement of personnel.

Variable	Workforce assigned
	$$Workforce\ assigned(t_0) = Initial\ Workforce\ assigned(t) \quad \{constant\}$$ $$Workforce\ assigned(t) = Workforce\ assigned(t-dt) \pm \int_{t-dt}^{t} Allocation\ Rate(t)dt \quad (5\text{-}62)$$
Type	Stock
Units	Workers
Description	Number of workers allocated to execute the maintenance tasks.

Variable	Workforce available
	$$Workforce\ available(t_0) = Initial\ Workforce\ Available(t) \quad \{constant\}$$ $$Workforce\ available(t) = Workforce\ available(t-dt) \pm \int_{t-dt}^{t} Allocation\ Rate(t)dt \qquad (5\text{-}63)$$
Type	Stock
Units	Workers
Description	Headcount available to be allocated to execute maintenance tasks.

Variable	Allocation rate
	$$Allocation\ rate(t) = Workforce\_Adjustment(t) \qquad (5\text{-}64)$$
Type	Rate
Units	Workers /Day
Description	Number of workers per day assigned to execute maintenance tasks.

Variable	Increasing rate
	$$Increasing\ rate(t) = Perceived\_Additional\_workforce\_required(t) / Time\_to\_increase\_\_the\_Workforce(t) \qquad (5\text{-}65)$$
Type	Rate
Units	Workers/Day
Description	Number of additional workers assigned to the project.

To study the relevance of delays and the perception of delays, a simple yet significant analysis was performed. This involved two scenarios. The first assumed ideal conditions, with neutral attitudes and no delays, whilst the second assumed the presence of delays and non-neutral attitudes towards them throughout the whole process. In this second scenario, the time taken to recognise backlog and the time taken to adjust the project accordingly were assigned using graphical functions, with a higher delay at the beginning of the project and almost no delay at the end. The experiment consisted of simulating the same disturbance in both scenarios. On day ten of the project, a signal was sent increasing perceived productivity, creating a false management perception that fewer resources were required. The results of the experiment are presented in the figures below. Figure 5-20, Figure 5-22, Figure 5-24 and Figure 5-26 show the results of the scenario with ideal conditions. Figure 5-21, Figure 5-23, Figure 5-25 and Figure 5-27 present the outcome of the second scenario.

Figure 5-20 and Figure 5-21 compare the number of maintenance tasks required over time. Line 1 represents the plan and line 2 represents real progress. In Figure 5-20, it can be seen that after the tenth day, when perceived productivity was modified, there is a slight difference between the plan and real progress. In Figure 5-21, however, the delay is higher, reducing at the end of the service, when more resources are assigned to reduce the backlog.

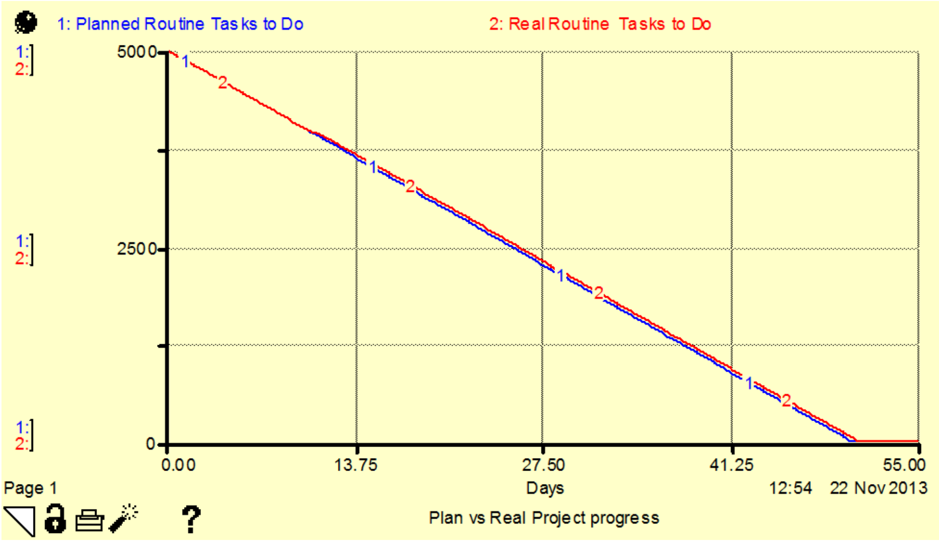


Figure 5-20 Remaining number of tasks per day (ideal conditions)

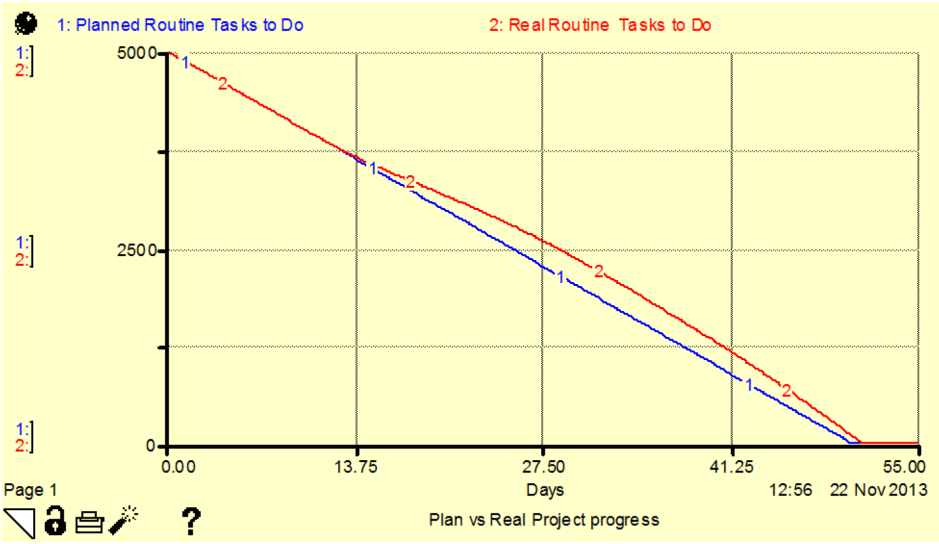


Figure 5-21 Remaining number of tasks per day (with delays)

Figure 5-22 and Figure 5-23 portray the change in productivity and how workforce allocation is affected by this perception. Line 1 represents estimated productivity, line 2 the workforce needed to meet the deadlines and line 3 the workforce that is assigned to execute maintenance tasks. Line 4 shows the real work progress. In Figure 5-22, it can be seen that after perceived productivity is adjusted, less workforce is required and consequently the level of workforce allocated to the process falls. However, as productivity has not actually changed, real progress also drops, sending a signal to adjust the workforce. Due to there being no delays in this scenario, the system is adjusted almost immediately and is operating as under the initial conditions after a couple of days. Figure 5-23 presents different behaviour. Here, when productivity is increased, there is a delay in perceiving that less workforce is needed and real progress gradually decreases. By the time the system has identified the false perception, the backlog of tasks is already significant. More resources are assigned to the process but because of the delay, the re-allocation of personnel takes time. By the end of the service, in an attempt to overcome this issue, a large volume of resources is assigned.

Figure 5-24 to Figure 5-27 depict the backlog of tasks and also show the relationship backlog has with the additional headcount required to meet work demand. In the first scenario (Figure 5-24 and Figure 5-26), the backlog remains almost without variation after the change in perceived productivity and the system itself regulates the workforce needed. In contrast, there is more fluctuation in the variables in the second scenario, shown in Figure 5-25 and Figure 5-27. Due to delays in the process, the backlog increases considerably and a point is reached where this cannot be corrected with the resources available. It then becomes necessary to increase the headcount.

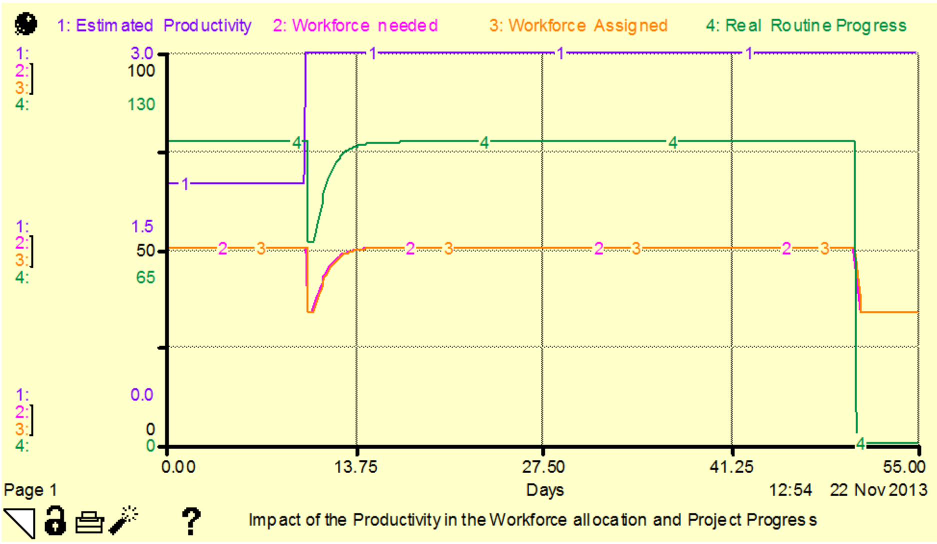


Figure 5-22 Workforce allocation per day (ideal conditions)

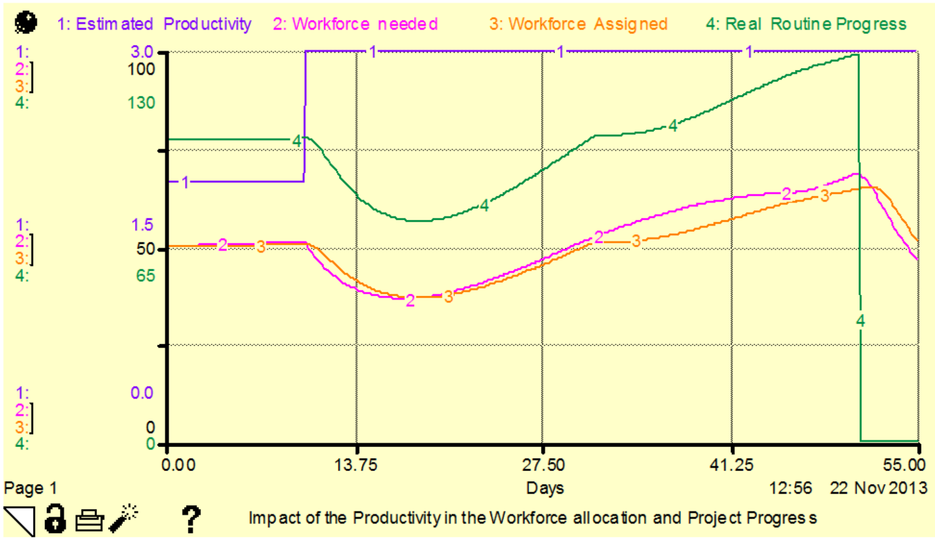


Figure 5-23 Workforce allocation per day (with delays)

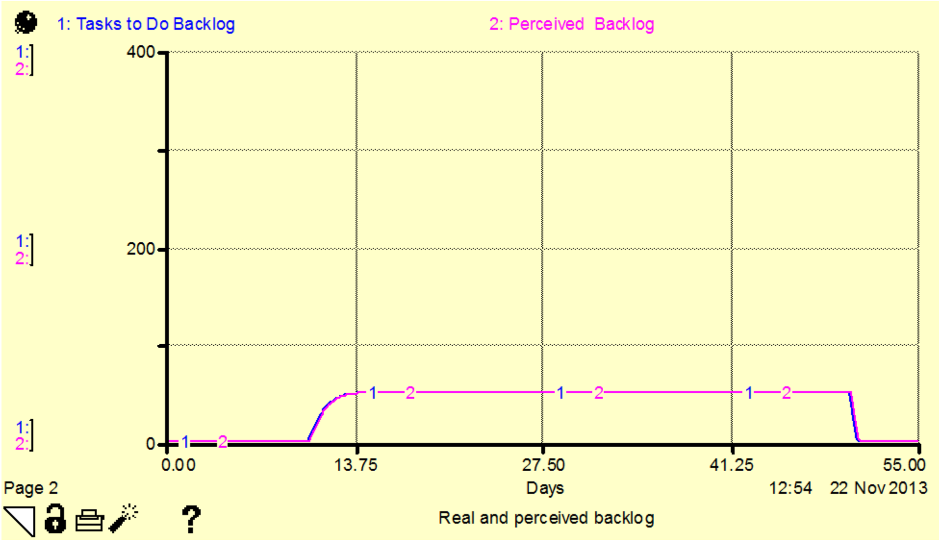


Figure 5-24 Project backlog (ideal conditions)

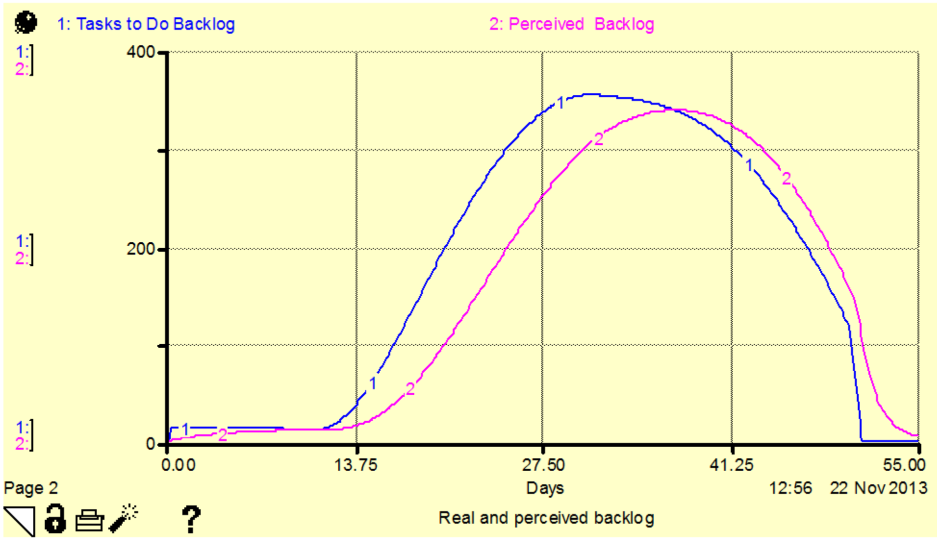


Figure 5-25 Project backlog (with delays)

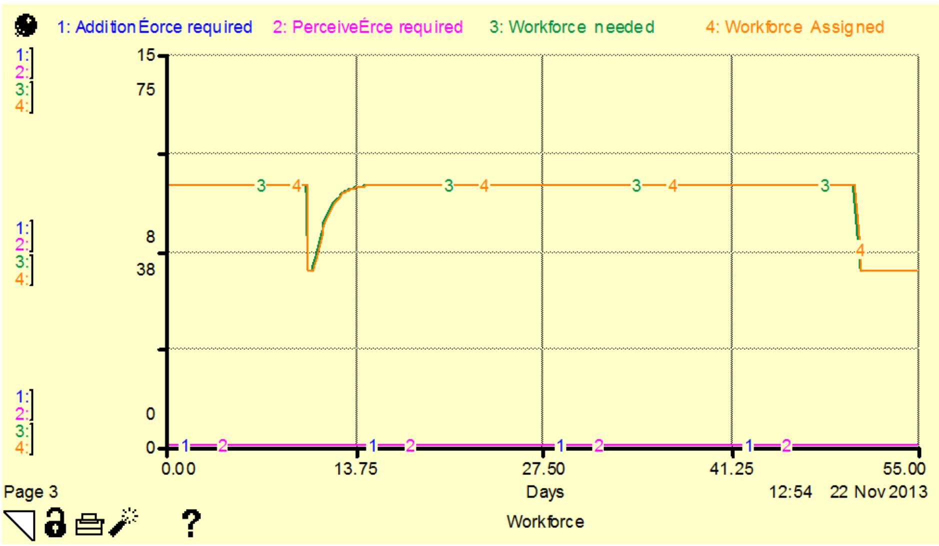


Figure 5-26 Additional workforce required (ideal conditions)

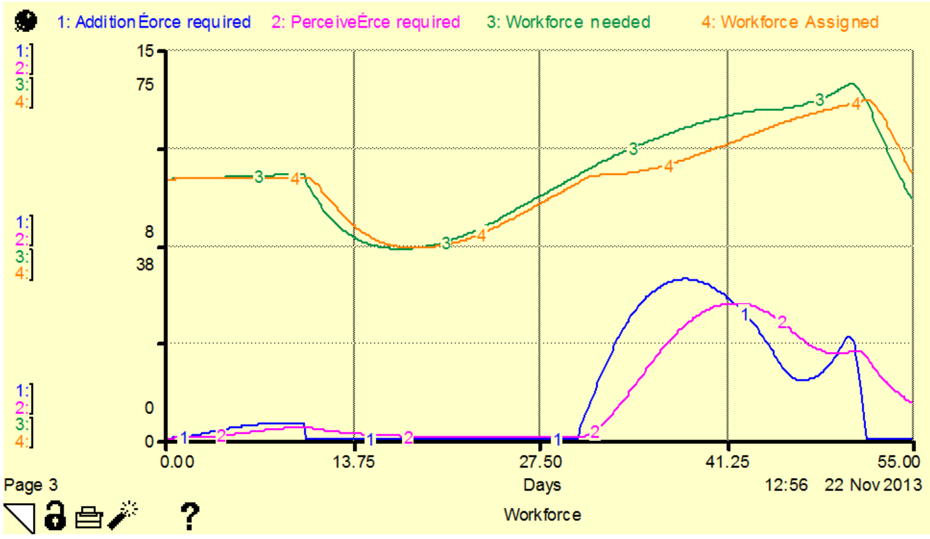


Figure 5-27 Additional workforce required (with delays)

This simple example shows how a slight change in the system has a significant impact, especially if the information does not flow properly in the system or if the decision makers take an incorrect approach.

5.4 Model validation

As previously discussed, validation in SD may be very difficult and subjective. The validation of the developed SD models was performed in several stages throughout the modelling process rather than as a single and isolated step. Firstly, through various feedback meetings, the coherence and structure of the conceptual model was reviewed and discussed with different experts in the industry to ensure that the problem was represented as accurately as possible. Secondly, for the quantitative models, the consistency of the mathematical equations was revised and a dimensional analysis was performed to verify that they were coherent in characterising the problem. Finally, several simulation scenarios were run to assess the general behaviour of the model and to identify unexpected results.

A natural step in the validation process would be to perform rigorous analytical tests to evaluate the effectiveness of the model in describing the real behaviour of the system. However, due to the sensitivity of the information it was difficult to gather enough data about the performance of process to conduct these analytical tests. Therefore, it was necessary to perform a validation of the model behaviour based on experts' experience regarding the observed behaviour of the maintenance process. For this, the quantitative models were discussed with experts who were then allowed to experiment by changing the values of certain variables related to particular maintenance strategies. Through developing and running different scenarios, the behaviour of the model was compared with the results they expected according to their experience.

In general, the experts were satisfied with the overall behaviour of the model, pointing out that the results obtained throughout the several scenarios were remarkably similar to the results they have encountered in real life, particularly when experimenting with the effects of non-routine rate and discovery rate on project duration. They also showed interest in the influence that management perceptions and attitudes have on project performance, agreeing that these are problems commonly faced during the execution of a maintenance check.

Notwithstanding the satisfactory results of the models, the experts raised serious concerns regarding their possible applicability. They noted that the models are very simple and do not take into account other relevant process factors, such as the skill of the personnel (which may affect the type of tasks they can perform), the number of inspectors available during the process (important as they find and evaluate non-routine tasks), or the availability of spare parts and tools, which may also affect the execution of the maintenance check. Additionally, they pointed out that the models make disputable assumptions that are seldom seen in real life, such as assuming constant personnel productivity and an average duration for tasks.

In general terms, the experts concluded that the models could be useful for gaining insight into the main challenges faced during heavy maintenance and as a learning instrument to explore the effects of particular maintenance strategies. However, they were not totally convinced that at this stage the models can be used to estimate the outcome of a maintenance service and therefore be used as a decision support tool.

5.5 Summary

In this chapter, the SD methodology was used to develop a qualitative model and two quantitative models aiming to analyse the delays and disruptions that occur during the execution of aircraft heavy maintenance services. The conceptual model was used to portray the feedback structure of the system and to analyse the complex interrelation between scheduled and unscheduled tasks. The quantitative models were used to describe particular aspects of the problem and to explore their effects on the duration of the maintenance service.

The conceptual model was developed based on experience and further enhanced through an iterative process with the help of experts in the field. In summary, this model describes how resources are allocated to perform scheduled tasks according to the plan and whether it is necessary to reduce any backlog of tasks. However, during the execution of routine tasks, unexpected damage, failures and discrepancies are discovered that require the programming of unscheduled tasks to correct them. The stochastic nature of unscheduled activities hinders the planning and control of tasks and resources. The main problem arises when available resources must be distributed and allocated between scheduled and unscheduled tasks, causing delays and disruptions during the process which might force a request for additional resources or an extension of the aeroplane delivery date.

The causal loop diagrams proved to be a highly useful tool to improve not only the understanding of the problem but also offer a wider vision of the process. They depict cause-and-effect relationships between variables and feedback loops in the system and illuminate the complex interactions between routine and non-routine tasks that hinder the management of resources and the control of the whole maintenance service. The conceptual model illustrates that although the uncertainty in the occurrence of unscheduled maintenance tasks is external to the process, the management of the process and the complex interaction between the elements in the system exacerbates the problem and causes delays and disruptions during the process. During the development of the conceptual model in the feedback meetings, the causal loop diagrams proved to be very valuable for facilitating communication and for promoting learning and discussion.

Despite the valuable findings obtained, the conceptual model does not have the ability to describe the dynamic behaviour of the system. For this reason, two quantitative models were developed using stock and flow diagrams. The first model analyses the effect of the arising of non-routines and the distribution of workforce between scheduled and non-scheduled tasks. Through simulation, three main issues were found. Firstly, after exploring the relation between the non-routine rate and the workload of the maintenance project and the allocation of the available resources, it was found that determining an accurate non-routine rate is essential to avoid shortage or excess of resources. Secondly, it was found that there is a direct relationship between discovery rate and the duration of the project, where an early discovery of discrepancies may increase the chances of completing the project on time. Finally, different distributions of workforce were explored to assess their impact on the service duration, demonstrating that a variable allocation is the most suitable strategy for reducing the project time.

The second model explores the effect that managers' perceptions have on the project performance. This model shows that even without the uncertainty factor, managerial attitudes towards delays have a significant impact on the decision-making process. It was shown that delays cause false perceptions about the real status of the project and make misguided decisions more likely. Delays are regulated by the speed at which information flows, and the sooner the information is updated, the less impact delays will have on overall performance. Attitudes are influenced by perception of project performance and the stage of execution and can aggravate fluctuations in the system, interfering with resource management.

The quantitative models helped to gain awareness of the complex interactions between the variables involved during aircraft maintenance and allowed experimentation with the dynamic behaviour of the system. Through the definition and comparison of different scenarios, it was possible to determine the relevance of certain variables for project performance, for instance the importance of using more rigorous methods to determine the non-routine rate and to decide optimal workforce allocation between the two types of maintenance task.

Chapter 6: Estimation of unscheduled maintenance tasks using the Evidential Reasoning rule

Uncertainty is inherent in almost any project and complex system and aircraft maintenance is no exception. Unscheduled maintenance tasks arise from unexpected damage, failures and discrepancies that are usually discovered during inspections performed as part of scheduled maintenance checks. The uncertainty triggered by the stochastic nature of these unforeseen events hinders the planning of non-routine tasks and the estimation and allocation of the resources required to accomplish them. As a consequence, managing these additional maintenance tasks is critical, as they might affect the cost, duration and quality of maintenance checks.

Usually, before starting a heavy maintenance check, the number of routine tasks to perform is well known, since they are clearly defined in the maintenance scheduled programme. In contrast, even when the occurrence of unscheduled activities is considered, the number of non-routine tasks is unknown before commencing the service. The exact number of unscheduled tasks is only determined once the maintenance check has been completed.

The number of non-routine tasks may vary from one service to another. It is highly related to the amount of damage and the number of failures, which depend on different factors such as environmental conditions, the utilisation and the age of the aircraft and the quality of previous maintenance checks, amongst others. Therefore, it is difficult to determine the total number of non-routine tasks in advance. Besides, this figure by itself does not represent the actual impact that additional unplanned activities have on the maintenance check as a whole, and also complicates comparison between different services. Consequently, it is proposed to use the *non-routine rate* as an indicator of the number of unplanned activities expressed as the ratio between the number of unscheduled and scheduled activities.

In this chapter, a novel approach is proposed to estimate unplanned maintenance activities by developing a model based on the evidential reasoning rule. The proposed model allows for more precise estimation of non-routine rate for a specific heavy maintenance check by analysing historical data relating to the usage and maintenance of an aeroplane. This approach attempts to analyse from a different perspective the uncertainty of unforeseen events that can cause delays in and disruption to complex projects.

The chapter has been organised in the following way. Firstly, the data and the variables used for building the model are described. Secondly, a brief statistical analysis is performed to better understand the variables and their interrelationships. Thirdly, to facilitate the modelling process, the data is discretised into different intervals. As a fourth step, to describe the ER rule model, a particular example is explained step by step. Then, an analysis of the effect of the interval size and limits is performed, and the impact of changing the number of variables is afterwards assessed. Subsequently, different scenarios are built by modifying the main characteristics of the pieces of evidence, namely dependency, reliability and importance, increasing the complexity of the model but also its meaningfulness. Finally, in the last section, a sensitivity analysis is carried out to explore the influence of some features of the variables in prediction accuracy.

6.1 Data and variables description

As explained in the second chapter, all scheduled tasks of a maintenance programme are planned and controlled according to specific intervals measured in three different ways: flight hours, cycles and calendar time. To simplify its management, these tasks are grouped into different scheduled packages according to their execution intervals, ranging from light inspections to exhaustive overhauls. Damage or failure is detected during an inspection and then corrected by programming non-routine tasks in addition to scheduled tasks. It is believed there are strong connections between an aircraft's age and operation to the number of unscheduled tasks required, i.e. the older the aeroplane and the higher its utilisation, the higher its deterioration and the expected amount of damage and number of failures. The maintenance package being performed might also have a significant effect on the estimation of the number of non-routine tasks.

It was also stressed in the second chapter that airlines are subject to very strict regulations and therefore are obliged to record systematically all their operational information and the maintenance history of their fleet. To build the proposed inference model, real operation and maintenance records of a commercial airline were used and a sample of ninety-one heavy maintenance services was collected. In order to maintain uniformity, the sample considers only one particular model of aeroplane, as different types of airliners might have different rates of damage and failure occurrence.

The data about the aircraft comprised its operations and its heavy maintenance records. Model, registration number and manufacturing date were used. For operation records, total flight hours and total cycles were collected (measured up to the beginning of the service). Finally, with regard to maintenance records, the data included the type of maintenance check, start and end date of the service, the total number of routine tasks, the total number of non-routine tasks, the total number of man-hours for routine tasks and the total number of man-hours for non-routine tasks.

It is proposed to use four variables, strongly related to aircraft utilisation and maintenance as the main inputs of the model, to estimate the non-routine rate. These variables were calculated based on the data collected and can be defined as follows:

1. Aeroplane's age (*a*): expresses the age (generally measured in years) from the manufacturing date to the starting date of the maintenance check.
2. Flight hours per year (*fhy*): the yearly average flight time measured since the aircraft manufacturing date to the starting date of the maintenance service.

3. Cycles per year (*cy*): the number of take-off and landing cycles counted from the manufacturing date to the start date of the maintenance check.
4. Type of maintenance service (*se*): the maintenance package being executed. Heavy maintenance services encompass C and D checks. Airlines generally organise C checks in different categories based on their frequency and elaborateness (e.g. C1, C2, C3, up to C8, which is equivalent to a D check or a major overhaul).

The variables were chosen as they represent the main elements for defining, structuring, managing and executing the scheduled maintenance programme, but especially for their relevance to the occurrence and discovery of damage, failures and discrepancies.

Non-routine rate is calculated as a relationship between the number of non-routine tasks over the number of routine tasks. For instance, a non-routine rate of 0.5 represents that for every 10 routine tasks, 5 non-routine tasks *will occur on average*. Additionally, the prior non-routine rate (*nrr*) distribution is defined to depict how the analysed maintenance services are scattered along the different non-routine rate intervals. In other words, it describes the non-routine rate distribution of the sample of maintenance checks.

The four variables (*a*, *fhy*, *cy* and *se*) along with the prior non-routine rate (*nrr*) distribution are considered as five pieces of evidence and combined by applying the ER rule to calculate the *expected non-routine rate (Enrr), as shown in* Figure 6-1. The model is used as an inference tool that allows the estimation of the number of unexpected maintenance activities caused by unforeseen events that are highly associated with the utilisation of the aircraft. In this way, rather than defining a single average non-routine rate with its associated standard deviation, the model determines a non-routine rate distribution for an aeroplane with certain operational features.

Table B-1 in Appendix B.1 presents the core information that is used for building the ER model. It presents a summarised version of the data collected, including a generic identifier for each maintenance service and the five pieces of evidence, namely, aircraft's age, flight hours per year, cycles per year, type of service and non-routine rate. For confidentiality reasons all sensitive data was disguised or not presented in the table, but this does not affect the model's development.

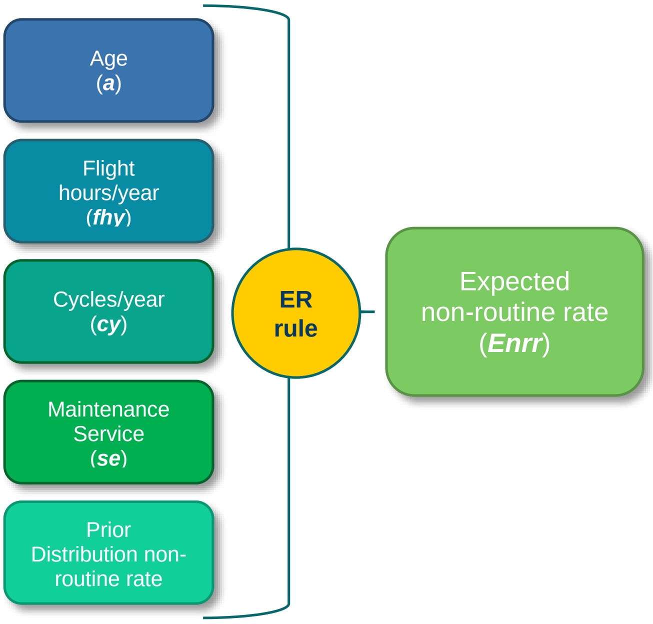


Figure 6-1 Main variables for estimating the non-routine rate

6.2 Data statistical analysis

To gain more insight into the variables and their relationships, a brief statistical analysis was performed. Firstly, basic descriptive statistical tools (such as mean, standard deviation, skewness, kurtosis, etc.) were used to summarise the general characteristics of the sample. Secondly, a correlation analysis was carried out to understand the linear relationship between the variables.

Table 6-1 summarises the results obtained from the descriptive statistics. It can be observed that the newest airliner of the analysed fleet is about 4.5 years while the oldest is around 12.5 years old. Annual operation ranges from approximately 2,400 to 4,500 flight hours and from around 950 to 2,700 cycles. The non-routine rate ranges from a minimum of almost 10% to a maximum of around 50%. On average, an aeroplane in the sample is almost eight years old, flying around 3,500 hours per year, completes nearly 1,700 take-offs and landings during the same period of time and has a maintenance ratio of approximately 2 non-routine tasks to every 10 routine tasks.

Regarding the dispersion of data, the coefficient of variation shows that amongst the four variables, non-routine rate is the most dispersed, followed by aircraft age, cycle and flight hours respectively. Considering the standard deviation and based on Chebyshev's rule, around 75% of the data is clustered between an age of 6 and 10 years, with a yearly utilisation of approximately 3,170 to 3,770 flight hours and 1,420 to 1,900 cycles and with a non-routine rate between 0.13 and 0.31.

Analysing the kurtosis and skewness, it can be noted that flight hours per year and cycles per year present a pointed distribution and nearly centred, i.e. most of the data is symmetrically clustered near to the mean. Although age and non-routine rate have a flat distribution and are skewed to the right, the former is flatter and slightly skewed whereas the latter is lightly flat but its skewness is larger.

Table 6-1 Descriptive statistics of the main variables

n = 91	Age (Years)	Flight Hours / Year	Cycles / Year	Non-routine rate
Min	4.56	2,417	959	0.096
Max	12.52	4,467	2,686	0.468
Mean	7.95	3,471	1,664	0.222
Standard deviation	1.97	300	240	0.088
Coefficient of Variation	25%	9%	14%	40%
Kurtosis	-0.83	1.70	1.70	-0.38
Skewness	0.26	-0.01	-0.01	0.60

Table 6-2 depicts the distribution of different maintenance service types within the sample. The most common maintenance checks were 2C, 3C+, 1C and 4C with around 20%, 18%, 15% and 13% respectively, followed by the 3C service with 10%. 4C+, 5C and 5C+ packages were carried out almost 7% of the time. With around 4%, the least common service was the 6C check.

Table 6-2 Relative frequency of heavy maintenance checks

Service Type	Relative frequency
1C	15.4%
2C	19.8%
3C	9.9%
3C+	17.6%
4C	13.2%
4C+	6.6%
5C	6.6%
5C+	6.6%
6C	4.4%

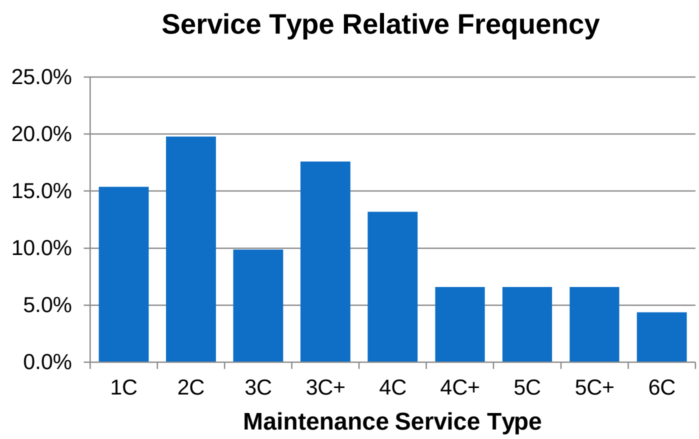


Table 6-3 is an excerpt from the correlation analysis presenting the most significant results. The close association between age, total flight hours and total cycles is evident: the older the aeroplane, the higher its accumulated usage as measured by flight hours and cycles. There is also a significant correlation between non-routine rate, age and total aeroplane operations: the older the aeroplane and the higher its utilisation, the larger the expected amount of damage and number of discrepancies. In order to reduce the effect of age over usage, it is proposed to use annual (i.e. flight hours per year and cycles per year) rather than accumulated utilisation. It is important to highlight the negative correlation that flight hours per year has with age and total flight hours and cycles. A possible explanation for this could be that over time aeroplanes are used less due to operation costs or maintenance requirements. Higher annual numbers of flight hours are found in newer aircraft and therefore the non-routine rate appears to decrease. However, due to the small correlation index, it seems that cycles per year by themselves do not play a relevant role in the occurrence of damage and discrepancies and also have a poor association with the other variables. The correlation analysis offers interesting and useful information regarding the linear relationship between variables. However, it is very likely that the interrelationships between the variables may be more complex than they look.

Table 6-3 Correlation between the variables

	Age (Years)	Total Flight Hours	Total Cycles	Flight Hours/year	Cycles / Year	Non-routine rate
Age (Years)	1.00	0.937**	0.804**	-0.649**	0.039	0.463**
Total Flight Hours		1.00	0.857**	-0.356**	0.189	0.417**
Total Cycles			1.00	-0.366**	0.615**	0.375**
Flight Hours/Year				1.00	0.186	-0.404**
Cycles/Year					1.00	0.045
Non-routine rate						1.00

**. Correlation is significant at the 0.01 level (2-tailed).

Figure 6-2 depicts the relationship between the non-routine rate and the type of maintenance check executed. Even though it appears that when the maintenance check increases in complexity and in-depth, the number of unscheduled tasks per scheduled task also rises, the positive correlation between the variables does not appear to be completely linear. Rather, from the data gathered, it looks that there is an increasing cyclical trend. A probable reason for this behaviour could be the way the maintenance packages are structured, varying the number and type of tasks with the aim of restoring the aircraft to its operational state.

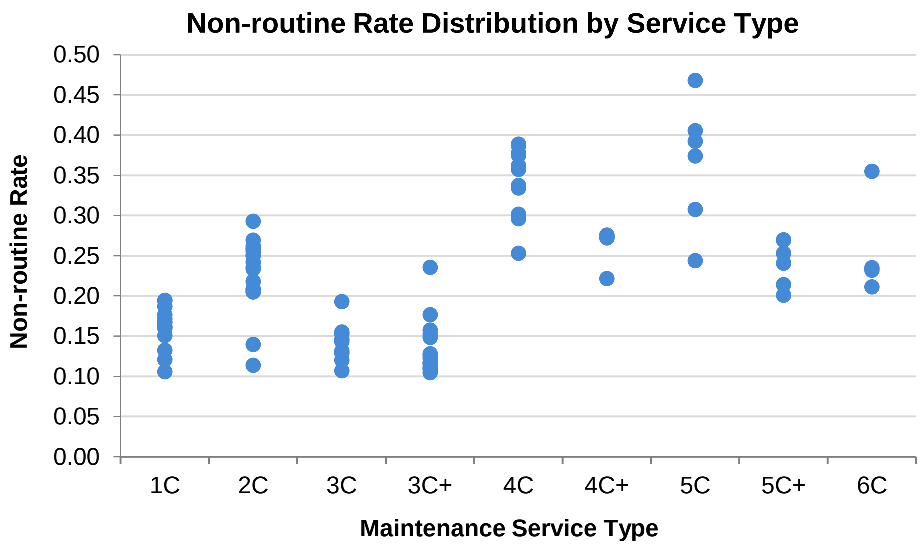


Figure 6-2 Scatterplot of non-routine rate by type of maintenance service

6.3 Data discretisation

Bearing in mind that four (a, fhy, cy and nrr) out of the five influencing variables are continuous, each of them was discretised, categorising them into different intervals. The reason for organising and classifying continuous data into discrete intervals is to facilitate the modelling process, as it helps to obtain the belief distributions (which will be described later). Moreover, by using discrete or categorised information it is easier to understand and analyse for practitioners and decision makers.

The discretisation process is carried out in three phases: 1) to properly categorise each variable, the number of intervals and their width is obtained using different rules. 2) The interval widths calculated in the previous step are reviewed and modified if necessary, to define intervals that meaningfully represent the variables for experts and practitioners by using appropriate class limits (e.g. it might be easier to conceptualise an age interval between 4 to 5 years than one of 4.37 to 5.66 years). 3) For each variable, different interval sizes are analysed comparing the shape of the estimated density function to avoid under- or over-smoothing and to determine which describes the data most precisely.

6.3.1 Determining the number of intervals and their width

Finding the correct number of intervals may not be an easy task. Doane (1976) raised the question of how to calculate and make a frequency classification with the "right" number of classes, "nice" class limits and "round" interval widths. In this regard, Guiasu (1986) explains that grouping data is a useful way of dealing with complexity. However, he further adds that a certain amount of information is lost when raw data is grouped in classes and that the larger the class interval, the greater the amount of information that is lost. On the other hand, if the number of classes is too large, the presentation of information can be misleading as it starts describing noises of the sample. Therefore, Guiasu (1986) concludes that a reasonable balance between information content and class homogeneity must be reached during the choice of a class interval.

Histograms are amongst the most important and useful graphical tools in statistical practice. They are exceptionally helpful for displaying and summarising data and for providing a consistent estimation and representation of any unknown density function (Scott, 2009). In the same vein, Birgé and Rozenholc (2006) and Freedman and Diaconis (1981) remark that although histograms may be considered an obsolete way of estimating densities, they are easy to produce and, unlike kernel estimators, are widely used in applied work.

For building a *regular histogram*[3], four basic parameters must be defined: 1) the lower and upper limits of the sample; 2) the number of intervals, also called classes or bins; 3) the width of each interval, or bin-width; and 4) the lower limit of the first interval.

The lower and upper limits of the sample (a, b) are used to define the range of the dataset ($R = b - a$) where the intervals are distributed. The number of classes (k) determines the intervals in which the raw data will be classified. The bin-width (h) represents the length of each interval. There is an inverse relation between the number of intervals and their width: the wider the bin, the fewer intervals required and vice versa. In this regard, Wand (1997) explains that bin-width

[3] A *regular histogram* is the one in which its intervals are of equal length (Birgé and Rozenholc, 2006). According to Scott (1992, 2009) to make the intervals comparable between them, they should have the same width, otherwise, if the interval widths are not of the same size, the shape of the histogram can be grossly misleading.

has a substantial effect on the appearance of the histogram with respect to the true distribution: a very small bin-width produces a jagged histogram with too much detail (*under-smoothing*) and conversely, a very large bin-width results in a histogram with not enough detail (*over-smoothing*).The lower limit of the first class (l_0) also affects the shape of the histogram, as it determines the position of the first bin and therefore the position of the others. Equation (6-1) shows the relation between the upper and lower limits, the bin-width and the number of intervals. As can be seen, the larger the range, the greater the number of intervals and the wider the bin the smaller the number of intervals.

$$k = \frac{b - a}{h} \tag{6-1}$$

A plethora of rules has been proposed for determining the number of intervals and the bin width for grouped data and the most common rules are described below.

6.3.1.1 Sturges' rule:

Sturges' rule (Sturges, 1926) is the most common rule for determining the number of intervals and their width for histogram density estimation. It is a normal-based rule as it assumes that the variable is normally distributed. The Gaussian variable is approximated to a binomial distribution in order to divide it into equal sized intervals. In other words, a normally distributed sample of size n can be divided in to k intervals, so that the class frequencies comprise a binomial series for all n which are powers of 2. Sturges' rule converts continuous, symmetric, normal data into discrete, symmetric, binomial classes (Doane, 1976). Equations (6-2) and (6-3) show Sturges' rule, where h is the class width or class interval, k represents the number of classes or bins, n is the number of elements of the sample and R is the range.

$$k = 1 + log_2 n \tag{6-2}$$

$$h = \frac{R}{1 + log_2 n} \tag{6-3}$$

Despite its wide usage, several authors have criticised Sturges' rule, stating that, particularly for large samples, severely skewed or multimodal distributions, it tends to over-smooth the histogram, failing to provide enough classes to reveal the real shape of the distribution (Doane, 1976; Scott, 1992; Scott, 2009; Wand, 1997).

6.3.1.2 Doane's rule:

Doane's rule (Doane, 1976) is a modification of Sturges' rule to overcome the problem of highly skewed distributions. As can be seen in eq. (6-4), this rule includes an additional term to reflect skewness. g_1 (6-5) represents the *moment coefficient of skewness* and σ_{g_1} (6-6) its standard deviation, which becomes smaller as sample size increases. If the sample is symmetrical, $g_1 = 0$, no extra intervals are added and the rule becomes the same as Sturges' rule.

$$k = 1 + log_2 n + log_2 \left(1 + \frac{|g_1|}{\sigma_{g_1}}\right) \tag{6-4}$$

$$g_1 = \frac{\sum(x-\bar{x})^3}{[\sum(x-\bar{x})^2]^{3/2}} \tag{6-5}$$

$$\sigma_{g_1} = \sqrt{\frac{6(n+2)}{(n+1)(n+3)}} \tag{6-6}$$

6.3.1.3 Scott's rule:

Scott's rule (Scott, 1979), like Sturges' rule, also assumes a Gaussian distribution of the sample. However, it is based on a different approach. This rule uses the integrated mean squared error (*IMSE*) as a global error measure of a histogram to calculate the optimal bin width. The *IMSE* compares the true density function, which is unknown, with the calculated histogram density. Therefore, Scott assumed a normal distribution and used the Gaussian density function as a reference. Eq. (6-7) presents Scott's rule, where n is the sample size and σ an estimate of the standard deviation. Scott (2014) explains that both Sturges' and Scott's rules are based on the normal distribution, but Sturges makes a deterministic calculation, whereas Scott's approach is based on optimising the global variance and squared bias of the histogram estimator.

Scott (1992) improved his rule (6-8) by adding two factors: the skewness and the kurtosis of the sample. The skewness factor (6-9) considers a lognormal density function where σ' represents the lognormal standard deviation. The kurtosis factor (6-10) uses a t distribution, where v are the degrees of freedom and Γ is the gamma distribution. These factors allow the rule to adjust the interval width for samples that are not completely normal.

$$h = 3.49\sigma n^{-1/3} \tag{6-7}$$

$$h = 3.49\sigma n^{-1/3}(SkFc)(KuFc) \tag{6-8}$$

$$SkFc = \frac{2^{1/3}\sigma'}{e^{5\sigma'^2/4}(\sigma'^2+2)^{1/3}(e^{\sigma'^2}-1)^{1/2}} \tag{6-9}$$

$$KuFc = \frac{\sqrt{v-2}}{2}\left[\frac{\Gamma(v+3)\Gamma\left(\frac{v}{2}\right)^2}{\Gamma\left(v+\frac{3}{2}\right)\Gamma\left(\frac{v+3}{2}\right)^2}\right]^{1/3} \tag{6-10}$$

6.3.1.4 Freedman-Diaconis' rule:

Similarly to Scott's rule, Freedman and Diaconis (1981) propose a formula to calculate the bin width based on the *IMSE*, but instead of using the estimated standard deviation, they replace it by utilising the interquartile range IQ resulting in a more robust rule (6-11). According to Scott (1992), the Freedman-Diaconis rule provides 35% more bins than Scott's rule, giving a rougher histogram. However, for highly skewed or spread samples this rule could be very useful.

$$h = 2IQn^{-1/3} \tag{6-11}$$

6.3.1.5 Rules results and comparison

Table 6-4 shows a comparison of the number of intervals and the bin-width using the rules described above. As can be seen, Sturges' rule suggests the same number of intervals (near 8) for the different variables, as the rule relies just on the size of the sample. Therefore, as the sample size is the same for all variables, the number of intervals remains the same. Doane's rule modifies Sturge's rule by adding the skewness effect. It can be seen that as age and non-routine rate are more skewed than flight hours and cycles, they require more intervals (around 9-10) than the latter (approx. 8). Scott's rule takes into account the dispersion of the data by using the standard deviation. According to the suggested number of intervals, the flight hours and cycles are more dispersed than age and non-routine rate. However, after considering the effect of skewness and kurtosis, it seems that these factors have greater impact on the number of intervals for age and non-routine rate (i.e. the enhanced Scott's rule suggests around 6-7 intervals for *a* and *nrr*, and 9-10 for *fhy* and *cy*). Similarly to Scott's rule, the Freedman-Diaconis rule also considers the data spread, but based on the interquartile range of the sample. His rule suggests more intervals for flight hours and cycles. In summary, these rules propose using a bin-width from 0.9 to 1.5 years for aircraft age, a bin-width of around 144 to 273 hours for flight hours, an interval-width between 74 and 230 cycles for take-off-landings and a class-width from 0.05 to 0.07 for non-routine rate.

Table 6-4 Number of intervals “*k*” and bin-widths “*h*” according to different rules.

Rules	**Age**		**Flight hours/year**		**Cycles/year**		**Non-routine rate**	
	h	***k***	***h***	***k***	***h***	***k***	***h***	***k***
Sturge's	1.1	7.5	273	7.5	230	7.5	0.05	7.5
Doane's	0.9	8.5	270	7.6	228	7.6	0.04	9.3
Scott's (Normal)	1.5	5.2	233	8.8	186	9.3	0.07	5.5
Scott's (w/Sk& Ku factors)	1.4	5.8	228	9.0	178	9.7	0.05	7.0
Freedman-Diaconis'	1.5	5.4	144	14.2	74	23.4	0.05	6.9

6.3.2 Intervals meaningfulness

The results obtained in Table 6-4 were reviewed and adjusted to take into account their meaningfulness and practicality for practitioners and experts. The class intervals are redefined to improve understanding and utilisation. Table 6-5 presents the suggested bin-widths for each variable. For aircraft age, it may be clearer to work with periods of one year or half a year and a bin width of 0.5, 1, 1.5 or 2 years of age is therefore recommended. For the interval-widths of flight hours and cycles, it is proposed to utilise multiples of 50 (i.e. 150, 200, 250 and 300). Non-routine rate is easier to handle with bin-widths of 0.025, 0.05 or 0.10.

Table 6-5 Proposed number of intervals and bin-widths

Option	Age		Flight hours/year		Cycles/year		Non-routine rate	
	h	***k***	***h***	***k***	***h***	***k***	***h***	***k***
1	0.5	15.9	150	13.7	150	11.5	0.025	14.9
2	1.0	8.0	200	10.3	200	8.6	0.05	7.4
3	1.5	5.3	250	8.2	250	6.9	0.10	3.7
4	2.0	4.0	300	6.8	300	5.8		

6.3.3 Data distribution

Using the bin-widths proposed in Table 6-5, several frequency distribution graphs were created for each variable with the aim of comparing their shapes and determining those that are most suitable for representing the data. The graphs were built using different interval sizes and by moving the lower limit of the first interval.

Figure 6-3 illustrates the frequency distribution of the sample for aircraft age when the bin size is half a year (i.e. 0.5). Figure 6-4 depicts the frequency distribution with a bin-width of two years (i.e. 2.0). These figures show the effect of the interval-width on the shape of the distribution. It can clearly be seen that in the first graph the distribution appears to be jagged, while the second has an over-smoothed shape. Using the same bin-width but with different interval limits can also affect the shape of the distribution. Figure 6-5 and Figure 6-6 present an example of this situation. Both histograms have a bin-width of 1.5 years, but the lower limit of the first graph is 2.5 years, whereas for the second it is 3.0 years. It is noticeable that the shape of the distribution is significantly different between the two histograms. Therefore, it is important to highlight the relevance of choosing the most appropriate bin-width and interval limits, as they might change the shape the frequency distribution and subsequently affect calculation of the belief distribution, so altering the results and performance of the ER model. In a similar way to this example, Table B-2, Table B-3, Table B-4 and Table B-5 in Appendix B.2 show the comparison of different frequency distributions for the four variables (age, flight hours, cycles and non-routine rate, respectively).

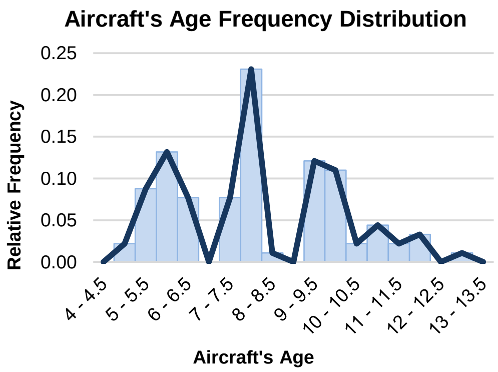


Figure 6-3 Aircraft's age frequency distribution with a bin-width of 0.5 years

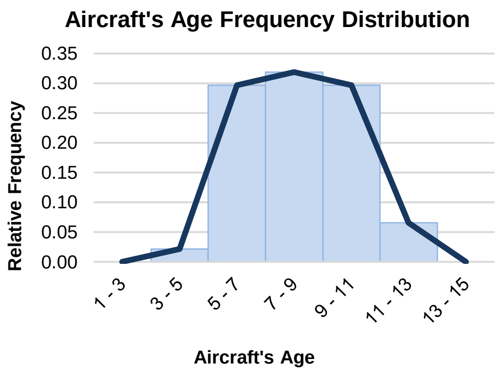


Figure 6-4 Aircraft's age frequency distribution with a bin-width of 2.0 years

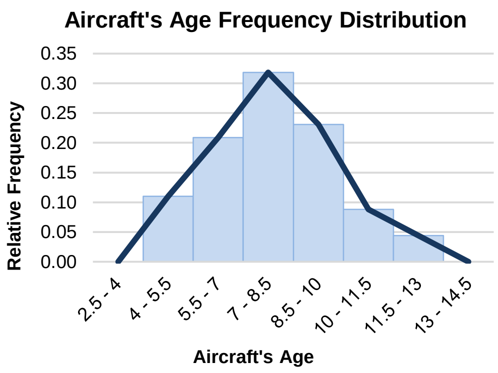


Figure 6-5 Aircraft's age frequency distribution with a bin-width of 1.5 year, lower limit 2.5

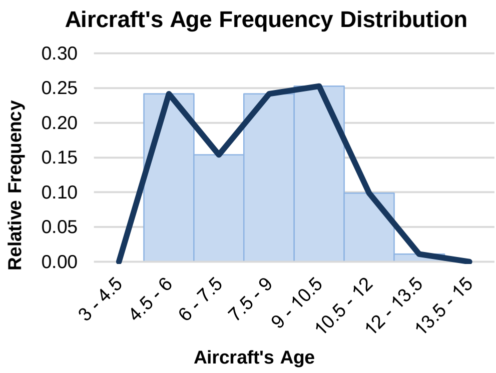


Figure 6-6 Aircraft's age frequency distribution with a bin-width of 1.5 year, lower limit 3.0

6.4 ER rule model

After performing this brief statistical analysis, and once the variables have been discretised into different intervals, the evidential reasoning (ER) rule, a generic Bayesian reasoning process for combining multiple pieces of independent evidence, is applied to develop an inference model for estimating unscheduled activities in aircraft maintenance. The procedure described below is based on Yang and Xu (2014) and is used for building the inference model.

1. Determine the frequency distribution between the intervals of the *predictor variables* and the classes of the observed outcome (i.e. non-routine rate) using a cross-tabulation. For example, given a specific age bin, determine the number of maintenance services that fit into a particular range of non-routine rate.
2. Based on the frequency distribution, the prior probabilities of the non-routine rate and the likelihoods for each interval of every variable are calculated.
   a. The prior probabilities are obtained by dividing the frequency of each non-routine interval by the total number of maintenance services. In other words, the prior probabilities represent the relative frequency of the non-routine rate.
   b. The likelihoods are determined for each non-routine rate interval, dividing the number of events for every interval of the *predictor variables* by the frequency of that particular non-routine rate interval.
3. Using the results of step 2, the belief distributions for each interval of the *predictor variables* (pieces of evidence) are obtained by dividing each likelihood by the sum of likelihoods in that class.
4. The ER rule is applied to aggregate two classes of different pieces of evidence, combining the non-routine rate belief distribution of the first variable with the non-routine rate belief distribution of the second variable. Then, the ER rule is used recursively to aggregate all the other pieces of evidence to finally obtain the belief distribution of the expected non-routine rate.

In order to clarify the described process, the subsequent paragraphs present an easy-to-follow example based on the collected maintenance records. An essential step before beginning this process is the definition of intervals for each variable. For the presented example, they are set as follows: 1) five classes for non-routine rate with a bin-width of 0.1, starting from 0 and ending in 0.5; 2) for age, five intervals of two years each with a lower limit of 4 years and an upper limit of 14; 3) nine bins of 250 flight hours each, ranging from 2250 to 4500; 4) eight classes with a size of 250 cycles, between 750 and 2750; and 5) services type, as a categorical variable, considers the nine different types of maintenance checks applied.

Firstly, each maintenance event in the sample is counted and allocated to its corresponding interval, building a matrix of frequencies between each variable and the non-routine rate. For instance, of the 35 aeroplanes aged between 6 and 8 years, 12 have a non-routine rate from 0.10-0.20, 14 a rate of 0.20-0.30 and nine a rate from 0.30-0.40 (shown in row 2 Table 6-6). The frequency distribution of the non-routine rate was obtained in the same way for the rest of the variables. The results can be seen in Table 6-6.

