POWER TRANSFORMER END-OF-LIFE MODELLING: LINKING STATISTICS WITH PHYSICAL AGEING

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List of Symbols and Abbreviations

Symbol	Meaning	Value/Unit
A	In-service transformer	
a	Parameter in exponential fit of transformer thermal hazard	
A	Chemical environment dependent factor	
A_0	Chemical environment dependent factor under referenced condition	
ACR	Average crude rate	
A_{ox}	Chemical environment dependent factor under oxidation	
A_{hy}	Chemical environment dependent factor under hydrolysis	
A_{py}	Chemical environment dependent factor under pyrolysis	
b	Parameter in exponential fit of transformer thermal hazard	
$\mathbb{C}$	In-service transformer compensation factor with errors considered	
C	Total number of censored transformers	
$C_{DP2}(T)$	2^{nd}-stage ageing rate factor	
CF	Compensation factor by curve fitting	
CF_0	Scrapped transformer compensation factor in accord with lowest DP determined thermal lifetime	
COD	Coefficient of determination	
D	National annual equivalent demand	GW
D_M^r	Critical value	
DP	Insulation paper degree of polymerisation	
DP_0	Initial DP when insulation is new	
$DP2_0$	DP after initial rapid ageing	
$DP_{av}(t)$	Average DP after t years service	
DP_{EoL}	Transformer thermal end-of-life criterion in this study	200
DP_{lowest}	Lowest DP	
$D_{s1}, D_{s2}, D_{s3}, \ldots, D_{sN}$	National summer demand at $t_{s2}, t_{s2}, t_{s3}, \ldots, t_{sN}$	GW
$D_{w1}, D_{w2}, D_{w3}, \ldots, D_{wN}$	National winter demand at $t_{w2}, t_{w2}, t_{w3}, \ldots, t_{wN}$	GW
E	Chemical reaction activation energy	J/mole
E_0	Activation energy under referenced condition	J/mole
EoL_0	Lowest DP determined thermal end-of-life	year
EoL	Thermal end-of-life calculated via IEC thermal model	year
E_{ox}	Activation energy under oxidation	J/mole
E_{hy}	Activation energy under hydrolysis	J/mole
E_{py}	Activation energy under pyrolysis	J/mole
Er_{CF}	Percentage error between CF_0 and CF	
Er_{CF}^L	Lower border of Er_{CF}	
Er_{CF}^U	Upper border of Er_{CF}	
Er_{EoL}	Percentage error between EoL_0 and EoL	
F	Failed transformer	
$f(t)$	Failure probability density function (PDF)	
$F(t)$	Cumulative distribution function (CDF)	
$F^0(t)$	Observed CDF at i^{th} failure	

$F_0(t)$	Derived CDF at i^{th} failure	
G_r	Gradient between average winding temperature and average oil temperature	K
G_r^*	IEC typically designed gradient between average winding temperature and average oil temperature	K
$h(t)$	Hazard function	
h_0	Constant random failure hazard rate	
h_{actual}	Transformer population actual failure hazard	
$h_{critical}$	Critical actual hazard rate	
$h_t(t)$	Thermal failure hazard as a function of age t	
$H(t)$	Cumulative hazard function	
HSF	Transformer winding hot-spot factor	
k	Ageing rate factor	Hour^{-1}
K	Transformer load factor	p.u.
$K\binom{i}{w}\binom{N-1}{N}$	i^{th} one minute load factor between winter load $K_{wN\text{-}1}$ and K_{wN}	p.u.
k_0	Reference ageing rate factor	Hour^{-1}
k_{10}	Constant	
k_{11}	Thermal model constant	
k_2	Constant	
k_{21}	Thermal model constant	
k_{22}	Thermal model constant	
k_{r}	Relative ageing rate factor	
$\boldsymbol{k}_{\mathbf{rr}}$	Reverse rank	
K-S test	Kolmogorov-Smirnov test of goodness-of-fit	
K_{s1}, K_{s2}, K_{s3}, …, K_{sN}	Transformer summer load factor at t_{s1}, t_{s2}, t_{s3}, …, $t_{s\mathrm{N}}$	p.u.
K_{w1}, K_{w2}, K_{w3}, …, K_{wN}	Transformer winter load factor at t_{w1}, t_{w2}, t_{w3}, …, $t_{w\mathrm{N}}$	p.u.
L'_{eq}	Transformer equivalent load at year of scrapping	MVA
L_1, L_2, L_3, …, L_N	Transformer load at time t_1, t_2, t_3, …, t_N	MVA
L_{eq}	Transformer annual equivalent load	MVA
LL	Transformer load losses	
LMA	Lowe molecular weight acid	
LoL	Transformer Loss-of-Life	hour
LoL_i	Initial Loss-of-Life	hour
LoL_{total}	Total loss-of-life of 1 year operation	hour
LSE	Least square estimator	
M	Total number of failures	
MLE	Maximum likelihood estimator	
N	Total number of transformer population	
$N_E(t)$	Number of exposed transformers at t	
$n_F(t)$	Number of failures within t	
$N_F(t)$	Total number of failed transformers up to t	
NLL	Transformer no-load losses	
$N_S(t)$	Number of survived transformers at the end of t	
OFAF	Oil forced and air forced cooling mode	
ONAN	Oil natural and air natural cooling mode	
r	Failure portion	
R	Gas constant	8.314J/mole/K

R	Ratio of load losses at rated current to no-load losses	
$R(t)$	Reliability function	
$R(t\|t-1)$	Conditional reliability at *t* by given transformer have survived at end of *t*-1	
R^*	IEC typically designed ratio of load losses at rated current to no-load losses	
t	Transformer service age	year
$\bar{t}$	Mean life of transformer population	year
TI	Tensile index	
t_{knee}	Knee point age	year
t_{op}	Transformer operation history	year
t_{re}	Transformer remaining life	year
TS	Tensile strength	
$t_{typical}$	Population median life, typical life or anticipated life	year
V	Transformer relative ageing rate	
var	Lifetime data variance	year^2
WTI	Winding temperature indicator	
X	Scrapped transformer	
x	Oil exponent	
x^*	IEC typically designed oil exponent	
y	Winding exponent	
y^*	IEC typically designed winding exponent	
$y_{1-\gamma}(r)$	Critical value corresponding to certain γ	
Y_X	Transformer scrapping year	year
α	Scale factor	
γ	Significant level	
$\Delta\theta_{avor}$	Average oil temperature rise over ambient	K
$\Delta\theta_{avwr}$	Average winding temperature rise over ambient	K
$\Delta\theta_{bor}$	Bottom oil temperature rise over ambient	K
$\Delta\theta_{h}$	Hot-spot temperature rise over top oil	K
$\Delta\theta_{h1}$	Hot-spot temperature rise component 1	K
$\Delta\theta_{h1i}$	Initial hot-spot temperature rise component 1	K
$\Delta\theta_{h2}$	Hot-spot temperature rise component 2	K
$\Delta\theta_{h2i}$	Initial hot-spot temperature rise component 2	K
$\Delta\theta_{hi}$	Initial temperature rise	K
$\Delta\theta_{hr}$	Hot-spot temperature rise over top oil at rated current	K
$\Delta\theta_{to}$	Top oil temperature rise over ambient	K
$\Delta\theta_{tor}$	Top oil temperature rise over ambient at rated current	K
$\Delta\theta_{tor}^*$	IEC typically designed transformer top oil temperature rise over ambient at rated current	K
θ_a	Ambient temperature	℃
θ_a^*	Constant ambient temperature	℃
θ_{ai}	1st recorded ambient temperature	℃
θ_h	Transformer hot-spot temperature	℃
θ_{h0}	Maximum allowed winding temperature	℃
θ_{hi}	Initial hot-spot temperature	℃
θ_{hr}	Hot-spot temperature at rated current	℃
θ_{to}	Top oil temperature	℃
θ_{toi}	Initial top oil temperature	℃
σ	Standard deviation of transformer lifetime	year
τ_o	Oil time constant	min
τ_w	Winding time constant	min

Abstract

Investment decisions on electric power networks have developed to balance network functionality and cost efficiency by analyzing the main risks associated with network operation. Ageing infrastructures, like large power transformers in particular, aggravate the stress of management, because the failure of a power transformer could cause power supply interruption, network reliability reduction, large economic losses and also environment impacts. Transformer asset management is therefore aimed to develop a cost-efficient replacement strategy to get the most usage of transformers. The main objective of this thesis is to understand how UK National Grid transformer assets failure trend can be used, as the engineering evidence to help make financial decisions related to transformer replacements.

The studies in this thesis are implemented via two main approaches. First statistical analyses methods are undertaken. This approach is realized to be non-optimal, because the transformer failure mechanism at the normal operation stage is different from that when transformers are aged. Secondly, the transformer physical ageing model is used to estimate thermal lifetimes under the ageing failure mechanism. In conjunction with the random hazard rate obtained by statistical analyses, the actual National Grid transformer population failure hazard with service age is derived.

Statistical analyses are carried out based on the ages of National Grid failed and in-service transformers. Transformer lifetime data are fitted into various distribution models by the least square estimator (LSE) and maximum likelihood estimator (MLE). Statistics are however powerless to suggest the population future failure trend due to their intrinsic limitations. National Grid operational experience actually indicates a stable and low value of the random failure hazard rate during the transformer early operation ages. The engineering knowledge however suggests an ageing failure mechanism exists which corresponds to an increasing hazard in the future.

Transformer lifetime under ageing failure mechanism is conservatively indicated by its thermal end-of-life corresponding to a specific level of insulation paper mechanical strength. By analyzing National Grid scrapped transformers' lowest degree of polymerization (DP), these transformers are estimated to have deteriorated at different rates and their thermal lifetimes distribute over a wide age range. The limited number of scrapped transformers cannot adequately indicate the ageing status of the whole population. A transformer's thermal lifetime is determined by its loading condition, thermal design characteristics and installation site ambient temperature. However, these input data are usually incomplete for an individual transformer.

A simplified approach is developed to predict the National Grid in-service transformer's thermal lifetime by using information from scrapped transformers. The in-service transformer population thermal hazard curve under ageing failure mechanism can thus be obtained.

Due to the independent effect from transformer random failure mechanism and ageing failure mechanism, the National Grid transformer population actual failure hazard curve with age is therefore derived as the superposition of the random failure hazard and the thermal hazard. Transformer asset managers are concerned about the knee point age, since aged transformer assets threaten network reliability and the transformer replacement strategy needs to be implemented effectively.

Declaration

No portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

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Chapter 1 Introduction

1.1 Overview

An electric power network is designed to transmit and distribute electricity with a high level of reliability and deliver the supply availability and quality expected by consumers. The equipment investment is conventionally driven by the technology and is orientated to ensure the safety and reliability of the network. The new economic circumstance, created by the liberalization of electricity markets and the introduction of competition, requires the investment to focus on network economic efficiency as well as reliability. The design and maintenance of electric power network have developed to the stage of balancing its functionality and investment/replacement efficiency by analyzing the main risks under which the network is operating and may operate in the future.

The aged assets aggravate the stress on management since they potentially lead to inefficient network performance, loose compliance to utilities' legislation standards and have a negative environmental impact [1, 2]. Surveys undertaken by EPRI and CIGRÉ show the large investment in generating capacity installed after World War II from the 1950's until the early 1970's, resulted in a large number of power transformers being commissioning particularly in North America and Europe [1-5]. Many of these transformers are now approaching or are beyond their designed operation life of 40 years; approximately 50% of the transformer assets currently operating are considered "old" or "aged" [2, 6, 7]. Although they are highly-reliable and long-life expectancy equipments, as the most important and cost-intensive assets in a power system, particularly at the transmission level, the failure of a power transformer could cause power supply interruption and reduce the reliability of the remaining system. The intensive expenditure on purchasing new transformers and the long-term replacement might not be avoided when failure occurred. Moreover the influences to substation surrounding environment should also be a concern [8].

According to a CIGRÉ survey in 2000, based on 300,000 substation assets among 13 countries [1], 93% of utilities had historically replaced less than 10% of their assets and 70% had replaced less than 5% [1]. The impact on system performance of few ageing-related failures has rarely been observed up to the present. This is probably related to the network *N*-1 or *N*-2 criterion undertaken at the design/planning stage [1]. However the low level of investment injected into the aged power system implies a proactive management strategy needs to be determined.

For a transmission and distribution business, asset management is defined by the UK Publicly Available Specification BSI-PAS 55 [9, 10]: -

> "*systematic and co-ordinated activities and practices through which an organization optimally manages its assets, and their associated performance, risks and expenditures over their lifecycle for the purpose of achieving the required level of service in the most cost effective manner*" [6, 8].

The objective of asset management is further indicated as: -

> "*to ensure (and be able to demonstrate) that the assets deliver the required function and level of performance in a sustainable manner, at an optimum whole-life cost without compromising health, safety, environmental performance, or the organization's reputation*" [1].

The above statements clearly identify the technical and economic stresses that the utilities are currently facing in response to the ageing assets and the uncertain operation/organization schemes subject to the sustainable and renewable power generations in the future.

More specifically in order to get the most usage of transformer assets CIGRÉ clarifies the power transformer life management as: -

> "*using existing body of knowledge and technologies, and looking into the future, develop guidelines with the objective to manage the life of transformers, to reduce failures, and to extend the life of transformers in order to produce a reliable and cost effective supply of electricity*" [11],

As a fundamental step towards transformer life management, population failure trend has been evaluated by statistics since 1980's and 1990's in North America. Utilities are now benefitting from the sharing and integrating of transformer lifetime data [6, 12-17]. Reliability models developed from other disciplines were applied in power transformer failure prediction. By estimating the population mean life and the failure risks, the age when the population reliability begins to reduce and thus a preventive replacement action needs to be implemented is determined [4]. Transformer age however is further realized as a deficient predictor of population failure as it is not the only determinant factor of failure [2]. Besides statistical prediction of population failure trend does not consider the transformer real operation condition and is consequently powerless to identify the specific transformers that need to be replaced.

Hence the monitoring of transformer defects and the tracking of operating condition are more and more concerned to identify the possible paths of defect evolutions and to help estimate the individual residual life since 1990's [4, 11, 17]. Transformer oil testing and the off-line power factor testing at reduced voltages are the most widely used methods of monitoring. The usage of other on-line and off-line testing methods are also gradually increasing in practice owing to the development of data interpretation tools, reliability and compatibility studies, and the need for cost reduction [3].

A proper maintenance strategy and replacement programme based on the condition monitoring data are now being adapted to optimize the assets life cycle, subject to the risks associated with long-term operation [11]. However the proactive replacement breaks down the actual population failure trends and statistical analysis is having to be suspended owing to a lack of failure data. However power transformer statistical analysis is still of great interest as it microscopically provides a general picture of population ageing and suggests for a utility its long-term capital investment strategy.

1.2 Statement of Problem

General statistical analysis on transformer lifetime data has clearly demonstrated that in North America and Europe there is little detectable increase in the transformer population failure risk with age. This is because the failure data are fairly limited and most transformer failures up to the present are caused by randomly occurring power system transient fault events [2, 4, 6].

The rise of failure risk and accordingly the increase of replacement cost caused by the transformer ageing failure mechanism however must eventually be seen according to the knowledge of insulation degradation [2, 4]. The decision on transformer replacement is important, however technically difficult to determine as the monitoring data, operational experience and deterioration knowledge are not straightforward to interpret and convert into valid information [4, 6].

Nevertheless if a transformer's remanent insulation strength could be assessed, it would be possible to evaluate the transformer's remaining lifetime [11]. Evidence from scrapping and replacing transformers can be used to suggest the design specifications, operational stresses, ageing status and even the lifetimes of those in-service transformers [4].

It is furthermore the asset manager's task to evaluate the impact of the increasing failure risk and to manage the ageing assets to ensure the power can be delivered reliably without excessive cost [4, 18]. Ideally, with an accurate ageing model, the number of failure can be predicted to allow the network to be operated at a sustainable stage, at which the number of transformer failures and the replacement costs remain constant so that an efficient, reliable and secure power supply is achieved [2].

1.3 Aims and Objectives of This Research

As the assets owner of the electricity transmission network in England and Wales and the transmission network operator across Great Britain, the National Grid Company has the obligation to predict outages, evaluate the impacts of outages and accordingly to develop a cost-efficient replacement strategy to safeguard the reliability of electricity transmission [4]. By studying the historical failures of National Grid power transformers and their replacement policy, this thesis will evaluate the National Grid in-service transformer population ageing status and to predict future National Grid transformer population failure risk using the engineering-based evidence to support the asset manager's financial decision.

Three main assumptions are made prior to the main contents in order to simplify the complex practical problems. These assumptions are:

- Failure risk generally includes both the likelihood of failure and the consequence of failure. In this thesis transformer failure risk is represented by the failure likelihood, or alternatively failure hazard only, since the influence of failure is difficult to quantify [1].

- Transformer paper insulation mechanical strength deterioration in the main winding is used as the indicator of transformer ageing-related failure. Other failures of transformer serviceabilities (i.e. transformer electromagnetic ability, integrity of current carrying and dielectric property) or outages of other subsystems (i.e. cooling system, bushing, on-load tap-changer, oil preservation and expansion system, protection and monitoring system) are not considered in this thesis [6].

- Transformer random failure and ageing-related failure are different failure mechanisms which independently contribute to a transformer's actual failure hazard.

The main objectives of this research are as follows:

- Undertake literature reviews on product lifetime statistical analysis and transformer paper insulation ageing.
- Analyze National Grid power transformers lifetime data via statistical approaches and summarize the advantages and drawbacks of statistics in a power transformer lifetime data study.
- Assess the paper insulation deterioration status of National Grid retired transformers in order to understand the discrepancies of transformer design, loading condition and installation site ambient temperature among National Grid transformers. Furthermore to clearly identify that transformer thermal lifetime is determined by multi-variables.
- Develop an effective approach to predict individual transformer thermal end-of-life by using information from retired transformers and then to produce the ageing failure hazard curve of National Grid power transformers.
- Generate a National Grid transformer population failure hazard curve by linking the statistically determined "random hazard" with the new approach developed "ageing failure hazard".

1.4 Methodologies of This Research

The methods used in this research can be summarized as statistical approaches and paper insulation ageing models.

- Statistical Approaches

Statistical analysis on product reliability has been developed since 1940's due to the demands of modern technologies [19-22]. Particularly in power systems, the lifetime prediction of generator windings, engine fans, turbine wheels, cables and transformers have been analyzed via statistics since the 1980's [23, 24].

Transformer lifetime statistical analysis is carried out based on the lifetime data obtained from both failed and in-service units, and is used to predict the population future failures in terms of failure hazard against age [25]. Steps associated with statistical analysis normally involves lifetime data collecting, distribution model selection, lifetime data fitting and finally goodness-of-fit testing. The above 4 steps however are not completely implemented in most engineering applications and consequently various approaches are further developed to simplify the complex analysis.

W. Nelson in 1972 proposed a systematic plotting method that involved fitting the lifetime data into traditional distribution models via a least square estimator (LSE) [26]. W. Y. Li modified Nelson's approach in 2004 and applied the refined approach in Canada BC Hydro 500kV reactors lifetime data analysis [27, 28]. Li's simplified approach is currently widely discussed and adopted by other researchers [29]. Meanwhile E. M. Gulachenski indicated the advantages of Bayesian method and used this method to analyze US New England Electric 115kV power transformers as early as 1990's [30]. Q. M. Chen furthermore employed Bayesian analysis to fit US PJM's transformer lifetime data into Iowa survival curves [31, 32].

Generally in engineering practice, transformer population failure hazard is exposited as the instantaneous failure probability at a specific age t, by knowing a certain amount of transformers have survived till age t-1. It is mathematically expressed as the ratio of the failure number within age t ($n_F(t)$) and the number of exposed transformers at age t ($N_E(t)$), as

$$h(t) = \frac{number\ of\ \ failed\ transformers\ within\ age\ t}{number\ of\ \ exp osed\ transformers\ at\ age\ t} = \frac{n_F(t)}{N_E(t)} \tag{1-1}$$

Transformer population failure hazard $h(t)$ obtained via statistics is thereafter used to predict the number of failures at each age. The number of transformer failure $n_F(t)$ at age t is determined as

$$n_F(t) = h(t) \times N_E(t) \tag{1-2}$$

However statistics is identified as powerless to suggest National Grid transformer population failure hazard in the future since transformer lifetime data are limited, distribution models are arbitrarily assigned and historical failures have not yet indicated the onset of population ageing-related failure. Statistical analysis can only evaluate National Grid transformer random failure hazard rate based on the operational experience up to the present. Transformer paper insulation ageing model is therefore used to predict individual lifetimes under ageing failure mechanism.

- Paper Ageing Models

A transformer’s end-of-life is eventually determined by its paper insulation useful lifetime subject to operational stresses, since insulation paper cannot be replaced once a winding is built-up. Vigorous material laboratory ageing tests undertaken for more than 60 years have revealed that paper degradation is a complex sequence of chemical reactions due to the effects from temperature, oxygen, water and acid contents. During degradation, the inter-fibre bonding is destroyed, the molecule chain is depolymerised and thus the paper mechanical strength is reduced. The collapse of insulation paper mechanical strength actually indicates the end of transformer useful life [33].

It is currently technically difficult to evaluate a transformer remaining life by using the monitored information from transformer winding temperature, oxygen, water and acids. However National Grid post-mortem analysis provides a unique access to examine its retired transformers paper ageing status by measuring the degree of polymerisation (DP) of the insulation paper. A widely used model presents 1/DP against transformer service age, between the DP values of 1000 and 200. Transformer thermal lifetime can therefore be estimated according to a linear model by presuming a threshold of paper mechanical property as transformer end-of-life criterion, normally at DP equals to 200. However the limited number of retired transformers is not an adequate sample that can directly indicate the ageing of the National Grid transformer population.

Equivalent to the DP reduction model, individual transformer thermal deterioration and end-of-life are also described in transformer thermal model explained by the IEC transformer loading guide [34]. This requires the unit’s real-time loading, ambient temperature, thermal design parameters and information about cooler switching.

In order to predict National Grid in-service transformer thermal lifetimes a simplified approach is proposed in this thesis based on the IEC transformer thermal model. The transformer ageing rate factor, conventionally calculated in the IEC thermal model, is first of all adjusted to involve the effects from oxygen, water and acids as well as winding temperature. The idea of hot-spot temperature compensation factor ($\mathbb{C}$) is developed to calibrate all the uncertainties of thermal model inputs. The functional relationship between $\mathbb{C}$ and the transformer equivalent load is then established based on the information available from National Grid retired transformers. By extrapolating this relationship to National Grid in-service transformers, individual thermal lifetime is obtained by a Monte Carlo iteration.

Statistics is finally turned to as a mathematical tool to generate the transformer population ageing-related failure hazard curve when transformers' thermal lifetimes are determined.

1.5 Project Overview

Aims and methodologies of this research are illustrated in Figure 1-1 as the overview of the whole thesis.

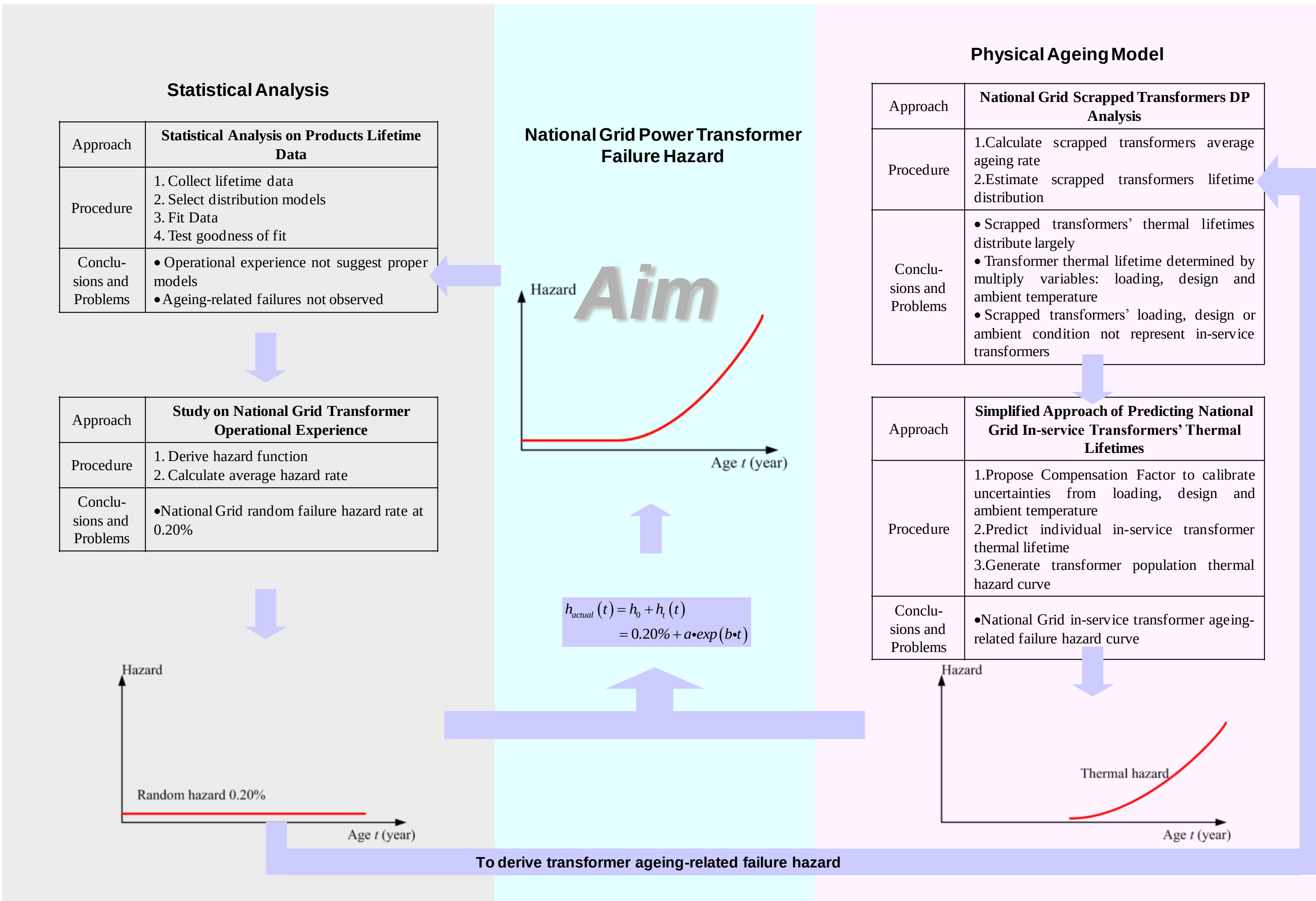


Figure 1-1 Project Overview

1.6 Outlines of This Thesis

Chapter 1 is a general introduction of the whole thesis.

Chapter 2 presents an intensive literature survey on statistics applied to product lifetime study. This chapter commences from a brief introduction of product failure and traditional failure hazard bathtub curve. Different types of lifetime data are classified; mathematical expressions of failure probabilities and commonly used distribution models are displayed. Steps of lifetime data statistical analysis and various statistical approaches are summarized from engineering applications. Advantages and disadvantages of these approaches are highlighted at the end of this chapter.

Chapter 3 implements the introduced statistical approaches by using National Grid transformer lifetime data. First of all National Grid transformer installation number, replacement history and the critical ageing status used at present are presented. Secondly as the prior step of statistical analysis, three types of transformers: active, failed and manually retired transformers are clearly classified and their service ages are derived accordingly. The least square estimator (LSE) approaches including Nelson's hazard plotting, CDF plotting method and Li's modified method are then carried to fit National Grid transformer lifetime data into commonly used normal, lognormal, Weibull [26] and smallest extreme value distribution. The maximum likelihood estimator (MLE) is also carried out for comparison purpose. The advantages and restrictions of lifetime data statistical analysis are thereafter concluded. Finally National Grid transformers random failure hazard is derived.

Chapter 4 summarizes the knowledge from transformer insulation paper laboratory ageing tests as the physical model to predict transformer ageing-related failure. The structure of insulation cellulose paper is firstly introduced; paper ageing process due to the effects from dispersed heat, water content, immersed oxygen and acids is described in terms of paper mechanical strength reduction. Transformer thermal ageing is specified to indicate transformer ageing-related failure and the thermal end-of-life criteria are thereafter discussed. Models to calculate transformer thermal lifetime are reviewed and the linear model of 1/DP against age is adopted in Chapter 5.

Chapter 5 evaluates National Grid scrapped transformers' ageing status by measuring their paper insulation lowest DP. The average ageing rate of scrapped transformers and their average thermal lifetime are calculated. The variation of scrapped transformer ageing rates and thermal lifetimes are further analyzed in respect of transformer loading condition, thermal design characteristics and their installation site ambient temperature. It is finally identified that transformer thermal lifetime is determined by multi-variables and the limited number of scrapped transformers should not be used to directly suggest the ageing status of National Grid in-service population.

Chapter 6 shows a simplified approach of predicting individual transformer thermal lifetime by using information from scrapped transformers. Transformer hot-spot compensation factor ($\mathbb{C}$) is firstly developed to replace the uncertain value of transformer winding hot-spot factor (*HSF*). Accordingly transformer annual equivalent load, national typical winter/summer daily demand profiles, IEC transformer thermal parameters and a unified yearly ambient temperature profile are designated as the inputs of transformer thermal lifetime calculation. National Grid 275/132kV in-service transformer thermal lifetimes are calculated and this group of transformers' cumulative failure probability curve and hazard curve are obtained thereafter.

Chapter 7 combines the statistically determined random failure hazard with the ageing model derived ageing-related failure hazard, to generate National Grid transformer population actual failure hazard curve against service age. The ideas of the knee point age t_{knee} and the corresponding critical hazard $h_{critical}$ are further proposed to clearly identify from the asset management point of view the onset of a dangerous period with too many failures which the system cannot afford.

Chapter 8 draws conclusions from this research and also presents further studies that may be carried out in the future.

Chapter 2

Literature Review on Product Lifetime Data Statistical Analysis

2.1 Introduction

A group of products are always designed and manufactured according to a typical specification. These products are hence expected to operate for a certain period of time which is called the product's designed end-of-life. A single number of designed end-of-life however does not tell a product's real operation experience as the operation condition varies among products.

Power transformer for instance, is designed to operate reliably for up to 40 years, however transformer may fail before age 40 as the real load may be heavier than expected or the short-circuit faults, lightning, over-voltage or other system transient events coming outside of the transformer may occur more frequently. Meanwhile transformer may well operate beyond 40 years if the load growth is slower or the system transient events are less frequent. Therefore transformer end-of-life should not be simply indicated by the designed 40 years; instead transformer lifetime is determined by its actual design, loading condition, experienced transient events and the maintenance scheme.

The bathtub curve [26] is a widely used end-of-life model to generally indicate the conditional probability of product failure against service age. The curve is always produced by traditional distribution models, for example normal distribution, Weibull distribution, gamma distribution and etc. Product lifetime data statistical analysis can be generally summarized as fitting the observed data into a presumed distribution model via optimisation approach; the population future trend of failure is therefore predicted. In various engineering practices, utilities or researchers have their own attitudes towards the classification of lifetime data, choices of distribution models, optimisation methods, and judgement on the fitting goodness.

Intensive reviews of statistic theories are presented in this chapter. Utilities' practices and applications especially in power electric systems are summarized and compared.

2.2 Overview of Product Failure

2.2.1 Transformer Failure and End-of-life Definition

Transformer failure and the definition of transformer end-of-life are specifically illustrated in this section to indicate the failure of a general product.

A transformer is designed to withstand the expected growing load and the system transient fault events. These transient fault events can be catalogued as short-circuit faults, lightning surge, switching surge and temporary over-voltages coming out of the transformers [6]. Initially a transformer does not fail as its insulation withstand strength is higher than the normal operating or the fault stress. As transformer ages, the insulation withstand strength gradually reduces due to its normal degradation and the cumulative effects from transient events; meanwhile transformer load increases with age. Once the insulation withstand strength cannot sustain the high operation stress, transformer fails. This process is illustrated in Figure 2-1.

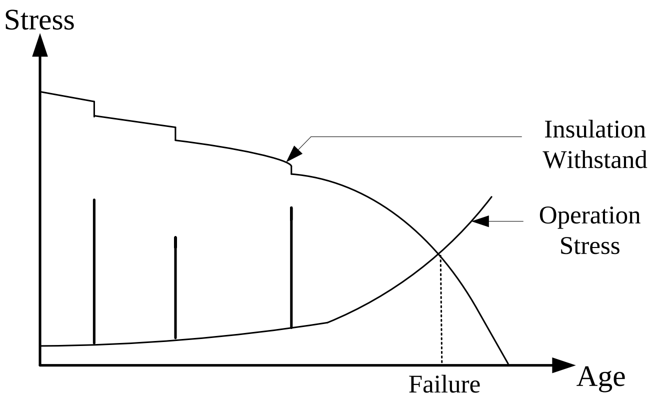


Figure 2-1 Transformer Failure Illustration [35]

In Figure 2-1, the impulses in the actual stress curve represent the sudden increased stresses from transient events. Such events occur randomly during transformer operation and they may result in the insulation withstand strength reduction. Each step change in the insulation withstand curve shows a slightly reduction of insulation strength. If the load increases, and/or a transient event occurs, then the insulation withstand strength reduces. The crossover point of insulation withstand curve and operation stress curve in Figure 2-1 indicates the expected operation lifetime of a transformer.

Transformer manufacturers and utilities have a common expectation that a power transformer should operate reliably for up to 40 years [6, 36, 37]. This number is derived via the laboratory accelerated ageing tests, and refers to the period when the transformer insulation is approaching an unacceptable condition under constant temperature and ideally dry condition [3, 37, 38]; the derivation is summarized by the IEEE and IEC loading guide for transformers under a constant winding temperature of 80 Celsius [38]. 40 years is thus considered as the transformer designed end-of-life.

If the load demand in Figure 2-1 is growing faster than expected, or the transient events occur more frequently, or the fault stress exceeds the insulation withstand strength, the transformer fails before age 40. A transformer pre-mature failure before the designed life, due to the significant effect from a transient event, is shown in Figure 2-2.

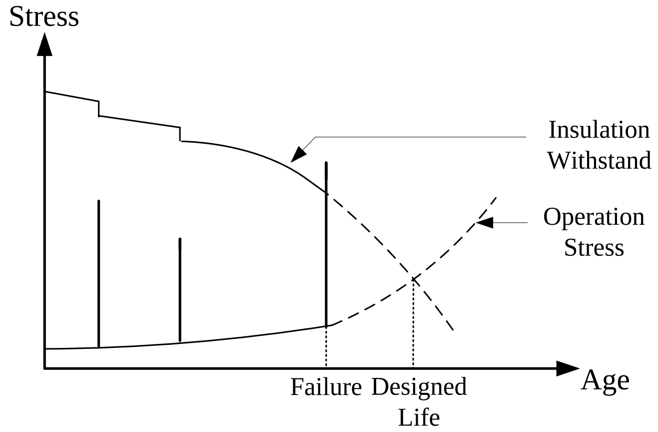


Figure 2-2 Transformer Failure before Designed End-of-Life [35]

However, operational experience shows that transformer insulation withstand strength may still sustain the actual operation and fault stress after commissioning for 40 years. This is because the load increased slower than expected, or transient events occurred less frequently [37]. The insulation strength reduces less than expected and the transformer will fail at an unknown age beyond 40. A transformer post-mature failure causing by less loaded condition is illustrated in Figure 2-3 in solid curves. The dash curves indicate the expected operation condition and the expected reduction of insulation strength.

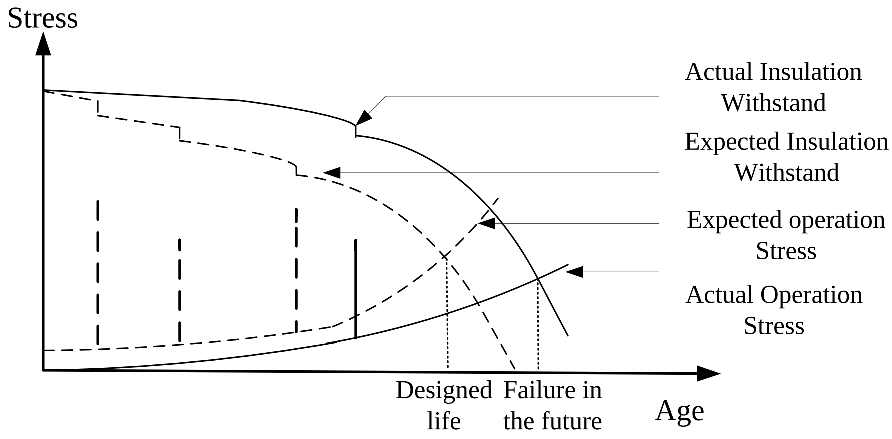


Figure 2-3 Transformer Failure beyond Designed End-of-Life

The reality of transformer pre-mature and post-mature failure reveals that the unique age of 40 years as transformer designed end-of-life does not indicate transformer failure. In fact transformer failure would depend on individual design, loading experience, maintenance scheme and even the condition of the installation site.

