# DESIGN AND ANALYSIS OF AD HOC COGNITIVE RADIO NETWORKS

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By

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### Abstract

This thesis describes how to design and analyze ad hoc cognitive radio networks. In a cognitive radio scenario which consists of primary users and secondary users, secondary users are allowed to opportunistically access the existing spectrum without adverse effect on primary users. A cognitive radio network is allowed to detect its communication environment, replace the parameters of its communication scheme to raise the quality of service for secondary users and decrease the interference to primary users. Alternatively, the other approach to implementing a cognitive radio network is to allow simultaneous transmission of primary users and secondary users, which is also termed as spectrum sharing. In this technique, a secondary transmitter can transmit while its maximum interference to the primary receiver is smaller than the predefined threshold. However, a secondary user must control its transmit power to get a reasonable transmission rate. In spectrum sharing approach, we maximize the ergodic capacity and minimize bit error rate under different constraints at the primary users. The effect of reducing channel side information at the secondary transmitter is discussed for both optimization problems. We will then extend the simple model to ad hoc cognitive radio network where higher number of links in primary and secondary networks exists. The interference from primary and other secondary transmitters were separately discussed. In a similar system model, we also analyze outage probability of secondary users under AWGN and Rayleigh fading channels, while applying Poisson distribution to accurately account for the spatial distribution of secondary users in a 2-dimensional plane. The explicit expressions are derived based on different parameters such as signal-to-interference-plus-noise ratio threshold, path loss exponent, signal-to-noise-ratio in the absence of interference and density of secondary interferes. The obtained results can be used to design and implementation of new protocols in ad hoc networks such that the highest data rate can be transmitted from a source node to a destination node with the lowest bit error rate.

The next part of this thesis is concerned with minimization of power transmission in co-channel femtocell networks in order to reduce the interference inflicted on the macrocell users while satisfying a target constraint on either the capacity or the BER. This minimization is applied in two different system models. We also prove that minimizing the transmit power can be utilized to further enhance energy efficiency in femtocell networks.

### Declaration

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# List of Abbreviations

AODV	ad hoc on-demand distance vector
AP	access point
ARPA	advanced research project agency
AWGN	additive white Gaussian noise
BS	base station
BER	bit error rate
CBR	cluster based routing
$\operatorname{CR}$	cognitive radio
DSDV	destination-sequenced distance vector
DSL	digital subscriber line
DSR	dynamic source routing
FCC	federal communications commission
IEEE	institute of electrical and electronics engineers
IET	institution of engineering and technology
MAC	media access control
OFDM	orthogonal frequency-division multiplexing
PDF	probability density function
PER	packet error rate

PR primary receiver

- PT primary transmitter
- PU primary user
- PRNET packet radio networks
- QoS quality of service
- RTS request to send
- RF radio frequency
- SINR signal to interference and noise ratio
- SNR signal to noise ratio
- SR secondary receiver
- ST secondary transmitter
- SU secondary user
- TORA temporally-ordered routing algorithm
- ZRP zone routing protocol

# List of Mathematical Notations

$\ln(.)$	natural logarithm
$\log_2(.)$	base-2 logarithm
$\exp(.)$	exponential function
$\Gamma(.)$	Gamma function
$rac{\partial}{\partial x}$	partial derivative respect to $x$
Pr(.)	probability of an event
$E(\cdot)$	expectation operator of a random variable
$_{m}F_{n}(.)$	hypergeometric function
$E_i(.)$	exponential integral
$\gamma$	Euler's constant
U(.,.,.)	confluent hypergeometric function
Q(.)	Q-function
erfc(.)	complementary error function
B(.)	beta function
$Li_{2}(.)$	polylogarithm function of order 2
$G_{p q}^{m n}(. _{(bq)}^{(a_p)})$	Meijer function
$\cos(.)$	cosine function
Ι	identity matrix

### **Publication List**

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To my wife Fatemeh Jahani

and

my dear son Mohammadarsin

### Chapter 1

### Introduction

#### 1.1 Wireless Systems

There is no doubt that wireless communication is one of the fastest-growing areas of the communications industry. During the last few decades, cellular networks have significantly developed and around two billion users are now affected by this industry. Mobile phones in most developed countries are a critical business tool and part of everyday life. Moreover, wireless networks in local areas, like homes, offices, or small group of buildings have replaced wired networks. Some new applications of wireless communications such as smart homes and appliances, automated highways and factories, sensor networks and cognitive radio networks have emerged from research ideas to concrete systems. The rapid development of wireless communication combined with the significant growth of laptop and palmtop computers reveal a bright future for wireless systems. However, there are still many challenges in designing wireless systems to support emerging applications [1]-[8].

The history of ad hoc networks is relatively old, it dates back to 1970s when packet radio networks (PRNETs) was developed by Advanced Research Project Agency (ARPA). Ad hoc networks are an infrastructureless network of mobile nodes and therefore are suitable when deploying an infrastructure would not be an easy task, or when the central control is destroyed by a natural disaster. Due to the absence of a base station, ad-hoc networks suffer from self interference generated by similar nodes in the same area. Cognitive radio is a technology to improve the spectral efficiency by sharing the spectrum that is originally allocated to the licensed user. Ad hoc cognitive radio takes the benefits of both ad hoc networks and cognitive radio techniques. Numerous performance evaluation and design techniques have been proposed in ad hoc cognitive radio, but there are still interesting unresolved problems. In this thesis, we present several original performance analyses and results for ad hoc cognitive radios in cellular network environments.

### **1.2** Contributions

The major contributions of this dissertation can be given as follows.

- Derive accurate expressions for evaluating the maximum capacity and minimum bit error rate under all different levels of channel side information which can be provided at the secondary transmitter. Then, these performance parameters are investigated by reducing different side information at the secondary transmitter.
- Derive expression for analyzing the total ergodic capacity of ad hoc cognitive radio networks under average and peak interference power constraints at the primary users. The total capacity of the secondary network highly depends on the number of secondary interference; however the primary transmitters have a negative impact.

- Derive explicit expressions for outage probability of secondary users based on spatial location of nodes, and in terms of the signal-to-interference-plusnoise ratio threshold, path loss exponent, signal-to-noise-ratio in the absence of interference and density of secondary interference under AWGN and Rayleigh fading channels.
- Derive new expressions for the required minimum downlink femtocell transmit power in the presence of co-channel interference from the macro base station while satisfying a target constraint on either the ergodic capacity or the average BER under Rayleigh fading.
- Analyze different approaches to find minimum transmit power in downlink communication over OFDM-based femtocell networks

#### **1.3** Thesis Organization

The rest of the thesis is organized as follows. Chapter 2 gives background on some of the important models and concepts in wireless communication that the thesis relies on. It introduces different networks including cellular networks, ad hoc networks, cognitive radio networks and femtocell networks that we refer to in the rest of the chapters. The characterization of radio channels is also described in this chapter. It does not contain any new contribution from the author and all information given is from the available literature.

The novel contributions of the author starts from Chapter 3, where two different optimization problems in a simple model of cognitive radio are discussed. First, a closed-form expression for evaluating the maximum capacity in cognitive radios under Rayleigh fading channels is derived. Furthermore, we minimized the bit error rate for the same system model under different modulation schemes used in wireless communication such as DPSK, BPSK/QPSK, MPSK and QAM. The effect of reducing channel side information at the secondary transmitter is discussed for both optimization problems.

In Chapter 4 we extend our system model to include the effect of higher number of links in primary and secondary networks. This chapter introduces a system model composed of a primary cellular network and a secondary ad hoc network, coexisting in the same area and sharing the spectrum. The total ergodic capacity of cognitive network under average and peak interference power constraint is studied in this chapter. The interference from primary and other secondary transmitters were separately discussed.

Chapter 5 employs a similar system model as given in chapter 4. In chapter 5, we assume that all secondary transmitters have the same transmission power and also there is no interference between primary and secondary networks and the only interference is amongst secondary users. This is because secondary users use the licensed spectrum when they are not occupied by primary systems. The main aim of this chapter is to derive expressions for outage probability of secondary users under AWGN and Rayleigh fading channels. Specifically, we employ the Poisson distribution to accurately account for the spatial distribution of secondary users in a 2-dimensional plane.

In chapter 6, we consider the downlink transmission of a femtocell network in two different system models under co-channel deployment. The minimized power at the femtocel AP in each of these models is analyzed. This chapter focuses on minimizing the transmit power from a femtocell AP in order to reduce the interference inflicted on the macro-cell users while satisfying a target constraint on either the ergodic capacity or the average BER. In the first system model, Lagrangian approach is applied to minimizing the transmission power since the optimization problem is convex, while we proposed an algorithm in the second system model to obtain minimum power. We also prove that minimizing the transmit power can be utilized to further enhance energy efficiency in femtocells.

Finally we conclude the thesis in Chapter 7 and propose future work.

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### Chapter 2

### Background

### 2.1 Main Characterization of Wireless Channels

The term wireless communication system refers to the transfer of data through electromagnetic waves over atmospheric space rather than using a wired system. The presence of the wireless channel as a medium is the main difference between these two systems. Unfortunately, this medium is hostile in regards to attenuating and delaying the transmitted signal. Thus, the wireless channel plays a vital role in having a reliable high-speed communication.

The transmitted signal varies over the distance due to path loss and shadowing. Path loss refers to reduction in signal power radiated by a transmitter as the transmitted signal propagates through the atmosphere or free space. Shadowing is caused by obstacles that affect signal power through reflection, scattering, and diffraction. For example, reflection occurs when a radio wave meets a very large object compared to the wave's wavelength, while scattering occurs when the radio wave collides with rough surfaces or small objects compared to the wavelength of the propagating wave. Path loss variation is related to very large distances which is around 100-1000 meters, whereas shadowing variation occurs over shorter distances which is proportional to the length of the obstructing object such as buildings and hills and is typically frequency independent. Therefore, path loss of signal as a function of distance and shadowing variation are known as large-scale propagation effects. The other category, known as small-scale propagation, refers to the constructive and destructive addition of multiple signal paths between the transmitter and receiver. This occurs over very short distances and is frequency dependent [1], [2].

#### 2.1.1 Channel Capacity and BER

#### Capacity in AWGN Channel

Channel capacity gives the information about the maximum amount of data which can be reliably transmitted over the channel between a transmitter and a receiver.

A discrete-time AWGN channel is considered with following relationship between input and output

$$y[i] = x[i] + n[i]$$
(2.1)

where x[i] and y[i] are the channel input and output at time *i*, respectively. n[i] denotes a white Gaussian noise random process.

In a band limited system, the channel capacity of a channel perturbed by AWGN is a function of the following parameters:

- P: transmit power,
- $N_0$ : the power spectral density of the noise,
- B: the channel bandwidth.

The channel SNR is constant and given by  $\gamma = \frac{P}{N_0 B}$ . The channel capacity is given by Shannon formula as

$$C = B \log_2\left(1+\gamma\right) \tag{2.2}$$

The above equation shows that the transmission rate is limited by the value of parameters  $P, N_0$  and B.

#### Capacity in Fading Channel

Capacity in a fading channel (Ergodic capacity) with receiver channel side information can be expressed as

$$C = \int_0^\infty B \log_2\left(1+\gamma\right) d\gamma \tag{2.3}$$

Since the above equation is a probabilistic average, i.e. Shannon capacity is averaged over the distribution of  $\gamma$ .

#### Bit Error Rate (BER)

In digital communication systems, due to noise or interference, number of received bits of a data stream have been altered. BER is calculated by the ratio of the number of incorrectly detected bits at the receiver to the total number of transmitted bits.



Figure 2.1: Comparing FDM with OFDM

# 2.2 Orthogonal Frequency Division Multiplexing (OFDM)

OFDM is a technique that splits a high-rate serial data stream into a set of lowrate substreams such that each of which is modulated on a separate subcarrier. As can be seen in Fig. 2.1 in this technique, due to orthogonal subchannels, subchannels are allowed to overlap and no interference will be present. Therefore, the bandwidth of each subcarriers becomes smaller than the coherence bandwidth of the channel. OFDM enhances the channel capacity and is more spectrum efficient than frequency division multiplexing (FDM). This technique is implemented by the fast Fourier transform (FFT) algorithm so it is easy to generate and demodulate.





Fig. 2.2 shows the implementation of OFDM. In transmitter side, the input data stream is first modulated by a QAM modulator and gives a symbol stream X[0], X[1], ..., X[N - 1]. After passing a serial-to-parallel converter, the N symbols output are the discrete frequency components of the OFDM modulator output s(t). These frequency components are converted into time samples by using the inverse fast Fourier transform (IFFT) algorithm. The IFFT gives

$$x[n] = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X[i] e^{j2\pi n i/N} \quad 0 \le n \le N-1$$
(2.4)

Then, the cyclic prefix will be added to the OFDM symbol, and the resulting time samples are passed through a D/A converter which will be then upconverted to frequency  $f_0$ .

The transmitted signal is filtered by the channel impulse response h(t) and added by noise. Therefore, we have

$$y(t) = x(t) * h(t) + n(t)$$
(2.5)

In receiver side, the received signal is downconverted to baseband and the resulting signal is filtered to remove the high frequency components. An analogue to digital converter samples the signal and then the prefix of samples are removed. After passing through a serial-to-parallel converter and an FFT, it is parallel-to-serial converted and passed through a QAM demodulator to obtain the original data.

#### 2.3 Cellular Systems

Cellular systems are widely-used around the world; these systems have emerged from the revolution in wireless communication. Data and voice communications are supported by cellular systems in local, national and international coverage. Mobile terminals inside vehicles with antennas fixed on the vehicle roof were the first purpose for designing cellular systems. However, these systems have recently been developed to support lightweight mobile terminals that operate everywhere at various speeds.

The main idea behind cellular systems is to reutilise the frequency, in which signal power decreases along with distance. The coverage area especially in cellular systems consists of non-overlapping cells, where each cell takes a set of channels. As can be seen in Fig. 2.3, this channel set is utilized in a different cell some distance away. Here,  $C_i$  represents the channel set that is utilized in a specific cell.

A central base station controls all operations in a cell. The separation between the cells that reutilise the same frequency must be as small as possible, leading to the reutilisation of frequencies as frequently as possible and the maximization of spectral efficiency. In order to specify the best distance of reutilisation and base station location, an accurate specification of signal propagation between the cells is needed.

When cellular systems were first used, the cost of base stations was around one million dollars and a small number of cells covered the whole area of a city. The base stations with high power covering several square miles were located on mountains or tall buildings. These cells are known as macrocells. Signals were uniformly propagated in all directions such that all mobiles that were at the same distance from the base station would have received constant power. Therefore, the circular area of constant power is considered as a hexagonal shape for the systems.

By contrast, nowadays smaller cells called microcells, picocells and femtocells are utilised for cellular networks in urban areas, where base stations transmit close to street level with lower power. There are two reasons to make cells smaller. The first reason is that areas with more user density may need a higher capacity; and the second reason is to reduce the cost and size of base stations. Moreover, less power is required at each mobile in microcellular networks because it is closer to the base station[1].

Although the system design is more complicated when smaller cells are used, mobiles can move between cells more quickly than a large cell. Furthermore, location management is more difficult due to there being more cells. In small cells, it is also more complicated to develop propagation models because signal propagation relies on the geometry of the reflectors and base station locations. More specifically, in microcells a hexagonal cell shape is not an accurate approximation for signal propagation. In the designing of microcell networks, triangular or square shapes are utilized, in which a large margin of error can occur[3]-[5]. Femtocells and their importance will be covered in section 2.5 and chapter 6.

### 2.4 Ad Hoc Networks

Ad hoc networks consist of mobile nodes connected via wireless links, without using a specific structure or centralized administration for either a short period of time or permanently. As can be seen in Fig. 2.4, ad hoc networks are known as infrastructure-less networks because a fixed infrastructure such as a base station is not required for operation. The nodes randomly move and co-operate with each


Figure 2.3: Cellular Systems [1]

other to communicate outside their neighborhood [6]-[11]. In ad hoc networks, a transmitter node A and a receiver node B can establish a wireless link when the power of the received signal to noise at the receiver is above a predefined threshold. In this case, A and B are neighbors. The wireless links between transmitter and receiver pairs would be bi-directional, however, unidirectional links may also be used due to differences in transmission power. Nodes outside each other's wireless range can communicate through the intermediate node as a relay node in multi-hop modes. However, the routing in multi-hop modes changes, when the topology of the network changes. Recently, several routing protocols in mobile ad hoc networks have been proposed, such as AODV [12], DSDV [13], DSR [14], ZRP [15], and TORA [16].

Ad hoc network routing protocols can be divided into proactive and reactive protocols. Proactive or table driven routing protocols try to update routing information between every pair of nodes in the network by propagating control packet, and the route updates at fixed time intervals so it causes an extra traffic throughout the network. A well-known example of proactive routing protocols is DSDV.

On the other hand, reactive or on-demand routing protocols create a route to a new destination only when there is a demand for it. This leads to reduction of the protocol overload. Examples of reactive routing protocols are AODV, DSR and TORA. The other type of protocols between these two groups, for example ZRP, adjusts the degree of reactivity/proactivity.

DSDV routing protocol is a protocol based on the Bellman-Ford Routing Algorithm with some changes like loop-free. The DSDV routing protocol is based on table-driven protocols, which need every node to broadcast updated information. The main purpose of DSDV is to make an algorithm without any loop. This protocol is a new version of distance vector protocols. An entry in the routing table includes an even sequence number, if there is a link; otherwise, the routing table uses an odd sequence number. The sequence numbers are recorded for each reachable destination. The mobile nodes are enabled by the sequence numbers to detect old routes from new ones. Each node throughout the network will periodically send the routing table. Other distance vector routing protocols, for example AODV, is based on the DSDV.

DSR is based on source routing so the sender is aware of the complete hopby-hop route to the destination. In this protocol, the complete sequence of nodes is carried by each data packet in order to send to the destination. Two main functions of DSR are route discovery and route maintenance. The role of the first function is to find new routes and the role of the second is to detect topology changes and inform other nodes throughout the network. There are many optimizations that have been proposed and simulated. The main improvements are salvaging, gratuitous route repair and promiscuous listening. AODV is combined of two previous protocols. This algorithm has improved the DSDV algorithm. The on-demand operation of route discovery and route maintenance used in DSR and the hop-by-hop routing and sequence number applied for DSDV are implemented in DSR protocol. Traditional routing tables are used in AODV and DSDV, but node cache is applied in DSR because of maintaining routing information. In addition, the sequence number is used by AODV in order to avoid previous routes and loops that may be created by routing. For management expired entries, which are in the routing table, timestamps fields is used in routing table. A source node produces a broadcast packet, Route Request (RREQ), and starts a Route Discovery operation at the time of demanding a path to a destination. A node broadcasts RREQ provided that it has not yet received RREQ. A node responds by a Route Reply (RREP) when it is the destination or it includes routing information about that destination.

TORA reduces the overhead by restricting routing messages to the neighborhood, which are close to the changes in order to responding to topological variations. This protocol uses three processes including: on-demand route create, route repair, and route erase. For route repair, TORA sends a BEACON-HELLO message sequence, a special message, between the sender and the receivers periodically since this protocol designs a reliable control message routing delivery. For route erase, this protocol removes the invalid routes and starts with a node that discovers network partition.

The main issue in an ad hoc network is to guarantee quality-of-service (QoS) in different applications. Most of the previous works have been directed towards QoS, with few research works interested in the estimation of capacity or other performance evaluation parameters. These parameters highly depend on the position of transmitter-receiver pair in the presence of other interferers. Uniform and Poisson distribution models are widely used for interferers' distribution [17]-[19].



Figure 2.4: Mobile ad hoc network [7]

Application	Possible scenarios	
Tactical networks	Military communication, Automated battlefields	
Education	Universities and campus settings	
Entertainment	Wireless P2P networking	
Sensor networks	Data tracking of environmental conditions	
Coverage extension	Extending cellular network access	
Emergency services	Policing and fire fighting	
Commercial	Electronic payments anytime and anywhere	

Table 2.1: Mobile ad hoc network applications

During the last few years, ad hoc networks have been employed in different applications such as military and commercial. Table 2.1 shows some of the main applications and possible scenarios [20].

## 2.5 Cognitive Radio Networks

In the last two decades, due to rapid deployment of new wireless applications and devices, spectral resource demand has increased. However, the frequency allocation chart highlights that most of the frequency bands have been occupied, under which a significant amount of spectrum is under-utilized. The inefficient usage of the licensed spectrum motivates using innovative techniques to exploit the available spectrum in a flexible and intelligent way. Therefore, in order to address the issue of spectrum efficiency, dynamic spectrum access techniques have been proposed.

The key enabling technology of dynamic spectrum access techniques is cognitive radio (CR) technology, which opportunistically shares the spectrum that is originally allocated to the licensed users. However, there is no agreement on the definition of cognitive radio as of now, various meanings in different contexts include the concept. Herein, we use the definition according to FCC: "Cognitive radio: A radio or system that senses its operational electromagnetic environment and can dynamically and autonomously adjust its radio operating parameters to modify system operation, such as maximize throughput, mitigate interference, facilitate interoperability, access secondary markets.". Therefore, the main aim of cognitive radio technology is to introduce new paths to spectrum access by autonomously exploiting a locally unused spectrum.

Cognitive radio can adjust its transmitter parameters reported by the environment in which it operates. In contrast to conventional radio devices, users in cognitive radio are equipped with reconfigurability and cognitive capability. Cognitive capability refers to sensing of the RF environment and gathering information about power, transmission frequency, modulation, bandwidth and etc. Reconfigurability refers to adapting the operational parameters according to the sensed information. Therefore, cognitive radio users sense the portion of the spectrum that is available, select the proper channel, coordinate with other users, and vacate the channel if a licensed user appears in that channel. By exploiting the opportunistic spectrum, particularly when licensed users coexist with unlicensed ones, conventional spectrum allocation techniques and spectrum access schemes may not be applicable. Meanwhile, new spectrum management techniques in cognitive radio, specifically in dynamic spectrum sharing and spectrum sensing need to be developed.

Unlicensed users, known as secondary users (SU), have a lower priority in using the spectrum and have to monitor in real-time, the band of the licensed spectrum to be used. As unlicensed users transmit data simultaneously with a licensed user, interference temperature limit should be less than a predefined threshold because their transmissions should not degrade licensed users' QoS. To manage the interference, the transmission power in unlicensed users should be carefully controlled, and also their competition in obtaining spectrum resources should be addressed.

Unlicensed users are only allowed to transmit while the spectrum is not occupied by licensed users. Therefore, they need to be aware of the licensed users' reappearance by different detection techniques, such as matched filtering, energy detection, feature detection, and coherent detection. However, detection performance in sensing techniques depends on shadowing, noise uncertainty, and multipath effect. With the aid of cooperative spectrum sensing, detection accuracy has been relatively improved [21]-[31].

#### 2.5.1 Cognitive Radio Functions

As can be seen in Fig 2.5, a duty cycle of cognitive radio contains detecting spectrum holes, capturing the best available frequency bands, coordinating with other users and vacating the frequency channel if a licensed user appears.