Table 6-6 Non-routine rate frequency distribution of each piece of evidence

Row	**Variable**	**Interval**	**Non-routine Rate (NRT/RT)**					**Total**
			0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***	
1	Age	4-6	1	20	1			22
2		6-8		12	14	9		35
3		8-10			16	4	2	22
4		10-12		7	3	1		11
5		12-14			1			1
6	Total Age		1	39	35	14	2	91
7	Flight hours / Year	2250-2500			1			1
8		2500-2750						0
9		2750-3000		2	3	1		6
10		3000-3250		2	1	1		4
11		3250-3500		10	16	11	2	39
12		3500-3750	1	12	11	1		25
13		3750-4000		11	3			14
14		4000-4250		1				1
15		4250-4500		1				1
16	Total Fh / Y		1	39	35	14	2	91
17	Cycles / Year	750-1000			1			1
18		1000-1250			3			3
19		1250-1500		9	5	1		15
20		1500-1750		19	18	8	2	47
21		1750-2000	1	10	5	5		21
22		2000-2250			2			2
23		2250-2500						0
24		2500-2750		1	1			2
25	Total Cy / Y		1	39	35	14	2	91

Row	Variable	Interval	Non-routine Rate (NRT/RT) *0-0.1*	*0.1-0.2*	*0.2-0.3*	*0.3-0.4*	*0.4-0.5*	Total
26	Maintenance Service Type	1C		14				14
27		2C		2	16			18
28		3C		9				9
29		3C+	1	14	1			16
30		4C			2	10		12
31		4C+			6			6
32		5C			1	3	2	6
33		5C+			6			6
34		6C			3	1		4
35	Total Services		1	39	35	14	2	91

The prior probabilities and likelihoods for each variable are then generated. Continuing with this example, the prior probability is calculated by dividing the frequency of the non-routine rate by the total number of events, i.e. 1/91, 39/91, 35/91, 14/91 and 2/91 correspondingly (rows 6, 16, 25 or 35 in Table 6-6 and the results in rows 6, 16, 25 or 35 in Table 6-7). The likelihoods are obtained by dividing the non-routine rate frequency of a particular piece of evidence by the total frequency of that specific non-routine rate interval. In the case of an aeroplane between 6 and 8 years old, this means dividing its specific non-routine frequency by the total non-routine frequency of each interval, namely 12/39, 14/35 and 9/14 (row 2 over row 6 of Table 6-6; results in row 2 Table 6-7). Table 6-7 presents the prior probabilities and likelihoods of all pieces of evidence, with the examples highlighted. The likelihood of aircraft aged between 6 and 8 years represent 31% of maintenance services with a non-routine rate of 0.10-0.20, 40% with a non-routine rate from 0.20-0.30 and 64% with a non-routine rate of 0.30-0.40. None of this class of aircraft have a non-routine rate between 0 and 0.10, or 0.40 and 0.50 (highlighted in row 2 Table 6-7).

Table 6-7 Prior probabilities and likelihoods of each piece of evidence

row	Variable	Interval	Non-routine Rate (NRT/RT) *0-0.1*	*0.1-0.2*	*0.2-0.3*	*0.3-0.4*	*0.4-0.5*
1	Age	4-6	1.000	0.513	0.029	-	-
2		6-8	-	0.308	0.400	0.643	-
3		8-10	-	-	0.457	0.286	1.000
4		10-12	-	0.179	0.086	0.071	-
5		12-14	-	-	0.029	-	-
6	Prior probability		0.011	0.429	0.385	0.154	0.022
7	Flight hours /Year	2250-2500	-	-	0.029	-	-
8		2500-2750	-	-	-	-	-
9		2750-3000	-	0.051	0.086	0.071	-
10		3000-3250	-	0.051	0.029	0.071	-
11		3250-3500	-	0.256	0.457	0.786	1.000
12		3500-3750	1.000	0.308	0.314	0.071	-
13		3750-4000	-	0.282	0.086	-	-
14		4000-4250	-	0.026	-	-	-

row	Variable	Interval	Non-routine Rate (NRT/RT)				
			0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
15		4250-4500	-	0.026	-	-	-
16	Prior probability		0.011	0.429	0.385	0.154	0.022
17	Cycles /Year	750-1000	-	-	0.029	-	-
18		1000-1250	-	-	0.086	-	-
19		1250-1500	-	0.231	0.143	0.071	-
20		1500-1750	-	0.487	0.514	0.571	1.000
21		1750-2000	1.000	0.256	0.143	0.357	-
22		2000-2250	-	-	0.057	-	-
23		2250-2500	-	-	-	-	-
24		2500-2750	-	0.026	0.029	-	-
25	Prior probability		0.011	0.429	0.385	0.154	0.022
26	Maintenance Service Type	1C	-	0.359	-	-	-
27		2C	-	0.051	0.457	-	-
28		3C	-	0.231	-	-	-
29		3C+	1.000	0.359	0.029	-	-
30		4C	-	-	0.057	0.714	-
31		4C+	-	-	0.171	-	-
32		5C	-	-	0.029	0.214	1.000
33		5C+	-	-	0.171	-	-
34		6C	-	-	0.086	0.071	-
35	Prior probability		0.011	0.429	0.385	0.154	0.022

The third step is to calculate the belief distributions for each piece of evidence, where each likelihood of a particular category is divided by the sum of likelihoods of that class. Following the example of aircraft aged between 6 and 8 years, the non-routine rate belief distribution of 0.228, 0.296 and 0.476 (shown in row 2 Table 6-8) is calculated by dividing 0.31 by the sum of 0.31, 0.40 and 0.64 (i.e. 1.35), 0.40 by 1.35 and 0.64 by 1.35 respectively (i.e. dividing each likelihood in row 2 Table 6-7 by the sum of those likelihoods). For this particular case, the belief distribution can be explained as the possibility of having a non-routine rate of 0.10 to 0.20 22.3% of the time, in 29.6% of the occasions a non-routine rate from 0.20 to 0.30 and a 47.6% chance of a non-routine rate between 0.30 and 0.40. Table 6-8 shows the non-routine rate belief distributions for each piece of evidence.

Table 6-8 Non-routine rate belief distribution of each piece of evidence

row	Variable	Interval	Non-routine Rate (NRT/RT)				
			0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
1	Age	4-6	0.649	0.333	0.019	-	-
2		6-8	-	0.228	0.296	0.476	-
3		8-10	-	-	0.262	0.164	0.574
4		10-12	-	0.533	0.255	0.212	-
5		12-14	-	-	1.000	-	-
6	Flight hours / Year	2250-2500	-	-	1.000	-	-
7		2500-2750	-	-	-	-	-

row	Variable	Interval	Non-routine Rate (NRT/RT)				
			0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
8		2750-3000	-	0.246	0.411	0.343	-
9		3000-3250	-	0.339	0.189	0.472	-
10		3250-3500	-	0.103	0.183	0.314	0.400
11		3500-3750	0.591	0.182	0.186	0.042	-
12		3750-4000	-	0.767	0.233	-	-
13		4000-4250	-	1.000	-	-	-
14		4250-4500	-	1.000	-	-	-
15	Cycles / Year	750-1000	-	-	1.000	-	-
16		1000-1250	-	-	1.000	-	-
17		1250-1500	-	0.519	0.321	0.160	-
18		1500-1750	-	0.189	0.200	0.222	0.389
19		1750-2000	0.569	0.146	0.081	0.203	-
20		2000-2250	-	-	1.000	-	-
21		2250-2500	-	-	-	-	-
22		2500-2750	-	0.473	0.527	-	-
23	Maintenance Service Type	1C	-	1.000	-	-	-
24		2C	-	0.101	0.899	-	-
25		3C	-	1.000	-	-	-
26		3C+	0.721	0.259	0.021	-	-
27		4C	-	-	0.074	0.926	-
28		4C+	-	-	1.000	-	-
29		5C	-	-	0.023	0.172	0.805
30		5C+	-	-	1.000	-	-
31		6C	-	-	0.545	0.455	-
32	Non-routine rate prior distribution		0.011	0.429	0.385	0.154	0.022

Once the belief distributions have been determined, the ER rule, as a generalised Bayesian inference process, is used to combine the five pieces of evidence in order to obtain the expected non-routine rate, taking into account their weights and reliabilities. In other words, the ER rule is applied to estimate the non-routine rate for an aeroplane, with specific operational and maintenance characteristics, by combining the belief distribution of the observed non-routine rate for age, flight hours, cycles, services and the prior distribution. Therefore, in the fourth step of the modelling procedure, the first couple of variables are aggregated and the ER rule is then applied recursively to combine the remaining pieces of evidence.

To simplify the presented practical case, weight and reliability were assumed equal to one, i.e. all the pieces of evidence are assumed to be fully reliable and highly important. The ER rule (equation (4-5) shown and described in Chapter 4:) is used to combine age (e_a) and flight hours per year (e_{fhy}). Using the recursive ER formula (4-6), the remaining pieces of evidence, i.e. cycles per year (e_{cy}), maintenance services (e_{se}) and prior distribution of the non-routine rate (e_{nrr}), were then aggregated to obtain the joint non-routine rate considering all the pieces of evidence (i.e. $Enrr_{e(5)}$).

The example continues by considering the non-routine rate distribution of the whole sample and an aeroplane aged between 6 and 8 years old, with a yearly utilisation of 3,250-3,500 flight hours and 1,500-1,750 cycles and which underwent a 4C maintenance check (rows 2, 10, 18, 27 and 32 in Table 6-8). The combined degree of belief of the five pieces of evidence is given in Table 6-9. Rows 2-6 present the belief distribution of the non-routine rate for each piece of evidence. In rows 7-11, the weighted belief distribution is calculated using $m_{\theta,j} = w_j p_{\theta,j}$, in this case as $w_j = 1$, $m_{\theta,j} = p_{\theta,j}$. In rows 12 and 13, the ER rule, eq. (4-5) is used to combine the first two pieces of evidence, i.e. age (e_a) and flight hours per year (e_{fhy}). Then, in rows 14 to 19 the ER rule is applied recursively, eq. (4-6), to aggregate the remaining variables. In the last row, the combined belief distribution of the non-routine rate, based on the information provided by the five pieces of evidence, is shown. As can be seen in row 20, for an aeroplane with the aforementioned characteristics, the expected belief distribution of the non-routine rate is a 6% chance for a rate between 0.20 and 0.30 and a 94% possibility for a rate from 0.3 to 0.4. Figure 6-7 depicts the aggregation process using the ER Rule for the example described above. Here, it is interesting to observe how the belief distribution changes shape with each iteration.

Table 6-9 ER Rule for an aeroplane aged 6-8, with 3,250-3,500 fh/y, 1500-1750 cy/y and a 4C service.

		Non-routine rate					
		0-0.1	0.1-0.2	0.2-0.3	0.3-0.4	0.4-0.5	P(Θ)
2	$p_{\theta,a}$	-	0.228	0.296	0.476	-	-
3	$p_{\theta,fhy}$	-	0.103	0.183	0.314	0.400	-
4	$p_{\theta,cy}$	-	0.189	0.200	0.222	0.389	-
5	$p_{\theta,se}$	-	-	0.074	0.926	-	-
6	$p_{\theta,nrr}$	0.011	0.429	0.385	0.154	0.022	-
7	$m_{\theta,a}$	-	0.228	0.296	0.476	-	-
8	$m_{\theta,fhy}$	-	0.103	0.183	0.314	0.400	-
9	$m_{\theta,cy}$	-	0.189	0.200	0.222	0.389	-
10	$m_{\theta,se}$	-	-	0.074	0.926	-	-
11	$m_{\theta,nrr}$	0.011	0.429	0.385	0.154	0.022	-
12	$\hat{m}_{\theta,e(2)}$	-	0.023	0.054	0.150	-	-
13	$m_{\theta,e(2)}$	-	0.103	0.238	0.659	-	-
14	$\hat{m}_{\theta,e(3)}$	-	0.019	0.048	0.146	-	-
15	$m_{\theta,e(3)}$	-	0.091	0.223	0.685	-	-
16	$\hat{m}_{\theta,e(4)}$	-	-	0.017	0.635	-	-
17	$m_{\theta,e(4)}$	-	-	0.025	0.975	-	-
18	$\hat{m}_{\theta,e(5)}$	-	-	0.010	0.150	-	-
19	$m_{\theta,e(5)}$	-	-	0.061	0.939	-	-
20	$Enrr_{\theta,e(5)}$	-	-	0.061	0.939	-	

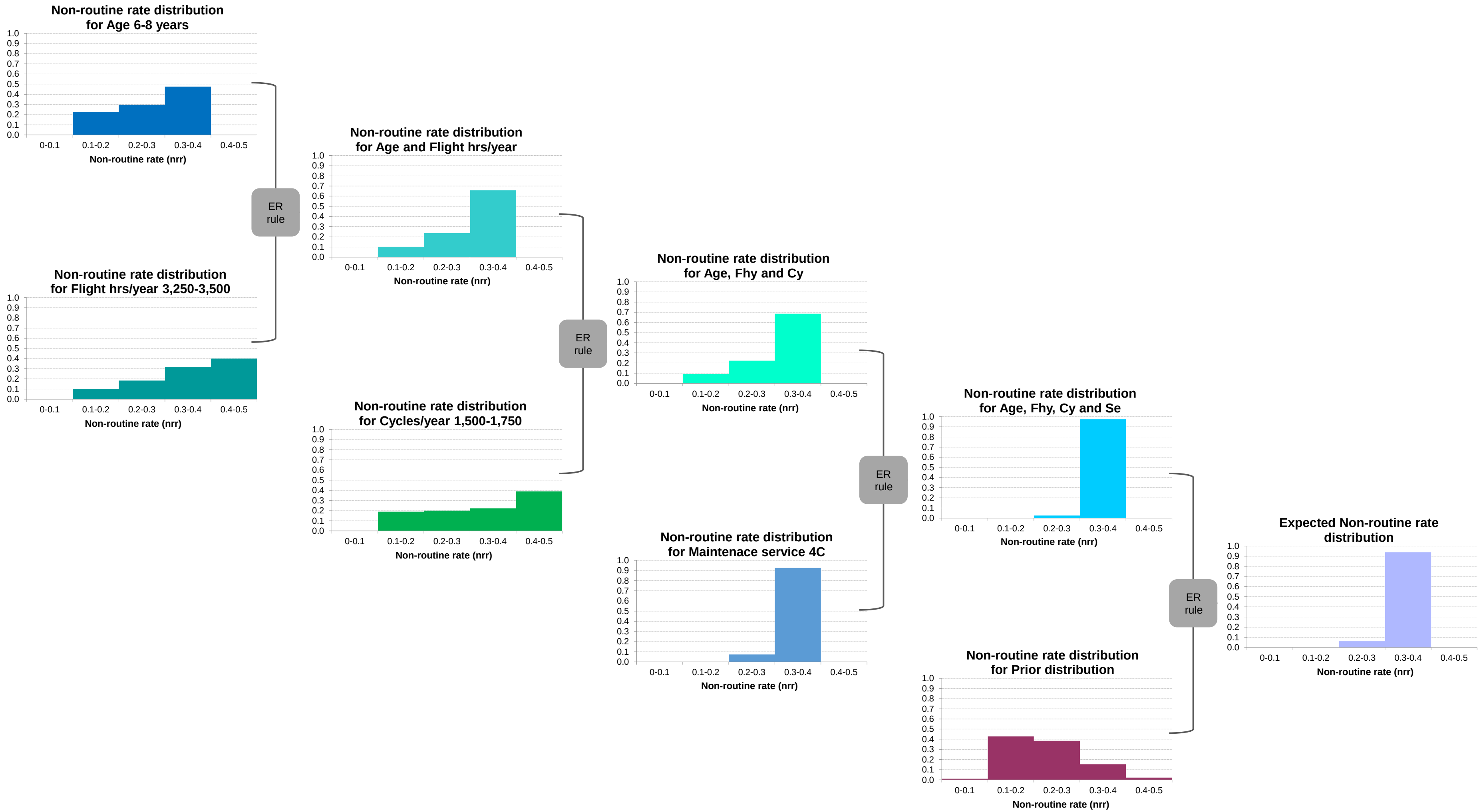


Figure 6-7 ER Rule for an aeroplane aged 6-8, with 3,250-3,500 fh/y, 1500-1750 cy/y and a 4C service.

Table 6-9 and Figure 6-7 describe the ER Rule aggregation process for a specific array of variables. There are 3,240 potential variable combinations, considering the intervals defined above (i.e. five classes for age, nine for flight hours per year, eight for cycles per year and nine for the type of maintenance service). The same process, therefore, should be applied 3,239 more times to obtain the expected belief distribution for all the possible cases. On the other hand, analysing the real data, just 43 real combinations are found, so in order to reduce computation time, the aggregation process is applied only for the actual cases. Table 6-10 presents the expected non-routine rate for the 43 real cases.

As can be noted from the description of the ER model, the aggregation process to estimate the non-routine rate for the different combinations of Age-Flight hours-Cycles-Services requires recurrent and extensive calculations even for the small sample used in the study. Performing all these mathematical operations manually would be tedious, inefficient and time consuming. Working with a traditional spreadsheet application would not be the best option as such software is not capable of working with a large amount of variables, especially for the optimisation models. For this reason, to reduce calculation time and to make the model more robust in handling a large number of variables and more responsive to changes, MATLAB is used to build the different aggregation models to address the scenarios that will be described in the following sections.

Table 6-10 Expected non-routine rate belief distribution for the real combinations

Age	**Hours**	**Cycles**	**Services**	**$Enrr_{\theta,e(5)}$ Estimated non-routine rate**				
				0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
4-6	3500-3750	1500-1750	3C	-	1.000	-	-	-
4-6	3500-3750	1500-1750	3C+	-	0.996	0.004	-	-
4-6	3500-3750	1750-2000	3C+	0.638	0.361	0.001	-	-
4-6	3750-4000	1250-1500	1C	-	1.000	-	-	-
4-6	3750-4000	1250-1500	2C	-	0.923	0.077	-	-
4-6	3750-4000	1250-1500	3C	-	1.000	-	-	-
4-6	3750-4000	1250-1500	3C+	-	0.999	0.001	-	-
4-6	3750-4000	1500-1750	1C	-	1.000	-	-	-
4-6	3750-4000	1500-1750	3C	-	1.000	-	-	-
4-6	4000-4250	1500-1750	3C+	-	1.000	-	-	-
4-6	4250-4500	1750-2000	3C	-	1.000	-	-	-
6-8	3250-3500	1500-1750	1C	-	1.000	-	-	-
6-8	3250-3500	1500-1750	4C	-	-	0.061	0.939	-
6-8	3250-3500	1750-2000	1C	-	1.000	-	-	-
6-8	3250-3500	1750-2000	4C	-	-	0.028	0.972	-
6-8	3250-3500	1750-2000	4C+	-	-	1.000	-	-
6-8	3500-3750	1500-1750	1C	-	1.000	-	-	-
6-8	3500-3750	1500-1750	2C	-	0.082	0.918	-	-
6-8	3500-3750	1500-1750	3C+	-	0.909	0.091	-	-
6-8	3500-3750	1500-1750	4C	-	-	0.330	0.670	-

6-8	3500-3750	1500-1750	4C+	-	-	1.000	-	-
6-8	3500-3750	1750-2000	4C+	-	-	1.000	-	-
6-8	3750-4000	1500-1750	2C	-	0.231	0.769	-	-
8-10	2250-2500	750-1000	2C	-	-	1.000	-	-
8-10	3000-3250	1500-1750	2C	-	-	1.000	-	-
8-10	3000-3250	1750-2000	5C	-	-	0.079	0.921	-
8-10	3250-3500	1000-1250	5C+	-	-	1.000	-	-
8-10	3250-3500	1500-1750	2C	-	-	1.000	-	-
8-10	3250-3500	1500-1750	4C	-	-	0.144	0.856	-
8-10	3250-3500	1500-1750	5C	-	-	0.043	0.154	0.802
8-10	3250-3500	2000-2250	5C+	-	-	1.000	-	-
8-10	3500-3750	1000-1250	5C+	-	-	1.000	-	-
8-10	3500-3750	1250-1500	5C+	-	-	1.000	-	-
10-12	2750-3000	1250-1500	1C	-	1.000	-	-	-
10-12	2750-3000	1250-1500	3C+	-	0.966	0.034	-	-
10-12	2750-3000	1250-1500	6C	-	-	0.896	0.104	-
10-12	3000-3250	1500-1750	1C	-	1.000	-	-	-
10-12	3000-3250	1500-1750	3C+	-	0.980	0.020	-	-
10-12	3250-3500	1750-2000	3C+	-	0.967	0.033	-	-
10-12	3500-3750	1750-2000	3C+	-	0.981	0.019	-	-
10-12	3750-4000	2500-2750	3C	-	1.000	-	-	-
10-12	3750-4000	2500-2750	6C	-	-	1.000	-	-
12-14	2750-3000	1000-1250	2C	-	-	1.000	-	-

To evaluate the performance of the model, four main indicators are used: 1) Mean Square Error (*MSE*); 2) Mean Absolute Error (*MAE*); 3) Mean Absolute Percentage Error (*MAPE*) and 4) Mean Accuracy Indicator (*MAI*).

Considering x_{est} as the estimated or predicted value by the model, x_{act} as the observed or real value and n as the number of elements, the *MSE* is the sum of the squared differences between the predicted and observed values, divided by the number of elements, as shown in eq. (6-12). The *MAE* is the sum of the absolute differences between the predicted and observed values, divided by the number of elements, as shown in eq. (6-13). The *MAPE* is expressed as the absolute value resulting from the difference between the predicted and the observed outcomes divided by the observed outcome. All the absolute values are summed and then divided by the number of elements and finally multiplied by 100 to express it as a percentage, as shown in eq. (6-14). The *MAI* is a proposed indicator to measure the efficiency of the model and is represented as the average of the accuracies of all maintenance services. The accuracy of each maintenance service is calculated by the complement of the relation between the absolute difference of the predicted and observed values and the maximum error amongst these two measures (i.e. the maximum possible difference), as shown in eq. (6-15).

$$MSE = \frac{\sum (x_{est} - x_{act})^2}{n} \tag{6-12}$$

$$MAE = \frac{\sum |x_{est} - x_{act}|}{n} \tag{6-13}$$

$$MAPE = \frac{\sum \left| \frac{x_{est} - x_{act}}{x_{act}} \right|}{n} * 100 \tag{6-14}$$

$$MAI = \frac{\sum \left(1 - \frac{|x_{est} - x_{act}|}{err_{MAX}} \right)}{n} * 100 \tag{6-15}$$

For the proposed model, the *MSE* is used to assess the performance in two stages. Firstly, it compares the estimated non-routine belief distribution with the actual distribution, based on real data. Secondly, it evaluates the difference between the estimated non-routine rate and the observed rate for each maintenance service in the sample, i.e. a one-to-one comparison. To differentiate these indicators, the former will be referred to as MSE_{DIST} and the latter as MSE_{SERV}. The other indicators (*MAE*, *MAPE* and *MAI*) are exclusively used to compare the estimated non-routine rate for each maintenance service against its recorded counterpart.

Table B-6 and Table B-7 are based on the estimated non routine rate presented in Table 6-10 and the actual rate observed in the sample (Table B-1 in Appendix B.1) and are displayed in Appendix B.3. Table B-6 compares the estimated non-routine rate distribution with its real counterpart. Here the MSE_{DIST} is used to assess the differences between the two distributions. Table B-7 shows the estimated average and the observed non-routine rates. In this case MSE_{SERV}, *MAE*, *MAPE* and *MAI* are used to compare these rates and to assess the model performance.

Comparing the estimated against the recorded values, the results of the indicators are as follows: $MSE_{DIST} = 0.0202$, $MSE_{SERV} = 0.0017$, $MAE = 0.0302$, $MAPE = 16.3\%$, $MAI = 93.3\%$. The average square error of the belief distribution is 0.0202, while the average square error of the non-routine rate is 0.0017. The MAPE indicates that the error of the model is 16.3% in average, whereas the MAE suggests that the model deviates ±0.0302 non-routine rate in average. The MAI expresses that the model predictions are in average 93.3% accurate when compared with the observed non-routine rates. As shown in this example, these indicators are very useful for assessing the performance of the model and will, therefore, be used in the following sections to compare different models.

6.4.1 Impact of limits and size of interval on model performance

This section aims to explore and analyse the effect of using different bin-widths and interval limits on model performance and to determine the arrangement of intervals where the efficiency of the model is the best. To accomplish this objective, the steps for building the ER rule model, described in the section above, are applied several times to run different trials of interval combinations and to compare their performance.

After the discretisation process as explained in section 6.3, different numbers of intervals and bin-widths were suggested for each of the proposed variables, based on the bin-width rules and meaningfulness for practitioners. For age, five class widths of 0.5, 1, 1.5, 2 and 2.5 years respectively are proposed. For the 1, 1.5 and 2 years bins, two different arrangements were defined by moving the interval limits, resulting in eight different options. Five bins of size 100, 150, 200, 250 and 300 each were proposed for flight hours per year as well as for cycles per year. For non-routine rate, two bin-widths are used, one of 0.05 and another of 0.10. As the type of maintenance service is a categorical variable the classes are well defined, so other arrangement options are unnecessary. Table 6-11 shows the interval and bin-width options for each variable.

Considering eight class widths alternatives for age, five for flight hours per year, five for cycles per year, two for the non-routine rate and one for the type of maintenance service, there are 400 different possible arrays for building the ER aggregation model. Therefore, to determine the best alternatives and to analyse the impact that bin size and interval limits have on the model, the model is run for these 400 different combinations. Table B-8 to Table B-23 in Appendix B.4 present the results of this analysis.

Figure 6-8 depicts an example where three different arrays of intervals are compared, using a class size for age of 0.5 years. For the first scenario, a bin-width of 0.1 for non-routine rate and a 250 interval size for both flight hours per year and cycles per year is used. In the second array, the non-routine rate remains the same, but the interval size for flight hours per year and cycles per year is reduced to 150 respectively. Finally in the last case, the bin-widths are shrunk to 0.05 for non-routine rate and to 100 for flight hours per year and cycles per year each.

Table 6-11 Bin-width and intervals options for each variable

Age	Bin-width	0.5	1	1	1.5	1.5	2	2	2.5
	No. Intervals	17	9	9	6	6	5	5	5
	Lower and upper limits	4.5 - 13.0	4.0 - 13.0	4.5 - 13.5	4.0 - 13.0	4.5 - 13.5	3.0 - 13.0	4.0 - 14. 0	2.5 - 15.0
Flight hours/year	Bin-width	100	150	200	250	300			
	No. Intervals	21	15	11	9	8			
	Lower and upper limits	2,400 - 4,500	2,300 - 4,550	2,400 - 4,600	2,250 - 4,500	2,300 - 4,700			

Cycles/year	Bin-width	100	150	200	250	300
	No. Intervals	18	12	10	8	6
	Lower and upper limits	900 - 2,700	900 - 2,700	800 - 2,800	750 - 2,750	900 - 2,700
Non-routine rate	Bin-width	0.05	0.1			
	No. Intervals	9	5			
	Lower and upper limits	0.05 - 0.50	0.0 - 0.5			
Maintenance Service	Bin-width	1				
	No. Intervals	9				
	Lower and upper limits	1C - 6C				

As might be expected, model performance improves when the number of intervals increases and the bin-width is reduced. Figure 6-8 clearly illustrates that when the size of the bin is reduced in each scenario, this leads to a decline in the Mean Absolute Percentage Error and in the Mean Absolute Error. For the MAPE, the fall goes from 16.7% to 15.5% and then to 10.1%; whereas for the MAE, the values drop from 0.0310 to 0.0281 and then to 0.0196. Accordingly, the accuracy of the model improves, increasing when the class size is diminished, rising from 93.1% to 93.7% and then to 95.4%. Supporting this argument, Figure 6-9 depicts the MAI for the 400 combinations of interval arrays. Despite variations, the general tendency of the graph is clear, and the accuracy of the model is enhanced significantly when the interval size is decreased.

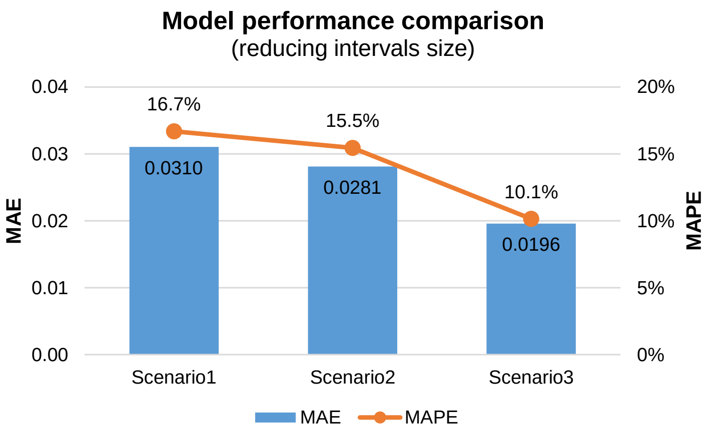


Figure 6-8 MAE and MAPE comparison across scenarios using different bin size

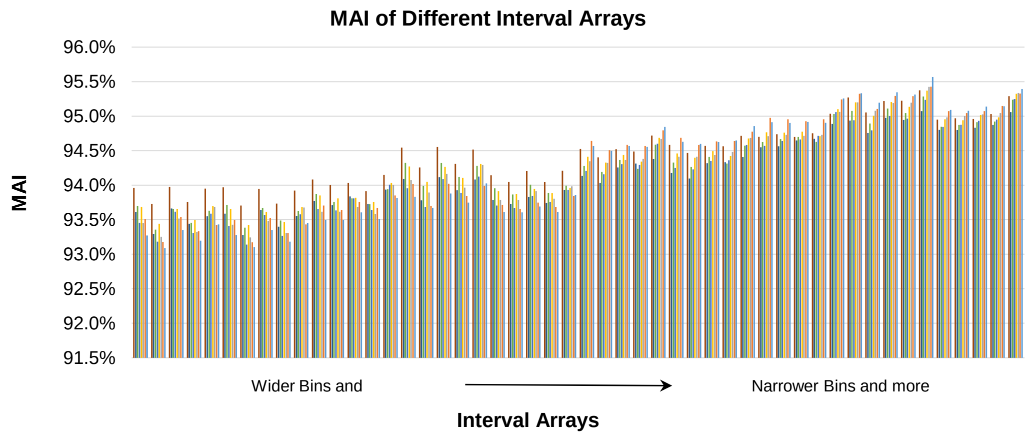
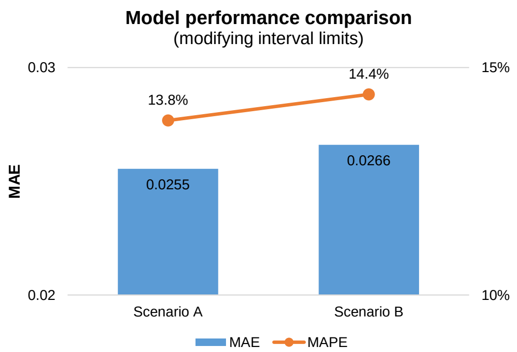


Figure 6-9 MAI for the 400 different combinations of interval arrays

Another interesting exercise is to explore how interval limits can affect model performance. Figure 6-10 describes two examples where the interval size remains the same but the interval limits are different. The first example consists of a non-routine rate bin of size 0.10, a bin of 150 flight hours per year, an interval size of 300 cycles per year and a class size of two years for age. The two scenarios differ in the interval limits for age: in the first case the lower limit is 3 years and the upper limit is 13 years, while for the second case, the limits start at 4 years and end at 14 years. The configuration for the second example is a bin-width for non-routine rate of 0.05, classes of 150 for flight hours per year and cycles per year and an age bin of one year. As in the first example, age intervals were changed to have one scenario with intervals from 4 to 13 and another with limits between 4.5 and 13.5.

From the examples presented in Figure 6-10, it can be noticed that despite the fact that the bin widths are the same, changing the position of the interval limits might alter the model accuracy. In the first example, using interval limits from 3 to 13 years had a slightly better performance in general compared with the 4 to 14 years case. Correspondingly, the first scenario (intervals limits from 4.0 to 13.0) of the second example has a slightly higher accuracy than the second (intervals limits from 4.5 to 13.5).



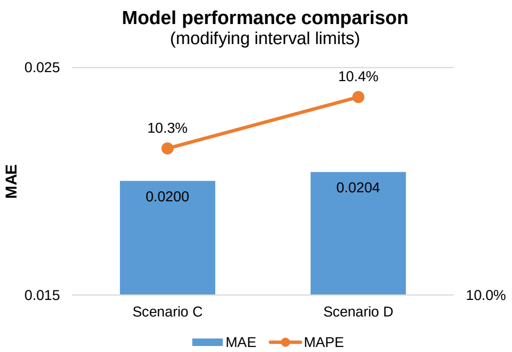


Figure 6-10 MAE and MAPE comparison across scenarios using different interval limits

As was demonstrated by the previous cases, in general choosing smaller class widths improves the model performance. Moreover, moving the interval limits can also vary efficiency. However, the performance of the model does not depend only on these two factors. Interestingly, there is another relevant element which also has influence on the results. This additional factor is the manner in which the interval sizes of each variable are arranged. In other words, it seems that there is a "*degree of affinity*" between some of the intervals and therefore, there are arrays performing better than others, notwithstanding their bin size and the limit position.

Figure 6-11 shows a representative example of how the arrangement of the intervals might enhance model accuracy. Assuming a bin-width of 0.10 for non-routine rate, two scenarios are used. The first scenario considers an interval size of 100 for flight hours per year and cycles per year, respectively and, for age, a class size of 0.5 years and interval limits between 3.0 and 13.5. The second scenario considers interval widths of 150 flight hours and 300 cycles per year and, for age, a bin size of 2.5 years with interval ranging from 2.5 to 15.0.

It would be expected that the first scenario achieves better performance as it has the smallest bin widths between the two cases. However, surprisingly, the second scenario shows an overall better efficiency. For this case, it seems that when the non-routine rate bin-width is 0.10, there is a greater "affinity" for larger class sizes with the other variables (flight hours, cycles and age). As a result, the accuracy of the second scenario is significantly higher compared with the first case, demonstrated by a MAI of 94.55% vs 93.85%.

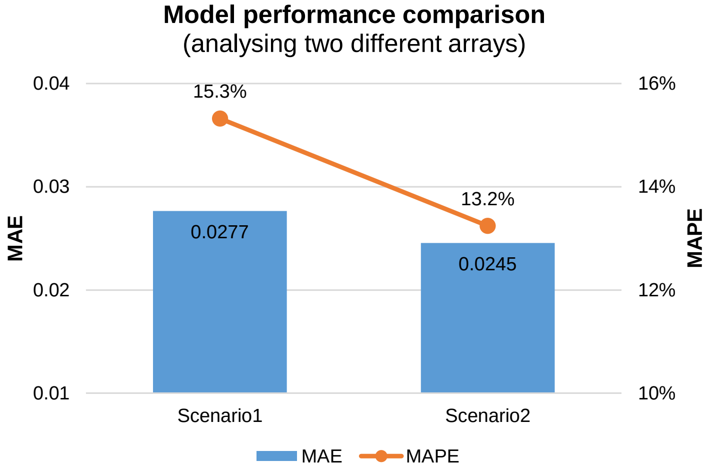


Figure 6-11 MAE and MAPE comparing two different arrays of intervals

In Table B-8 to Table B-23, in Appendix B.4, it can be seen that the best results of the model are obtained when the bin-width for flight hours per year is 150. Furthermore, when the class size of the non-routine rate is smaller (i.e. 0.05), the model has a better performance for narrower intervals for age and cycles per year. In contrast, when the non-routine rate interval is wider (i.e. 0.10) the model achieves better results for greater intervals for age and cycles per year. In other words, the performance is somewhat polarised, as when the non-routine rate interval size is small, there is a greater affinity between smaller bins for age and for cycles per year. When the non-routine rate interval is wider, the model works better for larger bins for age and cycles per year. However, the interval size for flight hours per year seems to be outside of this relation. In this manner, when the bin size of the non-routine rate is 0.05, the best performance of the model occurs when age has a bin-width of 0.5 years, flight hours per year a class size of 150 and an interval-width of 100 for cycles per year. Similarly, when the bin-width of non-routine rate is 0.10, the highest efficiency is achieved when the interval sizes are 2.5, 150 and 300 for age, flight hours per year and cycles per year, respectively (Appendix B.5, Table B-24 and Table B-25).

It can be argued that the findings explained in this section are interesting, but above all significant. It is suggested that defining and choosing the appropriate bin sizes and interval limits, as well as the proper arrangement between them, might lead to a higher performance of the model, resulting in a better estimation of non-routine rate. Therefore, based on these results, it is advisable for future researchers using a similar approach to consider these three elements in the design of their model to determine its sensitivity to changes in these variables.

Finally, as further research, it might be relevant and useful to develop an optimisation model for finding the best bin widths, interval limits and the combination arrays that maximise the model performance.

6.4.2 Impact of different number of variables combined

In this section, the ER rule model is run several times, changing the number of input variables (from five to two) and using different combinations amongst them. This allows for the relevance and impact of each of the five input variables (*a*, *fhy*, *cy*, *se* and *nrr*) on the estimation of the non-routine rate (*Enrr*) to be investigated. It also helps to determine whether using fewer pieces of evidence would result in better or at least similar results. There is just one possible array when the five variables are considered, but there are 5 different possible combinations of aggregating four of the five pieces of evidence. Similarly, using three and two variables leads to 10 arrays for each of these cases. Moreover, this analysis was conducted twice: once for the best combination of bin-widths when the non-routine rate interval size is 0.05 (i.e. bin-widths of 0.5, 150, 100, and 0.05 for *a*, *fhy*, *cy* and *nrr*, respectively (Table B-24)), and once for the best intervals when the non-routine rate bin is 0.1 (i.e. bin-widths of 2.5, 150, 300, and 0.1 for *a*, *fhy*, *cy* and *nrr*, respectively (Table B-25)). Henceforth the first case will be referred as *layout A* and the second as *layout B*.

One of the reasons for conducting this analysis was the concern that using five pieces of evidence would not necessarily improve the prediction accuracy, as greater number of variables could bring modelling noises and accumulated errors. Table 6-12 (based on the results from Table B-26 to Table B-31 in Appendix B.6) compares the performance of the best results from the different number of variables for either layout A or B. In both cases, it can be noticed that the best result is achieved when the five variables are used in the estimation of the non-routine rate. However, although including an additional piece of evidence in the model improves the performance, the increment on the performance is less significant with every new addition, as can be seen in Figure 6-12.

Table 6-12 Model performance using different numbers of variables

No. Variables	**Layout A (nrr of 0.05)**				**Layout B (nrr of 0.1)**			
	2	**3**	**4**	**5**	**2**	**3**	**4**	**5**
Age								
Fh/Y								
Cy/Y								
Services								
Nr rate								
MSE_{Dist}	0.0130	0.0144	0.0186	0.0175	0.0208	0.0021	0.0074	0.0076
MSE_{SERV}	0.0016	0.0009	0.0007	0.0007	0.0016	0.0013	0.0011	0.0010
MAE	0.0264	0.0215	0.0195	0.0188	0.0318	0.0267	0.0248	0.0245
MPAE	14.34%	11.95%	10.37%	9.98%	16.79%	14.50%	13.28%	13.24%
MAI	93.80%	94.93%	95.40%	95.57%	92.94%	94.06%	94.48%	94.55%

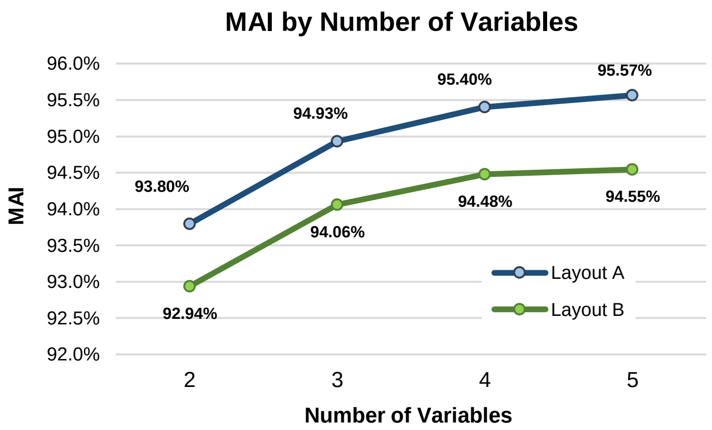


Figure 6-12 Mean Accuracy Indicator considering different numbers of variables

To analyse the particular relevance of each variable on the estimation of the non-routine rate, different combinations of four, three and two variables were tested. The results can be found in Table B-26 to Table B-31 in Appendix B.6.

As part of this analysis, when four variables are considered within layout A, it appears that the maintenance service type has a significant role in the model. In Figure 6-13-layout A, it is clear that the model performs better when this variable is included and that accuracy drops dramatically when it is omitted. The flight hours per year have a similar effect, even though their impact is not as relevant as that shown by the maintenance service type. In contrast, it seems that non-routine rate and, to a lesser extent, age do not have an important influence on the model performance. On the other hand, for layout B and using the same number of variables, the maintenance service type acts again as the major variable for estimating the non-routine rate, as the performance drops when is excluded but if it is considered, the accuracy improves. Surprisingly, in this case, non-routine rate is the variable that follows services in order of relevance and cycles per year is the variable that has least importance in the model, as can be observed in Figure 6-13-layout B.

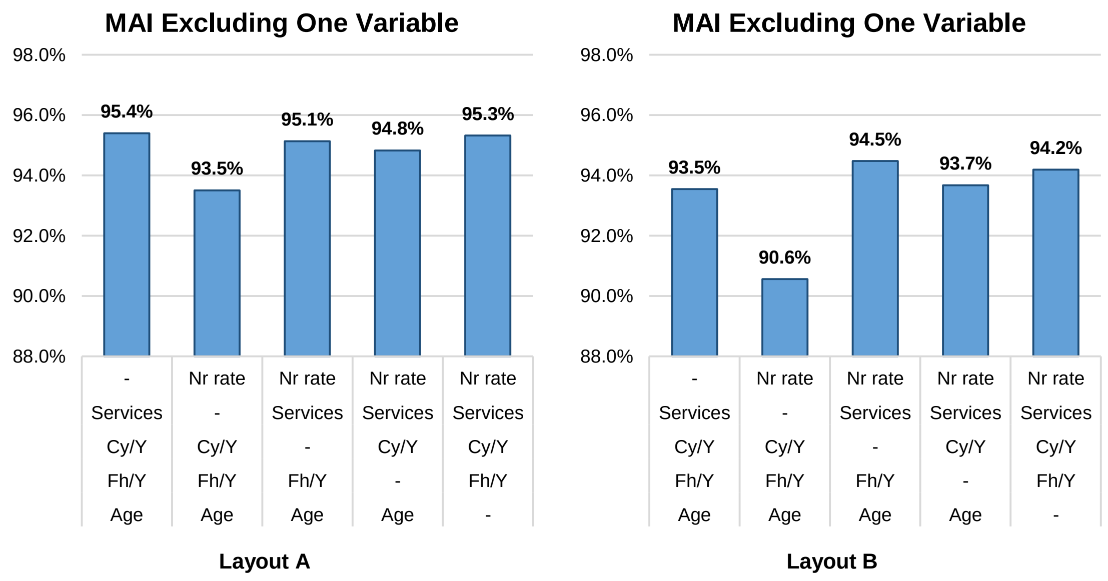


Figure 6-13 Mean Accuracy Indicator using four predictor variables

Figure 6-14-layout A presents the results for layout A when three variables are used. Supporting the results of the four variables analysis, all the combinations in which service type is not included had a deficient performance. Interestingly, however, the worst accuracy was obtained when neither age nor service type was considered. It appears that the absence of these two variables together has a strong effect on the model results, but the impact is less evident in all the other cases where the age along with other variable are considered. Moreover, service type works better in conjunction with the utilisation variables, as the best performance is achieved when service is used in combination with flight hours and cycles. Finally, the non-routine rate does not show particular significance after being included in the model. Likewise, in layout B (depicted in Figure 6-14-layout B), maintenance service type combined with age dramatically affect the performance of the model when they are taken out. Here, maintenance service type, in addition to the utilisation variables, produces the best results as well as that observed in the layout A. In contrast to layout A, in layout B the combination of maintenance service with non-routine rate also has a relevant role.

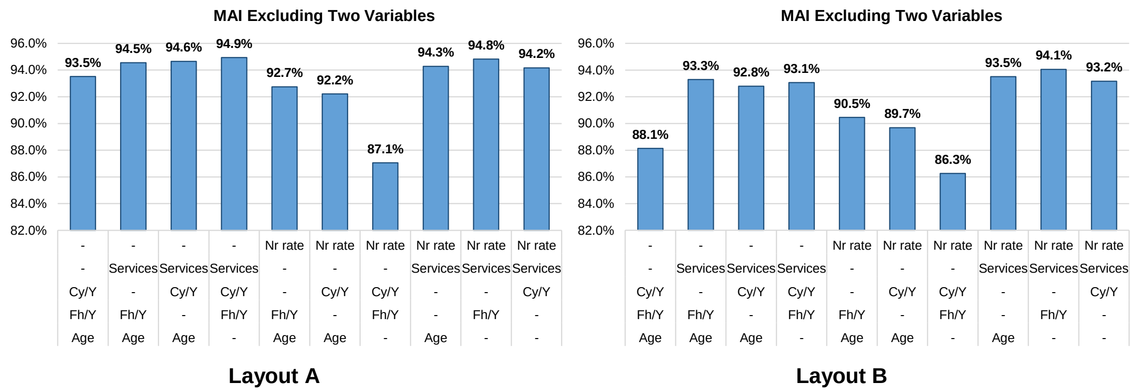


Figure 6-14 Mean Accuracy Indicator using three predictor variables

In the final test, as shown in figures Figure 6-15-A and Figure 6-15-B, only two variables are taken into account. Correspondingly, services remains as the most relevant variable for both layouts and the model achieves the highest MAI when this variable is combined with one operational variable i.e. cycles, flight hours or age. On the other hand, when cycles are used in combination with either non-routine rate or flight hours, the model reaches its lowest performance. The main difference between the two layouts is that while in layout A, in the same way as in the previous tests, the non-routine rate does not have a high significance in the results, layout B relies more on the interaction of this variable.

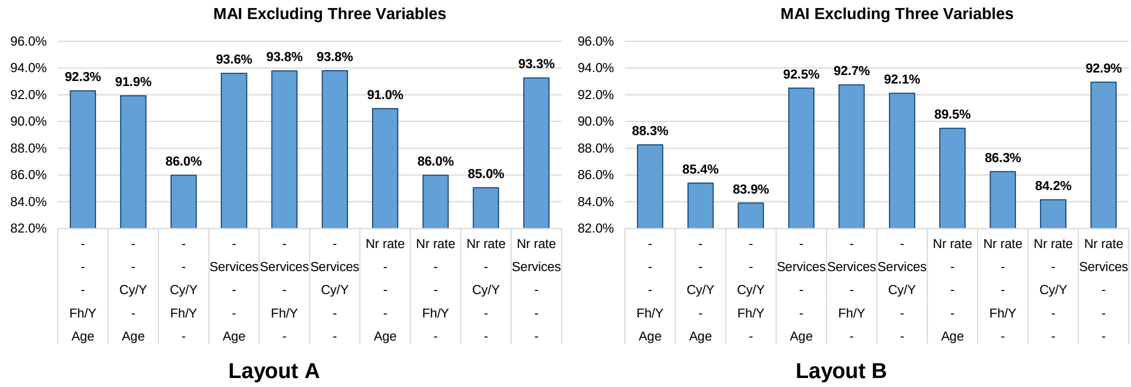


Figure 6-15 Mean Accuracy Indicator using two predictor variables

Interesting observations arise from the conducted analysis. Firstly, it is important to note the impact that the definition of bin-size and the way that the intervals are arranged have on the performance of the model. As a result of this, the two layouts differed, demonstrating that bin-width could affect the way the variables interact. Secondly, as was observed in all the tests, it is suggested that the essential variable for the estimation of the non-routine rate is the type of maintenance check. However, to improve prediction accuracy, this variable appears to need the support of at least one variable that expresses the usage of the aeroplane (e.g. age, flight hours or cycles). Nonetheless, and as a third finding, when the utilisation variables, i.e. flight hours and cycles, work together or in combination with the prior non-routine rate, the model experiences a considerable decline in accuracy. A likely explanation for this phenomenon is that these three variables provide similar evidence. Aircraft age acts in a similar way, giving the impression that this variable does not provide significant additional support when combined with the check type but, in contrast, enhances performance when used in combination with flight hours, cycles and the prior non-routine rate. A final observation is with regard to the dissimilar and opposite behaviour of the prior non-routine rate and aircraft age between the two layouts. In the first case, the former has a lesser influence in the model and the latter works more actively in the prediction accuracy. In the second layout, the roles are inverted. This discrepancy could be attributed to the bin-size effect: in the second layout the class-widths are too wide and the distributions are somewhat over-smoothed and aircraft age loses importance as its distribution becomes less meaningful. In contrast, the non-routine rate gains significance as its wider bins make it easier to predict the outcome for larger intervals. Therefore, the prior non-routine rate becomes a variable that summarises the behaviour of the phenomenon.

6.5 ER rule scenarios

It is worth bearing in mind that the development of the model has followed a systematic process, so that the results obtained in every step allow for a gradual increase in the complexity and robustness of the model. Therefore, as already mentioned, to simplify the preceding analysis and to have fewer variables involved, all the input variables were assumed to be completely independent. Moreover, all pieces of evidence were assumed to be fully reliable and highly important.

In this section, four different scenarios of the ER rule model are created using the two suggested layouts of bin-widths and considering the five pieces of evidence. Through these scenarios, the model continues increasing in complexity, but becomes less rigid, more meaningful and useful. For training and validation purposes, the sample of 91 maintenance checks was randomly divided into nine groups of ten each, keeping one extra record outside the groups. Hence, the ER model is built considering eighty maintenance services (eight random groups of ten), and eleven (the remaining group of ten and the extra service) are used for validation. Subsequently, the scenarios are executed using four different samples of eighty maintenance services each, i.e. each scenario is run eight times (four samples for each of the two interval layouts).

The first scenario is the simplest as it assumes that all the pieces of evidence are completely independent, fully reliable and have the same importance. Nonetheless, a significant drawback is that the model is not fully representing the actual conditions of the problem. The main characteristic of the second scenario is that the independence of the variables can be questioned. Therefore, the model is adjusted to assess the dependencies between the pieces of evidence, keeping their reliability and importance as in the first scenario. In the third scenario, and taking into account the results of the dependency assessment, the pieces of evidence are no longer considered fully reliable. Hence, based on the specific characteristics of each variable, reliability is adjusted, albeit the variables are still considered highly and equally important. Finally, in the last scenario, considering the results from the former scenarios, the importance of the pieces of evidence is reviewed and adjusted to maximise the performance of the model. To simplify the explanation, the four scenarios are described using one sample. However, the results of the four samples are shown for further detail in Appendix B.7 (from Table B-32 to Table B-59).

6.5.1 All pieces of evidence are independent, fully reliable and highly important

This is the basic scenario for aggregating the five predictor variables to estimate the non-routine rate. It considers that all pieces of evidence are completely independent, fully reliable and have the same importance; i.e. the alpha index, weight and reliability are all equal to one. Table 6-13 presents the results of this scenario for one of the samples. As already discussed, the performance of the model is better when smaller bins are used, i.e. layout A performs better than layout B. Contrastingly, it is interesting to discuss the MSE_{DIST} (representing the error between the real non-routine rate distribution and the one calculated by the model), compared with the other indicators, which has a better result for layout B. A possible explanation for this behaviour might be that for wider bins, the distribution smooths and therefore the error could decrease. It is noteworthy that this scenario shows the ideal conditions for all the variables. However, this does not fully represent the real conditions of the problem, hence, the following sections will attempt to address this weakness.

Table 6-13 Model performance when the variables are independent, fully reliable and highly important.

	Layout A	Layout B
Alpha Index	1	1
Reliability	1	1
Weight	1	1
MSE_{DIST}	0.0128	0.0049
MSE_{SERV}	0.0005	0.0010
MAE	0.0172	0.0244
MAPE	9.28%	12.78%
MAI	95.96%	94.57%

6.5.2 All pieces of evidence are fully reliable, highly important and their dependency is adjusted

The previous section considers that all the pieces of evidence are independent. However, in real life, this strong assumption does not commonly occur as generally there is a certain degree of dependency amongst the variables[4]. As mentioned before, Yang and Xu (2013) propose the ER rule as a conjunctive probabilistic reasoning process for combining multiple pieces of independent evidence. However, Yang and Xu (2015) have recently proposed a novel approach to cope with the independency premise by including a new term in the ER rule (referred to in this study as alpha-index) as shown in equations (4-7) and (4-8).

Analysing the dependency factor results is relevant, as considering the variables as totally independent might lead to unrealistic results since the pieces of evidence could provide the same information to some extent. Therefore in this section the variables are no longer considered completely independent, although their reliability and importance remains as in the first scenario. The dependency between the variables is adjusted by optimising the alpha-index, aiming to represent the real relationship between the variables. Thus, an optimisation model was built modifying the alpha-index to minimise the difference between the real non-routine rate distribution and the distribution calculated by the model (i.e. MSE_{DIST}). Table B-34 to Table B-57, in Appendix B.7.1, show the optimised alpha-index values resulting from the optimisation model.

[4] When two pieces of evidence are mutually exclusive, they should not be interrelated. However, if they are independent of each other, they are indeed interrelated, but conceptually their interrelationship should be such that the acquisition of one piece of evidence does not depend on whether the other piece of evidence is known or not (Yang and Xu 2013; 2015).

Table 6-14 compares the performance of the model when the variables are assumed independent against when the dependency has been adjusted. By optimising the alpha-index the MSE_{DIST} is reduced to almost zero for both layouts, which means that the estimated non-routine rate distribution is practically the same as the distribution observed in the sample. However, the other efficiency indicators remain almost the same as in scenario 6.5.1 (with slight improvements for layout A and minor retreats for layout B). Even though after optimising the alpha-index the overall performance of the model was not improved considerably, the dependency amongst the variables is now considered to closer represent the real association between the variables.

Table 6-14 Model performance when the variables are not completely independent, but are fully reliable and highly important.

	Layout A		**Layout B**	
	Original	**Optimising alphas**	**Original**	**Optimising alphas**
Alpha Index	1	Optimised	1	Optimised
Reliability	1	1	1	1
Weight	1	1	1	1
MSE_{DIST}	0.0128	0.0000	0.0049	0.0000
MSE_{SERV}	0.0005	0.0004	0.0010	0.0010
MAE	0.0172	0.0155	0.0244	0.0240
MAPE	9.28%	8.00%	12.78%	12.89%
MAI	95.96%	96.35%	94.57%	94.67%

6.5.3 All pieces of evidence are highly important, their dependency is adjusted and they are not fully reliable

Now that the dependency between the variables has been identified in section 6.5.2, another characteristic of the pieces of evidence can be analysed. In this section, the variables are still considered to be highly important, their dependency is given by the adjusted alpha-index, but they are not considered fully reliable. Therefore, each of the pieces of evidence is studied and their reliability is estimated based on their main features and considering their process of collection, recording and analysis. Gathering, recording, analysis and management are relevant due to the reliability of a piece of evidence being affected by all the elements involved in it, from the moment the data is generated until stored and used by the model. In other words, the reliability of a piece of evidence cannot be modified directly in the model, as it is an inherent feature of the quality of information and only by its improvement can reliability be increased.