Hence in order to precisely indicate transformer lifetime, transformer failure needs to be well defined rather than being defined as the single age of 40 years. Transformer failure is indicated by CIGRÉ as ***any outage when the withstand strength is exceeded by operational stress that requires the asset to be out of service*** [3, 11, 14, 39, 40], or in other words, ***a transformer reaches its end-of-life when the unit does not meet the operation requirements anymore*** [41, 42].

Based on the above definition, transformer failure should be distinguished from retirement. The latter is the manually determined replacement based on transformer unacceptable condition and it terminates a transformer end of useful life rather than reveals the transformer's actual end-of-life. Failure and manual retirement are strictly separated in the context of this thesis.

2.2.2 Failure Hazard Bathtub Curve

Discarding the single age of designed end-of-life, product failure is always considered in the form of probability: the likelihood of failure within a certain time interval given that this group of products have survived at the end of the last interval [23, 26, 28, 40]. This is called conditional failure probability or hazard rate as a function of age. A bathtub-shape curve of hazard is universally applicable to describe the product conditional failure probability. Figure 2-4 shows a traditional bathtub curve.

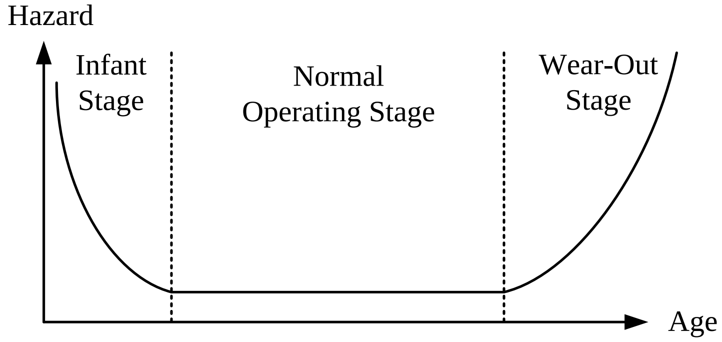


Figure 2-4 Traditional Bathtub Curve

The high infant mortality as the first stage of the bathtub curve is caused by the product defective workmanship, poor processing procedure or inherent material defects. Power transformers are subjected to vigorous manufacturing & commissioning procedure to weed out this stage, for example adequate specification and strict factory testing scheme prior to commissioning [4, 26, 43]. Besides, the shape of hazard at early ages varies significantly according to different transformer operation practice [11, 15-17].

The normal operating stage corresponds to a random failure period in which product failures randomly occurred. For power transformers particularly, the random failures are caused by system transient events independent of transformer age. The hazard rate during this period is low and constant. This normal operating stage is the most interesting period for asset managers.

As product ages, the failure hazard increases against age as indicated by the wear-out stage in the bathtub curve. For example transformers are more prone to fail due to their deteriorated insulation withstand strength. The wear-out stage indicates that the majority of transformers are approaching their end-of-life and a preventive replacement is necessary [26].

As the high infant mortality stage can not be seen among power transformers, the traditional 3-stage bathtub curve can be simplified into a 2-stage model including transformer normal operating stage and wear-out stage. This is shown in solid lines in Figure 2-5.

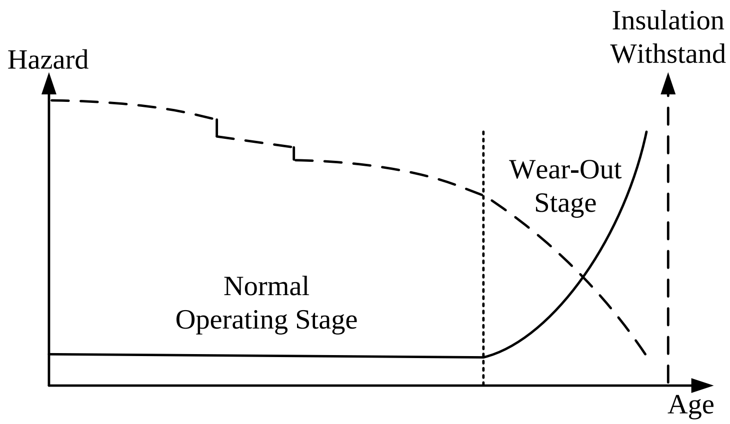


Figure 2-5 Transformer 2-Stage Hazard Model compared with Insulation Withstand Reduction

Especially for power transformers it is worth noticing that the increase of failure hazard from the normal operating stage to the wear-out stage is consistent with the reduction of insulation withstand strength shown in the dash line in Figure 2-5. In other words, transformer failure can be rare when the insulation strength is high; while failure is more likely to occur when the unit ages and the insulation degrades faster. Hence the normal operating stage infers the random failure mechanism underlying the constant low hazard rate and the wear-out stage implies the ageing failure mechanism with the significantly increasing hazard.

Except the failure mechanisms, “transformer failure” is a general concept in the above statement. Further concepts such as the component subject to failure (i.e. tap-changer, bushing, winding) and the characteristic of failure (i.e. repairable or non-repairable) are not classified; they will be further discussed in Chapter 3.

Again the bathtub curve is a generic product end-of-life model coincident with engineering prior knowledge. Statistical analyses are always carried out based on the observed product lifetime data to generate the bathtub curve.

2.3 Product Lifetime Data Mathematical Description

2.3.1 Types of Lifetime Data

Statistics only concerns the lifetime of individual product [14, 42] and lifetimes constitute the database for analysis. To undertake the statistical analyses, product lifetime data needs to be first of all “polished”. Generally if the failure time of each product is observed, the data are complete, otherwise they are incomplete [20, 26].

For incomplete data, when the observation period is pre-assigned, the data are considered time censored [20, 26]. Typically if the products' lifetimes are known to be beyond the pre-assigned time, these data are further clarified as censored on the right; otherwise if the units' lifetimes are known to be less than the pre-assigned time, they are censored on the left. If the observation is stopped when a specific number of failures are recorded, the data are failure censored [20, 26].

Additionally if all the censored products start to operate at the same time, they are "singly censored" [26]; if the operation starts differently and thus the incomplete observations are intermixed with the failure records, the data are called "multiply censored" [26]. Examples are given in Figure 2-6 to depict the singly censored data and multiply censored data respectively among two groups of right-time censored products.

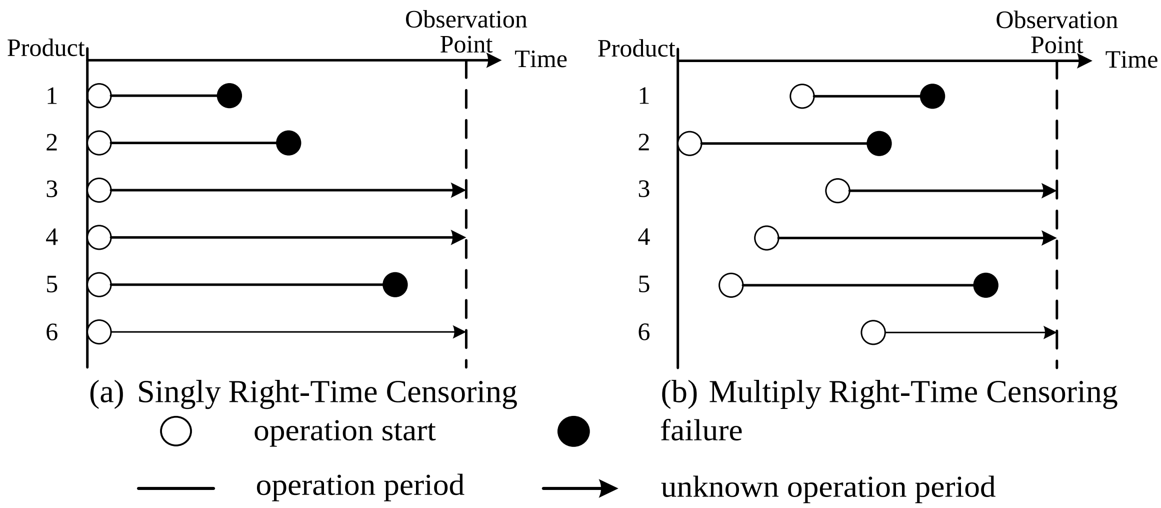


Figure 2-6 Singly Right-Time Censored Data and Multiply Right-Time Censored Data

The classification of failure data is summarized in Figure 2-7.

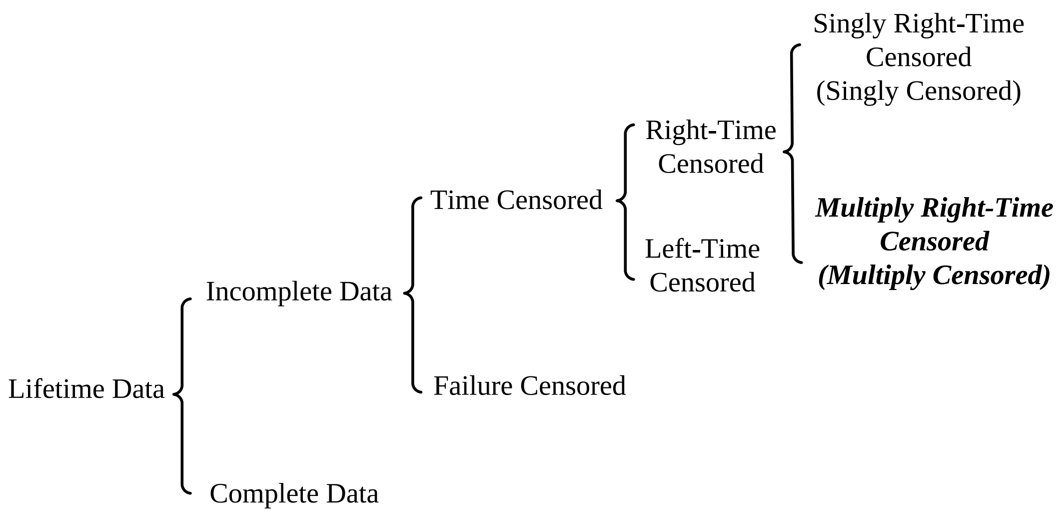


Figure 2-7 Lifetime Data Classification

In practice, when a utility operates multiple transformers: - some units are in operation and their lifetimes are only known to be beyond a certain age; new transformers are installed each year according to utility's lay out plan; and some transformers are determined to retire due to their bad condition. These are three main reasons to result in the transformer ***multiply right-time censored*** lifetime data. The ***multiply right-time censored*** type (in brief ***multiply censored***) is thus highlighted in Figure 2-7.

The type of product lifetime data determines the proper method for analyses. Moreover, either the survived or the removed products are censored units. The age of a failed product is derived as the year of its failure minus the year of commission and is equivalent to the product's lifetime. Lifetime of a censored product is not known because failure has not been observed. The age of a censored product is derived by the year of censoring minus the year of commission.

2.3.2 Probability Expressions of Product Lifetime Data

After classification, the data are formulated into probability form and proper distributions are chosen to describe the data. The following contents hereby present the commonly used probability functions of describing product lifetime data. Their applications specifically to complete data or incomplete data however are not classified.

- **Failure Probability Density Function**

The failure probability density function (PDF) $f(t)$ defines the product probability of failure within a short per unit time (i.e. within (t-Δt, t), and $\Delta t \to 0$) [44]. It can be calculated as the number of failed products within age t ($n_F(t)$) divided by the total number of product population (N) as

$$f(t) = \frac{Number\ of\ \ failed\ \ products\ within\ int\,erval\,(t - \Delta t, t)}{Total\ number\ of\ \ product\ \ population} = \frac{n_F(t)}{N} \qquad (2\text{-}1)$$

- **Cumulative Distribution Function**

The cumulative distribution function (CDF) or the cumulative failure probability function $F(t)$ indicates the proportion of cumulative number of failures until age t ($N_F(t)$) to the total number of products (N) [44]. $F(t)$ is mathematically expressed as

$$F(t) = \frac{Total\ number\ of\ \ failed\ \ products\ up\ to\ age\ t}{Total\ number\ of\ \ product\ \ population} = \frac{N_F(t)}{N} \qquad (2\text{-}2)$$

Besides $F(t)$ can be also derived as the integration of the instantaneous failure probability density $f(t)$ up to age t according to the definition, as

$$F(t) = \sum_{t_i=1}^{t} f(t_i) \tag{2-3}$$

where $f(t_i)$ indicates the PDF at age t_i and t_i should not be longer than t [44].

Moreover an alternative equation is always used to replace (2-2) or (2-3) by estimating CDF at each failure i, called median rank function $F(i)$ [45-47]. $F(i)$ is expressed as

$$F(i) = \frac{i}{N} \tag{2-4}$$

where the numerator i indicates the i^{th} failure by sorting failed products' lifetimes from smallest to largest and $i \le N$.

According to $F(i)$, each of individual failures is more precisely considered than $F(t)$ which summarises the number of failures at the same age. The idea of $F(i)$ is widely adopted in engineering practice and various calculation of $F(i)$ are well developed. Cacciari [25] and Jacquelin [45] summarized the commonly used unreliability rank functions respectively. Table 2-1 shows these functions.

Table 2-1 Estimation of Unreliability Rank Function

Source	Equation
IEC Standard 56 [25]	$F(t) = \frac{i-0.5}{N+0.25}$
Herd-Johnson [45]	$F(t) = \frac{i}{N+1}$
Hazen [25]	$F(t) = \frac{i-0.5}{N}$
Bernard [26]	$F(t) = \frac{i-0.3}{N+0.4}$
Filliben [48]	$F(t) = \frac{i-0.3175}{N+0.365}$

- **Reliability Function**

The reliability function $R(t)$ denotes the probability of products surviving up to age t. It is expressed as the number of survived units at the end of age t ($N_S(t)$) divided by the total number of products (N), shown as

$$R(t) = \frac{Number\ of\ survived\ products}{Total\ number\ of\ product\ population} = \frac{N_S(t)}{N} \tag{2-5}$$

Obviously at age t, ***Number of survived products*** + ***Number of failed products*** = ***Total number of product population***: $N_S(t) + N_F(t) = N$. The reliability function at age t can be further derived as

$$\begin{aligned} R(t) &= \frac{Total\ number\ of\ product\ population - Total\ number\ of\ failed\ products\ up\ to\ age\ t}{Total\ number\ of\ product\ population} \\ &= 1 - \frac{Total\ number\ of\ failed\ transformers\ up\ to\ age\ t}{Total\ number\ of\ transformer\ population} \\ &= 1 - F(t) \end{aligned} \tag{2-6}$$

As typically used in engineering practice, an actuarial method to calculate $R(t)$ via recursion was described by Chiang [26]. This method is suitable when the database is large and particularly for censored data analyses.

Similarly to the rank of unreliability $F(i)$, Kaplan and Meier [49] provided the product-limit estimation of reliability at each failure i as $R(i)$. Herd [26] subsequently suggested a similar method and Johnson developed a more complex equation [26]. These two methods concern the reliability at each failure moment and consequently are suitable for use when the sample size is relatively small.

The Kaplan-Meier method, Herd-Johnson method and the actuarial method of product reliability estimation will be further discussed in Chapter 3.

- **Hazard Function**

As mentioned in 2.2.2, a bathtub curve generally indicates the product conditional failure probability or hazard rate against age. The hazard function $h(t)$ describes the instantaneous failure probability at a specific age t.

By referring to the engineering practice such as [40, 42], if a certain amount of products have survived till age t-1, $h(t)$ can be estimated as

$$h(t) = \frac{number\ of\ failed\ products\ within\ interval\ (t-\Delta t, t)}{number\ of\ survived\ products\ at\ the\ end\ of\ t-1} = \frac{n_F(t)}{N_S(t-1)} = \frac{n_F(t)}{N_E(t)} \tag{2-7}$$

In transformer lifetime analyses, as implemented in this thesis, the ***number of survived transformers at the end of t*-1** is specifically named ***transformer exposed number at age t***, denoted by $N_E(t)$. $N_E(t)$ again implies the number of transformers aged t-1 years at the beginning of t.

Hazard $h(t)$ can be further related to PDF, CDF or reliability function. The expression is shown as

$$h(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1 - F(t)} = -\frac{d \ln R(t)}{dt} \tag{2-8}$$

In order to precisely concern the hazard rate at each failure, Nelson [26] introduced the observed instantaneous hazard $h(i)$ for the i^{th} failure, i=1, 2, 3, …, and $i \le N$ [26, 49]. According to Nelson $h(i)$ is calculated as

$$h(i) = \frac{1}{reverse\ rank\ \ for\ i^{\text{th}}\ \ failure} \tag{2-9}$$

where N products' lifetimes are ordered from low to high and the reverse rank is assigned to each product, as N to the first lifetime, N-1 to the second, …, and 1 to the N^{th} product. Hazard is only calculated at each failure according to (2-9).

By comparing the hazard calculation in (2-7) and (2-9), the former equation is a function of age t and the number of failed products at a particular age t is summarised, the latter is a function of failure rank i, in which the hazard at each failure is individually considered. Supposing 3 failures at age t with rank of i_1, i_2 and i_3 respectively, the failure hazards are calculated as $h(i_1)$, $h(i_2)$ and $h(i_3)$ according to (2-9). It can mathematically prove that when failure reverse ranks are high or failures are ranked at early ages, the sum of hazards at the same age t, as $h(i_1)+h(i_2)+h(i_3)$ gives approximately similar value of hazard $h(t)$ for that specific age according to (2-7).

Moreover, hazard is always calculated according to (2-7) for lifetime data general analysis while (2-9) is well utilized accompanying with the graphic plotting approaches developed by Nelson. Both of these equations will be further discussed when analyzing UK National Grid power transformer lifetime data in Chapter 3.

Other estimations of hazard rate are well developed especially in vital statistics. They are also widely used in product lifetime studies [20]. The average crude failure rate ($ACFR$) is calculated as

$$ACFR(t) = \frac{N_F(t)}{\sum_{i=1}^{t} n_i \bullet i} \tag{2-10}$$

where $N_F(t)$ is the total number of failures until age t, n_i represents the number of products aged for i years and $\sum_{i=1}^{t} n_i \cdot i$ thus indicates the sum of length of products operation periods, or the amount of product years until age t [20]. The word ***crude*** is to represent the calculation over the ***whole*** population rather than any ***specific*** subset [20].

(2-10) may yield different hazard values to those from (2-7) because product time-of-service is included in the denominator in (2-10), by which higher weightings are assigned to the early installed products.

A study on cable failure analysis via (2-10) was presented in EPRI EL-3501 [50], and it was concluded that this time-weighted *ACFR* can factor out the influence from different amount of installations from year to year and consequently the product failure hazard obtained from (2-10) is less fluctuant than that from (2-7). [50] also emphasized that (2-10) is suitable for hazard comparisons by utilities whose asset installation significantly increases or decreases in subsequent years, for example a utility may install its distribution transformers intensively during some continuous years while may have less installations within some other periods. The concept of *ACFR* was adopted in IEEE transformer reliability survey as early as 1970's [51], and it is also illustrated in CIGRÉ A2 committee report of a transformer reliability survey among Europe [39].

Another measurement of product hazard rate, presented in GE transformer reliability study in 1989 [52], is defined as

$$h(t) = \frac{Number\ of\ \ product\ \ failure\ up\ to\ age\ t}{Sum\ of\ exposed\ \ number\ of\ \ each\ age\ up\ to\ age\ t} = \frac{N_F(t)}{\sum_{i=1}^{t} N_E(t)} \qquad (2\text{-}11)$$

where $\sum_{i=1}^{t} N_E(t)$ summarizes the exposed numbers of products from age 1 to age t.

Compared to (2-7), (2-11) evaluates the hazard rate at each age in an average manner by taking into account the exposed products from previous ages. Although not many mathematical illustrations can be found for (2-11), this hazard calculation was also applied by electric engineers from US Carolina Power and Light System in 1970's [52].

Due to the various calculation of product failure hazard, any obtained hazard rate should be explained consistently with its own expression.

- **Cumulative Hazard Function**

Similarly to the definition of CDF, the cumulative hazard function $H(t)$ sums up all the outcomes of hazard $h(t)$ at each age ever before, which is expressed as

$$H(t)=\sum_{t_i=1}^{t} h(t_i) \tag{2-12}$$

where $h(t_i)$ indicates the hazard at age t_i and t_i should be no more than t.

By integrating (2-8) into (2-12), $H(t)$ can be further expressed as

$$H(t)=-ln\,R(t)=-ln\left[1-F(t)\right] \tag{2-13}$$

where $R(t)$ is the reliability function and $F(t)$ is the cumulative distribution function. Equation (2-13) yields the relationship between $H(t)$ and $F(t)$. In practical application, it is sometimes more convenient to work with $H(t)$ than with $F(t)$, for example in hazard plotting curve fitting approach which will be discussed in Chapter 3.

If product failure hazard is obtained as $h(i)$ via (2-9), the cumulative hazard function till N_F failures is thus determined as

$$H(N_F)=\sum_{i=1}^{N_F} h(i) \tag{2-14}$$

in which $h(i)$ is the observed instantaneous hazard for the i^{th} failed product and N_F indicates the total number of failures when analyzing.

The expressions of Failure Probability Density Function, Cumulative Distribution Function, Reliability Function, Hazard function and Cumulative Hazard Function are summarised in Table 2-2.

The Failure Probability Density Function, Cumulative Distribution Function, Reliability Function, Hazard function and Cumulative Hazard Function are equivalent expressions of a set of product lifetime data. The discrete forms as (2-1), (2-2), (2-5) and (2-7) indicate the observed instantaneous probability at each age and they are more convenient than the continuous forms in analysis [21]. Probability functions concerning individual failures as (2-4), (2-9), (2-14) and equations shown in Table 2-1 are more frequently applied when graphic plotting approaches are implemented.

Table 2-2 Summary of Probability Functions

	Failure Probability Density Function	Failure Cumulative Distribution Function	Reliability Function	Hazard Function	Cumulative Hazard Function
Basic expression	$f(t)=\frac{n_F(t)}{N}$	$F(t)=\frac{N_F(t)}{N}$	$R(t)=\frac{N_S(t)}{N}$	$h(t)=\frac{n_F(t)}{n_E(t)}$	$H(t)=\sum_{t_i=1}^{t} h(t_i)$
Practical application		$F(i)=\frac{i-0.5}{N}$ $F(i)=\frac{i-0.3}{N+0.4}$ and etc	By Actuarial method Kaplan-Meier method Herd-Johnson method	$h(i)=\frac{1}{reverse\ rank\ \ for\ i^{th}\ \ failure}$ $ACR(t)=\frac{N_F(t)}{N\bullet t}$ $h(t)=\frac{N_F(t)}{\sum_{i=1}^{t} N_E(t)}$	$H(N_F)=\sum_{i=1}^{N_F} h(i)$
Relation with other probability forms		$F(t)=\sum_{t_i=1}^{t} f(t_i)$	$R(t)=1-F(t)$	$h(t)=\frac{f(t)}{R(t)}$ $h(t)=\frac{f(t)}{1-F(t)}$ $h(t)=-\frac{d\,ln\,R(t)}{dt}$	$H(t)=-ln\,R(t)$ $H(t)=-ln[1-F(t)]$

$n_F(t)$: number of failed products at age t
$N_F(t)$: number of failure products up to age t
$N_S(t)$: number of survived products up to age t
$n_E(t)$: number of exposed products at age t
N: total number of products
i: the i^{th} failed product

- **Mean Life and Standard Deviation**

Particularly for a set of complete data, the population end-of-life can be simply modelled as mean life and the accompanying standard deviation.

The mean life of a group of failed products ($\bar{t}$) is calculated as the arithmetic average of lifetimes [22]. It is thus also called average life or expected life [26], derived as

$$\bar{t} = \frac{t_1 + t_2 + t_3 + ... + t_n}{N} \tag{2-15}$$

where N is the number of failed products, t_1, t_2, t_3 and t_n indicate the lifetime of unit 1, 2, 3 and n respectively.

The lifetime variance (*var*) is a measurement of data dispersion [22]. It is calculated as

$$var = \sum_{i=1}^{N} \frac{\left(t_i - \bar{t}\right)^2}{N} \tag{2-16}$$

in which $\bar{t}$ is the mean life of population derived by (2-15) and N is the number of products. The positive square root of variance is called standard deviation of product lifetime data (σ), as

$$\sigma = \sqrt{var} \tag{2-17}$$

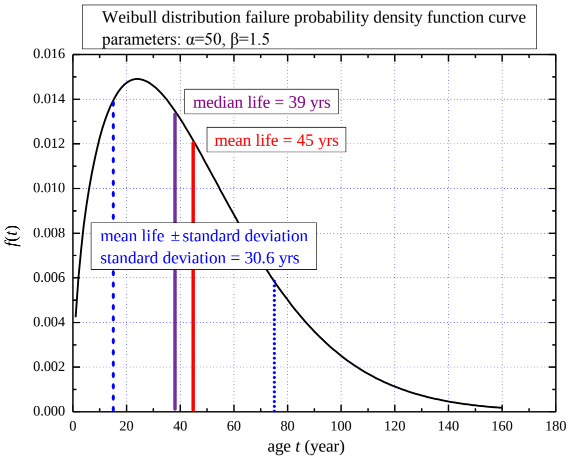
Typically standard deviation σ is more commonly to use than variance *var* in describing the data dispersion to the mean life $\bar{t}$, since σ has the same dimension of lifetime (i.e. in transformer lifetime study the unit of σ is year).

- **Population Median Life**

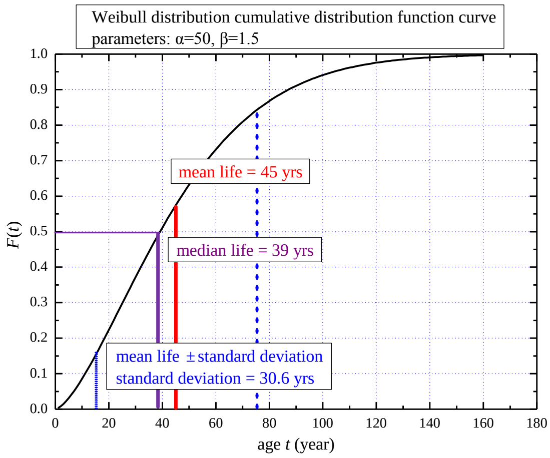
Median life t_{median}, also called population typical life or anticipated life, is the age up to which 50% of population has failed (CDF=50%) [26]. t_{median} thus can be obtained by reading the age corresponding to 50% failure from CDF curve.

According to the definition, products median life is not affected by the odd ages much longer than those of other products as the arithmetic average can be [53]; the value is only derived by considering the first 50% failed products within the population. Median life is often used by engineers in products lifetime analysis.

A comparison of population mean life ($\bar{t}$), median life (t_{median}) and the standard deviation (σ) is shown in Figure 2-8. The values of population mean life ($\bar{t}$), median life (t_{median}) and the range of one standard deviation in the centre of mean life ($\bar{t} \pm \sigma$) are presented in a Weibull distribution failure probability density function curve $f(t)$ and the Weibull cumulative distribution function curve $F(t)$.



(a) In Weibull Distribution Failure Probability Function Curve



(b) In Weibull Distribution Cumulative Distribution Function Curve

Figure 2-8 Comparison of Mean Life, Median Life and Standard Deviation of Weibull Distribution

For this specific group of lifetime data that follow a Weibull distribution, the population mean life ($\bar{t}$) is derived as 45 yrs and the standard deviation (σ) is obtained as 30.6 yrs. Population median life (t_{median}) is calculated as 45 yrs, which corresponds to the 50% in the cumulative distribution function curve $F(t)$. It is shown in Figure 2-8 that the values of population mean life ($\bar{t}$) and median life (t_{median}) are not equivalent, since they are different descriptions of data “average”.

The commonly used distribution models (for example, Weibull distribution) are presented in 2.3.3.

2.3.3 Commonly Used Distribution Models for Lifetime Data Analysis

In order to predict the future trend of failure, the historical failures need to be compared with specific distribution models. Those popularized distribution models are introduced in this section.

Exponential distribution, normal distribution, lognormal distribution, Weibull distribution and smallest extreme value distribution are the most commonly used continuous distribution models for product lifetime data analysis [19, 21]. The expressions of *f*(*t*), *F*(*t*), *h*(*t*), *H*(*t*), $\bar{t}$ and σ of each distribution model are presented; the characteristics of each model are also discussed.

- **Exponential Distribution**

The failure probability density function (PDF) of the exponential distribution is expressed as

$$f(t) = \frac{1}{\theta} exp\left(-\frac{t}{\theta}\right) \tag{2-18}$$

and the cumulative distribution function (CDF) is

$$F(t) = 1 - exp\left(-\frac{t}{\theta}\right) \tag{2-19}$$

where age *t* is the variable and θ is the parameter. θ indicates the population mean life $\bar{t}$ and also the standard derivation σ. The reciprocal of θ is the failure hazard.

$$h(t) = \frac{1}{\theta} \tag{2-20}$$

The simple expressions of exponential distribution result in its wide application. As the hazard remains constant as 1/θ, the exponential model infers that within a short period the failure probability of any unfailed products is the same as their failure probability within another short period [44]. The exponential model thus is used to describe the product population with failed units replaced and returned to service just in-time [46]. The model is also used to indicate the products subject to "chance failure" or "random failure" [23, 26], which corresponds to the normal operating stage of the bathtub curve.

(2-19) can be rewritten as

$$t = -\theta \bullet ln\left[1 - F(t)\right] \tag{2-21}$$

which shows a linear relationship between age *t* and *ln*[1-*F*(*t*)] with the slope of -θ. Considering the relationship between *F*(*t*) and *H*(*t*) as shown in (2-13), a linear equation between *t* and *H*(*t*) can be also derived as

$$t = \theta \bullet H(t) \tag{2-22}$$

Hence, by simply plotting *t* against *ln*[1-*F*(*t*)] or *t* against *H*(*t*), a straight line identifies that the data follow the exponential distribution and the slope indicates the value of θ. (2-21) and (2-22) thus suggest a simple graphic method for fitting the lifetime data into the exponential model. This method will be discussed in Section 2.4.2.3 and Chapter 3.

- **Normal Distribution**

Normal distribution or so-called Gaussian distribution has wide applications in describing product lifetime data. Normal distribution PDF is expressed as

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right] \tag{2-23}$$

in which μ is the mean and σ is the standard deviation. Both of the parameters are in the same unit as variable *t*. When μ=0 and σ=1 the distribution is specified as the standard normal distribution. Particularly in lifetime data study μ should be at least 2 or 3 times as great as σ, based on the fact that 97.5% of cumulative probabilities are within the range of μ±2σ or 99.86% within μ±3σ [26].

The normal distribution CDF is expressed as

$$F(t) = \int_0^t \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right] dy = \Phi\left(\frac{t-\mu}{\sigma}\right) = \Phi(Z) \tag{2-24}$$

where Φ( ) is the cumulative distribution function for the standard normal distribution (with μ=0 and σ=1). $Z = \frac{t-\mu}{\sigma}$ is defined as the standard normal distribution transfer factor, which transfers a general normal distribution to the standard normal distribution. Normal distribution hazard function can be derived based on (2-23) and (2-24) as

$$h(t) = \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right] \Bigg/ \left[1-\Phi\left(\frac{t-\mu}{\sigma}\right)\right] \tag{2-25}$$

The normal hazard is revealed to be a monotone increasing function against age; it thus corresponds to the wear-out stage in the bathtub curve. The reason that the normal distribution is popularized in practice is that many naturally-occurring variables distribute normally [22].

Similarly to the exponential distribution, normal distribution CDF in (2-24) can be rewritten as

$$t = \sigma \bullet \Phi^{-1}\left[F(t)\right] + \mu \tag{2-26}$$

and then (2-26) can further derive

$$t = \sigma \bullet \Phi^{-1}\left\{1 - exp\left[-H(t)\right]\right\} + \mu \tag{2-27}$$

where Φ^{-1}( ) is the inverse of the standard normal distribution CDF. According to (2-26) and (2-27) age *t* is linear with $\Phi^{-1}[F(t)]$ and $\Phi^{-1}\{1\text{-}exp[-H(t)]\}$ respectively.

- **Lognormal Distribution**

The lognormal distribution is an extension of the normal distribution, as the logarithm of lognormal distribution variable distributes normally. The PDF of the lognormal distribution is expressed as

$$f(t)=\frac{1}{\sqrt{2\pi}t\delta}exp\left\{\frac{-\left[ln(t)-\lambda\right]^2}{2\delta^2}\right\} \tag{2-28}$$

in which λ and δ are parameters and *t* is variable, *ln*( ) is the symbol of natural logarithm based on *e*. By comparing (2-28) with (2-23), λ is the mean value of *ln*(*t*) and is called the "log mean" of the lognormal distribution; δ is the standard deviation of *ln*(*t*) and is called the "log standard deviation" of the lognormal distribution. Therefore, according to the definition, if the variable *t* follows a lognormal distribution with parameters λ and δ, *ln*(*t*) follows a normal distribution with mean of λ (μ=λ) and standard derivation of δ (σ=δ) [26]. In other words, the plot of a lognormal distribution in a log scale shows the shape of a normal distribution.

The common logarithm based on 10, log_{10}( ), can also be used, according to which the PDF of the lognormal distribution is written as

$$f(t)=\frac{0.4343}{\sqrt{2\pi}t\delta'}exp\left\{\frac{-\left[log_{10}(t)-\lambda'\right]}{2\delta'^2}\right\} \tag{2-29}$$

where 0.4343=1/*ln*(10), parameter λ' is the mean of $log_{10}(t)$ and δ' is the standard deviation of $log_{10}(t)$. Because the natural logarithm *ln*(*t*) and the common logarithm $log_{10}(t)$ can be easily converted to each other by applying the relation of $log_{10}(t)=\frac{ln(t)}{ln(10)}$ and also the mean life and standard derivation of a certain group of lifetime data are deterministic values, there is not much difference between the natural or common logarithm being used throughout calculation [19]. The lognormal distribution based on the natural logarithm is to be used for the National Grid transformer lifetime analysis in this thesis.

The lognormal CDF is expressed as

$$F(t)=\Phi\left\{\frac{\left[ln(t)-\lambda\right]}{\delta}\right\} \tag{2-30}$$

where again Φ( ) shows the standard normal distribution CDF.

The lognormal hazard function is derived as

$$h(t)=\frac{1}{\sqrt{2\pi t}\delta}exp\left\{\frac{-[ln(t)-\lambda]^2}{2\delta^2}\right\}\Bigg/\left\{1-\Phi\left[\frac{ln(t)-\lambda}{\delta}\right]\right\} \tag{2-31}$$

Unlike the normal distribution, (2-31) is not monotone increasing against age. The shape of lognormal hazard varies depending on the value of δ: when δ is less than 0.2, *h*(*t*) increases against age *t*; when δ is around 0.5, *h*(*t*) is roughly constant over age; when δ is larger than 0.8, *h*(*t*) decreases over the most age. Therefore the lognormal distribution can describe different stages of the bathtub curve corresponding to increasing, constant or decreasing hazard rate.

Based on the relation between the normal distribution and lognormal distribution, the variable *t* in (2-26) and (2-27) can be substituted by *ln*(*t*) for the lognormal distribution as

$$ln(t)=\delta\bullet\Phi^{-1}\left[F(t)\right]+\lambda \tag{2-32}$$

$$ln(t)=\delta\bullet\Phi^{-1}\left\{1-exp\left[-H(t)\right]\right\}+\lambda \tag{2-33}$$

according to which *ln*(*t*) is linear with $\Phi^{-1}[F(t)]$ and $\Phi^{-1}\{1\text{-}exp[\text{-}H(t)]\}$ respectively.

Product mean life and standard derivation under the lognormal distribution are calculated respectively as [54]

$$\bar{t}=exp\left(\lambda+\frac{\delta^2}{2}\right) \tag{2-34}$$

$$\sigma=\sqrt{exp\left(2\delta^2+2\lambda\right)-exp\left(\delta^2+2\lambda\right)} \tag{2-35}$$

- **Weibull Distribution**

The Weibull distribution was firstly popularized in metallurgical failure analysis [46] and then applied among a variety of product life studies. The Weibull distribution PDF is expressed as

$$f(t)=\frac{1}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right] \tag{2-36}$$

in which α is the scale parameter and β is the shape parameter. α determines the spread of data; it also indicates the age corresponding to 63.2% CDF and thus is called the products characteristic life. β implies the shape of the distribution and it is dimensionless. Particularly when 3≤β≤4, the shape of the Weibull distribution is similar to that of a normal model.

The CDF is derived as

$$F(t) = 1 - exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right] \tag{2-37}$$

By applying twice natural logarithm in both sides of (2-37), this equation is rewritten as

$$ln(t) = \frac{1}{\beta} \bullet ln\left\{-ln\left[1 - F(t)\right]\right\} + ln\,\alpha \tag{2-38}$$

and furthermore

$$ln(t) = \frac{1}{\beta} \bullet ln\left[H(t)\right] + ln\,\alpha \tag{2-39}$$

Again the linear relationships shown in (2-38) and (2-39) are always applied when fitting the lifetime data into the Weibull distribution.