Therefore, the following functions can support a cognitive cycle:

1. Spectrum sensing



Figure 2.5: Cognitive cycle [23].

- 2. Spectrum management
- 3. Spectrum allocation and sharing.

Spectrum sensing refers to detecting the unused spectrum without causing any harmful interference to licensed users as illustrated in Fig 2.6. Spectrum management refers to capturing the best available frequency band and hoping among multiple frequency bands according to the time varying channel characteristics in order to satisfy the various QoS requirements. This means that if a licensed user appears on its frequency band, the unlicensed user can send the signal on other available frequency bands. In spectrum allocation and sharing, an unlicensed user may share the spectrum with licensed users such that the interference level at a licensed user should be limited by a tolerable threshold [23].



Figure 2.6: Illustration of spectrum holes [23]



Figure 2.7: Femtocell Networks [33]

Infrastructure	Expenses	Features
Femtocell	Providing a scalable archi-	Lower cost, Better indoor
	tecture to transport data	coverage
	over IP	
Distributed antennas	backhaul connection	Better coverage, Does not
		solve the indoor coverage
Microcells	Installing new cell towers,	System capacity gain from
	site lease and backhaul	smaller cell size, expensive

Table 2.2: Femtocell, Distributed antennas and Microcells

## 2.6 Femtocell Networks

An approach used to increase the capacity of wireless systems is to reduce the distance between transmitter and receiver. This present benefits such as higherquality links and more spatial re-use [32]. A less expensive infrastructure known as Femtocells or home base stations (BS) uses for this continued microization of cellular networks. Femtocells are short-range, low-power BSs and installed by home users to get better indoor voice and data reception. As shown in Fig 2.7, femtocells can connect to the cellular base stations via a seperate connection such as digital subscriber line (DSL), cable modem, or a separate radio frequency (RF) backhaul channel. In contrast to other approaches like microcells and distributed antenna, femtocells have very little upfront cost to the service provider. These three techniques are compared in Table 2.2 in more detail [35], [36].

According to the recent report on wireless usage, more than 70% of data traffic and 50% of voice calls originated from indoors [37]. Data networks require high signal quality due to transmission of multi-megabit per second data rates, while voice networks can tolerate lower signal quality because of low data rate in voice signals, around 10 kb/s or less. Achieving high data rates in indoor communication with high carrier frequencies is not an easy task to accomplish in wireless systems. Consequently, installing low power and short range links in such environments should be proposed, which leads to an encouraging femtocell approach. This technique provides transmission of high data rates and also reduces the traffic on the macrocell networks. Below we summarize the main features of femtocells.

- Reducing the costs: The deployment of femtocells minimizes the operating and capital expenditure costs for operators. For example, the cost of site leasing for a conventional urban macrocell is more than \$1000/month, which does not include electricity, backhaul and the operating expenses. Using femtocell approach also reduces the extra costs of adding macro base station towers [37], [38].
- 2. Improving macrocell reliability: The macrocell base station provides better reception for mobile users by redirecting its resources when femtocell networks absorb indoors traffic via IP backbone.
- 3. Increasing coverage and capacity: In femtocell networks, the distance between a transmitter and receiver pair is very short compared to a macrocell network, which results in transmissions with lower power, higher SINR and longer battery life for handsets. In addition, the number of active users that can be serviced into a certain area, known as area spectral efficiency, increases due to less interference [34].
- 4. Reducing subscriber turnover: Due to poor indoor coverage, some customers are encouraged to switch to other operators or use wired lines whenever they are in buildings. Thus, femtocell deployment that enhances indoor coverage causes home user satisfaction and avoiding to switch carriers.

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## Chapter 3

# **Optimization in Cognitive Radio**

In today's wireless systems, there is an increased demand for the wireless radio spectrum due to many new wireless communication networks such as wireless sensor networks, wireless local area networks, Bluetooth and so on. The frequency allocation chart of the Federal Communications Commission (FCC) shows that a severe under-utilization of the licensed spectrum has been observed. Cognitive radio networks have been recently proposed as an efficient method to reduce the problem by opportunistically accessing the spectrum. A cognitive radio network consists of the primary users (PU) and the secondary users (SU). A PU has the legacy priority access to the spectrum while an SU uses the spectrum when the interference to the PUs does not exceed a certain limit. The utilization of spectrum in traditional wireless networks is improved by cognitive radio technology such that it increases the number of applications and services in wireless systems. A cognitive radio network recognizes its communication environment and changes the parameters of its communication scheme to increase the quality of service of SUs [1]–[5].

Cognitive radios can be divided into two different types. In the first scheme, a spectrum sensing technique is required to detect spectrum opportunities and then transmit while the PU is absent [6], [7]. In the second approach, PUs and SUs employ spectrum sharing techniques while avoiding considerable interference to the primary receivers. In such systems, a medium access control layer protocol with ability to fairly allocate the spectrum between secondary users is required [8]. This chapter focuses on the second approach.

The ergodic and outage capacity offered by the dynamic spectrum sharing approach in a single-antenna fading primary network has been investigated in [9]-[11] under average and peak interference power constraints at the existing primary receiver (PR). These constraints at the PR belong to one of the following two types: the first one is the long-term constraint that regulates the average interference across all the fading state, and the other one is the short-term constraint that limits the instantaneous interference over each fading state. However, in [9]-[11], the interference from primary transmitter (PT) to secondary receiver (SR) is ignored and the capacity is evaluated based on the signal-to-noise ratio (SNR). A similar system model has been applied in recent works. For example, in [12], by employing Jensen's inequality on the objective function, the impact of the interference from PT to SR was assumed as a constant value and thus the ergodic capacity is approximated. The authors in [13] also considered the interference from PT to SR as a constant value. Furthermore, in all aforementioned works, it is assumed that perfect channel side information (CSI) is available at both the receiver and the transmitter. However, providing such side information in practice is very difficult. The transmitter requires a feedback path between the transmitter and receiver to get the side information [14], [15].

The major contribution of this chapter is that the three levels of channel side information available at the secondary transmitter (ST) are discussed, namely CSI between ST-SR, between ST-PR, and between PT-SR. Under average interference power constraint, the power at ST depends on CSI between ST-SR, between ST-PR, and between PT-SR. However, under peak interference power constraint, the power at ST only depends on CSI between ST-PR.

## 3.1 System Model

Fig. 3.1 shows a spectrum sharing scenario where a cognitive radio link consisting of a transmitter and a receiver uses the same bandwidth for transmission with an existing primary link consisting of a PT and a PR. A flat fading channel with perfect CSI at the receiver and transmitter of the secondary user is considered. The secondary link between ST and SR is characterized by instantaneous channel power gain  $g_1$  and the AWGN  $n_1$ . The noise  $n_1$  is an independent random variable with the distribution  $CN(0, N_0)$  (circularly symmetric complex Gaussian variable with mean zero and variance  $N_0$ ).

The channel between ST and PR with instantaneous channel power gain  $g_0$ has also been assumed. We consider the effect of the interference coming from the PT with constant power  $\rho$  on the SR. The instantaneous power at ST can be written as P. The instantaneous received signal-to-interference-plus-noise ratio (SINR) at the SR is

$$SINR = \frac{Pg_1}{N_0 + \rho h_1} \tag{3.1}$$

where  $h_1$  denotes the interference channel power gain between the PT and the SR.

In the following two sections, we derive expressions for evaluating the maximum capacity and minimum bit error rate under different constraints.



Figure 3.1: system model for spectrum sharing

### **3.2** Maximization of the Ergodic Capacity

#### 3.2.1 The Average Interference Power Constraint

We consider a third party's receiver in a fading environment, with a received average power constraint. Channel capacity can be obtained by optimal utilization of the transmitted power over time, in which the received power constraint is met.

In order to discuss the significance of having  $g_1, g_0$  and  $h_1$  at the ST, ergodic capacity is evaluated under different scenarios. In the first scenario, the optimum power allocation P is a function of  $g_1, g_0$  and  $h_1$ . In second scenario, the channel side information  $h_1$  at ST is reduced thus P becomes a function of  $g_1$  and  $g_0$ . In third scenario, we reduce the channel side information  $g_0$  at ST, consequently, P will be a function of  $g_1$  and  $h_1$ . In next scenario, channel side information  $g_1$ is not made available at ST so P becomes a function of  $g_0$  and  $h_1$ . Finally, all channel side information  $g_1, g_0$  and  $h_1$  at ST in last scenario are reduced then Psimplifies into a constant.

In what follows, we will explain these scenarios in more detail.

### **Full CSI** $[P(g_1, g_0, h_1)]$

The ergodic capacity of the secondary link can be found by solving the following optimization problem.

$$\max_{P \ge 0} \quad E\left[\ln\left(1 + \frac{P(g_1, g_0, h_1)g_1}{N_0 + \rho h_1}\right)\right]$$
(3.2a)

$$s.t \qquad E\left[P(g_1, g_0, h_1)g_0\right] \le Q_{average} \tag{3.2b}$$

where the transmit power of ST depends on all channel gains  $g_1, g_0$  and  $h_1$ . Equation (3.2b) represents the average interference power constraint which can be used to guarantee a long-term QoS of PU and  $Q_{average}$  is the maximum average received power limit at PR. The above optimization problem is equivalent to solving the following Lagrangian approach

$$L(P,\lambda) = E\left[\ln\left(1 + \frac{P(g_1, g_0, h_1)g_1}{N_0 + \rho h_1}\right)\right] - \lambda\left(E\left[P(g_1, g_0, h_1)g_0\right] - Q_{average}\right) \quad (3.3)$$

where  $\lambda$  is the nonnegative dual variable corresponding to the constraint (3.2b). Taking the derivative of the Lagrangian in (3.3) with respect to  $P(g_1, g_0, h_1)$  and letting the derivative equal to zero yields [16]

$$\frac{L(P,\lambda)}{\partial P} = E\left[\frac{g_1}{P(g_1,g_0,h_1)g_1 + N_0 + \rho h_1} - \lambda g_0\right] = 0$$
(3.4)

which results in

$$P(g_1, g_0, h_1) = \frac{1}{\lambda g_0} - \frac{N_0 + \rho h_1}{g_1}$$
(3.5)

Note that the optimum power allocation P, the instantaneous power at the ST, is a function of  $g_1, g_0$  and  $h_1$ . In (3.5) by considering the constraint  $P(g_1, g_0, h_1) \ge$  0, we get

$$\frac{g_0}{g_1} \le \frac{1}{\lambda \left(N_0 + \rho h_1\right)}$$
(3.6)

The parameter  $\lambda$  can be obtained by solving

$$Q_{average} = E\left[\frac{1}{\lambda} - \frac{g_0}{g_1}\left(N_0 + \rho h_1\right)\right].$$
(3.7)

which satisfies the Complementary Slackness Conditions [16]. We can get the maximum capacity by substituting (3.5) in (3.2a)

$$C = E\left[\ln\left(\frac{1}{\lambda}\frac{g_1}{g_0\left(N_0 + \rho h_1\right)}\right)\right].$$
(3.8)

We substitute  $x = \frac{g_0}{g_1}$  and  $y = (N_0 + \rho h_1)$  in (3.7), which yields

$$Q_{average} = \int_0^\infty \int_{x < \frac{1}{\lambda y}} \left(\frac{1}{\lambda} - xy\right) f_x(x) f_y(y) dx dy$$
(3.9)

In the case of Rayleigh fading, the probability density function (PDF) of the ratio between two exponential random variables  $\frac{g_0}{g_1}$  can be expressed as [17]

$$f_{\frac{g_0}{g_1}}(x) = \frac{1}{(1+x)^2} \tag{3.10}$$

and the PDF of the sum  $N_0 + \rho h_1$  becomes

$$f_{(N_0+\rho h_1)}(y) = \frac{1}{\rho} e^{-\frac{y-N_0}{\rho}}$$
(3.11)

By using (3.10) and (3.11), equation (3.9) becomes

$$Q_{average} = \frac{1}{\rho} \int_{N_0}^{\infty} \int_0^{\frac{1}{\lambda y}} \left(\frac{1}{\lambda} - xy\right) \frac{e^{-\frac{y - N_0}{\rho}}}{(1+x)^2} dx dy$$
(3.12)

Upon invoking [19, eq. (2.113), (4.222.8) and (4.331.2)], equation (3.12) reduces into the following form

$$Q_{average} = \frac{1}{\lambda} \left[ 1 + e^{\frac{1+\lambda N_0}{\lambda P}} (\lambda \rho - 1) E_i \left( -\frac{1+\lambda N_0}{\lambda \rho} \right) - e^{\frac{N_0}{\rho}} \lambda \rho E_i \left( -\frac{N_0}{\rho} \right) - \lambda (N_0 + \rho) \ln \left( 1 + \frac{1}{\lambda N_0} \right) \right] \quad (3.13)$$

where Ei (.) is the exponential integral function defined as Ei  $(x) = \int_{-\infty}^{x} \frac{e^{t}}{t} dt$  [19]. We can find  $\lambda$  for a given  $Q_{average}$  from equation (3.13). It is worth noting that determining  $\lambda$  from (3.13) requires the use of numerical integration.

Similarly, we obtain the channel capacity as

$$C = \int_0^\infty \int_{x < \frac{1}{\lambda y}} \ln\left(\frac{1}{\lambda}\frac{1}{xy}\right) f_x(x) f_y(y) dx dy$$
$$= \frac{1}{\rho} \int_{N_0}^\infty \int_0^{\frac{1}{\lambda y}} \ln\left(\frac{1}{\lambda}\frac{1}{xy}\right) \frac{e^{-\frac{y-N_0}{\rho}}}{(1+x)^2} dx dy$$
(3.14)

By changing the variable  $t = \frac{1}{x}$  and using [19, eq. (2.727.3), (4.337.1) and (4.331.2)] in (3.14), we obtain the following closed-form expression

$$C = e^{\frac{N_0}{p}} E_i \left(-\frac{N_0}{\rho}\right) - e^{\frac{1+\lambda N_0}{\lambda \rho}} E_i \left(-\frac{1+\lambda N_0}{\lambda \rho}\right) + \ln\left(1+\frac{1}{\lambda N_0}\right)$$
(3.15)

Equation (3.15) is a new closed-form expression for ergodic capacity when the ST knows all instantaneous channel gains  $g_1, g_0$  and  $h_1$ .

#### Partial CSI: Reduced only CSI $h_1 [P(g_1, g_0)]$

Here, we find the maximum capacity with a reduced side information where  $h_1$ is not made available at the ST. Hence, by disregarding  $h_1$ , the power of ST depends on  $g_1$  and  $g_0$ . The maximum capacity problem (3.2a) subject to (3.2b) changes into the following form

$$\max_{P \ge 0} \quad E\left[\ln\left(1 + \frac{P(g_1, g_0)g_1}{N_0 + E[\rho h_1]}\right)\right]$$
(3.16a)

s.t 
$$E[P(g_1, g_0)g_0] \le Q_{average}$$
 (3.16b)

where the transmit power is now only a function of  $(g_1, g_0)$  and independent of  $h_1$ . Following the same procedure used to derive (3.5) in the previous section, the optimal power allocation in the optimization problem (3.16a) subject to (3.16b) is given by

$$P(g_1, g_0) = \frac{1}{\lambda g_0} - \frac{N_0 + \rho}{g_1}$$
(3.17)

In (3.17) by considering the constraint  $P(g_1, g_0) \ge 0$ , we have

$$\frac{g_1}{g_0} \ge \lambda \left( N_0 + \rho \right) \tag{3.18}$$

Then, by replacing (3.17) into (3.16b) and considering equality we get

$$Q_{average} = E\left[\frac{1}{\lambda} - \frac{g_0}{g_1}\left(N_0 + \rho\right)\right].$$
(3.19)

The parameter  $\lambda$  can be obtained in terms of  $Q_{average}$  by using the nonlinear equation (3.19) and hence the maximum capacity becomes

$$C = E\left[\ln\left(\frac{1}{\lambda}\frac{g_1}{g_0\left(N_0 + \rho\right)}\right)\right]$$
(3.20)

By using (3.10), we find  $Q_{average}$  as follows

$$Q_{average} = \int_{0}^{\frac{1}{\lambda(N_{0}+\rho)}} \left(\frac{1}{\lambda} - x(N_{0}+\rho)\right) \frac{1}{(1+x)^{2}} dx$$
$$= \frac{\frac{1}{\lambda(N_{0}+\rho)} - (1+\lambda(N_{0}+\rho))\ln(1+\frac{1}{\lambda(N_{0}+\rho)}) + 1}{\frac{1}{(N_{0}+\rho)} + \lambda}$$
(3.21)

and the maximum capacity becomes

$$C = \int_{0}^{\frac{1}{\lambda(N_{0}+\rho)}} \ln(\frac{1}{\lambda} \frac{1}{x(N_{0}+\rho)}) \frac{1}{(1+x)^{2}} dx$$
  
=  $\ln\left(1 + \frac{1}{\lambda(N_{0}+\rho)}\right)$  (3.22)

which is a closed-form expression for ergodic capacity when ST is only dependent on  $g_0$  and  $g_1$ .

## Partial CSI: Reduced only CSI $g_0$ [ $P(g_1, h_1)$ ]

Here, the maximum capacity with a reduced side information  $g_0$  at the ST is computed. Therefore, we disregard the effect of  $g_0$  from the power allocation, resulting in the following optimization problem

$$\max_{P \ge 0} \quad E\left[\ln\left(1 + \frac{P(g_1, h_1)g_1}{N_0 + \rho h_1}\right)\right]$$
(3.23a)

$$s.t \qquad E\left[P(g_1, h_1)g_0\right] \le Q_{average} \tag{3.23b}$$

Note that the constraint is equivalent to

$$s.t \qquad E\left[P(g_1, h_1)\right] \le Q_{average} \tag{3.24a}$$

because  $g_0$  is independent of  $g_1$  and  $h_1$ . Similarly, by applying the Lagrangian approach, we get the optimal power allocation as

$$P(g_1, h_1) = \frac{1}{\lambda} - \frac{N_0 + \rho h_1}{g_1}$$
(3.25)

We can find  $Q_{average}$  as following

$$Q_{average} = \frac{1}{\rho} \int_{N_0}^{\infty} \int_{\lambda y}^{\infty} \left(\frac{1}{\lambda} - \frac{y}{g_1}\right) e^{-g_1} e^{-\frac{y-N_0}{\rho}} dg_1 dy \tag{3.26}$$

For  $N_0 = 0$ , (3.26) can be evaluated in the following closed-form

$$Q_{average} = \frac{1}{\lambda} + (-1 + \gamma) \rho - \rho \ln (1 + \lambda \rho)$$

The maximum capacity becomes

$$C = \frac{1}{\rho} \int_{N_0}^{\infty} \int_{\lambda y}^{\infty} \ln(\frac{1}{\lambda} \frac{g_1}{y}) e^{-g_1} e^{-\frac{y-N_0}{\rho}} dg_1 dy$$
  
=  $-E_i (-\lambda N_0) + e^{\frac{N_0}{\rho}} E_i \left(-\frac{N_0 + \lambda \rho N_0}{\rho}\right)$  (3.27)

Equation (3.27) is an expression for ergodic capacity when the ST knows only channel gains  $g_1$  and  $h_1$ .

#### Partial CSI: Reduced only CSI $g_1 [P(g_0, h_1)]$

In order to find the impact of having  $g_1$  at the ST, the maximum capacity with no  $g_1$  at the ST is computed. So, we ignore  $g_1$  from the optimization problem (3.2a) subject to (3.2b), yielding the equation

$$\max_{P \ge 0} \quad E\left[\int_0^\infty \ln\left(1 + \frac{P(g_0, h_1)g_1}{N_0 + \rho h_1}\right)e^{-g_1}dg_1\right]$$
(3.28a)

s.t 
$$E[P(g_0, h_1)g_0] \le Q_{average}$$
 (3.28b)

Following the same procedure by applying the Lagrangian approach, we find the optimal power allocation as

$$P(g_0, h_1) = \int_{(N_0 + \rho h_1)\lambda g_0}^{\infty} \left(\frac{1}{\lambda g_0} - \frac{N_0 + \rho h_1}{g_1}\right) e^{-g_1} dg_1$$
  
=  $\frac{e^{-(N_0 + \rho h_1)\lambda g_0}}{\lambda g_0} - (N_0 + \rho h_1)$   
×  $(\Gamma(0, (N_0 + \rho h_1)\lambda g_0) + \ln((N_0 + \rho h_1)\lambda g_0))$  (3.29)

Accordingly,  $Q_{average}$  becomes

$$Q_{average} = \frac{1}{\rho} \int_{N_0}^{\infty} \int_0^{\infty} \frac{1}{\lambda} (e^{-y\lambda g_0} - y\lambda g_0(\Gamma(0, y\lambda g_0) + \ln(y\lambda g_0))) e^{-g_0} e^{-\frac{y-N_0}{\rho}} dg_0 dy \quad (3.30)$$

For  $N_0 = 0$ , we can obtain the following closed form result

$$Q_{average} = \frac{1}{\lambda} + (\gamma - 1)\rho - \rho \ln(\lambda\rho) + \rho U(0, -1, \frac{1}{\lambda\rho})$$
(3.31)

where  $\gamma$  is the Euler's constant and U(a, b, z) is the confluent hypergeometric function. The parameter  $\lambda$  can be obtained in terms of  $Q_{average}$  and finally the maximum capacity is expressed as equation (3.32)

$$C = \frac{1}{\rho} \int_{0}^{\infty} \int_{N_{0}}^{\infty} \int_{y\lambda g_{0}}^{\infty} \ln\left(1 + g_{1}B\right) e^{-g_{1}} e^{-g_{0}} e^{-\frac{y - N_{0}}{\rho}} dg_{1} dy dg_{0}$$
  
$$= \frac{1}{\rho} \int_{N_{0}}^{\infty} \int_{0}^{\infty} \left(e^{\frac{1}{B}} \Gamma(0, y\lambda g_{0} + \frac{1}{B}) + e^{-y\lambda g_{0}} \ln\left(1 + y\lambda g_{0}B\right)\right) e^{-\frac{y - N_{0}}{\rho} - g_{0}} dg_{0} dy$$
  
(3.32)

where  $B = \frac{e^{-y\lambda g_0}}{y\lambda g_0} - (\Gamma(0, y\lambda g_0) + \ln(y\lambda g_0))$ . We observe that closed-form expressions are not obtainable for (3.32) and hence we need to solve the equation numerically.