As explained in chapter 2, to ensure airworthiness aircraft maintenance is structured in a rigorous system that relies heavily on accurate information. Nowadays most of the information is managed automatically, including flight hours and cycles. However, there is still some information that is recorded and analysed manually and this increases error margin and subjectivity, such as the number of non-routine tasks in a maintenance service. Additionally, there is information that is recorded automatically and is rigorously analysed, but contains some manual stages in the management process which could lead to inaccuracies. Age and service type are part of this cohort.

The non-routine tasks are programmed to correct the failures and discrepancies found during the execution of the scheduled tasks. However rework-tasks (tasks to correct errors or deficiencies in the quality) and miscellaneous activities (additional tasks not included in the plan, but necessary to properly execute it) are also commonly registered as non-routine tasks. These extra activities might bias the indicator of measuring the unexpected damage and failures during a maintenance check.

According to experience, it is believed that around 5% of non-routine tasks could be rework and miscellaneous tasks. Therefore, a reliability of 0.95 for non-routine rate is considered. Since the non-routine rate is the outcome of the model, its reliability must be considered for the final reliability of the predictor variables. For instance, for the non-routine rate prior distribution a reliability of 0.90 is assumed (i.e. the reliability of the non-routine rate is considered twice, 0.95 by 0.95).

Aircraft age is measured from the aircraft's manufacturing date to the start date of the maintenance check and this information is well registered in the maintenance records. Even so, one inconsistency was found in one of the records, where there was a typographical error in the fabrication year. Thus, the reliability of this variable had to be reduced to 0.95, reflecting other possible inconsistencies. The final reliability of this variable, after including the non-routine rate reliability, is 0.90.

Flight hours and cycles are two scrupulously managed variables as they are used for different areas in the airline. They are recorded and analysed automatically by sensors within the aircraft. Therefore, the quality of this piece of evidence could be considered significantly high. Nonetheless, to reduce the correlation between total flight hours and total cycles with age, annual utilisation was used instead. The main drawback of using this annualised value is that flight hours and cycles per year are an average indicator of the usage of the aeroplane. Therefore, the reliability for these two pieces of evidence is first assumed as 0.97 and then, considering the non-routine rate reliability, as 0.92.

Maintenance service type is the essential variable for organising and managing maintenance activities, as it is very well defined in the scheduled maintenance programme. Hence, this piece of evidence is considered fully reliable with a final reliability value of 0.95 after including the non-routine rate reliability.

After updating the reliability of the pieces of evidence based on the characteristics of the variables, it can be noticed in Table 6-15 that model performance drops considerably in comparison to the results from scenario 6.5.2. In a similar way to the adjustment of dependency, greater reliability values might produce better efficiency results in the model. However, if these values are not based on the real quality of the information, the model results would not be realistic. In other words, the main purpose of revising reliability is not to increase the model accuracy per se, but to reflect in the model the actual quality of the pieces of evidence and hence make the model more representative.

Even though reliability is estimated based on the main features of the pieces of evidence, the actual value is difficult to determine precisely, as on the one hand, the process itself is significantly reliable due to the strict normativity that regulates aircraft maintenance, but on the other hand, experience has shown that there are weaknesses caused by diverse factors, such as human error or fails in the systems, that impact upon the quality of the information. These factors are difficult to measure as they were not registered or reported. Therefore, a sensitivity analysis would be helpful to assess the impact that different reliability values have on the estimation accuracy of the non-routine rate.

Table 6-15 Model performance when the variables are not completely independent and reliable, but are highly important.

	Layout A			**Layout B**		
	Original	**Optimising alphas**	**Adjusting R**	**Original**	**Optimising alphas**	**Adjusting R**
Alpha Index	1	Optim	Optim	1	Optim	Optim
Reliability	1	1	R ≠ 1	1	1	R ≠ 1
Weight	1	1	1	1	1	1
MSE_{DIST}	0.0128	0.0000		0.0049	0.0000	
MSE_{SERV}	0.0005	0.0004	0.0010	0.0010	0.0010	0.0013
MAE	0.0172	0.0155	0.0220	0.0244	0.0240	0.0268
MAPE	9.28%	8.00%	10.95%	12.78%	12.89%	13.85%
MAI	95.96%	96.35%	94.83%	94.57%	94.67%	94.05%

6.5.4 All pieces of evidence are not fully reliable and their importance and dependency are adjusted

The final characteristic of the pieces of evidence can now be explored. Instead of considering all the variables highly important, their relevance is determined by optimising the model. In other words, the weight of each piece of evidence is modified by developing an optimisation model that minimises the Mean Absolute Error between the estimated non-routine rate for each maintenance service against its real counterpart.

In contrast to the results presented in Table 6-16, it can be seen from the different indicators that after optimising the weights, prediction accuracy improves significantly. In other words, by adjusting the importance of the distinct pieces of evidence, the performance of the model in estimating the non-routine rate is enhanced. Additionally, it can be noted that the accuracy of this scenario is very similar to the results under ideal conditions, i.e. when the pieces of evidence are completely independent, fully reliable and highly important. However, the main difference is that in this final scenario, alpha index, reliability and weight have been adjusted to represent as close as possible the actual conditions of the model.

Figure 6-16 illustrates the weight of each of the intervals for the different pieces of evidence for the different samples in layouts A and B. For instance, in Figure 6-16-layout A, it can be noted that for service type, intervals 1, 3, 6 and 9 that correspond to services 1C, 3C, 4C+ and 6 respectively have a relevant role for the estimation of non-routine rate. In contrast, the non-routine prior distribution has almost null importance for the model. Besides, it is also important to emphasise the noticeable similarities between the results of the different samples. Table B-58 and Table B-59, in Appendix B.7.2, present the optimised values of the weights that minimise prediction error.

Table 6-16 Model performance when the variables are not completely independent, reliable and important.

	Layout A				Layout B			
	Original	**Optimising alphas**	**Adjusting R**	**Optimising W**	**Original**	**Optimising alphas**	**Adjusting R**	**Optimising W**
Alpha Index	1	Optim	Optim	Optim	1	Optim	Optim	Optim
Reliability	1	1	R ≠ 1	R ≠ 1	1	1	R ≠ 1	R ≠ 1
Weight	1	1	1	Optim	1	1	1	Optim
MSE_{DIST}	0.0128	0.0000			0.0049	0.0000		
MSE_{SERV}	0.0005	0.0004	0.0010	0.0009	0.0010	0.0010	0.0013	0.0011
MAE	0.0172	0.0155	0.0220	0.0203	0.0244	0.0240	0.0268	0.0243
MAPE	9.28%	8.00%	10.95%	10.28%	12.78%	12.89%	13.85%	12.08%
MAI	95.96%	96.35%	94.83%	95.23%	94.57%	94.67%	94.05%	94.59%

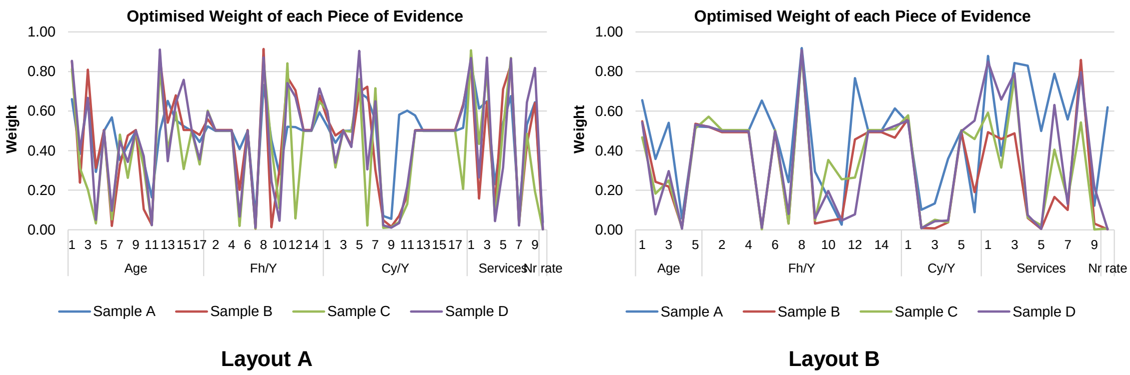


Figure 6-16 Importance degree (weight value) for each piece of evidence

In summary, to understand the general behaviour of the model, a simple scenario was created assuming perfect conditions. The dependency between the variables and their reliability were then adjusted to represent the actual characteristics of the information. Finally, the importance of the pieces of evidence was modified to improve the estimation of non-routine rate. However, these scenarios only show the performance for a given combination of dependency, reliability and weight. Therefore, it would be useful to deeply analyse the effect of these three evidence qualities on prediction accuracy. Hence, the next section aims to discover the aforementioned effect.

6.6 Sensitivity analysis of reliability

In addition to the scenarios presented in the section above, a sensitivity analysis was carried out to determine how reliability, importance and dependency of the pieces of evidence might impact upon the efficiency of estimating the non-routine rate, in other words, how a change in the values of reliability, weight or alpha-index might lead to different performance results. Hence, several experiments were conducted. The first assigned different reliability values to each of the predictor variables, but considered that the alpha index had already been adjusted and that the weights are equal to one. Secondly, the reliability of all the pieces of evidence was modified to assess their global impact, but keeping the previous assumptions. Finally, aiming to evaluate the influence of the dependency and relevance of the pieces of evidence, the final experiment changed the reliability and the values of the alpha-index and the weight, assuming these last two variables as equal to one and later using their optimised values.

All the sensitivity experiments were carried out twice for the four samples described in section 6.5, using the defined layouts A and B. For layout A, bin-widths of *0.5, 150, 100,* and *0.05* for *a, fhy, cy* and *nrr*, respectively were used and for layout B, bin-widths of 2.5, 150, 300, and 0.1 for *a, fhy, cy* and *nrr,* each.

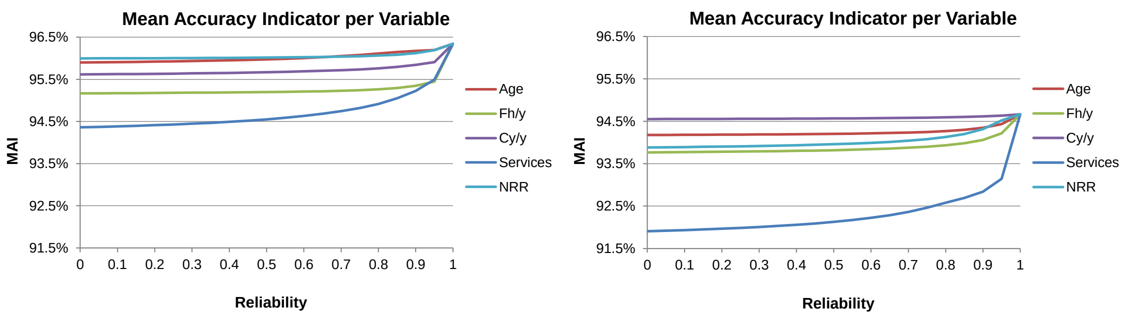
6.6.1 Alpha index has been optimised, weight is one and the reliability of one piece of evidence changes.

In order to assess the influence of the reliability on model performance and to address the relevance of each variable, in this scenario the accuracy is obtained by changing the reliability for one piece of evidence at a time, from being completely unreliable to a full reliability, while the other variables remain fully reliable. It also considers that the dependency of the variables has been adjusted by optimising the alpha-index, and assumes that all pieces of evidence are highly and equally important. Thus, the inference model is applied several times, changing in each round the reliability value of one variable from 0 to 1, considering the alpha index obtained in section 6.5.2. Weights are equal to 1. This process is repeated for every variable.

Figure 6-17 shows model accuracy when the reliability of one piece of evidence is changed in a range from 0 to 1, for layout A and B respectively. Each curve represents the behaviour of a specific variable towards a variation in the reliability value. From these figures, it can be observed, in both layouts, the remarkable sensitivity of the model towards changes in the reliability of the maintenance service type, emphasising with this the relevant role that this piece of evidence has on the estimation of the non-routine rate. In both layouts, it can be seen that when the reliability of the service type is low, the accuracy of the model is significantly lower too. On the other hand, when reliability increases, the performance of the model soars as well, showing a steady increment until the reliability almost reaches a value of 0.9, when it increases dramatically. It is noteworthy that the influence of this variable is more evident in layout B. Model performance is also sensitive to variations in the reliability of flight hours per year, meaning that the absence of this variable or its lack of reliability can impact considerably upon prediction accuracy. However, in layout B this sensitivity is not as pronounced as in layout A.

Estimation of the non-routine rate is less sensitive to changes in the reliability of the remaining three variables, as their fluctuations do not appear to have great effect on the results. Nonetheless, it is interesting that their role level is not the same between both layouts. For instance, cycles in layout A still have considerable repercussions for accuracy, while for layout B the prediction of the non-routine rate is almost neutral to changes to the cycles' reliability. Meanwhile, variations in the reliability for age and non-routine rate do not have a significant impact on the prediction accuracy for layout A whereas they still show an influence in layout B.

Figure 6-17 presents the behaviour for one of the samples analysed. The detailed results from this sensitivity test for the four samples along with their corresponding graphs are presented in Appendix B.8.1 (from Table B-60 to Table B-69). It is worth mentioning that the outcomes of the four samples are similar.



Layout A **Layout B**

Figure 6-17 Model sensitivity when the reliability of one variable is modified, for layouts A and B

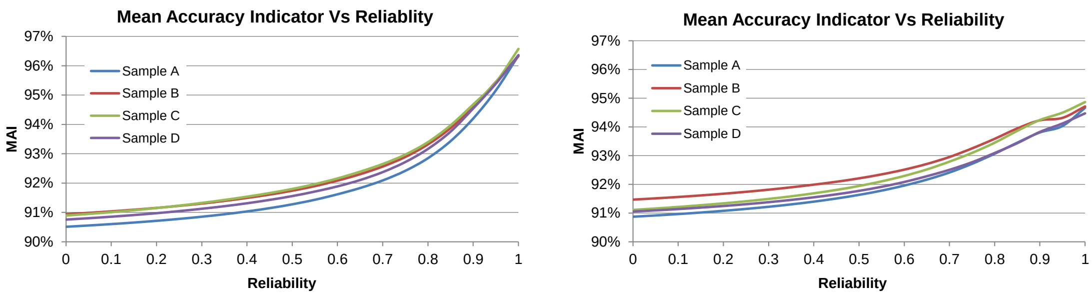
In summary, the results of this sensitivity test reinforce what was observed in the analysis presented in section 6.4.2, where it was concluded that maintenance service type is an essential piece of evidence for the estimation of the non-routine rate, followed by flight hours per year. Moreover, in order to have a higher accuracy prediction, these two variables have to be considered in the model, but also should be highly reliable. In addition to the outcomes of this test, it is necessary to analyse the global effect that the five variables have on the model efficiency. This analysis is carried out in the next section.

6.6.2 Alpha index has been optimised, weight is one and reliability is the same for all pieces of evidence.

This sensitivity test also aims to analyse the effect of reliability on model performance, but also considering the impact of all pieces of evidence as a whole. Hence, this scenario assumes that all the variables have the same reliability value, i.e. all of them are equally reliable, and this increases the global value from being completely unreliable to highly reliable. In other words, the ER model is run several times using the alpha index calculated in section 6.5.2; weights equal to 1, and reliability in all the variables growing from 0 to 1 in each run.

Figure 6-18 illustrates model accuracy when the reliability of all pieces of evidence varies equally. The behaviour of the four analysed samples is presented to show that both layouts have very similar patterns. The detailed results of this sensitivity test are presented in Appendix B.8.2 (from Table B-70 to Table B-72). From the figures, it can be noted that when the reliability of all pieces of evidence is almost null, the prediction accuracy of the model reaches its lowest value for both layouts, near 91%. In contrast, when reliability grows, the model performance is enhanced. However this improvement is considerably better for layout A in comparison to layout B, especially when it is closer to being fully reliable. Accuracy rises significantly, achieving almost 97%. In contrast, for layout B the increment is less noteworthy, rising to less than 95%. These results confirm that the layout with smaller bins has a better performance and strengthen the idea that layout A is more sensitive to changes in the reliability of the pieces of evidence.

After a comparison between the lowest global accuracy value (91% for both layouts) obtained in this test, against the lowest efficiency values from the experiment presented above (around 94.5% for layout A and less than 92% for layout B), it is interesting to note that in layout B the values are significantly close, which might suggest that for this layout the service type is the evidence that provides almost all of the information for estimating the non-routine rate.



Layout A **Layout B**

Figure 6-18 Model sensitivity when the reliability of all variables is modified, for layouts A and B

In the two sensitivity experiments described above, reliability was the main component analysed. Nonetheless, it is also necessary to explore the effect that dependency and weight have on prediction accuracy. Therefore, the following tests attempt to study the influence of these two components.

6.6.3 Reliability is the same for all pieces of evidence, weight and Alpha index are equal to one and then optimised.

The following sensitivity tests consider the variables equally reliable and increasing its value in each iteration, similarly as in the previous analysis. However, the tests are more comprehensive, considering also the effect of alpha-index and weights in model efficiency. In order to study these conditions, four different experiments were developed. The first two assume that all the pieces of evidence are completely independent. One test considers that all pieces of evidence are highly and equally important, while the other optimises the weights to reduce the prediction error. The remaining two experiments are based on the same weight scenarios, but use the adjusted dependency index. Summarising, one case considers alpha index and weight equal to one and the second assumes alpha index is one while the weight is optimised. In the third, alpha index is optimised and weight is equal to one and in the fourth, both alpha index and weight are optimised. For the four scenarios the model is run several times changing the reliability value from 0 to 1 for all the variables. The results of these tests are described using one of the samples and the detailed results for the four samples are presented along with their graphs in Appendix B.8.3 (from Table B-73 to Table B-82).

Considering a specific reliability, these four sensitivity experiments are created firstly, to assess the impact that optimising the weights might have on model performance by contrasting these results against accuracy when the weights are assumed to be equal to one and secondly, to compare the behaviour of the model when it is assumed that the variables are completely independent in contrast to when dependency has been adjusted by optimising the alpha-index values.

Figure 6-19 presents the case when it is assumed that all the pieces of evidence are completely independent, comparing the accuracy when the variables are considered highly important against the results when the importance has been adjusted, i.e. mathematically expressed as alpha index equal to one, weight equal to one for one of the curves and optimised for the other.

For both layouts, when the pieces of evidence are assumed to be completely independent and highly important (alpha-index = 1 and weight = 1), the model behaves in the same way as has already been explained. For inferior reliability values, the accuracy of the model is also low and when reliability increases, performance is also boosted but to a higher degree. However, when the importance of the pieces of evidence is adjusted to minimise the prediction error, the accuracy of the model enhances dramatically, even when the variables are considered unreliable. Nevertheless, it can be noted that this level of efficiency remains practically stable to changes in reliability, unless the variables are almost fully reliable where the model reaches its maximum accuracy point. Additionally, it appears that the optimisation of the weights can compensate for the lack of reliability of the variables by adjusting the importance of the pieces of evidence to obtain better accuracy results.

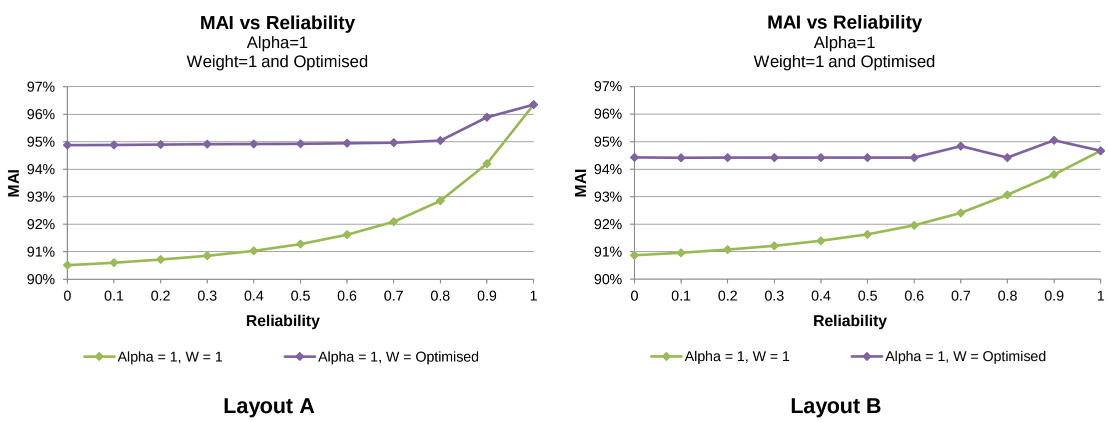
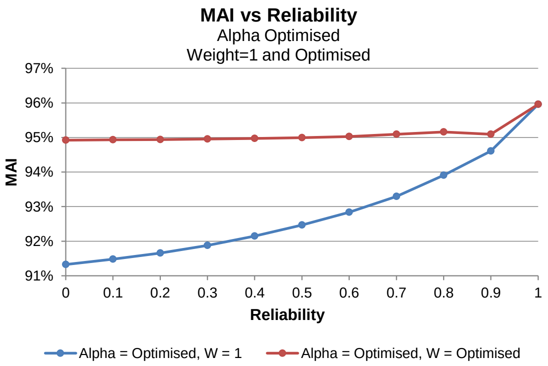
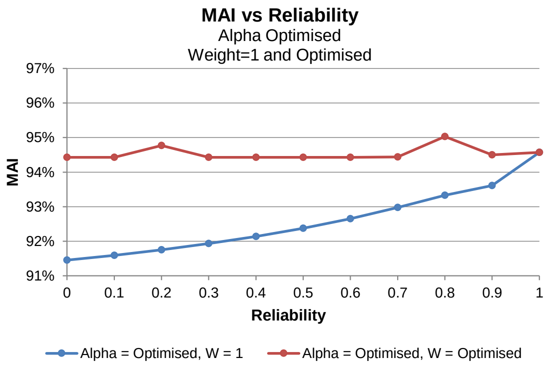


Figure 6-19 Model sensitivity when α=1, the reliability of all variables is modified and weight is optimised, for layouts A and B

Figure 6-20 compares the weights in the same way as in Figure 6-19, obtaining very similar results. The difference is that, for this case, dependency between the variables has been adjusted by optimising the alpha index. In this case, it can also be observed that the performance of the model is improved considerably by optimising the weights. Nevertheless, it seems that there is an upper limit of improvement that cannot be gone beyond by only adjusting the weights. To go further than that efficiency limit and enhance model performance, it is necessary to increase the reliability of the variables, i.e. improve the quality of the information.



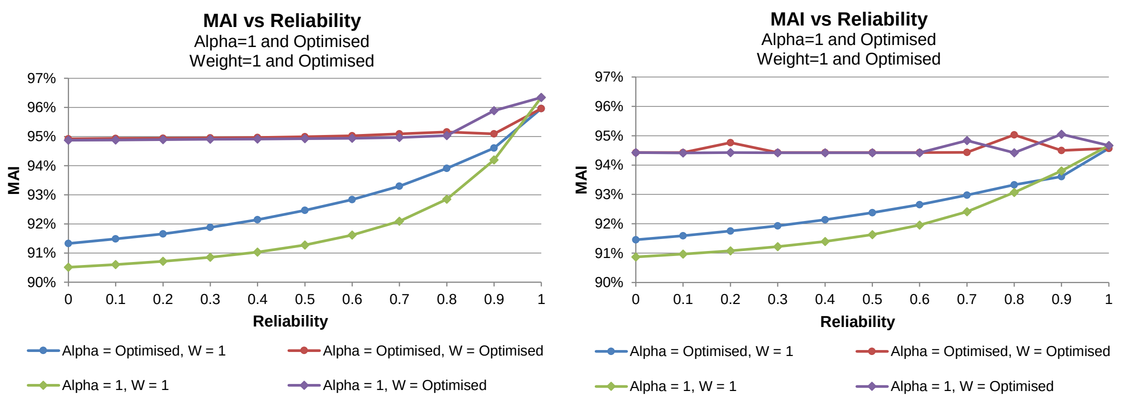
Layout A



Layout B

Figure 6-20 Model sensitivity when α has been adjusted, the reliability of all variables is modified and weight is optimised, for layouts A and B

Figure 6-21 compares the two cases described above where it can be observed that, for reliability values up to approximately 0.8, the model has a higher accuracy when the variables are assumed to be independent in comparison to when the dependency has been adjusted. However, this difference is more noticeable when the weight of the pieces of evidence is equal to one. It is only when the variables are almost fully reliable that this pattern is inverted, i.e. for higher reliability values, the prediction accuracy is better when the alpha index has been optimised. A possible explanation of this phenomenon could be that assuming the independence of the pieces of evidence might somehow duplicate or overlap the information provided by the pieces of evidence. Another interesting finding is that the performance of the model once the weights have been optimised is very similar, either when the alpha index is one or when it has been optimised, suggesting that this is the maximum accuracy level that can be reached by adjusting the importance of the pieces of evidence, i.e. by optimising the weights.

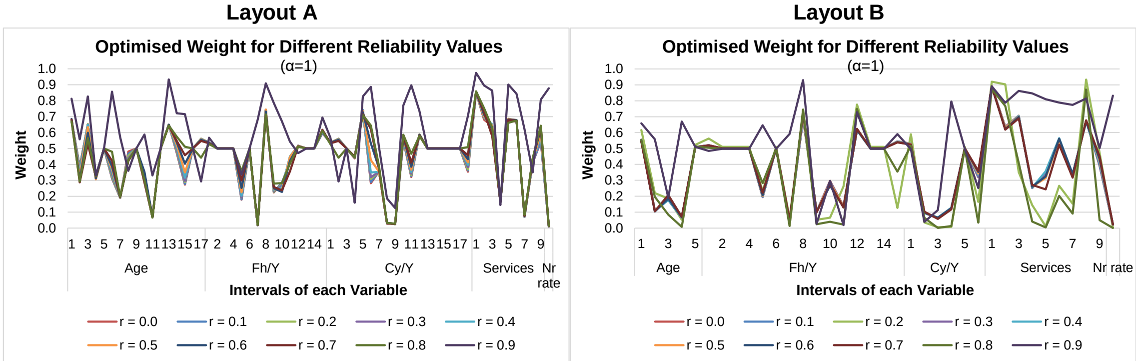


Layout A **Layout B**

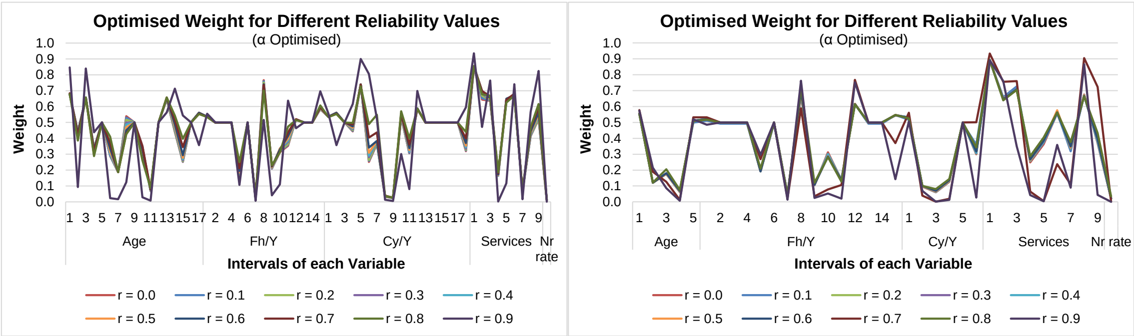
Figure 6-21 Model sensitivity comparison when α=1 and then is adjusted, and when weight=1 and then is optimised, for layouts A and B

Finally, it is interesting to observe in detail how the importance of the variables is adjusted for different reliability values. Each line in Figure 6-22 A, B, C and D represent the optimised weight for each interval of the input variables given a particular reliability (Figure A and D depict the weights for layout A when alpha index is one and when it has been optimised, respectively. Figures B and D are similar, but based on layout B). Interestingly, as can be seen in the figures, the values of the weight are very similar for the different reliability values (i.e. the lines are practically overlapping each other), with the exception of when reliability is 0.9. This might suggest that with small changes in the weight values, the model is capable of adjusting and maintaining accuracy for different reliability values. However, for higher reliability values, when the accuracy starts reaching its maximum value the changes in the weight are more remarkable, possibly as a result of the enhancement of accuracy.

Layout A **Layout B**



A) B)



C) D)

Figure 6-22 Importance degree (optimised weight values) for each piece of evidence for different reliability values, for layouts A and B

As this inference model is based on a small sample, the results from the sensitivity experiments cannot be considered as conclusive and therefore should not be used to generalise the behaviour of other ER models. Nevertheless, the presented sensitivity experiments were useful to gain insight into the influence that the reliability, dependency and importance of the pieces of evidence have on model performance. Firstly, they allow visualisation of the role that each of the variables have in prediction accuracy, which supports the idea that maintenance service type is a fundamental piece of evidence for this model. Secondly, it has been shown that for higher reliability values, the model performs considerably better. However, reliability cannot be modified directly and it is only when the quality of the information is improved that the reliability of a piece of evidence would increase. Thirdly, even when the accuracy of the model might be lower when the dependency of the pieces of evidence has been adjusted, this would offer a more representative model, abstracting reality in a better way. Finally, adjusting the importance of the pieces of evidence might lead to better prediction results. However, it is important to mention that there is a maximum limit for this improvement as a result of this modification. It is noteworthy that modifying or adjusting these features must be done with caution as increasing accuracy is not the only aim that should be taken into account. It is also important to consider the representativeness of these variables and the meaningfulness of the model.

6.7 Validation

As explained in section 6.5, from the original database of 91 maintenance checks, four different samples of 80 random maintenance services were used to build the model and the remaining eleven of each sample were used for validation purposes. The model was run on the validation group using the adjusted values for alpha index, reliability and weight obtained in sections 6.5.2, 6.5.3 and 6.5.4. Figure 6-23 compares the performance of the model for the training group, the validation fragment and also for the whole sample. Here, it can be seen that overall model performance is lower in the validation sample. However, the efficiency of the model is still satisfactory, as evidenced by the MAI values. For layout A, the accuracy for training is around 96% whilst accuracy for the validation fragment ranges from 90% to 94%. For layout B the accuracy for training is approximately 95%, whilst the validation accuracy varies from 90% to 95%. In other words, the ability of the model to estimate the non-routine rate is remarkable for both arrays of bins. The complete performance results are presented in Table B-83 and Table B-84 in Appendix B.9.

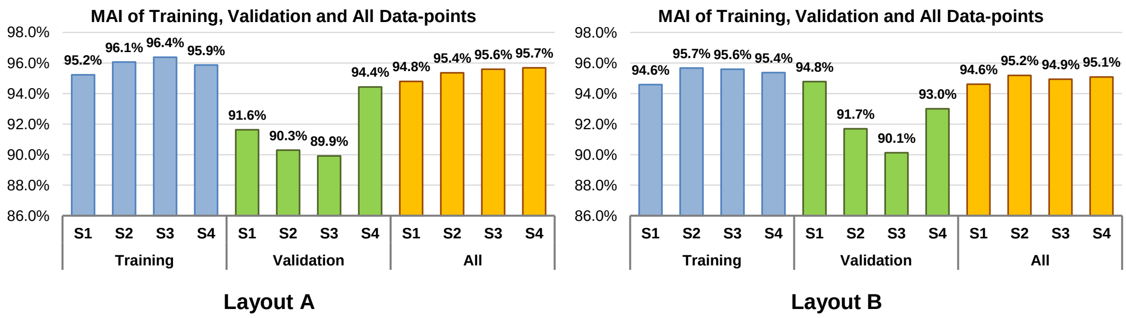


Figure 6-23 MAI comparison for training and validation groups and for the whole sample

As part of the validation stage, the model estimates the non-routine rate given particular operational and maintenance characteristics of the aeroplane, i.e. for an aeroplane with a specific age, flight hours and cycles per year, which will undergo certain maintenance service, the model estimates the non-routine rate based on the knowledge acquired during the training process. There are three possible ways in which the model can estimate the non-routine rate. Firstly, if the combination of maintenance and operational characteristics are already in the knowledge base, the model simply searches for the non-routine rate value of this specific arrangement and assigns it to the required combination. Secondly, if the combination is not in the original database but the particular pieces of evidence are available, the model estimates the non-routine rate based on the knowledge from the training phase, using the belief distributions, alpha-index, reliability and weight values. If the model does not have information about the qualities of the pieces of evidence (alpha-index, reliability and weight), it assumes the ideal case, i.e. completely independent, fully reliable and highly important, as appropriate. Finally, when one of the pieces of evidence needed to estimate the non-routine rate is not available in the knowledge database, the non-routine rate is calculated based on the information provided by the remaining pieces of evidence. It is worth noting that this prediction is considerably less accurate compared with that obtained using the complete information.

Despite the satisfactory results from validation, the model is not as accurate as it might be as the sample used to build the knowledge base is limited and small. Likewise, it is believed that a larger sample might have better estimation results. Moreover, the model is able to grow in knowledge and precision with every new addition to the sample, i.e. every new maintenance service that is appended to the database, and this would help to enrich the information and improve estimation accuracy.

6.8 Summary

The proposed model presents an innovative way for estimating non-routine rate in aircraft maintenance services based on the evidential reasoning rule, by using different but complementary pieces of evidence (aeroplane age, flight hours and cycles per year, service type and the non-routine rate of the whole sample). The model is used to estimate the unscheduled tasks that arise from unexpected damage and discrepancies. These unplanned additional tasks might alter the original plan, hindering its daily control, causing delays, overrunning costs and even risking the quality of the service. Therefore, the estimation of non-routine tasks might be useful for airlines and MROs to programme activities and to plan and allocate resources efficiently.

In this chapter, the effect on prediction accuracy of changing the interval size and the position of its limits, but also the impact of using a different arrangement of intervals amongst the variables, was explored. As a result of this analysis, it was found that, in general, the smaller the bins and the lesser the intervals, the better the estimation accuracy. Additionally, choosing the appropriate interval limits and the proper arrangement of classes between the variables might lead to better estimation of non-routine rate.

Furthermore, the influence that each of the different variables has on model performance was analysed. Firstly, it was noted that the arrangement of bin size could affect the way the variables interact, changing their importance in the model. Secondly, it appears that maintenance service type is the most relevant variable for the prediction of the non-routine rate, but to enhance model efficiency, this variable requires the support of at least one variable that describes aircraft utilisation (age, flight hours or cycles). Thirdly, it seems that some variables provide similar evidence for the estimation of non-routine tasks and hence, when they are combined, the model accuracy drops considerably. In contrast, when the variables are aggregated to others that provide supplementary information, prediction accuracy increases. Additionally, it was observed that when an array of small bins is chosen, the influence that the prior non-routine rate has on prediction accuracy is insignificant. Contrastingly, when larger bins are used, the role of this variable increases considerably.

Four scenarios were built to improve model accuracy, but more importantly its representativeness, by adjusting the qualities of the information. In the first scenario, dependency was modified by optimising the alpha-index. Then, in the second scenario, the reliability of the pieces of evidence was altered to represent the quality of the information. The main aim of these two scenarios was to shape the model to resemble reality as closely as possible. Finally, in the last scenario, the weights were optimised by regulating the importance of the pieces of evidence to enhance model performance.

In this chapter, different sensitivity experiments were carried out to analyse how the model reacts to changes in specific qualities of the information. First of all, the reliability of one variable was modified at a time, while the others remained fully reliable. Here, the service type emerged as the variable that most influenced the model, reinforcing former observations. Then, a second experiment was performed to analyse the global sensitivity of the model towards changes in reliability, showing that for greater reliability values the prediction accuracy improves dramatically. For the last experiment, not just reliability was modified, but also dependency and weights. It was found that prediction accuracy is slightly better when the pieces of evidence are assumed to be independent compared to when dependency has been adjusted. Moreover, when the weights are optimised, accuracy increases considerably even for lower reliability values, reaching a ceiling of performance.

After fine-tuning the model by modifying the alpha index, reliability and weights and analysing the sensitivity of the model towards changes in these parameters, it was necessary to test the real capacity of the model to estimate the non-routine rate. Thus, using the information and knowledge acquired from the training data, the model was used to estimate the non-routine rate from a different sample. The results were evaluated and compared against the performance from the training data. In general the model estimation accuracy shows satisfactory results having a MAI around 90% to 95%. It is believed that the model performance can improve if a larger sample of maintenance services is used. However, as described in the validation section, by adding each new maintenance service to the existing database, the model will adjust increasing in knowledge and precision.

Chapter 7: Discussion and conclusions

The main research goal was to analyse the effect of unscheduled maintenance tasks on the occurrence of delays and disruptions during the execution of aircraft heavy maintenance services. In order to achieve this objective, an exploratory case study was carried out based on empirical knowledge about the problem, supported by an extensive literature review and using real operational and maintenance records from an airline.

This final chapter aims to compile and discuss the main findings and results obtained throughout the development of this research as described in the previous chapters. Firstly, a summary of the empirical and theoretical bases of the problem is presented. The modelling process for building the system dynamics and the evidential reasoning rule models is then briefly described. After that, the research findings are discussed, linking them to the research questions. The main research limitations are examined next, followed by some suggestions for possible future research. Finally, the general conclusions of this research are presented.

7.1 Empirical and theoretical grounding

The airline industry is undoubtedly fascinating for its evident capacity to interconnect places and transport customers and goods and particularly because of its important role in globalisation, economic growth and social development. It is a very competitive and dynamic industry, characterised by large revenues and low-profit margins. In order to subsist in this hostile market, airlines and MROs are forced to implement strategies that improve their financial performance without compromising service quality. Some of the strategies have focused on aircraft maintenance as this is one of the main operating costs, having also great relevance for safety and service quality.

The maintenance of an aircraft is an unavoidable duty, organised in a systematic programme of scheduled tasks that is defined in conjunction with aviation authorities and manufacturers, and customised by airlines. Its key objective is to ensure the safety of aeroplane operations. It should be performed at the lowest possible cost with the highest quality standards to minimise impact on airline performance. Aircraft maintenance is grouped in a series of service checks organised by workload and periodicity, from the lighter and most frequent, commonly known as line maintenance, to the most exhaustive and less frequent, called heavy maintenance.

The literature suggests that heavy maintenance is one of the most critical processes within aircraft maintenance, where an aeroplane is out of service during a long period of time in order that a large number of tasks requiring also a great amount of specialised resources can be performed. Even when the tasks of the heavy maintenance check are rigorously defined in the scheduled maintenance programme, unexpected damage and failures are common occurrences and unscheduled activities must be performed to deal with these eventualities. These additional activities might have great impact on the overall schedule, leading to disruptions of the process and affecting delivery time and cost.

Specifically, as borne out both by personal experience and the literature review, the problem is the stochastic nature of non-routine tasks, hampering accurate planning and resource forecasts, triggering a complex interaction between scheduled and unscheduled tasks which are in a constant battle for resources. Therefore, due to continuous adjustments to the initial maintenance plan, managing the heavy maintenance process is not only challenging but may impact on scheduled times, cost and even service quality.

Due to its relevance for airlines, maintenance has been a common subject of study for many researchers. As the line maintenance process is directly linked to daily operations, it has been a preponderant topic of these studies, for example, to analyse workforce allocation and propose methods to recover from disruptions. Regarding heavy maintenance, valuable proposals have been made to address long-term planning of maintenance services and the scheduling of the maintenance tasks within this service. However, little attention has been given to the uncertainty caused by unscheduled maintenance tasks, assuming a predetermined tentative number of non-routine tasks.

Due to the aforementioned characteristics, some authors have defined Heavy Maintenance as a large, sophisticated and dynamic project. These features are normally related to the term complex project. However, even though the concept is widely used in the literature, few studies offer a precise definition, being frequently assumed as a synonym of a large project. After reviewing different interpretations, it can be claimed that complex projects are very dynamic, considering a great number of interconnected elements and activities, which are constrained in time and resources and where uncertainty exists throughout the whole process. Therefore, in summary it can be argued that Heavy Maintenance is a complex project.

Several authors argue that conventional project management tools by themselves fail to properly deal with the uncertainty and dynamism of complex projects, resulting in frequent cost overrun and missing deadlines. To overcome the limitations of the traditional methods, two main strands have been used as alternative and supportive approaches: mathematical optimisation models and modelling simulation. The former has been utilised to produce initial project plans that aim to optimise the duration of activities and improve resource allocation and usage, generating more accurate and robust project schedules. Unexpected events are normally tackled by buffering the project budget, resources or duration, taking into account statistical information and assuming probability distributions. Simulation has been widely applied to describe the dynamic behaviour of a project throughout time, to explore and analyse how the project responds to certain changes or unexpected events and to design and evaluate scenarios that improve project performance. Given the characteristics of Heavy Maintenance and its major challenges, simulation appears as a suitable supportive approach for representing the dynamic behaviour and the complex interaction between its elements.

Discrete Event Simulation (DES) and System Dynamics (SD) are the two main simulation approaches that have traditionally been used to study complex projects. DES is a stochastic modelling method that represents the transition of individual entities throughout a network of queues and activities, evaluated in discrete asynchronous periods of time (known as events) and where complexity arises from the randomness present in the system. DES allows a “microscopic” perspective of the system and has the ability to model with great detail each of the elements and their changes through time. For these reasons, DES has mainly been applied at operational and tactical levels for analysing system performance in detail. However, to achieve the expected results, DES requires large amounts of data and its outcome could be an extremely complicated model for large systems. System Dynamics is a deterministic simulation approach, where the system is represented as an array of interconnected stocks and flows modelled in small continuous time steps (known as *dt*). It takes a holistic perspective of the problem, with an “aerial” point of view of the system, emphasising that complexity arises from the feedback structure and the dynamic interrelationship between the elements. It is frequently utilised at strategic level as a decision support tool. Contrary to DES, SD models do not require a large amount of data and are able to handle qualitative and quantitative information. However, their interpretation and validation could be difficult and highly subjective. In particular, SD has proven its usefulness in project management for understanding and analysing complex projects through its capability to represent and model the dynamic behaviour of the system along with its sophisticated feedback structure.

Considering that heavy maintenance is a dynamic and complex project, restricted in time and budget, where a large amount of specialised resources are required to carry out a vast number of maintenance tasks, it would be difficult to collect detailed information of all the tasks and resources. It was found that a systemic vision of the process was more suitable for the purpose of this research, focussing on understanding the behaviour and feedback structure rather than an exhaustive description of the system and its elements. Therefore, System Dynamics was suggested as a methodology to study the heavy maintenance process and its main challenges.

However, the literature has pointed out the limitations of SD to cope with uncertainty, which is a basic characteristic of complex projects and is also a very relevant feature within heavy maintenance processes. Therefore, it was necessary to explore methodologies to supplement SD and overcome this limitation.

The uncertainty of an unexpected event, happening either before or during the execution of a project, and its associated consequences are the two key components of risk. For heavy maintenance, risk can be seen as the uncertainty derived from the latent occurrence of unexpected damage and failures, and the effect and severity of these events, which may affect the performance of the maintenance process.

Uncertainty refers to a deficiency, ambiguity or absence of information, knowledge or understanding about a particular situation and its possible outcomes. Uncertainty can be understood, managed, and even reduced, but cannot be completely eradicated. In the execution of a complex project there are different degrees of uncertainty depending on the availability of information and the knowledge about the system. In particular, for heavy maintenance it is difficult to objectively know the probability distribution of the unscheduled maintenance tasks. Most of the time, they are estimated based on subjective judgements.

Diverse theories have emerged to study uncertainty from different perspectives rather than the traditional probabilistic approach. These theories address particular aspects of uncertainty. Fuzzy set theory has been proposed to deal with vagueness of information, while evidence theory focusses on information ambiguity, formed by conflict and imprecision. Given highly regulated and strict conditions in which heavy maintenance is carried out, vagueness is not the most relevant quality of uncertainty, but it can be argued that there could be ambiguity in information. Therefore, the evidence theory seemed an appropriate perspective for analysing the uncertainty of unscheduled maintenance tasks. The core of evidence theory is Dempster's rule for combining independent pieces of evidence. However, several authors have stressed its counter-intuitive results when working in conflicting conditions. The evidential reasoning (ER) rule has been proposed as a generalised conjunctive probabilistic reasoning process for combining independent pieces of evidence, taking into account their weights and reliabilities, being capable of working under highly or completely conflicting conditions.

To complement the SD model that analyses the dynamic behaviour of and complex interaction between scheduled and unscheduled tasks, it was proposed to use the ER rule as a rigorous method capable of coping with variability and ambiguity for estimating the expected number of non-routine tasks considering their relationship with several operational and maintenance variables.

7.2 Models summary

The SD model was used to study how complex interaction between scheduled and unscheduled tasks hinders resource allocation during the execution of maintenance services, which might lead to delays and disruptions. The model was built using Vensim 6.3D for the causal loop diagrams and iThink 10.0.2 for the stock and flow diagrams, and for the mathematical modelling.

Qualitative and quantitative data were used for building the SD model. Based on the author's experience, an initial causal loop diagram depicting the problem was defined and after several discussions and feedback meetings with experts, it was improved and refined. In addition, real maintenance records considering maintenance services, time duration, number of tasks and man hours were utilised for developing stock and flow diagrams and for building the mathematical model.

Firstly, the conceptual model was developed by building several causal loop diagrams for understanding and describing the factors that cause delays during the execution of heavy maintenance services. The diagram depicts the management of scheduled task execution to ensure that the maintenance service is carried out according to the plan and illustrates the occurrence, evaluation and programming of unscheduled maintenance tasks, highlighting possible causes. The execution of unscheduled maintenance tasks is represented, emphasising the resulting conflict regarding resource allocation between the two different types of tasks. When the available resources are not sufficient to fulfil the project requirements, additional resources are requested, the last option being an extension of the maintenance service. The model also describes the delays in decision-making and the different attitudes taken by managers during the execution of the maintenance service.

The conceptual model was then transformed into a quantitative SD model. Due to the limited time frame of this research, only the workforce was analysed as the main resource of the system. The workforce is the most complex resource to manage during the process and with the most implications for the service. Therefore, the quantitative model remained representative and significant but simple at the same time. Two different models were developed. The first model explains and explores the impact of the occurrence and discovery of discrepancies on the maintenance service duration and the effect of resource allocation on the performance of the project. The second model describes and analyses the effect of delays on decision-making and also on the perceptions and attitudes of decision makers towards certain aspects of the project, for instance, backlog perception or allocation of additional labour. The two models were used to experiment and test different maintenance policies.

The ER rule was used to develop a novel approach for estimating non-routine tasks based on historical data regarding the usage and maintenance of an aircraft. The model was built using MATLAB R2015a to reduce calculation time and to make the model more robust and flexible.

Using real operational and maintenance records of an airline, four variables were identified as significant to estimate unscheduled maintenance tasks. Age, flight hours per year, cycles per year and the type of maintenance service to execute were chosen due to their relevance for defining and managing the maintenance programme and their relationship to the occurrence and discovery of damage, and failures. In addition to these four variables, the distribution of non-routine tasks in the sample of maintenance services analysed was used as further evidence to estimate unscheduled tasks. The non-routine rate was proposed as an indicator to measure the number of unscheduled tasks against the number of scheduled tasks and was used in the model to describe the expected number of unplanned tasks.

The continuous variables were discretised into different equal-size intervals to facilitate both the modelling process and interpretation for decision makers. To determine the most appropriate bin-widths, different discretisation rules were used, considering the usefulness for practitioners and the shape of the distributions. Once the data was discretised, the ER rule was applied recursively to aggregate each of the pieces of evidence to estimate the expected non-routine rate of an aeroplane undertaking a maintenance check.

Analysis was carried out to explore the effect that the class size, the position of the interval limits and the combination of different interval arrays have on the model prediction accuracy. After the analysis, two different arrays of intervals were chosen to conduct further experiments. In order to evaluate the role that each of the variables played in the model performance, different scenarios were executed varying the number and combination of variables.

Based on the results of the previous analyses, four different scenarios were developed to improve the robustness and meaningfulness of the inference model. The first scenario assumed ideal conditions by considering that all pieces of evidence are completely independent, fully reliable and have the same importance. Then, assuming reliability and importance as in the previous case, the dependency between the pieces of evidence was adjusted. In the third scenario, using the dependency values of the previous step, the reliability of the different variables was adapted by considering their characteristics while they were still assumed to be highly and equally important. Finally, using all the previous results, the importance of the pieces of evidence was modified to enhance model performance in the last scenario.

Several sensitivity analyses were performed to explore how changes in the reliability, importance and dependency of the pieces of evidence might influence the efficiency of estimating the non-routine rate. A small sample not used in the training process was utilised to test the estimation accuracy of the model.

7.3 Research findings

Four research questions were proposed to address particular aspects of the problem. The first two questions were answered by developing the SD models while the last two were approached with the ER rule model. In the following paragraphs, the questions are restated. The main research findings found throughout the development of this research are then briefly discussed.

1) *How does the interaction between scheduled and unscheduled tasks influence resource allocation throughout the maintenance process?*

As described in chapter 5 with the aid of several causal loop diagrams, a conceptual model illustrating the problem and its main features was developed. A very common condition observed in aircraft heavy maintenance, as in almost any project, is to assign more resources to increase progress when a backlog of activities is perceived. During the execution of the maintenance check, unexpected damage and failures are discovered and need to be corrected by programming additional maintenance tasks. However, these unscheduled activities require supplementary resources to be carried out. Due to the limited amount of resources available, there is a constant fight for them between scheduled and unscheduled tasks. If more resources are allocated to perform scheduled tasks, their progress increases but at the expense of unscheduled tasks and vice versa, creating delays and a backlog of tasks during execution Therefore, it becomes crucial to improve resource allocation management. When the backlog of tasks of either of the two types of tasks have increased considerably and hence created pressure to meet planned targets, and the total number of resources available is not sufficient to cope with the demand, there is pressure to increase the capacity of available resources. However, this strategy takes time to take effect as it is difficult to get extra specialised resources. If, despite all efforts to complete the project in time, there are still delays during its execution, the last option is to extend project duration beyond the point it was initially planned, to reduce backlog pressure and allow available resources to complete the maintenance tasks.

The initial causal loop diagrams were built based on the author's experience of the problem and further enhanced through an iterative process with the help of experts in the field. The causal loop diagrams proved to be a very effective tool. They helped to elucidate the complex interactions between routine and non-routine tasks that hinder the management of resources and the control of the whole maintenance service. Moreover, during the discussion sessions, the causal loop diagrams proved to be highly useful to explain the problem and encourage learning and debate.

In addition to the causal loop diagrams, through the simulation of the SD model, it was possible to explore the impact of workforce allocation and to test different policies of assigning personnel during the execution of the maintenance project. The simulation model enabled the examination of how, when non-routine tasks begin to appear and accumulate, the management of workforce allocation becomes more difficult. It was also interesting to experiment with how project duration is affected by assigning different proportions of headcount to scheduled and unscheduled tasks.

2) *How do the occurrence and discovery of damage and discrepancies affect the execution of the maintenance service?*

Like almost every machine, aircraft are prone to damage and failures. These may arise from different conditions such as environment, type of aeroplane, usage, age and type of operation, amongst others. Generally, damage and failures are not detected in their early stages: it is only when they start growing and spreading that they are discovered during a maintenance check. As explained in chapter 2, during the execution of scheduled maintenance, particularly during the inspection phase, damage and failures are found that need to be sorted out by programming additional maintenance activities. Several authors have argued that around thirty to sixty percent of the total workload in a heavy maintenance service is derived from unscheduled maintenance activities. However, due to their uncertain nature, it is difficult to programme in advance unscheduled tasks and to forecast the resources required to execute them.

The development of the conceptual model allowed for the comprehension and characterisation of the dynamic relationship between the execution of scheduled tasks and the discovery, programming and execution of unscheduled tasks. It also helped to elucidate the exogenous and endogenous aspects involved in the occurrence of non-routine tasks.

Once awareness of this relationship was achieved, the SD quantitative models were built to describe this behaviour and to visualise the effect of the occurrence and discovery of damage and failures on the duration of the maintenance service. Through evaluation of and comparison between different scenarios, it was possible to appreciate how a greater non-routine rate might extend the duration of a maintenance service. Similarly, late discovery of abnormalities and discrepancies can also force project duration to be longer than originally planned.

3) *What are the most relevant variables for estimating unscheduled maintenance tasks?*

As discussed in chapter six, age, flight hours per year, cycles per year and maintenance service type, along with non-routine tasks distribution were used as five different pieces of evidence to estimate the non-routine rate for a particular aeroplane undertaking a specific maintenance service. The variables were chosen firstly because of their significance in the definition and control of the aircraft maintenance programme and secondly, due to their close relationship with the occurrence and discovery of damage, and failures. These variables are normally considered by experts when providing a subjective estimation of the expected number of non-routine tasks. The impact that these variables have on non-routine rate estimation was explored by assessing model prediction performance using different numbers and combinations of variables.

It was found that the best results were achieved when the five variables were used. It was also found that the most relevant variable in the estimation process was the type of maintenance service, followed by the flight hours per year, although model accuracy improves further when they are utilised in combination with other variables. The variable with the least relevance is prior non-routine rate. However, this variable becomes more significant in the scenario where wider interval bins are used. It was also observed that there are variables that provide similar evidence for estimating the non-routine rate and therefore model performance decreases when they are combined without considering other variables. These results were also confirmed through a sensitivity analysis where the reliability of each variable was assumed from non-reliable to fully reliable, showing how susceptible the model is to changes in the quality of these five pieces of evidence. It was shown that model accuracy is very sensitive to changes in the reliability of the maintenance service type. With smaller bin-widths, in contrast, when the reliability of age and prior non-routine changes, model accuracy does not change significantly.

4) *How can operational and maintenance variables be used as different pieces of evidence for estimating the expected number of unscheduled maintenance tasks?*

The ER rule was applied to develop an inference model for estimating the non-routine rate of aircraft maintenance checks. In comparison with the estimates frequently used in the industry, the ER model proposed a formal and rigorous approach to calculate the expected number of unscheduled tasks by combining age, flight hours per year, cycles per year, maintenance service type, and prior non-routine tasks distribution, thus providing a novel approach to analyse the uncertainty of unexpected events that hinder the management of complex projects.

By recursively applying the ER rule, non-routine rate belief distributions provided by each of the five variables are aggregated to obtain the expected belief distribution of the non-routine rate for specific operational and maintenance characteristics of an aircraft. To show the effect of the interval size, during the analysis two different arrays of intervals were compared, the former with more intervals and narrow bins and the latter with fewer intervals and wider bins. The model was fine-tuned by adjusting dependency, reliability and importance, aiming to improve the model's accuracy and meaningfulness. Several sensitivity tests were then performed to investigate the influence of these parameters on the estimation performance.

The model was run several times using different training samples and was tested using small independent samples. For the training sample the model had a MAE of around 0.020 to 0.015 and a MAI of approximately 95.2% to 96.4%. For the validation sample, the results were lower but still satisfactory, achieving a MAE of 0.022 to 0.018 and a MAI between 94.8% and 95.7%.

The model could be a useful tool for airlines and MROs to estimate the expected number of non-routine tasks. Given a certain array of age, flight hours per year, cycles per year and service type, the model searches on the knowledge database obtained through the training process to combine those pieces of evidence and provide an estimation of non-routine rate.

7.3.1 Additional findings

In addition to the research findings discussed through addressing the research questions, other interesting results which are worth noting were obtained during the development of this research. Using the SD model, the impact of delays in decision-making, and the attitudes and perceptions of management during the execution of the maintenance service were analysed. Using the ER model, the impact of assigning different bin-widths, interval limits and interval arrays on the estimation of the non-routine rate was explored. The influence of dependency, reliability and weight of the pieces of evidence on model performance was also analysed.

The SD model allowed for the exploration of the impact of delays in the decision-making on the maintenance service duration. It was observed that even with the absence of the uncertainty created by the unscheduled maintenance tasks, delays can create fluctuations that hinder the management of the project: for instance, they can exacerbate the backlog of tasks or the requirement of additional resources. In a similar way, the attitudes of decision makers can affect significantly maintenance service performance. For example, in the early stages of the project, there is a reluctance to increase the resources available with the aim of improving the utilisation of resources. However, this strategy might affect the execution of activities and cause delays during the process.