The Weibull hazard function is obtained as

$$h(t) = \left(\frac{\beta}{\alpha}\right)\left(\frac{t}{\alpha}\right)^{\beta-1} \tag{2-40}$$

When β=1, the Weibull distribution is equivalent to an exponential model; when β>1, $h(t)$ increases against age; when β<1, $h(t)$ decreases. The variety shapes of hazard function by changing the value of β makes the Weibull model versatile to describe the lifetime data. Population mean life and the standard deviation are expressed as

$$\bar{t} = \alpha \bullet \Gamma\left[1 + \left(\frac{1}{\beta}\right)\right] \tag{2-41}$$

$$\sigma = \alpha\sqrt{\left\{\Gamma\left[1 + \left(\frac{2}{\beta}\right)\right] - \Gamma^{2}\left[1 + \left(\frac{1}{\beta}\right)\right]\right\}} \tag{2-42}$$

where again $\Gamma(\ )$ is the gamma function.

In engineering practice the Weibull model is applicable to describe the distribution of minimum value of a collection of independent observations from a normally distributed population [21]. For example, a transformer consists of various components and the transformer's lifetime is determined by the first failure among these components if the failure is non-repairable. Suppose components are independent and each of their lives distribute normally, the Weibull distribution is thus robust to describe the transformer lifetime consisting of those independent components [21].

Occasionally in practice, a third parameter ρ, named location parameter needs to be considered from (2-36) to (2-42). When the first failure is observed after age zero, the variable t should be substituted by t-ρ (typically ρ>0 for lifetime data analysis). ρ hence estimates the earliest failure age within the product population and the Weibull distribution is right-shifted if this location parameter is taken into account [47, 55]. It is concluded by quality/process engineers that when β is greater than 6, the location parameter ρ should always be considered in product lifetime analysis [47]. Particularly when analyzing transformer lifetime data, if the operational experience could justify that the onset of failure would not be less that a specific age, the location parameter ρ needs to be considered.

- **Smallest Extreme Value Distribution**

The smallest extreme value distribution is an extension of the Weibull distribution; it is used to describe certain extreme phenomena such as the temperature minima, material strength, as well as the first-failed-component determined product failure [26]. In the context of this thesis the name is in briefly quoted as "extreme value distribution".

The PDF of the extreme value distribution is expressed as

$$f(t) = \frac{1}{\delta} \bullet exp\left(\frac{t-\lambda}{\delta}\right) \bullet exp\left[-exp\left(\frac{t-\lambda}{\delta}\right)\right] \qquad (2\text{-}43)$$

and CDF is calculated as

$$F(t) = 1 - exp\left[-exp\left(\frac{t-\lambda}{\delta}\right)\right] \qquad (2\text{-}44)$$

where δ is the scale parameter and λ is the location parameter and both of them have the same unit as age t. λ is the age corresponding to 63.2% of CDF and it is called the population characteristic life [26]. When applied in product lifetime data analysis, λ is always suggested at least 4 times as great as δ [26].

According to (2-44), the relationship between age t and $F(t)$ can be derived as

$$t = \delta \bullet ln\left\{-ln\left[1 - F(t)\right]\right\} + \lambda \qquad (2\text{-}45)$$

and furthermore

$$t = \delta \bullet ln\left[H(t)\right] + \lambda \qquad (2\text{-}46)$$

which are used in the graphic plotting approaches of fitting lifetime data into the extreme value distribution.

The hazard function of the extreme value distribution is

$$h(t) = \frac{1}{\delta} \bullet exp\left(\frac{t-\lambda}{\delta}\right) \tag{2-47}$$

From the exponentially increasing value against age *t*, the extreme value distribution hazard function represents the wear-out period in the bathtub curve. The model therefore is similar to the normal distribution that reveals the units at older ages are prone to fail due to the ageing failure mechanism.

The hazard function *h*(*t*) and hazard curves of the exponential, normal, lognormal, 2-parameter Weibull distribution and extreme value distribution are listed in Table 2-3. Parameters of each distribution model are indicated as well.

Table 2-3 Hazard Function and Hazard Curve of Commonly Used Distribution Models

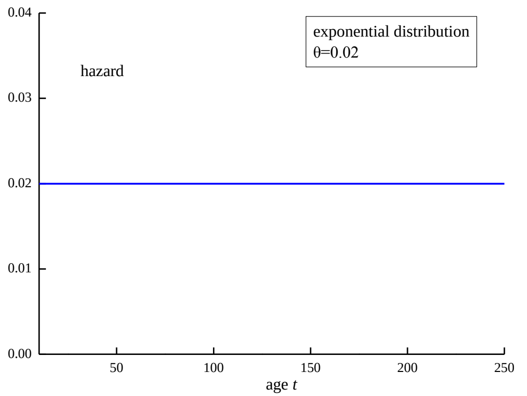
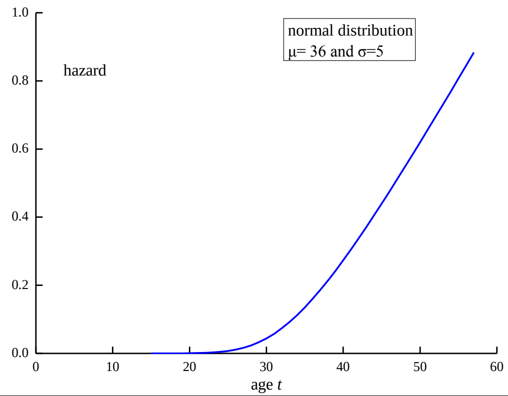
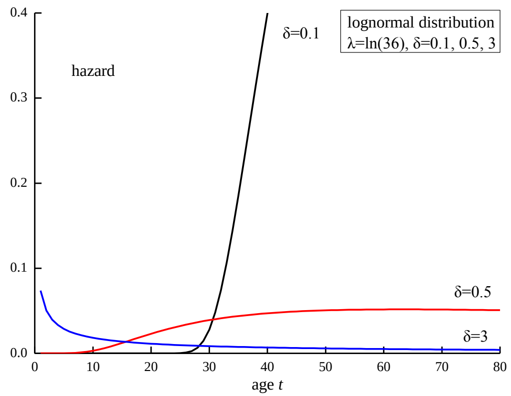
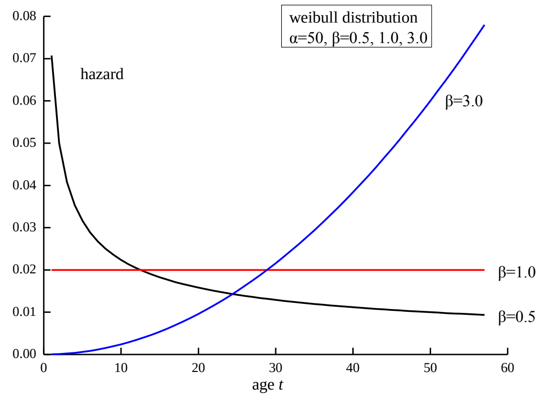
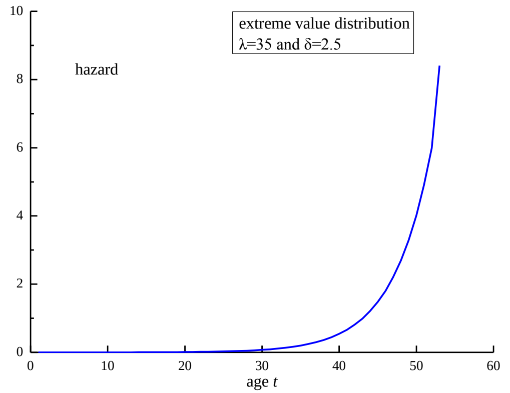
<table>
<tr><th>Distribution</th><th>exponential</th><th>normal</th><th>lognormal</th></tr>
<tr><td>Parameter</td><td>θ: mean and standard derivation</td><td>μ: mean
σ: standard deviation</td><td>λ: log mean
δ: log standard deviation</td></tr>
<tr><td>Hazard function</td><td>$h(t)=\frac{1}{\theta}$</td><td>$h(t)=\frac{\frac{1}{\sqrt{2\pi}\sigma}exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right]}{1-\Phi\left[\frac{t-\mu}{\sigma}\right]}$</td><td>$h(t)=\frac{\frac{1}{\sqrt{2\pi}t\delta}exp\left\{\frac{-[ln(t)-\lambda]^2}{2\delta^2}\right\}}{1-\Phi\left\{\frac{[ln(t)-\lambda]}{\delta}\right\}}$</td></tr>
<tr><td>Hazard curve</td><td>
</td><td>
</td><td>
</td></tr>
</table>

Table 2-3 Hazard Function and Hazard Curve of Commonly Used Distribution Models (Continued)

Distribution	Weibull	extreme value
Parameter	α: scale parameter β: shape parameter	λ: location parameter δ: scale parameter
Hazard function	$h(t)=\left(\frac{\beta}{\alpha}\right)\left(\frac{t}{\alpha}\right)^{\beta-1}$	$h(t)=\frac{1}{\delta}\bullet exp\left(\frac{t-\lambda}{\delta}\right)$
Hazard curve		

2.4 Literature Summary on Product Lifetime Data Statistical Analysis

The science of statistics has been dramatically developed since 1940's due to the growing requirement of modern technologies [4, 50, 51, 56]. Statistics application in areas of human life, medical research and military machine maintenance were much concerned in the early periods. Since 1950's and 1960's biomedical studies suggested advanced analysis methods; these methods were further developed by engineers in products' design and manufacturing [26]. Particularly in power systems, the lifetimes of generator windings, engine fans, turbine wheels, cables, transformers and insulation material strength are more and more concerned by statistics [26, 57].

The common procedure according to which the product lifetimes are analyzed statistically can be concluded in four steps as follows:

Step 1: collecting data;

Step 2: selecting the proper distribution model(s);

Step 3: fitting the data into the distribution model(s) and determining the best-fitted parameter(s) by optimal approach;

Step 4: carrying out the goodness-of-fit test to check the presumed distribution model(s).

The products population future failure trend in terms of hazard curve against age can be further predicted according to the presumed distribution and the obtained best-fitted parameters. The fundamental technique is to use an optimal approach to determine the values of parameters. The least square estimation (LSE), and in some occasions its extended form, the maximum likelihood estimation (MLE) are two frequently used optimal approaches [58].

Pioneering researchers on product lifetimes followed the above procedure; however differences exist in each step. Sections 2.4.1 to 2.4.4 summarize the engineering applications of statistics, especially in applications related to power systems. Following the four steps, arguments between applications are also presented.

2.4.1 Lifetime Data Collection

2.4.1.1 Complete Data

It was introduced in Section 2.3.1 that statistical analysis on product lifetimes only concerns the age of individual unit and in engineering practice, product lifetime data is often multiply right-time censored. However, in practice attention has not been paid to this type of data, especially in the field of power systems [59].

It can be found in many applications, the lifetimes of historical failed products are only considered in order to constitute a complete database. For example Hartford Steam Boiler (HSB) evaluated the US commerce transformers' average life based on all US utility transformer failure records published by US Commerce Department [24]; Meandering Regional Electric Company Iran implemented a reliability study on its low voltage transformers (62/20kV) according to the failure data only [60].

As highlighted previously for a set of complete data, the analysis is to simply calculate the failed products mean life ($\bar{t}$) and standard deviation (σ). Li pointed out the weakness of this method is that it only considers the information from failed products [27]. For power electric assets in particular, the simplified complete data may be workable for distribution level transformers or underground cables, it is not workable for large power transformers as these are fairly reliable and have not significant failure records.

2.4.1.2 Singly Censored Data

It can be seen from [61-63] that distribution transformers can be subdivided according to their installation years, such that the analyses on the multiply censored lifetime data can be simplified to the studies on transformers installed in the same year, which constitute groups of singly censored data.

Within a group of transformers installed in the same year, the transformer failure hazard is obtained via curve fitting. The number of transformer failure at each age can be predicted within that specific group; the sum of failure numbers from all the groups indicates the number of failure of the utility's entire transformer assets against age.

It can be mathematically proven that the transformer assets "number of failure against age" predicted among groups of singly censored data, is identical to that obtained among the entire multiply censored data. Although the analysis with singly censored data is relatively simple, it does not provide a generic failure trend for the entire population. Subdividing the population into groups of singly censored data is essentially suitable for products which are installed intensively within a short period.

2.4.1.3 Multiply Censored Data

Because specialized statistical techniques have been well developed to analyze multiply censored data in general reliability study, more engineering applications, particularly in power systems, are dealing with both the failed and survived products.

GE implemented a study on its transformer reliability in 1989, in which the survived transformers were taken into account as well as the failed units [52]. Li clarified the characteristics of the multiply censored lifetimes of Canada BC Hydro 500kV reactors and then predicted the population future failure hazard [27, 28]. Analysis on distribution transformers from US Utility Department presented in [61] was extended onto the overall assets after studies on transformers installed in the same year. Recent publications [1, 21, 42, 64-69] are prone to treat the multiply censored product lifetimes rather than the historical failures only or the simplified single censored type.

2.4.2 Distribution Model Selection

Proper distribution models are thereafter selected after the collection of product lifetime data. Traditional distribution models are commonly used, especially in power systems, because they are simple-mathematically expressed. Models developed from vital statistics are also adopted in engineering application. Utilities more often propose their own failure models, which have been proven suitable for their assets according to operational experience.

2.4.2.1 Traditional Distribution Models

The distributions presented in 2.3.3, and sometimes their extension forms, are commonly used in engineering practice since they have been proven to describe lifetimes in other areas. Particularly in power systems, applications of the exponential distribution [61, 70], the normal distribution [27, 28, 66] and the 2- or 3-parameter Weibull distribution [27, 28, 30, 60, 61, 64-70] can be found.

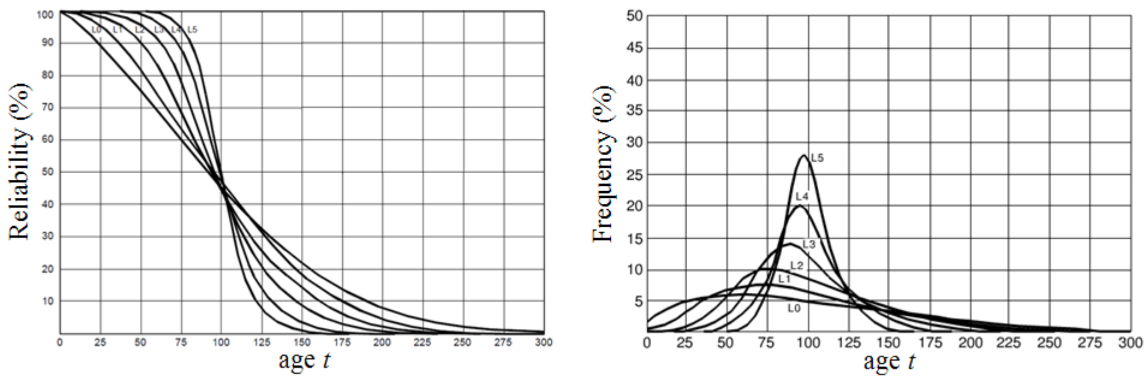
In respect of the 2- and 3-parameter Weibull model, Jongen suggested the use of a 3-parameter model when the first failure is observed significantly beyond age 1 [64, 68]. Cota-Felix compared the normal distribution and Weibull distribution by fitting the same lifetime data [69]; it indicates that although the traditional distributions are workable according to some utilities' practice, they may not provide the best description for any specific products.

2.4.2.2 Utility Defined Models

Products failure models are inclined to be defined according to utilities' operational experience, rather than by using the above traditional distributions.

- **Iowa Survivor Curves**

In an historical application of statistics, a family of empirical curves originally generated based on 176 groups of asset lives were named as Iowa curves after its production by Kurtz and Winfrey in Iowa State University [71]. These 18 basic curves are widely applied to model industrial products reliability in terms of survival function, typically after their validation and improvement in 1978 and 1992 [31]. Figure 2-9(a) presents a family of the Iowa survivor curves and Figure 2-9(b) shows the corresponding frequency curves.



(a) Left Mode of Iowa Survivor Curves

(b) Frequency Curves of Left Mode of Iowa Curves

Figure 2-9 Left Mode of Iowa Survivor Curves and Corresponding Frequency Curves [71]

The label **L** in Figure 2-9 indicates the left-mode group of Iowa Curves, of which the peak frequency in the corresponding PDF locate on the left as can be seen from Figure 2-9(b) [31]. The number **0-5** represents the relative heights of each frequency curve [71].

Iowa Curves are originally given by tables as there have been no fixed mathematical expressions for that family of curves. Fitting product lifetimes into Iowa Curves were historically implemented subjectively based on the utilities' prior judgement. According to [31] mathematical evidence is lacking to identify the best fit curve among the family of 18 curves.

- **Models Suggested from Vital Statistics**

Knowledge from vital statistics suggests mathematical models of mortality [72]. These models have been developed since the 18th century based on historical death records, from De Moivre's Law (1725), Gompertz Law (1825), Makeham's 1st (1860) and 2nd Law (1889) to Double Geometric Law (1867) [62, 72]. They are listed in Table 2-4.

Table 2-4 Commonly Used Human Death Mortality Models

De Moivre's Law (1725)	$h(t)=k(\rho-t)$
Gompertz Law (1825)	$h(t)=B\bullet c^t$
Makecham's 1st Law (1860)	$h(t)=A+B\bullet c^t$
Double Geometric Law (1867)	$h(t)=A+B\bullet c^t+C\bullet d^t$
Makecham's 2nd Law (1889)	$h(t)=A+D\bullet c+B\bullet c^t$
Perks' Law (1931)	$h(t)=\dfrac{A+B\bullet c^t}{1+E\bullet c^{-t}+D\bullet c^t}$
where *A*, *B*, *C*, *D*, *E*, and *k*, *ρ*, *c* are parameters in each model	

These models, especially the Gompertz Law, Makeham's 1st Law and Perks' Law are commonly adopted by finance and insurance industries to assess product reliability [32, 62]. The exponential term of Gompertz's Law was proposed based on the fact that human mortality is observed to increase exponentially [62]; it however neglected the accidental failure in term of constant mortality [62]. Makecham's 1st Law was thus developed by adding the random failures independent of age. These two models both imply increasing failure hazard rate at older ages. Perks further modified Makecham's 1st Law by adding $1+D\bullet c^t$ in the denominator to indicate the slower increase of mortality at older ages and $E\bullet c^{-t}$ to model the infant mortality [31, 32, 62].

Power system utilities further developed their own models based on the information shown in Table 2-4, and matched with their own historical equipment failure records.

As an extension of Gompertz Law, Cox suggested a proportional hazard model (PHM) for general statistics [73], expressed as

$$h(t;z) = h_0(t) \bullet exp\left(\beta_1 z_1 + \beta_2 z_2 + \ldots + \beta_q z_q\right) \tag{2-48}$$

In a transformer lifetime study, $h_0(t)$ is the unspecified hazard rate baseline dependent on equipment service age when effects from any external transient events are zero, z_j (j=1, …, q) simulate the effects from the external transient events and β_j (j=1, …, q) are parameters to measure these effects when transformers are in operation. This model is widely used in electric cable and transformer lifetime analysis [70, 73].

Hartford Steam Boiler (HSB) published a risk model based on Makeham's 1st Law for its transformer lifetime analysis, expressed as

$$h(t) = 0.005 + a \bullet exp(bt) \tag{2-49}$$

where a and b are constant parameters determined from historical failures, 0.005 represents the constant frequency of the randomly-occurred transient events and $a \cdot exp(bt)$ indicates the increasing hazard value due to transformer ageing [62, 63]. This hazard model can be further realized as combined from an exponential distribution (by hazard of 0.005) and a slow increase Weibull distribution (by hazard $a \cdot exp(bt)$). Application of this model can be found from insurance companies as well as power systems [42].

In transformer reliability study in particular, the infant mortality in Perks' Law should be neglected according to [31, 32, 72]. A simplified Perks' Law is proposed as

$$h(t) = \frac{A + c \bullet \exp(B \bullet t)}{1 + D \bullet \exp(B \bullet t)} \tag{2-50}$$

(2-50) was also adopted by [31, 32] to mathematically describe the Iowa Survivor Curves because the reliability function converted by (2-50) is similar to the family of Iowa Curves.

- **Piecewise Linear Models**

Except the models from vital statistics, hazard formulas should also be derived based on: the utilities' past operations, the understanding of product failure mechanisms and also in order to get rid of the comprehensive expressions.

For instance a piecewise linear model was presented by EPRI for underground cable reliability analysis. The model is expressed as

$$h(t) = \begin{cases} h_c, & 0 < t < T \\ h_c + m \bullet (t - T), & T \le t \end{cases} \quad \text{(2-51) [21]}$$

according to which the hazard rate remains h_c until the onset of burnout at age T and increases linearly by slope m after T.

According to an EPRI industrial survey on failures from substation power transformers, HV auto transformers and generation station transformers, generic values of h_c, T and m are summarized in [74]. The steady state hazard rate h_c is indicated to be 0.01, the burnout age T is assigned as 30 years, which is the mid age based on transformer unreliability records between 25 to 35 years. m is suggested to be 0.001 in order to model a doubled hazard rate 10 years after burnout T. Adjusted values of h_c, T and m can also be found in [74] for transformers under severe operation and extreme environments.

Similarly CIGRÉ published a table of transformer hazard change over service ages based on its survey on transmission/distribution transformers and generation transformers [18]. These typical hazards are shown in Table 2-5 and also graphically displayed in Figure 2-10.

Table 2-5 Transmission Hazard Rate by CIGRÉ Survey [18]

Age Range	<15yr	15-24yr	25-34yr	35-50yr	>50yr
Transmission/Distribution Transformer	0.5%	1.0%	1.5%	2.0%	3.0%
Generation Transformer	0.8%	1.5%	2.0%	2.5%	3.5%

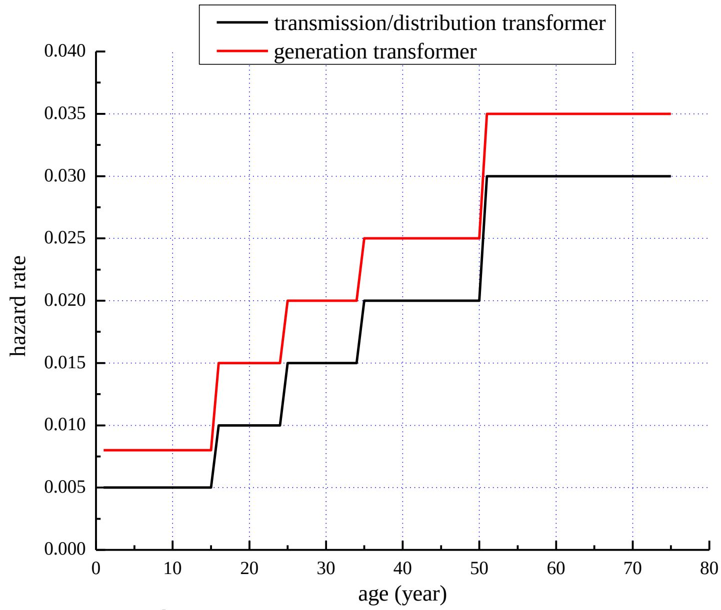


Figure 2-10 CIGRÉ Survey Transformer Hazard Rate Change by Age

It is also generally indicated in [18] that 0.5% hazard shows excellent condition, 2.0% infers acceptable condition and >2.0% presents unacceptable condition. When considering the generic transformer hazards, it must be recognized by [18] that the transformer hazard can vary significantly among utilities. The hazard depends on the units' loading practices, protection operations and oil quality maintenance policy. Expert opinions are important to identify any critical level of population hazard as well as the hazard definition adopted by the utility and their past operational experience.

2.4.2.3 UK National Grid Model

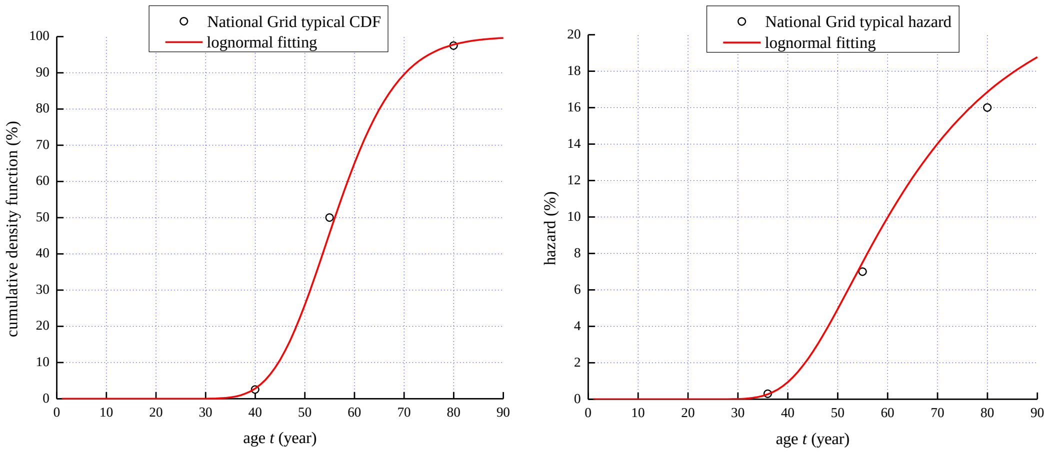
In particular UK National Grid Company proposes for planning purpose a general increasing hazard model, this is in accordance with its transformer operational experience [75]. This model is presented separately from the above industrial models, as this study is essentially intended to help National Grid predict failure of its transformer assets more precisely.

The UK National Grid transformer population failure model includes 3 typical ages of reaching the early onset of population significant unreliability (corresponding to 2.5% of CDF), the median lifetime (50% of CDF) and the late onset of population significant unreliability (97.5% of CDF). In addition these typical ages correspond to the hazard of 0.3%, 7% and 16% respectively based on the mathematical algorithm used to generate the failure model [75, 76]. These typical ages are listed in Table 2-6 as follows.

Table 2-6 UK National Grid Power Transformer Assets Failure Model

Expected CDF	Expected age t	Expected hazard	Expected age t
2.5%	40	0.3%	36
50%	55	7%	55
97.5%	80	16%	80

This discrete transformer population failure model can be continuously described by best fitted into the lognormal distribution. Figure 2-11(a) shows the fitted CDF and Figure 2-11(b) indicates the fitted hazard function.



(a) CDF Lognormal Fitting (b) Hazard Lognormal Fitting

Figure 2-11 National Grid Power Transformer Assets Failure Model Curve Fitting

This model is concluded to describe the failure data well [75]; however it does not consider any severe design deficiencies or operation variances throughout National Grid transformer population. The model needs to be reviewed when more failures are observed within the National Grid system.

2.4.3 Fitting Methods

The least square estimator (LSE), maximum likelihood estimator (MLE) and Bayesian method are three commonly used methods of fitting products lifetime data into the presumed distributions, especially into the traditional distribution models. They all can be applied on complete data, single censored data or multiply censored data; the least square estimator (LSE) in particular is most frequently adopted in engineering practice.

2.4.3.1 Least Square Estimator (LSE)

The principle of undertaking LSE is to minimise the sum of the squares of the residuals from the observed points to the fitted curve, and the minimum corresponds to the best fit under a certain model [22, 77]. For those picking up complete data for instance, the procedure of fitting data into a presumed distribution model by LSE is relatively easy. Product ages are organized against CDF $F(t)$ according to the linear relationship between expression of age t and $F(t)$ shown in (2-21), (2-26), (2-31), (2-37) and (2-44) for exponential, normal, lognormal, 2-parameter Weibull and extreme value distribution respectively; LSE is then used to determine the best fitting straight line. Appendix-I explains the procedure in much detail.

For those dealing with the singly censored data, for example products installed in the same year, the above classical fitting procedure for complete data is also applicable [61]. Failed products contribute to the plotting point only and it is thus criticized that using the classical fitting procedure for incomplete data enables more weighting to be applied to the failure samples while less weighting to the survived products, especially when the failure data is limited within the population.

The above classical fitting procedure is not straightforward for multiply censored data curve fitting because $F(t)$ at each age t is not obviously derived for multiply censored data. In order to implement LSE especially on multiply censored data, Nelson [26] proposed a graphic plotting method via actuarially estimating $H(t)$ or $F(t)$ at each age t; Li [27] further modified Nelson's method into an easier engineering solution.

- **Nelson's Graphic Plotting Method**

By identifying the characteristics of complete and incomplete product lifetime data, Nelson in 1972 presented a systematic method of fitting the complete data or the incomplete data in traditional distribution models [26].

One approach of Nelson's plotting method is hazard plotting. Similarly to the classical fitting procedure of complete data, age t or $ln(t)$ is plotted against expression of $H(t)$ according to (2-22), (2-27), (2-33), (2-39) or (2-46) under exponential distribution, normal distribution, lognormal distribution, 2-parameter Weibull distribution or extreme value distribution respectively. LSE determines the best fitting straight line and estimates the proper values of parameters under that specific distribution model.

CDF plotting is the other approach. By using Kaplan-Meier method, Herd-Johnson method or the actuarial method, the reliability function $R(t)$ is estimated, and then $F(t)$

can be derived as 1-$R(t)$. The subsequent steps follow those used in classical fitting procedure for complete data.

For application convenience, Nelson introduced various plotting papers for several distribution models, on which the linearity of plotting either by hazard plotting or CDF plotting can obviously be seen.

The main advantages of Nelson's method compared to the classical fitting are: **firstly** Nelson's method is applicable for both complete data and incomplete data; **secondly** for incomplete data in particular, the survived units contribute to the curve fitting as well as the failed units; **finally** the proper model can be easily selected by checking the linearity of plotting. However the graphic plotting method is very subjective: different people using the same plotting paper, based on the same data, may conclude differently. Nelson's method can not provide confidence limits for the parameters [26].

- **Li's Modified Method**

Li furthermore developed Nelson's method in his several successive publications based on Canada BC Hydro 500kV reactors data [27, 78]. He modified Nelson's CDF plotting approach involving actuarial estimation of $R(t)$ by artificially adding $F(t)$ of 0.001 at the age before the first failure and keeping the $F(t)$ of the last failure till the very last age. Although there is no clear mathematical evidence to support those modifications, Li's results are reasonable, as compared to the mean life derived by failed units only. Besides Li's method is more convenient to implement for engineering application, therefore this approach is being followed or discussed by other researchers [29, 59, 69].

2.4.3.2 Maximum Likelihood Estimator (MLE)

Compared to LSE, the maximum likelihood estimator (MLE) is more generally applied. MLE estimates "the best model that maximises the probability of seeing the observed failure" [79]. Hence the values of parameters derived by MLE are called the sample maximum likelihood estimate of their true values. MLE also provides the confidence limit(s) for the obtained distribution parameter(s). Although the estimation procedure may become computationally sophisticated especially when the number of parameters is large, MLE is fairly versatile for lifetime data analysis, particularly when the sample size is large. It has been proven by mathematicians that using MLE the mean value of an estimated parameter is close to its true value and its variance would be no greater than that by any other estimators [26, 67, 80].

Jongen compared MLE and LSE in [65, 67]. He applied Weibull fitting to a large epoxy resin bushing population in service in utilities in the Netherlands, and a small group of epoxy resin bushings at Nuon Energy Company (Netherlands), and concluded that MLE can handle heavily censored data better than LSE. However when the sample size was small, MLE may overestimate the parameter(s). Details of implementing MLE are introduced in Appendix-I.

2.4.3.3 Bayesian Method

Bayesian method is another powerful approach in product lifetime data analysis. Bayesian method firstly models parameter uncertainties in a prior distribution, it then builds the posterior probability based on the presumed distribution model and the parameter prior probability, and it finally estimates the values of parameters most satisfying the observed data. The procedure is similar to that of MLE because they both consider the most probable model which results in the observed data. Bayesian analysis however needs to incorporate parameter prior distribution [81]. Parameter prior distribution reflects the engineering judgement and prior knowledge for the product failure under study; it thus delivers more information for determining the values of parameters within the posterior probability [30, 81].

Gulachenski explained the advantages of Bayesian analysis as a probabilistic technique which could provide a complete picture of parameter uncertainties; he used this method to analyse US New England Electric 115kV power transformers [30]. Chen also employed Bayesian analysis to fit US PJM’s transformer lifetime data into the Iowa survival curves [31, 32]. The implement of Bayesian method is also illustrated in Appendix-I.

2.4.4 Goodness-of-Fit

In an entire fitting process of complete or incomplete data, the goodness-of-fit test is performed as the last step to verify the presumed distribution and its accompanying parameter value(s) [66]. The classical “Goodness-of-fit” tests, such as Kolmogorov-Smirnov test (briefly K-S test) [23], Cramér-von Mises test [82], Anderson-Darling test

[20], Chi-square test [23], and Hollander-Proschan test [49] concern the differences between the observed CDF and the fitted CDF at each failure [49]. A specific distribution is accepted only if the maximum difference is within a general standard criterion value [23]. Therefore, a distribution model and its parameters determined by the MLE, LSE or Bayesian method might be rejected by a goodness-of-fit test, because MLE for instance, estimates the most probabilistic parameters under the predetermined distribution model while the goodness-of-fit test questions this model to be used as a proper precondition.

In engineering practice however, the goodness-of-fit test is not usually carried out to simplify the lifetime data analysing process. In some applications related to power systems, the K-S test is used [30, 60, 61, 66, 70].

K-S test firstly orders the observations according to their ages from low to high, and then compares the observed CDF with the fitted CDF; if the maximum difference is no larger than the standard value under a certain significant level, the proposed model is acceptable at that significance level. The standard value is dependent on the sample size and the selected significant level. The standard values were tabulated by Hollander and Wolfe and particularly the extensive table for the multiply censored data is organized by Colosi [20]. The application of K-S test does not depend on the type of lifetime data or the specific distribution model being evaluated; however it tends to be more sensitive near the centre of the distribution and can only be used in continuous distributions [82].

Detail of how the K-S test is performed is illustrated in Appendix-II. The standard values under different population size and the significant level for both complete and multiply censored data are tabulated in Appendix-II.

2.4.5 Summary of Literatures on Lifetime Data Statistical Analysis

The four steps of product lifetime data curve fitting process are summarised in Table 2-7. Engineering publications related to each individual steps are listed; and their advantages and disadvantages highlighted.

Table 2-7 Lifetime Data Statistical Analysis Summary

<table>
<tr><th>step</th><th colspan="2">content</th><th>application example</th><th>advantage</th><th>disadvantage</th></tr>
<tr><td rowspan="3">1</td><td rowspan="3">data collection</td><td>complete data</td><td>• HSB [24]
• Meandering Reginal Electric [60]</td><td rowspan="2">• Simple for analysis</td><td rowspan="2">• Failure data of transformers are limited
• Information from censored units are not considered</td></tr>
<tr><td>singly censored data</td><td>• Jin [61]
• HSB [62, 63]</td></tr>
<tr><td>multiply censored data</td><td>• Jongen [64-68]
• EPRI [21]
• CIGRE [1, 42]
• Cota-Felix [69]</td><td>• Failed and survived transformers are both considered in analysis</td><td>• Need specific method for curve fitting</td></tr>
<tr><td rowspan="2">2</td><td rowspan="2">model collection</td><td>traditional statistical model</td><td>• Exponential distribution [61, 70]
• Normal distribution [27, 28, 66]
• 2/3-parameter Weibull distribution [27, 28, 30, 60, 61, 64-70]</td><td>• Theoretical distribution models are mathematically developed
• Specific applications are suggested from literatures</td><td>• Any specific distribution models are not suggested by very limited transformer failure data</td></tr>
<tr><td>utility defined model</td><td>• Iowa curves:
PJM [31, 32]
• Models from vital statistics:
HSB [62, 63]
PJM [31, 32]
CIGRE [42]
Prasad and Rao [70] and Kumar [73]
• Piecewise model:
EPRI [21, 74]
CIGRE [18]</td><td>• Models are well developed in human death mortality study
• Consistent with utilities' operational experience and expert judgement
• Results from large scale survey</td><td>• Hazard values are only valid accompanying with hazard definition
• Distribution model defined by one utility may not consist with other utilities' practical data</td></tr>
</table>

		UK National Grid model	National Grid Technical Guidance [75, 76]	• Consistent with National Grid operation experience up to present	• Design deficiencies or operation variances are not considered in the model • Needs to be refined when more failures are observed
3	curve fitting	LSE	Simple linearization method for complete and single censored data [61]	• Linearity can be determined by simple inspection	• Workable for complete data and single censored data • Only workable for some traditional distribution models
			Graphic plotting method [26]	• An entire systematic curve fitting technique • Suitable for both complete data and incomplete data • Easy to implement especially by using plotting papers	• Complicated to implement • Only workable for some traditional distribution models
			Li's modification [27, 29, 69]	• Relatively easy compared to graphic method	• Workable when very limited lifetime data are available • Mathematical argument on the modification continues
		MLE	Jongen [65, 67]	• Suitable for heavily censored data fitting • Provides parameter confidence limits	• Estimation procedure may become computationally sophisticated • May overestimate values of parameters when sample size is small

		Bayesian estimator	PJM [31, 32] Gulachenski and Besuner [30]	• Delivers more information from engineering prior knowledge for determining parameter values • Provides confidence limit for parameters	• Parameter prior distribution is not usually direct suggested • Mathematically complicated
4	goodness-of-fit testing	K-S test	Gulachenski and Besuner [30] Jin [61] Jongen [66] Prasad and Rao [70]	• Application does not depend on specific distribution model • Application does not require large sample size	• Much sensitive at the centre of the model • General standard values for multiply censored data are under investigation

2.5 Summary

Although products are assigned a certain end-of-life when they are designed and manufactured, this single age does not indicate the product's failure. Transformer failure for instance, relies on the unit's actual design, loading condition, the transient fault events it has experienced, and its installation environment. Transformer failure is hence realized as any outage that requires the transformer to be taken out of service rather than continue until its designed end-of-life (40 years).