#### Without CSI [constant P]

Here, all the channel side information which can be available at the ST are reduced and the power at the ST becomes constant. Hence, the maximum capacity is calculated by ignoring  $h_1$ ,  $g_0$  and  $g_1$  from the optimization problem, yielding the equation

$$\max_{P \ge 0} \quad E\left[\ln\left(1 + \frac{Pg_1}{N_0 + \rho h_1}\right)\right] \tag{3.33a}$$

$$s.t \qquad E\left[Pg_0\right] \le Q_{average} \tag{3.33b}$$

Here, we find the maximum capacity with no CSI available at ST, and therefore the power becomes a constant and independent of channel gains. We can simplify the above optimization problem into

$$\max_{P \ge 0} \quad E\left[\ln\left(1 + \frac{Pg_1}{N_0 + \rho h_1}\right)\right] \tag{3.34a}$$

$$s.t \qquad P \le Q_{average}$$
(3.34b)

which in the case of Rayleigh fading gives

$$C = E\left[\ln\left(1 + \frac{Q_{average}g_1}{N_0 + \rho h_1}\right)\right]$$
$$= \int_0^\infty \int_0^\infty \ln\left(1 + \frac{Q_{average}g_1}{N_0 + \rho h_1}\right) e^{-g_1} e^{-h_1} dg_1 dh_1$$
(3.35)

Therefore, the capacity in this case simplifies into

$$C = \frac{e^{\frac{N_0(1+\rho-Q_{average})}{Q_{average}}}E_i\left(-\frac{N_0(1+\rho)}{Q_{average}}\right) - e^{\frac{N_0}{\rho}}E_i\left(-\frac{N_0(1+\rho)}{\rho}\right)}{\frac{\rho}{Q_{average}} - 1}$$
(3.36)

which is a closed-form expression for capacity with no CSI available at ST. Numerical results which compares the ergodic capacities under different CSI will be given at the end of this section.

#### 3.2.2 The Peak Interference Power Constraint

The peak power constraint is more appropriate when the quality of service (QoS) is limited by the instantaneous SINR at the receiver. Therefore, we use the following optimization problem

$$\max_{P \ge 0} \quad E\left[\ln\left(1 + \frac{Pg_1}{N_0 + \rho h_1}\right)\right] \tag{3.37a}$$

$$s.t \qquad Pg_0 \le Q_{peak.} \tag{3.37b}$$

where the equation (3.37b) denotes peak interference power constraint and  $Q_{peak}$ is the peak received power limit at the existing PR. We obtain the maximum capacity if the power is replaced by

$$P(g_0) = \frac{Q_{peak}}{g_0} \tag{3.38}$$

where, in this case,  $P(g_0)$  is only a function of  $g_0$  and independent of  $g_1$  and  $h_1$ .

In order to investigate the impact of having  $g_0$  at the ST, ergodic capacity is evaluated under two different scenarios. In the first scenario, the optimum power allocation P is a function of  $g_0$  and in the second scenario, the channel side information  $g_0$  at ST is reduced and P is a constant.

#### Full CSI $[P(g_0)]$

In this case, we obtain the maximum capacity when the power at ST is a function of  $g_0$  as in the following expression

$$C = E\left[\ln\left(1 + \frac{Q_{peak}}{N_0 + \rho h_1} \frac{g_1}{g_0}\right)\right]$$
(3.39)

We substitute  $x = \frac{g_1}{g_0}$  and  $y = (N_0 + \rho h_1)$  in equation (3.39) resulting in,

$$C = \frac{1}{\rho} \int_{N_0}^{\infty} \int_0^{\infty} \ln\left(1 + \frac{Q_{peak}x}{y}\right) \frac{e^{-\frac{y-N_0}{\rho}}}{1+x^2} dx dy$$
(3.40)

The equation (3.40) can be simplified into (3.41)

$$C = \int_0^\infty \frac{1}{2\left(1+x^2\right)} \left( e^{\frac{N_0}{\rho}} \left( 2E_i \left(-\frac{N_0}{\rho}\right) + e^{\frac{Q_{peak}x}{\rho}} \left(-2E_i \left(-\frac{N_0 + Q_{peak}x}{\rho}\right) - 2\ln\left(\frac{Q_{peak}x}{\rho}\right) + \ln\left(\frac{Q_{peak}^2x^2}{\rho^2}\right) \right) \right) + 2\ln\left(1 + \frac{Q_{peak}x}{N_0}\right) dx \quad (3.41)$$

which can be calculated numerically.

#### Without CSI [constant P]

Here, we find the maximum capacity with a reduced side information where  $g_0$  is not provided at the ST. Thus, the power of ST becomes a constant. The maximum capacity problem changes into the following form

$$C = E\left[\ln\left(1 + \frac{Q_{peak}g_1}{N_0 + \rho h_1}\right)\right]$$
(3.42)

Likewise, in the case of Rayleigh fading the maximum capacity becomes

$$C = \frac{e^{\frac{N_0(1+\rho-Q_{peak})}{Q_{peak}}}E_i\left(-\frac{N_0(1+\rho)}{Q_{peak}}\right) - e^{\frac{N_0}{\rho}}E_i\left(-\frac{N_0(1+\rho)}{\rho}\right)}{\frac{\rho}{Q_{peak}} - 1}$$
(3.43)

Equation (3.43) is closed-form expression for capacity under peak interference power constraint when P is constant.

#### 3.2.3 Numerical Results

In this section, we present some numerical results for the maximum capacity under average/peak interference power constraints and different CSI levels. We assume that  $N_0 = 1$ .

#### The Average Interference Power Constraint

Fig. 3.2 and Fig. 3.3 display capacity versus  $Q_{average}$  under average interference power constraint for different values of  $\rho$ . Comparing Fig. 3.2 with Fig. 3.3 indicates that the interference from the PT can have a big impact on the capacity. As we can see, capacity in all cases increases with increasing  $Q_{average}$ . Further examination of Fig. 3.2 and Fig. 3.3 reveals that the highest capacity occurs when  $g_1, g_0$  and  $h_1$  at the power of ST are included while the lowest capacity occurs when  $g_1, g_0$  and  $h_1$  are excluded.

Another important observation is that the capacity difference between no reduced CSI and reducing only CSI  $g_1$  is very small such that having side information  $g_1$  at ST has negligible effect on the system performance. Again, we observe that when only CSI  $g_0$  is reduced, the secondary link loses all the capacity advantage that can be achieved by having all side information. Therefore,  $g_0$  has the highest impact on the capacity of the system, while having  $g_1$  has very minimal impact. This is because the channel side information  $g_0$  directly affects the optimization problem but channel side information  $g_1$  is inside the logarithmic function and has less impact. Furthermore, the effect of having only  $h_1$  is less than  $g_0$  and bigger than  $g_1$  such that by reducing only CSI  $h_1$ , we lose almost half of the obtainable capacity.

#### The Peak Interference Power Constraint

Fig. 3.4 shows the capacity under peak interference power constraint for two different values of  $\rho$ . In this case, the capacity almost linearly increases with increasing  $Q_{peak}$ . Comparing Fig. 3.4 against Fig. 3.2 and Fig. 3.3, we observe the performance of the channel under peak interference power constraint is almost the same as under average interference power constraint. As can be seen, the difference between the capacity in both cases when  $g_0$  is either included or ignored is considerable.

## **3.3** Minimization of the Bit Error Rate (BER)

The BER in most common types of digital modulation schemes in wireless communication takes one of the following forms [18]

$$BER = \begin{cases} \frac{1}{2} \exp\left(-SINR\right) & e.g. \ DPSK \\ aQ(\sqrt{bSINR}) & e.g. \ BPSK, QPSK \end{cases}$$
(3.44)

where Q(x) is the Q-function, and a, b > 0. The equation (3.44) applies to a wide class of modulation schemes. For example, exact results follow for quadrature



Figure 3.2: The impact of reducing CSI under average interference power constraint with  $\rho=5dB$ 



Figure 3.3: The impact of reducing CSI under average interference power constraint with  $\rho=10dB$ 



Figure 3.4: The impact of reducing CSI under peak interference power constraint

phase-shift keying (QPSK) and binary phase shift keying (BPSK) with (a, b) = (1, 2). Furthermore, in the case of M-PSK, (a, b) is  $\left(\frac{1}{\log_2 M}, \sin^2(\frac{\pi}{M}) \times \log_2 M\right)$  and for QAM (a, b) becomes  $\left(\frac{2}{\log_2 M}, \frac{3\log_2 M}{M-1}\right)$  to approximate the BER.

In the following two sections, we derive expressions for evaluating the minimum average BER under different constraints and different digital modulations.

## 3.3.1 Minimum BER under Average Interference Power Constraint

#### Minimization of Exp(-SINR)

The minimum BER under the average interference power constraint can be obtained by solving the following optimization problem

$$\min_{P \ge 0} \quad E\left[\frac{1}{2}\exp\left(-\frac{P(g_1, g_0, h_1)g_1}{N_0 + \rho h_1}\right)\right]$$
(3.45a)

s.t 
$$E[P(g_1, g_0, h_1)g_0] \le Q_{average}$$
 (3.45b)

Equation (3.45b) represents the average interference power constraint which can be used to guarantee a long-term QoS of PU.  $Q_{average}$  is the maximum average received power limit at PR. The optimal power allocation,  $P(g_1, g_0, h_1)$ , is obtained by forming the Lagrangian

$$L(P,\lambda) = E\left[\frac{1}{2}\exp\left(-\frac{P(g_1, g_0, h_1)g_1}{N_0 + \rho h_1}\right)\right] + \lambda\left(E\left[P(g_1, g_0, h_1)g_0\right] - Q_{average}\right) \quad (3.46)$$

where  $\lambda$  is the nonnegative dual variable. By applying the KKT conditions [16], the optimal power allocation satisfies the following equation

$$\frac{L(P,\lambda)}{\partial P} = E\left[-\frac{g_1}{N_0 + \rho h_1}\frac{1}{2}\exp(-\frac{P(g_1,g_0,h_1)g_1}{N_0 + \rho h_1}) + \lambda g_0\right] = 0$$
(3.47)

which results in

$$P(g_1, g_0, h_1) = \frac{N_0 + \rho h_1}{g_1} \ln\left(\frac{1}{N_0 + \rho h_1} \frac{1}{2\lambda} \frac{g_1}{g_0}\right).$$
(3.48)

Note that the optimum power allocation P, the instantaneous power at the ST, is a function of the channel gains  $g_1$ ,  $g_0$  and  $h_1$ . By considering the constraint  $P(g_1, g_0, h_1) \ge 0$  in (3.48) we get

$$\frac{1}{2}\frac{1}{N_0 + \rho h_1}\frac{g_1}{g_0} \ge \lambda \tag{3.49}$$

The parameter  $\lambda^*$ , which satisfies the Complementary Slackness Conditions [16], can be obtained by solving

$$Q_{average} = E\left[\frac{g_0}{g_1} \left(N_0 + \rho h_1\right) \ln\left(\frac{1}{N_0 + \rho h_1} \frac{1}{2\lambda^*} \frac{g_1}{g_0}\right)\right].$$
 (3.50)

We can get the minimum BER by substituting (3.48) in (3.45a)

$$BER = \lambda^* E\left[\frac{g_0}{g_1}\left(N_0 + \rho h_1\right)\right].$$
(3.51)

In order to find the impact of having  $g_0$  and  $h_1$  at the ST, *BER* is evaluated under two different special cases. In the first case, the interference from the PT is ignored ( $\rho = 0$ ) and then the optimum power allocation P becomes only a function of  $g_1$  and  $g_0$ . Thus, we can study the effect of having extra CSI  $g_0$  at the ST. In the second case, the interference from the PT is included and the optimum power allocation P is a function of  $g_1, g_0$  and  $h_1$ , which leads to study the effect of having extra CSI  $g_0$  and  $h_1$  at the ST.

#### Special Case 1: The effect of having extra CSI $g_0$ at ST

In order to focus on the effect of having  $g_0$  at the ST, we assume that  $\rho = 0$  and P is only a function of  $g_1$  and  $g_0$ .

In the case of a Nakagami-m fading model, the channel power gain is distributed as a gamma distribution [20]

$$f(x) = \frac{m^m}{\Gamma(m)} x^{m-1} \exp(-mx) \quad x \ge 0$$
 (3.52)

where  $\Gamma(x)$  is the Gamma function. Note that if  $g_1$  and  $g_0$  are independent gamma random variables with parameters  $m_1$  and  $m_0$  respectively, then the probability density function (PDF) of the ratio  $g_1/g_0$  becomes (e.g. [17, pp 695])

$$f_{g_1/g_0}(x) = \frac{x^{m_1 - 1}(1 + x)^{-m_1 - m_0}}{B(m_1, m_0)}$$
(3.53)

where  $B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$  is the beta function. By using (3.53) and assuming  $m_0 =$ 

 $m_1 = m, (3.50)$  becomes

$$Q_{average}\left(\lambda^{*}\right) = \frac{N_{0}}{B(m,m)} \int_{2\lambda^{*}N_{0}}^{\infty} \ln\left(\frac{x}{2\lambda^{*}N_{0}}\right) \frac{x^{m-1}}{x} (1+x)^{-2m} dx \qquad (3.54)$$

upon using [19, eq.(1.512.3) and eq.(3.197.2)], (3.54) can be rewritten as

$$Q_{average} \left(\lambda^{*}\right) = \frac{\left(\frac{1}{2\lambda^{*}N_{0}}\right)^{m}}{2\lambda^{*} \left(1+m\right)^{2} B(m,m)} \times {}_{3}F_{2} \left(2m, 1+m, 1+m; 2+m, 2+m; -\frac{1}{2\lambda^{*}N_{0}}\right) \quad (3.55)$$

in which  $_{3}F_{2}(a_{1}, a_{2}, a_{3}; b_{1}, b_{2}; z)$  is the hypergeometric function.

It is worth noting that we can find  $\lambda^*$  for a given  $Q_{average}$  from equation (3.55). Then, the minimum BER (3.51) is determined by

$$BER = \frac{\lambda^* N_0}{B(m,m)} \int_{2\lambda^* N_0}^{\infty} x^{m-2} (1+x)^{-2m} dx$$
(3.56)

which is reduced, upon the change of variable  $x = -\frac{1}{t}$ , to the simple closed-form

$$BER = -\lambda^* N_0 (-1)^{-m} \frac{B_{-\frac{1}{2\lambda^* N_0}} \left(1 + m, 1 - 2m\right)}{B(m, m)}$$
(3.57)

where  $B_a(c, d)$  is the incomplete Beta function. When m = 1 (Rayleigh fading), we obtain the following simplified equations instead of (3.55) and (3.57).

$$Q_{average}\left(\lambda^{*}\right) = -N_{0}\left(\ln\left(1 + \frac{1}{2\lambda^{*}N_{0}}\right) + Li_{2}\left(-\frac{1}{2\lambda^{*}N_{0}}\right)\right)$$
(3.58)

and

$$BER = \lambda^* N_0 B_{-\frac{1}{2\lambda^* N_0}} \left(2, -1\right) \tag{3.59}$$
where  $Li_2(.)$  is the polylogarithm function of order 2 [19].

## Special Case 2: The effect of having extra CSI $g_0$ and $h_1$ at ST

Here, the effect of interference coming from the PT on the SR is considered by studying the impact of providing channel gain  $g_0$  and  $h_1$  at the ST. We substitute  $x = \frac{g_1}{g_0}$  and  $y = (N_0 + \rho h_1)$  in (3.50), which yields

$$Q_{average}\left(\lambda^*\right) = \int_0^\infty \int_{x>2y\lambda^*} \frac{y}{x} \ln(\frac{x}{2\lambda^* y}) f_x(x) f_y(y) du dy \tag{3.60}$$

Closed-form result for (3.60) can be obtained in the special case of Rayleigh fading where  $m_0 = m_1 = 1$ . In this case the PDF of  $\frac{g_1}{g_0}$  becomes

$$f_{\frac{g_1}{g_0}}(x) = \frac{1}{(1+x)^2} \tag{3.61}$$

and the PDF of  $N_0 + \rho h_1$  for m = 1 in equation (3.52) can be expressed as

$$f_{(N_0+\rho h_1)}(y) = \frac{1}{\rho} e^{-\left(\frac{y-N_0}{\rho}\right)} \qquad y \ge N_0 \tag{3.62}$$

By using (3.61) and (3.62) in (3.60), we obtain

$$Q_{average}\left(\lambda^{*}\right) = \frac{1}{\rho} \int_{N_{0}}^{\infty} \int_{2y\lambda^{*}}^{\infty} \left(\frac{y}{x} \ln\left(\frac{x}{2\lambda^{*}y}\right)\right) \times \frac{1}{(1+x)^{2}} \left(e^{-\left(\frac{y-N_{0}}{\rho}\right)}\right) dudy. \quad (3.63)$$

In the special case of  $N_0 = 0$ , we can obtain closed-form results with the aid of [19, eq.(1.512.3), eq.(3.197.2), eq.(3.326.2), eq.(4.352.2), eq.(4.337.5), eq.(4.358.2)

and eq.(3.351.2)]. This can be expressed as

$$Q_{average} \left(\lambda^{*}\right) = \frac{1}{4\rho\lambda^{*2}} \left[\lambda^{*}\rho \left(2e^{\frac{1}{2\lambda^{*}\rho}} \left(2\lambda^{*}\rho - 1\right)E_{i}\left(\frac{-1}{2\lambda^{*}\rho}\right) + \lambda^{*}\rho(\pi^{2} + 2\ln^{2}2 + 2\gamma(\gamma - 4 - \ln 4) + \ln 256 + 2\ln\rho\ln(4\rho) - 4(\gamma - 2)\ln(\lambda^{*}\rho) + 2\ln\lambda^{*}\ln\left(4\lambda^{*}\rho^{2}\right)\right) - G_{34}^{44} \left(\frac{1}{2\lambda^{*}\rho}|_{-2,-2,-2,0}^{-2,-1,-1}\right)\right] (3.64)$$

where  $E_i(.)$  is the exponential integral function and  $\gamma$  is the Euler's constant value and  $G_{pq}^{mn}\left(.|_{(bq)}^{(a_p)}\right)$  is the Meijer function. Similarly, we substitute  $u = \frac{g_0}{g_1}$ and  $y = (N_0 + \rho h_1)$  in (3.51) to get

$$BER = \lambda^* \int_0^\infty \int_{u < \frac{1}{2y\lambda^*}} (uy) f_u(u) f_y(y) dudy$$
(3.65)

which results in

$$BER = \frac{\lambda^*}{\rho} \int_{N_0}^{\infty} \int_0^{\frac{1}{2y\lambda^*}} \frac{uy}{(1+u)^2} e^{-\left(\frac{y-N_0}{\rho}\right)} du dy.$$
(3.66)

By using [19, eq.(2.113.2), eq.(4.222.8), eq.(3.353.5) and eq.(4.352.2)], the double integration in (3.66) can be represented as equation (3.67)

$$BER = \frac{1}{4\rho\lambda^{*}} \left[ -e^{\frac{1+2\lambda^{*}N_{0}}{2\lambda^{*}\rho}} (1-2\lambda^{*}\rho+4\rho^{2}{\lambda^{*}}^{2}) E_{i}(-\frac{1+2\lambda^{*}N_{0}}{2\lambda^{*}\rho}) + 2\lambda^{*}\rho \left( 2e^{\frac{N_{0}}{p}}\rho\lambda^{*}E_{i}(-\frac{N_{0}}{\rho}) - 2\lambda^{*} \left(N_{0}+\rho\right) \ln(\frac{2\lambda^{*}N_{0}}{1+2\lambda^{*}N_{0}}) - 1 \right) \right]$$
(3.67)

# Minimization of $aQ(\sqrt{bSINR})$

The minimum BER under the average interference power constraint can be obtained by using  $Q(x) = \frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)$  and then solving the following optimization

problem

$$\min_{P \ge 0} \quad E\left[\frac{a}{2}\operatorname{erfc}\sqrt{\frac{b}{2}\frac{P(g_1, g_0, h_1)g_1}{N_0 + \rho h_1}}\right]$$
(3.68a)

s.t. 
$$E[P(g_1, g_0, h_1)g_0] \le Q_{average}$$
 (3.68b)

Equation (3.68b) represents the average interference power constraint which can be used to guarantee a long-term QoS of PU, where  $Q_{average}$  is the maximum average received power limit at PR. The error function can be written as the following identity which can be employed to simplify the BER analysis in fading environments [21, eq.(4.2)]

$$\operatorname{erfc}\sqrt{x} = \frac{2}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x}{\cos^2(\theta)}\right) d\theta.$$
(3.69)

Therefore (3.68a) becomes

$$\min_{P \ge 0} \quad E\left[\frac{a}{\pi} \int_0^{\pi/2} \exp\left(-\frac{b}{2} \frac{P(g_1, g_0, h_1)g_1}{N_0 + \rho h_1} \frac{1}{\cos^2(\theta)}\right) d\theta\right]$$
(3.70)

where the expectation is with respect to the channel gains  $g_1, g_0$  and  $h_1$ . Notice that (3.70) subject to (3.68b) is mathematically equivalent to the following problem

$$\min_{P \ge 0} \quad E\left[\frac{a}{2}\exp\left(-\frac{b}{2}\frac{P'(g_1, g_0, h_1, \theta)g_1}{N_0 + \rho h_1}\frac{1}{\cos^2(\theta)}\right)\right]$$
(3.71a)

s.t. 
$$E\left[P'(g_1, g_0, h_1, \theta)g_0\right] \le Q_{average}$$
 (3.71b)

where the expectation is with respect to  $g_1, g_0, h_1$  and  $\theta$ . Here, we regarded the integration in (3.70) with respect to  $\theta$  as expectation with respect to a dummy random variable  $\theta$ , which is uniformly distributed over  $(0, \pi/2)$ .  $P'(g_1, g_0, h_1, \theta)$ 

also represents a dummy power allocation which is a function of  $g_1, g_0, h_1$  and  $\theta$ .