The ER rule model was run several times with different bin sizes, moving the interval limits and using different arrays of intervals. From the analysis, interesting results were obtained. It was found that, in general, model estimation accuracy improves when the size of the bins is reduced. Model performance also changes when the bin-widths are the same, but the interval limits are changed. Finally, it was found that there are arrays of intervals that perform better than others, suggesting that there is a certain affinity between the bins of the different variables. In summary, the proper combination of bin size, interval limits and interval arrays is very important in improving the accuracy of the model.

The sensitivity analyses performed in the ER model allowed for the influence that the reliability, dependency and importance of pieces of evidence have on model accuracy to be identified. Reliability is an intrinsic characteristic of the pieces of evidence that represents its quality and should not be modified arbitrarily. However, the analysis allowed for the sensitivity of model accuracy to changes in the reliability of the variables to be assessed. It was also discovered that model performance is lower when the dependency of the pieces of evidence has been adjusted compared to when they are assumed to be independent. This is probably because in the latter case the information provided by the pieces of evidence is duplicated or overlapping. Finally, it was found that when the importance of the pieces of evidence is adjusted, model results improve considerably. However, it seems that there is a maximum point of enhancement that cannot be gone beyond by only adjusting the weights.

7.4 Research limitations

A model as a representation of reality cannot fully depict the real system that is being studied. Such is the case with this research and the models developed in it, due to the limited time frame and scope of this research and the availability of information, amongst other issues. In the following paragraphs, several important limitations of the research are discussed.

Some limitations are related to the information utilised in this research. For instance, the information and records obtained correspond to only one airline, and the experts consulted are from only one region. Therefore, the generalization of the results could be questioned, since there may be specific problems with that company that are not common throughout the industry. Some issues may be specific to the region under analysis. Another limitation is the narrow size of the dataset used for building the models, particularly the ER model. A larger database could lead to more reliable and robust results. However, it was difficult to obtain a larger dataset of maintenance services due to the sensitivity of the information and because the modest fleet size of the airline would require several years of maintenance information.

Despite the results and relevant findings obtained so far through the development of the SD quantitative model, it is important to state the main assumptions considered and discuss their limitations that may call into question the credibility of the research. Firstly, it has been explained that the dynamic and complex interaction of scheduled and unscheduled maintenance tasks and the required resources to accomplish them might affect project duration and overrunning costs. However, to facilitate the model development and due to its operational and financial implications, the SD model focused on analysing maintenance service duration time rather than studying maintenance costs. Secondly, although the conceptual model considered the three main types of resources involved in the execution of a maintenance service (parts and materials, tools and equipment and workforce), in the development of the quantitative model, only the workforce was used as the main resource to simulate. The workforce was chosen as is represents the most difficult resource to manage. Thirdly, regarding the duration and execution of tasks, productivity was assumed to be constant throughout simulation and the progress of tasks was considered to be linear, although in reality productivity tends to fluctuate and task progress is not precisely linear. The purpose of these assumptions was to facilitate the understanding of the development of the maintenance service and the interaction between the variables involved in the process. However, these assumptions might have negative implications on the modelling process, affecting and biasing the total duration of the maintenance service. Finally, the quantitative model focused on analysing the effect of the occurrence and discovery of discrepancies, the impact of workforce allocation policies, and the importance of perceptions and delays in decision-making, but the model did not consider the ripple and knock-on effects of increasing the available workforce.

Another limitation of the SD model is validation. It has been pointed out by several authors that validating an SD model is a real challenge that must be performed in various steps throughout the modelling process. In this research, different endeavours were made to validate the behaviour and performance of the SD model. However, further extensive tests are required to ensure the robustness of the model. Although the conceptual model was adjusted and improved considering the experts' suggestions and comments, subjectivity is always going to be present as it is based on experience and perception of the problem. It is highly likely that if the conceptual model is discussed with a different group of experts, they will make further modifications and improvements. Regarding the quantitative model, the general structure, mathematical equations and dimensional coherence was reviewed. However, due to the difficulty in obtaining sensitive information regarding the performance of maintenance services, the overall model performance was assessed comparing its behaviour and results with the experts' experience about the problem.

Regarding the ER model, for the estimation of the non-routine rate, flight hours per year and cycles per year were utilised. However, these measurements were obtained from the total flight hours and the total number of cycles, thus representing an average utilisation in relation to the age of the aircraft. It would have been interesting to also have flight hours per year and cycles per year measured from the immediately preceding maintenance service in order to have the exact utilisation of the aircraft prior to the maintenance service to analyse.

The benefits of grouping data have been stressed by different authors, but grouping data also has significant limitations, particularly the loss of information. During the data discretisation process, equal sized bins were used. It was also assumed that the information is equally distributed within the bins and that the aeroplanes inside a particular bin have the same qualities, even if they are at the extremes. For example: in an interval between 3,000 to 4,000 flight hours the expected non-routine rate is 0.35, hence, an aeroplane with 3,100 flight hours has the same expected non-routine rate as an airliner with 3,990 flight hours. These assumptions facilitated the development of the ER model and made it easier to understand for practitioners, but restrict the performance and accuracy of the model.

Assuming the variables to be fully independent might produce dubious or inaccurate results as this assumption is difficult to fulfil in real problems. Therefore, the dependency between the variables was analysed. It was adjusted by optimising the value of alpha-index. An optimisation model was developed to minimise the MSE_{DIST} between the real non-routine rate distribution and the distribution calculated by the model. However, using an optimisation approach to calculate the alpha-index has two major limitations. The first is that for large data sets the optimization model may require a considerable amount of time to obtain the results due to the large number of alpha-indices to optimise. The second is that the optimisation approach could alter the basic property of the ER rule to combine pieces of evidence in any order. As the optimised alpha-indices refer to a particular combination of variables, they cannot be used for a different combination. If a distinct array of variables is used, the alpha-index must be recalculated.

The reliability of a piece of evidence denotes the quality of the information that characterises that piece of evidence. Reliability should take into account the nature of the data, how it is collected, organised and stored. In the case of the ER model, the reliability of the different variables was estimated based on experience of the problem. However, this method of assigning reliability values can be deemed subjective and arbitrary. For this reason, a sensitivity analysis was carried out to examine the impact of reliability on the model estimation accuracy.

7.5 Suggestions for future research

Based on the development of this research and taking into account the results obtained, there are several aspects that would be worth continued investigation. The following paragraphs suggest possible lines of research.

Further research needs to be done to extend the simulation models by including other relevant resources and by strengthening the model assumptions regarding progress and productivity. Moreover, there has recently been increased interest in using hybrid models by combining two or more simulation approaches. It would be very interesting to develop a hybrid model based on SD and DES by combining their strengths. SD can be used to characterise the complex and dynamic interactions within the process, whilst DES can be utilised to describe the changes of certain elements during the maintenance service and to analyse the stochastic nature of the non-routine tasks.

As mentioned in the limitations section, the SD model focused on studying the duration of a maintenance service. Significant further research would be to incorporate the maintenance costs into the model and develop an optimisation model aiming to reduce both service time and maintenance costs.

The SD model allowed assessment of the impact of different resource allocation strategies on project duration. It was found that having a variable distribution of workforce produces better results compared with having a fixed allocation of workforce between routine and non-routine tasks. Further research to determine the best resource allocation between scheduled and unscheduled tasks during the execution of a maintenance service would be very interesting and valuable.

Regarding the data discretisation for the ER model, it is suggested that further research be undertaken in the following areas: 1) the utilisation and assessment of other methods for calculating the number of bins, such as kernel density distributions. 2) The use of unequal-size bins to evaluate their impact on model accuracy. 3) The development of a method to determine the optimal size of bins and the best array of intervals that maximises the model accuracy. 4) The exploration of the use of referential values to estimate the value of a specific element by determining its position between two referential values, rather than using intervals to determine the value of an element by allocating it into a particular interval.

In this research, the impact of bin size, the interval limits position and the array of intervals was explored. It would be interesting to perform similar analyses in a larger database to find out if the results and conclusions obtained in this research are still valid. Similarly, it would be worth carrying out these or comparable experiments in other ER rule models with similar characteristics to determine if they get similar results.

Regarding the analysis of the dependency between the variables, more research is required to explore alternative methods to calculate the values of alpha-index. Correspondingly, for the estimation of the reliability of the pieces of evidence, further investigation is recommended to explore more rigorous ways to determine the quality of a piece of evidence.

In chapter 3, it was explained that risk comprises two aspects: the uncertainty of the occurrence of a particular event and the consequences of that event. In this research, by developing the ER model, the first aspect was addressed, i.e. the belief distribution of the non-routine rate was estimated. Future research could therefore concentrate on the assessment of the impact of severity of damage and the occurrence of failures.

This thesis proposed the estimation of unscheduled maintenance tasks using a data-based model by applying the ER rule. An interesting advancement of this research would be to develop an estimation model based on experts' judgements regarding the occurrence of unscheduled maintenance tasks. This could then be integrated with the ER model to provide a more robust approach to analyse the uncertainty that may affect maintenance service performance.

Considerably more work will need to be done to enhance the validation of both models to ensure their robustness and usefulness. For the SD model, it would be appropriate to obtain more information about process performance and utilise analytical tools to assess the model estimation accuracy. Regarding the ER model, it would be interesting to develop other models using alternative methodologies such as artificial neural networks or support vector machine, and compare their results with the ER model to determine its efficiency in estimating the non-routine rate.

7.6 Conclusions

According to personal experience and borne out by the literature, it can be concluded that delays and disruptions are a recurring problem during aircraft heavy maintenance with significant economic and operational repercussions for airlines and MROs. It is suggested that missing deadlines and cost overruns are common during the execution of complex projects.

Based on the evidence presented thus far, it can be summarised that managing aircraft heavy maintenance service is challenging as there is a vast number of maintenance tasks to carry out that demand meticulous monitoring during their execution. These tasks require a great amount of limited and specialised resources that need to be carefully distributed and managed during the process. The project is also very likely to encounter eventualities mainly caused by the uncertain nature of unscheduled maintenance tasks. These additional activities make it even more difficult to manage the project, hindering resource planning and forcing continuous adjustments to the project schedule. All this may affect lead times, costs and service quality.

It can be argued that the present study contributes to existing knowledge of aircraft maintenance management by proposing the use of SD in combination with the ER rule to analyse the role of unscheduled tasks on delays and disruptions. SD was used to analyse and explain the dynamic behaviour of the system and the complex interrelation between the variables and to holistically explain the influence that unscheduled maintenance tasks have on service execution. The ER rule was applied as a rigorous method for estimating the expected number of unscheduled tasks within an aircraft maintenance project. An additional contribution was the definition of the non-routine rate as a measure to express the relationship between the number of unscheduled and programmed tasks. Furthermore, these contributions can be extended to the Project Management field due to the close affinity that heavy maintenance has with complex projects. Particularly, this work also extends our knowledge of SD by expanding its capabilities using the ER rule as a method for analysing the uncertainty present in the system. Finally, the research makes several contributions to the application of the ER rule by exploring the impact that bin size, interval limits and array of intervals have on the model accuracy and by investigating the role that dependency, reliability and weight have on the model performance.

Regardless of the limitations discussed above, using system dynamics in combination with the evidential reasoning rule provided significant and fruitful results that have helped to improve understanding of the impact that unscheduled tasks have on delays and disruptions during the execution of aircraft maintenance services. Additionally, this approach provided a platform to aid decision making by exploring and assessing different maintenance strategies to reduce project duration. It also provided a robust method for estimating the uncertain non-routine activities that hinder the execution of a maintenance check. This research was intended to bridge the gap identified in the literature review.

Despite the fact that this research concentrated on analysing a particular problem in the airline industry, the findings and conclusions obtained could be used to understand and address problems embodying similar characteristics, increasing awareness of possible causes and consequences. The models can be adapted and enhanced to improve and broaden their applicability to the management of complex projects.

The models, results and findings obtained during the development of this research are not intended to be an ultimate solution to the problems affecting aircraft maintenance. Neither do they seek to act as a substitute for current approaches in project management. Rather, they are proposed to be utilised in conjunction with other tools to tackle an intricate problem that must be addressed in a comprehensive manner.

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ANALYSING UNCERTAINTY AND DELAYS IN AIRCRAFT HEAVY MAINTENANCE

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Volume II of II: Appendixes

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Appendix A: SD models

A.1 Conceptual model

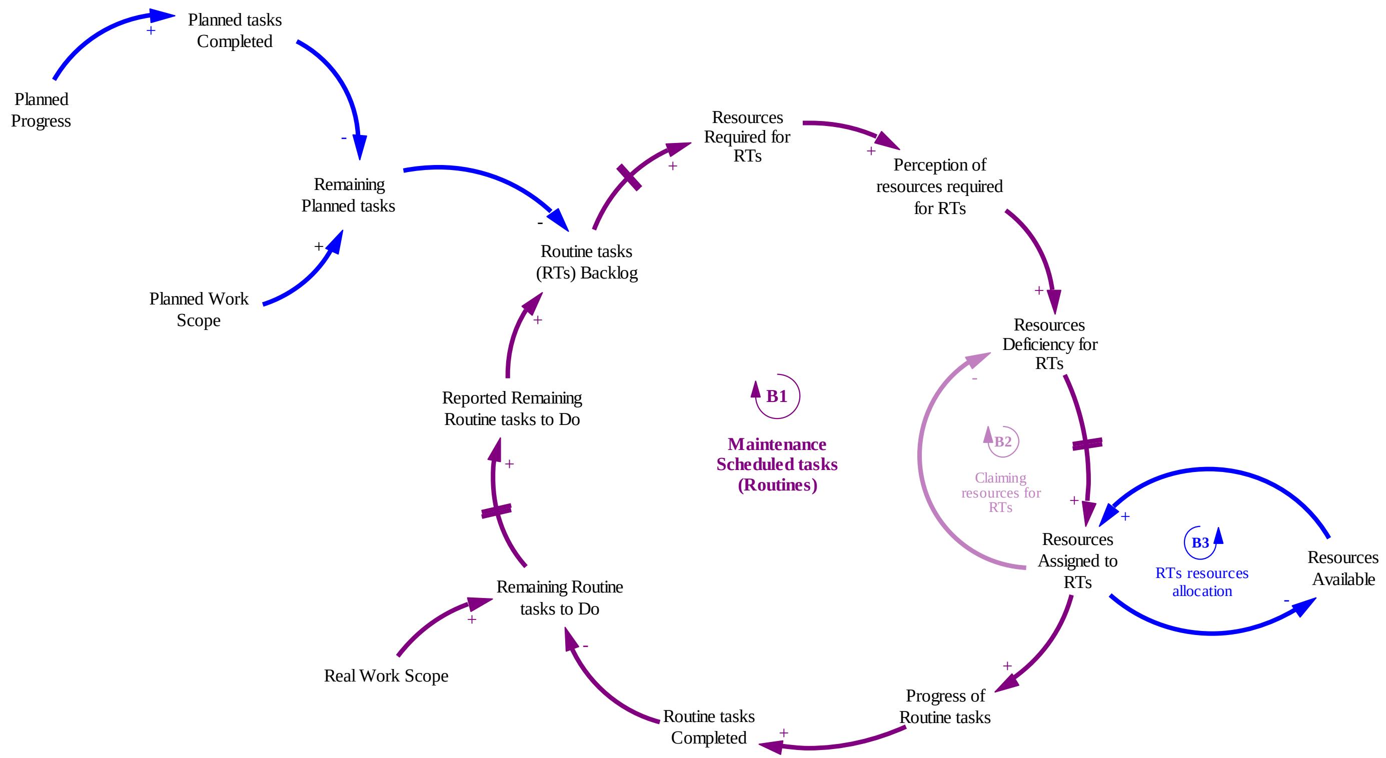


Figure A-1 Scheduled tasks and resources allocation

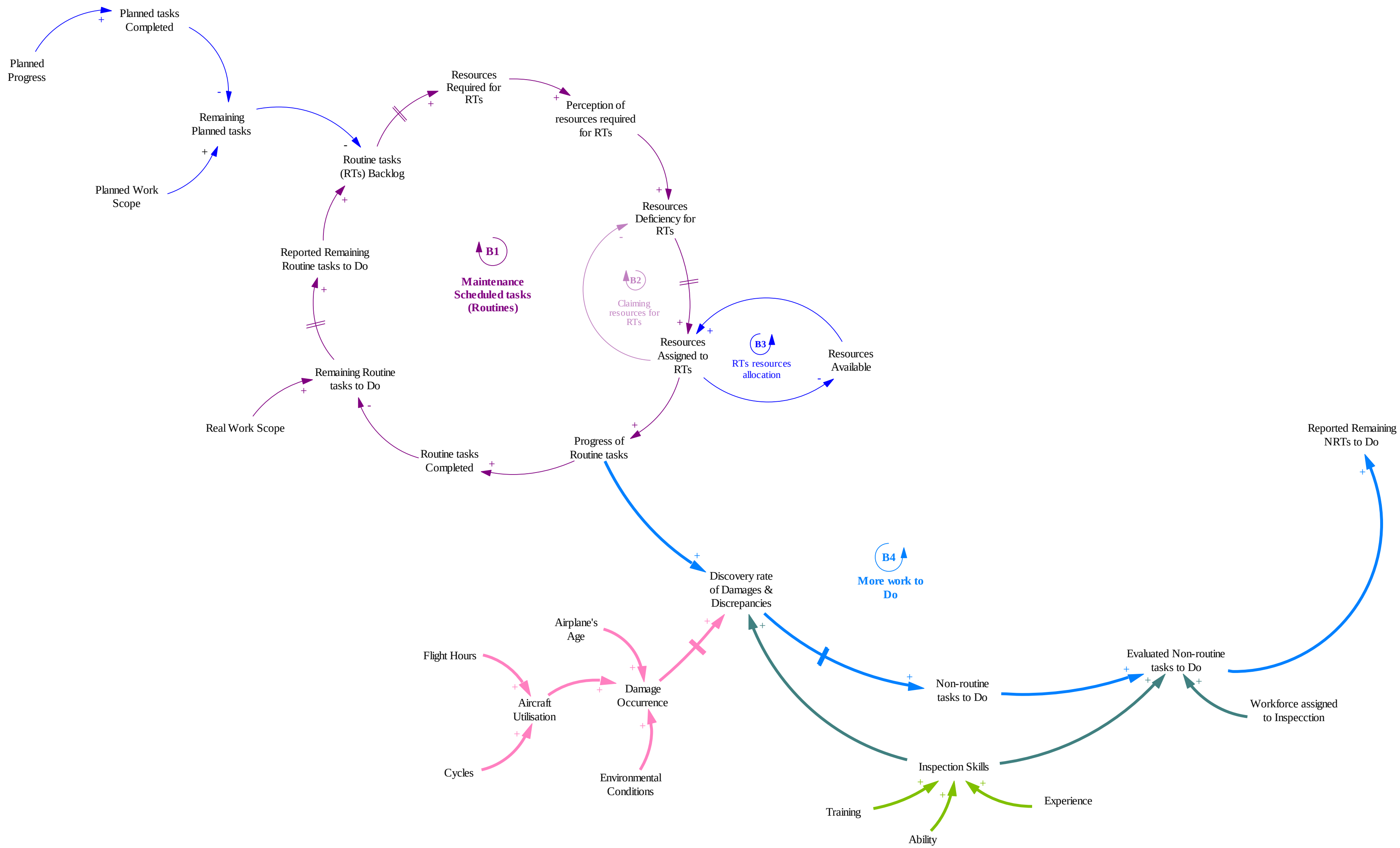


Figure A-2 Occurrence and discovery of discrepancies

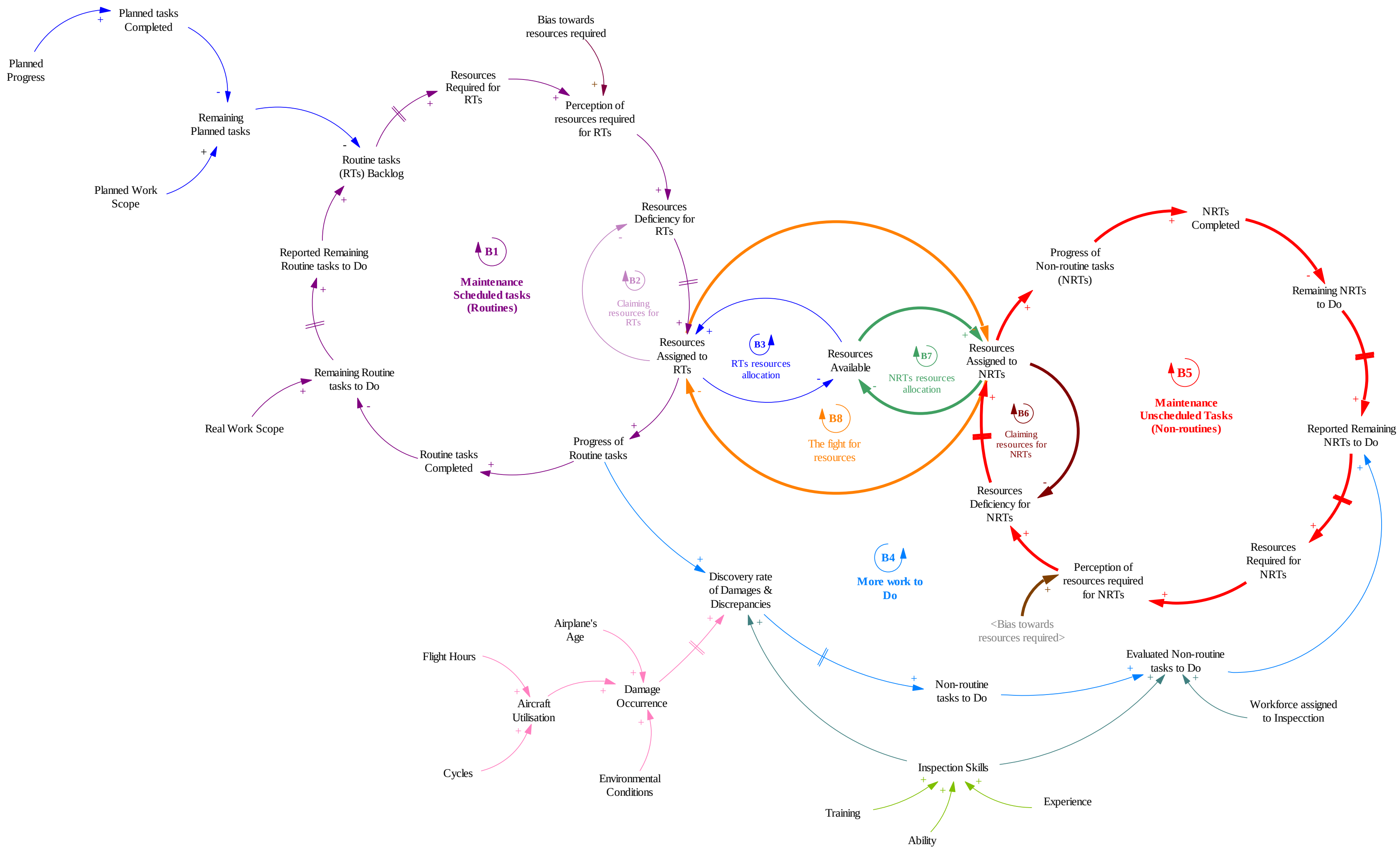


Figure A-3 Unscheduled tasks and the fight for resources

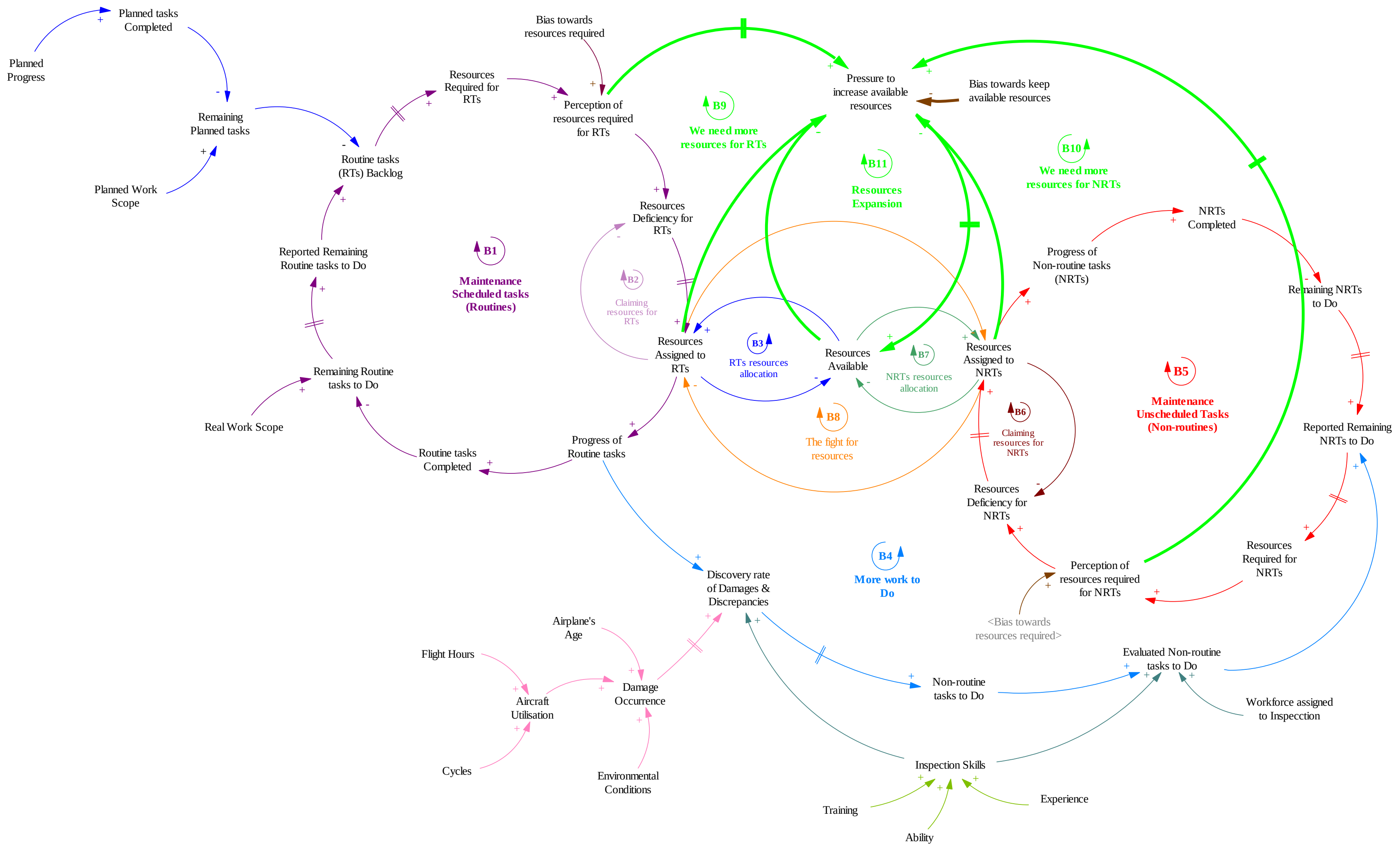


Figure A-4 Increase of available resources

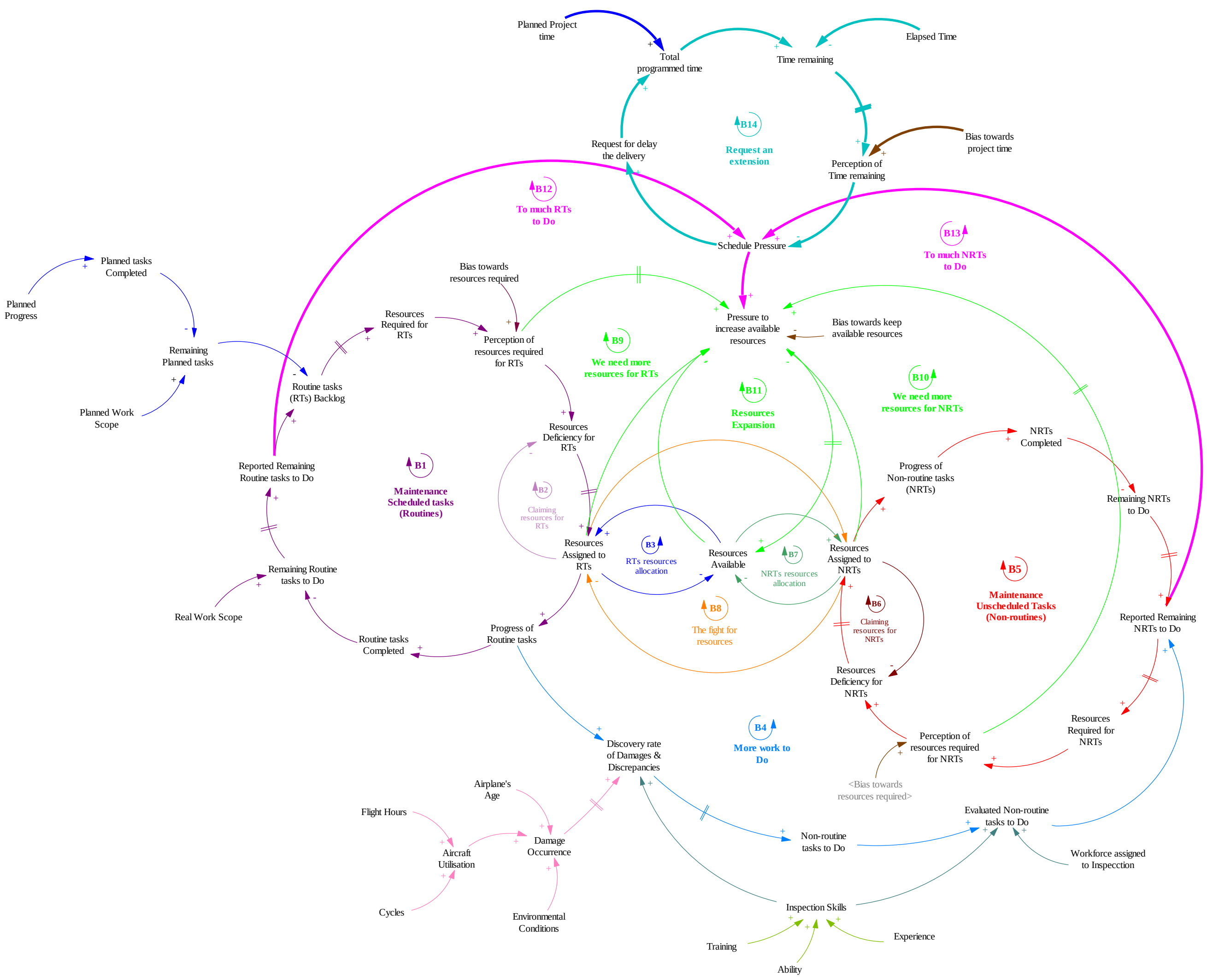


Figure A-5 Extend the project deadline

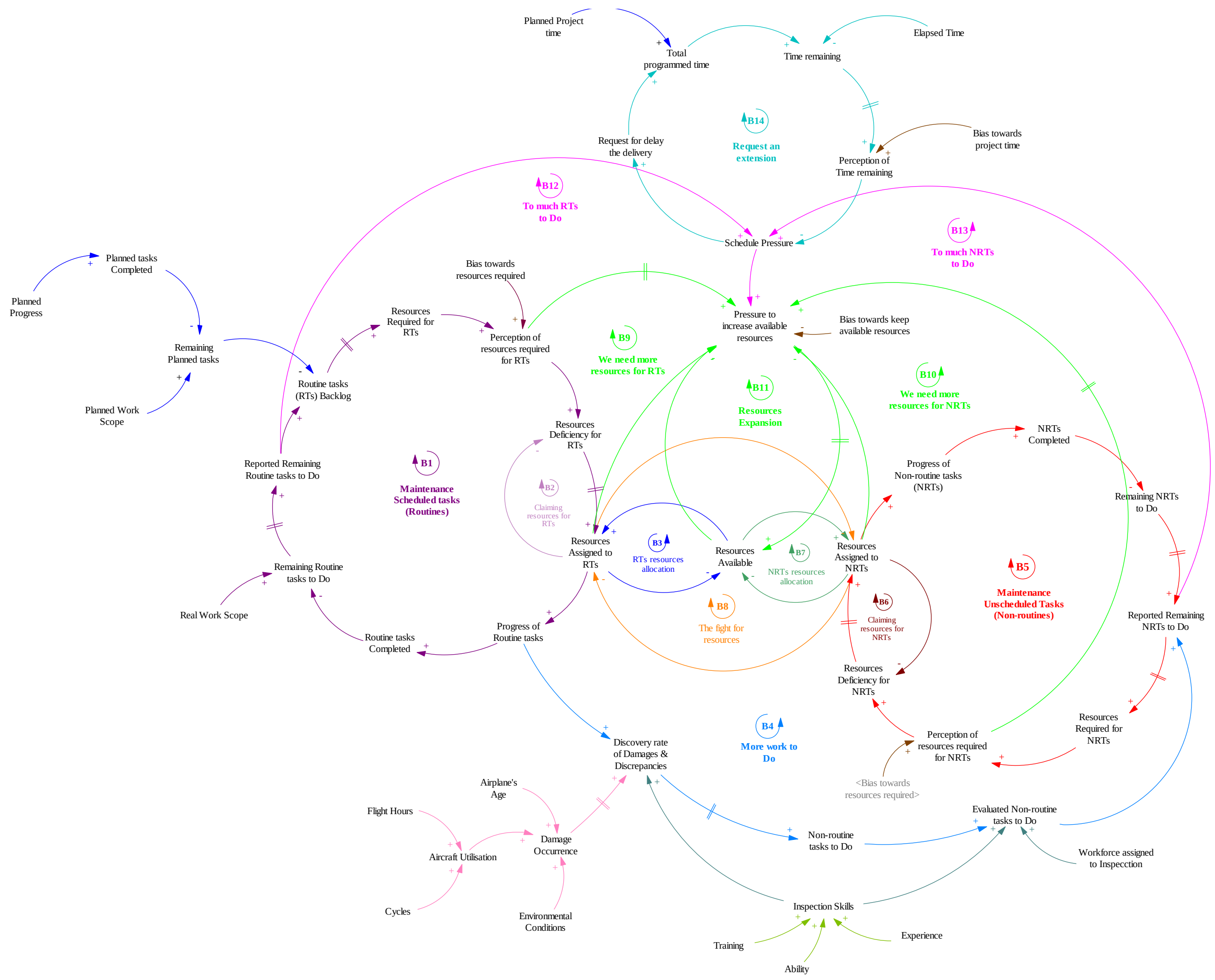


Figure A-6 Delays and disruptions in aircraft heavy maintenance process

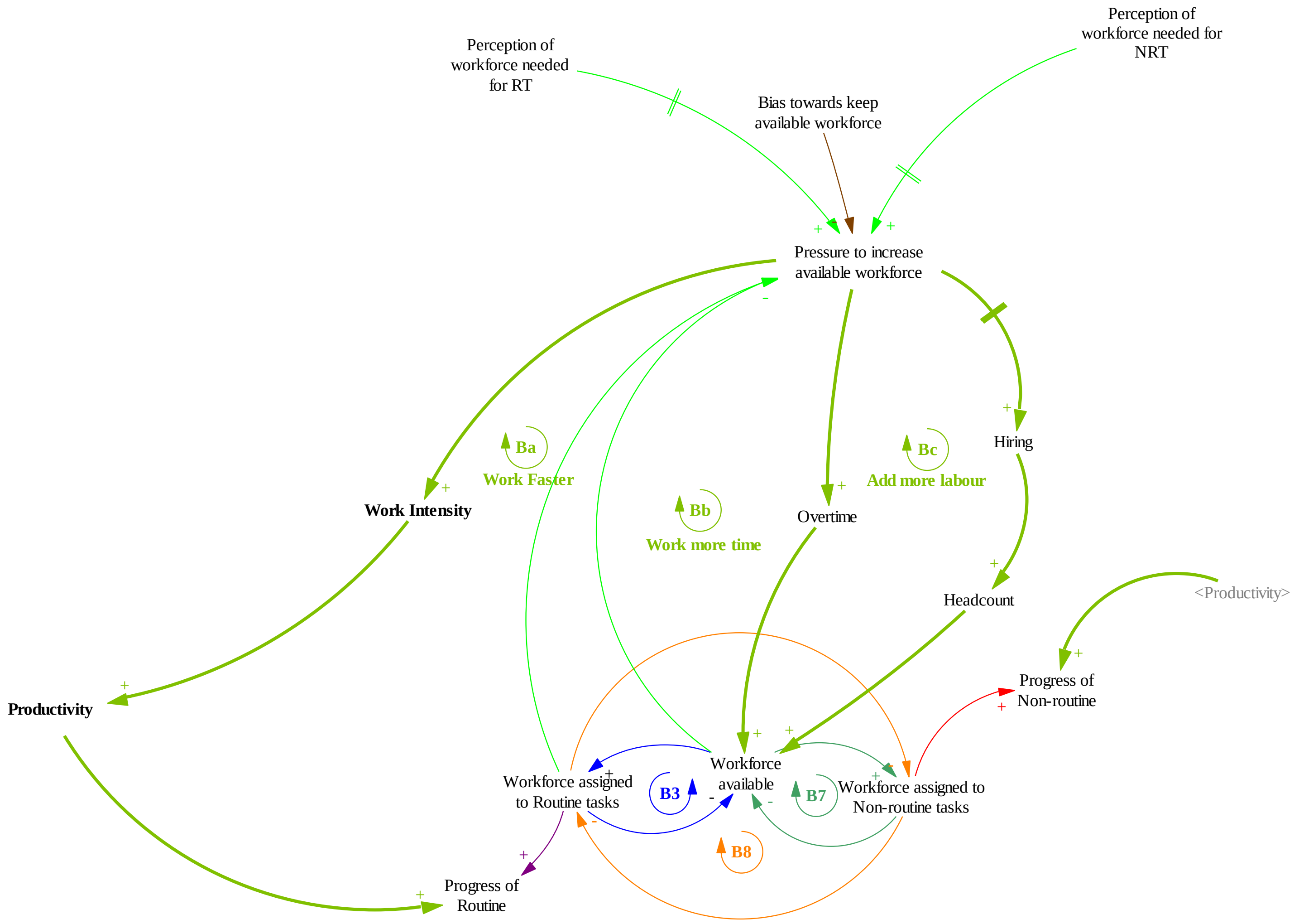


Figure A-7 Ways to increase workforce

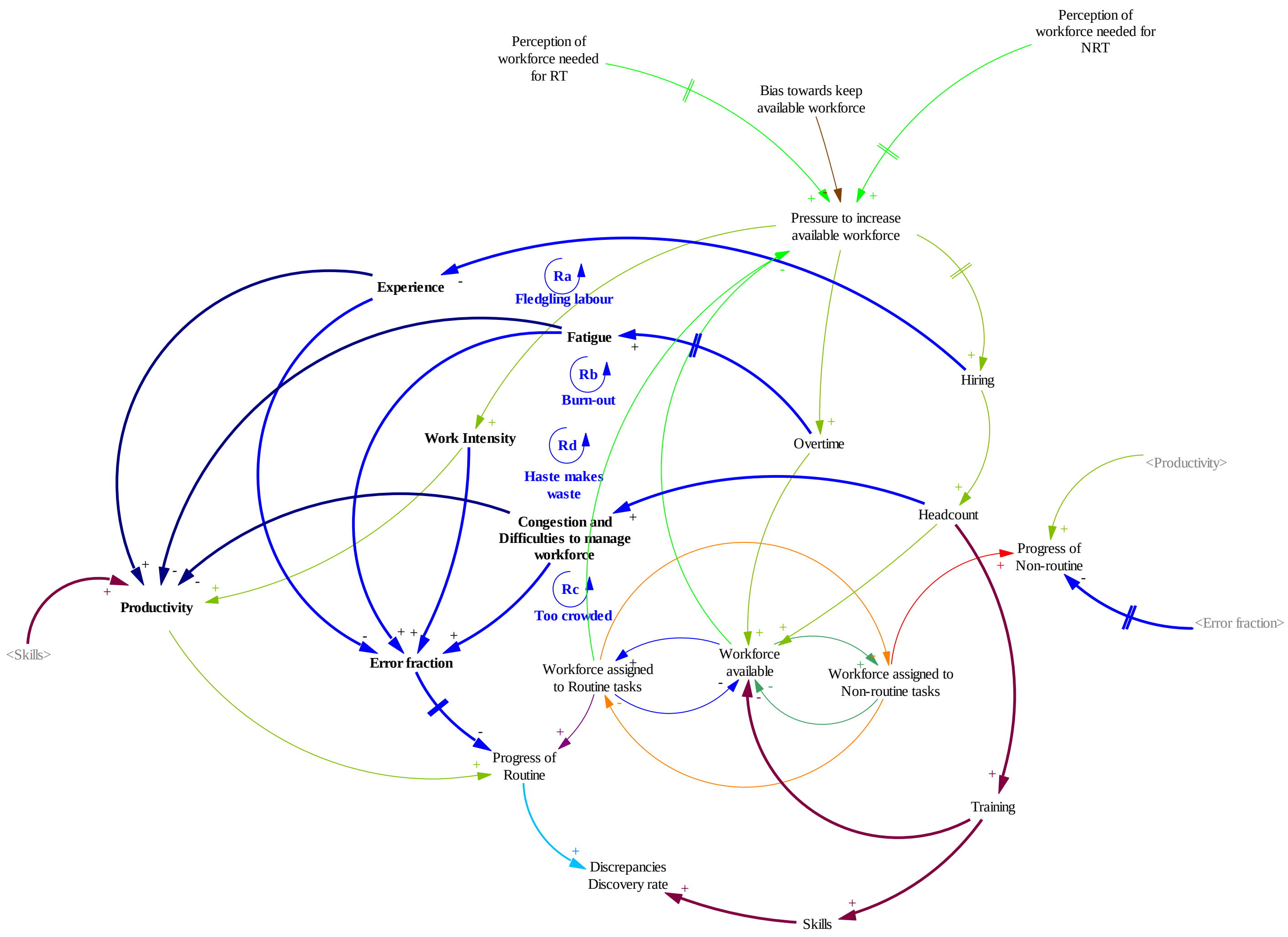


Figure A-8 Ripple and knock-on effects of increasing workforce

Appendix B: ER rule results

B.1 Main data-set

Table B-1 Main variables for building the ER rule model

ID	Groups	Age (Years)	Flight Hours/year	Cycles/year	Service Type	Non-Routine Rate
1	1	5.7	3,870.67	1,359	3C+	0.128
2	1	5.7	3,946.89	1,479	3C	0.155
3	1	10.0	3,294.95	1,704	5C	0.374
4	1	6.2	3,733.39	1,599	2C	0.205
5	1	9.8	3,499.54	1,205	5C+	0.270
6	1	7.8	3,365.26	1,797	4C+	0.272
7	1	10.0	3,346.04	1,664	5C	0.468
8	1	10.0	3,323.99	1,693	5C	0.406
9	1	6.4	3,581.72	1,746	2C	0.140
10	1	6.4	3,761.30	1,615	2C	0.236
11	2	7.8	3,550.80	1,679	4C	0.296
12	2	7.9	3,290.86	1,763	4C	0.387
13	2	10.7	3,206.17	1,750	3C+	0.157
14	2	7.5	3,309.39	1,793	1C	0.194
15	2	7.4	3,331.62	1,681	1C	0.160
16	2	9.4	3,395.93	1,682	2C	0.259
17	2	9.9	3,310.56	1,657	5C	0.392
18	2	5.5	3,607.77	1,753	3C+	0.117
19	2	12.5	2,987.85	1,046	2C	0.269
20	2	9.4	3,410.21	1,672	2C	0.255
21	3	5.7	4,024.75	1,527	3C+	0.149
22	3	9.6	3,702.64	1,273	5C+	0.201
23	3	11.8	2,954.58	1,381	6C+	0.232
24	3	10.6	3,251.48	1,900	3C+	0.177
25	3	7.7	3,321.49	1,746	4C	0.337
26	3	5.9	3,551.08	1,744	3C	0.120
27	3	5.3	3,666.47	1,606	3C	0.143
28	3	7.5	3,306.48	1,653	1C	0.151
29	3	9.3	3,205.10	1,690	2C	0.242

ID	Groups	Age (Years)	Flight Hours/year	Cycles/year	Service Type	Non-Routine Rate
30	3	7.6	3,346.97	1,709	1C	0.162
31	4	7.9	3,318.74	1,805	4C	0.357
32	4	5.6	3,873.21	1,498	3C	0.146
33	4	10.0	3,409.93	1,686	5C	0.244
34	4	7.9	3,508.23	1,814	4C+	0.276
35	4	4.6	3,814.47	1,505	1C	0.106
36	4	9.1	3,334.74	2,044	5C+	0.214
37	4	4.6	3,819.36	1,489	1C	0.121
38	4	7.2	3,290.43	1,759	1C	0.167
39	4	8.0	3,322.63	1,699	4C	0.334
40	4	9.9	3,241.95	1,905	5C	0.308
41	5	7.3	3,285.89	1,581	1C	0.172
42	5	6.0	3,621.21	1,784	3C+	0.153
43	5	5.4	4,467.20	1,919	3C	0.107
44	5	7.9	3,326.12	1,664	4C	0.359
45	5	9.3	3,363.37	1,702	2C	0.250
46	5	10.8	3,232.31	1,715	1C	0.167
47	5	11.5	2,906.01	1,471	1C	0.177
48	5	5.5	3,953.45	1,667	3C	0.129
49	5	6.4	3,612.33	1,730	2C	0.208
50	5	7.9	3,332.48	1,791	4C	0.378
51	6	5.8	3,906.89	1,498	3C	0.193
52	6	11.3	3,869.53	2,635	6C+	0.235
53	6	7.5	3,565.79	1,687	2C	0.250
54	6	7.7	3,299.48	1,769	4C	0.389
55	6	9.7	3,502.16	1,202	5C+	0.241
56	6	7.4	3,255.10	1,759	1C	0.187
57	6	9.6	3,694.48	1,282	5C+	0.253
58	6	6.3	3,754.93	1,625	2C	0.114
59	6	5.7	3,916.89	1,398	3C+	0.110
60	6	7.7	3,579.22	1,703	4C+	0.274
61	7	8.0	3,336.73	1,669	4C	0.375
62	7	5.6	3,574.64	1,639	3C+	0.122
63	7	11.9	2,929.50	1,473	6C+	0.355
64	7	6.1	3,704.26	1,609	2C	0.234

ID	Groups	Age (Years)	Flight Hours/year	Cycles/year	Service Type	Non-Routine Rate
65	7	5.4	3,543.50	1,722	3C	0.153
66	7	5.4	3,620.56	1,729	3C+	0.112
67	7	7.6	3,376.86	1,678	1C	0.195
68	7	10.8	3,875.07	2,686	3C	0.132
69	7	7.8	3,565.96	1,681	4C	0.301
70	7	8.0	3,324.20	1,768	4C+	0.221
71	8	10.4	3,628.94	1,909	3C+	0.126
72	8	8.0	3,280.24	1,779	4C+	0.273
73	8	5.1	3,587.45	1,648	3C+	0.116
74	8	5.2	3,636.00	1,808	3C+	0.116
75	8	5.5	3,889.59	1,383	3C+	0.105
76	8	6.0	3,961.20	1,476	2C	0.218
77	8	9.4	3,282.20	1,690	2C	0.262
78	8	9.3	3,332.37	1,680	2C	0.258
79	8	10.3	2,903.11	1,462	3C+	0.109
80	8	9.4	3,345.00	2,043	5C+	0.270
81	9	9.4	2,416.95	959	2C	0.205
82	9	7.8	3,380.84	1,678	4C	0.253
83	9	6.1	3,746.23	1,540	1C	0.132
84	9	11.9	2,945.41	1,379	6C+	0.211
85	9	7.6	3,559.61	1,684	3C+	0.236
86	9	7.4	3,290.91	1,753	1C	0.160
87	9	9.4	3,357.15	1,690	2C	0.293
88	9	7.9	3,350.86	1,805	4C+	0.275
89	9	7.8	3,346.15	1,712	4C	0.362
90	9	5.3	3,599.66	1,773	3C+	0.096
91	C	9.1	3,295.27	1,600	2C	0.208

B.2 Variables frequency distribution

Table B-2 Aircraft's age frequency distributions considering different bin-widths and interval limits

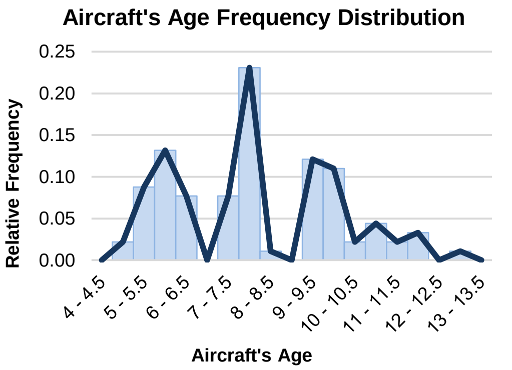


Figure B-9 Aircraft's age frequency distribution with a bin-width of 0.5 years and limits from 4 to 13.5

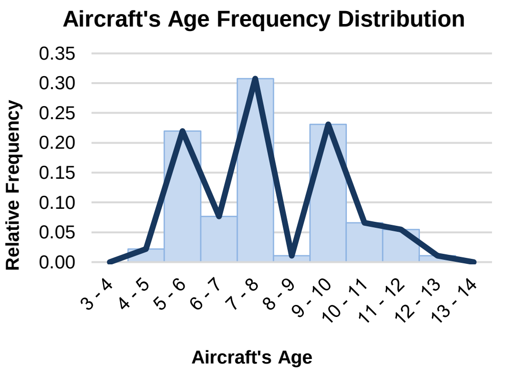


Figure B-10 Aircraft's age frequency distribution with a bin-width of 1 year and limits from 3 to 14

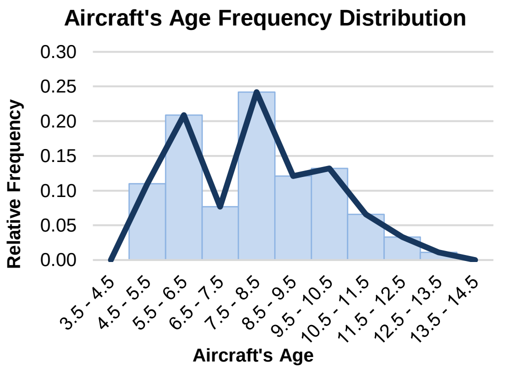


Figure B-11 Aircraft's age frequency distribution with a bin-width of 1 year and limits from 3.5 to 14.5

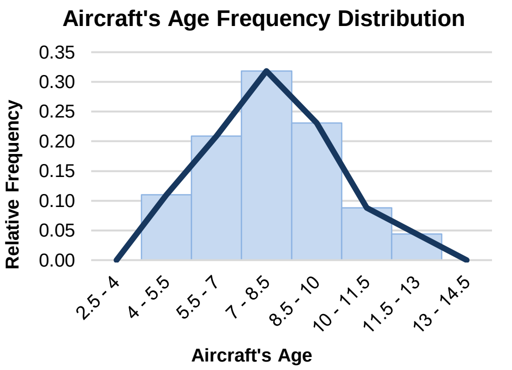


Figure B-12 Aircraft's age frequency distribution with a bin-width of 1.5 years and limits from 2.5 to 14.5

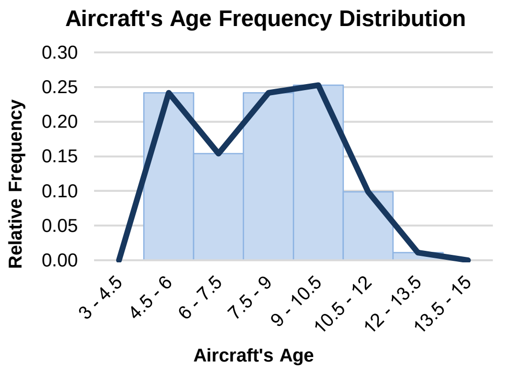


Figure B-13 Aircraft's age frequency distribution with a bin-width of 1.5 years and limits from 3 to 15

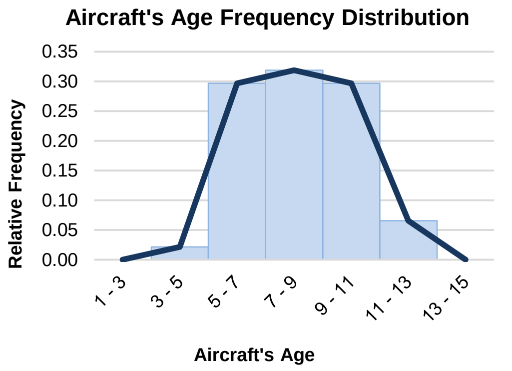


Figure B-14 Aircraft's age frequency distribution with a bin-width of 2 years and limits from 1 to 15

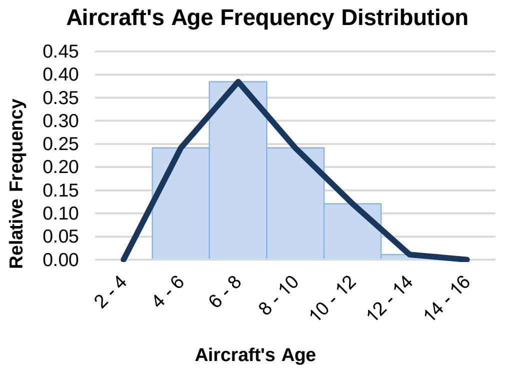


Figure B-15 Aircraft's age frequency distribution with a bin-width of 2 years and limits from 2 to 16

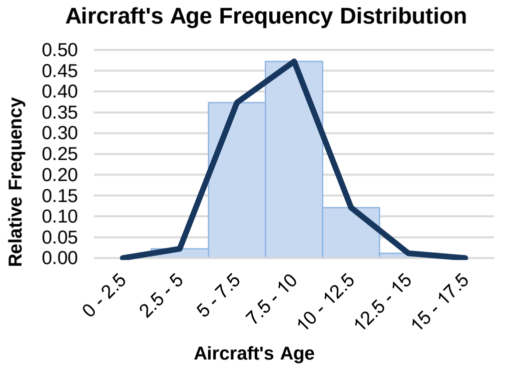


Figure B-16 Aircraft's age frequency distribution with a bin-width of 2.5 years and limits from 0 to 17.5

Table B-3 Flight hours per year frequency distributions considering different bin-widths and interval limits

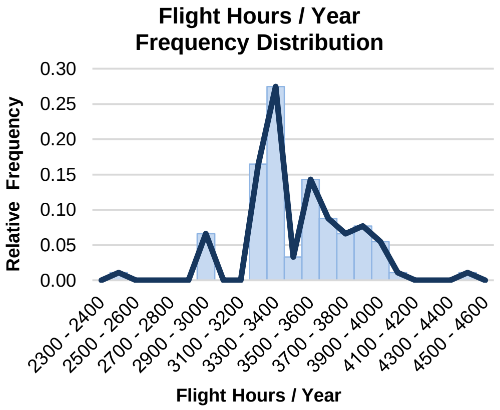


Figure B-17 Flight hours/year frequency distribution: bin-width of 100FH and limits from 2,300 to 4,600

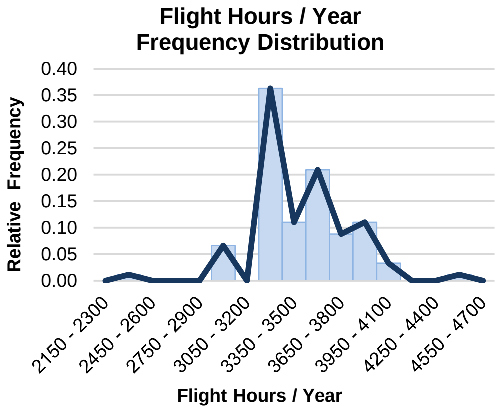


Figure B-18 Flight hours/year frequency distribution: bin-width of 150FH and limits from 2,150 to 4,700