The bathtub-shape curve is a widely used end-of-life model. It describes the likelihood of product failure, within a short time interval given that such product has survived at the end of the last time interval. Particularly with power transformers, the first stage in the bathtub curve is the normal operating stage with a relatively low hazard rate. This stage corresponds to a random failure period in which transformer failures are caused by randomly occurring transient fault events that are independent of transformer age. As the transformer ages, the hazard rate gradually increases as the unit is more and more prone to fail. This corresponds to the wear-out stage of the bathtub curve. In order to generate the hazard curve against age, statistical analyses are carried out.

Product lifetime is the only variable in a statistical study and the lifetime data needs to be polished as the first step of analysis. Transformer lifetime data in particular, are clarified as multiply right-time censored data, because transformers are commissioned in different years, some transformers do not fail when analysed, or some need to be taken out of service before failure. Proper distribution models are chosen to describe the classified lifetime data. An optimal approach is then carried out to determine the values of parameters under the presumed model(s). A goodness-of-fit test is sometimes undertaken as the last step.

Intensive literature, especially those involving applications in power systems and related to lifetime data curve fitting, has been reviewed in this chapter. Based on the literature, the following conclusions can be drawn:

- A variety of mathematical expressions have been developed as a measurement of hazard. Except for the traditional expressions developed according to statistical theories, utilities are prone to use their own defined hazard models based on their operational experience and expert opinion. The explanation of obtained hazard rate should be consistent with the utility's hazard expression.

- Although specialized methods are well developed for multiply censored lifetime data analyses in general reliability application, they have not been widely applied in power systems. In many applications the failed units are only used to derive the population mean life and standard derivation.
- The least-square estimator (LSE), maximum likelihood estimator (MLE) and Bayesian approach are three commonly used techniques to obtain the values of parameters for the presumed distribution model. In particular the key advantage of LSE as mathematically simple-to-implement results in its popularity among engineering applications.
- The goodness-of-fit test (i.e. K-S test), as the last step in the standard curve fitting procedure, is not always carried out to simplify the mathematical process.

Among the literature, Nelson first of all identified the characteristics of complete and incomplete lifetime data and developed a graphic plotting method for lifetime data curve fitting, this was essentially based on the LSE algorithm. Li simplified Nelson's graphic plotting method and verified his own modification by utilities' limited failure data. UK National Grid power transformer asset lifetime data are therefore analyzed statistically in Chapter 3 by Nelson's method and Li's modified method. The MLE approach is also carried out in Chapter 3 and used to compare with the LSE approach.

Chapter 3
National Grid Transformer Lifetime Data Statistical Analysis

3.1 Introduction

National Grid owns the high-voltage electricity transmission network (400kV and 275kV) in England and Wales and is the transmission network operator across Great Britain. National Grid has recorded the installation of its transformer assets including super-grid transformers (SGT), converters (COV), quadrature boosters (QB) etc since 1952. The analysis in this thesis only considers National Grid super-grid transformers (SGT) installed up to 2008, which includes the overwhelming majority of the National Grid transformer population. To simplify the statement, the word "transformer" or "power transformer" is used rather than "super-grid transformer". The number of transformers installed by year from 1952 to 2008 is shown in Figure 3-1 as follows.

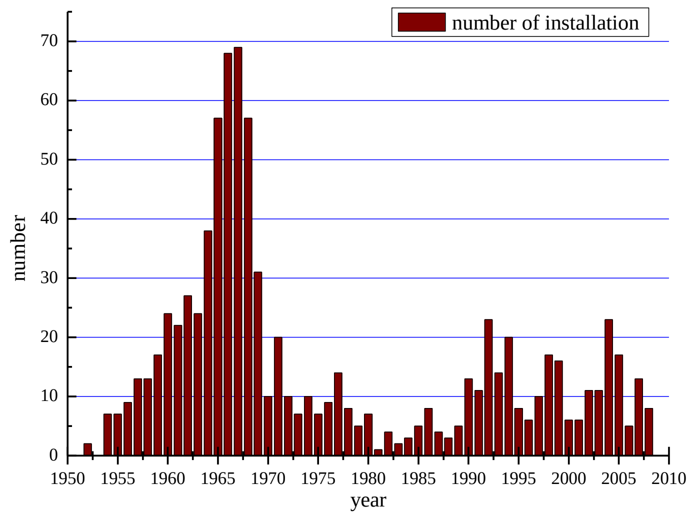


Figure 3-1 National Grid Power Transformers Installation Number against Year [43]

Figure 3-1 shows that significant numbers of power transformer were installed during the period 1960-1970, and particularly in year 1967 a peak capacity of 23 GVA was installed. Since then the installed capacity dramatically decreased and did not increase again until the 1990's, when higher level of electricity supplies and investment were required [43], and the gas generator stations started to replace the old and expensive coal stations.

Figure 3-2 shows the number of replaced transformers at each year until the end of 2008. The replaced transformers actually include those that have failed and those that are manually retired.

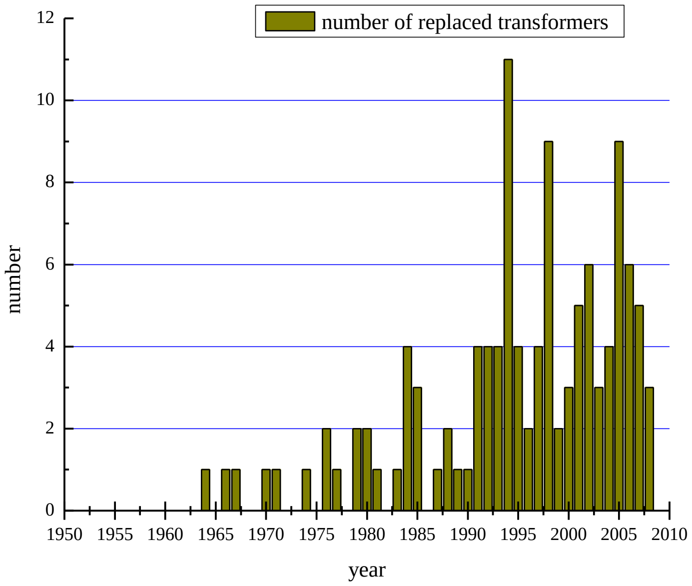


Figure 3-2 National Grid Power Transformers Number of Replacement against Year

According to the installation number in Figure 3-1 and the replacement number in Figure 3-2, the number of National Grid in-service transformers plotted against the year (1950-2008) can be derived, as shown in Figure 3-3.

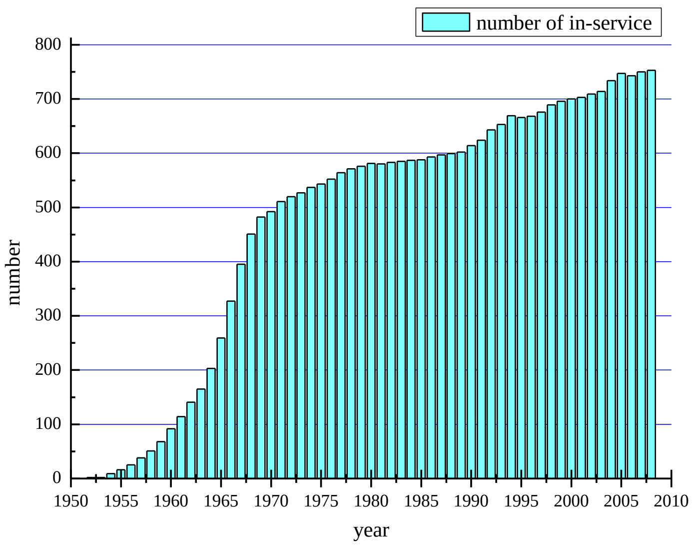


Figure 3-3 National Grid Power Transformers In-service Number against Year

A significant number of transformers were installed between 1960 and 1970, and as can be seen from Figure 3-3, most of these transformers are still operating on the National Grid transmission network.

Generally an in-service transformer is used in one of the three roles:

- an ***active transformer***, transmitting electric power from one voltage level to the other,
- a ***spare transformer***, disconnected from the network and used as a backup for emergency replacement [40],
- a ***redundant transformer***, disconnected from the network and waiting for removal [40, 76].

Assuming a transformer has operated for at least 1 year after installation, the National Grid transformer assets installed up to 2008 are actually observed in 2009. The age distribution of the 751 currently in-service transformers can be drawn in a pie chart as shown in Figure 3-4.

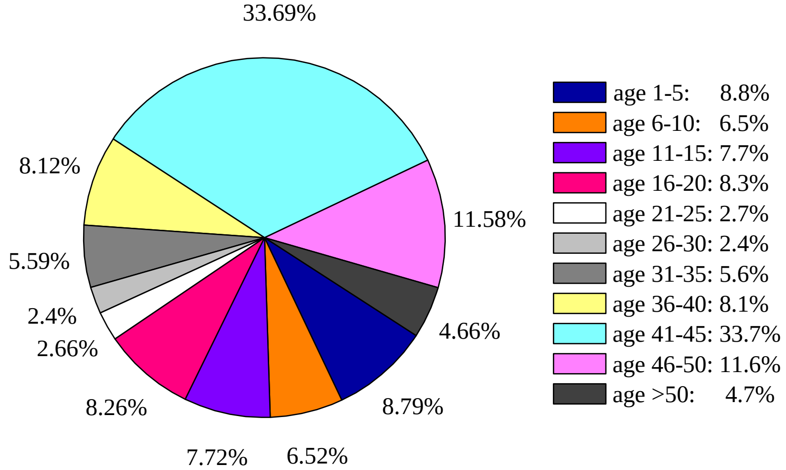


Figure 3-4 National Grid In-service Transformers Age Distribution

It can be seen from Figure 3-4, 8.1% transformers are approaching 40 years (age 36-40), which is the designed end-of-life of transformer. 50% of in-service transformers are aged beyond 40 years. Particularly there are 4.7% transformers have been in operating for more than 50 years.

A significant number of in-service transformers are aged more than 40 years; this verifies that the designed 40 years end-of-life should not be used to indicate actual transformer failure. Those “old” transformers, which remain in service, have benefited from relatively light loading and a proper maintenance policy. However to maintain network reliability it is important to understand and predict future failure based on present knowledge and information. Statistical analyses will be carried out in this chapter, using information about the transformers installed up to 2008. The statistical approaches introduced in Chapter 2 are also examined using real lifetime data from UK National Grid operational experience.

National Grid power transformer lifetimes are firstly derived, as they make up the database for analysis. Studies of fitting the lifetime data into normal distribution, lognormal distribution, 2-parameter Weibull distribution and extreme value distribution are carried out via the least square estimator (LSE) and the maximum likelihood estimator (MLE). The advantages and restrictions of transformer lifetime data statistical analysis are also addressed in this chapter.

3.2 National Grid Transformer Lifetime Data Derivation for Statistical Analysis

As summarized in Chapter 2, lifetime data collection is the first step in statistical analysis. In National Grid transformer operation practice, data collection includes transformer classification and transformer service age derivation. They are described in 3.2.1 and 3.2.2 respectively.

3.2.1 Transformer Classification

By tracing National Grid transformer operation history, several cases can be included within a transformer's service period. A transformer could:

- operate and fail;
- operate and be manually retired;
- operate and remain in service for an unknown future;
- operate, experience a fault, be repaired and returned to service; then fail or be manually retired, or remain in service for an unknown future;
- operate, experience a fault, be repaired typically having the winding rewound used new paper insulation, and returned to service; then fail or be manually retired, or remain in service for an unknown future.

In the above statement, the terminology ***fault*** is introduced in order to distinguish between a transformer that is repairable or has a reversible defect, from one which involves a severe un-repairable or irreversible failure. A transformer reversible defect or ***fault*** is defined as a transformer outage which can be repaired either in or out of the plinth, and then the transformer can be returned to service after the repair [11]. ***Failure*** in this context indicates a transformer that suffers a final un-repairable outage.

The above operational experiences are further illustrated in Figure 3-5. ○ and ● represent transformer commission start and failure respectively, x indicates manually retirement, | shows fault, (   ) indicates repair period after fault, — shows transformer operation period and → specifies a future that involves an unknown operation period for these in-service transformers.

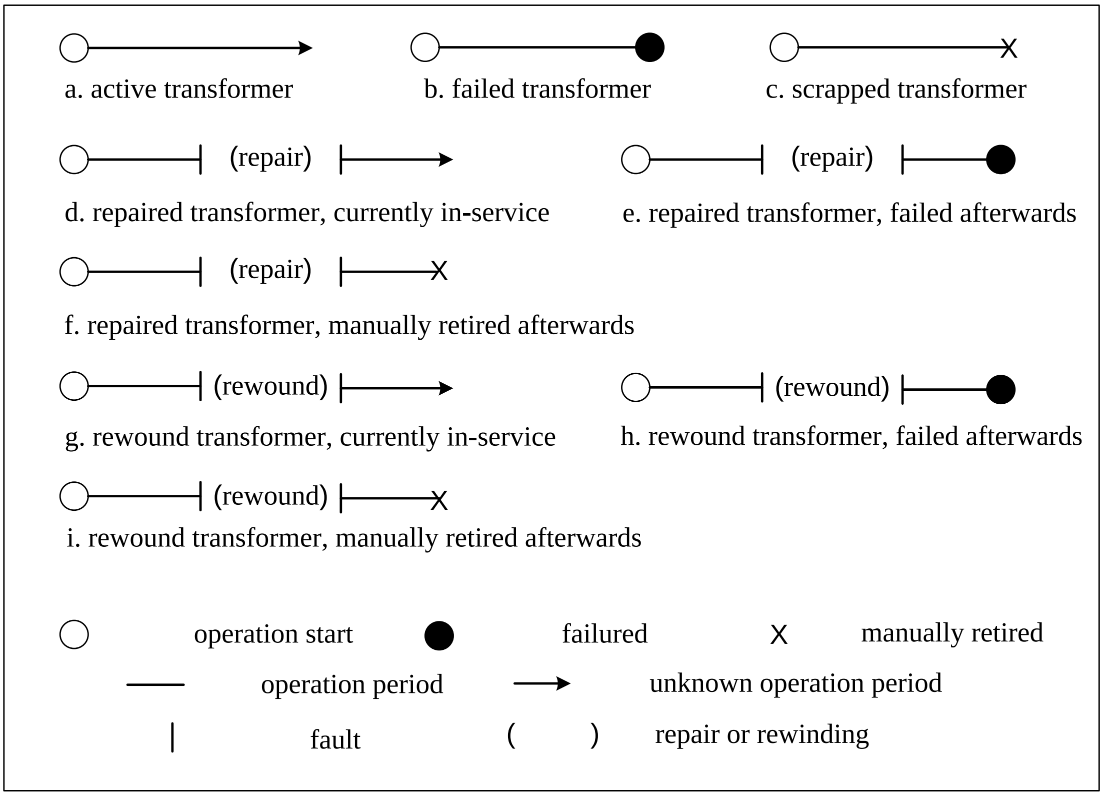


Figure 3-5 National Grid Transformer Operation Case

As indicated in Chapter 2 transformer failure is determined by the insulation withstand strength [40, 75]; hence, a rewound transformer with newly assembled winding insulation can be considered as a new unit once it is returned to service. The fault that caused a transformer rewinding should be treated as failure, because it implies the end-of-life of the old-insulated transformer. Meanwhile other repaired transformers, whose insulation system was not updated during repairing continue ageing after repair and cannot be considered as new transformers.

According to the above statement, the three types of transformers based on the operation status obtained when collecting data, are:

- Transformers currently in-service, known as active, spare or redundant transformers. Because the active transformers are the overwhelming majority of the in-service population, in-service transformers are represented by active units, denoted by **A**.
- Transformers failed after years of operation, denoted by **F**.
- Transformers manually retired according to their poor condition or design defects (identified by defects in similarly designed transformers). These transformers are usually scrapped afterwards and thus specified as scrapped transformers denoted by **X**.

In addition, failed transformers **F** and manually retired transformers **X** are generally named replaced transformers.

According to the definition of these 3 types, transformers experienced like **a** or **d** in Figure 3-5 are active units, **b** or **e** are failed transformers, and **c** or **f** are scrapped units. In particular transformers like **g**, **h** or **i** are considered as failed units firstly and start new lives after rewound which are further identified as **A**, **F** or **X** respectively.

In general therefore, each installed transformer can make a unique contribution to the population lifetime database except those rewound transformers. A rewound transformer is considered as a new unit once it is returned to service.

3.2.2 Transformer Service Age Derivation

Lifetimes of the failed transformers **F**, scrapped transformers **X** and currently in-service transformers **A** constitute the database for National Grid transformer lifetime statistical analysis.

As explained in 3.1, National Grid transformers are assumed to have operated for at least 1 year, Year 2009 is hereby used as the reference year in transformer age derivation. Transformer service ages till 2009 of the above 3 types of transformers are deduced as

- for an in-service transformer **A**: age = 2009 - year of installation
- for a failed transformer **F**: age = year of failure - year of installation
- for a scrapped transformer **X**: age = year of retirement - year of installation.

In addition it needs to be mentioned here that the period when a transformer is out of service for repair should be eliminated from its service age since transformer service age indicates the length of service period within the network.

The ages of National Grid transformers constitute a group of multiply right-time censored data, in which the age of a failed transformer **F** implies the lifetime of that particular unit; however either the active **A** or the scrapped transformers' (**X**) lifetimes have not been observed when collecting data. The reasons can be summarized as:

- installation of new transformers every year,
- retiring transformers before their actual failure, and
- collection of lifetime data while transformers are in service.

Up to the reference year 2009, UK National Grid has recorded 52 failed transformers and 62 manually retired units; 751 transformers are currently in service in the transmission network. Ages of these 865 transformers make up the database for statistical study. Lifetime analyses using the least-square estimator (LSE) are shown in 3.3 and using the maximum likelihood estimator (MLE) are shown in 3.4.

3.3 National Grid Transformer Lifetime Data Statistical Analysis by Least Square Estimator (LSE)

In this section, Nelson's graphic plotting approaches (including the hazarding plotting approach and CDF plotting approach) and Li's modified approach are conducted to fit National Grid transformer lifetime data. Again these approaches are all essentially based on the least-square estimation method (LSE). The normal distribution, lognormal distribution, 2-parameter Weibull distribution and smallest extreme value distribution are selected to model the lifetime data, because these commonly used distribution models can adequately describe the wear-out stage of the bathtub curve due to the ageing failure mechanism when transformers are operating at their older ages. Exponential distribution is not used because it describes the normal operating stage only and thus does not suggest transformer future failures due to ageing.

The hazard plotting approach, CDF plotting approach and Li's modified approach are presented in 3.3.1, 3.3.2 and 3.3.3 respectively. In each section, the contents are organized by:

- introducing the procedure of fitting according to a certain approach and showing the linear plot under the selected distribution models,
- displaying the estimates of parameters, the population mean life and the standard deviation under each distribution, and identifying the goodness-of-fit of each distribution,
- plotting the cumulative probability function under each distribution together with the observed data and comparing the estimated ages of reaching three typical cumulative probabilities under different distributions, and
- plotting the hazard function under each distribution together with the observed data.

3.3.1 By Hazard Plotting Approach

- **Curve Fitting Procedure**

The steps of implementing hazard plotting using the National Grid transformer lifetime data are listed as follows. Again, there are 865 lifetime data altogether, within which 52 are failed units and 813 are censored data.

Step 1: Order the ages of 865 transformers from smallest to largest in Column 1.

Step 2: Indicate failed transformers by "**1**" and censored transformers by "**0**" in Column 2. If failures and censorings both exist at the same age, it is generally assumed that transformer fails in the middle of that age.

Step 3: Assign the reverse rank ($\boldsymbol{k_{rr}}$) to each transformer. For example the reverse rank for the 1st transformer is 865, for the 2nd transformer is 864, …, and for the 865th is 1.

Step 4: For each failure i (i=1, 2, 3, …, 52), calculate the observed instantaneous hazard $h(i)$ as $h(i)=1/\boldsymbol{k_{rr}(i)}$, in which $\boldsymbol{k_{rr}(i)}$ is the reversed rank for the i^{th} failure and 1 considers the failures one by one even when they occurred at the same age. The calculation is discussed in detail in Chapter 2. In particular hazard is not calculated for censored transformers which are marked as "**0**" in Column 2.

Step 5: Calculate the cumulative hazard function $H(i)$ at each failure i. It is deduced as the sum of all the preceding observed instantaneous hazards and the hazard at the i^{th} failure.

Step 1 to **Step 5** are summarized in Table 3-1, which only shows a snapshot of the cumulative hazard function derivation on the "865" transformer lifetime data. An entire table is given in Table III-1 Appendix-III.

Table 3-1 Illustration of Hazard Plotting Approach Using National Grid Transformer Lifetime Data

age t	Censored/ Failed	Reversed Rank $\boldsymbol{k_{rr}}$	Hazard $h(i)$	Cumulative hazard $H(i)$
1	0	865		
1	0	864		
1	0	863		
1	0	862		
1	**1**	**861**	**0.001161**	**0.001161**
1	**1**	**860**	**0.001163**	**0.002324**
1	0	859		
1	0	858		
1	0	857		
1	0	856		
2	0	855		
2	0	854		
2	0	853		
2	0	852		
2	0	851		
2	0	850		
2	0	849		
2	**1**	848	**0.001179**	**0.003503**
2	0	847		
…	…	…	…	…
48	0	77		
48	0	76		
48	**1**	**75**	**0.013333**	**0.096664**
48	0	74		
…	…	…	…	…
55	0	3		
55	0	2		
57	0	**1**		

Step 6: Plot failure age *t* (in year) against the expression of cumulative hazard function *H*(*i*) according to the linear equation under the specific distribution model. These linear equations are

Normal distribution: $$t = \sigma \bullet \Phi^{-1}\left\{1 - exp\left[-H(t)\right]\right\} + \mu \quad (3\text{-}1)$$

Lognormal distribution: $$ln(t) = \delta \bullet \Phi^{-1}\left\{1 - exp\left[-H(t)\right]\right\} + \lambda \quad (3\text{-}2)$$

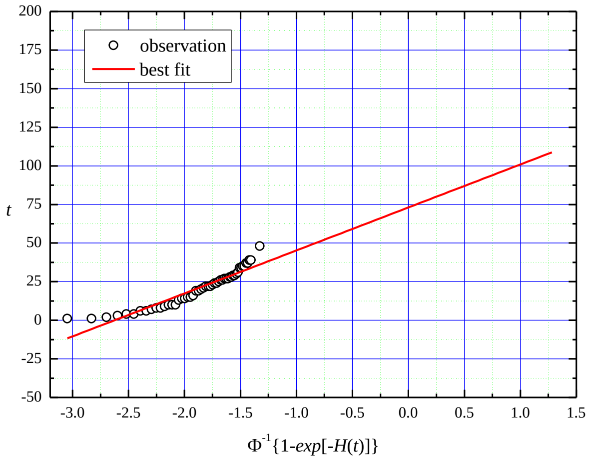
2-parameter Weibull distribution: $$ln(t) = \frac{1}{\beta} \bullet ln\left[H(t)\right] + ln\,\alpha \quad (3\text{-}3)$$

Extreme value distribution: $t = \delta \bullet ln\left[H(t)\right] + \lambda$ (3-4)

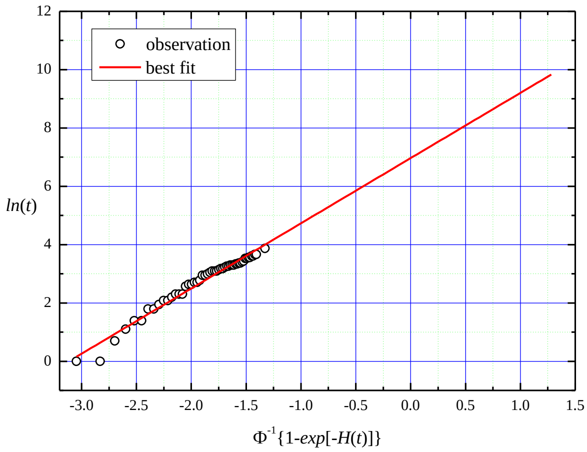
It can be noticed that several $H(i)$s may be plotted against an identical age t as more than 1 transformer fails at that particular age. As discussed in Chapter 2, the calculation of hazard function $h(i)$ and then the cumulative hazard function $H(i)$ precisely consider the contributions from individual failed transformers.

Step 7: Under a certain distribution model, use LSE to draw a straight line to best fit the plotted points in Step 6. The parameters of this distribution can be graphically estimated from the plotted straight line.

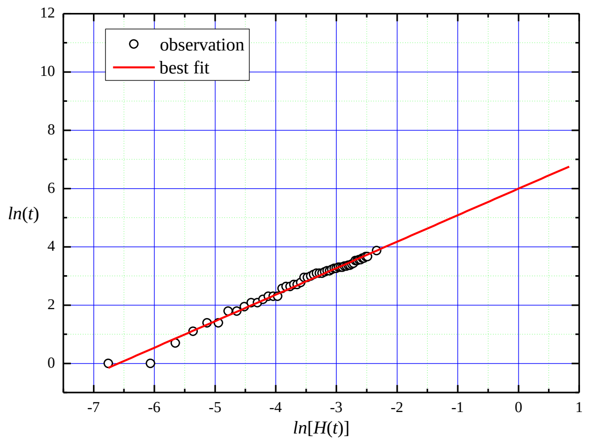
Details of implementing LSE are introduced in Appendix-I. The linear plotting under the 4 distribution models are shown in Figure 3-6, from (a) to (d).



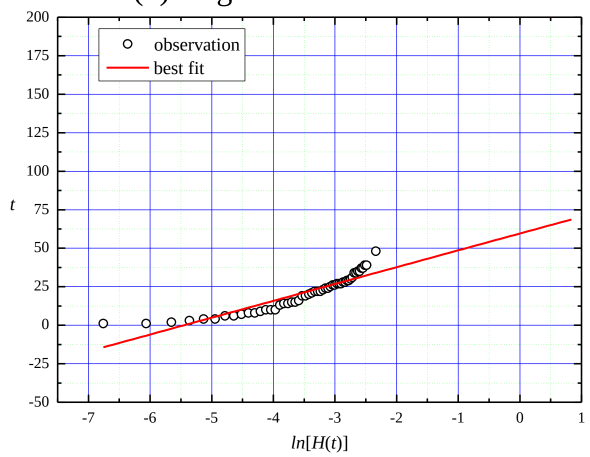
(a) Normal Distribution



(b) Lognormal Distribution



(c) 2-Parameter Weibull Distribution



(d) Extreme Value Distribution

Figure 3-6 Hazard Plotting under Traditional Distribution Models Using National Grid Transformer Lifetime Data

In each graph of Figure 3-6, the dots indicate 52 observed failures, and the red straight line is the best-fit curve according to LSE. The values of parameters under each distribution model are further obtained.

- **K-S Goodness of Fit Test**

As the last step of traditional curve fitting, Kolmogorov-Smirnov test (K-S test) is carried out to verify the fitting goodness of the presumed distribution models. The goodness-of-fit K-S test is not distribution-specific and it can reject any improper hypothesized distributions. The steps are summarized as follows by using the normal distribution as an example.

Step 1: Order the ages of 865 transformers from low to high. This is the same as the first step of hazard plotting.

Step 2: Designate "**0**" for each of the censored transformers and "**1**" for failed transformers, which is also the same as Step 2 in hazard plotting.

Step 3: List the reverse rank ($\boldsymbol{k_{rr}}$) for each transformer from 865 to 1. According to National Grid's record, two transformers failed within their first year of service (age t=1); they are assigned the reverse rank $\boldsymbol{k_{rr}}$=861 and 860 respectively.

Step 4: Calculate the reliability function $R(i)$ for the i^{th} failure, as

$$R(i) = \frac{k_{rr}(i)-1}{k_{rr}(i)} \bullet R(i-1) \tag{3-5}$$

in which i indicates the i^{th} failure, $k_{rr}(i)$ is the reversed rank for the i^{th} failure, $R(i$-1) is the reliability at the last failure i-1. Typically $R(0)$ is assigned to be 1 before the 1st failure. For example, for the 1st failure, the reliability function $R(1)$ is calculated as

$$R(1) = \frac{k_{rr}(1)-1}{k_{rr}(1)} \bullet R(0) = \frac{861-1}{861} \bullet 1 = 0.998839 \tag{3-6}$$

The process of reliability function recursive calculation at each failure is listed in Table 3-3 from Column 4 to Column 6.

This approach to the recursive calculation of reliability was developed by Kaplan and Meier in 1958. The process is also called the product-limit estimation of reliability and it was further adopted by Nelson in his CDF plotting approach. This recursive calculation will be further illustrated when implementing CDF plotting on National Grid transformer lifetime data in 3.3.2.

Step 5: Derive the cumulative probability function CDF $F(i)$ at failure i as

$$F(i) = 1 - R(i) \tag{3-7}$$

For the 1st failure, for example, $F(1)$ = 1-0.998839 = 0.001161. The value of $F(i)$ for each failure is shown in Column 7. Particularly in K-S test, these CDF are specified as observed CDF for the i^{th} failure and denoted as $F^0(i)$.

Step 6: At each failure i (i=1, 2, 3, …, 52) calculate CDF according to the presumed distribution and the obtained parameters. These are the theoretical CDF and denoted as $F_0(i)$for the i^{th} failure. Under the normal distribution for example, $F_0(i)$ is derived as

$$F_0(i) = \Phi\left[\frac{t(i)-\mu}{\sigma}\right] \tag{3-8}$$

where $t(i)$ is the age of the i^{th} failure, values of μ and σ are determined by previous hazard plotting. For example for the 1st failure, $F_0(1) = \Phi\left[\frac{t(1)-73.07}{27.84}\right] = 0.004821$, where μ=73.07 and σ=27.84 are hazard plotting determined parameters under the normal distribution model. The hazard plotting results will be presented in the following section.

Step 7: Compare observed $F^0(i)$ with theoretical $F_0(i)$ at each failure. For the 1st failure, for example, the absolute difference between $F^0(1)$ and $F_0(1)$ is calculated as $\left|F^0(1) - F_0(1)\right| = |0.001161 - 0.004821| = 0.003660$. The maximum absolute difference between $F^0(i)$ and $F_0(i)$ among M failures (M=52 in this specific case), denoted as $D_M(r)$, is recorded as

$$D_M(r) = max\left|F^0(i) - F_0(i)\right|, \quad i = 1,2,3,...,M \tag{3-9}$$

in which $r = \frac{M}{N}$ and N is the total number of transformers. $r = \frac{52}{865} = 0.060$ for National Grid transformer population, which indicates the portion of failures out of the whole population.

Step 8: According to the failure portion factor r and the prior assigned significance level γ, a critical value $y_{1-\gamma}(r)$ can be found from Table II-3 in Appendix-II. In engineering practice γ is always set 0.05, $y_{1-\gamma}(r)$ is thus found to be 0.5569 corresponding to r = 0.060 for National Grid transformer population. A tolerance factor denoted as $D_M^{\gamma}(r)$ is further derived as

$$D_M{}^{\gamma}(r) = \frac{y_{1-\gamma}(r)}{\sqrt{M}} = \frac{0.5569}{\sqrt{52}} = 0.077228 \qquad (3\text{-}10)$$

According to (3-10) the value of $D_M^{\gamma}(r)$ remains for a certain group of data (r and M unchanged) under a specific significance level γ. Hence $D_M^{\gamma}(r)$ = 0.077228 is valid to test the fitting of National Grid transformer lifetime data into various distribution models.

Step 9: If $D_M(r) \leq D_M^{\gamma}(r)$ = 0.077228, the distribution model is accepted as a suitable fit, unless it is rejected.

Step 1 to **Step 9** are summarized in Table 3-2. For illustration purpose information related to failed transformers is only shown in Table 3-2.

As highlighted in Table 3-2, the maximum absolute differences between the observed $F^0(i)$ and the normal distribution derived $F_0(i)$ is 0.091667. This value is larger than the critical value of 0.077228. Normal distribution with hazard plotting derived parameters is considered an inadequate model for the lifetime data.

The curve fitting results using the hazard plotting approach and the judgement on goodness-of-fit under other presumed distribution models will be discussed in the following section.

Table 3-2 Illustration of K-S Test involving Kaplan-Meier Method of Reliability Calculation Using National Grid Transformer Lifetime Data

Age t		Reversed Rank $k_{rr}(i)$	Reliability $R(i)=\frac{k_{rr}(i)-1}{k_{rr}(i)}\bullet R(i-1)$			Observed CDF $F^0(i)$	Normal distribution CDF $F^0(i)$	Absolute difference $\left\|F^0(i)-F_0(i)\right\|$
			$\frac{k_{rr}(i)-1}{k_{rr}(i)}$	$R(i-1)$	$R(i)$			
1	**1**	861	0.998839	1	0.998839	0.001161	0.004821	0.003660
1	**1**	860	0.998837	0.998839	0.997677	0.002323	0.004821	0.002498
2	**1**	848	0.998821	0.997677	0.996501	0.003499	0.005348	0.001849
3	**1**	838	0.998807	0.996501	0.995311	0.004689	0.005925	0.001237
4	**1**	826	0.998789	0.995311	0.994106	0.005894	0.006557	0.000664
4	**1**	825	0.998788	0.994106	0.992902	0.007098	0.006557	0.000541
6	**1**	787	0.998729	0.992902	0.991640	0.008360	0.008002	0.000358
6	**1**	786	0.998728	0.991640	0.990378	0.009622	0.008002	0.001620
7	**1**	774	0.998708	0.990378	0.989099	0.010901	0.008825	0.002077
8	**1**	765	0.998693	0.989099	0.987806	0.012194	0.009720	0.002474
8	**1**	764	0.998691	0.987806	0.986513	0.013487	0.009720	0.003767
9	**1**	758	0.998681	0.986513	0.985211	0.014789	0.010694	0.004095
10	**1**	745	0.998658	0.985211	0.983889	0.016111	0.011752	0.004360
10	**1**	744	0.998656	0.983889	0.982566	0.017434	0.011752	0.005682
10	**1**	743	0.998654	0.982566	0.981244	0.018756	0.011752	0.007004
13	**1**	703	0.998578	0.981244	0.979848	0.020152	0.015487	0.004665
14	**1**	695	0.998561	0.979848	0.978438	0.021562	0.016940	0.004622
14	**1**	694	0.998559	0.978438	0.977029	0.022971	0.016940	0.006031
15	**1**	680	0.998529	0.977029	0.975592	0.024408	0.018508	0.005900
15	**1**	679	0.998527	0.975592	0.974155	0.025845	0.018508	0.007337
16	**1**	662	0.998489	0.974155	0.972683	0.027317	0.020198	0.007118
19	**1**	614	0.998371	0.972683	0.971099	0.028901	0.026073	0.002828
19	**1**	613	0.998369	0.971099	0.969515	0.030485	0.026073	0.004412
20	**1**	602	0.998339	0.969515	0.967905	0.032095	0.028324	0.003771
21	**1**	597	0.998325	0.967905	0.966283	0.033717	0.030735	0.002982
22	**1**	593	0.998314	0.966283	0.964654	0.035346	0.033313	0.002033
22	**1**	592	0.998311	0.964654	0.963024	0.036976	0.033313	0.003662
22	**1**	591	0.998308	0.963024	0.961395	0.038605	0.033313	0.005292
23	**1**	584	0.998288	0.961395	0.959749	0.040251	0.036067	0.004184
24	**1**	576	0.998264	0.959749	0.958082	0.041918	0.039005	0.002913
24	**1**	575	0.998261	0.958082	0.956416	0.043584	0.039005	0.004579
25	**1**	571	0.998249	0.956416	0.954741	0.045259	0.042134	0.003125
26	**1**	567	0.998236	0.954741	0.953057	0.046943	0.045464	0.001479
26	**1**	566	0.998233	0.953057	0.951373	0.048627	0.045464	0.003163
27	**1**	562	0.998221	0.951373	0.949681	0.050319	0.049002	0.001318
27	**1**	561	0.998217	0.949681	0.947988	0.052012	0.049002	0.003011
27	**1**	560	0.998214	0.947988	0.946295	0.053705	0.049002	0.004703
28	**1**	557	0.998205	0.946295	0.944596	0.055404	0.052756	0.002648
28	**1**	556	0.998201	0.944596	0.942897	0.057103	0.052756	0.004347
29	**1**	552	0.998188	0.942897	0.941189	0.058811	0.056736	0.002075
29	**1**	551	0.998185	0.941189	0.939481	0.060519	0.056736	0.003784
30	**1**	543	0.998158	0.939481	0.937751	0.062249	0.060948	0.001302
31	**1**	533	0.998124	0.937751	0.935991	0.064009	0.065400	0.001391
34	**1**	492	0.997967	0.935991	0.934089	0.065911	0.080277	0.014366
34	**1**	491	0.997963	0.934089	0.932186	0.067814	0.080277	0.012463
35	**1**	482	0.997925	0.932186	0.930252	0.069748	0.085766	0.016019
35	**1**	481	0.997921	0.930252	0.928318	0.071682	0.085766	0.014085
37	**1**	461	0.997831	0.928318	0.926305	0.073695	0.097579	0.023884
37	**1**	460	0.997826	0.926305	0.924291	0.075709	0.097579	0.021871
39	**1**	429	0.997669	0.924291	0.922137	0.077863	0.110544	0.032681
39	**1**	428	0.997664	0.922137	0.919982	0.080018	0.110544	0.030526
48	**1**	75	0.986667	0.919982	0.907716	0.092284	0.183951	0.091667
Max								**0.091667**
Critical value								**0.077228**
Conclusion								**Rejected**

- **Curve Fitting Result**

The obtained values of parameters under the normal, lognormal, Weibull and smallest extreme value distribution are shown in Table 3-3. National Grid transformer population mean life ($\bar{t}$) and the accompanying standard deviation (σ) under each distribution model are thus derived. The failed transformers' arithmetic mean ($\bar{t}$) and standard deviation (σ) are also calculated according to (2-15) to (2-17); they are presented in Table 3-3 to compare with the mean life ($\bar{t}$) and standard deviation (σ) deduced from curve fitting.