The optimal power, P', is obtained by forming the Lagrangian

$$L(P',\lambda) = E\left[\frac{a}{2}\exp\left(-\frac{b}{2}\frac{P'(g_1,g_0,h_1,\theta)g_1}{N_0 + \rho h_1}\frac{1}{\cos^2(\theta)}\right)\right] + \lambda\left(E\left[P'(g_1,g_0,h_1,\theta)g_0\right] - Q_{average}\right) \quad (3.72)$$

where  $\lambda$  is the nonnegative dual variable. Taking the derivative of the Lagrangian with respect to  $P'(g_1, g_0, h_1, \theta)$  and letting the derivative equal to zero yields [16],

$$\frac{L(P',\lambda)}{\partial P'} = E\left[-\frac{g_1}{N_0 + \rho h_1} \frac{ab}{4\cos^2(\theta)} \exp\left(-\frac{b}{2\cos^2(\theta)} \frac{P'(g_1,g_0,h_1,\theta)g_1}{N_0 + \rho h_1}\right) + \lambda g_0\right] = 0$$
(3.73)

which results in

$$P'(g_1, g_0, h_1, \theta) = \frac{2}{b} \frac{N_0 + \rho h_1}{g_1} \cos^2(\theta) \ln[\frac{ab}{4\lambda g_0} \frac{g_1}{N_0 + \rho h_1} \frac{1}{\cos^2(\theta)}]$$
(3.74)

Then, we can find  $P(g_1, g_0, h_1)$  as the average of  $P'(g_1, g_0, h_1, \theta)$  over  $\theta$ 

$$P(g_1, g_0, h_1) = \frac{2 \times 2}{b\pi} \frac{N_0 + \rho h_1}{g_1} \int_0^{\pi/2} \cos^2(\theta) \ln[\frac{ab}{4\lambda g_0} \frac{g_1}{N_0 + \rho h_1} \frac{1}{\cos^2(\theta)}] d\theta \quad (3.75)$$

which gives

$$P(g_1, g_0, h_1) = \frac{1}{b} \frac{N_0 + \rho h_1}{g_1} \ln\left(\frac{1}{e} \frac{ab}{\lambda g_0} \frac{g_1}{N_0 + \rho h_1}\right)$$
(3.76)

In (3.76) by considering the constraint  $P(g_1, g_0, h_1) \ge 0$  we have

$$\frac{ab}{\lambda e}\frac{1}{N_0+\rho h_1} > \frac{g_0}{g_1}$$

The parameter  $\lambda^*$ , which satisfies the following Complementary Slackness Conditions, can be obtained by solving

$$Q_{average} = E\left[\frac{1}{b}\frac{g_0}{g_1}\left(N_0 + \rho h_1\right)\ln\left(\frac{1}{e}\frac{ab}{\lambda^* g_0}\frac{g_1}{N_0 + \rho h_1}\right)\right].$$
 (3.77)

We can get the minimum BER by substituting (3.76) in (3.68a) as

$$BER = E\left[\frac{a}{2}\operatorname{erfc}\sqrt{\frac{1}{2}\ln\left(\frac{1}{e}\frac{ab}{\lambda^*g_0}\frac{g_1}{N_0 + \rho h_1}\right)}\right].$$
(3.78)

In order to find the impact of having  $g_0$  and  $h_1$  at the ST, *BER* is evaluated under two different special cases. In the first case, the interference from the PT is ignored ( $\rho = 0$ ) and then the optimum power allocation P becomes only a function of  $g_1$  and  $g_0$ . Thus, we can study the effect of having extra CSI  $g_0$  at the ST. In the second case, the interference from the PT is included and the optimum power allocation P is a function of  $g_1, g_0$  and  $h_1$ , which leads to studying the effect of having extra CSI  $g_0$  and  $h_1$  at the ST.

Special Case 1: The effect of having extra CSI  $g_0$  at ST In order to focus on the effect of having  $g_0$  at the ST, we assume that  $\rho = 0$  and then P becomes a function of only  $g_1$  and  $g_0$ .

By invoking (3.53) and assuming  $m_0 = m_1 = m$ ,  $Q_{average}$  in the case of Nakagami fading becomes

$$Q_{average}\left(\lambda^*\right) = \frac{N_0}{B(m,m)} \frac{1}{b} \int_{\frac{\lambda^*e}{ab}N_0}^{\infty} \ln\left(\frac{ab}{\lambda^*e} \frac{x}{N_0}\right) \frac{x^{m-1}}{x} (1+x)^{-2m} dx \qquad (3.79)$$

The integral in (3.79) can be evaluated into a closed-form expression as follows

$$Q_{average} \left(\lambda^{*}\right) = \frac{N_{0}}{bB(m,m)(1+m)^{2}} \left(\frac{ab}{eN_{0}\lambda^{*}}\right)^{1+m} \times {}_{3}F_{2} \left(2m, 1+m, 1+m; 2+m, 2+m; -\frac{ab}{eN_{0}\lambda^{*}}\right) \quad (3.80)$$

in which  ${}_{3}F_{2}(a_{1}, a_{2}, a_{3}; b_{1}, b_{2}; z)$  is the hypergeometric function. Equation (3.80) can be simplified in the case of Rayleigh fading (m = 1) as

$$Q_{average}\left(\lambda^{*}\right) = \frac{N_{0}}{6b} \left(\pi^{2} - 6\ln\left(1 + \frac{ab}{eN_{0}\lambda^{*}}\right) + 3\ln^{2}\left(\frac{ab}{eN_{0}\lambda^{*}}\right) + 6Li_{2}\left(-\frac{eN_{0}\lambda^{*}}{ab}\right)\right) \quad (3.81)$$

where  $Li_2(.)$  is the polylogarithm function of order 2. It is worth noting that we can find  $\lambda^*$  for a given  $Q_{average}$  from equation (3.81). Likewise, the minimum *BER* can be expressed as

$$BER = \frac{a}{2B(m,m)} \int_{\frac{\lambda^* e}{ab} N_0}^{\infty} \operatorname{erfc} \sqrt{\frac{1}{2} \ln(\frac{ab}{\lambda^* e} \frac{x}{N_0})} x^{m-1} (1+x)^{-2m} dx.$$
(3.82)

(3.82) gives the minimum BER when  $\rho = 0$  and P is only a function of  $g_1$ and  $g_0$ .

Special Case 2: The effect of having extra CSI  $g_0$  and  $h_1$  at ST Here, the effect of interference on the SR coming from the PT is considered by studying the impact of providing channel gain  $g_0$  and  $h_1$  at the ST. We substitute  $x = \frac{g_1}{g_0}$ and  $y = (N_0 + \rho h_1)$  in equation (3.77) to find

$$Q_{average}\left(\lambda^{*}\right) = \int_{N_{0}}^{\infty} \int_{\frac{\lambda^{*}e}{ab}y}^{\infty} \frac{y}{bx} \ln\left(\frac{ab}{\lambda^{*}e}\frac{x}{y}\right) f_{x}(x) f_{y}(y) dx dy.$$
(3.83)

By using (3.61) and (3.62) in (3.83), we obtain

$$Q_{average}\left(\lambda^*\right) = \int_{N_0}^{\infty} \int_{\frac{\lambda^*e}{ab}y}^{\infty} \frac{y}{bx} \ln\left(\frac{ab}{\lambda^*e}\frac{x}{y}\right) \frac{e^{-\left(\frac{y-N_0}{\rho}\right)}}{(1+x)^2} dx dy.$$
(3.84)

In the special case of  $N_0 = 0$ , we obtain the closed-form result as equation

(3.85)

$$Q_{average} \left(\lambda^{*}\right) = \frac{1}{4be^{2}\lambda^{*^{2}}} \left[ e\lambda^{*}\rho \left(4e^{\frac{ab}{e\lambda^{*}\rho}} \left(e\lambda^{*}\rho - ab\right) E_{i}\left(\frac{-ab}{e\lambda^{*}\rho}\right) + e\lambda^{*}\rho \left(2\gamma^{2} - 8\gamma + \pi^{2} - 4\ln\frac{ab}{e\lambda^{*}} + 2\ln^{2}\rho - 4\ln\rho(\gamma - 2 + \ln\frac{ab}{e\lambda^{*}}) + \left(4\gamma - 4 + 2\ln\frac{ab}{e\lambda^{*}}\right)\ln\frac{ab}{e\lambda^{*}}\right) - 4a^{2}b^{2}G_{34}^{44}\left(\frac{ab}{e\lambda^{*}\rho}\right) \left[\frac{-2, -1, -1}{-2, -2, -2, 0}\right]$$
(3.85)

where  $E_i(.)$  is the exponential integral function and  $\gamma$  is Euler's constant value and  $G_{p q}^{m n}\left(.|_{(b_q)}^{(a_p)}\right)$  is the Meijer function [19]. Likewise, the following equation is obtained for the minimum *BER* 

$$BER = \int_{N_0}^{\infty} \int_{\frac{\lambda^* e}{ab}y}^{\infty} \frac{a}{2} \operatorname{erfc} \sqrt{\frac{1}{2} \ln\left(\frac{ab}{\lambda^* e} \frac{x}{y}\right)} \frac{e^{-\left(\frac{y-N_0}{\rho}\right)}}{(1+x)^2} dx dy.$$
(3.86)

We observe that closed-form expressions are not obtainable for (3.82) and (3.86), and hence we need to solve the equations numerically.

# 3.3.2 Minimum BER under Peak Interference Power Constraint

The peak power constraint is more appropriate when the quality of service (QoS) is limited by the instantaneous SINR at the receiver. Therefore, in this subsection, we replace the following equation denoting peak interference power constraint with (3.45b) and (3.68b)

$$s.t \quad Pg_0 \le Q_{peak.} \tag{3.87}$$

where  $Q_{peak}$  is the peak received power limit at the existing PR. We obtain the minimum bit error rate if the power is replaced by

$$P(g_0) = \frac{Q_{peak}}{g_0} \tag{3.88}$$

where, in this case,  $P(g_0)$  is only a function of  $g_0$  and independent of  $g_1$  and  $h_1$ 

#### Minimization of Exp(-SINR)

We can find the minimum BER in this case by substituting equation (3.88) in (3.45a)

$$BER = \frac{1}{2}E\left[\exp(-\frac{Q_{peak}}{N_0 + \rho h_1}\frac{g_1}{g_0})\right].$$
 (3.89)

We also substitute  $x = \frac{g_1}{g_0}$  and  $y = (N_0 + \rho h_1)$  in equation (3.89) resulting in,

$$BER = \frac{1}{2} \int_0^\infty \int_0^\infty \exp(-\frac{x}{y} Q_{peak}) f_x(x) f_y(y) du dy$$
(3.90)

We arrive at the following closed-form expression with the aid of [19, eq.(3.324.1), eq.(9.34.3) and eq.(7.811.5)] in the special case of m = 1 and the interference limited scenario ( $N_0 = 0$ ) by using the PDF of x and y in (3.61) and (3.62)

$$BER = \frac{1}{2\rho} \int_{N_0}^{\infty} \int_0^{\infty} \exp(-\frac{x}{y} Q_{peak}) \frac{e^{-\left(\frac{y}{\rho}\right)}}{(1+x)^2} dx dy$$
$$= \frac{1}{2} G_{1\,3}^{3\,1} \left(\frac{Q_{peak}}{\rho}|_{0,1,1}^0\right).$$
(3.91)

The above closed-form expression gives the minimum BER under exponential function.

## Minimization of $aQ(\sqrt{bSINR})$

We minimize the BER minimization problem as

$$BER = \frac{1}{2}E\left[\operatorname{erfc}\sqrt{\frac{Q_{peak}}{N_0 + \rho h_1}\frac{g_1}{g_0}}\right].$$
(3.92)

We substitute  $x = \frac{g_1}{g_0}$  and  $y = (N_0 + \rho h_1)$  in equation (3.92) and the BER can

be expressed as

$$BER = \frac{1}{\pi\rho} \int_0^{\pi/2} \int_{N_0}^{\infty} \int_0^{\infty} \exp\left(-\frac{xQ_{peak}}{y\cos^2(\theta)}\right) f_x(x) f_y(y) dx dy d\theta$$
(3.93)

We arrive at the following closed-form expression in the special case of m = 1and the interference limited scenario  $(N_0 = 0)$  by using the PDF of x and y in (3.61) and (3.62)

$$BER = \frac{1}{\pi\rho} \int_0^{\pi/2} \int_0^\infty \int_0^\infty \exp(-\frac{uQ_{peak}}{y\cos^2(\theta)}) \frac{e^{-\left(\frac{y}{\rho}\right)}}{(1+u)^2} du dy d\theta$$
$$= \frac{\Gamma(\frac{1}{2})}{2\pi} G_2^{4} \frac{1}{4} \left(\frac{Q_{peak}}{\rho}\Big|_{\frac{1}{2},0,1,1}^{0,1}\right)$$
(3.94)

where  $G_{pq}^{mn}\left(.|_{(bq)}^{(a_p)}\right)$  is the Meijer function.

#### 3.3.3 Numerical Results

In this section, we present some numerical results for the minimum BER under different scenarios. We also assume (a, b) = (1, 2) which means that BPSK or QPSK is considered. Throughout this section lines represent the results obtained from the analytical results and symbols represent the Monte Carlo simulation results. Both simulation and analytical results are closely matched which supports the validity of the presented analysis.

#### The Average Interference Power Constraint

Special Case 1: The effect of having extra CSI  $g_0$  at ST Fig. 3.5 and Fig. 3.6 display BER versus  $Q_{average}/N_0$  under average interference power constraint for different channel models without interference from the PT for DPSK and BPSK, respectively. The Nakagami parameter indicates the severity of fading,

such that for Rayleigh fading m=1 and for an AWGN channel without fading  $m = \infty$  [14]. As we can see, BER in all cases exponentially decreases with increasing  $Q_{average}/N_0$ . In order to discuss the significance of having channel gain  $g_0$  at the ST, we also include the minimum BER results with a reduced side information where  $g_0$  is not made available at the ST. Hence, by disregarding  $g_0$ , the power of ST depends only on  $g_1$ . The minimum BER problem (3.45a) subject to (3.45b) in DPSK or (3.68a) subject to (3.68b) in BPSK reduce into the simplified form

$$\min_{P \ge 0} \quad E\left[\frac{1}{2}\exp\left(-\frac{P(g_1)g_1}{N_0}\right)\right]$$
  
s.t  $E\left[P(g_1)\right] \le Q_{average}$ 

or

$$\min_{P \ge 0} \quad E\left[\frac{1}{2}\operatorname{erfc}\sqrt{\frac{P(g_1)g_1}{N_0}}\right]$$
  
s.t  $\quad E\left[P(g_1)\right] \le Q_{average}$ 

for which the expectation is only with respect to  $g_1$ . Examining Fig. 3.5 and Fig. 3.6, it can be seen that the BER when the power is a function of  $g_1$  and  $g_0$ , ST-SR and ST-PR CSI, is always lower than that when the power depends only on  $g_1$ , ST-SR CSI. Another important observation is that the difference between the BERs in both cases is very small such that the side information between the ST and the PR has negligible effect on the system performance. Due to difference between exponential function and complementary error function, we can observe that with the same  $Q_{average}/N_0$  the BER under BPSK is always lower than the BER under DPSK. Special Case 2: The effect of having extra CSI  $g_0$  and  $h_1$  at ST The behavior of BER versus  $Q_{average}/N_0$  considering the effect of interference coming from the PT on the SR under average interference power constraint in Rayleigh fading for DPSK and BPSK are shown in Fig. 3.7 and Fig. 3.8, respectively, with  $\rho = 10dB$ . Comparing these figures with Fig. 3.5 and Fig. 3.6 indicates that the interference from the PT can have a big impact on BER. In addition, the minimum BER results with a reduced side information  $g_0$  and  $h_1$  at the ST is also plotted. Therefore, we disregard the effect of  $g_0$  and  $h_1$  from the power allocation, resulting in the simplified optimization problem

$$\min_{P \ge 0} \quad E\left[\frac{1}{2}\exp\left(-\frac{P(g_1)g_1}{N_0 + E[\rho h_1]}\right)\right]$$

$$s.t \quad E\left[P(g_1)\right] \le Q_{average}$$

or

$$\min_{P \ge 0} \quad E\left[\frac{1}{2}\operatorname{erfc}\sqrt{\frac{P(g_1)g_1}{N_0 + E[\rho h_1]}}\right]$$
  
s.t  $\quad E\left[P(g_1)\right] \le Q_{average}$ 

where, in this case, the expectation is with respect to  $g_1$  only. This is in contrast

to (3.45a) and (3.68a) where the expectation is with respect to  $g_1, g_0$  and  $h_1$ . Fig. 3.7 and Fig. 3.8 show that the BER when the power is a function of  $g_1, g_0$  and  $h_1$ is always lower than that when the power depends only on  $g_1$ . Again we observe that the difference between the BERs in both cases is negligible such that having  $g_0$  and  $h_1$  at ST has little effect on the system performance.

#### The Peak Interference Power Constraint

The behavior of BER versus  $Q_{peak}/N_0$  considering the effect of interference coming from the PT on the SR under peak interference power constraint for Rayleigh fading over DPSK and BPSK are shown in Fig. 3.9 and Fig. 3.10, respectively, with  $\rho = 10dB$ . Likewise, we also plot the minimum BER results with a reduced side information  $g_0$  at the ST. Hence, we can ignore the effect of  $g_0$  from the optimization problem, resulting in the following

$$\min_{P \ge 0} E\left[\frac{1}{2}\exp\left(-\frac{Pg_1}{N_0 + \rho h_1}\right)\right]$$
  
s.t  $P \le Q_{peak}$ .

or

$$\min_{P \ge 0} E\left[\frac{1}{2}\operatorname{erfc}\sqrt{\frac{Pg_1}{N_0 + \rho h_1}}\right]$$
  
s.t  $P \le Q_{peak.}$ 

Similarly, the difference between the BERs in both cases when  $g_0$  is either included or ignored is very small.

# 3.4 Chapter Conclusion

In this chapter, we considered a spectrum-sharing system and evaluated the maximum ergodic capacity and minimum BER subject to either average or peak constraint on the interference power. We investigated the effect of different levels of channel side information which can be provided at the secondary transmitter. In most cases, closed-form results were derived for the capacity and BER. Using some



Figure 3.5: The effect of having  $g_0$  at the ST for different fading channel models under average interference power constraint in DPSK



Figure 3.6: The effect of having  $g_0$  at the ST for different fading channel models under average interference power constraint in BPSK.



Figure 3.7: The effect of having  $g_0$  and  $h_1$  at the ST under average interference power constraint for DPSK



Figure 3.8: The effect of having  $g_0$  and  $h_1$  at the ST under average interference power constraint for BPSK



Figure 3.9: The effect of having  $g_0$  at the ST for peak interference power constraint under DPSK considering the interference from PT



Figure 3.10: The effect of having  $g_0$  at the ST for peak interference power constraint under BPSK considering the interference from PT

results from the numerical analysis, the maximum capacity of the secondary link highly depends on having side information between secondary transmitter and primary receiver at the secondary transmitter. However the side information between secondary transmitter and secondary receiver at the secondary transmitter has negligible impact to the average capacity.

Evidently, with the same predefined parameters, minimum BER under average interference power constraints is lower than that under peak interference power constraints. One important observation made is that providing the extra side information between the secondary transmitter and primary receiver and also between primary transmitter and secondary receiver at the secondary transmitter requiring intersystem message-passing have negligible effect on the BER. Therefore, the results found in this chapter can be used as a trade off between performance and complexity.

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# Chapter 4

# Power Allocation in Adhoc Cognitive Radio

In the previous chapter, we considered a simple model for cognitive radio network where a secondary user can simultaneously transmit with a primary user on the same spectrum when the maximum interference offered to the primary receiver is below a predefined threshold. In this chapter, we extend our model to a general model with multiple primary and secondary users, instead of having one primary and secondary link. This scenario provides interesting results, especially when the number of secondary transmitters is large.

In ad hoc cognitive radio networks, the main issue is how to guarantee qualityof-service (QoS) in different applications. Capacity, varying as a function of the channel quality, is one of the major QoS requirements and is interference-limited in mobile communication systems.

In this chapter, we consider a primary cellular network where primary users communicate with the base station through the uplink transmission. The secondary users exist within the coverage area of the base station, and share the radio spectrum with the primary users and communicate with each other in an ad hoc fashion. The similar scenario has recently appeared and the sum throughput as a non-convex problem was solved [1]. Our main objective here is to maximize the ergodic capacity in a wireless ad hoc cognitive radio network when either average or peak interference at primary users is less than a predefined threshold, while assuming all channels experience Rayleigh fading.

Moreover, in [2]-[3], the ergodic capacity of fading channels under two different power constraints, and the corresponding optimal power allocations were given, but only secondary transmitter-secondary receiver and secondary transmitterprimary receiver links were assumed. Thus, the capacity is computed according to signal-to-noise ratio (SNR) and the interferences from other transmitters to the secondary receiver are ignored in [2]-[3].

First, we propose the special case where there is one secondary link, and therefore the only interference comes from primary transmitters. In this case, we obtain the expression for evaluating the ergodic capacity of the secondary link. Then, in general case, we consider the interference from primary and other secondary transmitters changing the optimization problem into a non-convex [4] [5]. Lagrangian technique cannot be applied as the duality gap may not be zero. In this case, we use two theorems recently proposed by [6], which prove that the duality gap of this optimization problem is zero. Thereafter, we form the Lagrangian dual problem and then present gradient method as an iterative algorithm to solve the dual problem.

# 4.1 System Model

As illustrated in Fig. 4.1, we introduce a scenario composed of a primary network with K users and a secondary ad hoc network with M links, coexisting in the same area and sharing the spectrum. In the secondary network, no centralized



Figure 4.1: Coexistence of a cognitive radio ad hoc network (red pairs) with primary users in cellular system (in blue)

authority is assumed to manage the network access for users. A point-to-point link is considered to communicate between a secondary transmitter and a secondary receiver. In order to avoid causing harmful interference to the primary users, secondary transmitters must control their transmit power. Active primary transmitters send information with constant power  $\rho$  and secondary transmitter *i* send signal with power  $P_i$  which is less than or equal to  $\rho$ .

In this system, the instantaneous received signal-to-interference-plus-noise ratio (SINR) at the secondary receiver i can be expressed as

$$SINR_{i} = \frac{P_{i}h_{ii}}{N_{0} + \sum_{k=1}^{K} \rho g_{ki} + \sum_{j=1, j \neq i}^{M} P_{j}h_{ji}}$$
(4.1)

where  $h_{ii}$  represents instantaneous channel power gain of the link between the secondary transmitter *i* and the secondary receiver *i*, which is assumed to be flat fading with additive white Gaussian noise (AWGN)  $n_0$ . Channel power gain  $h_{ji}$  denotes an interference channel between other secondary transmitters *j* and the secondary receiver *i*.  $g_{ki}$  denotes an interference channel power gain between an active primary transmitter k and the secondary receiver i. Furthermore, the noise  $n_0$  is assumed to be independent random variable (RV) with the distribution  $CN(0, N_0)$  (zero-mean circularly symmetric complex Gaussian noise with variance  $N_0$ ).

# 4.2 Ergodic Capacity Under Average Interference Power Constraint

We consider a fading environment with the received power constraint at a third party's receiver on average value. Channel capacity can be obtained by optimal utilization of the transmitted power over time, in which the received power constraint is met. Hence, the total ergodic capacity of secondary users can be found by solving the following optimization problem.

$$\max_{P_i} \quad \sum_{i=1}^{M} E\left[\ln(1 + SINR_i)\right] \tag{4.2a}$$

s.t. 
$$\sum_{i=1}^{M} E[P_i g_{ik}] \le Q \tag{4.2b}$$

where Q denotes the predefined average interference power threshold on a primary user. In addition,  $g_{ik}$  denotes the channel power gain of the link between the secondary transmitter i and the kth primary receiver.