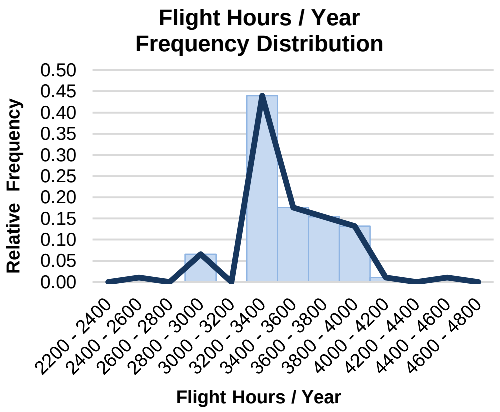


Figure B-19 Flight hours/year frequency distribution: bin-width of 200FH and limits from 2,200 to 4,800

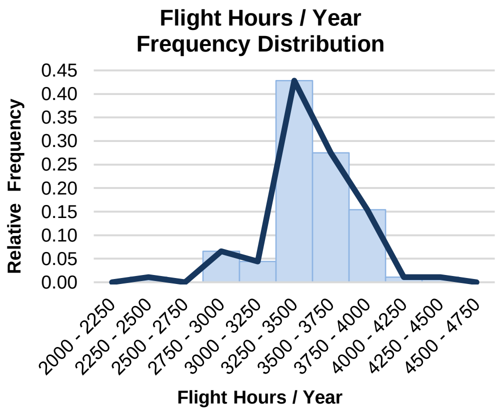


Figure B-20 Flight hours/year frequency distribution: bin-width of 250FH and limits from 2,000 to 4,750

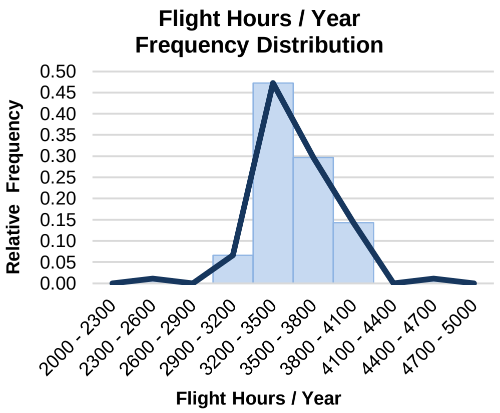


Figure B-21 Flight hours/year frequency distribution: bin-width of 300FH and limits from 2,000 to 5,000

Table B-4 Cycles per year frequency distributions considering different bin-widths and interval limits

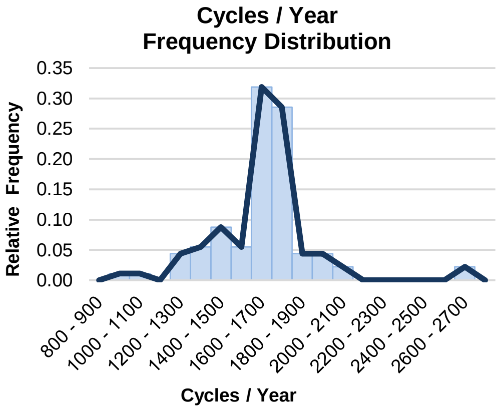


Figure B-22 Cycles/year frequency distribution: bin-width of 100Cy and limits from 800 to 2,700

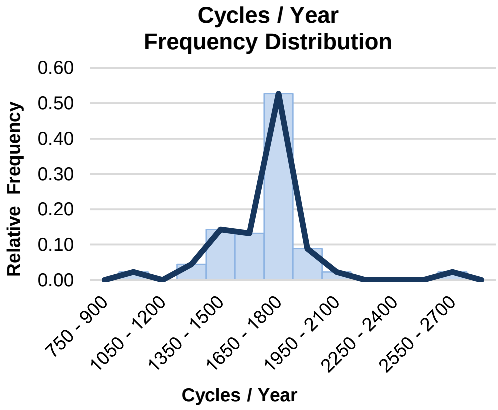


Figure B-23 Cycles/year frequency distribution: bin-width of 150Cy and limits from 750 to 2,700

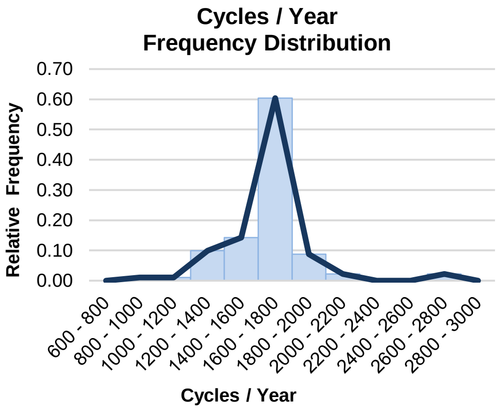


Figure B-24 Cycles/year frequency distribution: bin-width of 200Cy and limits from 600 to 3,000

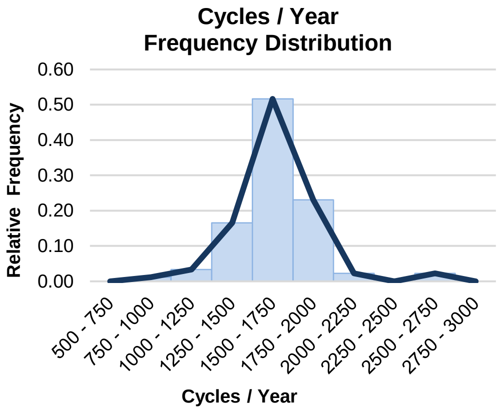


Figure B-25 Cycles/year frequency distribution: bin-width of 250Cy and limits from 500 to 3,000

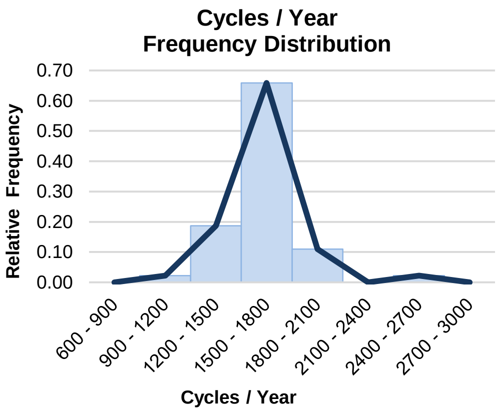


Figure B-26 Cycles/year frequency distribution: bin-width of 300Cy and limits from 600 to 3,000

Table B-5 Non-routine rate frequency distributions considering different bin-widths and interval limits

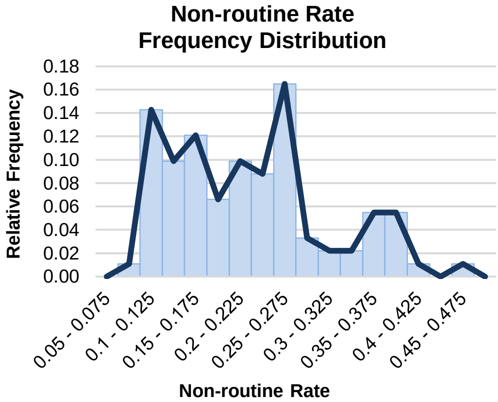


Figure B-27 Non-routine rate frequency distribution: bin-width of 0.025 and limits from 0.05 to 0.475

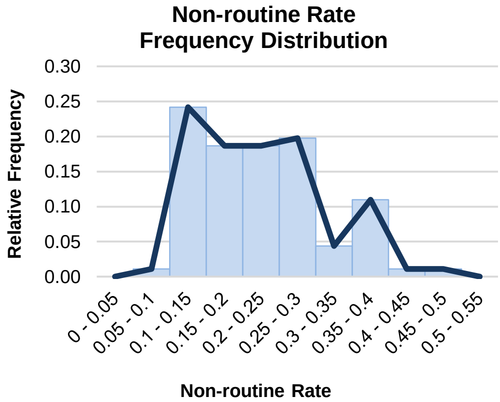


Figure B-28 Non-routine rate frequency distribution: bin-width of 0.05 and limits from 0.0 to 0.55

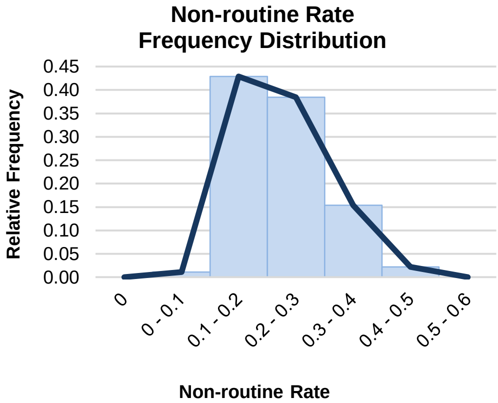


Figure B-29 Non-routine rate frequency distribution: bin-width of 0.10 and limits from 0.0 to 0.60

B.3 Results of the ER rule model example

Table B-6 Comparison between Non-routine rate real distribution and Estimated Non-routine rate belief distribution

Age	Hours	Cycles	Services	Non-routine rate Real distribution					Enrrθ,e(5) Estimated non-routine rate				
				0 - 0.1	***0.1 - 0.2***	***0.2 - 0.3***	***0.3 - 0.4***	***0.4 - 0.5***	***0 - 0.1***	***0.1 - 0.2***	***0.2 - 0.3***	***0.3 - 0.4***	***0.4 - 0.5***
4-6	3500-3750	1500-1750	3C	-	1.000	-	-	-	-	1.000	-	-	-
4-6	3500-3750	1500-1750	3C+	-	1.000	-	-	-	-	0.996	0.004	-	-
4-6	3500-3750	1750-2000	3C+	0.250	0.750	-	-	-	0.638	0.361	0.001	-	-
4-6	3750-4000	1250-1500	1C	-	1.000	-	-	-	-	1.000	-	-	-
4-6	3750-4000	1250-1500	2C	-	-	1.000	-	-	-	0.923	0.077	-	-
4-6	3750-4000	1250-1500	3C	-	1.000	-	-	-	-	1.000	-	-	-
4-6	3750-4000	1250-1500	3C+	-	1.000	-	-	-	-	0.999	0.001	-	-
4-6	3750-4000	1500-1750	1C	-	1.000	-	-	-	-	1.000	-	-	-
4-6	3750-4000	1500-1750	3C	-	1.000	-	-	-	-	1.000	-	-	-
4-6	4000-4250	1500-1750	3C+	-	1.000	-	-	-	-	1.000	-	-	-
4-6	4250-4500	1750-2000	3C	-	1.000	-	-	-	-	1.000	-	-	-
6-8	3250-3500	1500-1750	1C	-	1.000	-	-	-	-	1.000	-	-	-
6-8	3250-3500	1500-1750	4C	-	-	0.200	0.800	-	-	-	0.061	0.939	-
6-8	3250-3500	1750-2000	1C	-	1.000	-	-	-	-	1.000	-	-	-
6-8	3250-3500	1750-2000	4C	-	-	-	1.000	-	-	-	0.028	0.972	-
6-8	3250-3500	1750-2000	4C+	-	-	1.000	-	-	-	-	1.000	-	-
6-8	3500-3750	1500-1750	1C	-	1.000	-	-	-	-	1.000	-	-	-
6-8	3500-3750	1500-1750	2C	-	0.200	0.800	-	-	-	0.082	0.918	-	-
6-8	3500-3750	1500-1750	3C+	-	-	1.000	-	-	-	0.909	0.091	-	-
6-8	3500-3750	1500-1750	4C	-	-	0.500	0.500	-	-	-	0.330	0.670	-
6-8	3500-3750	1500-1750	4C+	-	-	1.000	-	-	-	-	1.000	-	-
6-8	3500-3750	1750-2000	4C+	-	-	1.000	-	-	-	-	1.000	-	-
6-8	3750-4000	1500-1750	2C	-	0.500	0.500	-	-	-	0.231	0.769	-	-

Age	Hours	Cycles	Services	Non-routine rate Real distribution					Enrrθ,e(5) Estimated non-routine rate				
				0 - 0.1	***0.1 - 0.2***	***0.2 - 0.3***	***0.3 - 0.4***	***0.4 - 0.5***	***0 - 0.1***	***0.1 - 0.2***	***0.2 - 0.3***	***0.3 - 0.4***	***0.4 - 0.5***
8-10	2250-2500	750-1000	2C	-	-	1.000	-	-	-	-	1.000	-	-
8-10	3000-3250	1500-1750	2C	-	-	1.000	-	-	-	-	1.000	-	-
8-10	3000-3250	1750-2000	5C	-	-	-	1.000	-	-	-	0.079	0.921	-
8-10	3250-3500	1000-1250	5C+	-	-	1.000	-	-	-	-	1.000	-	-
8-10	3250-3500	1500-1750	2C	-	-	1.000	-	-	-	-	1.000	-	-
8-10	3250-3500	1500-1750	4C	-	-	-	1.000	-	-	-	0.144	0.856	-
8-10	3250-3500	1500-1750	5C	-	-	0.200	0.400	0.400	-	-	0.043	0.154	0.802
8-10	3250-3500	2000-2250	5C+	-	-	1.000	-	-	-	-	1.000	-	-
8-10	3500-3750	1000-1250	5C+	-	-	1.000	-	-	-	-	1.000	-	-
8-10	3500-3750	1250-1500	5C+	-	-	1.000	-	-	-	-	1.000	-	-
10-12	2750-3000	1250-1500	1C	-	1.000	-	-	-	-	1.000	-	-	-
10-12	2750-3000	1250-1500	3C+	-	1.000	-	-	-	-	0.966	0.034	-	-
10-12	2750-3000	1250-1500	6C	-	-	0.667	0.333	-	-	-	0.896	0.104	-
10-12	3000-3250	1500-1750	1C	-	1.000	-	-	-	-	1.000	-	-	-
10-12	3000-3250	1500-1750	3C+	-	1.000	-	-	-	-	0.980	0.020	-	-
10-12	3250-3500	1750-2000	3C+	-	1.000	-	-	-	-	0.967	0.033	-	-
10-12	3500-3750	1750-2000	3C+	-	1.000	-	-	-	-	0.981	0.019	-	-
10-12	3750-4000	2500-2750	3C	-	1.000	-	-	-	-	1.000	-	-	-
10-12	3750-4000	2500-2750	6C	-	-	1.000	-	-	-	-	1.000	-	-
12-14	2750-3000	1000-1250	2C	-	-	1.000	-	-	-	-	1.000	-	-

Table B-7 Comparison between Real Non-routine rate and Estimated average Non-routine rate

Age (Years)		Flight Hours / Year		Cycles / Year		Service Type	Real Non-Routine Rate	Estimated Average Non-Routine Rate
Value	Interval	Value	Interval	Value	Interval			
5.7	4-6	3,870.67	3750-4000	1,359	1250-1500	3C+	0.128	0.150
5.7	4-6	3,946.89	3750-4000	1,479	1250-1500	3C	0.155	0.150
10.0	8-10	3,294.95	3250-3500	1,704	1500-1750	5C	0.374	0.426
6.2	6-8	3,733.39	3500-3750	1,599	1500-1750	2C	0.205	0.242
9.8	8-10	3,499.54	3250-3500	1,205	1000-1250	5C+	0.270	0.250
7.8	6-8	3,365.26	3250-3500	1,797	1750-2000	4C+	0.272	0.250
10.0	8-10	3,346.04	3250-3500	1,664	1500-1750	5C	0.468	0.426
10.0	8-10	3,323.99	3250-3500	1,693	1500-1750	5C	0.406	0.426
6.4	6-8	3,581.72	3500-3750	1,746	1500-1750	2C	0.140	0.242
6.4	6-8	3,761.30	3750-4000	1,615	1500-1750	2C	0.236	0.227
7.8	6-8	3,550.80	3500-3750	1,679	1500-1750	4C	0.296	0.317
7.9	6-8	3,290.86	3250-3500	1,763	1750-2000	4C	0.387	0.347
10.7	10-12	3,206.17	3000-3250	1,750	1500-1750	3C+	0.157	0.152
7.5	6-8	3,309.39	3250-3500	1,793	1750-2000	1C	0.194	0.150
7.4	6-8	3,331.62	3250-3500	1,681	1500-1750	1C	0.160	0.150
9.4	8-10	3,395.93	3250-3500	1,682	1500-1750	2C	0.259	0.250
9.9	8-10	3,310.56	3250-3500	1,657	1500-1750	5C	0.392	0.426
5.5	4-6	3,607.77	3500-3750	1,753	1750-2000	3C+	0.117	0.086
12.5	12-14	2,987.85	2750-3000	1,046	1000-1250	2C	0.269	0.250
9.4	8-10	3,410.21	3250-3500	1,672	1500-1750	2C	0.255	0.250
7.9	6-8	3,318.74	3250-3500	1,805	1750-2000	4C	0.357	0.347
5.6	4-6	3,873.21	3750-4000	1,498	1250-1500	3C	0.146	0.150
10.0	8-10	3,409.93	3250-3500	1,686	1500-1750	5C	0.244	0.426
7.9	6-8	3,508.23	3500-3750	1,814	1750-2000	4C+	0.276	0.250
4.6	4-6	3,814.47	3750-4000	1,505	1500-1750	1C	0.106	0.150
9.1	8-10	3,334.74	3250-3500	2,044	2000-2250	5C+	0.214	0.250
4.6	4-6	3,819.36	3750-4000	1,489	1250-1500	1C	0.121	0.150
7.2	6-8	3,290.43	3250-3500	1,759	1750-2000	1C	0.167	0.150
8.0	8-10	3,322.63	3250-3500	1,699	1500-1750	4C	0.334	0.336
9.9	8-10	3,241.95	3000-3250	1,905	1750-2000	5C	0.308	0.342

Age (Years)		Flight Hours / Year		Cycles / Year		Service Type	Real Non-Routine Rate	Estimated Average Non-Routine Rate
Value	Interval	Value	Interval	Value	Interval			
7.3	6-8	3,285.89	3250-3500	1,581	1500-1750	1C	0.172	0.150
6.0	4-6	3,621.21	3500-3750	1,784	1750-2000	3C+	0.153	0.086
5.4	4-6	4,467.20	4250-4500	1,919	1750-2000	3C	0.107	0.150
7.9	6-8	3,326.12	3250-3500	1,664	1500-1750	4C	0.359	0.344
9.3	8-10	3,363.37	3250-3500	1,702	1500-1750	2C	0.250	0.250
10.8	10-12	3,232.31	3000-3250	1,715	1500-1750	1C	0.167	0.150
11.5	10-12	2,906.01	2750-3000	1,471	1250-1500	1C	0.177	0.150
5.5	4-6	3,953.45	3750-4000	1,667	1500-1750	3C	0.129	0.150
6.4	6-8	3,612.33	3500-3750	1,730	1500-1750	2C	0.208	0.242
7.9	6-8	3,332.48	3250-3500	1,791	1750-2000	4C	0.378	0.347
5.8	4-6	3,906.89	3750-4000	1,498	1250-1500	3C	0.193	0.150
11.3	10-12	3,869.53	3750-4000	2,635	2500-2750	6C	0.235	0.250
7.5	6-8	3,565.79	3500-3750	1,687	1500-1750	2C	0.250	0.242
7.7	6-8	3,299.48	3250-3500	1,769	1750-2000	4C	0.389	0.347
9.7	8-10	3,502.16	3500-3750	1,202	1000-1250	5C+	0.241	0.250
7.4	6-8	3,255.10	3250-3500	1,759	1750-2000	1C	0.187	0.150
9.6	8-10	3,694.48	3500-3750	1,282	1250-1500	5C+	0.253	0.250
6.3	6-8	3,754.93	3750-4000	1,625	1500-1750	2C	0.114	0.227
5.7	4-6	3,916.89	3750-4000	1,398	1250-1500	3C+	0.110	0.150
7.7	6-8	3,579.22	3500-3750	1,703	1500-1750	4C+	0.274	0.250
8.0	6-8	3,336.73	3250-3500	1,669	1500-1750	4C	0.375	0.344
5.6	4-6	3,574.64	3500-3750	1,639	1500-1750	3C+	0.122	0.150
11.9	10-12	2,929.50	2750-3000	1,473	1250-1500	6C	0.355	0.260
6.1	6-8	3,704.26	3500-3750	1,609	1500-1750	2C	0.234	0.242
5.4	4-6	3,543.50	3500-3750	1,722	1500-1750	3C	0.153	0.150
5.4	4-6	3,620.56	3500-3750	1,729	1500-1750	3C+	0.112	0.150
7.6	6-8	3,376.86	3250-3500	1,678	1500-1750	1C	0.195	0.150
10.8	10-12	3,875.07	3750-4000	2,686	2500-2750	3C	0.132	0.150
7.8	6-8	3,565.96	3500-3750	1,681	1500-1750	4C	0.301	0.317
8.0	6-8	3,324.20	3250-3500	1,768	1750-2000	4C+	0.221	0.250
10.4	10-12	3,628.94	3500-3750	1,909	1750-2000	3C+	0.126	0.152
8.0	6-8	3,280.24	3250-3500	1,779	1750-2000	4C+	0.273	0.250

Age (Years)		Flight Hours / Year		Cycles / Year		Service Type	Real Non-Routine Rate	Estimated Average Non-Routine Rate
Value	Interval	Value	Interval	Value	Interval			
5.1	4-6	3,587.45	3500-3750	1,648	1500-1750	3C+	0.116	0.150
5.2	4-6	3,636.00	3500-3750	1,808	1750-2000	3C+	0.116	0.086
5.5	4-6	3,889.59	3750-4000	1,383	1250-1500	3C+	0.105	0.150
6.0	4-6	3,961.20	3750-4000	1,476	1250-1500	2C	0.218	0.158
9.4	8-10	3,282.20	3250-3500	1,690	1500-1750	2C	0.262	0.250
9.3	8-10	3,332.37	3250-3500	1,680	1500-1750	2C	0.258	0.250
10.3	10-12	2,903.11	2750-3000	1,462	1250-1500	3C+	0.109	0.153
9.4	8-10	3,345.00	3250-3500	2,043	2000-2250	5C+	0.270	0.250
9.4	8-10	2,416.95	2250-2500	959	750-1000	2C	0.205	0.250
7.8	6-8	3,380.84	3250-3500	1,678	1500-1750	4C	0.253	0.344
6.1	6-8	3,746.23	3500-3750	1,540	1500-1750	1C	0.132	0.150
11.9	10-12	2,945.41	2750-3000	1,379	1250-1500	6C	0.211	0.260
7.6	6-8	3,559.61	3500-3750	1,684	1500-1750	3C+	0.236	0.159
7.4	6-8	3,290.91	3250-3500	1,753	1750-2000	1C	0.160	0.150
9.4	8-10	3,357.15	3250-3500	1,690	1500-1750	2C	0.293	0.250
7.9	6-8	3,350.86	3250-3500	1,805	1750-2000	4C+	0.275	0.250
7.8	6-8	3,346.15	3250-3500	1,712	1500-1750	4C	0.362	0.344
5.3	4-6	3,599.66	3500-3750	1,773	1750-2000	3C+	0.096	0.086
5.7	4-6	4,024.75	4000-4250	1,527	1500-1750	3C+	0.149	0.150
9.6	8-10	3,702.64	3500-3750	1,273	1250-1500	5C+	0.201	0.250
11.8	10-12	2,954.58	2750-3000	1,381	1250-1500	6C	0.232	0.260
10.6	10-12	3,251.48	3250-3500	1,900	1750-2000	3C+	0.177	0.153
7.7	6-8	3,321.49	3250-3500	1,746	1500-1750	4C	0.337	0.344
5.9	4-6	3,551.08	3500-3750	1,744	1500-1750	3C	0.120	0.150
5.3	4-6	3,666.47	3500-3750	1,606	1500-1750	3C	0.143	0.150
7.5	6-8	3,306.48	3250-3500	1,653	1500-1750	1C	0.151	0.150
9.3	8-10	3,205.10	3000-3250	1,690	1500-1750	2C	0.242	0.250
7.6	6-8	3,346.97	3250-3500	1,709	1500-1750	1C	0.162	0.150
9.1	8-10	3,295.27	3250-3500	1,600	1500-1750	2C	0.208	0.250

B.4 Bin size and interval limits analysis

Table B-8 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.05 for non-routine rate and 0.5 for aircraft's age, with 4.5 and 13.0 years as lower and upper limits.

Bin-width		Model Performance Indicators				
Fh / Y	**Cy / Y**	**MSE_{DIST}**	**MSE_{SERV}**	**MAE**	**MAPE**	**MAI**
100	100	0.0211	0.0008	0.0196	10.1%	95.4%
100	150	0.0241	0.0009	0.0206	10.6%	95.1%
100	200	0.0281	0.0009	0.0207	10.7%	95.1%
100	250	0.0257	0.0009	0.0209	10.7%	95.1%
100	300	0.0264	0.0009	0.0209	10.7%	95.1%
150	100	0.0175	0.0007	0.0188	10.0%	95.6%
150	150	0.0193	0.0008	0.0199	10.3%	95.3%
150	200	0.0206	0.0008	0.0198	10.3%	95.3%
150	250	0.0218	0.0008	0.0204	10.6%	95.2%
150	300	0.0177	0.0008	0.0198	10.2%	95.3%
200	100	0.0207	0.0009	0.0202	10.4%	95.3%
200	150	0.0244	0.0010	0.0217	10.9%	94.9%
200	200	0.0285	0.0010	0.0216	11.0%	94.9%
200	250	0.0253	0.0010	0.0217	10.9%	94.9%
200	300	0.0261	0.0010	0.0216	10.9%	94.9%
250	100	0.0176	0.0012	0.0219	11.2%	94.9%
250	150	0.0191	0.0013	0.0227	11.4%	94.6%
250	200	0.0179	0.0013	0.0228	11.4%	94.6%
250	250	0.0166	0.0013	0.0230	11.5%	94.6%
250	300	0.0147	0.0013	0.0228	11.3%	94.6%
300	100	0.0184	0.0012	0.0219	11.2%	94.8%
300	150	0.0209	0.0013	0.0231	11.5%	94.6%
300	200	0.0199	0.0013	0.0231	11.5%	94.6%
300	250	0.0199	0.0014	0.0234	11.7%	94.5%
300	300	0.0170	0.0013	0.0231	11.4%	94.6%

Table B-9 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.05 for non-routine rate and 1.0 for aircraft's age, with 4.0 and 13.0 years as lower and upper limits.

Bin-width		**Model Performance Indicators**				
Fh / Y	**Cy / Y**	**MSE_{DIST}**	**MSE_{SERV}**	**MAE**	**MAPE**	**MAI**
100	100	0.0193	0.0008	0.0199	10.3%	95.3%
100	150	0.0211	0.0009	0.0206	10.6%	95.1%
100	200	0.0252	0.0009	0.0210	10.8%	95.1%
100	250	0.0236	0.0009	0.0211	10.7%	95.0%
100	300	0.0230	0.0009	0.0209	10.8%	95.1%
150	100	0.0153	0.0008	0.0194	10.3%	95.4%
150	150	0.0157	0.0008	0.0200	10.3%	95.3%
150	200	0.0193	0.0008	0.0200	10.4%	95.3%
150	250	0.0192	0.0009	0.0208	10.8%	95.1%
150	300	0.0158	0.0008	0.0199	10.3%	95.3%
200	100	0.0181	0.0009	0.0202	10.5%	95.2%
200	150	0.0203	0.0010	0.0214	10.8%	95.0%
200	200	0.0246	0.0010	0.0216	11.0%	94.9%
200	250	0.0221	0.0010	0.0214	10.8%	95.0%
200	300	0.0215	0.0010	0.0214	10.8%	95.0%
250	100	0.0148	0.0013	0.0222	11.3%	94.8%
250	150	0.0145	0.0013	0.0228	11.4%	94.6%
250	200	0.0187	0.0013	0.0228	11.4%	94.6%
250	250	0.0159	0.0014	0.0230	11.6%	94.6%
250	300	0.0152	0.0013	0.0226	11.3%	94.7%
300	100	0.0152	0.0013	0.0221	11.3%	94.8%
300	150	0.0161	0.0014	0.0231	11.5%	94.6%
300	200	0.0208	0.0014	0.0230	11.5%	94.6%
300	250	0.0193	0.0014	0.0234	11.7%	94.5%
300	300	0.0176	0.0014	0.0228	11.3%	94.6%

Table B-10 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.05 for non-routine rate and 1.0 for aircraft's age, with 4.5 and 13.5 years as lower and upper limits.

Bin-width		**Model Performance Indicators**				
Fh / Y	**Cy / Y**	**MSE_{DIST}**	**MSE_{SERV}**	**MAE**	**MAPE**	**MAI**
100	100	0.0208	0.0008	0.0198	10.2%	95.3%
100	150	0.0238	0.0009	0.0211	10.7%	95.0%
100	200	0.0280	0.0009	0.0212	10.8%	95.0%
100	250	0.0255	0.0009	0.0213	10.8%	95.0%
100	300	0.0264	0.0009	0.0213	10.9%	95.0%
150	100	0.0169	0.0007	0.0195	10.2%	95.4%
150	150	0.0187	0.0008	0.0204	10.4%	95.2%
150	200	0.0201	0.0008	0.0204	10.5%	95.2%
150	250	0.0212	0.0008	0.0209	10.8%	95.1%
150	300	0.0175	0.0008	0.0204	10.5%	95.2%
200	100	0.0205	0.0008	0.0210	10.7%	95.1%
200	150	0.0236	0.0009	0.0224	11.1%	94.7%
200	200	0.0284	0.0010	0.0225	11.3%	94.7%
200	250	0.0242	0.0010	0.0224	11.2%	94.7%
200	300	0.0253	0.0010	0.0225	11.3%	94.7%
250	100	0.0176	0.0012	0.0226	11.4%	94.7%
250	150	0.0187	0.0013	0.0235	11.5%	94.5%
250	200	0.0181	0.0013	0.0237	11.7%	94.4%
250	250	0.0160	0.0013	0.0238	11.9%	94.4%
250	300	0.0146	0.0013	0.0238	11.7%	94.4%
300	100	0.0185	0.0012	0.0226	11.5%	94.7%
300	150	0.0204	0.0013	0.0239	11.7%	94.4%
300	200	0.0207	0.0013	0.0240	11.8%	94.4%
300	250	0.0196	0.0013	0.0241	12.0%	94.3%
300	300	0.0171	0.0013	0.0240	11.9%	94.3%

Table B-11 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.05 for non-routine rate and 1.5 for aircraft's age, with 4.0 and 13.0 years as lower and upper limits.

Bin-width		**Model Performance Indicators**				
Fh / Y	**Cy / Y**	**MSE_{DIST}**	**MSE_{SERV}**	**MAE**	**MAPE**	**MAI**
100	100	0.0224	0.0008	0.0199	10.3%	95.3%
100	150	0.0270	0.0009	0.0213	10.8%	95.0%
100	200	0.0288	0.0009	0.0212	10.8%	95.0%
100	250	0.0289	0.0009	0.0215	10.9%	94.9%
100	300	0.0295	0.0009	0.0214	10.9%	95.0%
150	100	0.0193	0.0007	0.0197	10.4%	95.4%
150	150	0.0229	0.0008	0.0207	10.5%	95.1%
150	200	0.0217	0.0008	0.0204	10.5%	95.2%
150	250	0.0263	0.0008	0.0212	10.9%	95.0%
150	300	0.0215	0.0008	0.0204	10.4%	95.2%
200	100	0.0220	0.0008	0.0208	10.7%	95.1%
200	150	0.0270	0.0009	0.0225	11.1%	94.7%
200	200	0.0292	0.0009	0.0222	11.2%	94.8%
200	250	0.0281	0.0009	0.0223	11.1%	94.8%
200	300	0.0288	0.0009	0.0222	11.1%	94.8%
250	100	0.0200	0.0012	0.0226	11.5%	94.7%
250	150	0.0230	0.0013	0.0237	11.6%	94.4%
250	200	0.0196	0.0013	0.0234	11.6%	94.5%
250	250	0.0214	0.0013	0.0238	11.9%	94.4%
250	300	0.0190	0.0013	0.0235	11.6%	94.5%
300	100	0.0208	0.0012	0.0226	11.5%	94.7%
300	150	0.0248	0.0013	0.0240	11.8%	94.3%
300	200	0.0222	0.0013	0.0236	11.7%	94.4%
300	250	0.0252	0.0013	0.0241	12.0%	94.3%
300	300	0.0217	0.0013	0.0237	11.7%	94.4%

Table B-12 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.05 for non-routine rate and 1.5 for aircraft's age, with 4.5 and 13.5 years as lower and upper limits.

Bin-width		Model Performance Indicators				
Fh / Y	Cy / Y	MSE_{DIST}	MSE_{SERV}	MAE	MAPE	MAI
100	100	0.0198	0.0009	0.0202	10.6%	95.2%
100	150	0.0204	0.0010	0.0215	11.1%	95.0%
100	200	0.0239	0.0010	0.0215	11.1%	94.9%
100	250	0.0226	0.0010	0.0218	11.2%	94.9%
100	300	0.0226	0.0010	0.0219	11.4%	94.8%
150	100	0.0165	0.0008	0.0203	10.8%	95.2%
150	150	0.0165	0.0009	0.0214	11.2%	95.0%
150	200	0.0196	0.0009	0.0212	11.2%	95.0%
150	250	0.0197	0.0010	0.0221	11.7%	94.8%
150	300	0.0178	0.0009	0.0215	11.4%	94.9%
200	100	0.0191	0.0009	0.0210	11.1%	95.1%
200	150	0.0187	0.0011	0.0225	11.5%	94.7%
200	200	0.0240	0.0011	0.0227	11.8%	94.7%
200	250	0.0221	0.0011	0.0228	11.8%	94.6%
200	300	0.0234	0.0011	0.0231	12.1%	94.6%
250	100	0.0152	0.0013	0.0230	11.9%	94.6%
250	150	0.0152	0.0014	0.0240	12.1%	94.4%
250	200	0.0208	0.0014	0.0240	12.2%	94.4%
250	250	0.0184	0.0014	0.0245	12.6%	94.2%
250	300	0.0198	0.0014	0.0244	12.5%	94.3%
300	100	0.0152	0.0013	0.0230	11.9%	94.6%
300	150	0.0167	0.0014	0.0243	12.2%	94.3%
300	200	0.0228	0.0014	0.0242	12.3%	94.3%
300	250	0.0216	0.0015	0.0248	12.8%	94.2%
300	300	0.0222	0.0015	0.0246	12.7%	94.2%

Table B-13 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.05 for non-routine rate and 2.0 for aircraft's age, with 3.0 and 13.0 years as lower and upper limits.

Bin-width		**Model Performance Indicators**				
Fh / Y	**Cy / Y**	**MSE_{DIST}**	**MSE_{SERV}**	**MAE**	**MAPE**	**MAI**
100	100	0.0192	0.0008	0.0202	10.4%	95.2%
100	150	0.0237	0.0009	0.0216	10.9%	94.9%
100	200	0.0253	0.0009	0.0216	11.0%	94.9%
100	250	0.0257	0.0009	0.0218	11.0%	94.9%
100	300	0.0261	0.0009	0.0219	11.1%	94.8%
150	100	0.0148	0.0007	0.0200	10.4%	95.3%
150	150	0.0182	0.0008	0.0211	10.6%	95.0%
150	200	0.0193	0.0008	0.0208	10.6%	95.1%
150	250	0.0213	0.0009	0.0217	11.0%	94.9%
150	300	0.0191	0.0008	0.0209	10.6%	95.1%
200	100	0.0181	0.0008	0.0211	10.7%	95.0%
200	150	0.0230	0.0010	0.0228	11.2%	94.6%
200	200	0.0247	0.0010	0.0225	11.2%	94.7%
200	250	0.0236	0.0010	0.0227	11.3%	94.7%
200	300	0.0241	0.0010	0.0229	11.4%	94.6%
250	100	0.0154	0.0013	0.0231	11.5%	94.6%
250	150	0.0184	0.0014	0.0242	11.8%	94.3%
250	200	0.0201	0.0013	0.0238	11.6%	94.4%
250	250	0.0191	0.0014	0.0244	12.1%	94.3%
250	300	0.0199	0.0014	0.0241	11.8%	94.3%
300	100	0.0157	0.0013	0.0230	11.5%	94.6%
300	150	0.0197	0.0014	0.0245	11.9%	94.2%
300	200	0.0228	0.0013	0.0240	11.7%	94.4%
300	250	0.0229	0.0014	0.0247	12.2%	94.2%
300	300	0.0228	0.0014	0.0243	11.9%	94.3%

Table B-14 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.05 for non-routine rate and 2.0 for aircraft's age, with 4.0 and 14.0 years as lower and upper limits.

Bin-width		**Model Performance Indicators**				
Fh / Y	**Cy / Y**	**MSE_{DIST}**	**MSE_{SERV}**	**MAE**	**MAPE**	**MAI**
100	100	0.0227	0.0010	0.0210	11.1%	95.1%
100	150	0.0218	0.0011	0.0218	11.4%	94.9%
100	200	0.0254	0.0011	0.0220	11.5%	94.8%
100	250	0.0239	0.0011	0.0221	11.5%	94.8%
100	300	0.0235	0.0011	0.0221	11.6%	94.8%
150	100	0.0187	0.0010	0.0210	11.4%	95.1%
150	150	0.0164	0.0010	0.0215	11.3%	94.9%
150	200	0.0200	0.0010	0.0214	11.5%	95.0%
150	250	0.0204	0.0011	0.0223	11.9%	94.8%
150	300	0.0176	0.0011	0.0215	11.5%	94.9%
200	100	0.0232	0.0011	0.0217	11.6%	94.9%
200	150	0.0202	0.0012	0.0226	11.7%	94.7%
200	200	0.0261	0.0012	0.0227	12.0%	94.6%
200	250	0.0243	0.0012	0.0231	12.1%	94.6%
200	300	0.0249	0.0012	0.0232	12.3%	94.5%
250	100	0.0201	0.0015	0.0238	12.4%	94.4%
250	150	0.0168	0.0015	0.0241	12.3%	94.3%
250	200	0.0240	0.0015	0.0242	12.5%	94.3%
250	250	0.0224	0.0015	0.0251	13.0%	94.1%
250	300	0.0227	0.0015	0.0248	12.8%	94.2%
300	100	0.0202	0.0015	0.0239	12.6%	94.4%
300	150	0.0173	0.0015	0.0242	12.3%	94.3%
300	200	0.0250	0.0016	0.0244	12.7%	94.3%
300	250	0.0244	0.0016	0.0254	13.2%	94.0%
300	300	0.0240	0.0016	0.0249	13.0%	94.1%

Table B-15 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.05 for non-routine rate and 2.5 for aircraft's age, with 2.5 and 15.0 years as lower and upper limits.

Bin-width		Model Performance Indicators				
Fh / Y	**Cy / Y**	**MSE_{DIST}**	**MSE_{SERV}**	**MAE**	**MAPE**	**MAI**
100	100	0.0175	0.0008	0.0200	10.4%	95.3%
100	150	0.0172	0.0009	0.0211	10.8%	95.0%
100	200	0.0211	0.0009	0.0214	11.0%	95.0%
100	250	0.0192	0.0009	0.0214	10.9%	95.0%
100	300	0.0183	0.0009	0.0215	11.0%	94.9%
150	100	0.0136	0.0007	0.0197	10.3%	95.4%
150	150	0.0117	0.0008	0.0203	10.4%	95.2%
150	200	0.0157	0.0008	0.0203	10.4%	95.2%
150	250	0.0145	0.0008	0.0210	10.8%	95.1%
150	300	0.0114	0.0008	0.0201	10.3%	95.3%
200	100	0.0181	0.0009	0.0211	10.8%	95.0%
200	150	0.0159	0.0010	0.0223	11.2%	94.8%
200	200	0.0215	0.0010	0.0225	11.4%	94.7%
200	250	0.0174	0.0010	0.0224	11.3%	94.7%
200	300	0.0169	0.0010	0.0225	11.4%	94.7%
250	100	0.0125	0.0012	0.0225	11.3%	94.7%
250	150	0.0109	0.0012	0.0231	11.4%	94.6%
250	200	0.0162	0.0012	0.0231	11.4%	94.6%
250	250	0.0119	0.0012	0.0235	11.7%	94.5%
250	300	0.0112	0.0012	0.0230	11.3%	94.6%
300	100	0.0128	0.0012	0.0224	11.4%	94.7%
300	150	0.0120	0.0012	0.0234	11.5%	94.5%
300	200	0.0188	0.0012	0.0233	11.5%	94.5%
300	250	0.0157	0.0013	0.0238	11.9%	94.4%
300	300	0.0138	0.0012	0.0233	11.5%	94.5%

Table B-16 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.10 for non-routine rate and 0.5 for aircraft's age, with 4.5 and 13.0 years as lower and upper limits.

Bin-width		**Model Performance Indicators**				
Fh / Y	**Cy / Y**	**MSE_{DIST}**	**MSE_{SERV}**	**MAE**	**MAPE**	**MAI**
100	100	0.0173	0.0012	0.0277	15.3%	93.9%
100	150	0.0171	0.0013	0.0288	15.7%	93.6%
100	200	0.0221	0.0013	0.0284	15.7%	93.7%
100	250	0.0171	0.0013	0.0288	15.8%	93.6%
100	300	0.0183	0.0013	0.0288	15.8%	93.6%
150	100	0.0156	0.0011	0.0269	15.1%	94.0%
150	150	0.0162	0.0012	0.0281	15.5%	93.7%
150	200	0.0220	0.0012	0.0275	15.2%	93.9%
150	250	0.0168	0.0013	0.0285	15.8%	93.7%
150	300	0.0166	0.0012	0.0278	15.2%	93.8%
200	100	0.0203	0.0012	0.0278	15.4%	93.8%
200	150	0.0223	0.0013	0.0292	15.9%	93.5%
200	200	0.0285	0.0013	0.0288	15.8%	93.6%
200	250	0.0221	0.0014	0.0293	15.9%	93.5%
200	300	0.0239	0.0014	0.0293	15.9%	93.5%
250	100	0.0174	0.0016	0.0295	16.0%	93.4%
250	150	0.0174	0.0017	0.0307	16.3%	93.2%
250	200	0.0236	0.0016	0.0299	16.0%	93.3%
250	250	0.0179	0.0017	0.0310	16.7%	93.1%
250	300	0.0180	0.0016	0.0303	16.0%	93.3%
300	100	0.0176	0.0016	0.0296	16.1%	93.4%
300	150	0.0180	0.0017	0.0306	16.4%	93.2%
300	200	0.0242	0.0016	0.0299	16.0%	93.3%
300	250	0.0186	0.0017	0.0311	16.8%	93.1%
300	300	0.0190	0.0017	0.0303	16.1%	93.3%

Table B-17 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.10 for non-routine rate and 1.0 for aircraft's age, with 4.0 and 13.0 years as lower and upper limits.

Bin-width		Model Performance Indicators				
Fh / Y	**Cy / Y**	**MSE_{DIST}**	**MSE_{SERV}**	**MAE**	**MAPE**	**MAI**
100	100	0.0211	0.0012	0.0277	15.2%	93.8%
100	150	0.0203	0.0013	0.0284	15.5%	93.7%
100	200	0.0257	0.0013	0.0281	15.4%	93.7%
100	250	0.0203	0.0013	0.0285	15.6%	93.7%
100	300	0.0214	0.0013	0.0283	15.4%	93.7%
150	100	0.0183	0.0012	0.0271	15.0%	94.0%
150	150	0.0175	0.0012	0.0277	15.1%	93.8%
150	200	0.0236	0.0012	0.0269	14.6%	94.0%
150	250	0.0189	0.0013	0.0284	15.7%	93.7%
150	300	0.0175	0.0012	0.0269	14.6%	94.0%
200	100	0.0238	0.0012	0.0277	15.2%	93.9%
200	150	0.0244	0.0013	0.0285	15.5%	93.7%
200	200	0.0315	0.0013	0.0281	15.4%	93.8%
200	250	0.0250	0.0013	0.0286	15.6%	93.6%
200	300	0.0263	0.0013	0.0283	15.4%	93.7%
250	100	0.0209	0.0016	0.0296	15.9%	93.4%
250	150	0.0192	0.0016	0.0301	15.9%	93.3%
250	200	0.0260	0.0016	0.0291	15.4%	93.5%
250	250	0.0205	0.0017	0.0307	16.5%	93.2%
250	300	0.0197	0.0016	0.0293	15.4%	93.5%
300	100	0.0212	0.0016	0.0296	15.9%	93.4%
300	150	0.0199	0.0017	0.0300	15.9%	93.3%
300	200	0.0267	0.0016	0.0291	15.4%	93.5%
300	250	0.0213	0.0017	0.0307	16.5%	93.2%
300	300	0.0209	0.0016	0.0292	15.4%	93.5%

Table B-18 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.10 for non-routine rate and 1.0 for aircraft's age, with 4.5 and 13.5 years as lower and upper limits.

Bin-width		**Model Performance Indicators**				
Fh / Y	**Cy / Y**	**MSE_{DIST}**	**MSE_{SERV}**	**MAE**	**MAPE**	**MAI**
100	100	0.0166	0.0011	0.0271	15.0%	94.0%
100	150	0.0152	0.0012	0.0279	15.3%	93.8%
100	200	0.0212	0.0012	0.0274	15.2%	93.9%
100	250	0.0148	0.0013	0.0280	15.4%	93.8%
100	300	0.0157	0.0013	0.0280	15.4%	93.8%
150	100	0.0138	0.0010	0.0257	14.4%	94.3%
150	150	0.0127	0.0011	0.0272	14.9%	94.0%
150	200	0.0198	0.0011	0.0263	14.5%	94.2%
150	250	0.0132	0.0012	0.0275	15.3%	93.9%
150	300	0.0132	0.0011	0.0267	14.6%	94.1%
200	100	0.0201	0.0011	0.0270	15.0%	94.0%
200	150	0.0204	0.0013	0.0289	15.6%	93.6%
200	200	0.0284	0.0012	0.0284	15.6%	93.7%
200	250	0.0189	0.0013	0.0287	15.7%	93.6%
200	300	0.0204	0.0013	0.0287	15.7%	93.6%
250	100	0.0159	0.0015	0.0285	15.4%	93.7%
250	150	0.0153	0.0016	0.0301	15.9%	93.3%
250	200	0.0230	0.0015	0.0293	15.5%	93.5%
250	250	0.0143	0.0016	0.0304	16.3%	93.2%
250	300	0.0145	0.0015	0.0296	15.6%	93.4%
300	100	0.0169	0.0015	0.0284	15.4%	93.7%
300	150	0.0155	0.0016	0.0300	16.0%	93.3%
300	200	0.0247	0.0015	0.0292	15.5%	93.5%
300	250	0.0158	0.0016	0.0304	16.4%	93.3%
300	300	0.0161	0.0016	0.0295	15.6%	93.4%

Table B-19 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.10 for non-routine rate and 1.5 for aircraft's age, with 4.0 and 13.0 years as lower and upper limits.

Bin-width		Model Performance Indicators				
Fh / Y	Cy / Y	MSE_{DIST}	MSE_{SERV}	MAE	MAPE	MAI
100	100	0.0190	0.0012	0.0272	15.0%	94.0%
100	150	0.0165	0.0012	0.0275	15.1%	93.9%
100	200	0.0233	0.0012	0.0272	15.0%	93.9%
100	250	0.0160	0.0012	0.0276	15.2%	93.9%
100	300	0.0167	0.0012	0.0274	15.1%	93.9%
150	100	0.0157	0.0010	0.0256	14.3%	94.3%
150	150	0.0128	0.0011	0.0265	14.5%	94.1%
150	200	0.0208	0.0011	0.0258	14.2%	94.3%
150	250	0.0134	0.0012	0.0268	14.9%	94.1%
150	300	0.0126	0.0011	0.0258	14.1%	94.3%
200	100	0.0220	0.0011	0.0269	14.9%	94.0%
200	150	0.0202	0.0012	0.0281	15.2%	93.8%
200	200	0.0293	0.0012	0.0278	15.3%	93.8%
200	250	0.0192	0.0012	0.0279	15.2%	93.8%
200	300	0.0198	0.0012	0.0277	15.1%	93.8%
250	100	0.0187	0.0015	0.0284	15.3%	93.7%
250	150	0.0161	0.0015	0.0294	15.5%	93.5%
250	200	0.0248	0.0015	0.0288	15.2%	93.6%
250	250	0.0151	0.0015	0.0296	15.8%	93.4%
250	300	0.0145	0.0015	0.0285	15.0%	93.7%
300	100	0.0198	0.0015	0.0284	15.4%	93.7%
300	150	0.0163	0.0015	0.0293	15.6%	93.5%
300	200	0.0267	0.0015	0.0286	15.2%	93.7%
300	250	0.0168	0.0016	0.0295	15.9%	93.4%
300	300	0.0163	0.0015	0.0284	15.1%	93.7%

Table B-20 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.10 for non-routine rate and 1.5 for aircraft's age, with 4.5 and 13.5 years as lower and upper limits.

Bin-width		Model Performance Indicators				
Fh / Y	Cy / Y	MSE_{DIST}	MSE_{SERV}	MAE	MAPE	MAI
100	100	0.0195	0.0012	0.0273	15.1%	93.9%
100	150	0.0150	0.0013	0.0281	15.5%	93.8%
100	200	0.0205	0.0013	0.0277	15.3%	93.8%
100	250	0.0156	0.0013	0.0285	15.8%	93.7%
100	300	0.0174	0.0013	0.0283	15.6%	93.7%
150	100	0.0167	0.0011	0.0264	14.7%	94.1%
150	150	0.0162	0.0012	0.0275	15.1%	93.9%
150	200	0.0226	0.0012	0.0266	14.6%	94.1%
150	250	0.0179	0.0013	0.0284	15.9%	93.7%
150	300	0.0183	0.0012	0.0272	14.9%	94.0%
200	100	0.0217	0.0012	0.0270	14.9%	94.0%
200	150	0.0181	0.0013	0.0286	15.6%	93.6%
200	200	0.0256	0.0013	0.0279	15.3%	93.8%
200	250	0.0183	0.0013	0.0286	15.8%	93.6%
200	300	0.0213	0.0013	0.0286	15.8%	93.7%
250	100	0.0186	0.0015	0.0289	15.5%	93.6%
250	150	0.0182	0.0016	0.0303	16.0%	93.3%
250	200	0.0259	0.0015	0.0290	15.3%	93.6%
250	250	0.0199	0.0017	0.0309	16.7%	93.1%
250	300	0.0216	0.0016	0.0297	15.7%	93.4%
300	100	0.0189	0.0016	0.0289	15.5%	93.6%
300	150	0.0185	0.0016	0.0301	16.1%	93.3%
300	200	0.0272	0.0016	0.0287	15.3%	93.6%
300	250	0.0211	0.0017	0.0307	16.7%	93.2%
300	300	0.0233	0.0017	0.0294	15.8%	93.5%

Table B-21 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.10 for non-routine rate and 2.0 for aircraft's age, with 3.0 and 13.0 years as lower and upper limits.

Bin-width		Model Performance Indicators				
Fh / Y	**Cy / Y**	**MSE_{DIST}**	**MSE_{SERV}**	**MAE**	**MAPE**	**MAI**
100	100	0.0217	0.0012	0.0270	14.8%	94.0%
100	150	0.0189	0.0012	0.0275	15.0%	93.9%
100	200	0.0263	0.0012	0.0270	14.7%	94.0%
100	250	0.0183	0.0012	0.0276	15.2%	93.9%
100	300	0.0191	0.0012	0.0272	14.8%	94.0%
150	100	0.0179	0.0011	0.0257	14.2%	94.3%
150	150	0.0143	0.0011	0.0265	14.3%	94.1%
150	200	0.0231	0.0011	0.0256	13.8%	94.3%
150	250	0.0149	0.0012	0.0270	15.0%	94.0%
150	300	0.0140	0.0011	0.0255	13.8%	94.3%
200	100	0.0254	0.0011	0.0273	15.0%	93.9%
200	150	0.0237	0.0012	0.0282	15.2%	93.7%
200	200	0.0335	0.0012	0.0279	15.1%	93.8%
200	250	0.0222	0.0012	0.0281	15.3%	93.8%
200	300	0.0232	0.0012	0.0276	15.0%	93.9%
250	100	0.0213	0.0015	0.0287	15.2%	93.6%
250	150	0.0188	0.0015	0.0293	15.3%	93.5%
250	200	0.0283	0.0015	0.0285	14.8%	93.7%
250	250	0.0177	0.0016	0.0298	15.8%	93.4%
250	300	0.0178	0.0015	0.0283	14.7%	93.7%
300	100	0.0224	0.0015	0.0287	15.3%	93.6%
300	150	0.0188	0.0015	0.0295	15.5%	93.5%
300	200	0.0305	0.0015	0.0285	15.0%	93.7%
300	250	0.0198	0.0016	0.0299	16.0%	93.4%
300	300	0.0199	0.0015	0.0284	14.9%	93.7%

Table B-22 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.10 for non-routine rate and 2.0 for aircraft's age, with 4.0 and 14.0 years as lower and upper limits.

Bin-width		Model Performance Indicators				
Fh / Y	Cy / Y	MSE_{DIST}	MSE_{SERV}	MAE	MAPE	MAI
100	100	0.0209	0.0012	0.0273	15.1%	93.9%
100	150	0.0155	0.0013	0.0281	15.4%	93.7%
100	200	0.0208	0.0013	0.0278	15.2%	93.8%
100	250	0.0155	0.0013	0.0282	15.5%	93.7%
100	300	0.0163	0.0013	0.0280	15.3%	93.8%
150	100	0.0178	0.0011	0.0266	14.8%	94.1%
150	150	0.0163	0.0012	0.0273	14.9%	93.9%
150	200	0.0225	0.0012	0.0265	14.4%	94.1%
150	250	0.0177	0.0013	0.0280	15.5%	93.8%
150	300	0.0164	0.0012	0.0266	14.4%	94.1%
200	100	0.0238	0.0012	0.0273	15.1%	93.9%
200	150	0.0191	0.0013	0.0282	15.4%	93.7%
200	200	0.0260	0.0013	0.0277	15.2%	93.8%
200	250	0.0193	0.0013	0.0283	15.5%	93.7%
200	300	0.0203	0.0013	0.0280	15.3%	93.8%
250	100	0.0209	0.0016	0.0290	15.6%	93.6%
250	150	0.0180	0.0016	0.0297	15.7%	93.4%
250	200	0.0256	0.0015	0.0286	15.1%	93.6%
250	250	0.0202	0.0017	0.0302	16.3%	93.3%
250	300	0.0194	0.0016	0.0289	15.2%	93.6%
300	100	0.0209	0.0016	0.0290	15.6%	93.5%
300	150	0.0186	0.0016	0.0295	15.7%	93.4%
300	200	0.0264	0.0016	0.0285	15.1%	93.7%
300	250	0.0210	0.0017	0.0302	16.3%	93.3%
300	300	0.0205	0.0016	0.0287	15.2%	93.6%

Table B-23 Model performance comparison for different bin-widths of flight and cycles per year; considering interval-widths of 0.10 for non-routine rate and 2.5 for aircraft's age, with 2.5 and 15.0 years as lower and upper limits.