According to the implementation procedures introduced in the last section, K-S test is carried out for the normal distribution, lognormal distribution, Weibull distribution and extreme value distribution. The results are shown in Table 3-3. "**A**" represents the distribution is **acceptable**, while "**R**" means the distribution is **rejected**.

Besides, typically for curve fitting by the LSE, the coefficient of determination (COD) is calculated to indicate the goodness of the fitted straight line. Unlike the K-S test, COD evaluates the fitting goodness of the obtained parameters by presuming the selected distribution is proper. Generally a value of COD close to 1 indicates a good fit, i.e. a fit with COD=0.9 is generally considered to describe the data better than the other fit with COD=0.8. However, there is not a specific critical value of COD below which a fit can be rejected. The calculation of COD can be found in Part A of Appendix-I. The value of COD under each distribution is shown in Table 3-3.

Table 3-3 Hazard Plotting Results under Traditional Distribution Models Using National Grid Transformer Lifetime Data

	Normal distribution	Lognormal distribution	Weibull distribution	Extreme value distribution	From failures only
parameters	μ=73.07 σ=27.84	λ=6.97 δ=2.24	α=401.70 β=1.10	λ=59.55 δ=10.93	
mean $\bar{t}$ (year)	73	12932	387	53	20
std σ (year)	27.8	157030	353.1	14.0	11.5
K-S	**R**	**A**	**A**	**R**	
COD	**0.903**	**0.973**	**0.987**	**0.854**	

It shows in Table 3-3 that the population mean life ($\bar{t}$) is derived to be 73 yrs under the normal distribution and the standard deviation (σ) is 27.8 yrs; population mean life ($\bar{t}$)

is determined to be 12932 yrs with the standard deviation of 157030 yrs under the lognormal distribution; $\bar{t}$ is 387 yrs and σ is 353.1 yrs under the 2-parameter Weibull distribution; and $\bar{t}$ equals 53 yrs and σ is 14.0 yrs under the extreme value distribution. Any of these mean lives is much longer than the mean life ($\bar{t}$ =20 yrs) derived from 52 failed transformers and also the corresponding standard deviations are longer than the standard deviation (σ=11.6 yrs) obtained from the failed transformers. The prolonged transformer population mean life and the standard deviation, obtained from hazard plotting, are actually caused by the censored transformers. The reversed ranks of those censored transformers affect the denominator of the hazards that are calculated at individual failures; although 52 failures are only plotted, information from active units and scrapped transformers are also taken into account in curve fitting.

It needs to be noted from Table 3-3 that under the lognormal distribution, both the population mean life ($\bar{t}$ =12932 yrs) and the standard deviation (σ=157030 yrs) are very large, and typically the standard deviation σ is even larger than the population mean life $\bar{t}$. Although these values may not be engineering-significant, the value which is purely derived from a mathematical process actually indicates the high dispersion of the lifetime data, especially when the service ages of censored units are considered.

According to the COD value of fitting for each distribution model shown in Table 3-3, the lognormal and Weibull distribution better fit the transformer lifetime data than the other 2 models. It is also indicated in Table 3-3 that the lognormal distribution and Weibull distribution are verified to be good fitting models via the K-S test; while the normal distribution and extreme value distribution are both rejected by the K-S test, although their CODs are high (i.e. COD>0.9).

To clearly indicate the goodness-of-fit of each distribution model according to the K-S test, the theoretical cumulative probability function $F^0(t)$s are graphically compared with the observed $F_0(t)$s in Figure 3-7. The theoretical CDF under each distribution model can be derived according to

Normal distribution: $$F(t) = \Phi\left(\frac{t-\mu}{\sigma}\right) \quad (3\text{-}11)$$

Lognormal distribution: $$F(t) = \Phi\left\{\frac{\left[ln(t)-\lambda\right]}{\delta}\right\} \quad (3\text{-}12)$$

Weibull distribution: $$F(t)=1-exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right] \tag{3-13}$$

Extreme value distribution: $$F(t)=1-exp\left[-exp\left(\frac{t-\lambda}{\delta}\right)\right] \tag{3-14}$$

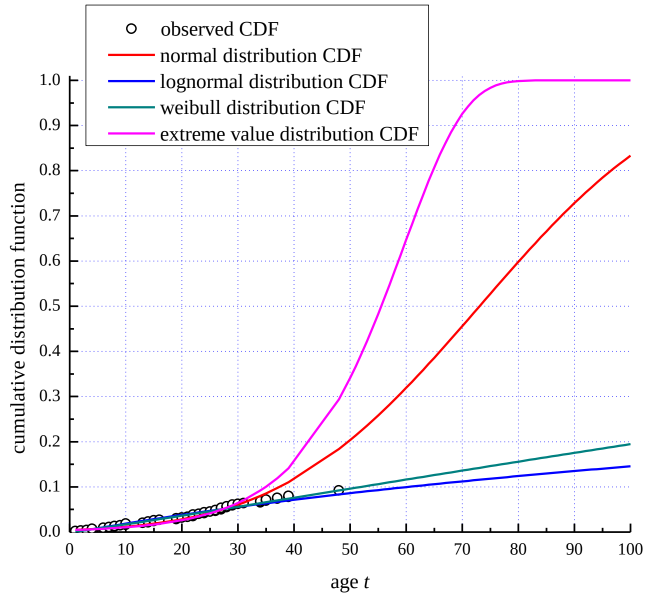


Figure 3-7 National Grid Transformer Lifetime Data Observed CDF vs. Hazard Plotting Derived CDF under Different Distribution Models

Figure 3-7 clearly indicates that the lognormal distribution and extreme value distribution are better to fit the observed lifetime data as compared to the other two distributions. However, as indicated in Table 3-2 and also shown in Figure 3-7, the absolute differences between $F^0(i)$ and $F_0(i)$ ($\left|F^0(i)-F_0(i)\right|$) when fitting the data into the normal distribution or the extreme value distribution, are less than the tolerance factor ($D_M^{\ \gamma}(r)$) of 0.077228 except the last failure at age 48. It can be imagined that the normal and extreme value distribution can also describe the National Grid transformer lifetime data well if the last failure is neglected, or a larger critical value $y_{1-\gamma}(r)$ and then a higher tolerance factor $D_M^{\ \gamma}(r)$ is used.

Besides, according to Figure 3-7 the age of reaching the early onset of population significant unreliability (corresponding to 2.5% of CDF), the age of reaching the median lifetime (50% of CDF) and the ageing of reaching the late onset of population significant unreliability (97.5% of CDF) derived under different distribution models can be obtained. It hereby compares in Table 3-4 the ages of reaching three typical CDFs under different distribution models.

Table 3-4 Hazard Plotting Derived Typical Ages in National Grid Transformer Lifetime Data Analysis

	Age *t*			
CDF *F*(*t*)	By normal	By lognormal	By Weibull	By extreme
2.5%	19	13	14	19
50%	73	>500	292	56
97.5%	128		>500	74

It can generally be seen from Table 3-4, from an asset manager's point of view, that the normal distribution and extreme value distribution might be proper to describe National Grid transformer assets lifetime data as the median life predicted under these two models are reasonable values and the estimated failure trend is more coincident with engineering prior knowledge.

Similarly the theoretical hazard function $h(t)$ under the normal, lognormal, Weibull and extreme value distribution can be calculated based on the obtained parameters. These hazard functions are rewritten here again as

Normal distribution: $$h(t)=\frac{\frac{1}{\sqrt{2\pi}\sigma}exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right]}{\Phi\left[\frac{t-\mu}{\sigma}\right]} \quad (3\text{-}15)$$

Lognormal distribution: $$h(t)=\frac{\frac{1}{\sqrt{2\pi}t\delta}exp\left\{\frac{-[ln(t)-\lambda]^2}{2\delta^2}\right\}}{\Phi\left\{\frac{[ln(t)-\lambda]}{\delta}\right\}} \quad (3\text{-}16)$$

Weibull distribution: $$h(t)=\left(\frac{\beta}{\alpha}\right)\left(\frac{t}{\alpha}\right)^{\beta-1} \quad (3\text{-}17)$$

Extreme value distribution: $$h(t)=\left(\frac{1}{\delta}\right)exp\left(\frac{t-\lambda}{\delta}\right) \quad (3\text{-}18)$$

Figure 3-8 compares the theoretical hazards with the observed hazard $h(i)$.

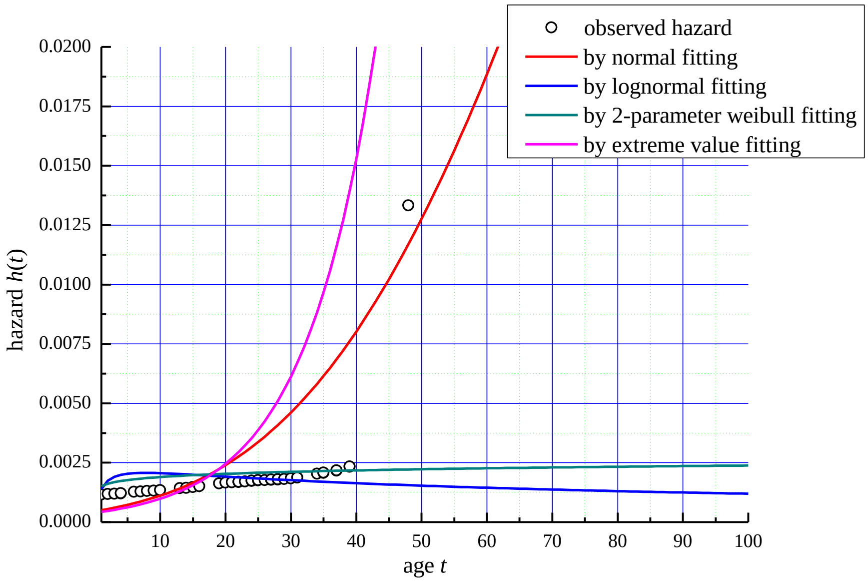


Figure 3-8 National Grid Transformer Lifetime Data Observed Hazard $h(i)$ vs. Hazard Plotting Derived Hazard $h(t)$ under Different Distribution Models

It would be also straightforward to compare the fittings by different models in Figure 3-8. It can be seen from Figure 3-8 that the normal distribution and extreme value distribution more consider the later observation, especially the last failure at age 48; the lognormal and Weibull distribution, on the other hand, fit the earlier failures well. The hazard function under lognormal distribution is derived to decrease and the Weibull model generates a roughly constant hazard. Although these two distributions are statistically accepted according to the K-S test as discussed previously, they are contrary to the engineers' prior knowledge that transformers will approach the wear-out stage with increasing hazard.

From a mathematical point of view, one can conclude that either the lognormal distribution or the 2-parameter Weibull distribution can effectively describe National Grid transformer lifetime data; mathematics however has little power to further identify the "best" distribution between the lognormal and Weibull model, because both of their CODs are comparably large (COD=0.973 under the lognormal distribution and COD=0.987 under the Weibull distribution), and according to the K-S test they are adequate models [83]. Asset managers, on the other hand, can easily overthrow these two models since the predicted population mean life ($\bar{\tau}$) of 12932 yrs under the lognormal distribution or 387 yrs under the Weibull distribution is extremely unconvincing according to the engineering judgement. In fact, engineering prior knowledge on future increasing failure hazard has to be suspended when statistically analyzing the product lifetime data.

The significantly large value of failure hazard derived from a single failure at age 48 ($h = \frac{1}{75} = 0.01333$, as shown in Figure 3-8) is due to the limited number of transformers that have the operational experience until that age. This failure was caused by the improper tap-changer maintenance under the random failure mechanism. Hence, the high value of failure hazard at age 48 does not indicate the onset of National Grid transformer increasing hazard, although the normal distribution or the extreme value distribution closely fits this last failure (as shown in Figure 3-8).

In fact, National Grid power transformer operational experience up to the present has not shown an adequate number of failures, especially failures at older ages (only one failure was recorded during age 40 and 50). The limited number of failures prevents any meaningful statistical analysis to be conducted [84]. Statistical curve fitting results would be distorted by the lifetime data of poor quality. To demonstrate this, the National Grid transformer lifetime data are now modified to arbitrarily add 10 more failures since age 48 until age 57, by which there is a bigger population of failure (62 failures) than actually recorded in National Grid's database (52 failures). The intentionally assumed 10 more failures is based on the technique referred to in Li's experimental test in [27]. The hazard plotting based on the Weibull distribution example is now implemented again. The re-produced linear plot based on the 62 failures is compared with the previous plot based on 52 failures in Figure 3-9.

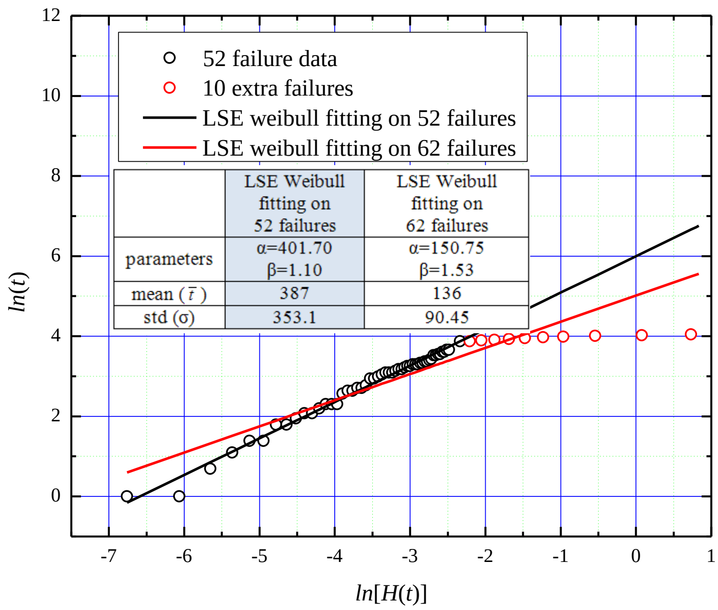


Figure 3-9 Hazard Plotting on 62 Arbitrary Failure Data under Weibull Distribution

It indicates in Figure 3-9 that if more failures were observed at later ages the population mean life ($\bar{\tau}$) and the standard deviation (σ) would reduce significantly. In fact National Grid power transformer operational experience up to the present has not shown adequate number of failures, especially failures at older ages. Moreover further studies reveal that historical failures are almost always caused by randomly-occurred transient events [40, 43, 75]. The normal distribution and the extreme value distribution models are not consistent with the fact that National Grid has not experienced a significant increase in the failure trend with age. Hence, curve fittings on the random failures cannot suggest future failures due to the ageing failure mechanism, and a decreasing hazard or constant hazard curve may be derived if the lognormal distribution or the Weibull distribution is used.

3.3.2 By CDF Plotting Approach

The CDF plotting approach is similar to hazard plotting, which also involves LSE for lifetime data curve fitting. CDF plotting calculates the reliability function $R(i)$ and then derives the cumulative probability function $F(i)$ at each failure i. $F(i)$ is written into the linear equation against the expression of age t under the presumed distribution model. LSE is then implemented to determine the parameters that best describe the linear relationship. Lifetime data curve fitting by the CDF plotting approach is undertaken in this section using National Grid operational data.

3.3.2.1 CDF Derivation

In order to calculate the reliability function $R(i)$ and then the CDF $F(i)$ at the i^{th} failure, Nelson introduced three methods in the CDF plotting approach. These are **Kaplan-Meier method**, **Herd-Johnson method** and the **actuarial method**.

- **Kaplan-Meier Method**

The Kaplan-Meier method or named product-limit method has been introduced when implementing the K-S goodness-of-fit test in the hazard plotting approach. The detailed derivation of $R(i)$ by Kaplan-Meier method can be found in Table 3-2 and an entire table for National Grid transformers is shown in Table III-2 Appendix-III. Again for the i^{th} failure, the reliability function $R(i)$ is calculated recursively by (3-5):

$$R(i)=\frac{k_{rr}(i)-1}{k_{rr}(i)}\bullet R(i-1) \tag{3-5}$$

where i indicates the i^{th} failure, $k_{rr}(i)$ is the reversed rank for failure i, $R(i\text{-}1)$ is the reliability at the last failure before i. $R(0)=1$ before the 1st failure. For a complete database including N failures, (3-5) can be simplified as the reliability at the i^{th} failure:

$$R(i) = \frac{N-i}{N} = 1 - \frac{i}{N} \tag{3-19}$$

and thus the CDF for the i^{th} failure is:

$$F(i) = 1 - R(i) = \frac{i}{N} \tag{3-20}$$

(3-20) is equivalent to (2-4) as the usual nonparametric estimation of CDF for a group of complete data. Hence the Kaplan-Meier method of estimating the reliability at each failure using the censored data, as shown in (3-5), is actually an extension of (3-19) using the complete data [26]. This method is widely used in engineering applications.

- **Herd-Johnson Method**

The Herd-Johnson method of reliability calculation for censored data was first suggested by Herd in 1960 and further developed by Johnson in 1964 [26]. The method is similar to the Kaplan-Meier method, which recursively calculates the reliability function $R(i)$ at the i^{th} failure as:

$$R(i) = \frac{k_{rr}(i)}{k_{rr}(i)+1} \bullet R(i-1) \tag{3-21}$$

where i indicates the i^{th} failure, $k_{rr}(i)$ is the reversed rank for failure i, $R(i\text{-}1)$ is the reliability at the last failure before i and $R(0)$ is assigned to be 1 before the 1st failure. For a group of complete data with N failures, (3-21) can be simplified to:

$$R(i) = \frac{N+1-i}{N+1} = 1 - \frac{i}{N+1} \tag{3-22}$$

And then $F(i)$ for the i^{th} failure is calculated as

$$F(i) = \frac{i}{N+1} \tag{3-23}$$

(3-23) is the Herd-Johnson estimation of the rank of unreliability at the i^{th} failure which has been shown in Table 2-1. The Herd-Johnson method of reliability calculation for censored data in (3-21) is consequently an extended form of reliability estimation for complete data.

- **Actuarial Method**

The procedure for using an actuarial method to derive the reliability function is summarized in the following steps using the lifetime data of National Grid transformers.

Step 1: List the ages of 865 transformers from low to high. In comparison with from the 1st step in hazard plotting, the same ages are marked only once.

Step 2: List the number of exposed transformers at each age, denoted as $N_E(t)$. Note: as in Chapter 2 the exposed transformers were defined as the units exposure to failure at a certain age, which includes the transformers failed or manually retired at that particular age.

Step 3: List the number of failures corresponding to the ages shown in Step 1, denoted as $n_F(t)$.

Step 4: Lists the number of censored transformers at the ages listed in Step 1, designated as $n_C(t)$. These censored transformers include the manually retired units at that particular age and also the in-service transformers which have not operated beyond that age.

Step 5: Calculate the number of survived transformers $N_S(t)$, still operational at age t, as $N_S(t) = N_E(t) - n_F(t) - n_C(t)$. Obviously the number of transformers that survived at the end of a certain age is equivalent to the number of exposed transformers of the next age, i.e. $N_S(t) = N_E(t+1)$. Column 5 shows the value of $N_S(t)$ at each age t.

Step 6: Calculate the reliability function $R(t)$ at failure age t recursively as

$$R(t) = R(t-1) \bullet R(t|t-1) \tag{3-24}$$

in which $R(t\text{-}1)$ is the reliability at the last age t-1, $R(t|t-1)$ is the conditional reliability at age t by the given transformers that have survived at the end of age t-1. Similarly to the Kaplan-Meier method and the Herd-Johnson method, $R(0)$=1 before the 1st failure. A simple derivation of the conditional reliability $R(t|t-1)$ is suggested by the actuarial method as

$$R(t|t-1) = 1 - \frac{n_F(t)}{N_E(t) - 0.5n_C(t)} \tag{3-25}$$

where $n_F(t)$ is the number of failures at age t, $N_E(t)$ is the exposed number of transformers at age t and $n_C(t)$ is the censored number at age t. $0.5n_C(t)$ is the adjustment for censored number by supposing those censored transformers have been operating till the middle of age t. If there is no failure at age t, the conditional reliability $R(t|t-1)=1$ at this specific age.

The process of deriving reliability $R(t)$ at age t is shown from Column 6 to Column 8.

Step 7: The cumulative failure probability $F(t)$ at age t is thereafter calculated as $1-R(t)$.

The above steps are summarized in Table 3-5. This table is a snapshot of the entire table of illustrating National Grid transformer lifetime data CDF; derived by the actuarial method. The entire table is shown in Table III-3 Appendix-III.

Table 3-5 Illustration of CDF Plotting Involving Actuarial Method Using National Grid Transformer Lifetime Data

Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7	Column 8	Column 9
Failure Age t	Number of Exposure $N_E(t)$	Number of Failure $n_F(t)$	Number of Censoring $n_C(t)$	Number of Surviving $N_S(t)$	$R(t-1)$	$1-\frac{n_F(t)}{N_E(t)-0.5n_C(t)}$	$R(t)$	$F(t)$
1	865	2	8	855	1	0.997677	0.997677	0.002323
2	855	1	13	841	0.997677	0.998821	0.996501	0.003499
3	841	1	5	835	0.996501	0.998807	0.995313	0.004687
4	835	2	17	816	0.995313	0.99758	0.992904	0.007096
5	816	0	23	793	0.992904	1	0.992904	0.007096
6	793	2	11	780	0.992904	0.99746	0.990383	0.009617
7	780	1	11	768	0.990383	0.998709	0.989104	0.010896
8	768	2	5	761	0.989104	0.997387	0.986520	0.013480
…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…
45	167	0	38	129	0.920022	1	0.920022	0.079978
46	129	0	19	110	0.920022	1	0.920022	0.079978
47	110	0	24	86	0.920022	1	0.920022	0.079978
48	86	1	22	63	0.920022	0.986667	0.907755	0.092245
49	63	0	16	47	0.907755	1	0.907755	0.092245
…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…
56	1	0	0	1	0.907755	1	0.907755	0.092245
57	1	0	1	0	0.907755	1	0.907755	0.092245

The cumulative failure probability function at each failure i, $F(i)$, derived by the Kaplan-Meier method and the Herd-Johnson method, and the cumulative probability as a function of age t, $F(t)$, derived by the actuarial method are compared in Figure 3-10 as follows.

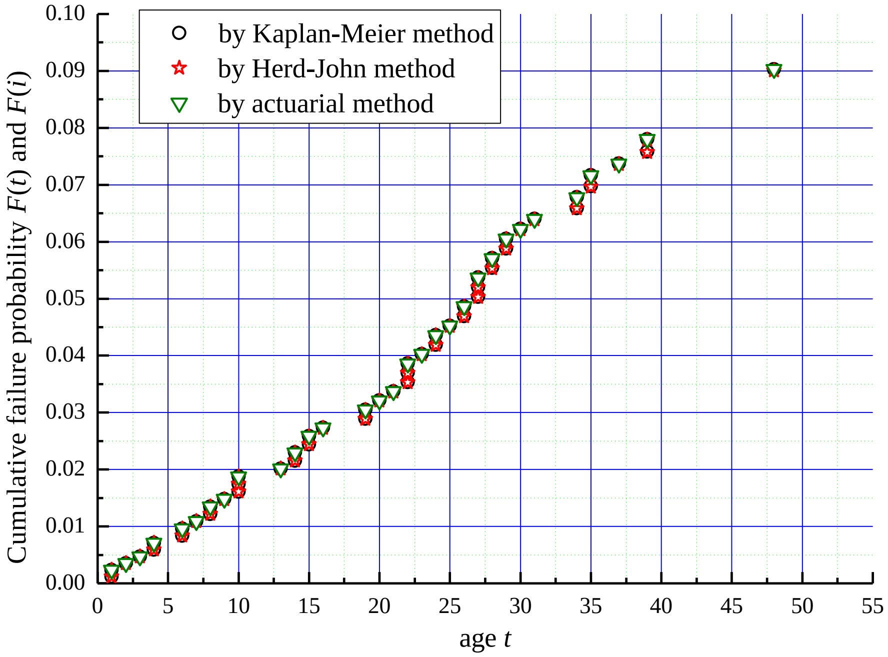


Figure 3-10 Comparisons between Kaplan-Meier Estimated $F(i)$, Herd-Johnson Estimated $F(i)$ and Actuarial Method Estimated $F(t)$ based on National Grid Transformer Lifetime Data

It can be seen from Figure 3-10 that the number of plotting points from the actuarial method is less than that from the other 2 methods. This is because transformers that failed at the same age are considered together in the actuarial method. However n failures at age t will result in n plotting points at that particular age according to the Kaplan-Meier method or the Herd-Johnson method. Hence CDF plotting involving either the Kaplan-Meier method or the Herd-Johnson method is easier to implement when the database is small. However, when the database is large, the actuarial method is better.

Generally, the cumulative probabilities at a certain age t as derived by the three methods are very similar, especially $F(i)$s from the Kaplan-Meier and Herd-Johnson method. In the following implementation of CDF plotting, the Kaplan-Meier method is used to calculate the cumulative probability $F(i)$ at the i^{th} failure. Values of $F(i)$ are shown in Table 3-2 as the observed CDF $F^0(i)$.

3.2.2.2 CDF Plotting via Kaplan-Meier Method

- **Curve Fitting Procedure**

Similarly to the hazard plotting, the CDF plotting approach plots the age of failure t or $ln(t)$ against the expressions of $F(i)$ according to a linear equation under a specific distribution model. These equations are written here as

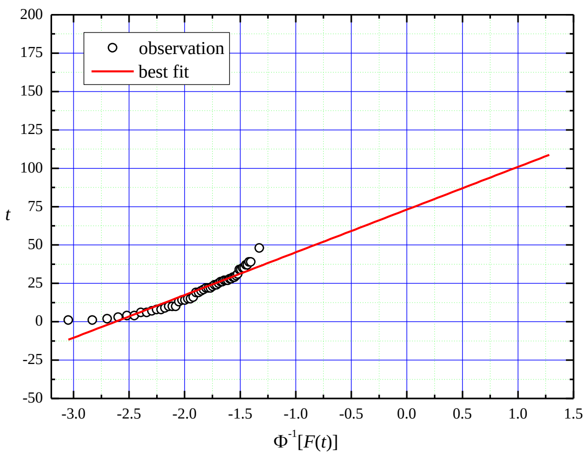
Normal distribution: $t = \sigma \bullet \Phi^{-1}\left[F(t)\right] + \mu$ (3-26)

Lognormal distribution: $ln(t) = \delta \bullet \Phi^{-1}\left[F(t)\right] + \lambda$ (3-27)

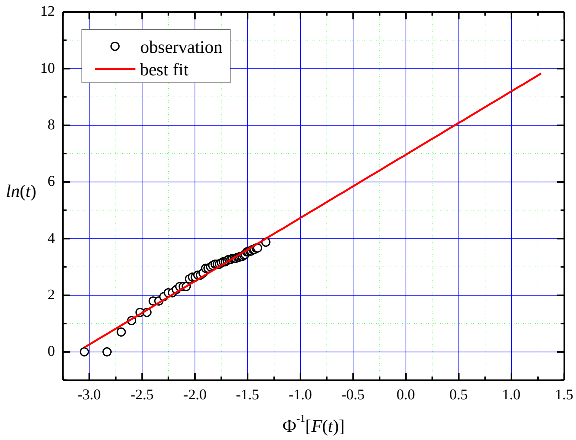
2-parameter Weibull distribution: $ln(t) = \frac{1}{\beta} ln\left\{-ln\left[1 - F(t)\right]\right\} + ln\,\alpha$ (3-28)

Extreme value distribution: $t = \delta \bullet ln\left\{-ln\left[1 - F(t)\right]\right\} + \lambda$ (3-29)

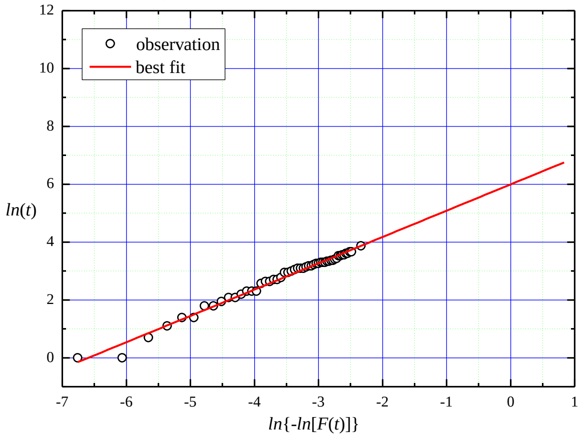
LSE is then carried out to draw a straight line to best fit the plotted data. The parameters of each distribution can be graphically estimated. The plotted points and the best-fitting linear curves under these distribution models are shown in Figure 3-11, from (a) to (d).



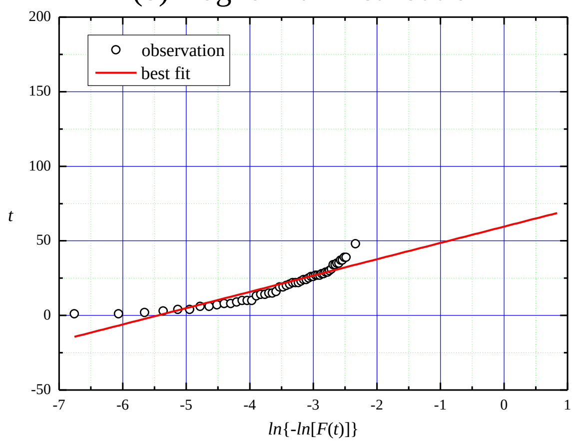
(a) Normal Distribution



(b) Lognormal Distribution



(c) 2-Parameter Weibull Distribution



(d) Extreme Value Distribution

Figure 3-11 CDF Plotting under Traditional Distribution Models Using National Grid Transformer Lifetime Data

- **Curve Fitting Result**

Values of parameters of the normal distribution, lognormal distribution, Weibull distribution and extreme value distribution are determined as shown in Table 3-6. National Grid transformer population mean life ($\bar{t}$) and the standard deviation (σ) are further calculated and presented in Table 3-6. The failed transformers' arithmetic mean

($\bar{t}$) and standard deviation (σ) are also shown. The K-S goodness-of-fit test results and the curve fitting determination coefficients (COD) are indicated in the table.

Table 3-6 CDF Plotting Results involving Kaplan-Meier Method in National Grid Transformer Lifetime Data Analysis

	Normal distribution	Lognormal distribution	Weibull distribution	Extreme value distribution	From failures only
parameters	μ=73.05 σ=27.84	λ=6.97 δ=2.24	α=401.30 β=1.10	λ=59.54 δ=10.93	
Mean $\bar{t}$ (year)	73	12897	387	53	20
Std σ (year)	27.8	156420	352.7	14.0	11.5
K-S	**R**	**A**	**A**	**R**	
COD	**0.903**	**0.973**	**0.987**	**0.854**	

The comparison of Table 3-6 with Table 3-3 shows that by using CDF plotting under the normal distribution, the values of the derived population mean life ($\bar{t}$) and the standard deviation (σ) are similar to the values obtained by the hazard plotting approach under the normal distribution ($\bar{t}$ =73 yrs and σ=27.8 yrs). Under the Weibull and extreme value distribution, the population mean life ($\bar{t}$) and the standard deviation (σ) are also similar to the corresponding values from the hazard plotting. Under the lognormal distribution, the values of population mean life ($\bar{t}$) and the standard deviation (σ) are less when using CDF plotting rather than hazard plotting. The similar results shown in Table 3-3 and Table 3-6 reveal that the hazard plotting approach and the CDF plotting approach involving the Kaplan-Meier method are essentially the same, because they both rely on LSE.

It is further verified that the plotting points used in these two approaches are similar. As introduced in Chapter 2, $F(i)$ used in CDF plotting can be converted into the cumulative hazard function $H(i)$ for failure i according to

$$H(i) = -ln\left[1-F(i)\right] \tag{3-30}$$

In order to generate similar plotting points, the $H(i)$s derived by the CDF plotting $F(i)$s are plotted against age t. The values are then compared with the cumulative hazards $H(i)$ used in the hazard plotting approach. Figure 3-12 shows the comparison.

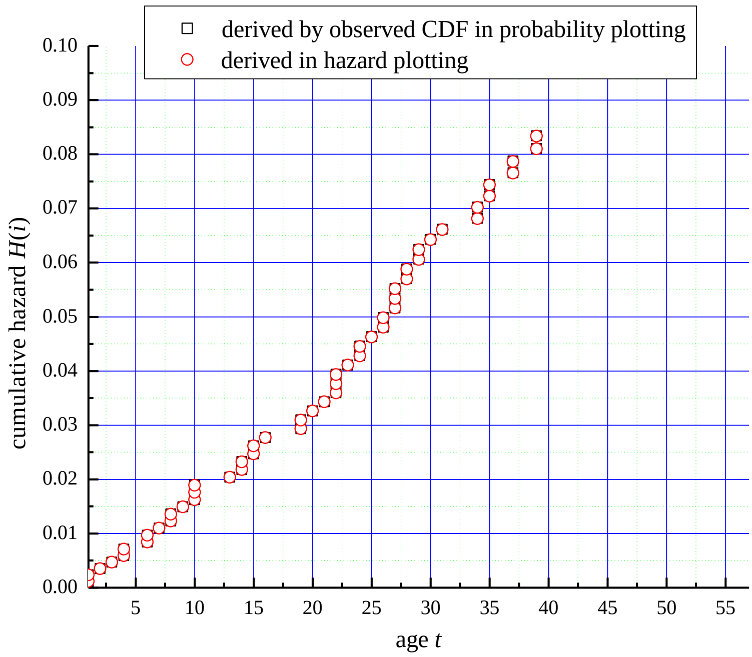


Figure 3-12 CDF Plotting Approach vs. Hazard Plotting Approach Using National Grid Transformer Lifetime Data

As can be seen from Figure 3-12, the value of cumulative hazard $H(i)$, converted by the observed $F(i)$ in the Kaplan-Meier CDF plotting approach, is almost the same as $H(i)$ in the hazard plotting approach. In order to implement LSE, the hazard plotting approach organizes the failure data in the linear form between age t and $H(i)$, while the CDF plotting uses the linear equation involving age t and $F(i)$. These two plotting approaches are essentially the same.

- **CDF Plotting via Actuarial Method**

In comparison, the plotting points in the actuarial method are different to the plotting points derived by either of the other two methods. The difference is illustrated in Figure 3-10. CDF plotting on National Grid transformer lifetime data involving the actuarial method is hereby carried out for comparison purpose. The results are shown in Table 3-7.