# 4.2.1 Special Case: One secondary Link (M=1)

Here, we assume that there is one secondary link, and hence, we discuss the impact of interference from only primary transmitters on the secondary link. In this case, optimization problem (4.2a) subject to (4.2b) is reduced as following

$$\max_{P_i} \quad E\left[\ln\left(1 + \frac{P_i h_{ii}}{N_0 + \sum_{k=1}^K \rho g_{ki}}\right)\right] \tag{4.3a}$$

s.t. 
$$E[P_i g_{ik}] \le Q$$
 (4.3b)

where the objective function (4.3a) is a concave in  $P_i$ . The above optimization problem is equivalent to solving the following Lagrangian problem.

$$L(P_{i},\mu) = E\left[\ln\left(1 + \frac{P_{i}h_{ii}}{N_{0} + \sum_{k=1}^{K}\rho g_{ki}}\right)\right] - \mu\left(E[P_{i}g_{ik}] - Q\right)$$
(4.4)

where  $\mu$  is the nonnegative dual variable corresponding to the constraint (4.3b). Taking the derivative of the Lagrangian in (4.4) with respect to  $P_i$  and letting the derivative equal to zero yields,

$$E\left[\frac{h_{ii}}{P_{i}h_{ii} + N_{0} + \sum_{k=1}^{K} \rho g_{ki}}\right] = \mu(E[g_{ik}])$$
(4.5)

which results in

$$P_{i} = \left(\frac{1}{\mu g_{ik}} - \frac{N_{0} + \sum_{k=1}^{K} \rho g_{ki}}{h_{ii}}\right)^{+}$$
(4.6)

where  $(.)^+$  denotes max(.,0). From the equation (4.6), we have

$$\frac{g_{ik}(N_0 + \sum_{k=1}^{K} \rho g_{ki})}{h_{ii}} < \frac{1}{\mu}$$
(4.7)

The parameter  $\mu$ , satisfying the following Complementary Slackness Conditions [7], can be obtained by

$$Q = \int_0^\infty \int_{x < \frac{1}{\mu y}} \left(\frac{1}{\mu} - xy\right) f_x(x) f_y(y) dx dy \tag{4.8}$$

where  $x = \frac{g_{ik}}{h_{ii}}$  and  $y = (N_0 + \sum_{k=1}^{K} \rho g_{ki})$ . In the case of Rayleigh fading, the channel power gains  $g_{ik}$ ,  $h_{ii}$  and  $g_{ki}$  follow exponential distribution. Furthermore, we assume that  $g_{ik}$ ,  $h_{ii}$  and  $g_{ki}$  are unit-mean and mutually independent. Therefore, the probability density function (PDF) of  $\frac{g_{ik}}{h_{ii}}$  can be expressed as [8]

$$f_{\frac{g_{ik}}{h_{ii}}}(x) = \frac{1}{(1+x)^2} \tag{4.9}$$

When  $g_{1i}, g_{2i}, ..., g_{Ki}$  are independent and identically distributed random variables, the PDF of  $(\sum_{k=1}^{K} g_{ki})$  is distributed as Gamma with parameter K. Accordingly, the PDF of  $N_0 + \sum_{k=1}^{K} \rho g_{ki}$  becomes

$$f_{(N_0 + \sum_{k=1}^{K} \rho g_{ki})}(y) = \frac{1}{\rho \Gamma(k)} e^{-\frac{y - N_0}{\rho}} \left(\frac{y - N_0}{\rho}\right)^{k-1}$$
(4.10)

In order to obtain the probability density function (PDF) of y ( $y = N_0 + \rho \sum_{k=1}^{K} g_{ki}$ ), the cumulative distribution function of the random variable Y is given by

$$F_Y\left(\frac{y-N_0}{\rho}\right) = \int_0^{\frac{y-N_0}{\rho}} \frac{x^{k-1}}{\Gamma(k)} e^{-x} dx \tag{4.11}$$

from (4.11) and using [9, eq. (2.33.10)], we have

$$F_Y\left(\frac{y-N_0}{\rho}\right) = \frac{1}{\Gamma(k)}\left(\Gamma(k) - \Gamma\left(k, \frac{y-N_0}{\rho}\right)\right)$$
(4.12)

where  $\Gamma(.)$  is Euler gamma function and  $\Gamma(.,.)$  is incomplete gamma function. Then, the probability density function  $f_Y(.)$  is given by taking the derivative of (4.12).

Now, by using (4.9) and (4.10), the equation (4.8) becomes

$$Q = \frac{1}{\rho\Gamma(k)} \int_{N_0}^{\infty} \int_0^{\frac{1}{\mu y}} \left(\frac{1}{\mu} - xy\right) \frac{1}{(1+x)^2} \left(e^{-\frac{y-N_0}{\rho}} \left(\frac{y-N_0}{\rho}\right)^{k-1}\right) dxdy \quad (4.13)$$

Similarly, we obtain the channel capacity as

$$C = \int_0^\infty \int_{x < \frac{1}{\mu y}} \ln\left(\frac{1}{\mu}\frac{1}{xy}\right) f_x(x) f_y(y) dx dy$$
  
=  $\frac{1}{\rho\Gamma(k)} \int_{N_0}^\infty \int_0^{\frac{1}{\mu y}} \ln(\frac{1}{\mu}\frac{1}{xy}) \frac{e^{-\frac{y-N_0}{\rho}} \left(\frac{y-N_0}{\rho}\right)^{k-1}}{(1+x)^2} dx dy$  (4.14)

For general value of K, the equations (4.13) and (4.14) do not admit closedform expressions and they need to be calculated numerically. However, for K =1 and upon invoking [9, eq. (2.113), (4.222.8) and (4.331.2)], equation (4.13) changes to the following closed-form

$$Q = \frac{1}{\mu} \left[ 1 + e^{\frac{1+\mu N_0}{\mu P}} (\mu \rho - 1) E_i \left( -\frac{1+\mu N_0}{\mu \rho} \right) - e^{\frac{N_0}{\rho}} \mu \rho E_i \left( -\frac{N_0}{\rho} \right) - \mu (N_0 + \rho) \ln \left( 1 + \frac{1}{\mu N_0} \right) \right]$$
(4.15)

where Ei(.) is the exponential integral function defined as Ei(x) =  $\int_{-\infty}^{x} \frac{e^{t}}{t} dt$ . We can find  $\mu$  for a given Q from the equation (4.15). It is worth noting that determining the  $\mu$  from (4.15) needs to use numerical integration.

By changing the variable  $t = \frac{1}{x}$  and using [9, eq. (2.727.3), (4.337.1) and (4.331.2)] in (4.14), a closed-form for the ergodic capacity can straightforwardly be obtained from (4.14) when k=1 as follows

$$C = \left[ e^{\frac{N_0}{p}} E_i \left( -\frac{N_0}{\rho} \right) - e^{\frac{1+\mu N_0}{\mu \rho}} E_i \left( -\frac{1+\mu N_0}{\mu \rho} \right) + \ln \left( 1 + \frac{1}{\mu N_0} \right) \right]$$
(4.16)

The above equation gives the ergodic capacity of one secondary link when interference from only primary transmitters exists.

## 4.2.2 General Case: M>1

In what follows, we study the general case where the impact of the interference from both primary and other secondary transmitters on the total ergodic capacity of secondary users are considered. In this case, the optimization problem can be expressed as follows

$$\max_{P_{i}} \sum_{i=1}^{M} E\left[ \ln\left( 1 + \frac{P_{i}h_{ii}}{N_{0} + \sum_{k=1}^{K} \rho g_{ki} + \sum_{j=1, j \neq i}^{M} P_{j}h_{ji}} \right) \right]$$
(4.17a)

s.t. 
$$\sum_{i=1}^{M} E[P_i g_{ik}] \le Q \tag{4.17b}$$

When the problem (4.17a) subject to (4.17b) is convex, we can solve the dual problem by forming the Lagrangian dual. Meanwhile, the convex structure guarantees that the solutions of the primal problem and dual problem are the same and the duality gap is zero. The main challenge in solving above problem is that the objective function (4.17a) is not concave in  $P_i$ , however, the concavity of the objective function is not a necessary condition for zero duality gap.

Here, we employ two theorems found in [6] which prove the duality gap of such optimization problem is zero, although the objective function (4.17a) is not concave.

The first theorem proves that the solution to the problem (4.17a) occurs at a point on the boundary of the feasible set created by the power constraint (4.17b). The second theorem shows that the solution is a concave function over the power constraint. Numerical results also confirm the concavity of (4.17a) over the power constraint (4.17b).

Using these two theorems, it can be concluded that the duality gap for problem (4.17a) subject to (4.17b) is zero. Then, we can form the Lagrangian as follows

$$L(P_{i},\mu) = \sum_{i=1}^{M} E\left[\ln\left(1 + \frac{P_{i}h_{ii}}{N_{0} + \sum_{k=1}^{K}\rho g_{ki} + \sum_{j=1, j\neq i}^{M}P_{j}h_{ji}}\right)\right] - \mu\left(\sum_{i=1}^{M} E\left[P_{i}g_{ik}\right] - Q\right) \quad (4.18)$$

where  $\mu$  is a Lagrangian dual variable. Consider  $D(\mu)$  as the dual objective function and unconstrained maximization of the Lagrangian

$$D(\mu) = \max_{P_i} L(P_i, \mu)$$
(4.19)

The dual optimization problem is to find  $\mu$ , which is

$$\mu^* = \arg\min_{\mu \ge 0} D(\mu) \tag{4.20}$$

The Lagrange dual problem (4.20) can be solved by an iterative algorithm such as the gradient method, where either the ellipsoid or subgradient methods can iteratively update  $\mu$  until the convergence criteria is met. The speed of convergence in the subgradient method highly depends on the step size, while the convergence in the ellipsoid method happens very fast [5]. The computational costs in each iteration of both methods are the same. In this chapter, due to simplicity we use the subgradient method to update  $\mu$ .

In the gradient method, we need to design a positive step size  $\alpha$  for updating  $P_i$  and  $\mu$ . Hence, the following iterations can be implemented:

$$P_i^{(n+1)} = P_i^{(n)} + \alpha \left( \frac{\partial}{\partial P_i^{(n)}} \sum_{i=1}^M E[\ln(1 + \frac{P_i^{(n)}h_{ii}}{N_0 + \sum_{k=1}^K \rho g_{ki} + \sum_{j=1, j \neq i}^M P_j^{(n)}h_{ji}})] - \mu^{(n)} \right)$$
(4.21)

$$\mu^{(n+1)} = \mu^{(n)} + \alpha \left( Q - \sum_{i=1}^{M} E\left[ P_i^{(n)} g_{ik} \right] \right)$$
(4.22)

where  $P_i^{(n)}$  and  $\mu^{(n)}$  are the values of  $P_i$  and  $\mu$  at stage n, respectively. In order to calculate the equation (4.21), we need to get at least M+K numerical integrations, for which the cost of computation is very high. To simplify the equation (4.21), we use the following Lemma proposed in [10].

Lemma 1: For any u, v > 0

$$\ln(1 + \frac{u}{v+1}) = \int_0^\infty (\frac{e^{-z}}{z})(e^{-zv} - e^{-z(v+u)})dz$$
(4.23)

Using the above Lemma, the expression (4.21) reduces into

$$P_{i}^{(n+1)} = P_{i}^{(n)} + \alpha \left( \sum_{i=1}^{M} \frac{\partial}{\partial P_{i}^{(n)}} \int_{0}^{\infty} \left(\frac{e^{-z}}{z}\right) \left( E\left[e^{-\frac{z}{N_{0}}\sum_{k=1}^{K} \rho g_{ki}}\right] E\left[e^{-\frac{z}{N_{0}}\sum_{j=1, j\neq i}^{M} P_{j}^{(n)} h_{ji}}\right] - E\left[e^{-\frac{z}{N_{0}}\sum_{k=1}^{K} \rho g_{ki}}\right] E\left[e^{-\frac{z}{N_{0}}\sum_{j=1, j\neq i}^{M} P_{j}^{(n)} h_{ji}}\right] E\left[e^{-\frac{z}{N_{0}}\sum_{k=1}^{K} \rho g_{ki}}\right] E\left[e^{-\frac{z}{N_{0}}\sum_{k=1}^{K} \rho g_{ki}}\right] E\left[e^{-\frac{z}{N_{0}}\sum_{j=1, j\neq i}^{K} P_{j}^{(n)} h_{ji}}\right] E\left[e^{-\frac{z}{N_{0}}\sum_{k=1}^{K} \rho g_{ki}}\right] \left(4.24\right)$$

Equation (4.24) can be simplified to give

$$P_{i}^{(n+1)} = P_{i}^{(n)} + \alpha \left( \left( \int_{0}^{\infty} \left( E \left[ \sum_{j=1}^{M} h_{ij} e^{-\frac{z}{N_{0}} (\sum_{k=1}^{K} \rho g_{ki} + \sum_{m=1}^{M} P_{m}^{(n)} h_{mj})} \right] \right) - E \left[ \sum_{\substack{j=1\\j \neq i}}^{M} h_{ij} e^{-\frac{z}{N_{0}} (\sum_{k=1}^{K} \rho g_{ki} + \sum_{\substack{m=1\\m \neq j}}^{M} P_{m}^{(n)} h_{mj})} \right] \right) e^{-z} dz - \mu^{(n)} \right) \quad (4.25)$$

In the case of Rayleigh fading, the channel power gains  $g_{ki}$ ,  $h_{ii}$  and  $h_{ji}$  are exponentially distributed with unit-mean. Finally, we arrive at the expression

Power control algorithm
1) Initialization $n = 0, \alpha, P_i^{(n)}$ and $\mu^{(n)}$
2) While not converged do
3) update $P_i^{(n+1)}$ by (4.26)
(4) update $\mu^{(n+1)}$ by (4.22)
5) calculate the Ergodic Capacity
$6) \qquad n=n+1$
7) End While

Table 4.1: Power control algorithm

(4.26) where the only integration is based on z.

$$P_{i}^{(n+1)} = P_{i}^{(n)} + \alpha \left( \int_{0}^{\infty} \left( \frac{N_{0}}{N_{0} + \rho z} \right)^{K} \left[ \Pi_{j=1}^{M} \frac{N_{0}}{N_{0} + P_{j}^{(n)} z} \sum_{j=1}^{M} \frac{1}{N_{0} + P_{j}^{(n)} z} - \frac{N_{0}}{\left(N_{0} + P_{i}^{(n)} z\right)^{2}} \sum_{\substack{j=1\\j \neq i}}^{M} \frac{N_{0}}{N_{0} + P_{j}^{(n)} z} \right] e^{-z} dz - \mu^{(n)} \right)$$
(4.26)

The steps for the power control algorithm are shown in Table 4.1, in which  $P_i^{(n+1)}$  and  $\mu^{(n+1)}$  are updated to maximize capacity.

Note that the complexity of this algorithm is the square of the number of the secondary transmitters [6]. Equation (4.26) is the new result for the optimum power allocation which maximizes ergodic capacity. To the best of our knowledge there is no similar result in the literature. It is to be emphasized that this was obtained by using non-direct method of lemma1 and duality gap theorems in [10] and [6].

# 4.3 Ergodic Capacity Under Peak Interference Power Constraint

An average received-power constraint as discussed in the previous section is reasonable when QoS of the primary network is determined by the average SINR. However, in many situations, the QoS of primary network would be limited by the instantaneous SINR at the receiver which renders a peak received-power constraint more appropriate. Therefore, we here use the following optimization problem

$$\max_{P_{i}} \sum_{i=1}^{M} E\left[\ln\left(1 + \frac{P_{i}h_{ii}}{N_{0} + \sum_{k=1}^{K} \rho g_{ki} + \sum_{j=1, j \neq i}^{M} P_{j}h_{ji}}\right)\right]$$
(4.27)

s.t. 
$$\sum_{i=1}^{M} [P_i g_{ik}] \le Q_.$$
 (4.28)

where (4.28) denotes peak interference power constraint at primary users and Q in this case is the peak received power limit.

## 4.3.1 Special Case: One secondary Link (M=1)

In the special case, M=1, the optimization problem (4.2a) subject to (4.28) can be changed as following

$$\max_{P_i} \quad E\left[\ln\left(1 + \frac{P_i h_{ii}}{N_0 + \sum_{k=1}^K \rho g_{ki}}\right)\right] \tag{4.29a}$$

s.t. 
$$P_i g_{ik} \le Q$$
 (4.29b)

We find the maximum capacity by replacing  $P_i$  as  $Q/g_{ik}$ . Thus,

$$\max_{P_i} \quad E\left[\ln\left(1 + \frac{Q}{N_0 + \sum_{k=1}^K \rho g_{ki}} \frac{h_{ii}}{g_{ik}}\right)\right] \tag{4.30}$$

by utilizing (4.9) and (4.10), equation (4.30) becomes

$$C = \frac{1}{\rho \Gamma(k)} \int_{N_0}^{\infty} \int_0^{\infty} \ln\left(1 + Q\frac{x}{y}\right) \frac{1}{(1+x)^2} \times \left(e^{-\frac{y-N_0}{\rho}} \left(\frac{y-N_0}{\rho}\right)^{k-1}\right) dxdy$$
(4.31)

which gives

$$C = \frac{1}{\rho \Gamma(k)} \int_{N_0}^{\infty} \frac{Q}{Q - y} \ln\left(\frac{Q}{y}\right) \left(e^{-\frac{y - N_0}{\rho}} \left(\frac{y - N_0}{\rho}\right)^{k - 1}\right) dy \tag{4.32}$$

We observe that a closed-form expression is not obtainable for (4.32), and hence we need to solve the equation numerically with different values of k.

## 4.3.2 General Case: M>1

In the general case, the effect of the interference from both primary and other secondary transmitters on the total capacity of secondary links is discussed under peak interference power constraint. In this case, the optimization problem can be expressed as follows

$$\max_{P_{i}} \sum_{i=1}^{M} E\left[ \ln\left( 1 + \frac{P_{i}h_{ii}}{N_{0} + \sum_{k=1}^{K} \rho g_{ki} + \sum_{j=1, j \neq i}^{M} P_{j}h_{ji}} \right) \right]$$
(4.33a)

s.t. 
$$\sum_{i=1}^{M} P_i g_{ik} \le Q \tag{4.33b}$$

where the objective function (4.33a) is not concave in  $P_i$ . Similarly, after applying two theorems in [6] and proving zero duality gap of the optimization problem, we can use the gradient method to solve (4.33a) subject to (4.33b). The power control algorithm is similar to the last section. The only difference is that the following equation is replaced by (4.22)

$$\mu^{(n+1)} = \mu^{(n)} + \alpha \left( Q - \sum_{i=1}^{M} P_i^{(n)} g_{ik} \right)$$
(4.34)

# 4.4 Numerical Results

In this section, we present the numerical results for ergodic capacity of the Rayleigh fading channels in the ad hoc cognitive radio under interference constraint. We assume that  $N_0 = 1$  and  $\rho = 5 \ dB$ .

## 4.4.1 Special Case: One secondary Link (M=1)

Fig. 4.2 and Fig. 4.3 show the plots for the ergodic capacity of one secondary link against average and peak interference power threshold (Q) in Rayleigh fading channels, respectively. These figures also show the effect of different number of primary interferers on the capacity. Evidently, as the number of primary interferers increases, the ergodic capacity decreases. Consequently, this indicates that the number of primary interferers is a dominant constraint on the achievable capacity in ad hoc cognitive radio. Furthermore, the capacity of the secondary link under average interference power threshold is always higher than that under peak interference power threshold. However, the difference between these two is very small.

### 4.4.2 General Case: M>1

In Figs. 4.4 and 4.5, the behavior of total capacity of ad hoc cognitive radio under average and peak interference power threshold versus the number of iterations in power control algorithm proposed in Table 1 are studied. In Figs. 4.4 and 4.5, we assume that M = 2 and K = 1. These figures indicates the convergence of the proposed algorithm for a step size  $\alpha = 0.05$ . We observe that almost the same iterations are required to get convergence for all values of Q in each figure. In addition, we observe that the convergence occurs very quickly under average interference power threshold.

Fig. 4.6 and Fig. 4.7 present the plots for total capacity of ad hoc cognitive radio versus the average and peak interference power threshold (Q) with K = 1and different number of M. The figures show that problem (4.17a) is concave in constraints (4.17b) and (4.33b) as discussed in the duality gap. We can see that as the number of secondary links increases, the total capacity decreases. For instance, under average interference power constraint and Q=10, the total capacity of two secondary links is almost 1.2 but the total capacity of four links is less than 1. Consequently, the secondary interference significantly decrease the total channel capacity reducing the performance of the secondary network.

# 4.5 Chapter Conclusion

We studied the channel capacity offered by spectrum sharing in time varying channels, motivated by the concept of cognitive radio networks. A primary network coexisting with an ad hoc cognitive radio network is considered. We particularly investigated the ergodic capacity of an ad hoc cognitive radio subject to either average or peak interference power constraint at the primary users. The interference from primary and other secondary transmitters were separately discussed. Using some results from the numerical analysis, the total capacity of the secondary network highly depends on the number of secondary interferers, however the primary transmitters have a negative impact. These results can be applied to evaluate and design efficient ad hoc networks coexisting with cellular network



Figure 4.2: The effect of different number of the primary interferers on the channel capacity of one secondary link under average interference power constraint.



Figure 4.3: The effect of different number of the primary interferers on the channel capacity of one secondary link under peak interference power constraint.



Figure 4.4: The capacity against number of iterations in power control algorithm under average interference power constraint.



Figure 4.5: The capacity against number of iterations in power control algorithm under peak interference power constraint.


Figure 4.6: The effect of different number of the secondary transmitters on the total capacity of secondary links under average interference power constraint.



Figure 4.7: The effect of different number of the secondary transmitters on the total capacity of secondary links under peak interference power constraint.

and sharing the spectrum bands.

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## Chapter 5

# Outage Probability in Adhoc Cognitive Radio

The utilization of spectrum in traditional wireless networks is improved by cognitive radio technology such that it increases the number of application and services in wireless systems. A cognitive radio network recognizes its communication environment and changes the parameters of its communication scheme to increase the quality of service of secondary users [1].

Cognitive radio networks in transmission model can be formed by either opportunistic spectrum access or spectrum sharing. In first transmission model, secondary users can opportunistically operate over the unused parts of licensed bands, while in second model secondary users are allowed to coexist with primary users as long as the interference from the secondary user to the primary user is less than an acceptable value [2]-[4].

A low outage probability is one of the major QoS requirements and is interferencelimited in mobile communication systems. The authors in [5] investigated the outage probability of SUs in a spectrum sharing network under Rayleigh fading channels. Furthermore, in [6], [7] similar studies were conducted under different power constraints. However, in the aforementioned works, the spatial distribution of the interfering nodes was not considered. We note that the Poisson process is a common model for the analysis of interference in communication systems. As shown in recent contributions ([8], [9]), the authors assumed that a set of interfering nodes are randomly located in a 2-dimensional Poisson point process with a specified spatial density.

Therefore, the main aim of this chapter is to analyze the outage probability of SUs based on spatial location of nodes under AWGN and Rayleigh fading channels. Specifically, we employ the Poisson point process to accurately account for the spatial distribution of SU nodes in a 2-dimensional plane.