Bin-width		Model Performance Indicators				
Fh / Y	**Cy / Y**	**MSE_{DIST}**	**MSE_{SERV}**	**MAE**	**MAPE**	**MAI**
100	100	0.0170	0.0011	0.0260	14.4%	94.2%
100	150	0.0109	0.0011	0.0268	14.6%	94.0%
100	200	0.0174	0.0011	0.0261	14.3%	94.2%
100	250	0.0098	0.0011	0.0268	14.7%	94.0%
100	300	0.0100	0.0011	0.0264	14.3%	94.1%
150	100	0.0128	0.0010	0.0247	13.7%	94.5%
150	150	0.0085	0.0010	0.0256	13.9%	94.3%
150	200	0.0170	0.0010	0.0245	13.3%	94.6%
150	250	0.0083	0.0011	0.0258	14.3%	94.3%
150	300	0.0076	0.0010	0.0245	13.2%	94.5%
200	100	0.0211	0.0011	0.0263	14.6%	94.1%
200	150	0.0149	0.0012	0.0274	14.8%	93.9%
200	200	0.0232	0.0011	0.0268	14.5%	94.0%
200	250	0.0117	0.0012	0.0270	14.8%	94.0%
200	300	0.0121	0.0011	0.0266	14.3%	94.1%
250	100	0.0150	0.0013	0.0273	14.6%	93.9%
250	150	0.0120	0.0013	0.0282	14.8%	93.7%
250	200	0.0210	0.0013	0.0272	14.2%	93.9%
250	250	0.0101	0.0014	0.0283	15.1%	93.7%
250	300	0.0096	0.0013	0.0271	14.1%	94.0%
300	100	0.0156	0.0013	0.0272	14.7%	93.9%
300	150	0.0111	0.0014	0.0281	14.8%	93.8%
300	200	0.0223	0.0013	0.0271	14.3%	94.0%
300	250	0.0112	0.0014	0.0282	15.1%	93.7%
300	300	0.0108	0.0013	0.0272	14.3%	94.0%

B.5 Bin size layouts

Table B-24 Layout A interval array

	No. Intervals	Bin size	Lower limit	Upper limit
Age	17	0.5	4.5	13
Fh/Y	15	150	2300	4550
Cy/Y	18	100	900	2700
Services	9	1	1C	6C
Nr rate	9	0.05	0.05	0.5

Table B-25 Layout B interval array

	No. Intervals	Bin size	Lower limit	Upper limit
Age	5	2.5	2.5	15
Fh/Y	15	150	2300	4550
Cy/Y	6	300	900	2700
Services	9	1	1C	6C
Nr rate	5	0.1	0	0.5

B.6 Number of predictor variables analysis

Table B-26 Layout A: Model performance comparison excluding one predictor variable

	Age Fh/Y Cy/Y Services -	**Age Fh/Y Cy/Y - Nr rate**	**Age Fh/Y - Services Nr rate**	**Age - Cy/Y Services Nr rate**	**- Fh/Y Cy/Y Services Nr rate**
MSE_{Dist}	0.019	0.030	0.018	0.018	0.012
MSE_{SERV}	0.001	0.002	0.001	0.001	0.001
MAE	0.020	0.028	0.021	0.022	0.020
MPAE	10.4%	13.8%	10.5%	11.0%	10.9%
MAI	95.4%	93.5%	95.1%	94.8%	95.3%

Table B-27 Layout B: Model performance comparison excluding one predictor variable

	Age Fh/Y Cy/Y Services -	**Age Fh/Y Cy/Y - Nr rate**	**Age Fh/Y - Services Nr rate**	**Age - Cy/Y Services Nr rate**	**- Fh/Y Cy/Y Services Nr rate**
MSE_{Dist}	0.013	0.062	0.007	0.008	0.003
MSE_{SERV}	0.001	0.003	0.001	0.001	0.001
MAE	0.029	0.042	0.025	0.028	0.026
MPAE	16.5%	21.7%	13.3%	15.1%	14.3%
MAI	93.5%	90.6%	94.5%	93.7%	94.2%

Table B-28 Layout A: Model performance comparison excluding two predictor variables

	Age Fh/Y Cy/Y - -	Age Fh/Y - Services -	Age - Cy/Y Services -	- Fh/Y Cy/Y Services -	Age Fh/Y - - Nr rate	Age - Cy/Y - Nr rate	- Fh/Y Cy/Y - Nr rate	Age - - Services Nr rate	- Fh/Y - Services Nr rate	- - Cy/Y Services Nr rate
MSE_{Dist}	0.033	0.021	0.018	0.014	0.040	0.028	0.046	0.016	0.012	0.011
MSE_{SERV}	0.002	0.001	0.001	0.001	0.002	0.002	0.005	0.001	0.001	0.001
MAE	0.028	0.023	0.023	0.022	0.031	0.033	0.055	0.024	0.022	0.025
MPAE	14.4%	12.0%	11.6%	12.0%	14.8%	15.5%	28.4%	11.9%	11.6%	13.4%
MAI	93.5%	94.5%	94.6%	94.9%	92.7%	92.2%	87.1%	94.3%	94.8%	94.2%

Table B-29 Layout B: Model performance comparison excluding two predictor variables

	Age Fh/Y Cy/Y - -	Age Fh/Y - Services -	Age - Cy/Y Services -	- Fh/Y Cy/Y Services -	Age Fh/Y - - Nr rate	Age - Cy/Y - Nr rate	- Fh/Y Cy/Y - Nr rate	Age - - Services Nr rate	- Fh/Y - Services Nr rate	- - Cy/Y Services Nr rate
MSE_{Dist}	0.088	0.015	0.015	0.009	0.066	0.065	0.083	0.009	0.002	0.003
MSE_{SERV}	0.006	0.002	0.002	0.002	0.003	0.003	0.005	0.001	0.001	0.002
MAE	0.053	0.030	0.032	0.031	0.043	0.046	0.062	0.029	0.027	0.031
MPAE	28.5%	17.0%	17.8%	17.6%	21.8%	23.2%	32.4%	15.3%	14.5%	16.6%
MAI	88.1%	93.3%	92.8%	93.1%	90.5%	89.7%	86.3%	93.5%	94.1%	93.2%

Table B-30 Layout A: Model performance comparison excluding three predictor variables

	Age Fh/Y - - -	Age - Cy/Y - -	- Fh/Y Cy/Y - -	Age - - Services -	- Fh/Y - Services -	- - Cy/Y Services -	Age - - - Nr rate	- Fh/Y - - Nr rate	- - Cy/Y - Nr rate	- - - Services Nr rate
MSE_{Dist}	0.046	0.032	0.056	0.020	0.016	0.013	0.037	0.052	0.050	0.036
MSE_{SERV}	0.002	0.002	0.007	0.002	0.001	0.002	0.003	0.005	0.007	0.002
MAE	0.033	0.034	0.060	0.027	0.026	0.026	0.038	0.060	0.064	0.029
MPAE	16.7%	17.1%	32.2%	13.6%	14.2%	14.3%	17.8%	30.6%	34.2%	14.9%
MAI	92.3%	91.9%	86.0%	93.6%	93.8%	93.8%	91.0%	86.0%	85.0%	93.3%

Table B-31 Layout B: Model performance comparison excluding three predictor variables

	Age Fh/Y - - -	Age - Cy/Y - -	- Fh/Y Cy/Y - -	Age - - Services -	- Fh/Y - Services -	- - Cy/Y Services -	Age - - - Nr rate	- Fh/Y - - Nr rate	- - Cy/Y - Nr rate	- - - Services Nr rate
MSE_{Dist}	0.093	0.095	0.108	0.025	0.011	0.011	0.062	0.088	0.102	0.021
MSE_{SERV}	0.005	0.007	0.010	0.002	0.002	0.002	0.003	0.005	0.007	0.002
MAE	0.053	0.066	0.072	0.034	0.033	0.035	0.047	0.062	0.071	0.032
MPAE	28.2%	33.5%	39.2%	18.2%	18.4%	19.4%	23.5%	32.2%	38.7%	16.8%
MAI	88.3%	85.4%	83.9%	92.5%	92.7%	92.1%	89.5%	86.3%	84.2%	92.9%

B.7 ER rule model scenarios adjusting alpha index, reliability and weight

Table B-32 Layout A (Four samples): Model performance comparison adjusting Alpha index, Reliability and Weight

	Sample A				**Sample B**				**Sample C**				**Sample D**			
	Original	Optimising alphas	R ≠ 1	Optimising W	Original	Optimising alphas	R ≠ 1	Optimising W	Original	Optimising alphas	R ≠ 1	Optimising W	Original	Optimising alphas	R ≠ 1	Optimising W
Alpha Index	1	Optim	Optim	Optim	1	Optim	Optim	Optim	1	Optim	Optim	Optim	1	Optim	Optim	Optim
Reliability	1	1	≠ 1	≠ 1	1	1	≠ 1	≠ 1	1	1	≠ 1	≠ 1	1	1	≠ 1	≠ 1
Weight	1	1	1	Optim	1	1	1	Optim	1	1	1	Optim	1	1	1	Optim
MSE_{DIST}	0.0128	0.0000			0.0119	0.0000			0.0167	0.0050			0.0159	0.0000		
MSE_{SERV}	0.0005	0.0004	0.0010	0.0009	0.0006	0.0004	0.0009	0.0006	0.0005	0.0003	0.0008	0.0005	0.0006	0.0004	0.0009	0.0007
MAE	0.0172	0.0155	0.0220	0.0203	0.0179	0.0156	0.0205	0.0167	0.0174	0.0146	0.0204	0.0154	0.0173	0.0155	0.0207	0.0176
MAPE	9.3%	8.0%	10.9%	10.3%	9.7%	8.1%	10.8%	9.1%	8.9%	7.5%	10.2%	7.8%	9.4%	8.1%	10.9%	9.6%
MAI	96.0%	96.3%	94.8%	95.2%	95.8%	96.3%	95.2%	96.1%	95.9%	96.6%	95.2%	96.4%	95.9%	96.4%	95.1%	95.9%

Table B-33 Layout B (Four samples): Model performance comparison adjusting Alpha index, Reliability and Weight

	Sample A				**Sample B**				**Sample C**				**Sample D**			
	Original	Optimising alphas	R ≠ 1	Optimising W	Original	Optimising alphas	R ≠ 1	Optimising W	Original	Optimising alphas	R ≠ 1	Optimising W	Original	Optimising alphas	R ≠ 1	Optimising W
Alpha Index	1	Optim	Optim	Optim	1	Optim	Optim	Optim	1	Optim	Optim	Optim	1	Optim	Optim	Optim
Reliability	1	1	≠ 1	≠ 1	1	1	≠ 1	≠ 1	1	1	≠ 1	≠ 1	1	1	≠ 1	≠ 1
Weight	1	1	1	Optim	1	1	1	Optim	1	1	1	Optim	1	1	1	Optim
MSE_{DIST}	0.0049	0.0000			0.0059	0.0000			0.0082	0.0000			0.0061	0.0000		
MSE_{SERV}	0.0010	0.0010	0.0013	0.0011	0.0009	0.0009	0.0011	0.0008	0.0009	0.0008	0.0011	0.0009	0.0010	0.0010	0.0012	0.0010
MAE	0.0244	0.0240	0.0268	0.0243	0.0242	0.0238	0.0252	0.0195	0.0231	0.0231	0.0244	0.0198	0.0252	0.0249	0.0262	0.0208
MAPE	12.8%	12.9%	13.8%	12.1%	13.5%	13.4%	13.9%	10.3%	12.1%	12.6%	12.8%	9.8%	13.7%	14.0%	14.0%	10.9%
MAI	94.6%	94.7%	94.0%	94.6%	94.6%	94.7%	94.4%	95.7%	94.9%	94.9%	94.6%	95.6%	94.4%	94.5%	94.2%	95.4%

B.7.1 Optimised alpha-index results

Table B-34 Layout A, Sample A: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year

Age	Fh / Y	Alpha-Index per Non-routine rate								
		0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
4.5-5	3800-3950	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3500-3650	0.240	0.625	0.513	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3650-3800	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	4400-4550	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3500-3650	0.500	0.627	0.294	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3800-3950	0.500	0.237	0.630	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3950-4100	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
6-6.5	3500-3650	0.500	0.541	0.500	0.449	0.500	0.500	0.500	0.500	0.500
6-6.5	3650-3800	0.500	0.428	0.500	0.557	0.500	0.500	0.500	0.500	0.500
7-7.5	3200-3350	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7-7.5	3500-3650	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3200-3350	0.500	0.500	0.500	0.500	0.500	0.373	0.584	0.500	0.500
7.5-8	3350-3500	0.500	0.500	0.500	0.464	0.534	0.500	0.500	0.500	0.500
7.5-8	3500-3650	0.500	0.500	0.101	0.529	0.658	0.468	0.500	0.500	0.500
8-8.5	3200-3350	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	2300-2450	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	3200-3350	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	3350-3500	0.500	0.500	0.500	0.144	0.659	0.500	0.500	0.500	0.500
9.5-10	3200-3350	0.500	0.500	0.500	0.254	0.500	0.057	0.791	0.349	0.349
9.5-10	3350-3500	0.500	0.500	0.500	0.182	0.638	0.500	0.500	0.500	0.500
9.5-10	3500-3650	0.500	0.500	0.500	0.640	0.131	0.500	0.500	0.500	0.500
9.5-10	3650-3800	0.500	0.500	0.500	0.312	0.606	0.500	0.500	0.500	0.500
10.5-11	3200-3350	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

Age	Fh / Y	Alpha-Index per Non-routine rate								
		0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
10.5-11	3800-3950	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11-11.5	2900-3050	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11-11.5	3800-3950	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11.5-12	2900-3050	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
12.5-13	2900-3050	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

Table B-35 Layout A, Sample A: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year with cycles per year

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate								
			0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
4.5-5	3800-3950	1400-1500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
4.5-5	3800-3950	1500-1600	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3500-3650	1700-1800	0.240	0.625	0.513	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3650-3800	1600-1700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	4400-4550	1900-2000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3500-3650	1600-1700	0.500	0.634	0.268	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3500-3650	1700-1800	0.500	0.490	0.506	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3800-3950	1300-1400	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3800-3950	1400-1500	0.500	0.237	0.630	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3950-4100	1500-1600	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3950-4100	1600-1700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
6-6.5	3500-3650	1700-1800	0.500	0.541	0.500	0.449	0.500	0.500	0.500	0.500	0.500
6-6.5	3650-3800	1500-1600	0.500	0.090	0.500	0.643	0.500	0.500	0.500	0.500	0.500
6-6.5	3650-3800	1600-1700	0.500	0.613	0.500	0.244	0.500	0.500	0.500	0.500	0.500
7-7.5	3200-3350	1500-1600	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7-7.5	3200-3350	1600-1700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate								
			0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
7-7.5	3200-3350	1700-1800	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7-7.5	3500-3650	1600-1700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3200-3350	1600-1700	0.500	0.500	0.500	0.500	0.500	0.096	0.643	0.500	0.500
7.5-8	3200-3350	1700-1800	0.500	0.500	0.500	0.500	0.500	0.592	0.338	0.500	0.500
7.5-8	3200-3350	1800-1900	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3350-3500	1600-1700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3350-3500	1700-1800	0.500	0.500	0.500	0.464	0.534	0.500	0.500	0.500	0.500
7.5-8	3350-3500	1800-1900	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3500-3650	1600-1700	0.500	0.500	0.101	0.644	0.527	0.468	0.500	0.500	0.500
7.5-8	3500-3650	1700-1800	0.500	0.500	0.500	0.275	0.646	0.500	0.500	0.500	0.500
7.5-8	3500-3650	1800-1900	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
8-8.5	3200-3350	1600-1700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	2300-2450	900-1000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	3200-3350	1600-1700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	3200-3350	2000-2100	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	3350-3500	1600-1700	0.500	0.500	0.500	0.348	0.601	0.500	0.500	0.500	0.500
9-9.5	3350-3500	1700-1800	0.500	0.500	0.500	0.366	0.595	0.500	0.500	0.500	0.500
9.5-10	3200-3350	1600-1700	0.500	0.500	0.500	0.406	0.500	0.336	0.706	0.349	0.349
9.5-10	3200-3350	1700-1800	0.500	0.500	0.500	0.368	0.500	0.309	0.678	0.500	0.500
9.5-10	3200-3350	1900-2000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9.5-10	3350-3500	1200-1300	0.500	0.500	0.500	0.182	0.638	0.500	0.500	0.500	0.500
9.5-10	3350-3500	1600-1700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9.5-10	3500-3650	1200-1300	0.500	0.500	0.500	0.640	0.131	0.500	0.500	0.500	0.500
9.5-10	3650-3800	1200-1300	0.500	0.500	0.500	0.312	0.606	0.500	0.500	0.500	0.500
10.5-11	3200-3350	1700-1800	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
10.5-11	3200-3350	1900-2000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate								
			0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
10.5-11	3800-3950	2600-2700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11-11.5	2900-3050	1400-1500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11-11.5	3800-3950	2600-2700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11.5-12	2900-3050	1300-1400	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11.5-12	2900-3050	1400-1500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
12.5-13	2900-3050	1000-1100	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

Table B-36 Layout A, Sample A: Optimised alpha-index per non-routine rate for combining aeroplane age, flight hours and cycles per year with service type

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate								
				0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
4.5-5	3800-3950	1400-1500	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
4.5-5	3800-3950	1500-1600	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3500-3650	1700-1800	3C	0.500	0.000	0.676	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3500-3650	1700-1800	3C+	0.240	0.726	0.101	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3650-3800	1600-1700	3C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	4400-4550	1900-2000	3C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3500-3650	1600-1700	3C+	0.500	0.634	0.268	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3500-3650	1700-1800	3C	0.500	0.658	0.044	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3500-3650	1700-1800	3C+	0.500	0.000	0.667	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3800-3950	1300-1400	3C+	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3800-3950	1400-1500	3C	0.500	0.237	0.630	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3950-4100	1500-1600	3C+	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3950-4100	1600-1700	3C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
6-6.5	3500-3650	1700-1800	2C	0.500	0.541	0.500	0.449	0.500	0.500	0.500	0.500	0.500
6-6.5	3650-3800	1500-1600	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate								
				0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
6-6.5	3650-3800	1500-1600	2C	0.500	0.090	0.500	0.643	0.500	0.500	0.500	0.500	0.500
6-6.5	3650-3800	1600-1700	2C	0.500	0.613	0.500	0.244	0.500	0.500	0.500	0.500	0.500
7-7.5	3200-3350	1500-1600	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7-7.5	3200-3350	1600-1700	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7-7.5	3200-3350	1700-1800	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7-7.5	3500-3650	1600-1700	2C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3200-3350	1600-1700	4C	0.500	0.500	0.500	0.500	0.500	0.096	0.643	0.500	0.500
7.5-8	3200-3350	1700-1800	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3200-3350	1700-1800	4C	0.500	0.500	0.500	0.500	0.500	0.592	0.338	0.500	0.500
7.5-8	3200-3350	1700-1800	4C+	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3200-3350	1800-1900	4C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3350-3500	1600-1700	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3350-3500	1600-1700	4C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3350-3500	1700-1800	4C+	0.500	0.500	0.500	0.464	0.534	0.500	0.500	0.500	0.500
7.5-8	3350-3500	1800-1900	4C+	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3500-3650	1600-1700	3C+	0.500	0.500	0.101	0.644	0.500	0.500	0.500	0.500	0.500
7.5-8	3500-3650	1600-1700	4C	0.500	0.500	0.500	0.500	0.527	0.468	0.500	0.500	0.500
7.5-8	3500-3650	1700-1800	4C+	0.500	0.500	0.500	0.275	0.646	0.500	0.500	0.500	0.500
7.5-8	3500-3650	1800-1900	4C+	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
8-8.5	3200-3350	1600-1700	4C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	2300-2450	900-1000	2C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	3200-3350	1600-1700	2C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	3200-3350	2000-2100	5C+	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	3350-3500	1600-1700	2C	0.500	0.500	0.500	0.348	0.601	0.500	0.500	0.500	0.500
9-9.5	3350-3500	1700-1800	2C	0.500	0.500	0.500	0.366	0.595	0.500	0.500	0.500	0.500
9.5-10	3200-3350	1600-1700	5C	0.500	0.500	0.500	0.406	0.500	0.336	0.706	0.349	0.349

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate								
				0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
9.5-10	3200-3350	1700-1800	5C	0.500	0.500	0.500	0.368	0.500	0.309	0.678	0.500	0.500
9.5-10	3200-3350	1900-2000	5C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9.5-10	3350-3500	1200-1300	5C+	0.500	0.500	0.500	0.182	0.638	0.500	0.500	0.500	0.500
9.5-10	3350-3500	1600-1700	5C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9.5-10	3500-3650	1200-1300	5C+	0.500	0.500	0.500	0.640	0.131	0.500	0.500	0.500	0.500
9.5-10	3650-3800	1200-1300	5C+	0.500	0.500	0.500	0.312	0.606	0.500	0.500	0.500	0.500
10.5-11	3200-3350	1700-1800	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
10.5-11	3200-3350	1700-1800	3C+	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
10.5-11	3200-3350	1900-2000	3C+	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
10.5-11	3800-3950	2600-2700	3C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11-11.5	2900-3050	1400-1500	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11-11.5	3800-3950	2600-2700	6C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11.5-12	2900-3050	1300-1400	6C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11.5-12	2900-3050	1400-1500	6C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
12.5-13	2900-3050	1000-1100	2C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

Table B-37 Layout A, Sample B: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year

Age	Fh / Y	Alpha-Index per Non-routine rate								
		0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
4.5-5	3800-3950	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5-5.5	3500-3650	0.299	0.609	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5-5.5	3650-3800	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5-5.5	4400-4550	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5.5-6	3500-3650	0.501	0.493	0.630	0.297	0.501	0.501	0.501	0.501	0.501
5.5-6	3800-3950	0.501	0.311	0.633	0.455	0.501	0.501	0.501	0.501	0.501
5.5-6	3950-4100	0.501	0.302	0.501	0.609	0.501	0.501	0.501	0.501	0.501
6-6.5	3500-3650	0.501	0.520	0.501	0.482	0.501	0.501	0.501	0.501	0.501
6-6.5	3650-3800	0.501	0.445	0.501	0.553	0.501	0.501	0.501	0.501	0.501
7-7.5	3200-3350	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7-7.5	3500-3650	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3200-3350	0.501	0.501	0.501	0.501	0.098	0.490	0.702	0.501	0.501
7.5-8	3350-3500	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3500-3650	0.501	0.501	0.130	0.651	0.501	0.501	0.501	0.501	0.501
8-8.5	3200-3350	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
9-9.5	2300-2450	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
9-9.5	3200-3350	0.501	0.501	0.501	0.586	0.360	0.501	0.501	0.501	0.501
9-9.5	3350-3500	0.501	0.501	0.501	0.241	0.667	0.501	0.501	0.501	0.501
9.5-10	3200-3350	0.501	0.501	0.501	0.437	0.501	0.040	0.751	0.354	0.355
9.5-10	3350-3500	0.501	0.501	0.501	0.147	0.650	0.501	0.501	0.501	0.501
9.5-10	3500-3650	0.501	0.501	0.501	0.650	0.146	0.501	0.501	0.501	0.501
9.5-10	3650-3800	0.501	0.501	0.501	0.288	0.611	0.501	0.501	0.501	0.501
10-10.5	2900-3050	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
10-10.5	3500-3650	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501

Age	Fh / Y	Alpha-Index per Non-routine rate								
		0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
10.5-11	3200-3350	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
11-11.5	2900-3050	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
11-11.5	3800-3950	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
11.5-12	2900-3050	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
12.5-13	2900-3050	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501

Table B-38 Layout A, Sample B: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year with cycles per year

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate								
			0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
4.5-5	3800-3950	1400-1500	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
4.5-5	3800-3950	1500-1600	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5-5.5	3500-3650	1600-1700	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5-5.5	3500-3650	1700-1800	0.299	0.609	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5-5.5	3500-3650	1800-1900	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5-5.5	3650-3800	1600-1700	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5-5.5	4400-4550	1900-2000	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5.5-6	3500-3650	1700-1800	0.501	0.493	0.630	0.297	0.501	0.501	0.501	0.501	0.501
5.5-6	3800-3950	1300-1400	0.501	0.545	0.501	0.455	0.501	0.501	0.501	0.501	0.501
5.5-6	3800-3950	1400-1500	0.501	0.218	0.633	0.501	0.501	0.501	0.501	0.501	0.501
5.5-6	3950-4100	1400-1500	0.501	0.032	0.501	0.664	0.501	0.501	0.501	0.501	0.501
5.5-6	3950-4100	1500-1600	0.501	0.588	0.501	0.395	0.501	0.501	0.501	0.501	0.501
5.5-6	3950-4100	1600-1700	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
6-6.5	3500-3650	1700-1800	0.501	0.520	0.501	0.482	0.501	0.501	0.501	0.501	0.501
6-6.5	3650-3800	1500-1600	0.501	0.094	0.501	0.652	0.501	0.501	0.501	0.501	0.501
6-6.5	3650-3800	1600-1700	0.501	0.629	0.501	0.206	0.501	0.501	0.501	0.501	0.501

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate								
			0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
7-7.5	3200-3350	1500-1600	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7-7.5	3200-3350	1600-1700	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7-7.5	3200-3350	1700-1800	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7-7.5	3500-3650	1600-1700	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3200-3350	1600-1700	0.501	0.501	0.501	0.501	0.352	0.148	0.693	0.501	0.501
7.5-8	3200-3350	1700-1800	0.501	0.501	0.501	0.501	0.426	0.645	0.304	0.501	0.501
7.5-8	3200-3350	1800-1900	0.501	0.501	0.501	0.501	0.414	0.501	0.574	0.501	0.501
7.5-8	3350-3500	1600-1700	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3350-3500	1700-1800	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3350-3500	1800-1900	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3500-3650	1600-1700	0.501	0.501	0.130	0.651	0.501	0.501	0.501	0.501	0.501
7.5-8	3500-3650	1700-1800	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3500-3650	1800-1900	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
8-8.5	3200-3350	1600-1700	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
9-9.5	2300-2450	900-1000	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
9-9.5	3200-3350	1600-1700	0.501	0.501	0.501	0.561	0.418	0.501	0.501	0.501	0.501
9-9.5	3200-3350	2000-2100	0.501	0.501	0.501	0.535	0.460	0.501	0.501	0.501	0.501
9-9.5	3350-3500	1600-1700	0.501	0.501	0.501	0.347	0.614	0.501	0.501	0.501	0.501
9-9.5	3350-3500	1700-1800	0.501	0.501	0.501	0.423	0.570	0.501	0.501	0.501	0.501
9.5-10	3200-3350	1600-1700	0.501	0.501	0.501	0.461	0.501	0.348	0.688	0.354	0.355
9.5-10	3200-3350	1700-1800	0.501	0.501	0.501	0.478	0.501	0.289	0.629	0.501	0.501
9.5-10	3200-3350	1900-2000	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
9.5-10	3350-3500	1200-1300	0.501	0.501	0.501	0.147	0.650	0.501	0.501	0.501	0.501
9.5-10	3350-3500	1600-1700	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
9.5-10	3500-3650	1200-1300	0.501	0.501	0.501	0.650	0.146	0.501	0.501	0.501	0.501
9.5-10	3650-3800	1200-1300	0.501	0.501	0.501	0.288	0.611	0.501	0.501	0.501	0.501

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate								
			0.05-0.1	0.1-0.15	0.15-0.2	0.2-0.25	0.25-0.3	0.3-0.35	0.35-0.4	0.4-0.45	0.45-0.5
10-10.5	2900-3050	1400-1500	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
10-10.5	3500-3650	1900-2000	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
10.5-11	3200-3350	1700-1800	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
10.5-11	3200-3350	1900-2000	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
11-11.5	2900-3050	1400-1500	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
11-11.5	3800-3950	2600-2700	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
11.5-12	2900-3050	1300-1400	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
12.5-13	2900-3050	1000-1100	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501

Table B-39 Layout A, Sample B: Optimised alpha-index per non-routine rate for combining aeroplane age, flight hours and cycles per year with service type

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate								
				0.05-0.1	0.1-0.15	0.15-0.2	0.2-0.25	0.25-0.3	0.3-0.35	0.35-0.4	0.4-0.45	0.45-0.5
4.5-5	3800-3950	1400-1500	1C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
4.5-5	3800-3950	1500-1600	1C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5-5.5	3500-3650	1600-1700	3C+	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5-5.5	3500-3650	1700-1800	3C+	0.299	0.609	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5-5.5	3500-3650	1800-1900	3C+	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5-5.5	3650-3800	1600-1700	3C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5-5.5	4400-4550	1900-2000	3C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
5.5-6	3500-3650	1700-1800	3C	0.501	0.660	0.017	0.501	0.501	0.501	0.501	0.501	0.501
5.5-6	3500-3650	1700-1800	3C+	0.501	0.001	0.732	0.297	0.501	0.501	0.501	0.501	0.501
5.5-6	3800-3950	1300-1400	3C+	0.501	0.545	0.501	0.455	0.501	0.501	0.501	0.501	0.501
5.5-6	3800-3950	1400-1500	3C	0.501	0.218	0.633	0.501	0.501	0.501	0.501	0.501	0.501
5.5-6	3950-4100	1400-1500	2C	0.501	0.032	0.501	0.664	0.501	0.501	0.501	0.501	0.501
5.5-6	3950-4100	1500-1600	3C+	0.501	0.588	0.501	0.395	0.501	0.501	0.501	0.501	0.501

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate								
				0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
5.5-6	3950-4100	1600-1700	3C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
6-6.5	3500-3650	1700-1800	2C	0.501	0.520	0.501	0.482	0.501	0.501	0.501	0.501	0.501
6-6.5	3650-3800	1500-1600	1C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
6-6.5	3650-3800	1500-1600	2C	0.501	0.094	0.501	0.652	0.501	0.501	0.501	0.501	0.501
6-6.5	3650-3800	1600-1700	2C	0.501	0.629	0.501	0.206	0.501	0.501	0.501	0.501	0.501
7-7.5	3200-3350	1500-1600	1C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7-7.5	3200-3350	1600-1700	1C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7-7.5	3200-3350	1700-1800	1C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7-7.5	3500-3650	1600-1700	2C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3200-3350	1600-1700	4C	0.501	0.501	0.501	0.501	0.352	0.148	0.693	0.501	0.501
7.5-8	3200-3350	1700-1800	1C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3200-3350	1700-1800	4C	0.501	0.501	0.501	0.501	0.426	0.645	0.304	0.501	0.501
7.5-8	3200-3350	1700-1800	4C+	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3200-3350	1800-1900	4C	0.501	0.501	0.501	0.501	0.414	0.501	0.574	0.501	0.501
7.5-8	3350-3500	1600-1700	4C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3350-3500	1700-1800	4C+	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3350-3500	1800-1900	4C+	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3500-3650	1600-1700	3C+	0.501	0.501	0.130	0.651	0.501	0.501	0.501	0.501	0.501
7.5-8	3500-3650	1600-1700	4C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3500-3650	1700-1800	4C+	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
7.5-8	3500-3650	1800-1900	4C+	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
8-8.5	3200-3350	1600-1700	4C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
9-9.5	2300-2450	900-1000	2C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
9-9.5	3200-3350	1600-1700	2C	0.501	0.501	0.501	0.561	0.418	0.501	0.501	0.501	0.501
9-9.5	3200-3350	2000-2100	5C+	0.501	0.501	0.501	0.535	0.460	0.501	0.501	0.501	0.501
9-9.5	3350-3500	1600-1700	2C	0.501	0.501	0.501	0.347	0.614	0.501	0.501	0.501	0.501

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate								
				0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
9-9.5	3350-3500	1700-1800	2C	0.501	0.501	0.501	0.423	0.570	0.501	0.501	0.501	0.501
9.5-10	3200-3350	1600-1700	5C	0.501	0.501	0.501	0.461	0.501	0.348	0.688	0.354	0.355
9.5-10	3200-3350	1700-1800	5C	0.501	0.501	0.501	0.478	0.501	0.289	0.629	0.501	0.501
9.5-10	3200-3350	1900-2000	5C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
9.5-10	3350-3500	1200-1300	5C+	0.501	0.501	0.501	0.147	0.650	0.501	0.501	0.501	0.501
9.5-10	3350-3500	1600-1700	5C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
9.5-10	3500-3650	1200-1300	5C+	0.501	0.501	0.501	0.650	0.146	0.501	0.501	0.501	0.501
9.5-10	3650-3800	1200-1300	5C+	0.501	0.501	0.501	0.288	0.611	0.501	0.501	0.501	0.501
10-10.5	2900-3050	1400-1500	3C+	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
10-10.5	3500-3650	1900-2000	3C+	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
10.5-11	3200-3350	1700-1800	1C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
10.5-11	3200-3350	1700-1800	3C+	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
10.5-11	3200-3350	1900-2000	3C+	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
11-11.5	2900-3050	1400-1500	1C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
11-11.5	3800-3950	2600-2700	6C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
11.5-12	2900-3050	1300-1400	6C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501
12.5-13	2900-3050	1000-1100	2C	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501	0.501

Table B-40 Layout A, Sample C: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year

Age	Fh / Y	Alpha-Index per Non-routine rate								
		0.05-0.1	**0.1-0.15**	**0.15-0.2**	**0.2-0.25**	**0.25-0.3**	**0.3-0.35**	**0.35-0.4**	**0.4-0.45**	**0.45-0.5**
4.5-5	3800-3950	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
5-5.5	3500-3650	0.341	0.419	0.638	0.499	0.499	0.499	0.499	0.499	0.499
5-5.5	3650-3800	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
5-5.5	4400-4550	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
5.5-6	3500-3650	0.499	0.370	0.592	0.497	0.499	0.499	0.499	0.499	0.499
5.5-6	3800-3950	0.499	0.468	0.527	0.499	0.499	0.499	0.499	0.499	0.499
5.5-6	3950-4100	0.499	0.349	0.499	0.590	0.499	0.499	0.499	0.499	0.499
6-6.5	3500-3650	0.499	0.564	0.499	0.412	0.499	0.499	0.499	0.499	0.499
6-6.5	3650-3800	0.499	0.496	0.499	0.501	0.499	0.499	0.499	0.499	0.499
7-7.5	3200-3350	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
7.5-8	3200-3350	0.499	0.499	0.499	0.559	0.408	0.458	0.544	0.499	0.499
7.5-8	3350-3500	0.499	0.499	0.499	0.498	0.499	0.499	0.499	0.499	0.499
7.5-8	3500-3650	0.499	0.499	0.344	0.592	0.555	0.428	0.499	0.499	0.499
8-8.5	3200-3350	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
9-9.5	2300-2450	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
9-9.5	3200-3350	0.499	0.499	0.499	0.551	0.434	0.499	0.499	0.499	0.499
9-9.5	3350-3500	0.499	0.499	0.499	0.492	0.505	0.499	0.499	0.499	0.499
9.5-10	3200-3350	0.499	0.499	0.499	0.497	0.499	0.462	0.555	0.485	0.485
9.5-10	3350-3500	0.499	0.499	0.499	0.453	0.537	0.499	0.499	0.499	0.499
9.5-10	3650-3800	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
10-10.5	2900-3050	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
10-10.5	3500-3650	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
10.5-11	3200-3350	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
10.5-11	3800-3950	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499

Age	Fh / Y	Alpha-Index per Non-routine rate								
		0.05-0.1	**0.1-0.15**	**0.15-0.2**	**0.2-0.25**	**0.25-0.3**	**0.3-0.35**	**0.35-0.4**	**0.4-0.45**	**0.45-0.5**
11-11.5	2900-3050	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
11.5-12	2900-3050	0.499	0.499	0.499	0.347	0.499	0.499	0.591	0.499	0.499
12.5-13	2900-3050	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499

Table B-41 Layout A, Sample C: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year with cycles per year

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate								
			0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
4.5-5	3800-3950	1400-1500	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
4.5-5	3800-3950	1500-1600	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
5-5.5	3500-3650	1600-1700	0.499	0.501	0.497	0.499	0.499	0.499	0.499	0.499	0.499
5-5.5	3500-3650	1700-1800	0.341	0.416	0.639	0.499	0.499	0.499	0.499	0.499	0.499
5-5.5	3500-3650	1800-1900	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
5-5.5	3650-3800	1600-1700	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
5-5.5	4400-4550	1900-2000	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
5.5-6	3500-3650	1600-1700	0.499	0.507	0.491	0.497	0.499	0.499	0.499	0.499	0.499
5.5-6	3500-3650	1700-1800	0.499	0.356	0.596	0.498	0.499	0.499	0.499	0.499	0.499
5.5-6	3800-3950	1300-1400	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
5.5-6	3800-3950	1400-1500	0.499	0.468	0.527	0.499	0.499	0.499	0.499	0.499	0.499
5.5-6	3950-4100	1400-1500	0.499	0.348	0.499	0.591	0.499	0.499	0.499	0.499	0.499
5.5-6	3950-4100	1500-1600	0.499	0.499	0.499	0.498	0.499	0.499	0.499	0.499	0.499
5.5-6	3950-4100	1600-1700	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
6-6.5	3500-3650	1700-1800	0.499	0.564	0.499	0.412	0.499	0.499	0.499	0.499	0.499
6-6.5	3650-3800	1500-1600	0.499	0.496	0.499	0.501	0.499	0.499	0.499	0.499	0.499
6-6.5	3650-3800	1600-1700	0.499	0.498	0.499	0.499	0.499	0.499	0.499	0.499	0.499
7-7.5	3200-3350	1500-1600	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate								
			0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
7-7.5	3200-3350	1600-1700	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
7-7.5	3200-3350	1700-1800	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
7.5-8	3200-3350	1600-1700	0.499	0.499	0.499	0.499	0.492	0.430	0.559	0.499	0.499
7.5-8	3200-3350	1700-1800	0.499	0.499	0.499	0.559	0.423	0.521	0.474	0.499	0.499
7.5-8	3200-3350	1800-1900	0.499	0.499	0.499	0.499	0.494	0.499	0.504	0.499	0.499
7.5-8	3350-3500	1600-1700	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
7.5-8	3350-3500	1700-1800	0.499	0.499	0.499	0.498	0.499	0.499	0.499	0.499	0.499
7.5-8	3350-3500	1800-1900	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
7.5-8	3500-3650	1600-1700	0.499	0.499	0.344	0.592	0.555	0.428	0.499	0.499	0.499
7.5-8	3500-3650	1800-1900	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
8-8.5	3200-3350	1600-1700	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
9-9.5	2300-2450	900-1000	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
9-9.5	3200-3350	1600-1700	0.499	0.499	0.499	0.515	0.481	0.499	0.499	0.499	0.499
9-9.5	3200-3350	2000-2100	0.499	0.499	0.499	0.537	0.454	0.499	0.499	0.499	0.499
9-9.5	3350-3500	1600-1700	0.499	0.499	0.499	0.495	0.502	0.499	0.499	0.499	0.499
9-9.5	3350-3500	1700-1800	0.499	0.499	0.499	0.495	0.502	0.499	0.499	0.499	0.499
9.5-10	3200-3350	1600-1700	0.499	0.499	0.499	0.499	0.499	0.500	0.522	0.485	0.485
9.5-10	3200-3350	1700-1800	0.499	0.499	0.499	0.497	0.499	0.460	0.534	0.499	0.499
9.5-10	3200-3350	1900-2000	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
9.5-10	3350-3500	1200-1300	0.499	0.499	0.499	0.453	0.537	0.499	0.499	0.499	0.499
9.5-10	3350-3500	1600-1700	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
9.5-10	3650-3800	1200-1300	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
10-10.5	2900-3050	1400-1500	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
10-10.5	3500-3650	1900-2000	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
10.5-11	3200-3350	1700-1800	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
10.5-11	3200-3350	1900-2000	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate								
			0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
10.5-11	3800-3950	2600-2700	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
11-11.5	2900-3050	1400-1500	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
11.5-12	2900-3050	1300-1400	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
11.5-12	2900-3050	1400-1500	0.499	0.499	0.499	0.347	0.499	0.499	0.591	0.499	0.499
12.5-13	2900-3050	1000-1100	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499

Table B-42 Layout A, Sample C: Optimised alpha-index per non-routine rate for combining aeroplane age, flight hours and cycles per year with service type

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate								
				0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
4.5-5	3800-3950	1400-1500	1C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
4.5-5	3800-3950	1500-1600	1C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
5-5.5	3500-3650	1600-1700	3C+	0.499	0.501	0.497	0.499	0.499	0.499	0.499	0.499	0.499
5-5.5	3500-3650	1700-1800	3C	0.499	0.073	0.638	0.499	0.499	0.499	0.499	0.499	0.499
5-5.5	3500-3650	1700-1800	3C+	0.341	0.598	0.500	0.499	0.499	0.499	0.499	0.499	0.499
5-5.5	3500-3650	1800-1900	3C+	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
5-5.5	3650-3800	1600-1700	3C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
5-5.5	4400-4550	1900-2000	3C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
5.5-6	3500-3650	1600-1700	3C+	0.499	0.507	0.491	0.497	0.499	0.499	0.499	0.499	0.499
5.5-6	3500-3650	1700-1800	3C	0.499	0.569	0.418	0.499	0.499	0.499	0.499	0.499	0.499
5.5-6	3500-3650	1700-1800	3C+	0.499	0.106	0.637	0.498	0.499	0.499	0.499	0.499	0.499
5.5-6	3800-3950	1300-1400	3C+	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
5.5-6	3800-3950	1400-1500	3C	0.499	0.468	0.527	0.499	0.499	0.499	0.499	0.499	0.499
5.5-6	3950-4100	1400-1500	2C	0.499	0.348	0.499	0.591	0.499	0.499	0.499	0.499	0.499
5.5-6	3950-4100	1500-1600	3C+	0.499	0.499	0.499	0.498	0.499	0.499	0.499	0.499	0.499
5.5-6	3950-4100	1600-1700	3C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate								
				0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
6-6.5	3500-3650	1700-1800	2C	0.499	0.564	0.499	0.412	0.499	0.499	0.499	0.499	0.499
6-6.5	3650-3800	1500-1600	1C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
6-6.5	3650-3800	1500-1600	2C	0.499	0.496	0.499	0.501	0.499	0.499	0.499	0.499	0.499
6-6.5	3650-3800	1600-1700	2C	0.499	0.498	0.499	0.499	0.499	0.499	0.499	0.499	0.499
7-7.5	3200-3350	1500-1600	1C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
7-7.5	3200-3350	1600-1700	1C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
7-7.5	3200-3350	1700-1800	1C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
7.5-8	3200-3350	1600-1700	4C	0.499	0.499	0.499	0.499	0.492	0.430	0.559	0.499	0.499
7.5-8	3200-3350	1700-1800	1C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
7.5-8	3200-3350	1700-1800	4C	0.499	0.499	0.499	0.499	0.499	0.521	0.474	0.499	0.499
7.5-8	3200-3350	1700-1800	4C+	0.499	0.499	0.499	0.559	0.423	0.499	0.499	0.499	0.499
7.5-8	3200-3350	1800-1900	4C	0.499	0.499	0.499	0.499	0.494	0.499	0.504	0.499	0.499
7.5-8	3350-3500	1600-1700	1C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
7.5-8	3350-3500	1600-1700	4C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
7.5-8	3350-3500	1700-1800	4C+	0.499	0.499	0.499	0.498	0.499	0.499	0.499	0.499	0.499
7.5-8	3350-3500	1800-1900	4C+	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
7.5-8	3500-3650	1600-1700	3C+	0.499	0.499	0.344	0.592	0.499	0.499	0.499	0.499	0.499
7.5-8	3500-3650	1600-1700	4C	0.499	0.499	0.499	0.499	0.555	0.428	0.499	0.499	0.499
7.5-8	3500-3650	1800-1900	4C+	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
8-8.5	3200-3350	1600-1700	4C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
9-9.5	2300-2450	900-1000	2C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
9-9.5	3200-3350	1600-1700	2C	0.499	0.499	0.499	0.515	0.481	0.499	0.499	0.499	0.499
9-9.5	3200-3350	2000-2100	5C+	0.499	0.499	0.499	0.537	0.454	0.499	0.499	0.499	0.499
9-9.5	3350-3500	1600-1700	2C	0.499	0.499	0.499	0.495	0.502	0.499	0.499	0.499	0.499
9-9.5	3350-3500	1700-1800	2C	0.499	0.499	0.499	0.495	0.502	0.499	0.499	0.499	0.499
9.5-10	3200-3350	1600-1700	5C	0.499	0.499	0.499	0.499	0.499	0.500	0.522	0.485	0.485

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate								
				0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
9.5-10	3200-3350	1700-1800	5C	0.499	0.499	0.499	0.497	0.499	0.460	0.534	0.499	0.499
9.5-10	3200-3350	1900-2000	5C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
9.5-10	3350-3500	1200-1300	5C+	0.499	0.499	0.499	0.453	0.537	0.499	0.499	0.499	0.499
9.5-10	3350-3500	1600-1700	5C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
9.5-10	3650-3800	1200-1300	5C+	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
10-10.5	2900-3050	1400-1500	3C+	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
10-10.5	3500-3650	1900-2000	3C+	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
10.5-11	3200-3350	1700-1800	1C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
10.5-11	3200-3350	1700-1800	3C+	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
10.5-11	3200-3350	1900-2000	3C+	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
10.5-11	3800-3950	2600-2700	3C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
11-11.5	2900-3050	1400-1500	1C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
11.5-12	2900-3050	1300-1400	6C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499
11.5-12	2900-3050	1400-1500	6C	0.499	0.499	0.499	0.347	0.499	0.499	0.591	0.499	0.499
12.5-13	2900-3050	1000-1100	2C	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499	0.499

Table B-43 Layout A, Sample D: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year

Age	Fh / Y	Alpha-Index per Non-routine rate								
		0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
4.5-5	3800-3950	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3500-3650	0.284	0.683	0.442	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	4400-4550	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3500-3650	0.500	0.546	0.650	0.165	0.500	0.500	0.500	0.500	0.500
5.5-6	3800-3950	0.500	0.337	0.610	0.477	0.500	0.500	0.500	0.500	0.500
5.5-6	3950-4100	0.500	0.096	0.500	0.652	0.500	0.500	0.500	0.500	0.500
6-6.5	3500-3650	0.500	0.580	0.500	0.379	0.500	0.500	0.500	0.500	0.500
6-6.5	3650-3800	0.500	0.466	0.500	0.539	0.500	0.500	0.500	0.500	0.500
7-7.5	3200-3350	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7-7.5	3500-3650	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3200-3350	0.500	0.500	0.500	0.567	0.060	0.197	0.697	0.500	0.500
7.5-8	3350-3500	0.500	0.500	0.500	0.420	0.571	0.500	0.500	0.500	0.500
7.5-8	3500-3650	0.500	0.500	0.128	0.518	0.665	0.461	0.500	0.500	0.500
8-8.5	3200-3350	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	2300-2450	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	3200-3350	0.500	0.500	0.500	0.434	0.562	0.500	0.500	0.500	0.500
9-9.5	3350-3500	0.500	0.500	0.500	0.193	0.674	0.500	0.500	0.500	0.500
9.5-10	3200-3350	0.500	0.500	0.500	0.315	0.500	0.179	0.772	0.351	0.351
9.5-10	3350-3500	0.500	0.500	0.500	0.243	0.638	0.500	0.500	0.500	0.500
9.5-10	3500-3650	0.500	0.500	0.500	0.651	0.104	0.500	0.500	0.500	0.500
9.5-10	3650-3800	0.500	0.500	0.500	0.087	0.654	0.500	0.500	0.500	0.500
10-10.5	2900-3050	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
10-10.5	3500-3650	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
10.5-11	3200-3350	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

Age	Fh / Y	Alpha-Index per Non-routine rate								
		0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
10.5-11	3800-3950	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11-11.5	2900-3050	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11-11.5	3800-3950	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11.5-12	2900-3050	0.500	0.500	0.500	0.124	0.500	0.500	0.649	0.500	0.500
12.5-13	2900-3050	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

Table B-44 Layout A, Sample D: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year with cycles per year

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate								
			0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
4.5-5	3800-3950	1400-1500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
4.5-5	3800-3950	1500-1600	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3500-3650	1600-1700	0.500	0.587	0.397	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3500-3650	1700-1800	0.284	0.619	0.529	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3500-3650	1800-1900	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	4400-4550	1900-2000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3500-3650	1600-1700	0.500	0.682	0.235	0.413	0.500	0.500	0.500	0.500	0.500
5.5-6	3500-3650	1700-1800	0.500	0.026	0.723	0.303	0.500	0.500	0.500	0.500	0.500
5.5-6	3800-3950	1300-1400	0.500	0.522	0.500	0.477	0.500	0.500	0.500	0.500	0.500
5.5-6	3800-3950	1400-1500	0.500	0.300	0.610	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3950-4100	1400-1500	0.500	0.096	0.500	0.652	0.500	0.500	0.500	0.500	0.500
5.5-6	3950-4100	1600-1700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
6-6.5	3500-3650	1700-1800	0.500	0.580	0.500	0.379	0.500	0.500	0.500	0.500	0.500
6-6.5	3650-3800	1500-1600	0.500	0.164	0.500	0.647	0.500	0.500	0.500	0.500	0.500
6-6.5	3650-3800	1600-1700	0.500	0.620	0.500	0.188	0.500	0.500	0.500	0.500	0.500
7-7.5	3200-3350	1500-1600	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate								
			0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
7-7.5	3200-3350	1600-1700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7-7.5	3200-3350	1700-1800	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7-7.5	3500-3650	1600-1700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3200-3350	1600-1700	0.500	0.500	0.500	0.500	0.436	0.197	0.672	0.500	0.500
7.5-8	3200-3350	1700-1800	0.500	0.500	0.500	0.567	0.337	0.500	0.503	0.500	0.500
7.5-8	3200-3350	1800-1900	0.500	0.500	0.500	0.500	0.452	0.500	0.543	0.500	0.500
7.5-8	3350-3500	1600-1700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3350-3500	1700-1800	0.500	0.500	0.500	0.420	0.571	0.500	0.500	0.500	0.500
7.5-8	3350-3500	1800-1900	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3500-3650	1600-1700	0.500	0.500	0.128	0.648	0.533	0.461	0.500	0.500	0.500
7.5-8	3500-3650	1700-1800	0.500	0.500	0.500	0.239	0.651	0.500	0.500	0.500	0.500
7.5-8	3500-3650	1800-1900	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
8-8.5	3200-3350	1600-1700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	2300-2450	900-1000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	3200-3350	1600-1700	0.500	0.500	0.500	0.150	0.649	0.500	0.500	0.500	0.500
9-9.5	3200-3350	2000-2100	0.500	0.500	0.500	0.616	0.272	0.500	0.500	0.500	0.500
9-9.5	3350-3500	1600-1700	0.500	0.500	0.500	0.369	0.600	0.500	0.500	0.500	0.500
9-9.5	3350-3500	1700-1800	0.500	0.500	0.500	0.369	0.600	0.500	0.500	0.500	0.500
9.5-10	3200-3350	1600-1700	0.500	0.500	0.500	0.445	0.500	0.179	0.721	0.351	0.351
9.5-10	3200-3350	1700-1800	0.500	0.500	0.500	0.381	0.500	0.500	0.601	0.500	0.500
9.5-10	3200-3350	1900-2000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9.5-10	3350-3500	1200-1300	0.500	0.500	0.500	0.243	0.638	0.500	0.500	0.500	0.500
9.5-10	3350-3500	1600-1700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9.5-10	3500-3650	1200-1300	0.500	0.500	0.500	0.651	0.104	0.500	0.500	0.500	0.500
9.5-10	3650-3800	1200-1300	0.500	0.500	0.500	0.087	0.654	0.500	0.500	0.500	0.500
10-10.5	2900-3050	1400-1500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate								
			0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
10-10.5	3500-3650	1900-2000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
10.5-11	3200-3350	1700-1800	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
10.5-11	3800-3950	2600-2700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11-11.5	2900-3050	1400-1500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11-11.5	3800-3950	2600-2700	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11.5-12	2900-3050	1300-1400	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11.5-12	2900-3050	1400-1500	0.500	0.500	0.500	0.124	0.500	0.500	0.649	0.500	0.500
12.5-13	2900-3050	1000-1100	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

Table B-45 Layout A, Sample D: Optimised alpha-index per non-routine rate for combining aeroplane age, flight hours and cycles per year with service type

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate								
				0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
4.5-5	3800-3950	1400-1500	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
4.5-5	3800-3950	1500-1600	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3500-3650	1600-1700	3C+	0.500	0.587	0.397	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3500-3650	1700-1800	3C	0.500	0.001	0.664	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3500-3650	1700-1800	3C+	0.284	0.713	0.174	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	3500-3650	1800-1900	3C+	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5-5.5	4400-4550	1900-2000	3C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3500-3650	1600-1700	3C+	0.500	0.682	0.235	0.413	0.500	0.500	0.500	0.500	0.500
5.5-6	3500-3650	1700-1800	3C+	0.500	0.026	0.723	0.303	0.500	0.500	0.500	0.500	0.500
5.5-6	3800-3950	1300-1400	3C+	0.500	0.522	0.500	0.477	0.500	0.500	0.500	0.500	0.500
5.5-6	3800-3950	1400-1500	3C	0.500	0.300	0.610	0.500	0.500	0.500	0.500	0.500	0.500
5.5-6	3950-4100	1400-1500	2C	0.500	0.096	0.500	0.652	0.500	0.500	0.500	0.500	0.500
5.5-6	3950-4100	1600-1700	3C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate								
				0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
6-6.5	3500-3650	1700-1800	2C	0.500	0.580	0.500	0.379	0.500	0.500	0.500	0.500	0.500
6-6.5	3650-3800	1500-1600	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
6-6.5	3650-3800	1500-1600	2C	0.500	0.164	0.500	0.647	0.500	0.500	0.500	0.500	0.500
6-6.5	3650-3800	1600-1700	2C	0.500	0.620	0.500	0.188	0.500	0.500	0.500	0.500	0.500
7-7.5	3200-3350	1500-1600	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7-7.5	3200-3350	1600-1700	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7-7.5	3200-3350	1700-1800	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7-7.5	3500-3650	1600-1700	2C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3200-3350	1600-1700	4C	0.500	0.500	0.500	0.500	0.436	0.197	0.672	0.500	0.500
7.5-8	3200-3350	1700-1800	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3200-3350	1700-1800	4C	0.500	0.500	0.500	0.500	0.497	0.500	0.503	0.500	0.500
7.5-8	3200-3350	1700-1800	4C+	0.500	0.500	0.500	0.567	0.342	0.500	0.500	0.500	0.500
7.5-8	3200-3350	1800-1900	4C	0.500	0.500	0.500	0.500	0.452	0.500	0.543	0.500	0.500
7.5-8	3350-3500	1600-1700	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3350-3500	1600-1700	4C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3350-3500	1700-1800	4C+	0.500	0.500	0.500	0.420	0.571	0.500	0.500	0.500	0.500
7.5-8	3350-3500	1800-1900	4C+	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
7.5-8	3500-3650	1600-1700	3C+	0.500	0.500	0.128	0.648	0.500	0.500	0.500	0.500	0.500
7.5-8	3500-3650	1600-1700	4C	0.500	0.500	0.500	0.500	0.533	0.461	0.500	0.500	0.500
7.5-8	3500-3650	1700-1800	4C+	0.500	0.500	0.500	0.239	0.651	0.500	0.500	0.500	0.500
7.5-8	3500-3650	1800-1900	4C+	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
8-8.5	3200-3350	1600-1700	4C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	2300-2450	900-1000	2C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9-9.5	3200-3350	1600-1700	2C	0.500	0.500	0.500	0.150	0.649	0.500	0.500	0.500	0.500
9-9.5	3200-3350	2000-2100	5C+	0.500	0.500	0.500	0.616	0.272	0.500	0.500	0.500	0.500
9-9.5	3350-3500	1600-1700	2C	0.500	0.500	0.500	0.369	0.600	0.500	0.500	0.500	0.500