Table 3-7 CDF Plotting Results involving Actuarial Method in National Grid Transformer Lifetime Data Analysis

	Normal distribution	Lognormal distribution	Weibull distribution	Extreme value distribution	From failures only
parameters	μ=75.27 σ =29.11	λ=7.07 δ=2.30	α=448.05 β=1.06	λ=61.91 δ=11.65	
Mean $\bar{t}$ (year)	75	16725	438	55	20
Std σ (year)	29.1	236540	415.6	14.9	11.5
K-S	**R**	**A**	**A**	**R**	
COD	**0.914**	**0.970**	**0.987**	**0.873**	

The comparison of Table 3-3 with Table 3-6 and Table 3-7, shows that by using the actuarial method, the population mean life ($\bar{t}$) and the standard deviation (σ) under a specific distribution are slightly larger than the values obtained by using the hazard plotting or the CDF plotting involving the Kaplan-Meier method. For example, under the normal distribution, the population mean life is shown to be 75 yrs and the standard deviation is 29.1 yrs according to Table 3-7 by using the actuarial method ($\bar{t}$=75 yrs and σ=29.1 yrs), as compared to $\bar{t}$=73 yrs and σ=27.8 yrs presented in Table 3-6 when using the Kaplan-Meier method. This is because in either the Kaplan-Meier or the Herd-Johnson method failures even at the same age are individually considered; i.e. ages with more than one failure are assigned higher weightings as compared to ages with just one failure. These highly weighted ages essentially pull the population mean life backward to early ages. The differences are not significant and the K-S test draws the same conclusion to each distribution model when involving the Kaplan-Meier method and the actuarial method.

Section 3.3.1 and 3.3.2 indicate that a large amount of effort is required to organize the lifetime data in either the hazard plotting or the CDF plotting approach. The data collection may become complicated especially when the database is large and heavily censored. Li therefore modified these classical approaches in order to simplify lifetime data statistical analysis for an engineering application. However restrictions need to be borne in mind when using Li's approach. Li's approach is introduced in the following section and the restrictions are emphasised at the end of the section.

3.3.3 By Li's Approach

3.3.3.1 Introduction of Li's Approach

- **Curve Fitting Procedure**

As a modified approach to the hazard plotting and CDF plotting approach, Li's approach is also based on LSE. The procedure of implementing Li's modified approach is summarized as follows, and applied to the analysis of the National Grid transformer lifetime data.

Step 1: List ages of transformers from lowest to highest in Column 1. Similarly to the actuarial method, the same ages are marked only once.

Step 2: Calculate the number of failures $n_F(t)$ at age t in Column 2.

Step 3: Calculate the number of exposed transformers $N_E(t)$ at age t in Column 3.

The above three steps are the same as the first three steps in the actuarial reliability calculation in the CDF plotting.

Step 4: Calculate the ratio of the failure number $n_F(t)$ to the exposed number $N_E(\mathrm{t})$ at age t, i.e. $\frac{n_F(t)}{N_E(t)}$. As discussed in Chapter 2, $\frac{n_F(t)}{N_E(t)}$ actually calculates the hazard value at age t. Li however designates this expression as ***failure probability f*(*t*)** at age t and further calculates the ***cumulative failure probability F*(*t*)** in the next step.

Step 5: Calculate the ***cumulative failure probability F*(*t*)** at age t as the sum of the failure probability $f(t)$ at each age up to age t.

The above 5 steps are summarized in Table 3-8.

Table 3-8 Li's Modified Approach Illustration

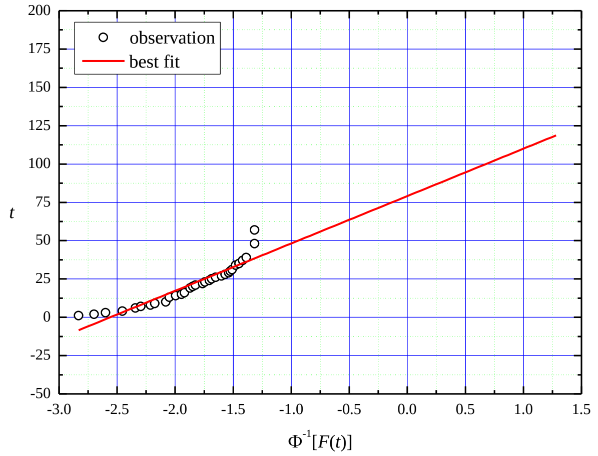
Age t	Number of failure	Number of exposed transformer	***Failure probability $f(t)$***	***Cumulative failure probability $F(t)$***
1	2	865	0.002312	0.002312
2	1	855	0.001170	0.003482
3	1	841	0.001189	0.004671
4	2	835	0.002395	0.007066
5	0	816	0	0.007066
6	2	793	0.002522	0.009588
7	1	780	0.001282	0.010870
8	2	768	0.002604	0.013474
9	1	761	0.001314	0.014788
10	3	754	0.003979	0.018767
11	0	733	0	0.018767
12	0	717	0	0.018767
13	1	707	0.001414	0.020182
14	2	699	0.002861	0.023043
15	2	689	0.002903	0.025946
16	1	669	0.001495	0.027440
17	0	654	0	0.027440
18	0	633	0	0.027440
19	2	622	0.003215	0.030656
20	1	605	0.001653	0.032309
21	1	599	0.001669	0.033978
22	3	595	0.005042	0.039020
23	1	589	0.001698	0.040718
24	2	579	0.003454	0.044172
25	1	572	0.001748	0.045920
26	2	569	0.003515	0.049435
27	3	564	0.005319	0.054754
28	2	558	0.003584	0.058339
29	2	555	0.003604	0.061942
30	1	547	0.001828	0.063770
31	1	538	0.001859	0.065629
32	0	527	0	0.065629
33	0	510	0	0.065629
34	2	497	0.004024	0.069653
35	2	486	0.004115	0.073769
36	0	476	0	0.073769
37	2	467	0.004283	0.078051
38	0	453	0	0.078051
39	2	434	0.004608	0.082660
40	0	423	0	0.082660
41	0	398	0	0.082660
42	0	340	0	0.082660
43	0	276	0	0.082660
44	0	220	0	0.082660
45	0	167	0	0.082660
46	0	129	0	0.082660
47	0	110	0	0.082660
48	1	86	0.011628	0.094287
49	0	63	0	0.094287
50	0	47	0	0.094287
51	0	35	0	0.094287
52	0	26	0	0.094287
53	0	19	0	0.094287
54	0	13	0	0.094287
55	0	8	0	0.094287
56	0	1	0	0.094287
57	0	1	0	0.094287

Step 6: According to Li, a ***cumulative failure probability F*(*t*)** as small as 0.001 or 0.0001 is inserted at the age before the 1st failure. This is aimed to point out that the CDF before the 1st failure is negligible. In this specific case of National Grid transformer lifetime data analysis, however, this does not need to be concerned, as two transformers failed at age 1.

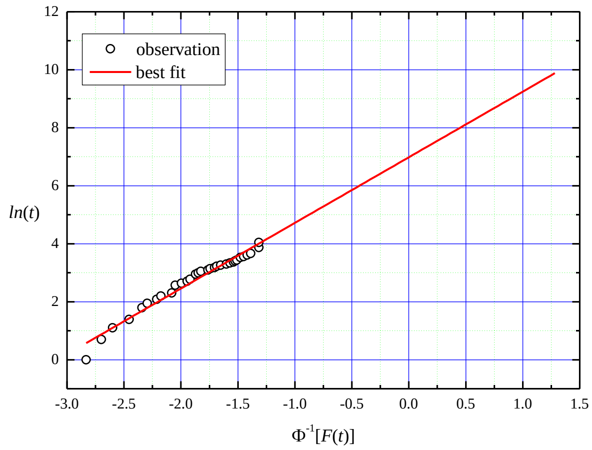
Step 7: Because there are no failures between *t*=49 and until the last age *t*=57 in National Grid's database, the ***cumulative failure probability F*(*t*)** at age 57 is the same as the value derived at age 48 (***F*(57)**=***F*(48)**=0.094287). According to Li, the last age 57 and the corresponding ***cumulative failure probability F*(57)** are also considered in analysis.

Step 8: In Li's work the failure age *t*, including the age before the 1st failure and the last age without failure, is plotted against $\Phi^{-1}\left[F(t)\right]$ under the normal distribution; whilst *ln*(*t*) is plotted against $ln\left\{-ln\left[1-F(t)\right]\right\}$ under the Weibull distribution. LSE is used to fit the plotted data and then the values of the parameters are estimated.

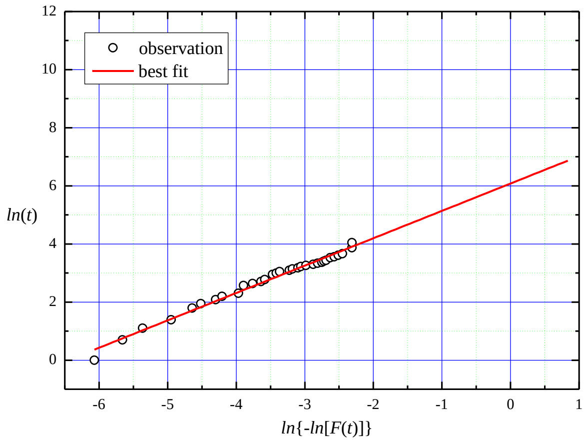
When using Li's approach for National Grid transformer lifetime data curve fitting, the lognormal distribution and the extreme value distribution are used according to the linear equations (3-26) and (3-28). The linear plots under the normal, lognormal, Weibull and extreme value distribution are illustrated in Figure 3-13 as follows.



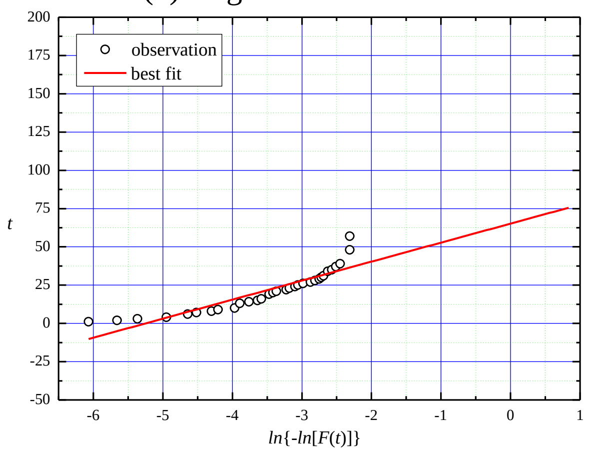
(a) Normal Distribution



(b) Lognormal Distribution



(c) 2-Parameter Weibull Distribution



(d) Extreme Value Distribution

Figure 3-13 Li's Modified Approach under Traditional Distribution Models Using National Grid Transformer Lifetime Data

- **Curve Fitting Result**

The estimates of the parameters under each model, derived population mean life ($\bar{t}$) and the standard deviation (σ) are shown in Table 3-9. The K-S test results and curve fitting COD are also indicated in Table 3-9.

Table 3-9 Results of Li's Approach in National Grid Transformer Lifetime Data Analysis

	Normal distribution	Lognormal distribution	Weibull distribution	Extreme value distribution	From failures only
parameters	μ=79.09 σ =30.90	λ=6.98 δ=2.26	α=437.25 β=1.06	λ=65.18 δ=12.42	
Mean $\bar{t}$ (year)	79	13942	427	58	20
Std σ (year)	30.9	179830	402.610	15.9	11.5
K-S	**R**	**A**	**A**	**R**	
COD	**0.870**	**0.969**	**0.986**	**0.825**	

Li's approach also reveals that the lognormal distribution and the Weibull distribution are both a better fit to the National Grid transformer lifetime data, when assessed according to the K-S goodness-of-fit test results and the higher COD values. The estimates of parameters, the population mean life ($\bar{\tau}$) and the standard deviation (σ) under a certain distribution are more or less similar to the values derived by CDF plotting involving the actuarial method. The reasons can be further clarified as:

- ***Firstly*** Li's and CDF plotting approach, involving the actuarial method, have approximately the same number of plotting points, except that Li considers the last age up to the present without failure.
- ***Secondly*** the ***cumulative failure probability F(t)*** at age *t* in Li's approach is similarly developed as the CDF at the same age *t* by the actuarial method.

The above two reasons can be illustrated by comparing the ***cumulative failure probability F(t)*** in Li's approach with the CDF from the actuarial method. Figure 3-14 shows the comparison plot.

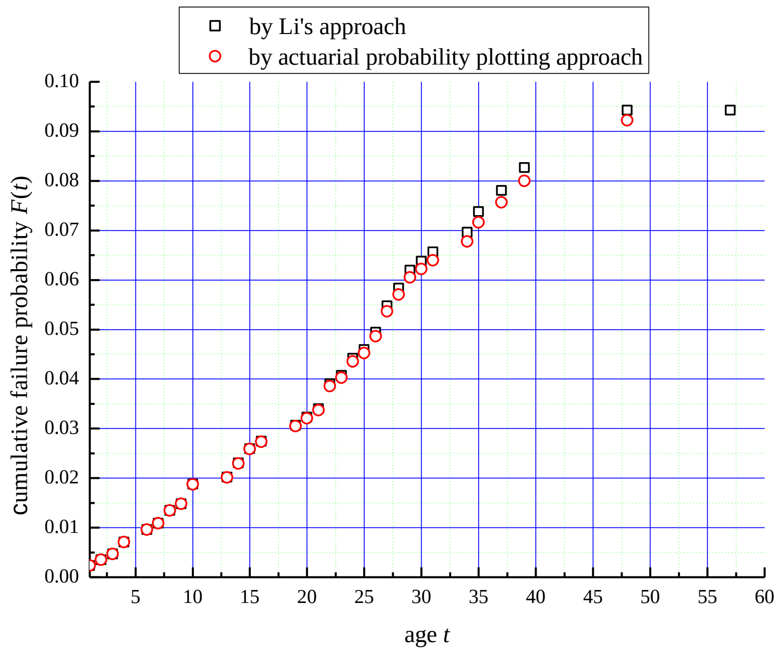


Figure 3-14 Comparison of $F(t)$ in Li's Approach and $F(t)$ in Actuarial Probability Approach Using National Grid Transformer Lifetime Data

According to Figure 3-14, the ***cumulative failure probability F(t)*** at age *t* developed by Li is slightly higher than the $F(t)$ derived using the CDF plotting approach by the actuarial method, especially after age 25. These slightly higher values and the artificially added point at the last age actually result in the difference in the population mean life between Li's approach and the actuarial CDF plotting approach.

3.3.3.2 Limitations of Li's Approach

The lifetime data curve fitting approach developed by Li and based on LSE is easier to implement, especially for a large group of censored data, as compared to classical plotting approaches. Li's approach has been verified by applying it to BC Hydro 500kV reactors with limited failure [27, 28]; however several restrictions need to be kept in mind when using Li's approach.

- **Firstly, the cumulative failure probability *F*(*t*) is misleading-defined by Li.**

The ***failure probability function f*(*t*)** and the ***cumulative failure probability F*(*t*)** developed in Li's approach are actually the observed failure hazard *h*(*t*) at age *t* and the cumulative hazard *H*(*t*) up to age *t*, according to their definitions as explained in Chapter 2.

According to Taylor Series[i],

$$-ln\left[1-F(t)\right]=F(t)+\frac{F(t)^2}{2}+\frac{F(t)^3}{3}+...+\frac{F(t)^\infty}{\infty} \tag{3-31}$$

Once *F*(*t*) is small, the sum of the 2nd, 3rd, … components of the right-hand side of (3-31) are significantly small as compared to the 1st component *F*(*t*). (3-31) can thus be simplified as -*ln*[1-*F*(*t*)]=*F*(*t*). Additionally *H*(*t*)=-*ln*[1-*F*(*t*)], *H*(*t*)=*F*(*t*) is only satisfied when *F*(*t*) is close to zero. In this specific study of National Grid transformer lifetimes, the *F*(*t*) derived by the actuarial method is as small as 0.092245 at the last age, while the cumulative hazard *H*(*t*) calculated by Li's approach is 0.094287. *F*(*t*) and *H*(*t*) are close to each other, and the estimates of the parameters under a certain distribution are similar using these two approaches. However, a small *F*(*t*) cannot always be satisfied especially when failures occur at older ages.

For example suppose an extra transformer fails at age 57, the cumulative failure probability *F*(*t*) according to the actuarial method is derived to be **1** at age 57; while the

---

[i] According to Taylor Series $ln\left[1-x\right]=-\sum_{n=1}^{\infty}\frac{x^n}{n}$, for -1≤*x*≤1.

cumulative failure probability F(t) (actually the cumulative hazard $H(t)$) according to Li's approach is deduced as **1.1292**. $F(t) \neq H(t)$ in this hypothesized case. Additionally the value 1.1292 for $F(t)$ according to Li is outside the definition $0 \leq F(t) \leq 1$.

- **Secondly, a small value of 0.001 or 0.0001 is arbitrarily inserted for *F*(*t*) at the age before the 1st failure**, although this is not considered in National Grid transformer lifetime data analysis since two transformers failed at age 1. Even though this small number can be mathematically proven to have negligible effect on the curve fitting result as the age corresponding to either 0.001 or 0.0001 is outside the range of $\mu \pm 3\sigma$[ii]. Note: - it is illogical to assume that CDF jumps from a negligible value at age *t*-1 to a certain value at age *t*. According to the theory of a continuous distribution, the best estimation for CDF at age *t*-1 should not be very different to the value of CDF at age *t*.

- **Thirdly, the inserted *F*(*t*) at the last age provides extra information for curve fitting.** Li proposed this step as it emphasises the fact that up to the last age there are no more failures observed and thus the cumulative failure probability has no increase since the last failure occurred at an early age. Essentially however, since the lifetime data are multiply right-time censored, transformer assets in particular have too little operational experience of failure, rather than they will not fail. If the failure number is significantly small and the data are heavily censored since early ages, adding $F(t)$ at the last age can have large influence on the fitting results.

As introduced in Chapter 2, the application of LSE is straightforward, but this optimization method is mathematically robust only when the sample size is small and the data are complete or at least not heavily censored [67]. For the heavily censored data, for example National Grid transformer assets lifetime data, the maximum likelihood estimator (MLE) will be a more suitable method for curve fitting. MLE is therefore carried out in the following section.

[ii] For example, for a standard normal distribution, the CDF of 0.001 corresponds to the age t=-3.09023. This value falls outside the region of $\pm 3\sigma = \pm 3$. Because the region of $\pm 3\sigma$ covers 99.86% cumulative probabilities, the ages (t) out of this region have insignificant effect in lifetime data analysis.

3.4 National Grid Transformer Lifetime Data Statistical Analysis by Maximum Likelihood Estimator (MLE)

- **Curve Fitting Procedure**

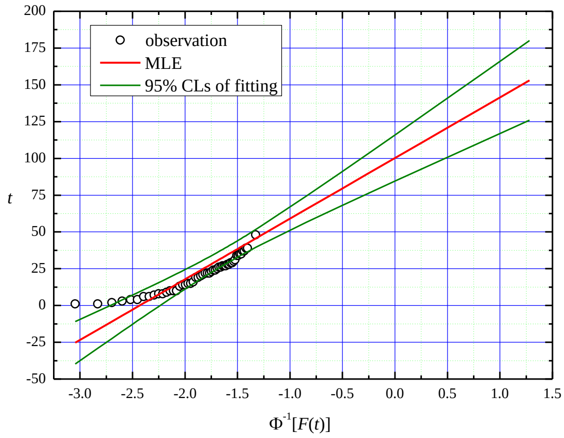
The application of MLE on National Grid transformer lifetime data is carried out in MALAB using the built-in function MLE. Information of 865 transformers including 52 failed units, 62 manually retired units and 751 in-service transformers are organized into 2 columns:

- 1st column categorises the ages of 865 transformers from low to high,
- 2nd column indicates the status of each transformer: "**0**" represents a manually retired or in-service transformer and "**1**" indicates a failed unit.

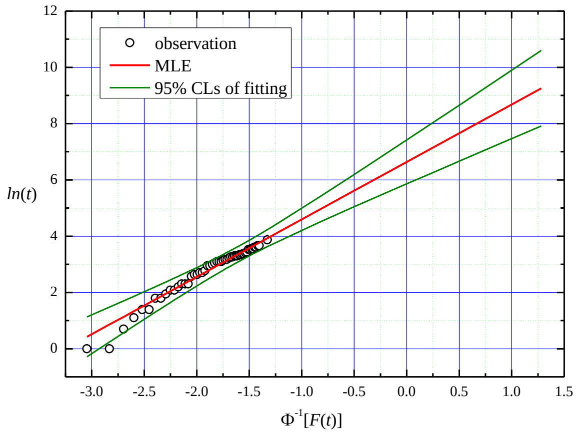
These two columns are the inputs of MLE, and they are the same as the first two columns of Table 3-1. Again an entire table is shown in Table III-1 Appendix-III.

The normal distribution, lognormal distribution, 2-parameter Weibull distribution and extreme value distribution are selected to fit the lifetime data. Based on a presumed distribution model, MLE is used to estimate the most likely values of parameters which can maximize the likelihood of the observed data. Associated with MLE, the initial values of the distribution parameters are provided by the estimation engine "garchfit" [85], which is also a built-in function in MATLAB; however the discussion about this estimation engine is beyond the scope of the thesis. The detailed algorithm of MLE can be found in Appendix-I. The most advanced property of MLE is that except for the estimates of the parameters under a presumed distribution, it also provides the confidence limits for parameters and the confidence limit for the age corresponding to a certain value of CDF.

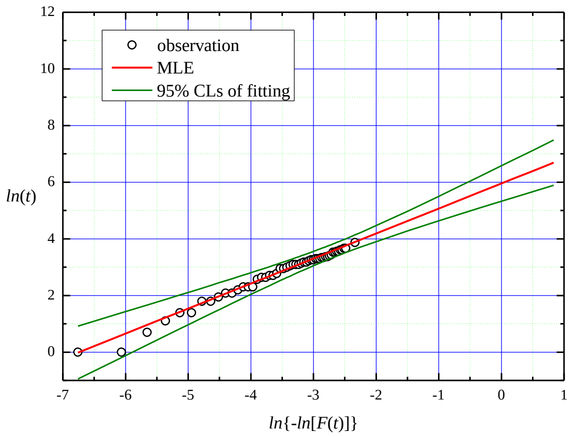
Figure 3-15 (a) to (d) shows National Grid transformer lifetime data probability plots under the normal, lognormal, Weibull and extreme distribution determined by MLE.



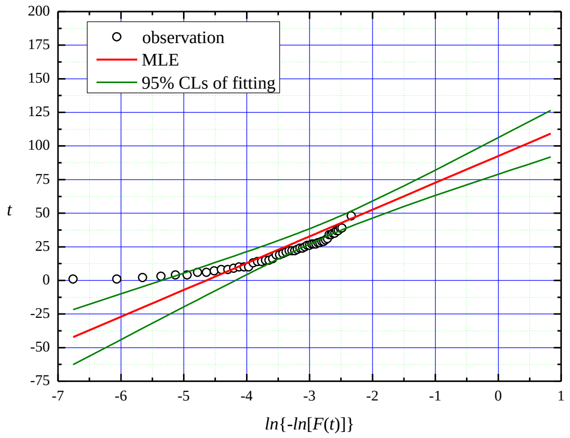
(a) Normal Distribution



(b) Lognormal Distribution



(c) 2-Parameter Weibull Distribution



(d) Extreme Value Distribution

Figure 3-15 Probability Plots of MLE Fitting under Traditional Distribution Models Using National Grid Transformer Lifetime Data

In each graph of Figure 3-15, the black circles of observed cumulative failure probabilities CDF are derived by the Kaplan-Meier method introduced in 3.3.2, the red line shows the linear curve best fit to the plotted points by MLE under each presumed distribution, and the green lines are the upper and lower 95% confidence limits accompanying the red curve.

- **Curve Fitting Result**

The best estimates of parameters, population mean life ($\bar{t}$), standard deviation (σ) and K-S goodness-of-fit test results are displayed in Table 3-10. For the MLE algorithm the coefficient of determination (COD) is not a measurement of fitting goodness as for the LSE algorithm and consequently it is not listed in Table 3-10.

Table 3-10 MLE Results in National Grid Transformer Lifetime Data Analysis

	Normal distribution	Lognormal distribution	Weibull distribution	Extreme value distribution	From failures only
parameters	μ=100.24 σ =41.23	λ=6.64 δ=2.04	α=384.94 β=1.13	λ=92.55 δ=19.94	
Mean $\bar{\tau}$ (year)	100	6152	368	81	20
Std σ (year)	41.2	49088	325.7	25.6	11.5
K-S	**A**	**A**	**A**	**A**	

According to Table 3-10, transformer population mean life ($\bar{\tau}$) and the standard deviation (σ) under the normal distribution or extreme value distribution by MLE are derived longer than the values given by LSE; while the mean life ($\bar{\tau}$) and standard deviation (σ) by MLE when fitting into the lognormal and Weibull distribution are less than the values determined by LSE. The transformer population mean life of 6152 yrs under the lognormal distribution model is too large to be convinced and the mean life of 368 yrs under the Weibull distribution is also lack of engineering evidence to be relied on.

By comparing the corresponding graphs in Figure 3-11 and Figure 3-15, and the results shown in Table 3-6 and Table 3-10 based on the same lifetime data, the differences between MLE and LSE can be concluded as:

- the MLE determined the best-fit curve (shown in red lines in Figure 3-15), under a certain distribution model, that fits the later failures better than the earlier failures, while LSE treats the plotted data equally and thus the best-fit curve locates in the middle of the plotted data.
- Using MLE, all of the presumed distribution models are suitable for the National Grid transformer lifetime data, as identified by the K-S test results with the same tolerance factor $D_M^{\gamma}(r)$ = 0.077228. Hence, in engineering applications, K-S test is not usually carried out when using MLE for product lifetime data curve fitting.

The theoretical CDFs under the above distributions are derived according to the obtained parameters and are compared with the observed hazard CDFs in Figure 3-16.

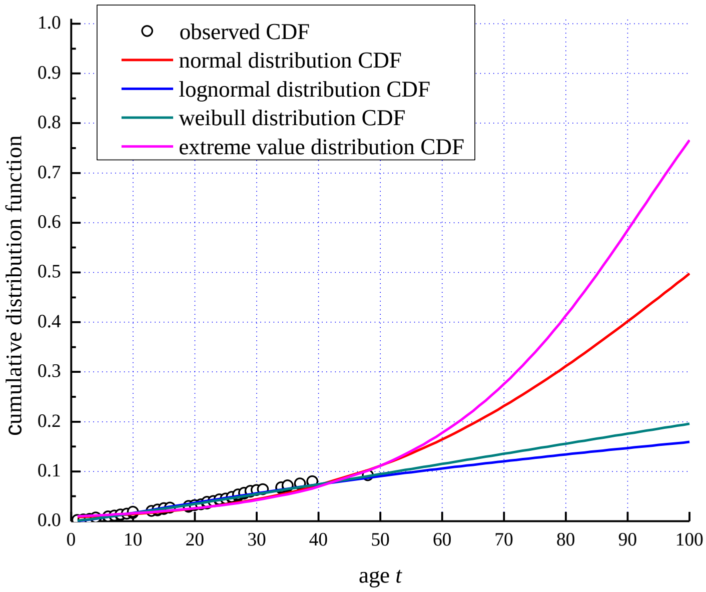


Figure 3-16 National Grid Transformer Lifetime Data Observed CDF vs. MLE Derived CDF under Different Distribution Models

Figure 3-16 shows that all the selected distribution models can fit National Grid transformer lifetime data properly, which is consistent with the K-S test judgement. The CDF curves under the normal distribution and extreme value distribution however increases significantly at later ages. Furthermore according to Figure 3-16 the ages of reaching three typical cumulative failure probabilities under different distribution models can be obtained and are compared in Table 3-11 as follows.

Table 3-11 MLE Derived Typical Ages in National Grid Transformer Lifetime Data Analysis

	Age *t*			
CDF *F*(*t*)	By normal	By lognormal	By Weibull	By extreme
2.5%	20	14	15	19
50%	101	>500	279	85
97.5%	181		>500	119

It can be seen from Table 3-11 that the typical ages derived under the normal distribution and the extreme value distribution are more reasonable values than under the other two distribution models.

Moreover the theoretical hazards under the above four distributions are also compared with the observed hazards as plotted in Figure 3-17.

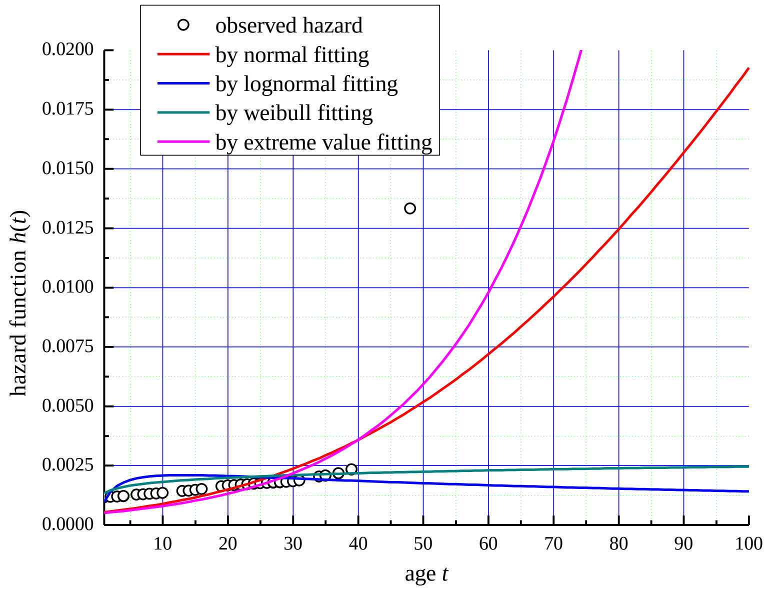
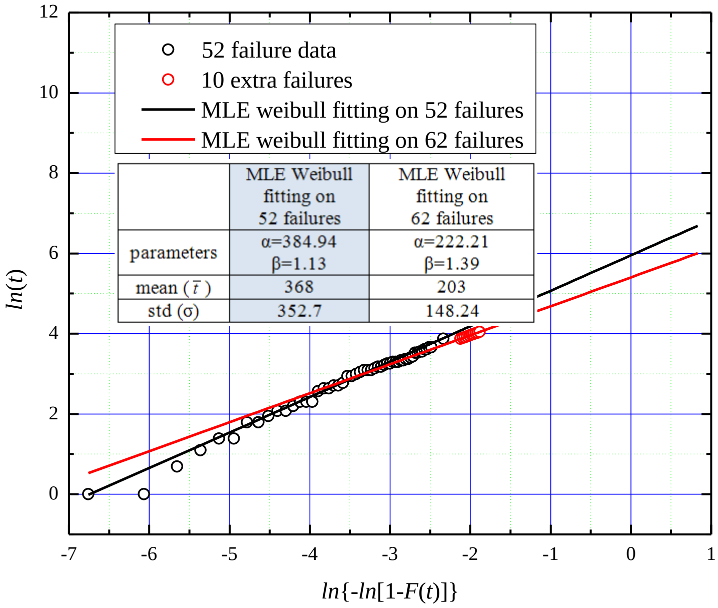


Figure 3-17 National Grid Transformer Lifetime Data Observed Hazard $h(i)$ vs. MLE Derived Hazard $h(t)$ under Different Distribution Models

Similarly to the results from LSE hazard plotting, the hazard curve under the Weibull model shows a roughly constant value. The normal hazard and the extreme value hazard are both increasing against age and the hazard values under these two models are similar before age 40. The lognormal distribution shows a decreasing hazard and the values before age 30 are similar to those derived under the Weibull model; these two models both fit the early failures well. Moreover it can be generally found by comparing Figure 3-17 with Figure 3-8 that the hazard curves derived by MLE under the lognormal and Weibull distribution are fairly similar to the hazard curves estimated by LSE under these two models.

The estimated failure trend under the normal distribution and the extreme value distribution, as shown in Figure 3-17, is coincident with the engineering prior knowledge that transformers failure hazard will develop to the wear-out stage in accordance with an increasing hazard against age. Consequently the normal distribution and extreme value distribution are better fits for National Grid transformer lifetime data from an asset manager's point of view. However, as explained when using the hazard plotting approach, the high value of failure hazard at the last failure at age 48 (again $h=\frac{1}{75}=0.01333$, as shown in Figure 3-17) is obtained based on the small number of transformers that have been operated at that age, and this high value of hazard at age 48 does not indicate the onset of National Grid transformer ageing-related failures with an increasing hazard. The validity of curve fitting under the normal distribution or the extreme value distribution, which presents an increasing hazard curve with age, is debatable.

In other words, as discussed in hazard plotting, National Grid transformer operational experience has not yet revealed a significant increasing failure hazard with age. In order to verify the influence of data quality on the curve fitting results, MLE analysis is carried out on 62 transformer failure data with 10 more failures arbitrarily added since age 48 until age 57. The re-produced probability plot under Weibull distribution based on 62 failures is compared with the previous fitting in Figure 3-18.



	MLE Weibull fitting on 52 failures	MLE Weibull fitting on 62 failures
parameters	α=384.94 β=1.13	α=222.21 β=1.39
mean ($\bar{\tau}$)	368	203
std (σ)	352.7	148.24

Figure 3-18 MLE Fitting on 62 Arbitrary Failure Data under Weibull Distribution

According to the hypothesised case with 10 more failures at later ages, as indicated in Figure 3-18, National Grid transformer population mean life ($\bar{\tau}$) and the corresponding standard deviation (σ) would be significantly reduced. Since the ageing-related failures have not been observed up to the present, distribution models with increasing hazard, for example the Weibull distribution, have little power to predict the increasing hazard based on constant random hazards.

Generally lifetime data curve fitting by LSE, MLE or the Bayesian approach, not implemented in this thesis, is a purely mathematical description of product lifetime data. The engineering prior knowledge is difficult to be included in the statistical process. A good fitting from a mathematical point of view is therefore very likely to be overthrown or suspended by engineering judgement. In particular, for National Grid transformer lifetime data analysis, limited failure data and an underlying random failure mechanism should not be used to generate ageing-related failures in the future. The advantages and restrictions of statistical analysis are further summarized in the following section.

3.5 Advantages and Restrictions of Statistical Analysis on Lifetime Data

3.5.1 Advantages

As can be seen from 3.3 and 3.4, product lifetimes, for instance power transformer lifetime data, can be easily analyzed statistically using LSE or MLE once the failure record is available.

Statistics concerning the age of individual products, for example transformers, because they only involve one variable, are simple to analyse. Traditional distributions provide empirical models which the observed lifetime data are expected to fit. The population mean life can thereafter be derived. The LSE, MLE and Bayesian approach are well developed in general statistics and they are also verified in engineering application. Analysis softwares have been well produced and released for commercial use in order to simplify the sophisticated calculation especially when the sample size is large or the data are heavily censored [26].

However, mathematical analysis may mislead the conclusion in an engineering application, since it is based on several false suppositions, as discussed in 3.5.2.

3.5.2 Restrictions

The restrictions of implementing statistical analysis on product lifetime data based on the use of National Grid transformer lifetime data analysis are listed as follows.

- Although the service age of each individual transformer is deduced as the only variable, it is not the only determinant factor to transformer failure. According to National Grid operational experience, transformer design, loading condition, maintenance strategy and even the installation site environment can affect the transformer’s lifetime more than its service age.

- The distribution models are arbitrarily chosen, although some distributions are verified as proper models according to experience with other items of high voltage plant. This is the reason utilities prefer to define their own asset failure models, based on operational experience or engineering prior knowledge, instead of using a traditional distribution. However for power transformers in particular, there is little operational experience that can be used to suggest any appropriate models. Hence, a mathematical good fit, can sometimes be rejected by asset managers.

- The quality of lifetime data and the underlying failure mechanism would essentially affect the fitting results. According to National Grid transformer operational experience, limited failures are recorded and typically they occur at early ages. In addition these failures almost always occur randomly. National Grid transformers are thus in the normal operating stage of the bathtub curve up to the present and the prediction of future failures due to the ageing failure mechanism cannot be relied on, even though curve fitting is statistically approved.

National Grid transformer random failures are more clearly indicated when discussing operational experience in 3.6.

3.6 National Grid Transformer Historical Failure Analysis and Evidence of Random Failure Mechanism

National Grid transformer failure hazard function is first of all derived and the underlying random failure mechanism further identified. The drawback of undertaking statistical analysis on random failure samples to predict future ageing-related failure is again emphasized.

As indicated in 3.2.2, 52 transformers are failed units recorded in the National Grid system. The number of failures against age is shown in Figure 3-19.