#### 5.1 System Model

As illustrated in Fig. 5.1, we introduce a scenario composed of a primary cellular network and a secondary ad-hoc network, coexisting in the same area. Secondary transmitter and receiver pairs are randomly located on a plane. In the secondary network, no centralized authority is assumed to manage the network access for users. A point-to-point link is considered to communicate between a secondary transmitter and a secondary receiver.

Here we analyze the behaviour of a single reference secondary transmitterreceiver pair, separated by a distance r in the presence of M secondary interfering transmitters. We also assume that the channel power gain between this secondary transmitter-receiver pair is g. The locations of secondary interfering nodes form a Poisson process of intensity  $\lambda$ . The reference secondary transmitter sends information with constant power P while the transmission power of the mth interfering secondary transmitter is  $P_m$ . The distance from the mth secondary transmitting node to the reference receiver is denoted by  $X_m$ .



Figure 5.1: Coexistence of a cognitive radio ad hoc network (red pairs) with primary users in cellular system (in blue)

We further denote the interference channel power gain between the mth interfering secondary transmitter and the secondary reference receiver as  $h_m$ . Therefore, we define the SINR as

$$SINR = \frac{Pgr^{-\beta}}{\sum_{m=1}^{M} P_m X_m^{-\beta} h_m + \eta}$$
(5.1)

where  $\eta$  represents the noise power and  $\beta$  is the path loss exponent which takes a value between 2 and 6 [10]. In this chapter, we derive general results for  $\beta > 2$ . Moreover, we assume that secondary users use the licensed spectrum when it is not occupied by primary system. Consequently, there is no interference between primary and secondary networks and the only interference is amongst secondary users. Therefore, we focus on spectrum sensing with ideal scenario which is referred to as perfect sensing.

### 5.2 Outage Probability under AWGN Channels

An outage occurs when the signal-to-interference-plus-noise ratio (SINR) drops below the threshold  $\alpha$  or equivalently the received mutual information is less than  $\log_2(1 + \alpha)$ . Thus, the outage probability for secondary users in the presented model can be written as:

$$P_{out} = \Pr\left(SINR \le \alpha\right). \tag{5.2}$$

From (5.2), we can obtain the following equation

$$P_{out} = \Pr\left(\sum_{m=1}^{M} P_m X_m^{-\beta} \ge \frac{Pr^{-\beta}}{\alpha} - \eta\right).$$
(5.3)

According to Markov's Inequality (Limit Theorems), we have:

$$\Pr\left(X \ge a\right) \le \frac{E[X]}{a} \tag{5.4}$$

which allows the outage probability to be expressed as:

$$P_{out} \leq \frac{E[\sum_{m=1}^{M} P_m X_m^{-\beta}]}{\frac{Pr^{-\beta}}{\alpha} - \eta}$$
(5.5)

we condition on M which is distributed as a Poisson random variable [11] with average  $\lambda \pi D^2$ . In order to solve (5.5), we involve the following lemma.

Lemma: The Compound Poisson Identity. Let

$$S = \sum_{i=1}^{N} X_i \tag{5.6}$$

be a compound Poisson random variable with Poisson parameter  $\lambda$  and component distribution F, and let X be a random variable having distribution F that is independent of S. Then, for any function h(x)

$$E[Sh(S)] = E[N]E[Xh(s+X)]$$
(5.7)

by using this lemma and assuming h(S) = 1, equation (5.5) becomes

$$P_{out} \le \frac{\lambda \pi D^2}{\frac{Pr^{-\beta}}{\alpha} - \eta} E(P_1 X_1^{-\beta}).$$
(5.8)

By using the assumption that the reference receiver is located at the centre of a disc with radius D such that the distances  $\{X_m, m = 1, 2, ...M\}$  of the interfering SU nodes from the centre are independent and distributed by the pdf:

$$f(x) = \begin{cases} \frac{2x}{D^2} & R < x < D\\ 0 & Otherwise \end{cases}$$
(5.9)

where R is a very small value representing the minimum radius between an interfering node and the reference receiver. Utilizing (5.9) in (5.8), we arrive at

$$P_{out} \leq \frac{\lambda \pi}{\frac{Pr^{-\beta}}{\alpha} - \eta} \int_{R}^{D} 2P_1 X_1^{1-\beta} dX_1 \tag{5.10}$$

Finally, as  $D \to \infty$ , equation (5.10) reduces into

$$P_{out} \le \frac{2P_1 \lambda \pi}{P(\frac{r-\beta}{\alpha} - \frac{\eta}{P})} \frac{R^{2-\beta}}{\beta - 2}$$
(5.11)

(5.11) is an upper bound for outage probability of secondary users under AWGN channel.

# 5.3 Outage Probability under Rayleigh Fading Channels

In this case, the resulting outage probability in secondary networks can be written as:

$$P_{out} = \Pr\left(\frac{Pgr^{-\beta}}{\sum_{m=1}^{M} P_m X_m^{-\beta} h_m + \eta}\alpha\right).$$
(5.12)

Equation (5.12) is simplified to give

$$P_{out} = \Pr\left(g\frac{\alpha}{Pr^{-\beta}}\left(\sum_{m=1}^{M}P_m X_m^{-\beta} h_m + \eta\right)\right)$$
(5.13)

Here, the link between the reference secondary transmitter and receiver pair is assumed to be a Rayleigh fading channel with unit mean. For a Rayleigh faded channel, g is exponentially distributed. Thus, we have

$$P_{out} = 1 - \exp\left(-\frac{\alpha}{Pr^{-\beta}} \left(\sum_{m=1}^{M} P_m X_m^{-\beta} h_m + \eta\right)\right)$$
(5.14)

We first condition on M similar to the analysis in section III. Therefore,

$$P_{out} = 1 - \exp\left(-\frac{\alpha}{r^{-\beta}}\frac{\eta}{P}\right) \times E\left[\exp\left(-\frac{\alpha}{Pr^{-\beta}}\left(\sum_{m=1}^{M}P_m X_m^{-\beta}h_m\right)\right)|M\right]$$
(5.15)

this gives

$$P_{out} = 1 - \exp\left(-\frac{\alpha}{r^{-\beta}}\frac{\eta}{P}\right) \times \prod_{m=1}^{M} E\left[\exp\left(-\frac{\alpha}{Pr^{-\beta}}(P_m X_m^{-\beta} h_m)\right)\right]$$
(5.16)

with some changes, the following equation can be obtained,

$$P_{out} = 1 - \exp\left(-\frac{\alpha}{r^{-\beta}}\frac{\eta}{P}\right) \times E\left[\exp\left(-\frac{\alpha}{Pr^{-\beta}}(P_1X_1^{-\beta}h_1)\right)\right]^M$$
(5.17)

on averaging out M, we remove the condition to arrive at the following equation

$$P_{out} = 1 - \exp\left(-\frac{\alpha}{r^{-\beta}}\frac{\eta}{P}\right) \left(\sum_{m=0}^{\infty} \frac{e^{-\lambda\pi D^2}(\lambda\pi D^2)}{m!}\right) \times E\left[\exp\left(-\frac{\alpha}{Pr^{-\beta}}(P_1X_1^{-\beta}h_1)\right)\right]^m$$
(5.18)

With some manipulation, (5.19) can be obtained

$$P_{out} = 1 - \exp\left(-\frac{\alpha}{r^{-\beta}}\frac{\eta}{P}\right) \times \exp\left(-\lambda\pi D^2\left(1 - E\left[\exp\left(-\frac{\alpha}{Pr^{-\beta}}\left(P_1X_1^{-\beta}h_1\right)\right)\right]\right)\right)$$
(5.19)

by utilizing (5.9) in (5.19), (where  $R \to 0$ ) we arrive at

$$P_{out} = 1 - \exp\left(-\frac{\alpha}{r^{-\beta}}\frac{\eta}{P}\right) \exp(-\lambda\pi D^2 (\int_0^D [1 - \exp\left(-\frac{\alpha}{Pr^{-\beta}}(P_1X_1^{-\beta}h_1)\right)] \frac{2X_1}{D^2} dX_1)).$$
(5.20)

When  $D \to \infty$ , (5.20) becomes

$$P_{out} = 1 - \exp\left(-\frac{\alpha}{r^{-\beta}}\frac{\eta}{P}\right) \times \exp\left(-\lambda\pi\Gamma(1-\frac{2}{\beta})\left(\frac{\alpha}{Pr^{-\beta}}P_{1}h_{1}\right)^{\frac{2}{\beta}}\right).$$
(5.21)

In (5.21) we have utilized the following integral which is valid for any  $\beta > 2$ and a > 0.

$$\int_{0}^{\infty} (1 - e^{-ar^{-\beta}}) 2r dr = a^{\frac{2}{\beta}} \int_{0}^{\infty} \beta r^{-\beta+1} e^{-r^{-\beta}} dr = a^{\frac{2}{\beta}} \Gamma(1 - \frac{2}{\beta}).$$
(5.22)

Here, we find the average on  $h_1$  for the case where  $h_1$  experiences Rayleigh



Figure 5.2: Outage probability vs. SINR threshold ( $\alpha$ ) for fixed density of secondary interferers ( $\lambda = 10^{-4}$ ) and fixed SNR = 25 dB

fading by using following equation:

$$E[h_1^{\frac{2}{\beta}}] = \int_0^\infty h_1^{\frac{2}{\beta}} e^{-h_1} dh_1 = \Gamma(1 + \frac{2}{\beta})$$
(5.23)

where  $\Gamma(.)$  is the Gamma function [12]. Finally, we find the following expression for outage probability

$$P_{out} = 1 - \exp\left(-\frac{\alpha}{r^{-\beta}}\frac{\eta}{P}\right) \times \exp\left(-\lambda\pi\Gamma(1-\frac{2}{\beta})\left(\frac{\alpha P_1}{Pr^{-\beta}}\right)^{\frac{2}{\beta}}\Gamma(1+\frac{2}{\beta})\right).$$
(5.24)

### 5.4 Numerical Results

In this section, the numerical results for outage probabilities are presented. For simplicity, we choose the distance between the reference secondary transmitter and receiver pair as r = 1. The outage probabilities are computed in terms of  $\alpha$  (SINR threshold),  $\beta$  (path loss exponent),  $SNR = \frac{P}{\eta}$  (signal-to-noise-ratio in the absence of interference) and  $\lambda$  (the density of secondary interference). We further simplify by assuming that the transmission power of all secondary nodes are equal. Fig. 5.2 contains plots of the numerically computed outage probability versus  $\alpha$  for path loss  $\beta = 3$  under Rayleigh fading and AWGN channels. As shown in Fig. 5.2, the outage probability increases when  $\alpha$  changes from 0 through 20. The reason for this behaviour can be discerned from (5.2) and (5.12), where increasing the value of  $\alpha$  while keeping other parameters constant results in larger values of the outage probability.

Fig. 5.3 shows that the level of outage probability depends on the path loss  $\beta$  such that when the path loss is increasing the value of SINR becomes bigger resulting in a bigger outage probability. The outage probability is very high for  $\beta$  close to 2 but then decreases exponentially when the value of  $\beta$  is increasing. We further observe that the outage probability is much more sensitive to  $\beta$  in AWGN channels compared to Rayleigh fading channels.

In Fig. 5.4, the outage probability is plotted against the SNR over both Rayleigh fading and AWGN channels with fixed path loss  $\beta = 3$ . This figure reveals that the outage probability is decreasing when SNR changes from 0 through 40 dB. The figure also shows that at large values of SNR, the outage probability becomes independent of the SNR, such that for a fixed interferer density it converges to the same value for both Rayleigh fading and AWGN channels. This is expected from equation (5.1) because  $\frac{1}{SNR} <<\frac{1}{P} \sum_{m=1}^{M} P_m X_m^{-\beta} h_m$  when SNR increases. Therefore SNR in (5.1) can be ignored at large value of SNR.

Furthermore, we observe that an increased interferer density results in a larger value of outage probability in both channels. This is obvious, the higher density of secondary interferers leads the system to have higher interference and thus



Figure 5.3: Outage probability vs. path loss exponent for fixed density of secondary interferers ( $\lambda = 10^{-4}$ ), fixed SNR = 25 dB and SINR threshold ( $\alpha = 10$ )



Figure 5.4: Outage probability vs. SNR  $(SNR = \frac{P}{\eta})$ 

according to (5.2) and (5.12) the outage probability becomes larger. Finally, a common observation in all results is that the outage probability in the Fading channels is higher than that obtained in the AWGN channel. This is because SINR under AWGN channel is always bigger than SINR under Rayleigh fading channels.

### 5.5 Chapter Conclusion

We have proposed an analytical model for probability of outage in a cognitive ad-hoc network where a number of interfering transmitters in secondary networks with arbitrary locations are distributed according to a homogeneous Poisson point process (PPP) with a specified intensity. We have derived new expression for upper bound of outage probability based on AWGN channel and also exact expression under Rayleigh fading. These new expressions are numerically evaluated in terms of different parameters. Numerical results have shown that in all cases outage probability under AWGN channels is less than that under Rayleigh fading channels. As expected, we observed that SINR threshold and the density of secondary interferers are directly proportional to the value of outage probability, while the path loss exponent and SNR exhibit an inverse proportional relationship with the outage probability.

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## Chapter 6

# Minimum Transmit Power in Femtocell

Femtocell network has emerged as an approach to increase the capacity and coverage of mobile cellular systems by getting the transmitter and receiver closer to each other. Femtocell network operates in the licensed spectrum bands assigned to macrocell netwoks. This technology further reduces the operational expenses and capital cost. A femtocell consists of a low power, short range access point (AP) to provide in-building coverage to home users over the internet based IP backhaul such as cable modem or digital subscriber line (DSL). The base station in this technique serves mainly indoor users requiring a much lower transmit power compared to the macrocell users [1], [2].

Frequency configuration as an important issue in femtocells has been discussed in many recent publications. In order to increases spectral efficiency per area, femtocell and existing networks use the same frequency band in an arrangement known as co-channel deployment. However, due to the finite number of licensed spectrum bands, the capacity of femtocell networks under this approach is limited. Furthermore, femtocell AP and femtocell users in co-channel approach may cause high interference to the macrocell users and vice versa, reducing the performance of the both networks.

In [3], two kinds of interference amongst nodes are discussed, namely cross-tier interference and intra-tier interference. In cross-tier interference, the interference and the victim of interference belong to different tiers, while in the intra-tier interference, the interferer and the victim are in the same tier. Possible interference mitigation techniques are also studied in [3]. The performance of femtocell networks with co-channel deployment while considering cross-tier interference is discussed in [5]. The authors in [6] have mainly focused on designing optimal power allocation schemes to maximize the downlink capacity in femtocell APs, where the femtocell users are allowed to access the same frequency band when the macrocells are not active. In [7], different power control techniques have been employed at femtocells in order to handle the dominant interference. These techniques reduce the transmission power of femtocells to improve the performance of victim macrocell users. However, this sacrifices the total throughput of the femtocell users. Furthermore, in [8], [9], the optimization of ergodic, outage and the delay-limited capacity of secondary users in cognitive radio explaining a dynamic spectrum sharing approach have been investigated under average and peak interference power constraints at the existing primary receiver.

In this chapter, we present new original result on minimization of transmit power in femtocell network. We consider the downlink transmission in a femtocell network under co-channel deployment. Then, we obtain the minimum power required at a femtocell AP to meet target QoS constraints such that when the power is less than the minimum power, the capacity of the network becomes low and the BER is too high, consequently, the femtocell network is not efficient. This work shows that minimizing the transmit power can be utilized to further enhance energy efficiency in femtocells. Our main focus in this chapter is to shed light into the effect of the femtocell interference into the macrocell users (which are the primary users) and discuss the possible gains in the energy efficiency.

In what follows, two different system models are proposed, and the minimized power at femtocel AP in each of these models is analyzed. In first system model, we minimize the average transmit power subject to either ergodic capacity or average BER, while in second system model the instantaneous power is minimized subject to either capacity or BER in OFDM femtocell.

### 6.1 System Model for Non-OFDM Femtocell

Consider the scenario depicted in Fig.6.1 where a single macrocell overlaps with a femtocell. The femtocell user attempts to access the macrocell base station but fails because the received signal is too weak. Hence, it connects to the stronger signal from the femtocell AP. We assume that the femtocell adjusts its downlink transmit power level based on the interference power that is measured at the femtocell as in [3], [4]. A flat fading channel between the femtocell AP and the femtocell receiver with instantaneous channel power gain  $h_F$  and additive white Gaussian noise (AWGN) have been assumed. The interference between the macrocell base station and femtocell receiver is assumed to be a flat fading channel characterized by instantaneous fading state  $h_M$ . The transmit power at femtocell AP is  $P_F$  normalized to the AWGN power (that is  $P_F$  is the signal-tonoise ratio (SNR)). The instantaneous received signal to interference and noise ratio (SINR) of a femtocell user can be expressed using the following equation

$$SINR_F = \frac{P_F h_F}{1 + \rho h_M} \tag{6.1}$$



Figure 6.1: Coexistence between femtocells and macrocells network

where  $\rho$  is a normalizing constant such that  $\rho h_M$  represents the normalized macrocell interference power measured at the desired user. In this chapter, we assume Rayleigh fading, so both  $h_F$  and  $h_M$  are independent and exponentially distributed random variables.

#### 6.1.1 Minimization Under Capacity Constraint

In this section, the minimum transmit power of femtocell AP over fading channels can be achieved by solving the following optimization problem, in which the ergodic capacity between the femtocell AP and desired receiver is bigger than a predefined threshold

$$\min_{P_F > 0} E[P_F] \tag{6.2a}$$

subject to  $E\left[\ln\left(1 + \text{SINR}_F\right)\right] \ge R_{\min}$  (6.2b)

where  $R_{\min}$  denotes the minimum capacity required in downlink communication in nats/sec/Hz (1 bit =  $\log_2 e \approx 0.693$  nats). The constraint in above optimization problem is convex so we are allowed to use the Lagrangian approach, which is commonly used in almost all recent papers on resource allocations over wireless networks. The above optimization problem is equivalent to solving the following Lagrangian approach [11]

$$L(P_F,\lambda) = E[P_F] - \lambda \left( E\left[ \ln\left(1 + \frac{P_F h_F}{1 + \rho h_M}\right) \right] - R_{\min} \right)$$
(6.3)

where  $\lambda$  is the nonnegative dual variable corresponding to the constraint (6.43c). Taking the derivative of the Lagrangian in (6.3) with respect to  $P_F$  gives

$$\frac{\partial L(P_F,\lambda)}{\partial P_F} = 1 - \frac{\lambda h_F}{1 + \rho h_M + P_F h_F}.$$
(6.4)

Letting the derivative equal to zero yields

$$P_F = \lambda - \frac{1 + \rho h_M}{h_F}.$$
(6.5)

In (6.5) by considering the constraint  $P_F > 0$  we have

$$\frac{1+\rho h_M}{\lambda} \le h_F. \tag{6.6}$$

The parameter  $\lambda^*$  that minimizes (6.2a) can be obtained by solving (6.43c) and (6.5)

$$R_{\min} = E \left[ \ln \left( 1 + \frac{h_F}{1 + \rho h_M} P_F \right) \left| \frac{1 + \rho h_M}{\lambda^*} \le h_F \right] \right]$$
$$= E \left[ \ln \left( 1 + \frac{h_F}{1 + \rho h_M} \left( \lambda^* - \frac{1 + \rho h_M}{h_F} \right) \right) \left| \frac{1 + \rho h_M}{\lambda^*} \le h_F \right].$$
(6.7)

For the sake of simplicity and mathematical tractability (which otherwise becomes too difficult if real propagation models are involved), we opt for the simple Rayleigh fading model. In these cases we were able to obtain simple closed-form results for the required minimum power, which allow us to gain some insight into this problem. This is also very common in many key papers on the topic (e.g. [12]-[19]).

The channel power gains  $h_F$  and  $h_M$  in the case of Rayleigh fading follow exponential distribution. Furthermore, we assume that  $h_F$  and  $h_M$  are unit-mean and mutually independent. Then, the probability density function (PDF) in this case becomes

$$f(x) = \exp(-x). \tag{6.8}$$

By using (6.8), equation (6.7) reduces into

$$R_{\min} = \iint_{\frac{1+\rho x}{\lambda^*} \le y} \ln\left(\frac{\lambda^* y}{1+\rho x}\right) e^{-x-y} dx dy.$$
(6.9)

The double integral in (6.9) can be straightforwardly evaluated to the following closed-form under Rayleigh fading by using [20, eq.(8.212.16)], the integration by parts and [20, eq.(3.352.2)]

$$R_{\min} = e^{\frac{1}{\rho}} \operatorname{Ei} \left( -\frac{1}{\lambda^*} - \frac{1}{\rho} \right) - \operatorname{Ei} \left( -\frac{1}{\lambda^*} \right)$$
(6.10)

where Ei(.) is the exponential integral function defined as Ei(x) =  $\int_{-\infty}^{x} \frac{e^{t}}{t} dt$ . We can find  $\lambda^{*}$  for a given  $R_{\min}$  from equation (6.10). It is worth noting that determining the  $\lambda^{*}$  from (6.10) needs to be calculated numerically. Likewise, the minimum (average) transmit power can be computed as

$$\min E[P_F] = \iint_{\substack{\frac{1+\rho x}{\lambda^*} \le y}} \left(\lambda^* - \frac{1+\rho x}{y}\right) e^{-x-y} dx dy$$
(6.11)

By using [20, eq.(3.351.5)] and eq.(5.221.5), then integrating by parts and [20,

eq.(3.352.2)], (6.11) can be simplified as

$$\min E[P_F] = \lambda^* e^{-\frac{1}{\lambda^*}} + (1+\rho) \operatorname{Ei}\left(-\frac{1}{\lambda^*}\right) - \rho e^{\frac{1}{\rho}} \operatorname{Ei}\left(-\frac{1}{\lambda^*} - \frac{1}{\rho}\right).$$
(6.12)

#### 6.1.2 Minimization Under BER Constraint

We know the BERs in various digital modulation schemes used in wireless communication requires computing average of

$$BER = \begin{cases} \frac{1}{2} \exp(-SINR) & DPSK\\ \frac{1}{2} \operatorname{erfc} \sqrt{SINR} & BPSK, QPSK. \end{cases}$$
(6.13)

In following two subsections, we find the minimum of the average power under average bit error rate (BER) constraint for different digital modulations.