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate								
				0.05-0.1	***0.1-0.15***	***0.15-0.2***	***0.2-0.25***	***0.25-0.3***	***0.3-0.35***	***0.35-0.4***	***0.4-0.45***	***0.45-0.5***
9-9.5	3350-3500	1700-1800	2C	0.500	0.500	0.500	0.369	0.600	0.500	0.500	0.500	0.500
9.5-10	3200-3350	1600-1700	5C	0.500	0.500	0.500	0.445	0.500	0.179	0.721	0.351	0.351
9.5-10	3200-3350	1700-1800	5C	0.500	0.500	0.500	0.381	0.500	0.500	0.601	0.500	0.500
9.5-10	3200-3350	1900-2000	5C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9.5-10	3350-3500	1200-1300	5C+	0.500	0.500	0.500	0.243	0.638	0.500	0.500	0.500	0.500
9.5-10	3350-3500	1600-1700	5C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
9.5-10	3500-3650	1200-1300	5C+	0.500	0.500	0.500	0.651	0.104	0.500	0.500	0.500	0.500
9.5-10	3650-3800	1200-1300	5C+	0.500	0.500	0.500	0.087	0.654	0.500	0.500	0.500	0.500
10-10.5	2900-3050	1400-1500	3C+	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
10-10.5	3500-3650	1900-2000	3C+	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
10.5-11	3200-3350	1700-1800	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
10.5-11	3200-3350	1700-1800	3C+	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
10.5-11	3800-3950	2600-2700	3C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11-11.5	2900-3050	1400-1500	1C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11-11.5	3800-3950	2600-2700	6C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11.5-12	2900-3050	1300-1400	6C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
11.5-12	2900-3050	1400-1500	6C	0.500	0.500	0.500	0.124	0.500	0.500	0.649	0.500	0.500
12.5-13	2900-3050	1000-1100	2C	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500

Table B-46 Layout B, Sample A: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year

Age	Fh / Y	Alpha-Index per Non-routine rate				
		0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
2.5-5	3800-3950	0.500	0.500	0.500	0.500	0.500
5-7.5	3200-3350	0.500	0.500	0.500	0.500	0.500
5-7.5	3500-3650	0.386	0.597	0.462	0.500	0.500
5-7.5	3650-3800	0.500	0.480	0.518	0.500	0.500
5-7.5	3800-3950	0.500	0.524	0.476	0.500	0.500
5-7.5	3950-4100	0.500	0.500	0.500	0.500	0.500
5-7.5	4400-4550	0.500	0.500	0.500	0.500	0.500
7.5-10	2300-2450	0.500	0.500	0.500	0.500	0.500
7.5-10	3200-3350	0.500	0.166	0.335	0.746	0.595
7.5-10	3350-3500	0.500	0.490	0.510	0.500	0.500
7.5-10	3500-3650	0.500	0.094	0.644	0.440	0.500
7.5-10	3650-3800	0.500	0.500	0.500	0.500	0.500
10-12.5	2900-3050	0.500	0.500	0.283	0.590	0.500
10-12.5	3200-3350	0.500	0.679	0.201	0.500	0.500
10-12.5	3800-3950	0.500	0.500	0.500	0.500	0.500
12.5-15	2900-3050	0.500	0.500	0.500	0.500	0.500

Table B-47 Layout B, Sample A: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year with cycles per year

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate				
			0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
2.5-5	3800-3950	1200-1500	0.500	0.500	0.500	0.500	0.500
2.5-5	3800-3950	1500-1800	0.500	0.500	0.500	0.500	0.500
5-7.5	3200-3350	1500-1800	0.500	0.500	0.500	0.500	0.500
5-7.5	3500-3650	1500-1800	0.386	0.597	0.462	0.500	0.500
5-7.5	3650-3800	1500-1800	0.500	0.480	0.518	0.500	0.500
5-7.5	3800-3950	1200-1500	0.500	0.524	0.476	0.500	0.500
5-7.5	3950-4100	1500-1800	0.500	0.500	0.500	0.500	0.500
5-7.5	4400-4550	1800-2100	0.500	0.500	0.500	0.500	0.500
7.5-10	2300-2450	900-1200	0.500	0.500	0.500	0.500	0.500
7.5-10	3200-3350	1500-1800	0.500	0.166	0.512	0.613	0.595
7.5-10	3200-3350	1800-2100	0.500	0.500	0.281	0.663	0.500
7.5-10	3350-3500	1200-1500	0.500	0.500	0.500	0.500	0.500
7.5-10	3350-3500	1500-1800	0.500	0.490	0.510	0.500	0.500
7.5-10	3350-3500	1800-2100	0.500	0.500	0.500	0.500	0.500

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate				
			0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
7.5-10	3500-3650	1200-1500	0.500	0.500	0.500	0.500	0.500
7.5-10	3500-3650	1500-1800	0.500	0.094	0.644	0.440	0.500
7.5-10	3500-3650	1800-2100	0.500	0.500	0.500	0.500	0.500
7.5-10	3650-3800	1200-1500	0.500	0.500	0.500	0.500	0.500
10-12.5	2900-3050	1200-1500	0.500	0.500	0.283	0.590	0.500
10-12.5	3200-3350	1500-1800	0.500	0.582	0.405	0.500	0.500
10-12.5	3200-3350	1800-2100	0.500	0.622	0.327	0.500	0.500
10-12.5	3800-3950	2400-2700	0.500	0.500	0.500	0.500	0.500
12.5-15	2900-3050	900-1200	0.500	0.500	0.500	0.500	0.500

Table B-48 Layout B, Sample A: Optimised alpha-index per non-routine rate for combining aeroplane age, flight hours and cycles per year with service type

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate				
				0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
2.5-5	3800-3950	1200-1500	1C	0.500	0.500	0.500	0.500	0.500
2.5-5	3800-3950	1500-1800	1C	0.500	0.500	0.500	0.500	0.500
5-7.5	3200-3350	1500-1800	1C	0.500	0.500	0.500	0.500	0.500
5-7.5	3500-3650	1500-1800	2C	0.500	0.273	0.640	0.500	0.500
5-7.5	3500-3650	1500-1800	3C	0.500	0.500	0.500	0.500	0.500
5-7.5	3500-3650	1500-1800	3C+	0.386	0.706	0.260	0.500	0.500
5-7.5	3650-3800	1500-1800	1C	0.500	0.500	0.500	0.500	0.500
5-7.5	3650-3800	1500-1800	2C	0.500	0.480	0.518	0.500	0.500
5-7.5	3650-3800	1500-1800	3C	0.500	0.500	0.500	0.500	0.500
5-7.5	3800-3950	1200-1500	3C	0.500	0.500	0.500	0.500	0.500
5-7.5	3800-3950	1200-1500	3C+	0.500	0.524	0.476	0.500	0.500
5-7.5	3950-4100	1500-1800	3C	0.500	0.500	0.500	0.500	0.500
5-7.5	3950-4100	1500-1800	3C+	0.500	0.500	0.500	0.500	0.500
5-7.5	4400-4550	1800-2100	3C	0.500	0.500	0.500	0.500	0.500
7.5-10	2300-2450	900-1200	2C	0.500	0.500	0.500	0.500	0.500
7.5-10	3200-3350	1500-1800	1C	0.500	0.500	0.500	0.500	0.500
7.5-10	3200-3350	1500-1800	2C	0.500	0.166	0.671	0.500	0.500
7.5-10	3200-3350	1500-1800	4C	0.500	0.500	0.357	0.621	0.500
7.5-10	3200-3350	1500-1800	4C+	0.500	0.500	0.500	0.500	0.500
7.5-10	3200-3350	1500-1800	5C	0.500	0.500	0.405	0.490	0.595
7.5-10	3200-3350	1800-2100	4C	0.500	0.500	0.436	0.558	0.500
7.5-10	3200-3350	1800-2100	5C	0.500	0.500	0.357	0.617	0.500
7.5-10	3200-3350	1800-2100	5C+	0.500	0.500	0.500	0.500	0.500

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate				
				0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
7.5-10	3350-3500	1200-1500	5C+	0.500	0.500	0.500	0.500	0.500
7.5-10	3350-3500	1500-1800	1C	0.500	0.500	0.500	0.500	0.500
7.5-10	3350-3500	1500-1800	2C	0.500	0.490	0.510	0.500	0.500
7.5-10	3350-3500	1500-1800	4C	0.500	0.500	0.500	0.500	0.500
7.5-10	3350-3500	1500-1800	4C+	0.500	0.500	0.500	0.500	0.500
7.5-10	3350-3500	1500-1800	5C	0.500	0.500	0.500	0.500	0.500
7.5-10	3350-3500	1800-2100	4C+	0.500	0.500	0.500	0.500	0.500
7.5-10	3500-3650	1200-1500	5C+	0.500	0.500	0.500	0.500	0.500
7.5-10	3500-3650	1500-1800	3C+	0.500	0.094	0.619	0.500	0.500
7.5-10	3500-3650	1500-1800	4C	0.500	0.500	0.548	0.440	0.500
7.5-10	3500-3650	1500-1800	4C+	0.500	0.500	0.500	0.500	0.500
7.5-10	3500-3650	1800-2100	4C+	0.500	0.500	0.500	0.500	0.500
7.5-10	3650-3800	1200-1500	5C+	0.500	0.500	0.500	0.500	0.500
10-12.5	2900-3050	1200-1500	1C	0.500	0.500	0.500	0.500	0.500
10-12.5	2900-3050	1200-1500	6C	0.500	0.500	0.283	0.590	0.500
10-12.5	3200-3350	1500-1800	1C	0.500	0.500	0.500	0.500	0.500
10-12.5	3200-3350	1500-1800	3C+	0.500	0.582	0.405	0.500	0.500
10-12.5	3200-3350	1800-2100	3C+	0.500	0.622	0.327	0.500	0.500
10-12.5	3800-3950	2400-2700	3C	0.500	0.500	0.500	0.500	0.500
10-12.5	3800-3950	2400-2700	6C	0.500	0.500	0.500	0.500	0.500
12.5-15	2900-3050	900-1200	2C	0.500	0.500	0.500	0.500	0.500

Table B-49 Layout B, Sample B: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year

Age	Fh / Y	Alpha-Index per Non-routine rate				
		0-0.1	0.1-0.2	0.2-0.3	0.3-0.4	0.4-0.5
2.5-5	3800-3950	0.498	0.498	0.498	0.498	0.498
5-7.5	3200-3350	0.498	0.498	0.498	0.498	0.498
5-7.5	3500-3650	0.412	0.636	0.392	0.498	0.498
5-7.5	3650-3800	0.498	0.498	0.498	0.498	0.498
5-7.5	3800-3950	0.498	0.503	0.493	0.498	0.498
5-7.5	3950-4100	0.498	0.378	0.493	0.498	0.498
5-7.5	4400-4550	0.498	0.498	0.498	0.498	0.498
7.5-10	2300-2450	0.498	0.498	0.498	0.498	0.498
7.5-10	3200-3350	0.498	0.244	0.221	0.757	0.627
7.5-10	3350-3500	0.498	0.498	0.498	0.498	0.498
7.5-10	3500-3650	0.498	0.092	0.589	0.498	0.498
7.5-10	3650-3800	0.498	0.498	0.498	0.498	0.498
10-12.5	2900-3050	0.498	0.647	0.283	0.498	0.498
10-12.5	3200-3350	0.498	0.705	0.201	0.498	0.498
10-12.5	3500-3650	0.498	0.650	0.285	0.498	0.498
10-12.5	3800-3950	0.498	0.498	0.498	0.498	0.498
12.5-15	2900-3050	0.498	0.498	0.498	0.498	0.498

Table B-50 Layout B, Sample B: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year with cycles per year

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate				
			0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
2.5-5	3800-3950	1200-1500	0.498	0.498	0.498	0.498	0.498
2.5-5	3800-3950	1500-1800	0.498	0.498	0.498	0.498	0.498
5-7.5	3200-3350	1500-1800	0.498	0.498	0.498	0.498	0.498
5-7.5	3500-3650	1500-1800	0.412	0.501	0.543	0.498	0.498
5-7.5	3500-3650	1800-2100	0.498	0.643	0.323	0.498	0.498
5-7.5	3650-3800	1500-1800	0.498	0.498	0.498	0.498	0.498
5-7.5	3800-3950	1200-1500	0.498	0.503	0.493	0.498	0.498
5-7.5	3950-4100	1200-1500	0.498	0.036	0.593	0.498	0.498
5-7.5	3950-4100	1500-1800	0.498	0.638	0.327	0.498	0.498
5-7.5	4400-4550	1800-2100	0.498	0.498	0.498	0.498	0.498
7.5-10	2300-2450	900-1200	0.498	0.498	0.498	0.498	0.498
7.5-10	3200-3350	1500-1800	0.498	0.244	0.482	0.602	0.627
7.5-10	3200-3350	1800-2100	0.498	0.498	0.249	0.674	0.498
7.5-10	3350-3500	1200-1500	0.498	0.498	0.498	0.498	0.498
7.5-10	3350-3500	1500-1800	0.498	0.498	0.498	0.498	0.498
7.5-10	3350-3500	1800-2100	0.498	0.498	0.498	0.498	0.498
7.5-10	3500-3650	1200-1500	0.498	0.498	0.498	0.498	0.498
7.5-10	3500-3650	1500-1800	0.498	0.092	0.589	0.498	0.498

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate				
			0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
7.5-10	3500-3650	1800-2100	0.498	0.498	0.498	0.498	0.498
7.5-10	3650-3800	1200-1500	0.498	0.498	0.498	0.498	0.498
10-12.5	2900-3050	1200-1500	0.498	0.647	0.283	0.498	0.498
10-12.5	3200-3350	1500-1800	0.498	0.604	0.374	0.498	0.498
10-12.5	3200-3350	1800-2100	0.498	0.618	0.353	0.498	0.498
10-12.5	3500-3650	1800-2100	0.498	0.650	0.285	0.498	0.498
10-12.5	3800-3950	2400-2700	0.498	0.498	0.498	0.498	0.498
12.5-15	2900-3050	900-1200	0.498	0.498	0.498	0.498	0.498

Table B-51 Layout B, Sample B: Optimised alpha-index per non-routine rate for combining aeroplane age, flight hours and cycles per year with service type

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate				
				0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
2.5-5	3800-3950	1200-1500	1C	0.498	0.498	0.498	0.498	0.498
2.5-5	3800-3950	1500-1800	1C	0.498	0.498	0.498	0.498	0.498
5-7.5	3200-3350	1500-1800	1C	0.498	0.498	0.498	0.498	0.498
5-7.5	3500-3650	1500-1800	2C	0.498	0.312	0.623	0.498	0.498
5-7.5	3500-3650	1500-1800	3C	0.498	0.498	0.498	0.498	0.498
5-7.5	3500-3650	1500-1800	3C+	0.412	0.615	0.399	0.498	0.498
5-7.5	3500-3650	1800-2100	3C+	0.498	0.643	0.323	0.498	0.498
5-7.5	3650-3800	1500-1800	1C	0.498	0.498	0.498	0.498	0.498
5-7.5	3650-3800	1500-1800	2C	0.498	0.498	0.498	0.498	0.498
5-7.5	3650-3800	1500-1800	3C	0.498	0.498	0.498	0.498	0.498
5-7.5	3800-3950	1200-1500	3C	0.498	0.498	0.498	0.498	0.498
5-7.5	3800-3950	1200-1500	3C+	0.498	0.503	0.493	0.498	0.498
5-7.5	3950-4100	1200-1500	2C	0.498	0.036	0.593	0.498	0.498
5-7.5	3950-4100	1500-1800	3C	0.498	0.498	0.498	0.498	0.498
5-7.5	3950-4100	1500-1800	3C+	0.498	0.638	0.327	0.498	0.498
5-7.5	4400-4550	1800-2100	3C	0.498	0.498	0.498	0.498	0.498
7.5-10	2300-2450	900-1200	2C	0.498	0.498	0.498	0.498	0.498
7.5-10	3200-3350	1500-1800	1C	0.498	0.498	0.498	0.498	0.498
7.5-10	3200-3350	1500-1800	2C	0.498	0.244	0.702	0.498	0.498
7.5-10	3200-3350	1500-1800	4C	0.498	0.498	0.319	0.644	0.498
7.5-10	3200-3350	1500-1800	4C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3200-3350	1500-1800	5C	0.498	0.498	0.402	0.441	0.627
7.5-10	3200-3350	1800-2100	4C	0.498	0.498	0.409	0.577	0.498
7.5-10	3200-3350	1800-2100	5C	0.498	0.498	0.360	0.611	0.498
7.5-10	3200-3350	1800-2100	5C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3350-3500	1200-1500	5C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3350-3500	1500-1800	2C	0.498	0.498	0.498	0.498	0.498
7.5-10	3350-3500	1500-1800	4C	0.498	0.498	0.498	0.498	0.498
7.5-10	3350-3500	1500-1800	4C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3350-3500	1500-1800	5C	0.498	0.498	0.498	0.498	0.498

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate				
				0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
7.5-10	3350-3500	1800-2100	4C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3500-3650	1200-1500	5C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3500-3650	1500-1800	3C+	0.498	0.092	0.589	0.498	0.498
7.5-10	3500-3650	1500-1800	4C	0.498	0.498	0.498	0.498	0.498
7.5-10	3500-3650	1500-1800	4C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3500-3650	1800-2100	4C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3650-3800	1200-1500	5C+	0.498	0.498	0.498	0.498	0.498
10-12.5	2900-3050	1200-1500	1C	0.498	0.498	0.498	0.498	0.498
10-12.5	2900-3050	1200-1500	3C+	0.498	0.647	0.283	0.498	0.498
10-12.5	2900-3050	1200-1500	6C	0.498	0.498	0.498	0.498	0.498
10-12.5	3200-3350	1500-1800	1C	0.498	0.498	0.498	0.498	0.498
10-12.5	3200-3350	1500-1800	3C+	0.498	0.604	0.374	0.498	0.498
10-12.5	3200-3350	1800-2100	3C+	0.498	0.618	0.353	0.498	0.498
10-12.5	3500-3650	1800-2100	3C+	0.498	0.650	0.285	0.498	0.498
10-12.5	3800-3950	2400-2700	6C	0.498	0.498	0.498	0.498	0.498
12.5-15	2900-3050	900-1200	2C	0.498	0.498	0.498	0.498	0.498

Table B-52 Layout B, Sample C: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year

Age	Fh / Y	Alpha-Index per Non-routine rate				
		0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
2.5-5	3800-3950	0.500	0.500	0.500	0.500	0.500
5-7.5	3200-3350	0.500	0.500	0.500	0.500	0.500
5-7.5	3500-3650	0.405	0.637	0.387	0.500	0.500
5-7.5	3650-3800	0.500	0.203	0.645	0.500	0.500
5-7.5	3800-3950	0.500	0.500	0.500	0.500	0.500
5-7.5	3950-4100	0.500	0.408	0.561	0.500	0.500
5-7.5	4400-4550	0.500	0.500	0.500	0.500	0.500
7.5-10	2300-2450	0.500	0.500	0.500	0.500	0.500
7.5-10	3200-3350	0.500	0.240	0.148	0.728	0.596
7.5-10	3350-3500	0.500	0.499	0.502	0.500	0.500
7.5-10	3500-3650	0.500	0.071	0.675	0.402	0.500
7.5-10	3650-3800	0.500	0.500	0.500	0.500	0.500
10-12.5	2900-3050	0.500	0.647	0.285	0.484	0.500
10-12.5	3200-3350	0.500	0.683	0.205	0.500	0.500
10-12.5	3500-3650	0.500	0.621	0.354	0.500	0.500
10-12.5	3800-3950	0.500	0.500	0.500	0.500	0.500
12.5-15	2900-3050	0.500	0.500	0.500	0.500	0.500

Table B-53 Layout B, Sample C: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year with cycles per year

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate				
			0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
2.5-5	3800-3950	1200-1500	0.500	0.500	0.500	0.500	0.500
2.5-5	3800-3950	1500-1800	0.500	0.500	0.500	0.500	0.500
5-7.5	3200-3350	1500-1800	0.500	0.500	0.500	0.500	0.500
5-7.5	3500-3650	1500-1800	0.405	0.560	0.497	0.500	0.500
5-7.5	3500-3650	1800-2100	0.500	0.594	0.395	0.500	0.500
5-7.5	3650-3800	1500-1800	0.500	0.203	0.645	0.500	0.500
5-7.5	3800-3950	1200-1500	0.500	0.500	0.500	0.500	0.500
5-7.5	3950-4100	1200-1500	0.500	0.066	0.652	0.500	0.500
5-7.5	3950-4100	1500-1800	0.500	0.638	0.318	0.500	0.500
5-7.5	4400-4550	1800-2100	0.500	0.500	0.500	0.500	0.500
7.5-10	2300-2450	900-1200	0.500	0.500	0.500	0.500	0.500
7.5-10	3200-3350	1500-1800	0.500	0.240	0.457	0.641	0.596
7.5-10	3200-3350	1800-2100	0.500	0.500	0.228	0.644	0.500
7.5-10	3350-3500	1200-1500	0.500	0.500	0.500	0.500	0.500
7.5-10	3350-3500	1500-1800	0.500	0.499	0.502	0.500	0.500
7.5-10	3350-3500	1800-2100	0.500	0.500	0.500	0.500	0.500
7.5-10	3500-3650	1500-1800	0.500	0.071	0.675	0.402	0.500
7.5-10	3500-3650	1800-2100	0.500	0.500	0.500	0.500	0.500

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate				
			0-0.1	*0.1-0.2*	*0.2-0.3*	*0.3-0.4*	*0.4-0.5*
7.5-10	3650-3800	1200-1500	0.500	0.500	0.500	0.500	0.500
10-12.5	2900-3050	1200-1500	0.500	0.647	0.285	0.484	0.500
10-12.5	3200-3350	1500-1800	0.500	0.597	0.383	0.500	0.500
10-12.5	3200-3350	1800-2100	0.500	0.611	0.356	0.500	0.500
10-12.5	3500-3650	1800-2100	0.500	0.621	0.354	0.500	0.500
10-12.5	3800-3950	2400-2700	0.500	0.500	0.500	0.500	0.500
12.5-15	2900-3050	900-1200	0.500	0.500	0.500	0.500	0.500

Table B-54 Layout B, Sample C: Optimised alpha-index per non-routine rate for combining aeroplane age, flight hours and cycles per year with service type

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate				
				0-0.1	*0.1-0.2*	*0.2-0.3*	*0.3-0.4*	*0.4-0.5*
2.5-5	3800-3950	1200-1500	1C	0.500	0.500	0.500	0.500	0.500
2.5-5	3800-3950	1500-1800	1C	0.500	0.500	0.500	0.500	0.500
5-7.5	3200-3350	1500-1800	1C	0.500	0.500	0.500	0.500	0.500
5-7.5	3500-3650	1500-1800	2C	0.500	0.475	0.525	0.500	0.500
5-7.5	3500-3650	1500-1800	3C	0.500	0.500	0.500	0.500	0.500
5-7.5	3500-3650	1500-1800	3C+	0.405	0.586	0.472	0.500	0.500
5-7.5	3500-3650	1800-2100	3C+	0.500	0.594	0.395	0.500	0.500
5-7.5	3650-3800	1500-1800	1C	0.500	0.500	0.500	0.500	0.500
5-7.5	3650-3800	1500-1800	2C	0.500	0.203	0.645	0.500	0.500
5-7.5	3650-3800	1500-1800	3C	0.500	0.500	0.500	0.500	0.500
5-7.5	3800-3950	1200-1500	3C	0.500	0.500	0.500	0.500	0.500
5-7.5	3800-3950	1200-1500	3C+	0.500	0.500	0.500	0.500	0.500
5-7.5	3950-4100	1200-1500	2C	0.500	0.066	0.652	0.500	0.500
5-7.5	3950-4100	1500-1800	3C	0.500	0.500	0.500	0.500	0.500
5-7.5	3950-4100	1500-1800	3C+	0.500	0.638	0.318	0.500	0.500
5-7.5	4400-4550	1800-2100	3C	0.500	0.500	0.500	0.500	0.500
7.5-10	2300-2450	900-1200	2C	0.500	0.500	0.500	0.500	0.500
7.5-10	3200-3350	1500-1800	1C	0.500	0.500	0.500	0.500	0.500
7.5-10	3200-3350	1500-1800	2C	0.500	0.240	0.692	0.500	0.500
7.5-10	3200-3350	1500-1800	4C	0.500	0.500	0.317	0.635	0.500
7.5-10	3200-3350	1500-1800	4C+	0.500	0.500	0.500	0.500	0.500
7.5-10	3200-3350	1500-1800	5C	0.500	0.500	0.382	0.504	0.596
7.5-10	3200-3350	1800-2100	4C	0.500	0.500	0.412	0.571	0.500
7.5-10	3200-3350	1800-2100	5C	0.500	0.500	0.354	0.599	0.500
7.5-10	3200-3350	1800-2100	5C+	0.500	0.500	0.500	0.500	0.500
7.5-10	3350-3500	1200-1500	5C+	0.500	0.500	0.500	0.500	0.500
7.5-10	3350-3500	1500-1800	1C	0.500	0.500	0.500	0.500	0.500
7.5-10	3350-3500	1500-1800	2C	0.500	0.499	0.502	0.500	0.500
7.5-10	3350-3500	1500-1800	4C	0.500	0.500	0.500	0.500	0.500
7.5-10	3350-3500	1500-1800	4C+	0.500	0.500	0.500	0.500	0.500
7.5-10	3350-3500	1500-1800	5C	0.500	0.500	0.500	0.500	0.500

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate				
				0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
7.5-10	3350-3500	1800-2100	4C+	0.500	0.500	0.500	0.500	0.500
7.5-10	3500-3650	1500-1800	3C+	0.500	0.071	0.638	0.500	0.500
7.5-10	3500-3650	1500-1800	4C	0.500	0.500	0.566	0.402	0.500
7.5-10	3500-3650	1800-2100	4C+	0.500	0.500	0.500	0.500	0.500
7.5-10	3650-3800	1200-1500	5C+	0.500	0.500	0.500	0.500	0.500
10-12.5	2900-3050	1200-1500	1C	0.500	0.500	0.500	0.500	0.500
10-12.5	2900-3050	1200-1500	3C+	0.500	0.647	0.294	0.500	0.500
10-12.5	2900-3050	1200-1500	6C	0.500	0.500	0.515	0.484	0.500
10-12.5	3200-3350	1500-1800	1C	0.500	0.500	0.500	0.500	0.500
10-12.5	3200-3350	1500-1800	3C+	0.500	0.597	0.383	0.500	0.500
10-12.5	3200-3350	1800-2100	3C+	0.500	0.611	0.356	0.500	0.500
10-12.5	3500-3650	1800-2100	3C+	0.500	0.621	0.354	0.500	0.500
10-12.5	3800-3950	2400-2700	3C	0.500	0.500	0.500	0.500	0.500
12.5-15	2900-3050	900-1200	2C	0.500	0.500	0.500	0.500	0.500

Table B-55 Layout B, Sample D: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year

Age	Fh / Y	Alpha-Index per Non-routine rate				
		0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
2.5-5	3800-3950	0.498	0.498	0.498	0.498	0.498
5-7.5	3200-3350	0.498	0.498	0.498	0.498	0.498
5-7.5	3500-3650	0.400	0.665	0.377	0.498	0.498
5-7.5	3650-3800	0.498	0.526	0.471	0.498	0.498
5-7.5	3800-3950	0.498	0.503	0.494	0.498	0.498
5-7.5	3950-4100	0.498	0.111	0.637	0.498	0.498
5-7.5	4400-4550	0.498	0.498	0.498	0.498	0.498
7.5-10	2300-2450	0.498	0.498	0.498	0.498	0.498
7.5-10	3200-3350	0.498	0.260	0.207	0.744	0.596
7.5-10	3350-3500	0.498	0.495	0.501	0.498	0.498
7.5-10	3500-3650	0.498	0.098	0.650	0.452	0.498
7.5-10	3650-3800	0.498	0.498	0.498	0.498	0.498
10-12.5	2900-3050	0.498	0.605	0.323	0.525	0.498
10-12.5	3200-3350	0.498	0.632	0.326	0.498	0.498
10-12.5	3500-3650	0.498	0.637	0.313	0.498	0.498
10-12.5	3800-3950	0.498	0.498	0.498	0.498	0.498
12.5-15	2900-3050	0.498	0.498	0.498	0.498	0.498

Table B-56 Layout B, Sample D: Optimised alpha-index per non-routine rate for combining aeroplane age and flight hours per year with cycles per year

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate				
			0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
2.5-5	3800-3950	1200-1500	0.498	0.498	0.498	0.498	0.498
2.5-5	3800-3950	1500-1800	0.498	0.498	0.498	0.498	0.498
5-7.5	3200-3350	1500-1800	0.498	0.498	0.498	0.498	0.498
5-7.5	3500-3650	1500-1800	0.400	0.536	0.544	0.498	0.498
5-7.5	3500-3650	1800-2100	0.498	0.648	0.298	0.498	0.498
5-7.5	3650-3800	1500-1800	0.498	0.526	0.471	0.498	0.498
5-7.5	3800-3950	1200-1500	0.498	0.503	0.494	0.498	0.498
5-7.5	3950-4100	1200-1500	0.498	0.111	0.637	0.498	0.498
5-7.5	3950-4100	1500-1800	0.498	0.498	0.498	0.498	0.498
5-7.5	4400-4550	1800-2100	0.498	0.498	0.498	0.498	0.498
7.5-10	2300-2450	900-1200	0.498	0.498	0.498	0.498	0.498
7.5-10	3200-3350	1500-1800	0.498	0.260	0.452	0.635	0.596
7.5-10	3200-3350	1800-2100	0.498	0.498	0.260	0.660	0.498
7.5-10	3350-3500	1200-1500	0.498	0.498	0.498	0.498	0.498
7.5-10	3350-3500	1500-1800	0.498	0.495	0.501	0.498	0.498
7.5-10	3350-3500	1800-2100	0.498	0.498	0.498	0.498	0.498
7.5-10	3500-3650	1200-1500	0.498	0.498	0.498	0.498	0.498
7.5-10	3500-3650	1500-1800	0.498	0.098	0.650	0.452	0.498

Age	Fh / Y	Cy / Y	Alpha-Index per Non-routine rate				
			0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
7.5-10	3500-3650	1800-2100	0.498	0.498	0.498	0.498	0.498
7.5-10	3650-3800	1200-1500	0.498	0.498	0.498	0.498	0.498
10-12.5	2900-3050	1200-1500	0.498	0.605	0.323	0.525	0.498
10-12.5	3200-3350	1500-1800	0.498	0.632	0.326	0.498	0.498
10-12.5	3500-3650	1800-2100	0.498	0.637	0.313	0.498	0.498
10-12.5	3800-3950	2400-2700	0.498	0.498	0.498	0.498	0.498
12.5-15	2900-3050	900-1200	0.498	0.498	0.498	0.498	0.498

Table B-57 Layout B, Sample D: Optimised alpha-index per non-routine rate for combining aeroplane age, flight hours and cycles per year with service type

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate				
				0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
2.5-5	3800-3950	1200-1500	1C	0.498	0.498	0.498	0.498	0.498
2.5-5	3800-3950	1500-1800	1C	0.498	0.498	0.498	0.498	0.498
5-7.5	3200-3350	1500-1800	1C	0.498	0.498	0.498	0.498	0.498
5-7.5	3500-3650	1500-1800	2C	0.498	0.298	0.638	0.498	0.498
5-7.5	3500-3650	1500-1800	3C	0.498	0.498	0.498	0.498	0.498
5-7.5	3500-3650	1500-1800	3C+	0.400	0.657	0.375	0.498	0.498
5-7.5	3500-3650	1800-2100	3C+	0.498	0.648	0.298	0.498	0.498
5-7.5	3650-3800	1500-1800	1C	0.498	0.498	0.498	0.498	0.498
5-7.5	3650-3800	1500-1800	2C	0.498	0.526	0.471	0.498	0.498
5-7.5	3800-3950	1200-1500	3C	0.498	0.498	0.498	0.498	0.498
5-7.5	3800-3950	1200-1500	3C+	0.498	0.503	0.494	0.498	0.498
5-7.5	3950-4100	1200-1500	2C	0.498	0.111	0.637	0.498	0.498
5-7.5	3950-4100	1500-1800	3C	0.498	0.498	0.498	0.498	0.498
5-7.5	4400-4550	1800-2100	3C	0.498	0.498	0.498	0.498	0.498
7.5-10	2300-2450	900-1200	2C	0.498	0.498	0.498	0.498	0.498
7.5-10	3200-3350	1500-1800	1C	0.498	0.498	0.498	0.498	0.498
7.5-10	3200-3350	1500-1800	2C	0.498	0.260	0.677	0.498	0.498
7.5-10	3200-3350	1500-1800	4C	0.498	0.498	0.322	0.636	0.498
7.5-10	3200-3350	1500-1800	4C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3200-3350	1500-1800	5C	0.498	0.498	0.388	0.496	0.596
7.5-10	3200-3350	1800-2100	4C	0.498	0.498	0.416	0.570	0.498
7.5-10	3200-3350	1800-2100	5C	0.498	0.498	0.355	0.611	0.498
7.5-10	3200-3350	1800-2100	5C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3350-3500	1200-1500	5C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3350-3500	1500-1800	1C	0.498	0.498	0.498	0.498	0.498
7.5-10	3350-3500	1500-1800	2C	0.498	0.495	0.501	0.498	0.498
7.5-10	3350-3500	1500-1800	4C	0.498	0.498	0.498	0.498	0.498
7.5-10	3350-3500	1500-1800	4C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3350-3500	1500-1800	5C	0.498	0.498	0.498	0.498	0.498
7.5-10	3350-3500	1800-2100	4C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3500-3650	1200-1500	5C+	0.498	0.498	0.498	0.498	0.498

Age	Fh / Y	Cy / Y	Service	Alpha-Index per Non-routine rate				
				0-0.1	***0.1-0.2***	***0.2-0.3***	***0.3-0.4***	***0.4-0.5***
7.5-10	3500-3650	1500-1800	3C+	0.498	0.098	0.628	0.498	0.498
7.5-10	3500-3650	1500-1800	4C	0.498	0.498	0.547	0.452	0.498
7.5-10	3500-3650	1500-1800	4C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3500-3650	1800-2100	4C+	0.498	0.498	0.498	0.498	0.498
7.5-10	3650-3800	1200-1500	5C+	0.498	0.498	0.498	0.498	0.498
10-12.5	2900-3050	1200-1500	1C	0.498	0.498	0.498	0.498	0.498
10-12.5	2900-3050	1200-1500	3C+	0.498	0.605	0.375	0.498	0.498
10-12.5	2900-3050	1200-1500	6C	0.498	0.498	0.470	0.525	0.498
10-12.5	3200-3350	1500-1800	1C	0.498	0.498	0.498	0.498	0.498
10-12.5	3200-3350	1500-1800	3C+	0.498	0.632	0.326	0.498	0.498
10-12.5	3500-3650	1800-2100	3C+	0.498	0.637	0.313	0.498	0.498
10-12.5	3800-3950	2400-2700	3C	0.498	0.498	0.498	0.498	0.498
10-12.5	3800-3950	2400-2700	6C	0.498	0.498	0.498	0.498	0.498
12.5-15	2900-3050	900-1200	2C	0.498	0.498	0.498	0.498	0.498

B.7.2 Optimised weights results

Table B-58 Layout A (Four samples): Optimised weights for each piece of evidence

Variable	No. Interval	Interval	Sample A	Sample B	Sample C	Sample D
Age	1	4.5 - 5	0.660	0.852	0.813	0.854
	2	5 - 5.5	0.384	0.238	0.312	0.402
	3	5.5 - 6	0.662	0.809	0.204	0.667
	4	6 - 6.5	0.293	0.313	0.032	0.051
	5	6.5 - 7	0.500	0.504	0.500	0.502
	6	7 - 7.5	0.568	0.019	0.053	0.096
	7	7.5 - 8	0.353	0.328	0.481	0.449
	8	8 - 8.5	0.424	0.477	0.263	0.343
	9	8.5 - 9	0.500	0.504	0.500	0.502
	10	9 - 9.5	0.347	0.103	0.315	0.372
	11	9.5 - 10	0.164	0.024	0.023	0.023
	12	10 - 10.5	0.500	0.868	0.816	0.912
	13	10.5 - 11	0.652	0.541	0.405	0.346
	14	11 - 11.5	0.555	0.680	0.594	0.641
	15	11.5 - 12	0.524	0.504	0.307	0.758
	16	12 - 12.5	0.500	0.504	0.500	0.502
	17	12.5 - 13	0.444	0.479	0.330	0.355
Fh/Y	1	2300 - 2450	0.523	0.559	0.602	0.598
	2	2450 - 2600	0.500	0.504	0.500	0.502
	3	2600 - 2750	0.500	0.504	0.500	0.502
	4	2750 - 2900	0.500	0.504	0.500	0.502
	5	2900 - 3050	0.408	0.201	0.020	0.067
	6	3050 - 3200	0.500	0.504	0.500	0.502
	7	3200 - 3350	0.068	0.004	0.006	0.011
	8	3350 - 3500	0.732	0.914	0.860	0.871
	9	3500 - 3650	0.454	0.013	0.416	0.246
	10	3650 - 3800	0.281	0.306	0.109	0.046
	11	3800 - 3950	0.521	0.771	0.842	0.741
	12	3950 - 4100	0.519	0.705	0.058	0.671
	13	4100 - 4250	0.500	0.504	0.500	0.502
	14	4250 - 4400	0.500	0.504	0.500	0.502
	15	4400 - 4550	0.594	0.679	0.648	0.715
Cy/Y	1	900 - 1000	0.523	0.559	0.602	0.598
	2	1000 - 1100	0.440	0.476	0.315	0.338
	3	1100 - 1200	0.500	0.504	0.500	0.502
	4	1200 - 1300	0.501	0.426	0.496	0.419
	5	1300 - 1400	0.696	0.695	0.762	0.905
	6	1400 - 1500	0.667	0.723	0.021	0.306
	7	1500 - 1600	0.555	0.303	0.716	0.650

Variable	No. Interval	Interval	Sample A	Sample B	Sample C	Sample D
	8	1600 - 1700	0.070	0.049	0.010	0.022
	9	1700 - 1800	0.055	0.015	0.014	0.011
	10	1800 - 1900	0.582	0.071	0.040	0.034
	11	1900 - 2000	0.603	0.173	0.130	0.219
	12	2000 - 2100	0.578	0.504	0.500	0.502
	13	2100 - 2200	0.500	0.504	0.500	0.502
	14	2200 - 2300	0.500	0.504	0.500	0.502
	15	2300 - 2400	0.500	0.504	0.500	0.502
	16	2400 - 2500	0.500	0.504	0.500	0.502
	17	2500 - 2600	0.500	0.504	0.500	0.502
	18	2600 - 2700	0.515	0.633	0.207	0.623
Services	1	1C	0.843	0.835	0.908	0.868
	2	2C	0.613	0.157	0.434	0.264
	3	3C	0.649	0.646	0.802	0.871
	4	3C+	0.230	0.128	0.051	0.044
	5	4C	0.541	0.711	0.513	0.315
	6	4C+	0.676	0.834	0.868	0.866
	7	5C	0.089	0.030	0.025	0.022
	8	5C+	0.532	0.431	0.485	0.644
	9	6C+	0.645	0.641	0.195	0.818
Nr rate	1		0.037	0.003	0.002	0.002

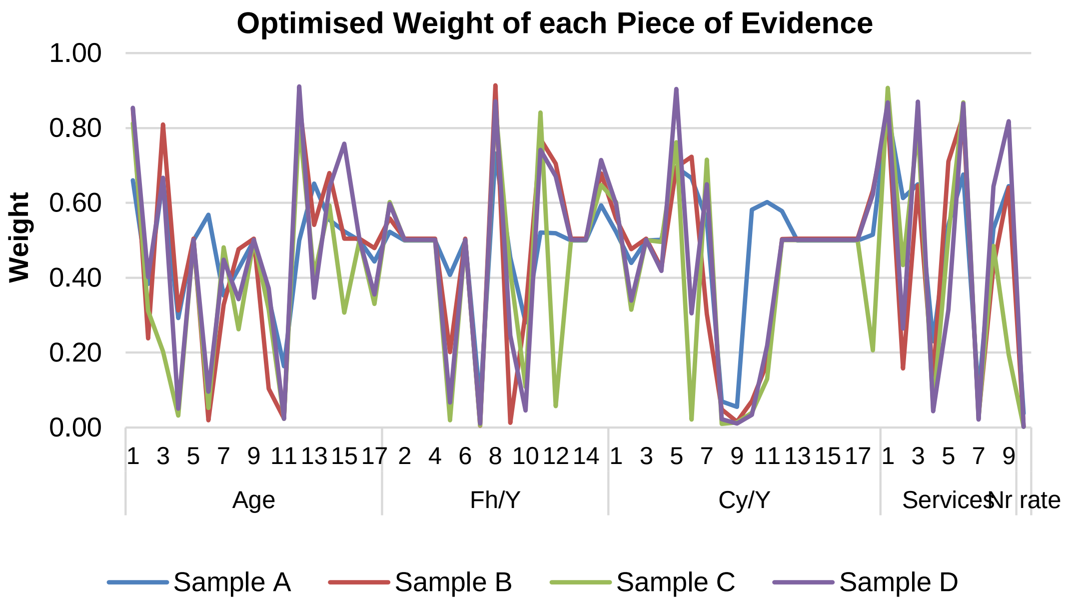


Figure B-30 Layout A: Optimised weights for each piece of evidence

Table B-59 Layout B (Four samples): Optimised weights for each piece of evidence

Variable	No. Interval	Interval	Sample A	Sample B	Sample C	Sample D
Age	1	2.5 - 5	0.6555	0.5483	0.4678	0.5425
	2	5 - 7.5	0.3584	0.2424	0.1832	0.0781
	3	7.5 - 10	0.5413	0.2184	0.2496	0.2973
	4	10 - 12.5	0.0564	0.0109	0.0070	0.0060
	5	12.5 - 15	0.5227	0.5363	0.5126	0.5312
Fh/Y	1	2300 - 2450	0.5194	0.5205	0.5720	0.5213
	2	2450 - 2600	0.5009	0.4935	0.5039	0.4980
	3	2600 - 2750	0.5009	0.4935	0.5039	0.4980
	4	2750 - 2900	0.5009	0.4935	0.5039	0.4980
	5	2900 - 3050	0.6530	0.0090	0.0029	0.0107
	6	3050 - 3200	0.5009	0.4935	0.5039	0.4980
	7	3200 - 3350	0.2411	0.0318	0.0312	0.0797
	8	3350 - 3500	0.9184	0.9039	0.8935	0.9085
	9	3500 - 3650	0.2943	0.0321	0.0484	0.0592
	10	3650 - 3800	0.1618	0.0450	0.3532	0.1962
	11	3800 - 3950	0.0259	0.0570	0.2555	0.0471
	12	3950 - 4100	0.7664	0.4564	0.2636	0.0782
	13	4100 - 4250	0.5009	0.4935	0.5039	0.4980
	14	4250 - 4400	0.5009	0.4935	0.5039	0.4980
	15	4400 - 4550	0.6137	0.4652	0.5095	0.5254
Cy/Y	1	900 - 1200	0.5413	0.5613	0.5789	0.5529
	2	1200 - 1500	0.1011	0.0092	0.0089	0.0081
	3	1500 - 1800	0.1338	0.0077	0.0515	0.0431
	4	1800 - 2100	0.3589	0.0371	0.0386	0.0468
	5	2100 - 2400	0.5009	0.4935	0.5039	0.4980
	6	2400 - 2700	0.0884	0.1903	0.4592	0.5514
Services	1	1C	0.8790	0.4932	0.5920	0.8501
	2	2C	0.3745	0.4587	0.3143	0.6577
	3	3C	0.8428	0.4875	0.7707	0.7912
	4	3C+	0.8295	0.0601	0.0622	0.0745
	5	4C	0.4993	0.0047	0.0232	0.0049
	6	4C+	0.7892	0.1666	0.4053	0.6306
	7	5C	0.5577	0.1004	0.1534	0.1303
	8	5C+	0.8048	0.8584	0.5431	0.7970
	9	6C+	0.1214	0.0317	0.0013	0.2160
Nr rate	1	1	0.6190	0.0024	0.0063	0.0029

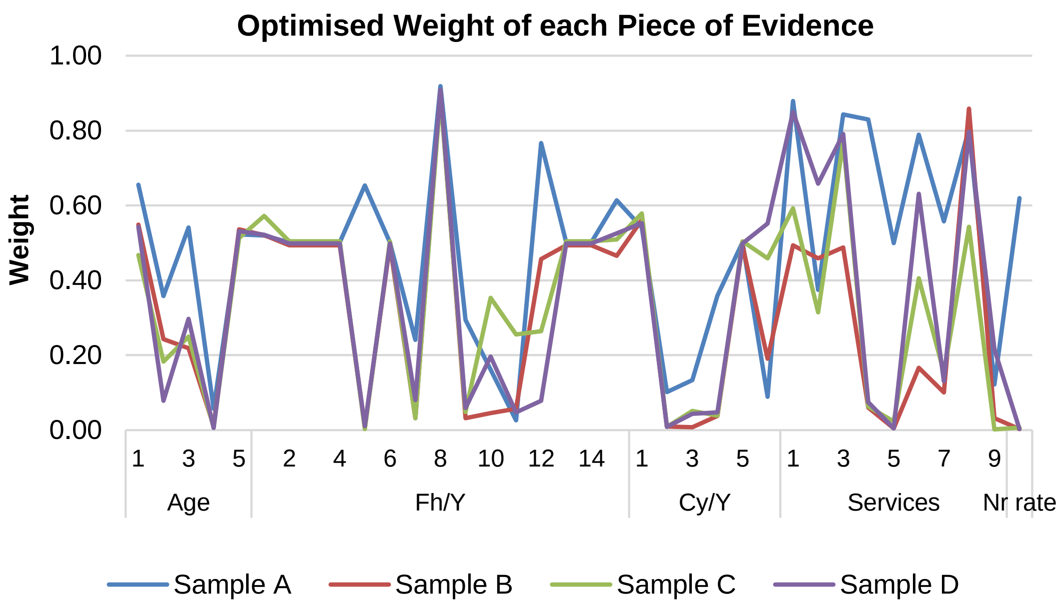


Figure B-31 Layout B: Optimised weights for each piece of evidence

B.8 Sensitivity analysis results

B.8.1 Sensitivity analysis when the reliability of one variable is modified at a time

Table B-60 Layout A, Sample A: Sensibility analysis when alpha index has been optimised, weight is one and the reliability of one variable is modified at a time

	Reliability	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
Age	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.016	0.016	0.016	0.016
	MPAE	9.4%	9.4%	9.4%	9.4%	9.4%	9.4%	9.3%	9.3%	9.3%	9.3%	9.2%	9.2%	9.2%	9.1%	9.0%	9.0%	8.9%	8.7%	8.6%	8.5%	8.0%
	MAI	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.1%	96.1%	96.1%	96.1%	96.2%	96.2%	96.3%
Fh/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.021	0.021	0.021	0.021	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.019	0.016
	MPAE	9.9%	9.9%	9.9%	9.9%	9.9%	9.9%	9.9%	9.9%	9.9%	9.9%	9.9%	9.9%	9.9%	9.8%	9.8%	9.8%	9.8%	9.8%	9.7%	9.5%	8.0%
	MAI	95.2%	95.2%	95.2%	95.2%	95.2%	95.2%	95.2%	95.2%	95.2%	95.2%	95.2%	95.2%	95.2%	95.2%	95.2%	95.2%	95.3%	95.3%	95.3%	95.4%	96.3%
Cy/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.017	0.016
	MPAE	9.1%	9.1%	9.1%	9.1%	9.1%	9.1%	9.1%	9.1%	9.1%	9.0%	9.0%	9.0%	9.0%	9.0%	8.9%	8.9%	8.8%	8.8%	8.7%	8.6%	8.0%
	MAI	95.6%	95.6%	95.6%	95.6%	95.6%	95.6%	95.6%	95.6%	95.7%	95.7%	95.7%	95.7%	95.7%	95.7%	95.7%	95.7%	95.8%	95.8%	95.8%	95.9%	96.3%
Services	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.023	0.023	0.023	0.023	0.023	0.023	0.022	0.022	0.022	0.021	0.020	0.019	0.016
	MPAE	11.9%	11.9%	11.9%	11.9%	11.9%	11.8%	11.8%	11.8%	11.7%	11.7%	11.6%	11.6%	11.5%	11.4%	11.3%	11.1%	11.0%	10.7%	10.4%	9.8%	8.0%
	MAI	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.5%	94.5%	94.5%	94.5%	94.6%	94.6%	94.7%	94.7%	94.8%	94.9%	95.1%	95.2%	95.5%	96.3%
NRR	MSE_{SERV}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	MAE	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.016	0.016	0.016
	MPAE	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.8%	8.8%	8.8%	8.8%	8.8%	8.8%	8.8%	8.8%	8.8%	8.7%	8.7%	8.6%	8.6%	8.4%	8.0%
	MAI	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.1%	96.1%	96.1%	96.2%	96.3%

Table B-61 Layout A, Sample B: Sensibility analysis when alpha index has been optimised, weight is one and the reliability of one variable is modified at a time

	Reliability	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
Age	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.019	0.019	0.019	0.019	0.019	0.019	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.017	0.017	0.017	0.016
	MPAE	10.4%	10.4%	10.4%	10.3%	10.3%	10.3%	10.3%	10.2%	10.2%	10.2%	10.1%	10.1%	10.0%	9.9%	9.9%	9.7%	9.6%	9.4%	9.1%	8.8%	8.1%
	MAI	95.6%	95.6%	95.6%	95.6%	95.6%	95.6%	95.7%	95.7%	95.7%	95.7%	95.7%	95.7%	95.7%	95.8%	95.8%	95.8%	95.9%	95.9%	96.0%	96.1%	96.3%
Fh/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.018	0.016
	MPAE	9.5%	9.5%	9.5%	9.5%	9.5%	9.5%	9.5%	9.5%	9.5%	9.5%	9.5%	9.5%	9.4%	9.4%	9.4%	9.4%	9.4%	9.3%	9.3%	9.2%	8.1%
	MAI	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.6%	95.6%	95.6%	95.6%	95.7%	96.3%
Cy/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.016
	MPAE	9.0%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.8%	8.8%	8.8%	8.8%	8.7%	8.7%	8.5%	8.1%
	MAI	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.1%	96.3%
Services	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.021	0.021	0.021	0.021	0.021	0.021	0.020	0.020	0.019	0.018	0.016
	MPAE	11.4%	11.4%	11.4%	11.4%	11.4%	11.3%	11.3%	11.3%	11.2%	11.2%	11.1%	11.1%	11.0%	10.9%	10.8%	10.7%	10.5%	10.3%	10.0%	9.4%	8.1%
	MAI	94.8%	94.8%	94.8%	94.8%	94.9%	94.9%	94.9%	94.9%	94.9%	94.9%	95.0%	95.0%	95.0%	95.1%	95.1%	95.2%	95.2%	95.3%	95.5%	95.7%	96.3%
NRR	MSE_{SERV}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	MAE	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.016	0.016	0.016	0.016	0.016
	MPAE	8.8%	8.8%	8.8%	8.8%	8.8%	8.7%	8.7%	8.7%	8.7%	8.7%	8.7%	8.7%	8.7%	8.7%	8.7%	8.7%	8.7%	8.6%	8.6%	8.5%	8.1%
	MAI	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.2%	96.2%	96.3%

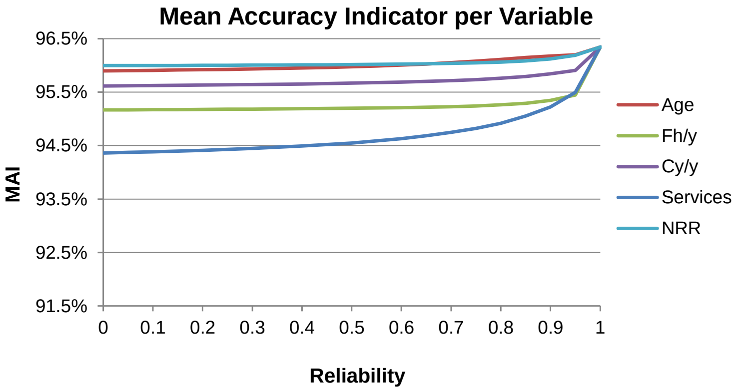
Table B-62 Layout A, Sample C: Sensibility analysis when alpha index has been optimised, weight is one and the reliability of one variable is modified at a time

	Reliability	**0**	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
Age	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	MAE	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.015
	MPAE	8.9%	8.9%	8.9%	8.9%	8.9%	8.8%	8.8%	8.8%	8.8%	8.7%	8.7%	8.7%	8.6%	8.6%	8.5%	8.4%	8.4%	8.3%	8.2%	8.0%	7.5%
	MAI	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.2%	96.2%	96.2%	96.2%	96.2%	96.3%	96.3%	96.3%	96.3%	96.6%
Fh/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.018	0.018	0.015
	MPAE	9.4%	9.4%	9.4%	9.4%	9.4%	9.4%	9.4%	9.3%	9.3%	9.3%	9.3%	9.3%	9.3%	9.3%	9.3%	9.2%	9.2%	9.2%	9.1%	8.9%	7.5%
	MAI	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.5%	95.6%	95.6%	95.6%	95.7%	95.8%	96.6%
Cy/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000
	MAE	0.018	0.018	0.018	0.018	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.016	0.016	0.015
	MPAE	8.5%	8.5%	8.5%	8.5%	8.5%	8.5%	8.5%	8.4%	8.4%	8.4%	8.4%	8.4%	8.4%	8.3%	8.3%	8.3%	8.2%	8.1%	8.0%	7.8%	7.5%
	MAI	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	95.9%	96.0%	96.0%	96.0%	96.0%	96.0%	96.1%	96.1%	96.3%	96.6%
Services	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.024	0.024	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.022	0.022	0.022	0.022	0.021	0.021	0.020	0.019	0.018	0.015
	MPAE	11.7%	11.7%	11.7%	11.7%	11.6%	11.6%	11.6%	11.5%	11.5%	11.4%	11.3%	11.3%	11.2%	11.1%	11.0%	10.8%	10.6%	10.3%	9.8%	9.0%	7.5%
	MAI	94.5%	94.5%	94.5%	94.5%	94.5%	94.5%	94.6%	94.6%	94.6%	94.6%	94.7%	94.7%	94.8%	94.8%	94.9%	95.0%	95.1%	95.2%	95.5%	95.8%	96.6%
NRR	MSE_{SERV}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	MAE	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.015	0.015	0.015
	MPAE	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.3%	8.2%	8.2%	8.2%	8.1%	8.0%	7.5%
	MAI	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.4%	96.4%	96.6%

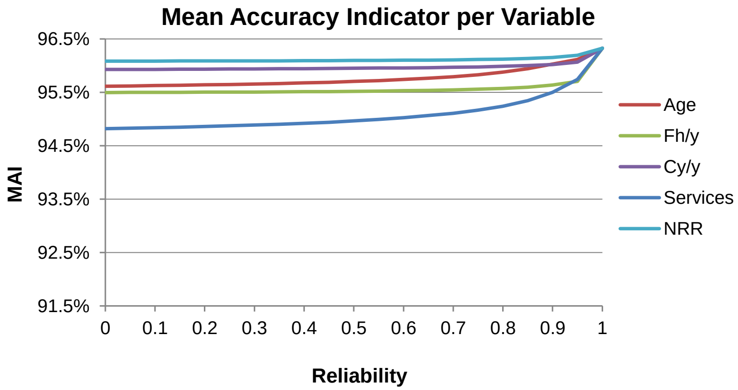
Table B-63 Layout A, Sample D: Sensibility analysis when alpha index has been optimised, weight is one and the reliability of one variable is modified at a time