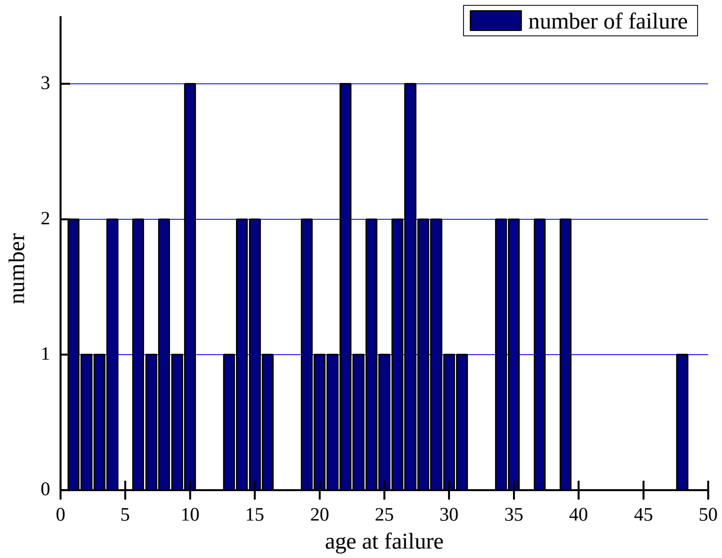


Figure 3-19 National Grid Transformers Number of Failure against Age

The exposed number of National Grid transformers against service age can also be derived. Again as defined in Chapter 2, the product exposed number is the number of products exposure to failure at a certain age; product exposed number indicates the total number of products which have had service experience until a particular age [84]. The exposed number of National Grid transformers at each age is obtained based on information about installation and replacement, i.e.:

$$\text{transformer exposed number @ age } t = \text{total installation number} - \sum_{i=1}^{t-1} \text{failed number} - \sum_{i=1}^{t-1} \text{scrapped number} \quad (3\text{-}32)$$

The upper limit *t*-1 in the sum notation $\sum_{i=1}^{t-1}$ for either the failed transformers or the scrapped transformers, indicates that the transformers that failed or were scrapped at age *t* are actually exposed to failure before their replacement. Hence, both the failed transformers and the scrapped transformers at age *t* are taken into account in the exposed number at that specific age. The National Grid transformers exposed number against age is shown in Figure 3-20.

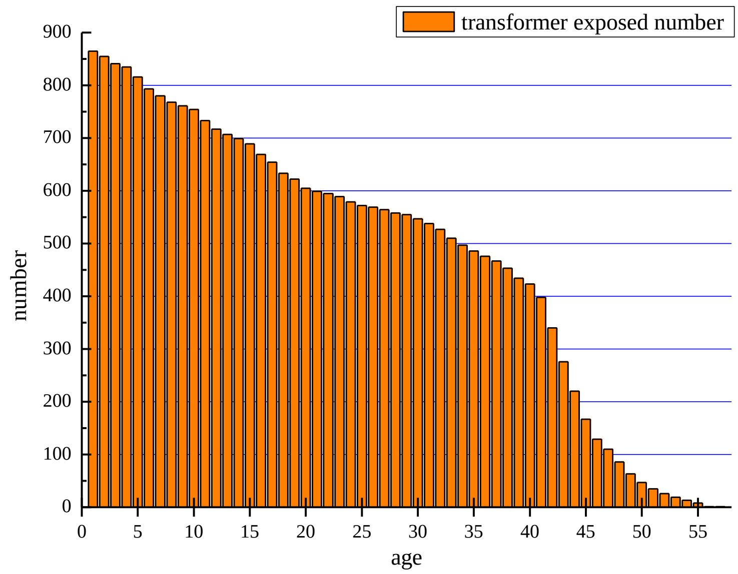


Figure 3-20 National Grid Transformers Exposed Number

Moreover as explained in Chapter 2 for general analysis, National Grid transformers failure hazard can be deduced according to (2-7). This equation is rewritten here as

$$h(t)=\frac{number\ of\ \ failed\ transformers\ within\ interval\ (t-\Delta t,t)}{number\ of\ \ exposed\ transformers\ at\ the\ beginning\ of\ \ t}=\frac{n_F(t)}{N_E(t)} \qquad (3\text{-}33)$$

according to which National Grid transformers hazard function can be obtained as the quotient of failure number $n_F(t)$ at age t (shown in Figure 3-19) divided by the exposed number $N_E(t)$ (shown in Figure 3-20). Figure 3-21 shows the National Grid transformers hazard curve.

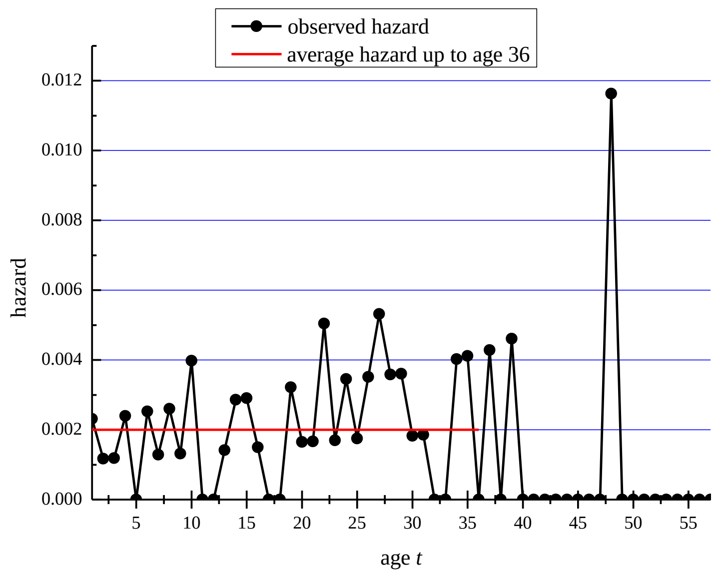


Figure 3-21 National Grid Transformers Failure Hazard against Age

It can be found by comparing the hazard curve shown in Figure 3-21 with the traditional bathtub curve in Figure 2-4, that the high infant mortality in the traditional bathtub curve cannot be seen from National Grid transformers failure hazard curve. This reality verifies that the adequate factory testing was performed before the commissioning of each transformer.

The red line drawn in Figure 3-21 shows the arithmetic average hazard rate up to age 36. "Age 36" is selected because National Grid proposes in its technical guidance [75, 76], that a hazard of 0.30% until age 36 corresponds to the early onset of transformer population significant unreliability, see Chapter 2. The arithmetic average hazard rate up to age 36 is evaluated as 0.20% according to the study in this thesis, and the value is even less than National Grid generally predicted hazard rate of 0.30% up to this specific age. The figure of 0.20% is sufficiently low and stable and tends to indicate the normal operating stage for National Grid transformers up to age 36. Generally, the proper operation schemes that National Grid executes on its transformers and the mild weather in most part of England and Wales result in the random occurrence of transformer failures.

Additionally at older ages, a single failure yields a high value of hazard, which can be seen from the distinct hazard rate of 1.2% at age 48 in Figure 3-21. Note: - 1.2% is derived as one failure divided by the exposed number of 86 at age 48, as $\frac{1}{86} \bullet 100\% = 1.2\%$. A study was undertaken in [86] to further analyze this high value of failure hazard by using a Bayesian method. The results indicate a large uncertainty of observing the hazard rate of 1.2% at age 48, by given one failure at that particular age [86]. A binomial distribution was also used in [86] to describe the probability of observing one failure out of 86 exposed transformers at age 48. The probability is determined to be a low value as compared to the values derived in previous ages [86]. Therefore this significantly higher hazard rate is due to the limited number of exposed transformers at the old age of 48.

Because new transformers are installed in the National Grid transmission system every year and an increasing number of installations have occurred during recent years, large numbers of transformers are operating at their early ages. Meanwhile transformers with poor condition are removed from service before their actual failure, this is necessary to maintain transmission network security and reliability. Hence UK National Grid has little experience of operating transformers at old ages.

Moreover, National Grid failure records show that the failed transformer, with 48 years' operation, was installed in 1956 and failed in 2004 due to improper tap-changer maintenance. The post-mortem investigation further indicates the moderate ageing status of this transformer's main insulation [87]. Either the failure reason or the post-mortem analysis suggests a random failure for this specific transformer. It must be noted, that except for one transformer that failed in 2007, after 37 years' operation, when the failure was caused by severe solid insulation ageing, most other failures resulted from the effects of power system transient events or problems of transformer design [40, 75, 88]. Observed random failures should not be used to predict ageing-related failures in the future.

It is concluded here that statistical analysis on lifetime data, does not work properly for National Grid transformers because a limited number of failures have occurred and the historical failure records indicate most can be considered random failures.

In addition since the high value of hazard at age 48 does not provide sufficient evidence to indicate the onset of transformer population ageing-related failure, the normal operating stage or the random failure period could extend much longer than the observed age 36. Further questions thereafter concern:

- how long will this random failure period or transformer normal operating stage last,
- is the average hazard of 0.20% adequately stable to describe the hazard rate within the random failure period, and
- when will the transformer population ageing-related failure period start.

To answer the first two questions involves intensive mathematical analysis and this is not considered in the thesis. It is consequently assumed that:

- the random failure mechanism works constantly during the entire period of the transformer operation by assuming National Grid network is well-maintained,
- National Grid transformers fail randomly with a constant hazard rate of 0.20%.

In order to predict future transformer population ageing-related failure in terms of increasing hazard. Transformer lifetime under the ageing failure mechanism must be predicted via a physical ageing model. Scrapped transformers can be examined by measuring their paper insulation mechanical strength and this is used to suggest the ageing status of other in-service transformers.

3.7 Summary

Due to significant installation of power transformers during 1960-1970 and after 1990's, 751 transformers were operating in the National Grid transmission network at the end of 2008. Within those in-service transformers, 8.1% are approaching the designed end-of-life of 40 years and more than 40% are beyond this critical age. Although National Grid transformer operational experience have benefited from a proper commissioning scheme and maintenance strategy, it is of great importance to predict transformer population future failures based on present knowledge and information. Statistical approaches are therefore carried out in this chapter on National Grid transformer lifetime data.

Transformers are firstly classified according to their operational experience: - active transformers (**A**), failed transformers (**F**) and scrapped transformers (**X**). The service ages of these three types of transformers are thereafter derived and constitute the database for statistical analysis. National Grid transformer lifetime data are heavily censored as only 52 transformers failed.

The normal distribution, lognormal distribution, 2-parameter Weibull distribution and extreme value distribution are selected to model transformer lifetime data, as they are commonly used to describe the lifetimes of other high voltage equipment.

The least square estimator (LSE) including the hazard plotting approach and the CDF plotting approach is first of all implemented using National Grid transformer lifetime data. The result of fitting lifetime data under a specific distribution model, using hazard plotting is similar to that using CDF plotting. Transformer population mean life is deduced to be much longer than that calculated only from failure samples. In particular the lognormal distribution and the 2-parameter Weibull distribution fit the National Grid transformer lifetime data well from a mathematical point of view. However the normal distribution and extreme value distribution are revealed as more reliable than the other two models since the derived population mean life is reasonable and the obtained hazard curve is more consistent with engineering prior knowledge, i.e. transformer failure will develop to the wear-out stage with increasing hazard. However, these two models are statistically identified improper according to K-S test of curve fitting goodness.

Li's simplified approach based on LSE is also implemented on National Grid transformer lifetime data. The result is revealed more or less similar to that by the CDF plotting. Although Li's approach has been applied to BC Hydro 500kV reactors, this simplified approach would be improper when dealing with limited failure data due to assumptions that are not valid.

The maximum likelihood estimator (MLE) used to analyse the National Grid transformer lifetime data and is also compared with LSE. Results indicate that MLE can fit the later failures better than the earlier failures, while LSE treats the observed data equally. Similarly to LSE, the normal distribution and extreme value distribution are determined as better models to describe National Grid transformer lifetime data because they are in accordance with engineering prior knowledge.

By using the analysis of National Grid transformer lifetime data as an example, it concludes that although the statistical analysis on product lifetime data can be easily implemented, in engineering practice they may not provide reliable prediction for future failure. The only variable considered in the analysis, for instance transformer service age, is not the determinant factor for individual failure, the operational experience or engineering prior knowledge does not suggest any proper distribution models to best describe the data and especially for National Grid transformer lifetime analysis, limited failures are observed and they do not indicate future failures properly. The historical failures of National Grid transformers indicate a random failure mode, which derives an average random hazard rate of 0.20% up to age 36. The value could be assumed to last much longer. Future failures due to the ageing-related mechanism thereby should not be predicted by the observed random failures.

In order to predict National Grid transformer future failures under the ageing failure mechanism, a physical ageing model based on transformer insulation paper deterioration will be used. National Grid scrapped transformers' paper mechanical strength can be examined and these transformers constitute a unique sample to suggest the ageing status of other in-service transformers.

Chapter 4 introduces the ageing process of transformer paper insulation and also describes commonly used models for estimating transformer lifetimes based on paper ageing.

Chapter 4

Literature Summary on Transformer Paper Insulation Ageing

4.1 Introduction

Cellulose paper in combination with mineral oil has been used as the main insulation in a transformer since the late 1920's and the early 1930's. Cellulose paper-mineral oil combined insulation has provided sufficient mechanical and dielectric strength and therefore continues to be the main insulation for power transformers up to 1500MVA, and for voltage levels up to 1000kV [89, 90].

Circulating oil inside the transformer provides a cooling medium that dissipates the heat produced from the windings and core structure. It also insulates the metallic parts at different electrical potentials [36]. Cellulose paper located around the copper conductors prevents voltage breakdown and corona discharge between windings or different parts of the same winding [91]. A pressboard barrier, made from cellulose material, is used to subdivide the oil gap and raise the breakdown strength of the gap [91].

Mineral oil can become acidic after many years of operation due to the effects of atmospheric oxygen and water absorbed during operation or left in the mineral oil or winding structure during the manufacturing process [40, 91]. Additionally oil can form a sludge when it deteriorates, which gradually reduces its cooling ability. Mineral oil, which is in a poor condition tends to be reclaimed or changed based on routine oil analysis, and followed by adding the oil inhibitor [92]. Experience shows the oil condition can improve markedly when the oil is changed, however good performance after an oil change has not been reported within the National Grid system, which is probably due to the contamination introduced during oil change, or the poor quality of the reclaimed oil [40, 92].

On the other hand, although it shows in National Grid's records that several 400/275kV and 400/132kV transformers were rewound and returned to service [40], the pressboard and especially the winding insulation paper cannot be replaced once a winding is built-up. Hence, the lifetime of transformer paper insulation indicates the transformers useful life. In other words a transformer's end-of-life is eventually determined by its paper insulation; the useful lifetime is subject to its operating condition.

The effect on transformer cellulose paper ageing of high temperatures, water, oxygen and acids is first of all presented in this chapter. The definition of transformer thermal end-of-life is thereafter proposed. The criteria of transformer thermal end-of-life based on the reduction of the degree of polymerisation (DP) and tensile strength (TS) of cellulose paper are presented. Models describing paper degradation against transformer operation age are summarized and the DP reduction model used in this thesis is highlighted.

4.2 Ageing of Cellulose Paper

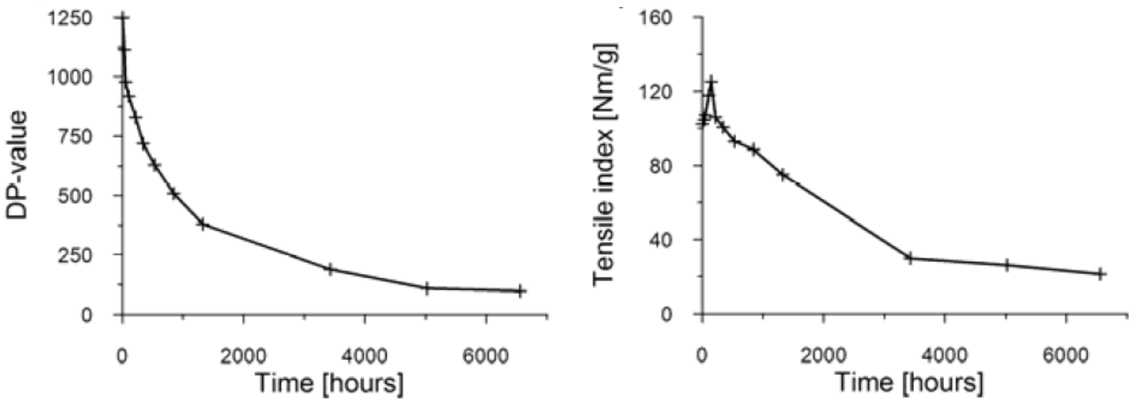
Cellulose ageing is a complex sequence of chemical reactions. During degradation, the inter-fibre bonding is destroyed, the chain is depolymerised and the cellulose mechanical strength is reduced [33]. The reduction of cellulose dielectric property is impossible to measure, unless the insulation is mechanically destroyed [78, 93]. In addition according to National Grid post-mortem investigations on its failed transformers, the dielectric property of conductor insulation does not change significantly even when the transformer is going through ageing [93]. In fact the reduction of cellulose paper mechanical strength may result in a transformer winding turn-to-turn short circuit and produce the paper fragments or fibres existing in the oil duct. Finally this might rupture the paper dielectric property [33, 90]. Transformer ageing should be identified through the reduction of the insulation paper's mechanical strength instead of the dielectric property.

4.2.1 Cellulose Paper Structure

Transformer insulation paper and pressboard are mostly made from unbleached softwood pulp cellulose material. A cellulose molecule consists of a long chain of glucose rings [78]. The average number of the glucose rings in a cellulose molecule is denoted as the degree of polymerisation (DP). The value of DP indicates the length and the mechanical strength of the cellulose fibre, and thus DP is a valid indicator of paper ageing [78, 94].

Tensile strength (TS) describes the cellulose mechanical properties, and is an indicator of the paper fibre strength and the inter-fibre bonds strength. As TS is recognized to be more sensitive than other mechanical properties such as stretching, bursting, folding and tearing strengths [93, 95], it is thereby widely adopted to indicate paper ageing.

DP and TS, as the indicators of cellulose paper mechanical strength, will reduce while the transformer goes through ageing. Figure 4-1 (a) and (b) show the DP and TS reduction obtained during a cellulose paper accelerated ageing test under 3% water content[iii] and at a constant 110 ℃ temperature [94]. Typically in Figure 4-1(b), TS is expressed by the tensile index (TI) which is the breaking load per unit width.



(a) DP Reduction against Time (b) TS Reduction against Time

Figure 4-1 Development of DP and TS Reduction with Paper Ageing [94]

The reductions of DP and TS are generally related to each other. This can be seen from Figure 4-2 according to [92].

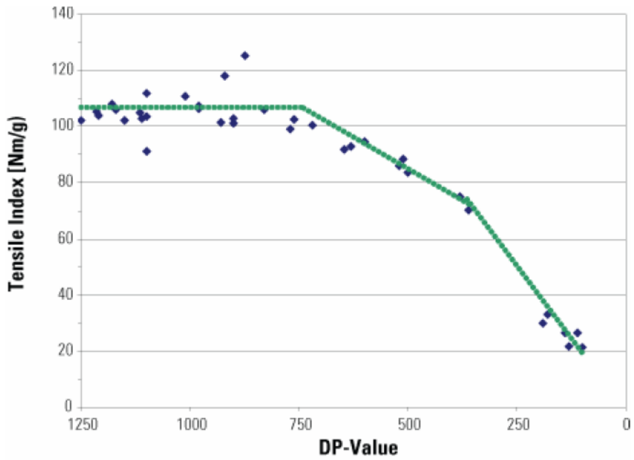


Figure 4-2 Cellulose Paper Tensile Strength (TS) against Degree of Polymerisation (DP) [92]

[iii] The percentage value is the mass ratio between the water and the dry non-oily paper.

In the following discussion about cellulose paper ageing under various agents, DP is adopted to indicate the paper mechanical strength reduction.

4.2.2 Cellulose Paper Ageing Agents

4.2.2.1 Primary Ageing Agent

It was identified by Montsinger that cellulose mechanical strength deteriorates in accordance with a thermal effect, in which heat or temperature is the degradation agent [93, 95]. Figure 4-3 shows the DP reduction against time (in hours) when conducting the paper accelerated ageing test under different temperatures whilst keeping the environment unchanged [94].

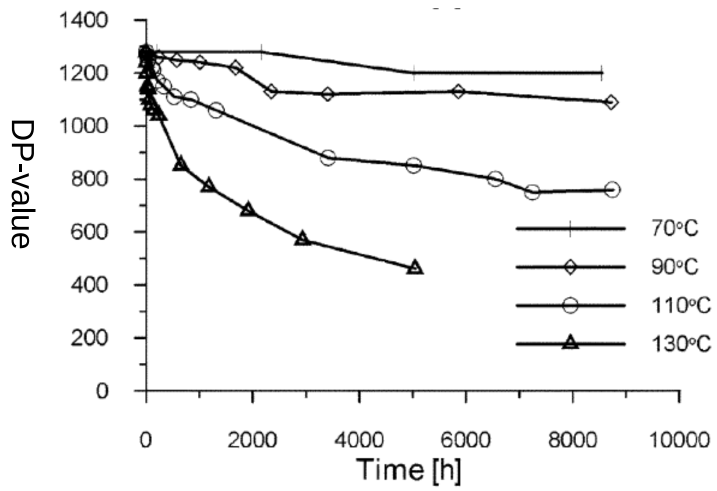


Figure 4-3 DP Reduction against Time with 3% Moisture at Different Constant Temperatures [94]

The effect of temperature on cellulose ageing can be seen from Figure 4-3. Deteriorating from 1300, the DP value of cellulose paper remains almost around the initial value after 9000 hrs under the constant temperature of 70 ℃ or 90 ℃; while if the temperature increases to 130 ℃, DP reduces to less than half of the initial value after 5000 hrs.

According to Montsinger's experimental data, reported in the 1920's [95]: - each 5-10 ℃ increase in temperature doubles the rate of cellulose ageing. In 1944, Montsinger specified this simple rule of thumb that in the vicinity 100 ℃ to 110 ℃, a 6 ℃ increase in temperature can result in doubling the ageing rate [36, 78, 96]. This simple rule of 6 ℃ is adopted in IEC transformer loading guide [34, 97], in which the transformer relative ageing rate *V* is obtained via comparing the hottest winding temperature with the reference temperature, as

$$V = 2^{\left(\frac{\theta_h - 98}{6}\right)} \tag{4-1}$$

where *V* is the relative ageing rate, θ_h is the winding hottest temperature or hot-spot temperature in Celsius and 98 is the reference temperature in Celsius. (4-1) is adopted in IEC loading guide for transformer loss of life calculation.

4.2.2.2 Secondary Ageing Agents

It was further indicated by Dakin according to his observation during cellulose paper accelerated ageing that the cellulose degradation is a complex sequence of chemical reactions which involves the chain scission (also called depolymerisation) and the release of hydrogen, short chain hydrocarbons, carbon monoxide and carbon dioxide [33, 98, 99]. The commonly accepted ageing processes are oxidation, hydrolysis and pyrolysis and accordingly the action agents are water, oxygen and acids except for temperature [100]. Experiments have revealed that the higher the content of water, oxygen and acids in the cellulose molecule, the more significant the level of degradation that can be observed [33, 94, 99, 101-103]. In order to distinguish temperature as the primary agent of cellulose ageing; water, oxygen and acids are classified as the secondary ageing agents or in some occasion the accelerating agents [104].

- **Effect of Water**

The presence of water in paper insulation comes from 3 sources: - residual moisture that remained in the structural components, ingress from the surrounding air and decomposition of the cellulose and oil [11]. Excessive water may shorten and weaken the fibre molecules and lead to decreased dielectric strength [105]. Additionally the effect of water is also strongly related to the acids dissolved in it. It is generally concluded that within the early stage of cellulose ageing, the water content in paper is proportional to its degradation rate; while during the later stage, the water content is proportional to the logarithm of degradation rate [33]. [33] demonstrates the degradation at the later stages as: -

$$ln(k) = ln(\alpha) + \beta \bullet H \qquad \text{(4-2) [33]}$$

where *k* is the paper ageing rate factor, related to the number of broken bonds, α and β are constants and *H* represents the paper moisture percentage content.

Moreover Shroff and Stannett reported their measurement of DP reduction against time of ageing under different moisture levels in [102]. The data was reassessed by Emsley in [33] by plotting the reciprocal of DP against time as shown in Figure 4-4.

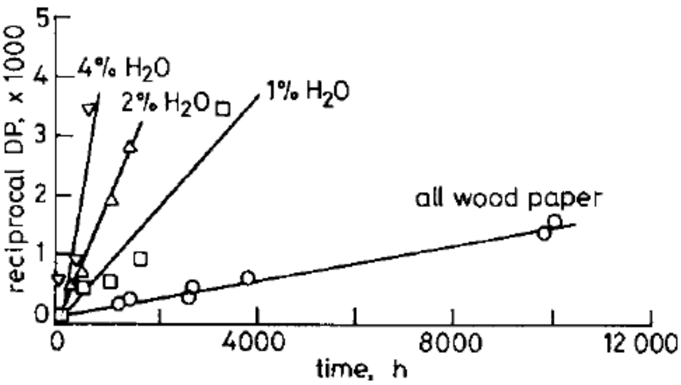


Figure 4-4 Cellulose Paper Degradation as a Function of Time under Different Water Content [33]

Again the percentage figure (%) in Figure 4-4 shows the water content as the mass ratio between the water and the dry non-oily paper. In engineering practice, the water content in transformer paper insulation would always be controlled to be less than 0.5% by the factory drying process [33, 36]. Transformers in the U.S. and many European countries are delivered to their installation site with a sealing system involving either a nitrogen cushion or a rubber membrane in the expansion tank in order to prevent the ingress of water from air [92].

When in operation, transformers are also protected from absorbing the moisture from surrounding atmosphere. Most transformers in Europe and South America are equipped with the silica gel water absorbers at the inlet of the expansion tank, since transformers in these countries are free breathing type with an open conservator for oil preservation [78, 92]. However as compared to the oxygen free transformers in the U.S. and Japan, based on a hermetically sealed container or other means to prevent water ingress, the moisture level in free breathing transformers are significantly higher according to the data published in [78].

In addition water is one of the by-products from paper degradation, thus the moisture level within transformer insulation gradually increases with paper ageing. CIGRÉ Task Force D1.01.10 reports the moisture level of 3-4% within the free breathing transformers according to the operational experiences among European countries [92]. Higher moisture level of 4-8% was also reported in some extreme cases as shown by

Fallou [33]. Typically at 4% moisture in the paper, the cellulose deterioration rate is observed as 20 times greater than at a moisture level of 0.5% [33]. According to Lundgaard [94], at the moisture level of 4%, the cellulose operation life would reduce to 2.5% at normal service temperatures. The experiments by Lundgaard [94] shows that for example, a 40 yrs lifetime of cellulose paper at 0.5% moisture would reduce to 1 yr if the moisture level increases to 4% and the temperature remains.

- **Effect of Oxygen**

Oxygen can always dissolve in transformer oil and is another key factor that accelerates cellulose paper degradation. Oxygen can attack the carbon atoms in the cellulose molecule, products such as carbon monoxide, carbon dioxide, aldehydes, acids and water are released from the destroyed fibre bonds. Cellulose paper mechanical strength is therefore weakened due to oxidation.

As generally indicated by Fabre and Pichon, and also re-tested by Emsley and Lundgaard respectively, the higher the level of oxygen contained in the oil, the more significant the degradation of the paper insulation [33, 94, 105, 106]. Experimental study reveals that due to the presence of oxygen, the rate of cellulose degradation might increase by 2-3 times, as compared with the vacuum circumstance [92]. Figure 4-5 [106] shows the effect of oxygen on the aggravation of ageing via plotting paper DP reduction against time under different oxygen contents. The varying oxygen contents are measured by different oxygen pressure in mBar; the abbreviation of Cu and Naph represents the copper granules and copper naphthenate (Cu^{2+}/ Cu^{+}) respectively which are two catalysts used in the test. Adding these two catalysts does not have a significantly different effect on DP reduction [106].

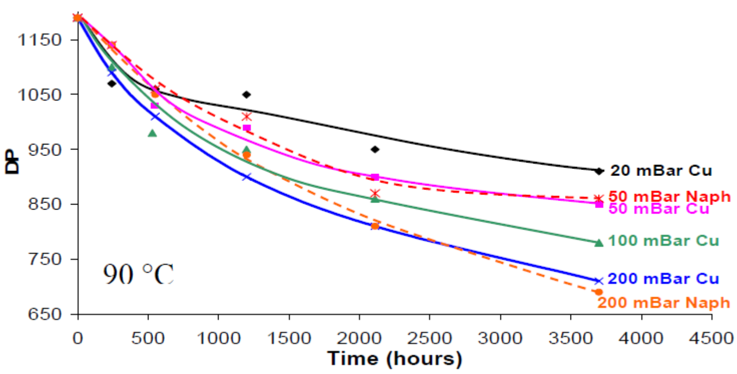


Figure 4-5 Cellulose Paper Degradation as a Function of Time under Different Oxygen Pressure [106]

Laboratory data shows the oxygen level can reach up to approximately 30,000ppm[iv] in fully saturated oil, however operational experiences from the free breathing transformers indicate the level would be around 20,000ppm due to the oxygen dissolution effect of free-breathing [92, 105, 106]. Experiment implies if the dissolved oxygen could be maintained below 2,000ppm, the degradation rate would be 1/5 of that derived from a free breathing transformer (with 20,000ppm oxygen dissolved in oil) [92]; if it could further decrease to below 300ppm, the rate would be 1/16 of that in a saturated oil condition (with 30,000ppm oxygen dissolved in oil) [33].

Additionally the water released during oxidation would again promote the rate of cellulose ageing. Clark indicated that the effect of oxidation could be accelerated by the presence of water content [101], later experiments by Emsley however implied an antagonistic interaction involving oxygen and water when the temperature is below 120 ℃, especially when the contents of water and oxygen are low [105]. According to Emsley, cellulose paper ageing rate reduces at a low water level and low oxygen concentration [105]. Except for the produced water, the acids generated by oxidation would also accelerate the cellulose degradation.

- **Effect of Acid**

The acidity involved in the insulation is another important factor for cellulose ageing because acids would initiate the cellulose hydrolysis together with water. Hydrolysis is the dominant degradation process at the transformer normal operating temperature or overload temperature (i.e. the hot-spot temperature <140 ℃[v]) [92, 100] and the rate of hydrolysis is determined by the content of acids. Additionally cellulose hydrolysis produces water and acids, by which the reaction is self-catalyzed.

According to the experimental studies the low molecular weight acids (LMA), such as formic, acetic and levulinic, tend to accelerate hydrolysis process more effectively, because they are easily dissolved in water. On the other hand, the large oil-like carboxylic acids, for instance stearic and naphthenic acids have minimal effects on cellulose ageing. The effects of these acids are displayed in Figure 4-6.

[iv] ppm: parts per million, concentration of oxygen in oil.

[v] 140^{o}C is given by IEC Loading Guide for Oil-Immersed Power Transformers, as the critical temperature that the hot-spot temperature should not be exceeded under short-time emergency loading to avoid bubble existence.

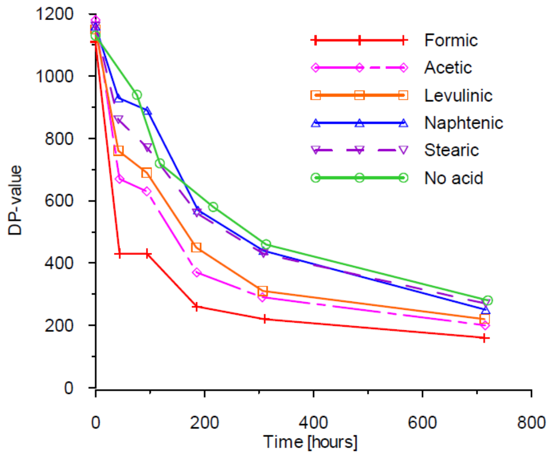


Figure 4-6 Cellulose Paper Degradation as a Function of Time under Different Acids [100]

Particularly low and large molecular weight acids can both be generated from mineral oil oxidation, from which the produced LMA would also participate in the paper hydrolysis [100]. Except for the produced LMA, the oxidation of mineral oil would gradually cumulate sludge, in terms of suspended impurities or semi-conductive sediment, either of which could significantly reduce the oil dielectric strength and accelerate the ageing of oil and paper insulation. In fact the synergistic ageing involving oil and paper insulation has been revealed by the laboratory accelerated ageing test.

4.2.2.3 Kraft Paper and Thermal Upgrade Paper

Transformer insulation paper and pressboard as indicated previously are made from unbleached cellulose material by the kraft chemical process. This type of kraft paper and kraft pressboard has been used as transformer solid insulation since the 1960's [36, 89]. Up to the present, most power transformers in the UK and European continent countries are designed to use kraft paper as solid insulation since kraft paper is cheap while can meet the specific physical, chemical and electrical requirements [36, 89].

However, in the U.S., insulation paper has been thermally upgraded and especially used in the high-rated transformers since the mid of 1960's in order to enhance the transformer insulation life [89]. The process of thermally upgrading is to neutralize the acids and increase the thermal resistance, due to which paper hydrolysis would be less catalyzed by LMA. The insulation ageing rate is thereby reduced by the factor of 1.5-3 [38, 92].

Thermal upgrade paper has also been proven to be more reliable, in terms of higher tensile strength and bursting strength, under elevated temperatures than the non-upgrade kraft paper. The hottest winding temperature that can be tolerated is 110 ℃ for the thermal upgrade paper while it is 98 ℃ for the kraft paper [89].

4.2.3 Calculation of Transformer Ageing Rate

Transformer ageing rate factor *k* can be expressed by a first-order Arrhenius equation [33] by considering the effects from temperature, water, oxygen and acids, which is

$$k = A \bullet exp\left[-\frac{E}{\mathrm{R}\bullet(\theta_h + 273)}\right] \tag{4-3}$$

where *A* is the pre-exponential factor or the contamination factor in hour^{-1}, depending on the paper chemical environment including the water, acidity, and oxygen content, *E* is the activation energy in J/mole, R is the gas constant of 8.314J/mole/K and θ_h is the hot-spot temperature in Celsius.

By considering dry kraft paper in oil (<0.5% water), an ideal constant operating condition of 98 ℃ and no oxygen accessed, the reference ageing rate factor k_0 can be obtained as:

$$k_0 = A_0 \bullet exp\left[-\frac{E_0}{\mathrm{R}\bullet(\theta_{h0} + 273)}\right] \tag{4-4}$$

in which again A_0 and E_0 are the pre-exponential factor and activation energy respectively under the reference condition and θ_{h0} indicates the reference temperature 98 ℃. Transformer relative ageing rate factor k_r can be further developed as the ratio of a certain ageing rate *k* to the reference ageing rate factor k_0 as

$$k_r = \frac{k}{k_0} = \frac{A \bullet exp\left[-\frac{E}{\mathrm{R}\bullet(\theta_h + 273)}\right]}{A_0 \bullet exp\left[-\frac{E_0}{\mathrm{R}\bullet(\theta_{h0} + 273)}\right]} = \frac{A}{A_0} \bullet exp\left[\frac{E_0}{\mathrm{R}\bullet(\theta_{h0} + 273)} - \frac{E}{\mathrm{R}\bullet(\theta_h + 273)}\right] \tag{4-5}$$

If the insulation paper chemical environment changes are neglected during the transformer operation, *A* and *E* in (4-5) are equivalent to the referenced A_0 and E_0 respectively. (4-5) hence is simplified to be:

$$k_r = exp\left[\frac{E_0}{\mathrm{R}\bullet(\theta_{h0} + 273)} - \frac{E_0}{\mathrm{R}\bullet(\theta_h + 273)}\right] \tag{4-6}$$

in which the winding hot-spot temperature (θ_h) remains as the only variable to transformer ageing. By using the mean value of activation energy E_0 of 111×10^3J/mole according to Emsley [33], the transformer relative ageing rate factor k_r, as a function of the transformer hot-spot temperature θ_h, is derived as

$$k_r = exp\left[\frac{13351}{98+273} - \frac{13351}{\theta_h + 273}\right] \tag{4-7}$$

By taking logarithm base 2 on both sides of (4-1) and (4-7) respectively, values of $\log_2(k_r)$ and $\log_2(V)$ from the hot-spot temperature θ_h=20℃ to θ_h=110℃ are compared in Figure 4-7.

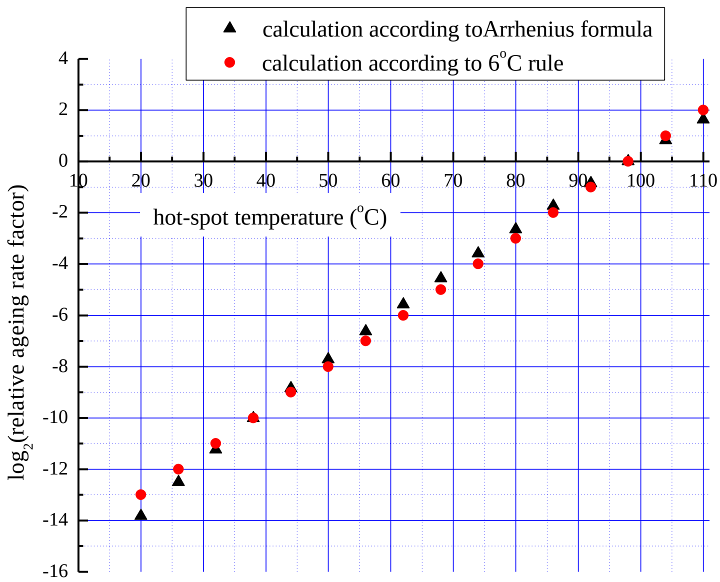


Figure 4-7 Comparison between Arrhenius' Form of Relative Ageing Rate Factor k_r and Montsinger's 6℃ Rule Ageing Rate Factor V

It can be seen from Figure 4-7 that within the transformer normal operation temperatures, the relative ageing rate factor derived according to Montsinger's 6℃ rule, V by (4-1) is similar to k_r according to the Arrhenius' formula shown in (4-7), It is hereby determined that the Arrhenius' form of relative ageing rate factor k_r expressed in (4-7) is used to replace Montsinger's 6℃ equation (4-1) in the following calculation of transformer ageing.