#### BER Constraint under DPSK

In DPSK, the optimization problem under minimum BER constraints can be expressed as

$$\min_{P_F > 0} E[P_F] \tag{6.14a}$$

subject to 
$$E\left[\frac{1}{2}\exp\left(-\mathrm{SINR}_{F}\right)\right] \leq BER_{\mathrm{max}}.$$
 (6.14b)

Equation (6.14b) represents the average BER constraint and  $BER_{max}$  is the maximum BER for downlink communication in the femtocell network. Similarly, the above minimization problem is equivalent to solving the following Lagrangian

$$L(P_F,\lambda) = E[P_F] + \lambda \left(\frac{1}{2}E\left[\exp\left(-\frac{P_F h_F}{1+\rho h_M}\right)\right] - BER_{\max}\right)$$
(6.15)

where  $\lambda$  is the nonnegative dual variables corresponding to the constraint (6.14b). Forming the Lagrangian and applying the first-order Karush–Kuhn–Tucker (KKT) conditions, the minimum power,  $P_F$ , can be found

$$P_F = \frac{(1+\rho h_M)}{h_F} \ln\left(\frac{\lambda h_F}{2(1+\rho h_M)}\right).$$
(6.16)

By considering the constraint  $P_F \ge 0$ , (6.16) is equivalent to

$$\frac{2(1+\rho h_M)}{\lambda} \le h_F. \tag{6.17}$$

The parameter  $\lambda^*$  can be obtained by solving the constraint (6.14b) for  $\lambda$ . Substitute (6.16) into (6.14b) to get

$$BER_{\max} = E\left[\frac{1}{2}\exp\left(-\frac{h_F}{1+\rho h_M}\frac{(1+\rho h_M)}{h_F} \ln(\frac{\lambda^* h_F}{2(1+\rho h_M)})\right) \left|\frac{2(1+\rho h_M)}{\lambda^*} \le h_F\right] \\ = E\left[\frac{1+\rho h_M}{\lambda^* h_F} \left|\frac{2(1+\rho h_M)}{\lambda^*} \le h_F\right].$$
(6.18)

We employ the equation (6.8) for Rayleigh fading to get

$$BER_{\max} = \iint_{\substack{\frac{2(1+\rho y)}{\lambda^*} \le x}} \frac{1+\rho y}{\lambda^* x} e^{-y-x} dx dy.$$
(6.19)

Upon invoking [20, eq.(3.351.5)] and eq.(5.221.5)], then integrating by parts and using [20, eq.(3.352.2)], we can evaluate the integral in (6.19) as following

$$BER_{\max} = \frac{\rho e^{\frac{1}{\rho}}}{\lambda^*} \operatorname{Ei}\left(-\frac{2}{\lambda^*} - \frac{1}{\rho}\right) - \frac{1+\rho}{\lambda^*} \operatorname{Ei}\left(-\frac{2}{\lambda^*}\right) - \frac{\rho}{(\lambda^* + 2\rho)} e^{-2/\lambda^*}.$$
 (6.20)

The minimum average transmit power under BER constraints can be calculated from

$$\min E[P_F] = \iint_{\substack{\frac{2(1+\rho y)}{\lambda^*} \le x}} \frac{(1+\rho y)}{x} \ln\left(\frac{\lambda^* x}{2(1+\rho y)}\right) e^{-x-y} dx dy.$$
(6.21)

In an interference limited scenario,  $\text{SINR}_F$  in (6.14b) reduces into  $\frac{P_F h_F}{\rho h_M}$  and (6.21) can be evaluated in a closed-form by using [20, eq.(1.512.1), eq.(3.351.2) and eq.(6.455.2)]

$$\min E[P_F] = \rho \log \left(\frac{2\rho}{2\rho + \lambda^*}\right) - \rho \operatorname{Li}_2\left(-\frac{\lambda^*}{2\rho}\right)$$
(6.22)

where  $\text{Li}_2(.)$  is the polylogarithm function of order 2 given by  $\text{Li}_2(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^2}$ . In order to calculate the average minimum transmit power, we find  $\lambda^*$  for a given  $BER_{\text{max}}$  from the equation (6.20) and then substitute in equation (6.22).

#### BER Constraint under BPSK/QPSK

In this case, the optimization problem can be changed as following formula

$$\min_{P_F > 0} \qquad E[P_F] \tag{6.23}$$

subject to 
$$E\left[\frac{1}{2}\operatorname{erfc}\sqrt{\operatorname{SINR}_F}\right] \leq BER_{\max}.$$
 (6.24)

The optimal power allocation,  $P_F$ , is obtained by forming the Lagrangian

$$L(P_F, \lambda) = E[P_F] + \lambda \left( E\left[\frac{1}{2}\operatorname{erfc}\sqrt{\frac{P_F h_F}{1+\rho h_M}}\right] - BER_{\max} \right)$$
(6.25)

where  $\lambda$  is the nonnegative dual variable. The error function can be define as

$$\operatorname{erfc}\sqrt{x} = \frac{2}{\pi} \int_0^{\pi/2} \exp(-\frac{x}{\cos^2(\theta)}) d\theta$$
(6.26)

We have used the assumption that  $\theta$  is a dummy random variable in  $(0, \pi/2)$ such that

$$E[g(\theta)] = \frac{2}{\pi} \int_0^{\pi/2} g(\theta) d\theta \tag{6.27}$$

By using (6.27) and applying the KKT conditions, the optimal power allocation must satisfy the following equation:

$$\frac{\partial L(P'_F, \lambda)}{\partial P'_F} = E\left[\left(1 + \frac{2\lambda}{\pi^2} \left(-\frac{1}{\cos^2(\theta)} \frac{h_F}{1 + \rho h_M}\right) \times \exp\left(-\frac{1}{\cos^2(\theta)} \frac{P'_F h_F}{1 + \rho h_M}\right)\right)\right] = 0 \quad (6.28)$$

where  $P'_F$  represents as a dummy power allocation which is a function of  $h_F$ ,  $h_M$ and  $\theta$ . The above equation gives

$$P_{F}^{'} = \frac{(1+\rho h_{M})\cos^{2}(\theta)}{h_{F}} \ln\left(\frac{2\lambda h_{F}}{\pi^{2}\cos^{2}(\theta)(1+\rho h_{M})}\right).$$
 (6.29)

Then, we can find  $P_F$  as the average of  $P'_F$  over  $\theta$ 

$$P_F = \frac{(1+\rho h_M)}{h_F} \frac{2}{\pi} \int_0^{\pi/2} \cos^2(\theta) \ln\left(\frac{2\lambda h_F}{\pi^2 \cos^2(\theta) (1+\rho h_M)}\right) d\theta$$

which gives

$$P_F = \frac{1}{2} \frac{(1+\rho h_M)}{h_F} \left(-1 + \ln\left(\frac{8\lambda h_F}{\pi^2 \left(1+\rho h_M\right)}\right)\right)$$

By considering the constraint  $P_F \ge 0$  we have

$$h_F > \frac{\pi^2 \left(1 + \rho h_M\right) e}{8\lambda}$$

where e is the Euler's constant. The parameter  $\lambda$ , which satisfies the following Complementary Slackness Conditions, can be obtained by

$$BER_{\max} = \int_0^\infty \int_{\frac{\pi^2(1+\rho y)e}{8\lambda}}^\infty \left(\frac{1}{2}\operatorname{erfc}\sqrt{\frac{\pi}{4}\left(\ln\left(\frac{8\lambda x}{\pi^2(1+\rho y)}\right) - 1\right)}\right) e^{-x}e^{-y}dxdy$$
(6.30)

We observe that closed-form expressions are not obtainable for the above equation and hence we need to solve the equation numerically.

Similarly, the minimum of average transmission power becomes

$$E[P_F] = \int_0^\infty \int_{\frac{\pi^2(1+\rho y)e}{8\lambda}}^\infty \frac{1}{2} \frac{(1+\rho y)}{x} \left( -1 + \ln\left(\frac{8\lambda x}{\pi^2(1+\rho y)}\right) \right) e^{-x} e^{-y} dx dy \quad (6.31)$$

We find the following closed form equation in the interference limited scenario

$$E[P_F] = \frac{\rho}{2} \left( 1 - \ln\left(e + \frac{8\lambda}{\pi^2 \rho}\right) - Li_2(\frac{-8\lambda}{e\pi^2 \rho}) \right)$$
(6.32)

which is the minimum transmit power subject to average BER under BPSK/QPSK modulation.

#### 6.1.3 Numerical Results

In this section, we present some numerical results to illustrate the minimum transmit power required at femtocell AP under different constraints. In order to evaluate the performance of the proposed method, we compare the minimized  $P_F$  which is a function of  $h_M$  and  $h_F$  with a constant power  $P_{\text{F-constant}}$  at femtocell AP.



Figure 6.2: Minimum SNR  $(P_F)$  and constant SNR  $(P_{\text{F\_constant}})$  against a minimum given transmission rate  $R_{\min}$ 

#### **Under Capacity Constraint**

We can find  $P_{\text{F-constant}}$  by following equation for a given  $R_{\min}$ 

$$R_{\min} = E\left[\ln\left(1 + \frac{P_{\text{F-constant}} \times h_F}{1 + \rho h_M}\right)\right]$$
(6.33)

In the case of Rayleigh fading, we have

$$R_{\min} = \int_0^\infty \int_0^\infty \ln\left(1 + \frac{P_{\text{F\_constant}} \times h_F}{1 + \rho h_M}\right) \times e^{-h_F} e^{-h_M} dh_F dh_M.$$
(6.34)

We find the above double integral by using [20, eq.(8.212.16)], the integration by parts and [20, eq.(3.352.2)] as following expression

$$R_{\min} = \frac{P_{\text{F\_constant}}}{P_{\text{F\_constant}} - \rho} \left( e^{\frac{1}{\rho}} \operatorname{Ei} \left( -\frac{1}{\rho} \right) - e^{\frac{1}{P_{\text{F\_constant}}}} \operatorname{Ei} \left( -\frac{1}{P_{\text{F\_constant}}} \right) \right). \quad (6.35)$$

Fig. 6.2 plots the minimum and constant transmit power for different values



Figure 6.3: Minimum SNR  $(P_F)$  and constant SNR  $(P_{\rm F\_constant})$  against a maximum given  $BER_{\rm max}$  under DPSK

of interference levels  $\rho = 5,10$  dBs. Fig. 6.2 shows that the minimum required power depends on the minimum rate. Furthermore, the difference between  $P_F$ and  $P_{\text{F-constant}}$  becomes small at high data rates ( $R_{\min} >> 2$ ).

#### Under BER Constraint

BER Constraint under DPSK

In this case, we obtain  $P_{\text{F-constant}}$  for a given  $BER_{\text{max}}$  by

$$BER_{\max} = E\left[\frac{1}{2}\exp\left(-\frac{P_{\text{F\_constant}} \times h_F}{1+\rho h_M}\right)\right]$$
(6.36)

Similarly, in the case of Rayleigh fading and using [20, eq.(3.353.5), n=1], it yields

$$BER_{\max} = \frac{1}{2} + \frac{\operatorname{Ei}\left(-\frac{1}{\rho}\left(1 + P_{\mathrm{F\_constant}}\right)\right)e^{\frac{1}{\rho}\left(1 + P_{\mathrm{F\_constant}}\right)}}{2\rho} \times P_{\mathrm{F\_constant}} \qquad (6.37)$$



Figure 6.4: Minimum SNR  $(P_F)$  and constant SNR  $(P_{\rm F\_constant})$  against a maximum given  $BER_{\rm max}$  under BPSK/QPSK

BER Constraint under BPSK/QPSK

In BPSK/QPSK, we find  $P_{\text{F-constant}}$  for a given  $BER_{\text{max}}$  from

$$BER_{\max} = E\left[\frac{1}{2}\operatorname{erfc}\sqrt{\frac{P_{\text{F-constant}} \times h_F}{1 + \rho h_M}}\right]$$
(6.38)

$$= \frac{1}{2} \int_0^\infty \int_0^\infty \operatorname{erfc} \sqrt{\frac{P_{\text{F-constant}} \times h_F}{1 + \rho h_M}} e^{-h_F} e^{-h_M} dh_F dh_M$$
(6.39)

which gives

$$BER_{\rm max} = \frac{1}{2} - \frac{P_{\rm F\_constant}}{\frac{P_{\rm F\_constant}}{\pi\rho}} \operatorname{erfc} \sqrt{\frac{\pi + P_{\rm F\_constant}}{\pi\rho}}}{2\rho P_{\rm F\_constant}}$$
(6.40)

The behavior of minimum and constant transmit power versus BERs constraint under Rayleigh fading is shown in Fig. 6.3 and Fig. 6.4. The impact of



Figure 6.5: Energy efficiency against minimum SNR and constant SNR

different  $\rho$  is also shown. The results indicate that the difference between minimized power  $P_F$  and constant power  $P_{\text{F-constant}}$  in terms of BER is considerable. For instance, when  $BER_{\text{max}} = 3 \times 10^{-3}$  and  $\rho = 5$  dB, the power under DPSK decreases from 28 dB to nearly 18 dB, resulting in 10 dB reduction.

#### **Energy Efficiency**

Energy efficiency, as a performance measure, is defined as following equation

$$U = \frac{E \left[ \ln \left( 1 + \text{SINR}_F \right) \right]}{E[P_F]}.$$
(6.41)

which is applied for energy efficiency in recent publications, e.g., [21],[22]. The behavior of energy efficiency versus minimized and constant transmit power for different values of  $\rho = 5,10$  dBs is shown in Fig. 6.5. As can be seen, energy efficiency difference for low transmit power is very large, while this difference decreases as the transmit power increases. This figure also indicates that in addition to improve the performance of victim macrocell users, the proposed approach brings further energy efficiency enhancement to femtocells.

### 6.2 System Model for OFDM Femtocell

In this system model, a single macrocell serving a region is considered and the femtocell users constitute the femtocell network, which is laid over cellular networks in the cell. Each femtocell user adjusts its power level in downlink transmission considering the interference caused by the other femtocell and macrocell users. The downlink of OFDM-based femtocell networks is considered where M femtocells numbered 1,...,M are employing N subchannels. We also assume that subchannels in OFDM are orthogonal and femto base stations estimate instantaneous channel state information perfectly. The signal to interference and noise ratio (SINR) in a femtocell m over subchannel n can be expressed using the following equation:

$$SINR_{m}^{n} = \frac{P_{m}^{(n)}H_{mm}^{(n)}}{a_{m}^{(n)} + \sum_{\substack{j=1\\j \neq m}}^{M} P_{j}^{(n)}H_{jm}^{(n)}}$$
(6.42)

where  $P_m^{(n)}$  and  $H_{mm}^{(n)}$  are the downlink transmission power and channel gain in a femtocell m over the subchannel n respectively. Furthermore,  $a_m^{(n)}$  is the sum of the thermal noise and the interference from a macrocell base station in a femtocell m over the subchannel n. The similar system model has been used in [23] and the total capacity of the system was maximized.

#### 6.2.1 Capacity constraint

The minimum power for femtocell networks in downlink communication over fading channels can be achieved by solving the following optimization problem, in which the channel capacity between the femtocell base station m and desired receiver is bigger than a predefined threshold  $R_m^{(n)}$  over the subchannel n and also the total interference caused by femtocell base stations on macrocell users does not exceed certain threshold Q.

$$\min_{P_m^{(n)}} \sum_{m=1}^M \sum_{n=1}^N P_m^{(n)}$$
(6.43a)

s.t. 
$$\sum_{n=1}^{N} \omega \log_2 \left[1 + SINR_m^n\right] \ge R_m^{(n)} \qquad m = 1, 2, \dots M$$
(6.43b)

$$\sum_{n=1}^{N} P_m^{(n)} G_m^{(n)} \le Q \tag{6.43c}$$

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Due to disjoint subchannel constraint, we can decompose the general problem (6.43a) subject to (6.43b) and (6.43c) into N sub-problem for each subchannel. Thus, we consider a single subchannel problem and corresponding optimization problem can be expressed as

$$\min_{P_m^{(n)}} \qquad \sum_{m=1}^M P_m^{(n)} \tag{6.44a}$$

s.t. 
$$\omega \log_2 \left[1 + SINR_m^n\right] \ge R_m^{(n)} \quad m = 1, 2, ...M$$
 (6.44b)

$$\sum_{n=1}^{N} P_m^{(n)} G_m^{(n)} \le Q \tag{6.44c}$$

The equation (6.44b) is changed as follows:

$$P_m^{(n)}H_{mm}^{(n)} - (2^{R_m^{(n)}/\omega_m} - 1)\left(\sum_{\substack{j=1\\j\neq m}}^M P_j^{(n)}H_{jm}^{(n)}\right) \ge a_m^{(n)}(2^{R_m^{(n)}/\omega} - 1)$$
(6.45)

We formulate the minimization problem of power allocation as the following

linear  $\operatorname{programming}(LP)$ 

$$\min_{P_m^{(n)}} \sum_{m=1}^{M} P_m^{(n)}$$
(6.46a)
  
s.t.  $P_m^{(n)} H_{mm}^{(n)} - \left(2^{R_m^{(n)}/\omega_m} - 1\right) \left(\sum_{\substack{j=1\\j\neq m}}^{M} P_j^{(n)} H_{jm}^{(n)}\right) \ge a_m^{(n)} (2^{R_m^{(n)}/\omega} - 1)$ 
  
(6.46b)

$$m = 1, 2, ...M$$

$$\sum_{n=1}^{N} P_m^{(n)} G_m^{(n)} \le Q$$
(6.46c)

#### 6.2.2 Feasibility Check

Let us address the question of whether it is feasible to find a vector of positive transmit power levels  $P^n = (P_1^{(n)}, P_2^{(n)}, ..., P_M^{(n)})^T$  such that all the data rate constraints of the M femto base stations are met and the interference caused to macrocell users does not exceed the acceptable threshold.

Here, we define an  $M \times 1$  vector  $U^n$  as:

$$\mathbf{U}^{n} = \left(\frac{a_{1}^{(n)}(2^{R_{1}^{(n)}/\omega} - 1)}{H_{11}^{(n)}}, \frac{a_{2}^{(n)}(2^{R_{2}^{(n)}/\omega} - 1)}{H_{22}^{(n)}}, \dots, \frac{a_{M}^{(n)}(2^{R_{M}^{(n)}/\omega} - 1)}{H_{MM}^{(n)}}\right)^{T}$$
(6.47)

where the notation  $x^T$  is the transpose of the vector x and an  $M \times M$  matrix  $\mathbf{F}^n$  has the following entries as:

$$F_{jm}^{n} = \begin{cases} 0 & \text{if } p = q \\ \frac{(2^{R_{1}^{(n)}/\omega} - 1)H_{jm}^{(n)}}{H_{mm}^{(n)}} & \text{if } p \neq q \end{cases}$$
(6.48)

In matrix form, these data rate constraints of M femtocell base stations can

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be expressed as:

$$(\mathbf{I} - \mathbf{F}^n)\mathbf{P}^n \ge \mathbf{U}^n \tag{6.49}$$

where **I** is the  $M \times M$  identity matrix. According the Perron-Frobenious theorem [24], the equation (6.49) has a component-wise nonnegative solution for  $P^n$  only when  $\lambda_{\max}^n < 1$  where  $\lambda_{\max}^n$  is the maximum eigenvalue of  $\mathbf{F}^n$ . In this case, the Pareto optimal solution is:

$$\mathbf{P}^n = (\mathbf{I} - \mathbf{F}^n)^{-1} \mathbf{U}^n \tag{6.50}$$

Thus, each component of vector  $\mathbf{P}^n$  is a function of  $\{R_1^{(n)}, R_2^{(n)}, ..., R_M^{(n)}\}$ , which means the transmission power in a subchannel depends on its own rate in that subchannel and all other links sharing that subchannel.

In what follow we present an algorithm to find minimized power for optimization problem (6.46a)-(6.46c) since the linear programming belongs to NP-hard problems.

#### 6.2.3 Proposed Algorithm

Determine matrix  $\mathbf{U}^n$  and  $\mathbf{F}^n$  by equation (6.47) and (6.48) for a subchannel

- If  $\lambda_{\max}^c$  (the maximum eigenvalue of  $\mathbf{F}^n$ ) <1 then
  - Find  $\mathbf{P}^{n*}$  (the Pareto optimal solution) by equation (6.50)
  - If  $\sum_{n=1}^{N} P_m^{(n)} G_m^{(n)} \leq Q$  then

Mark  $\mathbf{P}^{n*}$  as a Pareto optimal solution

Else Break

Else Break

Stop
As described above, the algorithm checks the simultaneous power allocation of M femtocells on subchannel n. In the first step, we determine whether the maximum eigenvalue of matrix  $\mathbf{F}^n$  is less than one. Next step checks Pareto optimal power  $\mathbf{P}^{n*}$  as in (6.50). Then, all the determined powers must satisfy the constraints for protecting the macrocell users by equation (6.44c).

#### 6.2.4 Distributed Algorithm

In the previous section, it was assumed that the knowledge of channel gains for the entire network and rate requirements is given, and then we described centralized algorithm to find optimal transmission power.

Here, we describe a distributed and efficient iterative algorithm which is independently executed on the transmitting node of each link by considering the cooperation with the receiver and no coordination with other links. Here, the transmitter running the algorithm is supposed to have no knowledge of channel gains for whole the network and only knows about the channels at the receiver. Power control is periodically performed in each link and its subcarrier. Let  $P^n(t)$ and  $P^n(t+1)$  denote the power transmitted by one link on one subcarrier in two consecutive time instant t and t+1, respectively. The power of each link at time t+1 can be updated as following expression:

$$P^{n}(t+1) = \frac{R_{m}^{(n)}}{\omega \log_{2}\left[1 + SINR_{m}^{n}\right]}P^{n}(t)$$
(6.51)

It is shown in [25] and [26] that when the maximum eigenvalue of  $F^n$  is less than 1 ( $\lambda_{\max}^n < 1$ ), the power control algorithm in (6.51) exponentially converges to the Pareto optimal.

#### 6.2.5 Bit Error Rate(BER) Constraint

The minimum of transmission power under bit error rate (BER) constraint can be obtained by solving the following optimization problem.

$$\min_{P_m^{(n)}} \sum_{m=1}^M P_m^{(n)}$$
(6.52a)

s.t 
$$\exp\left[-SINR_{m}^{n}\right] \le B_{m}^{(n)}$$
  $m = 1, 2, ...M$  (6.52b)

$$\sum_{n=1}^{N} P_m^{(n)} G_m^{(n)} \le Q \tag{6.52c}$$

Note that the equation (6.52b) is replaced by erfc  $\left[\sqrt{SINR_m^n}\right] \leq B_m^{(n)}$  in terms of BPSK and QPSK. We can change the equation (6.52b) as follows:

$$P_m^{(n)} H_{mm}^{(n)} + \ln(B_m^{(n)}) \sum_{\substack{j=1\\j \neq m}}^M P_j^{(n)} H_{jm}^{(n)} \ge -a_m^{(n)} \ln(B_m^{(n)})$$
(6.53)

Following linear programming(LP) can be formulated to minimize power allocation in femtocell downlink communication

$$\min_{P_m^{(n)}} \sum_{m=1}^M P_m^{(n)}$$
(6.54a)

s.t. 
$$P_m^{(n)}H_{mm}^{(n)} + \ln(B_m^{(n)})\sum_{\substack{j=1\\j\neq m}}^M P_j^{(n)}H_{jm}^{(n)} \ge -a_m^{(n)}\ln(B_m^{(n)}) \quad m = 1, 2, ...M$$

$$\sum_{n=1}^{N} P_m^{(n)} G_m^{(n)} \le Q \tag{6.54c}$$

Accordingly, we can have feasibility check and same algorithm to solve this optimization problem.