	Reliability	**0**	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
Age	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.016	0.016	0.016	0.016	0.015
	MPAE	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.6%	9.6%	9.6%	9.6%	9.5%	9.5%	9.4%	9.4%	9.3%	9.2%	9.0%	8.9%	8.7%	8.4%	8.1%
	MAI	95.9%	95.9%	95.9%	95.9%	95.9%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.0%	96.1%	96.1%	96.1%	96.2%	96.2%	96.3%	96.4%
Fh/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.019	0.015
	MPAE	9.6%	9.6%	9.6%	9.6%	9.6%	9.6%	9.6%	9.6%	9.6%	9.6%	9.6%	9.6%	9.6%	9.6%	9.5%	9.5%	9.5%	9.5%	9.4%	9.3%	8.1%
	MAI	95.4%	95.4%	95.4%	95.4%	95.4%	95.4%	95.4%	95.4%	95.4%	95.4%	95.4%	95.4%	95.4%	95.4%	95.4%	95.5%	95.5%	95.5%	95.5%	95.6%	96.4%
Cy/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.017	0.015
	MPAE	9.3%	9.2%	9.2%	9.2%	9.2%	9.2%	9.2%	9.2%	9.2%	9.2%	9.2%	9.2%	9.1%	9.1%	9.1%	9.1%	9.0%	8.9%	8.8%	8.6%	8.1%
	MAI	95.7%	95.7%	95.7%	95.7%	95.7%	95.7%	95.7%	95.7%	95.7%	95.7%	95.7%	95.7%	95.7%	95.8%	95.8%	95.8%	95.8%	95.8%	95.9%	96.0%	96.4%
Services	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.022	0.022	0.022	0.022	0.021	0.021	0.020	0.015
	MPAE	12.1%	12.1%	12.0%	12.0%	12.0%	12.0%	12.0%	11.9%	11.9%	11.9%	11.8%	11.8%	11.7%	11.6%	11.5%	11.4%	11.3%	11.1%	10.8%	10.3%	8.1%
	MAI	94.5%	94.5%	94.5%	94.5%	94.5%	94.5%	94.5%	94.6%	94.6%	94.6%	94.6%	94.6%	94.7%	94.7%	94.8%	94.8%	94.9%	95.0%	95.1%	95.4%	96.4%
NRR	MSE_{SERV}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	MAE	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.016	0.016	0.015
	MPAE	9.0%	9.0%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	8.8%	8.8%	8.7%	8.6%	8.4%	8.1%
	MAI	96.0%	96.0%	96.0%	96.0%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.1%	96.2%	96.4%

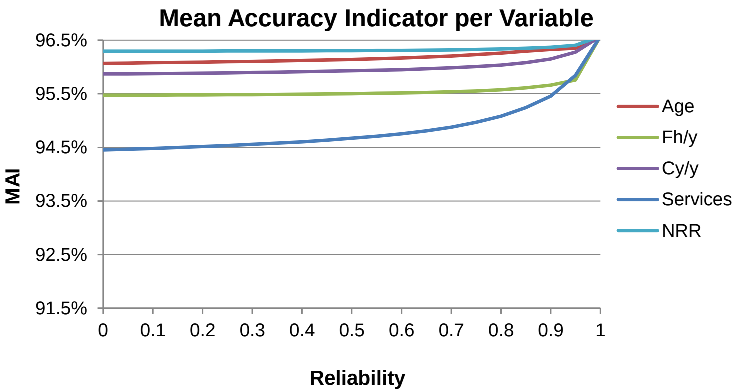
Table B-64 Layout A (Four samples): MAI when alpha index has been optimised, weight is one and the reliability of one variable is modified at a time



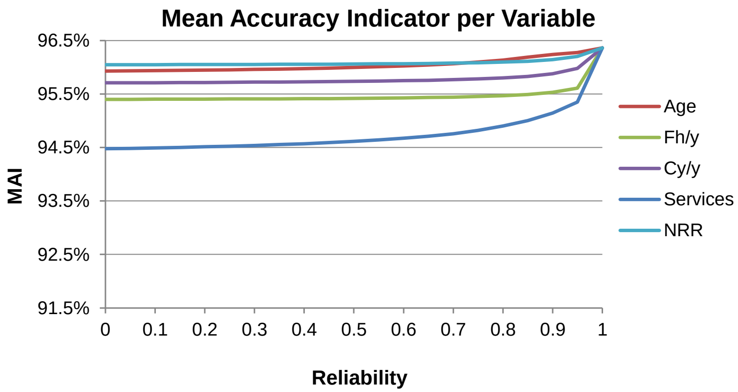
Sample A



Sample B



Sample C



Sample D

Table B-65 Layout B, Sample A: Sensibility analysis when alpha index has been optimised, weight is one and the reliability of one variable is modified at a time

	Reliability	**0**	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
Age	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.025	0.025	0.024
	MPAE	14.0%	14.0%	14.0%	14.0%	14.0%	14.0%	14.0%	14.0%	14.0%	14.0%	14.0%	14.0%	13.9%	13.9%	13.9%	13.9%	13.8%	13.8%	13.6%	13.4%	12.9%
	MAI	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.3%	94.3%	94.3%	94.4%	94.7%
Fh/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.027	0.027	0.027	0.027	0.026	0.024
	MPAE	14.6%	14.6%	14.6%	14.6%	14.6%	14.6%	14.6%	14.6%	14.6%	14.5%	14.5%	14.5%	14.5%	14.5%	14.4%	14.4%	14.3%	14.2%	14.0%	13.7%	12.9%
	MAI	93.8%	93.8%	93.8%	93.8%	93.8%	93.8%	93.8%	93.8%	93.8%	93.8%	93.8%	93.8%	93.8%	93.9%	93.9%	93.9%	93.9%	94.0%	94.1%	94.2%	94.7%
Cy/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.025	0.025	0.025	0.025	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024
	MPAE	13.1%	13.1%	13.1%	13.1%	13.1%	13.1%	13.1%	13.1%	13.1%	13.1%	13.1%	13.1%	13.1%	13.1%	13.1%	13.1%	13.0%	13.0%	13.0%	13.0%	12.9%
	MAI	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.6%	94.7%
Services	MSE_{SERV}	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.001
	MAE	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.035	0.035	0.035	0.035	0.034	0.034	0.033	0.033	0.032	0.031	0.024
	MPAE	18.7%	18.7%	18.6%	18.6%	18.6%	18.5%	18.5%	18.4%	18.4%	18.3%	18.2%	18.1%	18.0%	17.9%	17.7%	17.5%	17.3%	17.1%	16.8%	16.2%	12.9%
	MAI	91.9%	91.9%	91.9%	91.9%	92.0%	92.0%	92.0%	92.0%	92.1%	92.1%	92.1%	92.2%	92.2%	92.3%	92.4%	92.5%	92.6%	92.7%	92.8%	93.1%	94.7%
NRR	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.028	0.028	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.026	0.026	0.026	0.025	0.024
	MPAE	15.1%	15.1%	15.1%	15.1%	15.0%	15.0%	15.0%	15.0%	14.9%	14.9%	14.9%	14.8%	14.8%	14.7%	14.6%	14.5%	14.3%	14.1%	13.7%	13.1%	12.9%
	MAI	93.9%	93.9%	93.9%	93.9%	93.9%	93.9%	93.9%	93.9%	93.9%	93.9%	94.0%	94.0%	94.0%	94.0%	94.0%	94.1%	94.1%	94.2%	94.3%	94.5%	94.7%

Table B-66 Layout B, Sample B: Sensibility analysis when alpha index has been optimised, weight is one and the reliability of one variable is modified at a time

	Reliability	**0**	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
Age	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.025	0.024
	MPAE	14.8%	14.8%	14.8%	14.8%	14.8%	14.8%	14.8%	14.8%	14.8%	14.7%	14.7%	14.7%	14.7%	14.7%	14.7%	14.6%	14.6%	14.5%	14.4%	14.2%	13.4%
	MAI	94.1%	94.1%	94.1%	94.1%	94.1%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.3%	94.3%	94.4%	94.7%
Fh/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.026	0.026	0.026	0.026	0.026	0.025	0.024
	MPAE	14.9%	14.9%	14.9%	14.9%	14.9%	14.9%	14.8%	14.8%	14.8%	14.8%	14.8%	14.8%	14.7%	14.7%	14.7%	14.6%	14.6%	14.5%	14.3%	14.0%	13.4%
	MAI	94.0%	94.0%	94.0%	94.0%	94.0%	94.0%	94.0%	94.0%	94.1%	94.1%	94.1%	94.1%	94.1%	94.1%	94.1%	94.1%	94.2%	94.2%	94.3%	94.4%	94.7%
Cy/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024
	MPAE	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%	13.4%
	MAI	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%
Services	MSE_{SERV}	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001
	MAE	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.036	0.035	0.035	0.035	0.035	0.034	0.034	0.033	0.031	0.024
	MPAE	19.0%	19.0%	19.0%	19.0%	19.0%	18.9%	18.9%	18.9%	18.9%	18.8%	18.8%	18.7%	18.7%	18.6%	18.5%	18.4%	18.2%	18.0%	17.6%	16.8%	13.4%
	MAI	92.0%	92.0%	92.0%	92.0%	92.0%	92.0%	92.0%	92.0%	92.0%	92.1%	92.1%	92.1%	92.1%	92.2%	92.2%	92.3%	92.4%	92.5%	92.7%	93.1%	94.7%
NRR	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.027	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.025	0.025	0.024
	MPAE	15.2%	15.2%	15.2%	15.2%	15.2%	15.1%	15.1%	15.1%	15.1%	15.1%	15.1%	15.0%	15.0%	15.0%	14.9%	14.8%	14.7%	14.6%	14.3%	13.8%	13.4%
	MAI	94.1%	94.1%	94.1%	94.1%	94.1%	94.1%	94.1%	94.1%	94.1%	94.1%	94.1%	94.2%	94.2%	94.2%	94.2%	94.2%	94.2%	94.3%	94.4%	94.5%	94.7%

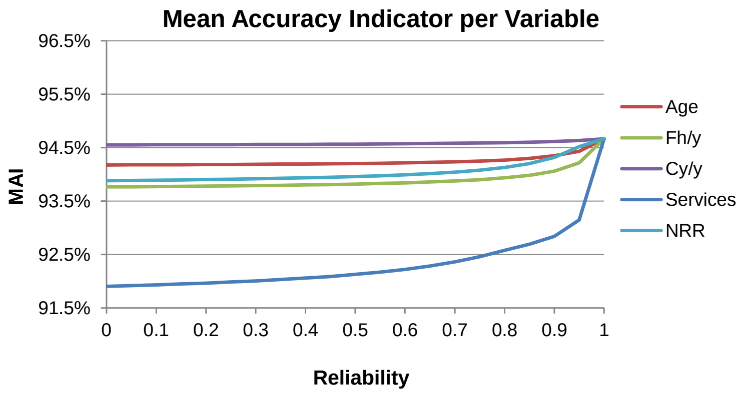
Table B-67 Layout B, Sample C: Sensibility analysis when alpha index has been optimised, weight is one and the reliability of one variable is modified at a time

	Reliability	**0**	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
Age	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.024	0.023
	MPAE	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.8%	13.7%	13.7%	13.7%	13.7%	13.7%	13.7%	13.6%	13.6%	13.5%	13.4%	13.2%	12.6%
	MAI	94.3%	94.3%	94.3%	94.3%	94.3%	94.3%	94.3%	94.3%	94.3%	94.3%	94.3%	94.3%	94.3%	94.4%	94.4%	94.4%	94.4%	94.4%	94.5%	94.6%	94.9%
Fh/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.026	0.026	0.026	0.025	0.023
	MPAE	14.5%	14.5%	14.5%	14.5%	14.5%	14.5%	14.5%	14.5%	14.4%	14.4%	14.4%	14.4%	14.3%	14.3%	14.3%	14.2%	14.1%	14.0%	13.8%	13.4%	12.6%
	MAI	94.0%	94.0%	94.0%	94.0%	94.0%	94.0%	94.0%	94.0%	94.0%	94.0%	94.0%	94.0%	94.0%	94.0%	94.1%	94.1%	94.1%	94.2%	94.3%	94.4%	94.9%
Cy/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.024	0.024	0.024	0.024	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023	0.023
	MPAE	12.7%	12.7%	12.7%	12.7%	12.7%	12.7%	12.7%	12.7%	12.7%	12.7%	12.7%	12.7%	12.7%	12.7%	12.7%	12.7%	12.7%	12.6%	12.6%	12.6%	12.6%
	MAI	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.9%
Services	MSE_{SERV}	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.002	0.001
	MAE	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.038	0.038	0.038	0.038	0.038	0.037	0.037	0.036	0.036	0.035	0.032	0.023
	MPAE	20.2%	20.2%	20.2%	20.2%	20.1%	20.1%	20.1%	20.0%	20.0%	19.9%	19.9%	19.8%	19.7%	19.6%	19.4%	19.2%	18.9%	18.6%	18.0%	16.8%	12.6%
	MAI	91.3%	91.3%	91.3%	91.4%	91.4%	91.4%	91.4%	91.4%	91.4%	91.5%	91.5%	91.5%	91.6%	91.6%	91.7%	91.8%	91.9%	92.1%	92.3%	92.9%	94.9%
NRR	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.025	0.025	0.024	0.023	0.023
	MPAE	14.9%	14.8%	14.8%	14.8%	14.8%	14.8%	14.7%	14.7%	14.7%	14.6%	14.6%	14.5%	14.4%	14.3%	14.2%	14.1%	13.9%	13.6%	13.1%	12.4%	12.6%
	MAI	94.1%	94.1%	94.1%	94.1%	94.1%	94.1%	94.1%	94.1%	94.1%	94.1%	94.2%	94.2%	94.2%	94.2%	94.3%	94.3%	94.4%	94.5%	94.6%	94.8%	94.9%

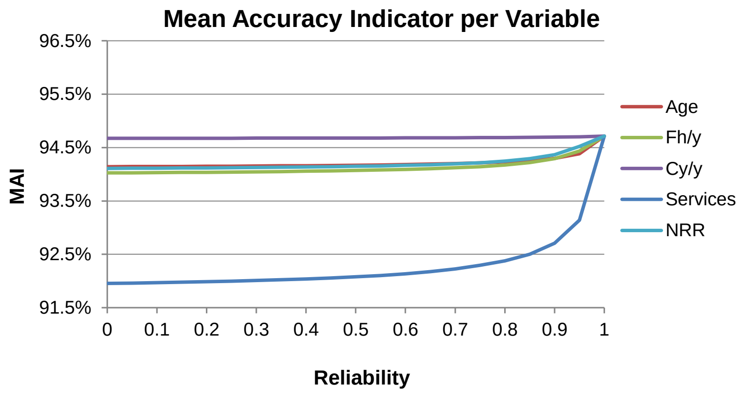
Table B-68 Layout B, Sample D: Sensibility analysis when alpha index has been optimised, weight is one and the reliability of one variable is modified at a time

	Reliability	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
Age	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.027	0.026	0.026	0.025
	MPAE	15.3%	15.3%	15.3%	15.3%	15.3%	15.3%	15.3%	15.3%	15.2%	15.2%	15.2%	15.2%	15.2%	15.2%	15.1%	15.1%	15.0%	15.0%	14.8%	14.6%	14.0%
	MAI	93.9%	93.9%	93.9%	93.9%	93.9%	93.9%	93.9%	93.9%	93.9%	93.9%	93.9%	94.0%	94.0%	94.0%	94.0%	94.0%	94.0%	94.1%	94.1%	94.2%	94.5%
Fh/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.028	0.028	0.028	0.028	0.028	0.027	0.025
	MPAE	15.8%	15.8%	15.8%	15.8%	15.7%	15.7%	15.7%	15.7%	15.7%	15.7%	15.7%	15.7%	15.6%	15.6%	15.6%	15.5%	15.4%	15.3%	15.2%	14.9%	14.0%
	MAI	93.6%	93.6%	93.6%	93.6%	93.6%	93.6%	93.6%	93.6%	93.6%	93.6%	93.6%	93.6%	93.6%	93.7%	93.7%	93.7%	93.7%	93.8%	93.9%	94.0%	94.5%
Cy/y	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025
	MPAE	14.1%	14.1%	14.1%	14.1%	14.1%	14.1%	14.1%	14.1%	14.1%	14.1%	14.1%	14.1%	14.1%	14.1%	14.1%	14.1%	14.1%	14.1%	14.1%	14.0%	14.0%
	MAI	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.5%	94.5%
Services	MSE_{SERV}	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.001
	MAE	0.039	0.039	0.039	0.039	0.039	0.038	0.038	0.038	0.038	0.038	0.038	0.038	0.037	0.037	0.037	0.036	0.036	0.035	0.034	0.031	0.025
	MPAE	20.0%	20.0%	20.0%	19.9%	19.9%	19.8%	19.8%	19.7%	19.7%	19.6%	19.5%	19.5%	19.4%	19.2%	19.1%	18.9%	18.6%	18.2%	17.7%	16.7%	14.0%
	MAI	91.4%	91.4%	91.4%	91.4%	91.4%	91.5%	91.5%	91.5%	91.5%	91.6%	91.6%	91.6%	91.7%	91.7%	91.8%	91.9%	92.0%	92.2%	92.5%	93.0%	94.5%
NRR	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.027	0.027	0.027	0.027	0.026	0.025	0.025
	MPAE	16.2%	16.1%	16.1%	16.1%	16.1%	16.1%	16.0%	16.0%	16.0%	15.9%	15.9%	15.8%	15.7%	15.7%	15.5%	15.4%	15.2%	14.9%	14.4%	13.8%	14.0%
	MAI	93.7%	93.7%	93.7%	93.7%	93.7%	93.7%	93.8%	93.8%	93.8%	93.8%	93.8%	93.8%	93.8%	93.9%	93.9%	93.9%	94.0%	94.1%	94.2%	94.4%	94.5%

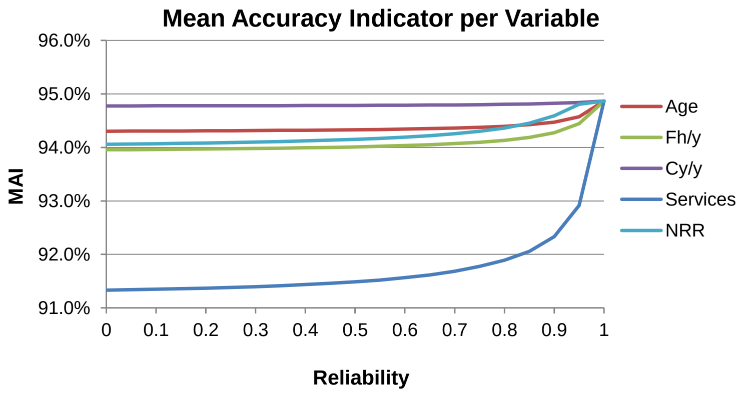
Table B-69 Layout B (Four samples): MAI when alpha index has been optimised, weight is one and the reliability of one variable is modified at a time



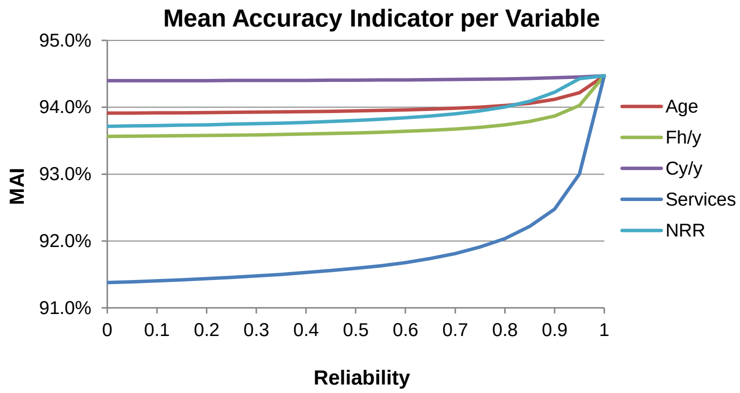
Sample A



Sample B



Sample C



Sample D

B.8.2 Sensitivity analysis when the reliability of all variables is modified equally at the same time

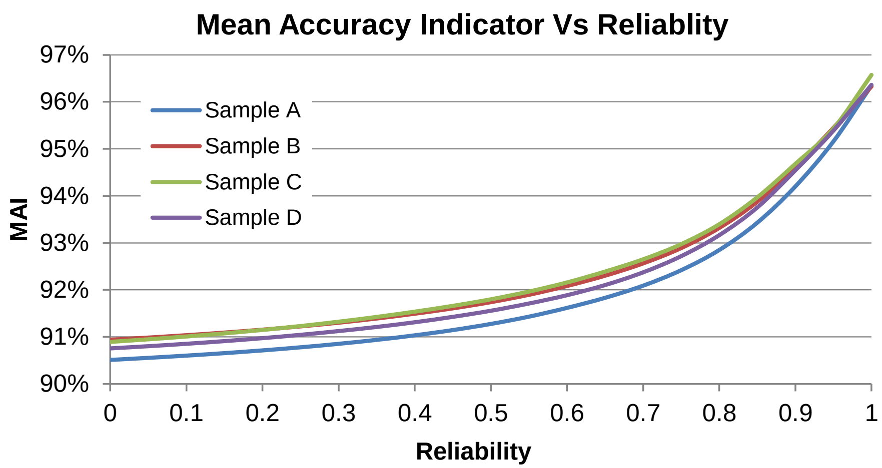
Table B-70 Layout A (Four samples): Sensibility analysis when alpha index has been optimised, weight is one and the reliability of all variables is modified equally at the same time

	Reliability	**0**	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
Sample A	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.000
	MAE	0.040	0.040	0.040	0.040	0.039	0.039	0.039	0.039	0.038	0.038	0.037	0.036	0.036	0.035	0.034	0.032	0.030	0.028	0.025	0.021	0.016
	MPAE	21.4%	21.3%	21.2%	21.0%	20.9%	20.7%	20.5%	20.3%	20.1%	19.8%	19.4%	19.1%	18.6%	18.0%	17.4%	16.6%	15.5%	14.1%	12.3%	10.3%	8.0%
	MAI	90.5%	90.6%	90.6%	90.7%	90.7%	90.8%	90.9%	90.9%	91.0%	91.1%	91.3%	91.4%	91.6%	91.8%	92.1%	92.4%	92.8%	93.4%	94.2%	95.2%	96.3%
Sample B	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.039	0.038	0.038	0.038	0.038	0.037	0.037	0.037	0.036	0.036	0.035	0.034	0.034	0.033	0.032	0.030	0.028	0.026	0.023	0.019	0.016
	MPAE	21.4%	21.3%	21.1%	21.0%	20.8%	20.6%	20.4%	20.2%	19.9%	19.6%	19.3%	18.9%	18.4%	17.8%	17.1%	16.3%	15.2%	13.8%	12.0%	10.1%	8.1%
	MAI	90.9%	91.0%	91.0%	91.1%	91.2%	91.2%	91.3%	91.4%	91.5%	91.6%	91.7%	91.9%	92.1%	92.3%	92.6%	92.9%	93.3%	93.9%	94.6%	95.4%	96.3%
Sample C	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.039	0.038	0.038	0.038	0.038	0.037	0.037	0.036	0.036	0.035	0.035	0.034	0.033	0.032	0.031	0.030	0.028	0.026	0.023	0.019	0.015
	MPAE	21.0%	20.9%	20.7%	20.5%	20.4%	20.1%	19.9%	19.6%	19.3%	19.0%	18.6%	18.2%	17.7%	17.1%	16.4%	15.6%	14.5%	13.1%	11.4%	9.7%	7.5%
	MAI	90.9%	90.9%	91.0%	91.1%	91.1%	91.2%	91.3%	91.4%	91.5%	91.7%	91.8%	92.0%	92.2%	92.4%	92.7%	93.0%	93.4%	94.0%	94.7%	95.4%	96.6%
Sample D	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.000
	MAE	0.039	0.039	0.039	0.039	0.038	0.038	0.038	0.037	0.037	0.036	0.036	0.035	0.034	0.034	0.032	0.031	0.029	0.027	0.023	0.020	0.015
	MPAE	21.7%	21.6%	21.5%	21.3%	21.2%	21.0%	20.8%	20.6%	20.3%	20.0%	19.7%	19.3%	18.8%	18.2%	17.5%	16.6%	15.5%	14.1%	12.2%	10.3%	8.1%
	MAI	90.8%	90.8%	90.9%	90.9%	91.0%	91.0%	91.1%	91.2%	91.3%	91.4%	91.6%	91.7%	91.9%	92.1%	92.4%	92.7%	93.2%	93.8%	94.5%	95.4%	96.4%

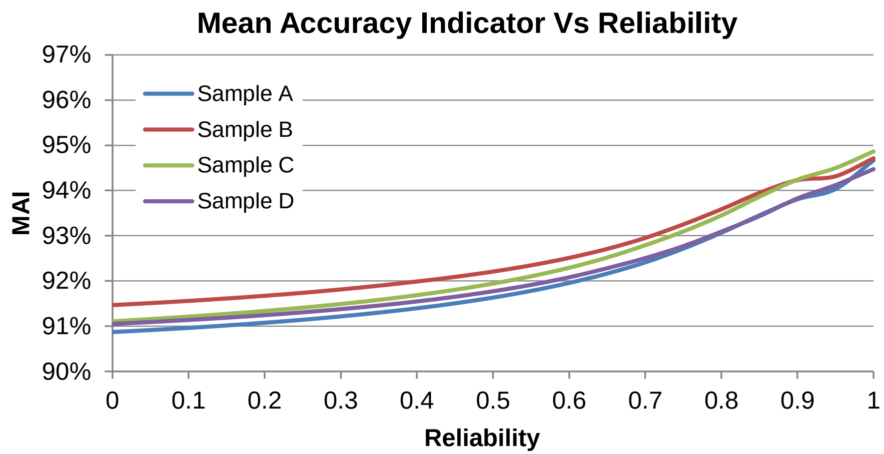
Table B-71 Layout B (Four samples): Sensibility analysis when alpha index has been optimised, weight is one and the reliability of all variables is modified equally at the same time

Samples	**Reliability**	**0**	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**	**0.8**	**0.85**	**0.9**	**0.95**	**1**
Sample A	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001
	MAE	0.041	0.041	0.041	0.040	0.040	0.040	0.040	0.039	0.039	0.038	0.038	0.037	0.036	0.035	0.034	0.033	0.031	0.029	0.028	0.027	0.024
	MPAE	21.3%	21.2%	21.1%	20.9%	20.8%	20.6%	20.4%	20.2%	20.0%	19.7%	19.4%	19.1%	18.6%	18.1%	17.6%	16.9%	16.1%	15.2%	14.4%	13.9%	12.9%
	MAI	90.9%	90.9%	91.0%	91.0%	91.1%	91.1%	91.2%	91.3%	91.4%	91.5%	91.6%	91.8%	92.0%	92.2%	92.4%	92.7%	93.1%	93.5%	93.8%	94.0%	94.7%
Sample B	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.038	0.038	0.038	0.038	0.037	0.037	0.037	0.036	0.036	0.036	0.035	0.034	0.034	0.033	0.032	0.030	0.029	0.027	0.026	0.026	0.024
	MPAE	20.8%	20.7%	20.6%	20.4%	20.3%	20.1%	19.9%	19.7%	19.5%	19.3%	19.0%	18.7%	18.3%	17.8%	17.2%	16.6%	15.8%	14.9%	14.3%	14.2%	13.4%
	MAI	91.5%	91.5%	91.6%	91.6%	91.7%	91.7%	91.8%	91.9%	92.0%	92.1%	92.2%	92.3%	92.5%	92.7%	93.0%	93.3%	93.6%	93.9%	94.2%	94.3%	94.7%
Sample C	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001
	MAE	0.040	0.040	0.040	0.039	0.039	0.039	0.038	0.038	0.037	0.037	0.036	0.036	0.035	0.034	0.032	0.031	0.029	0.028	0.026	0.025	0.023
	MPAE	21.3%	21.2%	21.0%	20.9%	20.7%	20.5%	20.3%	20.1%	19.8%	19.5%	19.1%	18.7%	18.3%	17.7%	17.0%	16.3%	15.5%	14.5%	13.5%	13.0%	12.6%
	MAI	91.1%	91.2%	91.2%	91.3%	91.3%	91.4%	91.5%	91.6%	91.7%	91.8%	91.9%	92.1%	92.3%	92.5%	92.8%	93.1%	93.4%	93.9%	94.2%	94.5%	94.9%
Sample D	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001
	MAE	0.040	0.040	0.040	0.040	0.039	0.039	0.039	0.038	0.038	0.038	0.037	0.036	0.036	0.035	0.034	0.033	0.031	0.030	0.028	0.026	0.025
	MPAE	21.6%	21.5%	21.4%	21.2%	21.1%	20.9%	20.8%	20.6%	20.3%	20.1%	19.8%	19.4%	19.0%	18.5%	18.0%	17.3%	16.6%	15.7%	14.8%	14.1%	14.0%
	MAI	91.1%	91.1%	91.1%	91.2%	91.2%	91.3%	91.4%	91.5%	91.5%	91.7%	91.8%	91.9%	92.1%	92.3%	92.5%	92.8%	93.1%	93.4%	93.8%	94.1%	94.5%

Table B-72 Layout A and B (Four samples): MAI when alpha index has been optimised, weight is one and the reliability of all variables is modified equally at the same time



Layout A



Layout B

B.8.3 Sensitivity analysis modifying alpha-index, weight and reliability

Table B-73 Layout A (Four samples): Sensibility analysis when alpha index is one, weight is one and the reliability of all variables is modified equally at the same time

	Reliability	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
Sample A	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001
	MAE	0.037	0.036	0.035	0.035	0.033	0.032	0.030	0.028	0.026	0.023	0.017
	MPAE	19.3%	18.9%	18.4%	17.9%	17.2%	16.4%	15.6%	14.6%	13.2%	11.8%	9.3%
	MAI	91.3%	91.5%	91.7%	91.9%	92.1%	92.5%	92.8%	93.3%	93.9%	94.6%	96.0%
Sample B	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001
	MAE	0.035	0.035	0.034	0.033	0.032	0.031	0.029	0.027	0.024	0.022	0.018
	MPAE	19.3%	18.9%	18.5%	17.9%	17.2%	16.5%	15.5%	14.3%	12.7%	11.6%	9.7%
	MAI	91.7%	91.9%	92.0%	92.3%	92.5%	92.8%	93.2%	93.7%	94.4%	94.9%	95.8%
Sample C	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.035	0.035	0.034	0.033	0.032	0.030	0.029	0.027	0.024	0.022	0.017
	MPAE	18.9%	18.4%	17.9%	17.3%	16.6%	15.8%	14.9%	13.7%	12.3%	11.0%	8.9%
	MAI	91.7%	91.8%	92.0%	92.3%	92.5%	92.8%	93.2%	93.7%	94.3%	94.8%	95.9%
Sample D	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.000
	MAE	0.039	0.039	0.038	0.038	0.037	0.036	0.034	0.032	0.029	0.023	0.015
	MPAE	21.7%	21.5%	21.2%	20.8%	20.3%	19.7%	18.8%	17.5%	15.5%	12.2%	8.1%
	MAI	90.8%	90.9%	91.0%	91.1%	91.3%	91.6%	91.9%	92.4%	93.2%	94.5%	96.4%

Table B-74 Layout A (Four samples): Sensibility analysis when alpha index is one, weight is optimised and the reliability of all variables is modified equally at the same time

	Reliability	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
Sample A	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.022	0.022	0.022	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.017
	MPAE	11.3%	11.2%	11.2%	11.2%	11.1%	11.0%	10.9%	10.8%	10.6%	10.7%	9.3%
	MAI	94.9%	94.9%	94.9%	95.0%	95.0%	95.0%	95.0%	95.1%	95.2%	95.1%	96.0%
Sample B	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.016	0.016	0.016	0.020	0.016	0.016	0.016	0.020	0.020	0.017	0.018
	MPAE	8.9%	8.9%	8.8%	11.0%	8.8%	8.7%	8.7%	10.8%	10.7%	9.0%	9.7%
	MAI	96.2%	96.2%	96.2%	95.3%	96.2%	96.2%	96.2%	95.3%	95.4%	96.1%	95.8%
Sample C	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.016	0.017	0.016	0.016	0.016	0.019	0.017	0.019	0.019	0.015	0.017
	MPAE	8.0%	8.6%	8.1%	8.2%	8.0%	9.8%	8.5%	9.6%	9.6%	7.8%	8.9%
	MAI	96.3%	96.1%	96.3%	96.3%	96.3%	95.6%	96.1%	95.6%	95.6%	96.4%	95.9%
Sample D	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.018	0.021	0.019	0.015
	MPAE	11.6%	11.6%	11.6%	11.6%	11.6%	11.6%	11.6%	9.7%	11.5%	10.1%	8.1%
	MAI	94.9%	94.9%	94.9%	94.9%	94.9%	94.9%	94.9%	95.7%	94.9%	95.6%	96.4%

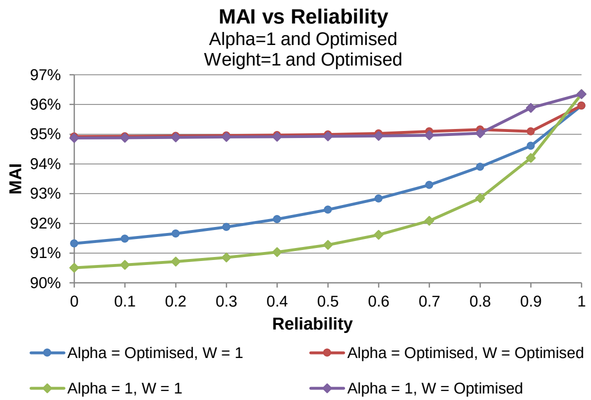
Table B-75 Layout A (Four samples): Sensibility analysis when alpha index is optimised, weight is one and the reliability of all variables is modified equally at the same time

	Reliability	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
Sample A	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.000
	MAE	0.040	0.040	0.039	0.039	0.038	0.037	0.036	0.034	0.030	0.025	0.016
	MPAE	21.4%	21.2%	20.9%	20.5%	20.1%	19.4%	18.6%	17.4%	15.5%	12.3%	8.0%
	MAI	90.5%	90.6%	90.7%	90.9%	91.0%	91.3%	91.6%	92.1%	92.8%	94.2%	96.3%
Sample B	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.000
	MAE	0.039	0.038	0.038	0.037	0.036	0.035	0.034	0.032	0.028	0.023	0.016
	MPAE	21.4%	21.1%	20.8%	20.4%	19.9%	19.3%	18.4%	17.1%	15.2%	12.0%	8.1%
	MAI	90.9%	91.0%	91.2%	91.3%	91.5%	91.7%	92.1%	92.6%	93.3%	94.6%	96.3%
Sample C	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.000
	MAE	0.039	0.038	0.038	0.037	0.036	0.035	0.033	0.031	0.028	0.023	0.015
	MPAE	21.0%	20.7%	20.4%	19.9%	19.3%	18.6%	17.7%	16.4%	14.5%	11.4%	7.5%
	MAI	90.9%	91.0%	91.1%	91.3%	91.5%	91.8%	92.2%	92.7%	93.4%	94.7%	96.6%
Sample D	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001
	MAE	0.036	0.035	0.034	0.033	0.032	0.031	0.029	0.027	0.025	0.023	0.017
	MPAE	19.5%	19.1%	18.6%	18.0%	17.3%	16.4%	15.4%	14.3%	13.0%	11.8%	9.4%
	MAI	91.6%	91.7%	91.9%	92.1%	92.4%	92.7%	93.1%	93.6%	94.1%	94.7%	95.9%

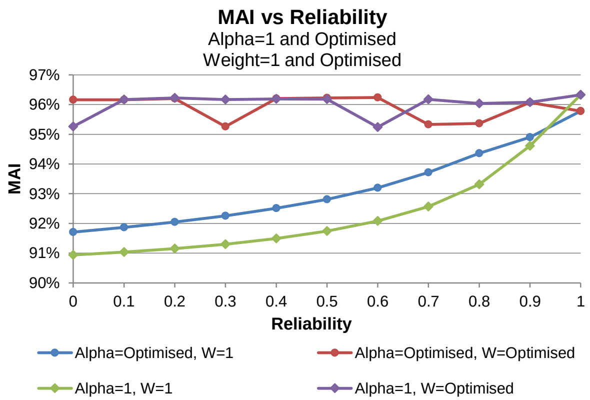
Table B-76 Layout A (Four samples): Sensibility analysis when alpha index is optimised, weight is optimised and the reliability of all variables is modified equally at the same time

	Reliability	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
Sample A	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.021	0.021	0.017	0.016
	MPAE	11.4%	11.4%	11.3%	11.3%	11.3%	11.2%	11.2%	11.1%	10.9%	8.9%	8.0%
	MAI	94.9%	94.9%	94.9%	94.9%	94.9%	94.9%	94.9%	95.0%	95.0%	95.9%	96.3%
Sample B	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.020	0.016	0.016	0.016	0.016	0.016	0.020	0.016	0.017	0.017	0.016
	MPAE	11.0%	8.8%	8.7%	8.8%	8.8%	8.8%	11.1%	8.8%	9.3%	9.1%	8.1%
	MAI	95.3%	96.2%	96.2%	96.2%	96.2%	96.2%	95.2%	96.2%	96.0%	96.1%	96.3%
Sample C	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	MAE	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.019	0.015
	MPAE	8.1%	8.1%	8.2%	8.1%	8.1%	8.0%	8.1%	8.0%	8.0%	9.5%	7.5%
	MAI	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.3%	96.4%	96.3%	95.6%	96.6%
Sample D	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.022	0.022	0.022	0.022	0.022	0.022	0.022	0.018	0.018	0.017	0.017
	MPAE	11.7%	11.7%	11.7%	11.7%	11.7%	11.7%	11.7%	9.8%	9.6%	9.3%	9.4%
	MAI	94.8%	94.8%	94.8%	94.8%	94.8%	94.8%	94.9%	95.7%	95.8%	95.9%	95.9%

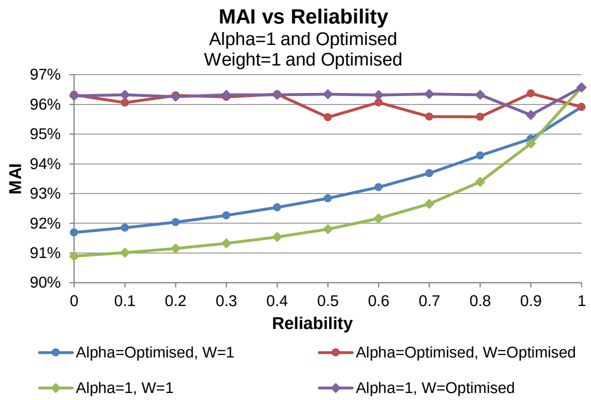
Table B-77 Layout A (Four samples): MAI comparison when alpha index and weight are one and then optimised, and the reliability of all variables is modified equally at the same time



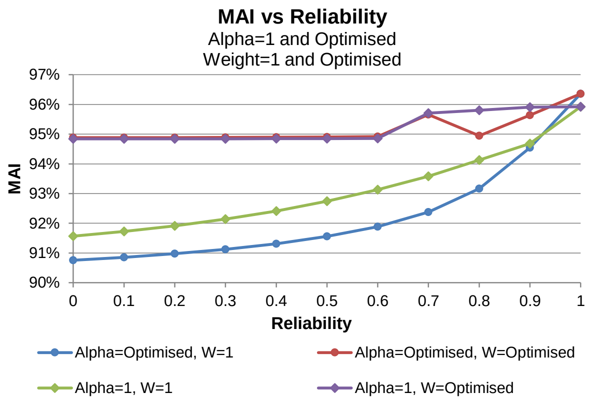
Layout A Sample A



Layout A Sample B



Layout A Sample C



Layout A Sample D

Table B-78 Layout B (Four samples): Sensibility analysis when alpha index is one, weight is one and the reliability of all variables is modified equally at the same

	Reliability	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
Sample A	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
	MAE	0.038	0.038	0.037	0.036	0.035	0.034	0.033	0.032	0.030	0.029	0.024
	MPAE	19.6%	19.2%	18.8%	18.3%	17.8%	17.3%	16.8%	16.2%	15.5%	15.1%	12.8%
	MAI	91.5%	91.6%	91.8%	91.9%	92.1%	92.4%	92.7%	93.0%	93.3%	93.6%	94.6%
Sample B	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001
	MAE	0.036	0.036	0.035	0.034	0.033	0.032	0.031	0.029	0.028	0.027	0.024
	MPAE	19.3%	19.0%	18.6%	18.2%	17.7%	17.2%	16.6%	15.9%	15.2%	14.8%	13.5%
	MAI	91.9%	92.1%	92.2%	92.4%	92.6%	92.8%	93.1%	93.5%	93.8%	94.1%	94.6%
Sample C	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001
	MAE	0.037	0.037	0.036	0.035	0.034	0.033	0.031	0.030	0.028	0.026	0.023
	MPAE	19.4%	18.9%	18.5%	18.0%	17.4%	16.8%	16.2%	15.5%	14.6%	13.8%	12.1%
	MAI	91.7%	91.9%	92.1%	92.3%	92.5%	92.7%	93.0%	93.4%	93.9%	94.3%	94.9%
Sample D	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
	MAE	0.038	0.037	0.036	0.036	0.035	0.033	0.032	0.031	0.030	0.028	0.025
	MPAE	19.8%	19.5%	19.0%	18.5%	18.0%	17.4%	16.9%	16.4%	15.9%	15.4%	13.7%
	MAI	91.6%	91.7%	91.9%	92.1%	92.3%	92.6%	92.8%	93.1%	93.4%	93.7%	94.4%

Table B-79 Layout B (Four samples): Sensibility analysis when alpha index is one, weight is optimised and the reliability of all variables is modified equally at the same time

	Reliability	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
Sample A	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.025	0.025	0.024	0.025	0.025	0.025	0.025	0.025	0.022	0.025	0.024
	MPAE	12.6%	12.6%	12.1%	12.6%	12.6%	12.6%	12.6%	12.7%	11.5%	12.4%	12.8%
	MAI	94.4%	94.4%	94.8%	94.4%	94.4%	94.4%	94.4%	94.4%	95.0%	94.5%	94.6%
Sample B	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.020	0.020	0.020	0.022	0.020	0.020	0.020	0.020	0.020	0.020	0.024
	MPAE	10.6%	10.7%	10.4%	11.7%	10.7%	10.4%	10.7%	10.5%	10.6%	10.6%	13.5%
	MAI	95.6%	95.5%	95.6%	95.1%	95.6%	95.6%	95.6%	95.6%	95.6%	95.6%	94.6%
Sample C	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.020	0.020	0.020	0.023	0.020	0.027	0.020	0.020	0.023	0.023	0.023
	MPAE	10.0%	9.9%	9.9%	11.5%	9.8%	13.5%	10.1%	9.9%	11.5%	11.6%	12.1%
	MAI	95.5%	95.6%	95.5%	94.9%	95.6%	94.0%	95.5%	95.6%	95.0%	94.9%	94.9%
Sample D	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.024	0.021	0.026	0.025
	MPAE	12.2%	12.2%	12.2%	12.2%	12.2%	12.2%	12.2%	12.1%	11.4%	13.7%	13.7%
	MAI	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	94.8%	95.3%	94.3%	94.4%

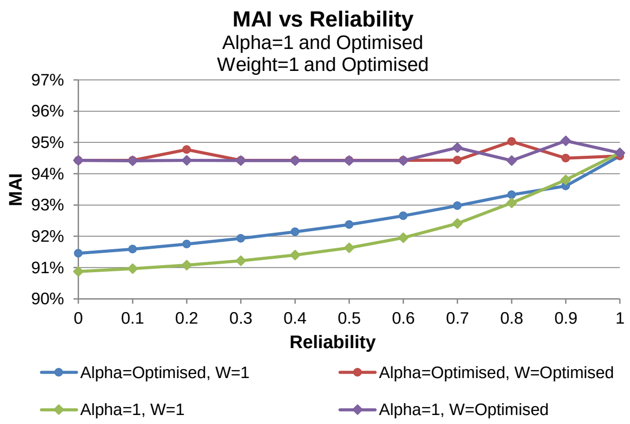
Table B-80 Layout B (Four samples): Sensibility analysis when alpha index is optimised, weight is one and the reliability of all variables is modified equally at the same time

	Reliability	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Sample A	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001
	MAE	0.041	0.041	0.040	0.040	0.039	0.038	0.036	0.034	0.031	0.028	0.024
	MPAE	21.3%	21.1%	20.8%	20.4%	20.0%	19.4%	18.6%	17.6%	16.1%	14.4%	12.9%
	MAI	90.9%	91.0%	91.1%	91.2%	91.4%	91.6%	92.0%	92.4%	93.1%	93.8%	94.7%
Sample B	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001
	MAE	0.038	0.038	0.037	0.037	0.036	0.035	0.034	0.032	0.029	0.026	0.024
	MPAE	20.8%	20.6%	20.3%	19.9%	19.5%	19.0%	18.3%	17.2%	15.8%	14.3%	13.4%
	MAI	91.5%	91.6%	91.7%	91.8%	92.0%	92.2%	92.5%	93.0%	93.6%	94.2%	94.7%
Sample C	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
	MAE	0.040	0.040	0.039	0.038	0.037	0.036	0.035	0.032	0.029	0.026	0.023
	MPAE	21.3%	21.0%	20.7%	20.3%	19.8%	19.1%	18.3%	17.0%	15.5%	13.5%	12.6%
	MAI	91.1%	91.2%	91.3%	91.5%	91.7%	91.9%	92.3%	92.8%	93.4%	94.2%	94.9%
Sample D	MSE_{SERV}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001
	MAE	0.040	0.040	0.039	0.039	0.038	0.037	0.036	0.034	0.031	0.028	0.025
	MPAE	21.6%	21.4%	21.1%	20.8%	20.3%	19.8%	19.0%	18.0%	16.6%	14.8%	14.0%
	MAI	91.1%	91.1%	91.2%	91.4%	91.5%	91.8%	92.1%	92.5%	93.1%	93.8%	94.5%

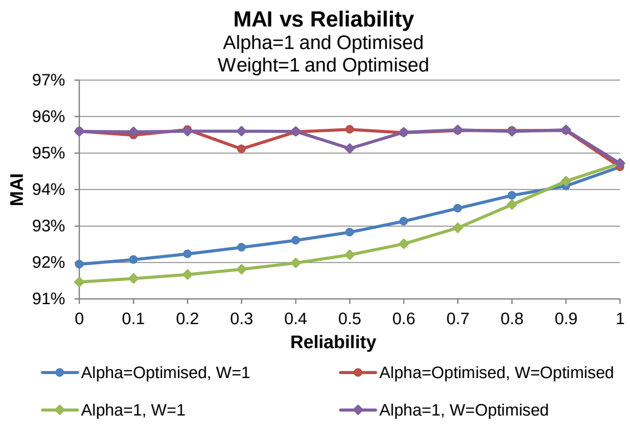
Table B-81 Layout B (Four samples): Sensibility analysis when alpha index is optimised, weight is optimised and the reliability of all variables is modified equally at the same time

	Reliability	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Sample A	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.023	0.025	0.022	0.024
	MPAE	12.6%	12.6%	12.6%	12.6%	12.6%	12.6%	12.6%	12.0%	12.6%	11.5%	12.9%
	MAI	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.4%	94.8%	94.4%	95.1%	94.7%
Sample B	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.020	0.020	0.020	0.020	0.020	0.022	0.020	0.020	0.020	0.020	0.024
	MPAE	10.6%	10.7%	10.6%	10.6%	10.6%	11.7%	10.6%	10.4%	10.6%	10.5%	13.4%
	MAI	95.6%	95.6%	95.6%	95.6%	95.6%	95.1%	95.6%	95.6%	95.6%	95.6%	94.7%
Sample C	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.020	0.021	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.023
	MPAE	9.8%	10.6%	9.7%	9.7%	9.8%	9.8%	9.8%	9.8%	9.8%	9.8%	12.6%
	MAI	95.7%	95.2%	95.6%	95.6%	95.6%	95.6%	95.6%	95.6%	95.6%	95.6%	94.9%
Sample D	MSE_{SERV}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	MAE	0.024	0.024	0.024	0.024	0.024	0.024	0.021	0.024	0.024	0.021	0.025
	MPAE	12.3%	12.2%	12.2%	12.3%	12.2%	12.2%	11.0%	12.2%	12.2%	10.9%	14.0%
	MAI	94.7%	94.7%	94.7%	94.7%	94.7%	94.7%	95.3%	94.7%	94.7%	95.3%	94.5%

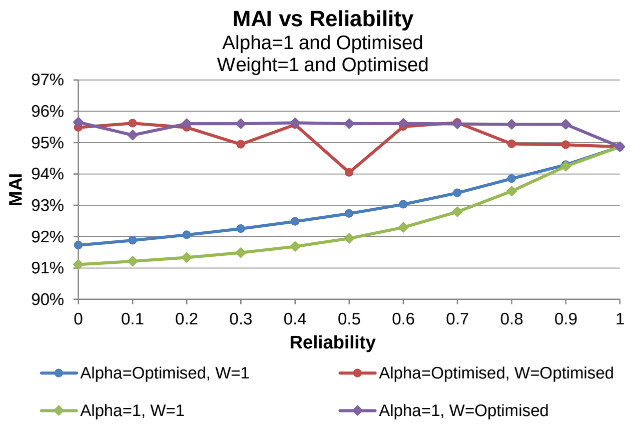
Table B-82 Layout B (Four samples): MAI comparison when alpha index and weight are one and then optimised, and the reliability of all variables is modified equally at the same time



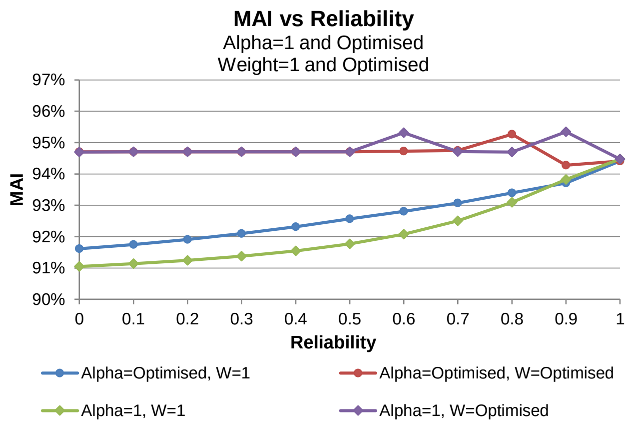
Layout B Sample A



Layout B Sample B



Layout B Sample C



Layout B Sample D

B.9 Validation results

Table B-83 Layout A: Model performance comparison of the training and validation groups and all the dataset

	Training Group				Validation Group				All Data			
	Sample A	**Sample B**	**Sample C**	**Sample D**	**Sample A**	**Sample B**	**Sample C**	**Sample D**	**Sample A**	**Sample B**	**Sample C**	**Sample D**
MSE_{SERV}	0.0009	0.0006	0.0005	0.0007	0.0021	0.0027	0.0030	0.0009	0.0010	0.0009	0.0008	0.0007
MAE	0.0203	0.0167	0.0154	0.0176	0.0356	0.0413	0.0428	0.0237	0.0221	0.0197	0.0187	0.0183
MAPE	10.3%	9.1%	7.8%	9.6%	24.1%	18.6%	24.3%	13.3%	12.0%	10.3%	9.8%	10.0%
MAI	95.2%	96.1%	96.4%	95.9%	91.6%	90.3%	89.9%	94.4%	94.8%	95.4%	95.6%	95.7%

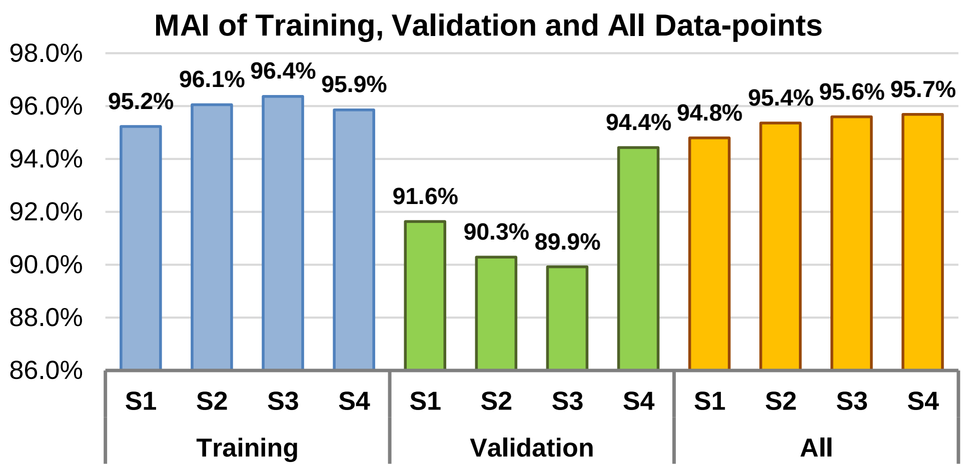


Figure B-32 Layout A: MAI comparison for training and validation groups and for the whole sample

Table B-84 Layout B: Model performance comparison of the training and validation groups and all the dataset

	Training Group				Validation Group				All Data			
	Sample A	**Sample B**	**Sample C**	**Sample D**	**Sample A**	**Sample B**	**Sample C**	**Sample D**	**Sample A**	**Sample B**	**Sample C**	**Sample D**
MSE_{SERV}	0.0011	0.0008	0.0009	0.0010	0.0011	0.0024	0.0031	0.0013	0.0011	0.0010	0.0011	0.0010
MAE	0.0243	0.0195	0.0198	0.0208	0.0235	0.0374	0.0444	0.0315	0.0242	0.0216	0.0228	0.0221
MAPE	12.1%	10.3%	9.8%	10.9%	15.5%	17.3%	24.8%	16.8%	12.5%	11.2%	11.6%	11.6%
MAI	94.6%	95.7%	95.6%	95.4%	94.8%	91.7%	90.1%	93.0%	94.6%	95.2%	94.9%	95.1%

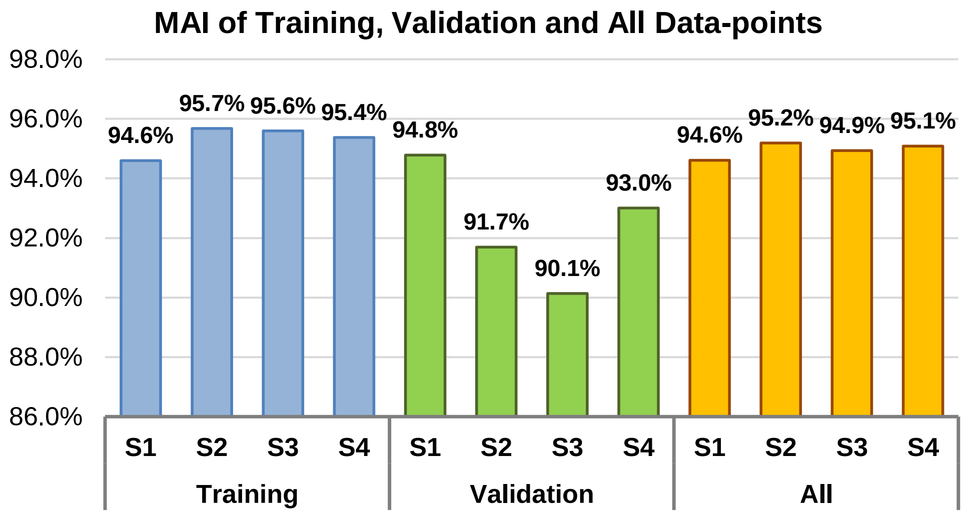


Figure B-33 Layout B: MAI comparison for training and validation groups and for the whole sample