Figure 6.6: Total transmit power by two femtocell base stations versus given  $R_2$  for  $H_{11}^{(n)} = 0.7$ ,  $H_{22}^{(n)} = 0.63$ ,  $H_{21}^{(n)} = 0.2$ ,  $H_{12}^{(n)} = 0.02$ ,  $G_1^{(n)} = 0.25$ ,  $G_1^{(n)} = 0.25$ .

#### 6.2.6 Numerical Simulation

In this section, we present and discuss the numerical results to illustrate the value of power transmission in downlink communication of the channels associated with gain under different constraints. Here, without any loss of generality, we assume the two user served by different femtocell base stations and single subchannel.

Fig. 6.6 contains plots of the numerically computed the total transmission power from two femtocell base stations with  $a_m^{(n)} = 1$  and Q = 10. This figure also shows that proposed an distributed algorithms have the same results as discussed in distributed algorithm. Evidently, as the value of capacity increases, the value of total transmission powers increases.

The behavior of the total transmission power versus BER in different modulation schemes under channels associated with gain is shown in Fig.6.7 and Fig.6.7. These plots indicates that the transmission power under DPSK modulation is



Figure 6.7: Total transmit power by two femtocell base stations versus given BER<sub>2</sub> under DPSK for  $H_{11}^{(n)} = 0.7$ ,  $H_{22}^{(n)} = 0.63$ ,  $H_{21}^{(n)} = 0.2$ ,  $H_{12}^{(n)} = 0.02$ ,  $G_1^{(n)} = 0.25$ ,  $G_1^{(n)} = 0.25$ .



Figure 6.8: Total transmit power by two femtocell base stations versus given BER<sub>2</sub> Under QPSK/BPSK for  $H_{11}^{(n)} = 0.7$ ,  $H_{22}^{(n)} = 0.63$ ,  $H_{21}^{(n)} = 0.2$ ,  $H_{12}^{(n)} = 0.02$ ,  $G_1^{(n)} = 0.25$ ,  $G_1^{(n)} = 0.25$ .

higher than that under BPSK and QPSK at the same BER.

### 6.3 Chapter Conclusion

In this chapter, we considered femtocell networks with a co-channel deployment under two different system model. In first system model, we solved the minimization problem for the transmit power in a downlink femtocell subjected to two different types of constraints; minimum ergodic capacities and maximum bit error rate. In both cases, we derived some expressions for evaluating the minimum transmit power. The proposed method was compared with the constant power at femtocell AP. The proposed technique also demonstrates the solution for improving the energy-efficiency of femtocells.

In second system model, OFDM femtocell networks are considered where femtocells are employing orthogonal subchannels. In order to find minimum transmit power under two different performance parameters, we proposed an algorithm since the obtained linear programming belongs to NP-hard problems.

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# Chapter 7

## **Conclusions and Future Works**

### 7.1 Conclusion

In this thesis, the performance of ad hoc cognitive radio networks is studied. We use capacity and bit error rate as the performance measuring criterion. In a cognitive radio, in terms of spectrum sharing, secondary users can access the existing spectrum when the interference to the primary users is not above a predefined threshold.

Chapter 1 gives an introduction to wireless systems. In chapter 2, we briefly introduce different networks employed in this thesis including cellular network, ad hoc network, cognitive radio network and femtocell network.

The performance of a simplified model in cognitive radio over time varying channels is investigated first in chapter 3. The effect of different levels of channel side information is also analysed. It can be shown that the maximum capacity of the secondary link highly depends on providing side information between secondary transmitter and primary receiver at the secondary transmitter. However the side information between secondary transmitter and secondary receiver at the secondary transmitter has negligible impact on the ergodic capacity. The other important observation made in this chapter is that having the side information between the secondary transmitter and primary receiver and also between primary transmitter and secondary receiver at the secondary transmitter have negligible effect on the BER. The results found in this chapter are as a trade off between the performance and complexity.

In Chapter 4 we extend our system model to ad hoc cognitive radio networks where primary network and a secondary ad hoc network, coexisting in the same area and sharing the spectrum. We separately discussed the impact of interference from primary and secondary interferers on the cognitive radio performance. We have also shown that the total capacity of the secondary network reduces as the number of secondary interferers increases. The results give us a complete picture of the performance in ad hoc cognitive radio networks that can be designed in practice. In chapter 5, we analysed the outage probability of the similar system model where secondary interferers with arbitrary locations are distributed according to a homogeneous Poisson point process. Based on AWGN channel and Rayleigh fading, new expressions are derived. We have discussed the relation between outage probability and different parameters such as density of secondary interferers, path loss exponent and SNR.

The minimization of the power in cognitive radio particularly in femtocell networks is an important research area, which is studied in chapter 6. In this chapter, femtocell networks with co-channel deployment are considered, and minimum transmit power in two different system models are investigated. In first system model, we solved the minimization problem for the transmit power in a downlink femtocell subjected to two different types of constraints; minimum ergodic capacities and maximum bit error rate. New expressions in both cases for evaluating the minimum transmit power are derived. Moreover, It can be shown that minimizing the transmit power brings further energy efficiency enhancement to femtocell networks. In second system model, we consider OFDM-based femtocell network where orthogonal subchannels are employed by femtocells. We proposed an algorithm to find minimum power in this case under two different constraints.

### 7.2 Future Works

There are a number of directions where the research work can be extended as future work. Throughout this thesis we have focused on ad hoc cognitive radio networks and used the assumptions that would be valid therein. However, the analyses can be extended in the future to other applications like cooperation in wireless networks, relaying and etc. Some specific areas are listed below where future work can be carried out to extend our work.

- Extending the models presented for cognitive radio networks to include relay nodes and applying optimization methods to solve them.
- Optimal power allocation strategies for spectrum sensing and sharing over cognitive radio networks.
- Game theory: A mathematical theory of decision-making in which a competitive situation is analyzed. It attempts to mathematically determine the optimal course of action. Game theory is already considered in radio resource management of wireless networks as well as in the routing techniques and access control development.
- Designing protocols for multiple channels over dynamic spectrum access and cognitive radio networks.

# Appendix A

## A.1 Convex Optimization

The mathematical optimization problem is in the following form:

min 
$$f_0(x)$$
 (A.1a)

subject to 
$$f_i(x) < b_i, \ i = 1, ..., m$$
 (A.1b)

where the vector  $x = (x_1, ..., x_n)$  is the optimization variable,  $f_0$  represents the objective function,  $f_i$  are the constraint functions and the constants  $b_1, ..., b_m$  are the predefined limits for constraint functions. The optimal solution of the above problem is a vector  $x^*$  when it has the smallest value of  $f_0$  among all vectors satisfying the constraints.

In a convex optimization problem, the objective and constraint functions are convex, which means they satisfy the following condition

$$f_i(\alpha x + \beta y) \le \alpha f_i(x) + \beta f_i(x) \tag{A.2}$$

for all  $x, y \in \mathbf{R}^n$  and all  $\alpha, \beta \in \mathbf{R}$  with  $\alpha + \beta = 1, \alpha \ge 0, \beta \ge 0$ .



Figure A.1: Some simple convex and nonconvex sets. Left. The hexagon is convex. Middle. The kidney shaped set is not convex. Right. The square is not convex.

**Convex Set**: A set *C* is convex when the line segment between any two points in *C* lies in *C*. In other words, for any  $\theta$  with  $0 \le \theta \le 1$ , any  $x_1, x_2 \in C$ we have

$$\theta x_1 + (1 - \theta) x_2 \in C. \tag{A.3}$$

Fig A.1 shows some examples of convex sets.

**Convex function:** A function  $f : \mathbf{R}^n \to \mathbf{R}$  is called convex if domain f is a convex set and we have

$$f(\theta x + (1 - \theta)y) \le \theta f(x) + (1 - \theta)f(y) \tag{A.4}$$

for all  $x, y \in \text{domain } f$ , and  $\theta$  with  $0 \le \theta \le 1$ .

As shown in Fig A.2, the above inequality geometrically means that the line segment between any two points (x, f(x)) and (y, f(y)) lies above the graph of f. Note that f is concave if -f is convex. Table A.1 gives some examples of convex and concave functions over the domain **R**.



Figure A.2: Graph of a convex function

Convex functions	Concave functions
exponential: $e^{ax}$ , for any $a \in \mathbf{R}$	affine: $ax + b$ on <b>R</b> , for any $a, b \in \mathbf{R}$
power: $x^{\alpha}$ on $\mathbf{R}_{++}$ for $\alpha \ge 1$ or $\alpha \le 0$	powers: $x^{\alpha}$ on $\mathbf{R}_{++}$ , for $0 \leq \alpha \leq 1$
negative entropy: $x \log x$ on $\mathbf{R}_{++}$	logarithm: $logx$ on $\mathbf{R}_{++}$

Table A.1: Convex functions and Concave functions

**2nd-order conditions:** For twice differentiable function f over convex domain, f is convex if and only if

$$\nabla^2 f(x) \succeq 0 \text{ for all } x \in \text{domain } f$$
 (A.5)

where  $\nabla^2 f(x)$  is Hessian or second derivative and given by

$$\nabla^2 f(x)_{i,j} = \frac{\partial^2 f(x)}{\partial x_i \partial y_j} \qquad i, j = 1, \dots, n$$
(A.6)

**Example:** Quadratic over-linear,  $f(x, y) = \frac{x^2}{y}$  (Fig A.3)

$$\nabla^2 f(x,y) = \frac{2}{y^3} \begin{bmatrix} y & y \\ -x & -x \end{bmatrix}^T \succeq 0$$
(A.7)

which is convex for any y > 0



Figure A.3: Graph of  $f(x, y) = x^2/y$ .

Solution for the convex problem: For the standard optimization problem in (A.1a) subject to (A.1b), the Lagrangian is defined as

$$L(x,\lambda) = f_0(x) + \sum_{i=1}^m \lambda_i f_i(x)$$
(A.8)

where linear combinations of constraints are added to the objective function with  $\lambda_i$  which are Lagrange Multipliers. Vector  $\lambda$  is called dual variable or Lagrange multiplier vector. Accordingly, the Lagrange dual function is the minimum value of the Lagrangian over x:

$$L(x,\lambda) = g(\lambda) = \inf L(x,\lambda) = \inf \left( f_0(x) + \sum_{i=1}^m \lambda_i f_i(x) \right)$$
(A.9)

# Appendix B

### **B.1** Random Variables

It frequently occurs that in performing an experiment we are mainly interested in some functions of the outcome as opposed to the outcome itself. As an example, we are interested to know that the sum of the two dice in tossing dice is seven which means it is not concerned whether the actual outcomes was (6, 1) or (5, 2) or (4, 3) or (3, 4) or (2, 5) or (1, 6). These quantities of interest which is defined on the sample space, are random variables. The value of a random variable is determined by the outcome of the experiment and therefore we assign probabilities to the possible values of the random variable.

The cumulative distribution function (cdf): F(.) of the random variable X is defined for any real number  $b, -\infty < b < \infty$ , by

$$F(b) = P\{X \le b\} \tag{B.1}$$

F(b) is the probability that the random variable X can take on a value less than or equal to b. The main properties of the cumulative distribution function F are

1. F(b) is a nondecreasing function of b,

- 2.  $\lim_{b\to\infty} F(b) = F(\infty) = 1,$
- 3.  $\lim_{b \to -\infty} F(b) = F(-\infty) = 0.$

#### **B.1.1** Discrete Random Variables

A random variable that can take on at most a countable number of possible values is said to be discrete. For a discrete random variable X, we define the probability mass function p(a) of X by

$$p(a) = P\{X = a\} \tag{B.2}$$

The probability mass function p(a) is positive for a countable number of values of a.

If X takes one of the values  $x_1, x_2, ...,$  we have

$$p(x_i) > 0, \qquad i = 1, 2, \dots$$
 (B.3)

$$p(x) = 0$$
, all other values of  $x$  (B.4)

and

$$\sum_{i=1}^{\infty} p(x_i) = 1 \tag{B.5}$$

According to probability mass functions, discrete random variables are classified. Some of random variables are:

- 1. The Bernoulli Random Variable
- 2. The Binomial Random Variable
- 3. The Geometric Random Variable

4. The Poisson Random Variable

#### B.1.2 Continuous Random Variables

Here, we are interested in the random variables whose set of possible values is uncountable. X is a continuous random variable if there exists a nonnegative function f(x) which has the following property for any set B of real numbers

$$P\{X \in B\} = \int_{B} f(x)dx \tag{B.6}$$

The function f(x) is known as the probability density function. For example, if B = [a, b], we have

$$P\{a \le X \le b\} = \int_{a}^{b} f(x)dx \tag{B.7}$$

The cumulative distribution F(.) and the probability density function f(.) has the following relationship

$$F(a) = P\{X \in (-\infty, a]\} = \int_{-\infty}^{b} f(x)dx$$
 (B.8)

Differentiating both sides of the above equation gives

$$\frac{d}{da}F(a) = f(a) \tag{B.9}$$

There are several continuous random variables such as

- 1. The Uniform Random Variable
- 2. Exponential Random Variables
- 3. Gamma Random Variables

4. Normal Random Variables

### **B.2** Poisson Distribution and Poisson Process

#### B.2.1 Poisson Distribution

Let X be a random variable which can take on one of the values 0,1,2,... X is called a Poisson random variable (RV) with parameter  $\lambda$ , when for some  $\lambda > 0$ , we have

$$p(k) = P\{X = k\} = e^{-\lambda} \frac{\lambda^k}{k!}, \quad k = 0, 1, \dots$$
 (B.10)

The above equation defines a probability mass function since

$$\sum_{k=0}^{\infty} p(k) = e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = e^{-\lambda} e^{\lambda} = 1$$
 (B.11)

Note that the mean and variance of Poisson RV are

$$E[X] = \lambda \tag{B.12}$$

$$Var[X] = \lambda \tag{B.13}$$

The Poisson RV has been widely used in communication networks, as it is used to model the number of customers arriving during various intervals of time.

**Example:** Suppose that the number of accidents occurring on a street has Poisson distribution with parameter  $\lambda = 1$ , what is the probability that there is at least one accident today?

$$P\{X \ge 1\} = 1 - P\{X = 0\} = 1 - e^{-1} \approx 0.633 \tag{B.14}$$

#### **B.2.2** Poisson Process

A stochastic process  $\{N(t), t \ge 0\}$  is called a counting process when N(t) is the total number of "events" that have occurred up to time t. For example, if N(t) represents the number of goals scored by a given soccer player up to time t, then  $\{N(t), t \ge 0\}$  is a counting process. Thus, whenever this soccer player scores a goal, an event of this process will occur.

From above definition, for a counting process N(t) must satisfy

- 1.  $N(t) \ge 0$ .
- 2. N(t) is integer valued.
- 3. If s < t, then  $N(s) \le N(t)$ .
- 4. For s < t, N(t) N(s) is the number of events occurring in the interval (s, t].

A counting process is called independent increments if the numbers of events occurring in disjoint time intervals are independent.

#### Definition of the Poisson Process:

The counting process  $\{N(t), t \ge 0\}$  is called a Poisson process with rate  $\lambda, \lambda > 0$ , if

- 1. N(0) = 0.
- 2. The process has independent increments.
- 3. In any interval of length t, the number of events is Poisson distributed with mean  $\lambda t$ . So, for all  $s, t \ge 0$  we have

$$P\{N(t+s) - N(s) = n\} = e^{-\lambda t} \frac{(\lambda t)^n}{n!}, \qquad n = 0, 1, \dots$$
(B.15)

# Appendix C

### C.1 THE PROPAGATION MODEL

The target of propagation model is to determine the probability of satisfactory performance of wireless systems which is based on the radio wave propagation. Several propagation models have been developed in which the received signal is a function of distance. The free space propagation model as an ideal propagation condition has been considered by H. T. Friis. In this model, there is only one line of sight (LOS) path between the transmitter and receiver pair. The following equation, known as Friis equation, gives the received power at the receiver

$$P_r = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2 \tag{C.1}$$

where  $P_r$  and  $P_T$  represent the received and transmitted power, respectively;  $G_T$  and  $G_R$  represent the transmitter and receiver antenna gains, respectively; dis the distance between the transmitter and the receiver pair; and finally  $\lambda$  is the wavelength. The path loss is also defined as follows

$$PL(dB) = L_p(dB) = -10 \log\left(\frac{\lambda}{4\pi d}\right)^2$$
 (C.2)

#### C.1.1 Log-Distance Path Loss Model

Propagation models should estimate the average received power at a certain distance from the transmitter. The average path loss for a transmitter-receiver separation can be expressed as

$$\overline{PL}(d) = \overline{PL}(d_0) + 10n \log\left(\frac{d}{d_0}\right)$$
(C.3)

where *n* is the path loss exponent;  $d_0$  is the reference distance, and  $\overline{PL}(d_0)$  is the mean path loss at  $d_0$ . The value of  $\overline{PL}(d_0)$  usually is free-space path loss at a distance 1m from the transmitter.

The received power at the distance d becomes

$$P_r(d) = P_r(d_0) \left(\frac{d}{d_0}\right)^n \tag{C.4}$$

where  $P_r(d_0)$  represents the received power at the reference distance  $d_0$ .

#### C.1.2 Log Normal Shadowing

Shadowing is the attenuation caused by change of environment in different locations but at the same transmitter-receiver separation distance. This is because any obstacles between a transmitter and a receiver. Studies show that the effect of shadowing can be modelled as a log-normal distribution. To include the shadowing into above equations,  $X_{\sigma}$  is added as follows

$$PL(d) = \overline{PL}(d) + X_{\sigma}$$
$$= \overline{PL}(d_0) + 10n \log\left(\frac{d}{d_0}\right) + X_{\sigma}$$
(C.5)

where  $\overline{PL}$  represents the mean path loss at distance d;  $X_{\sigma}$  is the log-normal shadowing effect with zero mean and variance  $\sigma$ .

## C.2 Multipath Channel Model

In this model, the transmitted signal arrives at the receiver through different paths. Received signal from each path has own phase, gain and delay which may be different from other paths. The multipath channel is modelled as a filter. In this model, it is assumed that x(t) is the transmitted bandpass waveforms,  $h(t,\tau)$  is the impulse response of the time varying multipath channel and y(t)represents the received waveform. The variable t is the variations of time and  $\tau$  is the multipath delay for a specific time t. The received signal y(t) is a convolution of x(t) and  $h(t,\tau)$  as

$$y(t) = \int_{-\infty}^{\infty} x(t)h(t,\tau)d\tau$$
 (C.6)

The impulse response of the time varying multipath channel can be expressed as  $N_{-1}$ 

$$h(t,\tau) = \sum_{i=0}^{N-1} a_i(t,\tau) \exp[j\theta_i(t,\tau)] \,\delta\left(\tau - \tau_i\left(t\right)\right)$$
(C.7)

where  $a_i(t, \tau)$ ,  $\theta_i \tau_i(t)$  represent amplitude, phase and the excess delay of each paths. N is the total number of paths and  $\delta$  is the unit impulse function. Since channel impulse response is continuous time, multipath delay is divided into discrete segments, known as excess delay bins, as

$$\tau_i = i\Delta\tau \quad \forall i = \{0, ..., L-1\}$$
(C.8)

where  $\Delta \tau$  is the delay bin width. Therefore (C.7) changes into

$$h(t,\tau) = \sum_{i=0}^{L-1} a_i(t,k\Delta\tau) \exp\left[j\theta_i(t,k\Delta\tau)\right] \delta\left(\tau - k\Delta\tau\right)$$
(C.9)

where L is the maximum delay path.

In the following, we explain some multipath channel characteristics in more detail.

 Time Dispersion Parameters: mean excess delay and RMS delay spread are two parameters to measure the time dispersion. Mean excess delay can be expressed as

$$\overline{\tau} = E[\tau] = \sum_{k} prob(k)\tau_k = \sum_{k} \left(\frac{E[a_k^2]}{\sum_k E[a_k^2]}\right)\tau_k \tag{C.10}$$

and the RMS delay spread is

$$\delta_{\tau} = \sqrt{\overline{\tau^2} - \overline{\tau}^2} \tag{C.11}$$

2. Coherence bandwidth: This parameter represents the range of the frequencies over which the multipath channel is flat. Coherence bandwidth for frequency correlation bigger than 0.9 can be obtained by

$$B_c \approx \frac{1}{50\delta_\tau} \tag{C.12}$$

3. Doppler spread and Coherence time: These two parameters explain the time varying nature of a channel. Doppler spread determines an estimation of spectral widening and is expressed as

$$B_D = 2f_{d-\max} \tag{C.13}$$

where  $f_{d-\max} = \frac{v}{\lambda}$  is the maximum Doppler shift. Coherence time indicates the time duration in which the fading parameters and channel impulse response are constant. This parameter for correlation bigger than 0.5 is

$$T_c \approx \frac{9}{16\pi f_{d-\max}} \tag{C.14}$$

#### C.2.1 Fading Channels

Constructive or destructive interference between several versions of a transmitted signal arriving at the receiver causes fading. Channel fadings are different based on the relationship between the signal parameters like bandwidth and symbol period, and channel parameters like doppler spread and RMS delay spread. A received signal suffer from flat fading when the coherence bandwidth of the channel is greater than the transmitted signal's bandwidth  $(B_c > B_s)$ . In terms of time domain, flat fading occurs when the symbol period of the transmitted signal is greater than the delay spread of the channel  $(T_s > \delta_{\tau})$ .

#### **Rayleigh Fading**

The channel gain at the k - th bin with  $N_k$  arriving path of a no line-of-sight (N-LOS) multipath channel is considered as

$$\widetilde{a} = a_k e^{j\theta_k} = \sum_{i=0}^{N_k - 1} a_{k,i} e^{j\theta_{k,i}} = \sum_{i=0}^{N_k - 1} a_{k,i}^I + ja_{k,i}^Q = a_k^I + ja_k^Q$$
(C.15)

where I and Q are in-phase and quadrature phase component of channel

gain, respectively. Here, it is also assumed that all path gains are independent and,  $a_k^I$  and  $a_k^Q$  are Gaussian distributed with zero mean and equal variance  $\sigma^2$ . By applying central limit theorem, it can be shown that  $a_k = \sqrt{a_k^{I^2} + a_k^{Q^2}}$  is Rayleigh-distributed with distribution

$$p(a_k) = \frac{a_k}{\sigma^2} \exp\left(-\frac{a_k^2}{2\sigma^2}\right) \qquad 0 \le a_k \le \infty \tag{C.16}$$

and phase  $\theta_k = \tan^{-1}(\frac{a_k^Q}{a_k^I})$  is uniformly distributed over  $2\pi$ .

#### **Rician Fading**

In this model, a channel with a dominant path and several weak paths is considered. The channel fading statistics is Rician distributed with pdf

$$p(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right) \quad A \ge 0, \ r \ge 0$$
(C.17)

where A denotes the peak amplitude of the dominant signal and  $I_0(.)$  represents zero-order Bessel function of the first kind.