Moreover according to the laboratory tests, oxygen, water and acidity content immersed in the paper insulation are changing with the transformer operating condition. The change of transformer paper insulation chemical environment and the relative ageing rate k_r, are reflected by the change of the pre-exponential factor A and activation energy E corresponding to different deterioration processes.

Emsley summarized a generic value of activation energy E independent of deterioration processes as 111×10^3J/mole with the 95% confidence bounds from 105×10^3J/mole to 117×10^3J/mole [33, 92, 107]. Other researchers however found that kraft paper oxidation has the activation energy E ranges from 80 - 90×10^3J/mole in early experiment and 50 - 65×10^3J/mole later, the normal hydrolysis has the activation energy E in the range of 115 - 130×10^3J/mole and E increases over 250×10^3J/mole above 140 ℃ when the insulation deteriorates under pyrolysis [92, 100]. Lundgaard also verified the different activation energies under different ageing processes, in which 96×10^3J/mole is the typical value for oxidation and 125×10^3J/mole for hydrolysis [92].

The pre-exponential factor A also affects the value of relative ageing rate. A increases with the aggressively increased acids and also the water content that dissociates the acids [92]. A significantly higher pre-exponential factor A is assigned in cellulose hydrolysis process.

It is further reported that kraft paper oxidation is dominant when the temperature is no more than 60 ℃ [100]. Oxidation is attenuated with time as it produces low molecular weight acids (LMA) which can effectively promote the process of hydrolysis. Hydrolysis is revealed to be most significant in the temperatures of 70 - 130 ℃ [100]. Once the temperature increases to over 150 ℃, heat directly destroys the cellulose molecule rings and thus pyrolysis is dominant at high temperatures [100].

The equation for the transformer relative ageing rate factor k_r (4-5), can be further expressed as a piecewise function corresponding to the three deterioration processes within different temperature ranges. The piecewise function is written as

$$k_r = \begin{cases} \dfrac{A_{ox}}{A_0}\bullet exp\left[\dfrac{E_0}{\mathrm{R}\bullet(\theta_{h0}+273)} - \dfrac{E_{ox}}{\mathrm{R}\bullet(\theta_h+273)}\right], & when\ \theta_h \le 60^0C \\ \dfrac{A_{hy}}{A_0}\bullet exp\left[\dfrac{E_0}{\mathrm{R}\bullet(\theta_{h0}+273)} - \dfrac{E_{hy}}{\mathrm{R}\bullet(\theta_h+273)}\right], & when\ 60^0C < \theta_h \le 150^0C \\ \dfrac{A_{py}}{A_0}\bullet exp\left[\dfrac{E_0}{\mathrm{R}\bullet(\theta_{h0}+273)} - \dfrac{E_{py}}{\mathrm{R}\bullet(\theta_h+273)}\right], & when\ 150^0C < \theta_h \end{cases} \tag{4-8}$$

where A_{ox}, A_{hy} and A_{py} are the pre-exponential factor of oxidation, hydrolysis and pyrolysis respectively, E_{ox}, E_{hy} and E_{py} indicate the activation energy under these three deterioration processes, and again A_0 and E_0 represent the pre-exponential factor and activation energy for the referenced dry and no oxygen accessed kraft paper.

However, recent experimental evidence is lacking to support the exact values of *A*'s and *E*'s under various chemical conditions. According to unconfirmed data given in [100], typical values of the pre-exponential factor *A* and the activation energy *E* under the oxidation and hydrolysis deterioration process are summarized in Table 4-1.

Table 4-1 Pre-Exponential Factor (*A*) and Activation Energy (*E*) for Cellulose Oxidation and Hydrolysis Based on Experiment Results [100]

	Dry Kraft no oxygen (reference)	Dry oxygen accessed (oxidation)	1.5% Moisture (hydrolysis)	Pyrolysis
Temperature (℃)	98	$\theta_h \leq 60$	$60 < \theta_h < 150$	$\theta_h > 150$
E ($\times 10^3$J/mole)	128	$89\times(1+1.5\%)=90.335$	$128\times(1-1.5\%)=126.08$	
A (hr^{-1})	4.1×10^{10}	4.6×10^{5}	1.5×10^{11}	

It needs to be mentioned here that the values of *A* and *E* under 1.5% moisture are chosen as typical values for cellulose hydrolysis, since transformers with 1.5% moisture in the paper are commonly considered wet by National Grid [87]. The +/-1.5% incremental bar is considered in the activation energies and used to generate a continuous function of k_r from oxidation to hydrolysis. Note: +/-1.5% is considered reasonable by Emsley [33]. Values of *E* and *A* for pyrolysis are not given in Table 4-1, and k_r under pyrolysis is not considered, because the transformer hot-spot temperature should not exceed 140℃ in normal operation unless the short-time emergency loading condition rarely occurs [34].

Based on the values of the parameters given by Table 4-1, the transformer relative ageing rate k_r in (4-8) can be obtained. It compares k_r with Montsinger's relative ageing rate *V* in Figure 4-8 by plotting k_r and *V* against winding hot-spot temperature.

It can be seen from Figure 4-8 that a transformer deteriorates much faster when the effects of secondary ageing agents are taken into account, especially when the hot-spot temperature is high. In fact the higher relative ageing rate is closer to transformer operation practice and therefore the modified piecewise relative ageing rate k_r, calculated by (4-8), is used to determine transformer ageing in the rest of this thesis.

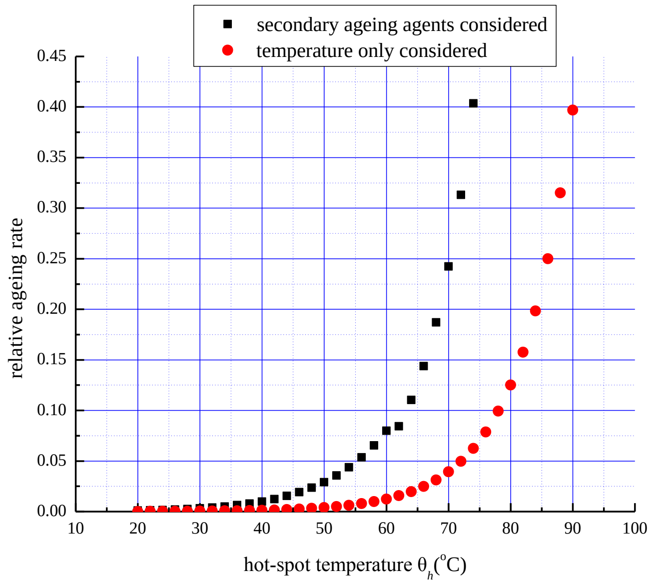


Figure 4-8 Comparison between Modified Piecewise Relative Ageing Rate Factor k_r and Montsinger's 6 ℃ Ageing Rate Factor V

4.3 Transformer Thermal End-of-Life Criterion

4.3.1 Transformer Thermal End-of-Life Definition

The process that leads to the degradation of the mechanical properties of transformer paper insulation was referred to as transformer thermal ageing before oxygen, water and acids were recognized as other degradation agents [95, 103]. The concept of transformer "***thermal ageing***" is now used to describe insulation mechanical property degradation under primary and secondary agents. Transformer "***thermal end-of-life***" is thus used to indicate the end of paper insulation useful life, i.e. when the paper insulation is prejudiced since its mechanical strength reduces to an unacceptable value. Accordingly, the transformer "***thermal lifetime***" represents the transformer operational period, i.e. the transformer has reached a physical condition (DP=200) below which the transformer is considered not suitable for operation [78]. The well adopted physical threshold is called transformer "***thermal end-of-life criterion***".

It is also recognized that transformer paper insulation mechanical strength can also be influenced by short-circuit faults, lightning, over-voltages or other system related transient events [78, 108-111]. McNutt concluded the progressive steps of transformer ageing-related failure since the start of degradation: firstly after years of service, the mechanical strength of paper insulation reduces due to the thermal ageing effects, and the paper becomes brittle and weak; secondly continuous transient events, i.e. short-circuit faults further weaken the paper insulation; finally, the dielectric strength is ultimately destroyed due to the mechanical motion during a transient event [51, 112]. Hence the thermal ageing effects and the external transient forces work synthetically for the insulation degradation that leads to transformer ageing-related failure.

According to McNutt, by considering the effect of system transient events, a transformer actually approaches its functional end-of-life rather than its thermal end-of-life [112]. It is also inferred that a transformer may not actually fail when its useful mechanical property has deteriorated to an unacceptable value; however, the transformer is vulnerable to failure. Alternatively a transformer could stay in service with its brittle insulation as no external transient forces triggered its failure. National Grid post-mortem analysis on its retired transformers finds certain transformers have operated well beyond their thermal lifetimes when replaced. Therefore transformer thermal end-of-life criterion is established only based on the insulation paper physical condition rather than the transformer actual breakdown when its functional life is finished.

However a quantitative model for directly predicting a transformer’s functional lifetime or ageing-related failure is still under investigation. Transformer ageing-related failure is considered as transformer paper insulation thermal ageing in this thesis [1, 3]. Hence, transformer thermal lifetime, expressed as the operational period that results in a loss of the insulation paper mechanical strength, under a particular operation conditions, is used as the basis of transformer end-of-life. Furthermore transformer thermal hazard is thus the baseline of the transformer ageing-related failure hazard. The transformer thermal failure mechanism essentially indicates the ageing failure mechanism in the context of this thesis.

4.3.2 Transformer Thermal End-of-Life Criterion

In order to assign the critical condition of transformer paper insulation mechanical strength as thermal end-of-life criterion, the cellulose tensile strength (TS) or the degree of polymerisation (DP) can both be used as they are measurable properties [78]. The length of time, for reaching a specific criterion measured under a well-dried, oxygen-free and 110℃-constant-temperature condition is called the normal insulation life corresponding to the assigned criterion [78].

4.3.2.1 Based on Percentage of Remaining TS

Transformer insulation TS is measured according to IEEE "Standard Test Procedure for Sealed Tube Ageing of Liquid-Immersed Transformer Insulation" under low oxygen and moisture conditions [90, 96]. The percentage of remaining TS is used to indicate transformer thermal ageing.

As early as the 1930's, Montsinger [95] published his observation on cellulose TS reduction during the paper accelerated ageing test and used the 50% remaining TS as the transformer thermal end-of-life criterion. This criterion was further adopted by Shroff [102] in the 1980's and McNutt in the 1990's [78]. In fact, 50% remaining TS, which corresponds to 65,000hr normal insulation life, was commonly adopted as the transformer thermal end-of-life criterion in early ages (1930's – 1980's). However, this percentage value is controversial, as it is not considered reliable for insulation that was brittle during commissioning (i.e. low TS) [78]. More importantly according to tests, respectively undertaken by General Electric and Westinghouse U.S. during 1978-1982, the transformer insulation mechanical strength was in a satisfactory condition when 50% TS is retained [78]. Hence, 50% remaining TS is a rather conservative transformer thermal end-of-life criterion.

As a response to the 50% remaining TS, Malmlöw, Dakin, Sumner and Lawson suggested using 20% or even 10% retained TS as the transformer thermal end-of-life criterion [103, 113]. Note: 50% retained TS was adopted in the 1981 IEEE loading guide for power transformers (C57.92-1981), this was upgraded to 25% remaining TS in the 1995 loading guide, version C57.91-1995 [38, 96]. However, the above transformer thermal end-of-life criteria, based on percentage of remaining TS, are not used in this thesis.

4.3.2.2 Based on Absolute DP Value

The insulation paper DP value is also used to indicate transformer thermal end-of-life. According to EPRI research on transformer loading, undertaken in the 1980's [96, 114], the absolute DP value is a better indicator of the mechanical characteristics of paper than the percentage remaining TS, since the measurement of DP is simple and the results are less dispersed.

It is found in kraft paper that DP is more than 2,000 in its natural state and is between 1,000 and 1,400 after a transformer has been manufactured; the actual value depends on the manufacturing techniques [33, 102, 114]. DP might further reduce to around 950 after drying and some typical factoring drying regimes result in a value around 750-850 [102]. Kraft paper DP value starts to decrease exponentially after the commissioning of the transformer.

As briefly revealed by paper accelerated ageing tests, at the range of DP 950-600 or 500, paper mechanical strength does not change significantly; while from DP 500 to 200 the strength reduces, proportional to DP reduction [33, 92, 102]. Transformer thermal end-of-life criterion defined by an absolute DP value however has continued to be debated since the 1960's [78]. Independent investigations suggested using various DP values between 250 and 100 to indicate transformer thermal end-of-life. It is further summarized by McNutt in [78] that, Bozzini [115] suggested a value of 100-150, Lampe [116] selected 200, Fabre and Pichon [117] preferred 100-200 and Shroff [102] used 250 when conducting their individual accelerated ageing tests.

4.3.2.3 Contend of DP and TS as Thermal End-of-Life Criterion

In fact the remaining 50% initial TS is equivalent to a DP value around 350 [100]. It is also indicated by Shroff, Fabre and Pichon, that paper insulation may retain 40%-20% TS when the DP is approaching 150-200 [102, 118]. Typically below a particular DP, 150-200, when the paper is measured, it has no mechanical strength.

Transformer thermal end-of-life criterion, based either on the percentage remaining TS or on the absolute DP value, has never been uniformly applied; hence the 50% and 25% remaining TS and a DP of 200 are all referred to as the power transformer thermal end-of-life criteria in both the IEEE and IEC transformer loading guide [34, 96]. These transformer thermal end-of-life criteria are shown in Table 4-2.

Table 4-2 Power Transformer Thermal End-of-Life Criteria and Normal Insulation Life at 110 ℃ [34, 96]

Basis	Normal Insulation Life	
	Hours	Years
50% retained tensile strength (TS) of insulation	65,000	7.42
25% retained tensile strength (TS) of insulation	135,000	15.41
200 retained degree of polymerization (DP) in insulation	150,000	17.12

The transformer normal insulation life, corresponding to each of these criteria, is also presented in Table 4-2. The normal insulation life for reaching a specific thermal end-of-life criterion, is measured in a paper accelerated ageing test under a well-dried, oxygen-free and 110 ℃-constant-temperature condition [78]. Moreover recent laboratory studies on insulation ageing, tends to use a DP of 200 as the thermal end-of-life criterion [33, 92, 94, 119, 120] because it yields a reasonable longer insulation normal life of 150,000hrs compared to the 65,000hrs based on the 50% remained TS. In addition, a transformer thermal degradation model based on DP reduction is well established. In addition, in recent engineering applications, for example transformer lifetime prediction, a DP of 1000 is used to represent new paper, and a DP of 200 to indicate exhausted insulation [43, 84, 121-124]. In particular, insulation paper with a DP of 200 and the corresponding 150,000hrs are adopted in this thesis as the transformer thermal end-of-life criterion and corresponds to the normal insulation life.

4.4 Models of Transformer Thermal Lifetime Calculation via DP Reduction

DP values can be obtained from paper samples taken from a transformer that was retired and scrapped. Experiences from the U.S. and U.K. have proved that the measurement of scrapped transformers' DP values provides a useful tool to suggest the in-service population end-of-life [78]. The lowest DP value obtained from the transformer main winding insulation is always used to indicate the transformer thermal degradation because the lowest DP represents the weakest part of the insulation paper due to thermal ageing and it eventually determines the transformer's thermal lifetime.

4.4.1 DP Reduction Model Development

As early as 1940's, the reduction of DP was formulated as

$$DP_{lowest}(t) = DP_0 \bullet (1-r)^{-t} \tag{4-9) [114}$$

where $DP_{lowest}(t)$ is the DP value after the transformer operation period t, and is always the lowest DP, DP_0 is the initial DP when the insulation is new, always set at 1000, r is a negative constant parameter and t represents the service age. The equation "$\eta = \frac{DP_0}{DP_{lowest}(t)} - 1$" is called the number of cellulose chain scissions which directly describes paper ageing [114]. (4-9) can be furthermore expressed as

$$\eta = (1-r)^t - 1 \tag{4-10}$$

according to which the number of cellulose chain scissions, increases exponentially with the transformer service age t.

The DP reduction against transformer service age was further described as a simplified first-order chemical kinetic process [120]. By Kuhn in 1930, Ekamstam in 1936, then Sharples in 1957 and further verified by Zou and Emsley in 1990's. The model is expressed as:

$$\frac{1}{DP_{lowest}(t)} - \frac{1}{DP_0} = k \bullet t \tag{4-11) [120}$$

where $DP_{lowest}(t)$ is the lowest insulation DP after the transformer service age t, DP_0 is the initial DP, k is the ageing rate factor and t is the elapsed time of transformer service.

As indicated previously k is dependent on the transformer operation condition and can be described by a first-order Arrhenius equation as shown in (4-3). Between DP 1000 and 200 however, k can be roughly considered as a constant value since the degradation process involving this range of DPs is related to the cellulose amorphous regions or the inhomogeneous composition according to Emsley and Lundgaard [33, 94, 107]. Cellulose amorphous regions are firstly attacked and degrade faster than the crystalline regions due to their large permeability [33, 94, 107]. Figure 4-9 hereby shows a red line to indicate the linear increase of 1/DP against t up to DP approaching 200. This line hence corresponds to a constant ageing rate factor k.

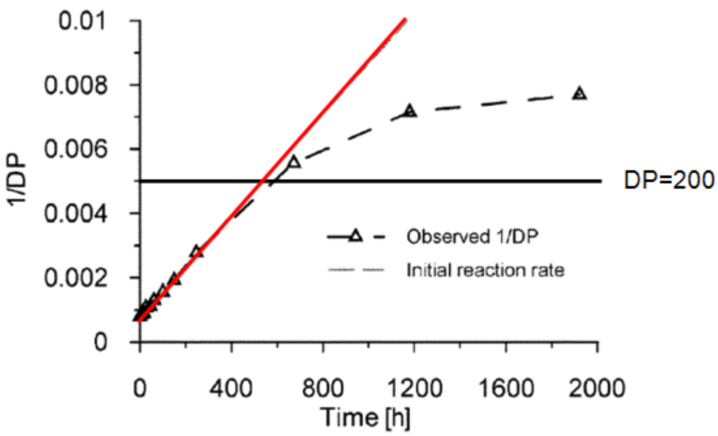


Figure 4-9 1/DP Increase against Time (hr) [94]

However according to Zou, the constant ageing rate factor k is roughly considered which leads to an over-simplified model as shown in (4-11) [119]. Emsley subsequently modified (4-11) by changing k to ensure it exponentially decreased with transformer service, since the paper inhomogeneous composition is progressively destroyed [125, 126]. The refined model is expressed as

$$\frac{1}{DP_{lowest}(t)} - \frac{1}{DP_0} = \frac{k_{10}}{k_2} \bullet \left(1 - e^{-k_2 \bullet t}\right) \qquad \text{(4-12) [125]}$$

where k_{10} and k_2 are constant figures. The exponentially decreasing ageing rate in (4-12) is closer to the practical degradation of paper, than a constant ageing rate k [125]. Typically when k_2 is small, (4-12) is identical to (4-11) with $k_{10} = k$ [125]. Moreover (4-12) can describe the insulation ageing process within the full range of measurable DP values, for example until DP=100. Whilst the linear increase of 1/DP shown in (4-11) is only valid within a range of DP=1000 to 200 [126]. The DP decrease model shown in (4-12) is suitable especially when DP 150 or 100 is used as transformer thermal end-of-life criterion.

With the exception of the above models derived from paper accelerated ageing tests, utilities develop the empirical models of DP reduction based on their operational experiences. Engineers in Weidmann [126] responded to Emsley's refined model by comparing (4-12) with their own model which describes paper ageing as a 2-stage process of DP reduction as

$$DP_{lowest}(t) = DP2_0 \bullet exp\left(-C_{DP2} \bullet t\right) \qquad \text{(4-13) [126]}$$

in which $DP_{lowest}(t)$ is the measured lowest DP after age t, $DP2_0$ is the DP value after an initial rapid ageing process (this is also the start DP of the 2nd-stage long-term ageing process), C_{DP2} is the 2nd-stage ageing rate factor, and t is the transformer in-service period. [126] also discusses the drawbacks of (4-13), i.e. this empirical model does not consider the elapsed time of the 1st-stage of rapid ageing and it is necessary to empirically assign the value of $DP2_0$. Whereas in transformer operation practice, the early stage of paper insulation ageing can be critical to a transformer end-of-life.

Additionally engineers in Hitachi, Japan proposed a model that described the change of the paper insulation mean DP value against transformer service age, according to their operation data. This is expressed as

$$DP_{av}(t) = (1 - 0.014t) \bullet DP_0 \qquad \text{(4-14) [127]}$$

where $DP_{av}(t)$ is the average DP of the transformer paper insulation after age t, instead of the lowest DP considered in the above models. Since the empirical models are built up according to utilities' operation data, they may not be applicable to describe DP reduction in other utilities.

4.4.2 DP Reduction Model Used in This Thesis

As the most general and widely used model in transformer thermal lifetime estimation, the model of DP reduction against transformer service age, expressed in (4-11), is used in Chapter 5 for the estimation of the National Grid scrapped transformers' ageing rates and their thermal lifetimes. However, it is important to assume:

- DP equals 1000 when a transformer starts to operate and DP equals 200 when the transformer reaches its thermal end-of-life,
- between a DP of 1000 and 200, the transformer paper insulation degrades with a constant rate k, and
- the change of transformer chemical environment, including the oxygen, moisture and acids contents, is not taken into account.

Furthermore (4-11) can be rewritten as

$$k = \frac{\dfrac{1}{DP_{lowest}} - \dfrac{1}{DP_0}}{t_{op}} \qquad \text{(4-15)}$$

where, DP_{lowest} indicates the lowest insulation DP after the transformer service age t_{op} at scrapping. According to (4-15), an individual transformer ageing rate factor k is derived. By simply assuming transformer ageing at the same rate k, the unit's remaining life t_{re} is estimated as

$$t_{re} = \frac{\dfrac{1}{DP_{EoL}} - \dfrac{1}{DP_{lowest}}}{k} \qquad \text{(4-16)}$$

where, DP_{EoL} is the transformer thermal end-of-life criterion and is 200 in this study. A scrapped transformer average ageing rate k during service and its thermal lifetime $EoL = t_{op} + t_{re}$ can be calculated according to (4-15) and (4-16) based on the information of its lowest DP when scrapping and its service age t_{op} until scrapping.

It can be derived from (4-16) that, for a scraped transformer whose lowest DP was less than 200 (DP_{lowest}<200), its remaining life would be negative. It suggests for this case that the thermal lifetime of this particular transformer has ended before scrapping and its thermal life (EoL) should have been reached when DP equals 200. Details of the calculation will be given in Chapter 5.

4.5 Summary

The insulation of medium and large power transformers is provided by the cellulose paper-mineral oil combined system. Due to the irreplaceability of paper insulation, a transformer’s end-of-life is eventually determined by the useful lifetime of its insulation paper subject to the unit’s operation condition.

Transformer insulation paper is made from cellulose material, with a molecular structure composed of long chains of glucose rings. The molecular degree of polymerisation (DP) and tensile strength (TS), decreases with cellulose degradation. Hence, both are valid indicators of cellulose ageing.

The ageing of cellulose paper is represented by the deterioration of the papers mechanical properties due to the effects of heat, water, oxygen and low molecular weight acids. A simple 6 ℃ rule is widely accepted to express the effect of heat, as the transformer ageing rate doubles when the winding hottest temperature increases by 6 ℃. The presence of water in paper can shorten and weaken the fibre molecules and the effect is strongly related to the acids dissolved. Oxygen can attack the carbon atoms in the cellulose molecule and the released acids and water will initiate and then catalyze the cellulose hydrolysis process. The oxygen, water and acids immersed in the paper insulation are changing with the transformer operating conditions. The change of the chemical environment associated with the paper insulation significantly influences the transformer ageing rate.

Chapter 4 Literature Summary on Transformer Paper Insulation Ageing

As a concept, transformer thermal ageing describes the paper insulation mechanical property degradation due to the effects of heat, water, oxygen and acids. Consequently transformer thermal lifetime suggests that the transformer operation period, is the time required to reach a presumed physical condition below which the transformer is not suitable to operate. Transformer thermal end-of-life is used as the basis of transformer actual failure; transformer ageing failure mechanism discussed in this thesis is essentially the failure mechanism related to thermal ageing.

Either absolute DP, or the percentage remaining TS of insulation paper, can be used to indicate the critical level of the insulation mechanical property. Transformer normal insulation life thus varies corresponding to different DPs or percentage TSs used as the thermal end-of-life criterion. This thesis assumes the transformer paper insulation DP of 200 implies the end of thermal life and a DP of 1000 indicates new insulation.

The development of DP reduction models is introduced in this Chapter. A widely used model presents 1/DP, the linear increase against transformer service age between DP 1000 and 200, by a first-order kinetic equation. This model is used to estimate the thermal lifetimes and ageing rate factors of National Grid scrapped transformers in Chapter 5.

Chapter 5

National Grid Scrapped Transformer Thermal End-of-Life Analysis

5.1 Introduction

UK National Grid has been carrying out post-mortem analysis on its retired transformers since about 1993 [40]. Those transformers were taken out-of-service due to failure, substation upgrading, convinced design problems, or the suggested advanced ageing status according to previously scrapped transformers. The condition of insulation paper can be assessed through transformer scrapping; paper samples are sent to the laboratory to measure the DP value [40].

DP values of paper insulation are usually measured at more than one location on the transformer winding and often several measurements are taken at one location. The DP values of a transformer normally scatter from the bottom to the top of the winding and also from the outer layer and the inner layer of one disc. The dispersion of DP represents the nonuniform distribution of temperature, oxygen, water and acids within the transformer main tank. The lowest DP from the transformer main winding insulation is used to indicate the transformer thermal ageing because it is usually from the hottest spot of the transformer winding and indicates the weakest part of the insulation which is the most vulnerable to breakdown.

In this chapter the ageing rates of National Grid scrapped transformers and their thermal lifetimes are estimated according to the widely used model involving transformer lowest DP and service age. It is intended to use the thermal failure trend derived from the scrapped transformers to represent the future failure of the National Grid in-service population. Scrapped transformers' representativeness is furthermore discussed in terms of loading conditions, thermal design parameters and installation ambient conditions since these are the determinant factors of transformer thermal lifetime.

5.2 National Grid Scrapped Transformer Thermal Ageing Analysis

Until 2009, 77 power transformers owned by UK National Grid have been scrapped and their paper insulation lowest DP values obtained. These 77 scrapped transformers make up a unique sample to suggest the ageing status of National Grid in-service transformer population. The DP values of these scrapped transformers can be analyzed by either of the following two ways.

5.2.1 Analysis 1: Calculate Scrapped Transformers Average Ageing Rate

As presented in Chapter 4, transformer ageing rate factor *k* can be roughly considered as a constant between DP 1000 and 200. *k* is estimated as

$$k = \frac{\frac{1}{DP_{lowest}} - \frac{1}{DP_0}}{t_{op}} \qquad (5\text{-}1)$$

where DP_{lowest} is the transformer lowest DP value when scrapping, DP_0 shows the initial DP of 1000 and t_{op} indicates the transformer service age. It hereby calculates the ageing rate factor of each of the scrapped transformers according to (5-1). Figure 5-1 further shows the reciprocal of lowest DP ($1/DP_{\text{lowest}}$) of each scrapped transformer against the individual service age t_{op} until scrapping. The reciprocal of DP is considered and compared with the 1/DP linear model shown in (5-1).

In Figure 5-1, each scrapped transformer is represented by one red square. The red line shows the linear fitting of 77 $1/DP_{lowest}$ dots; the interception of 0.001 was fixed at t=0 since the initial DP equals to 1000 and $1/DP_0$=0.001. The slope of this line actually indicates the average ageing rate k_{av} of these 77 scrapped transformers. The corresponding average thermal lifetime of the 77 scrapped transformers was determined as 67 yrs ($\frac{\frac{1}{DP_{EoL}} - \frac{1}{DP_0 = 1000}}{k_{av}} = 67$, where DP_{EoL}=200, as the thermal end-of-life criterion).

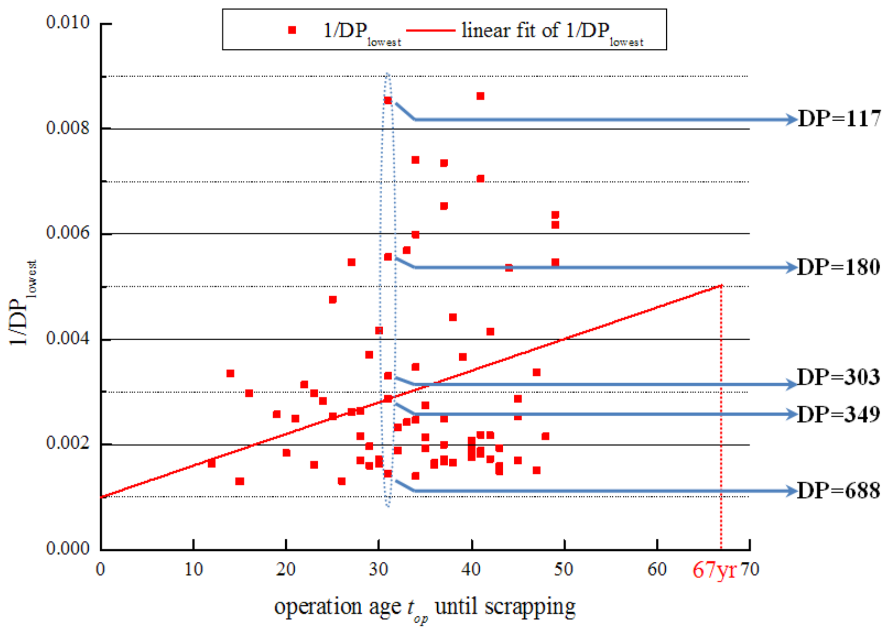


Figure 5-1 National Grid Scrapped Transformers 1/DP_{lowest} against Service Age t_{op} until Scrapping

It also shows in Figure 5-1 that the scrapped transformers' ageing rates vary significantly. It is highlighted in Figure 5-1 that 5 transformers all scrapped after 31 years' service represent low, moderate and advanced ageing by the large dispersed DP values, from less than 200 to more than 600 [87]. The transformer at the bottom is revealed to have DP as high as 688, which indicates minimal ageing of this failed transformer. Further study shows that the transformer was less frequently used, and its failure was caused by the mechanical damage due to overheating of the LV winding when supplying a steelworks with arc furnaces [87]. The DP of two transformers of moderate ageing were 349 and 303 respectively. One transformer failed due to a suspected bushing or tap-changer problem which caused dielectric rupture according to [87] while the other was replaced by a higher rating transformer as the substation was upgraded [87]. One advanced-aged transformer with the DP of 180 was retired due to substation upgrading when no obvious problem was observed during the monitoring of its main tank [87]. The oldest transformer among these 5 units is revealed to have the DP of 117. It was out-of-service due to core and frame circulating current and over-heated shield rings, this was recognized as a common problem among identically designed transformers [87].

It is suggested, based on DP values from these two advanced-aged transformers, that transformer thermal lifetime according to the criterion of DP=200 does not indicate a transformer actual failure. However, a DP of 200 substantially defines a critical physical condition below which a transformer would be at an unacceptable high risk of failure. These two transformers have consumed their useful thermal lifetimes; however neither of them did fail, since no system transient triggered their failure.

It can further be concluded, according to those 5 transformers that since transformers may have operated and/or been designed differently, they deteriorate at different rates which results in significantly different DP values, even when they were scrapped after the same period of operation.

5.2.2 Analysis 2: Calculate Scrapped Transformers Average Thermal lifetime

As introduced in Chapter 4, scrapped transformer's lowest DP can be used to predict the transformer's thermal lifetime *EoL* by assuming the transformer would deteriorate at the same ageing rate *k* if it stayed in service. The calculation steps for each scrapped transformer are again summarized as follows.

Step 1: Estimate the ageing rate factor (k) of a scrapped transformer based on its lowest DP (DP_{lowest}) when scrapping and the service age (t_{op}), as

$$k = \frac{\frac{1}{DP_{lowest}} - \frac{1}{DP_0}}{t_{op}} \tag{5-2}$$

where DP_0=1000 shows new transformer insulation.

Step 2: Assuming this transformer would continue ageing at this estimated ageing rate k, calculate the residual year to reach the critical DP of 200, as the end of its thermal lifetime. The residual years of operation, also called the transformer remaining life (t_{re}), is calculated as

$$t_{re} = \frac{\frac{1}{DP_{EoL}} - \frac{1}{DP_{lowest}}}{k} \tag{5-3}$$

where, DP_{EoL}=200 is the thermal end-of-life criterion.

Step 3: A transformer's thermal lifetime (EoL) is then the sum of its operation life (t_{op}) and remaining life (t_{re}), expressed as

$$EoL = t_{op} + t_{re} \tag{5-4}$$

As mentioned in Chapter 4, for a certain transformer whose lowest DP was less than 200 when scrapped, its predicted remaining life (t_{re}) according to (5-3) would be negative, which suggests the useful thermal lifetime of this particular transformer has finished before replacement. The thermal lifetime (*EoL*) of this particular transformer therefore needs to be calculated backward to the moment when DP equals 200 and the value obtained would be less than its actual operation history (t_{op}).

Repeat Step 1 to 3 for each scrapped transformer. The distribution of the National Grid 77 scrapped transformers predicted thermal lifetimes are displayed in Figure 5-2.

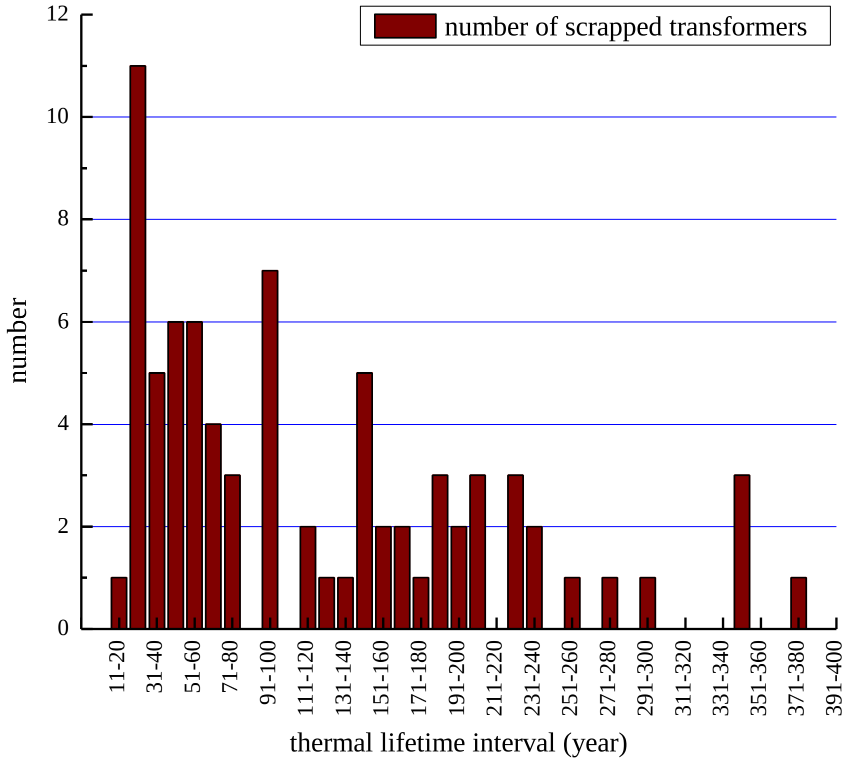


Figure 5-2 Distribution of National Grid 77 Scrapped Transformers Predicted Thermal Lifetimes

Figure 5-2 shows that the mean value of 77 scrapped transformers' thermal lifetimes is 120yrs; however, the data are widely distributed from 16yrs to 373yrs. This large variation of transformer thermal lifetimes once again infers different designs and the various loading conditions that the scrapped transformers have experienced.

77 scrapped transformers' predicted thermal lifetimes can be further expressed into a cumulative distribution curve as shown in Figure 5-3. Among this small group of transformers, the ages of reaching three typical cumulative failure probabilities; the early onset of population significant unreliability (corresponding to 2.5% of CDF), the

median lifetime (50% of CDF) and the late onset of population significant unreliability (97.5% of CDF), are calculated and indicated in Table 5-1.

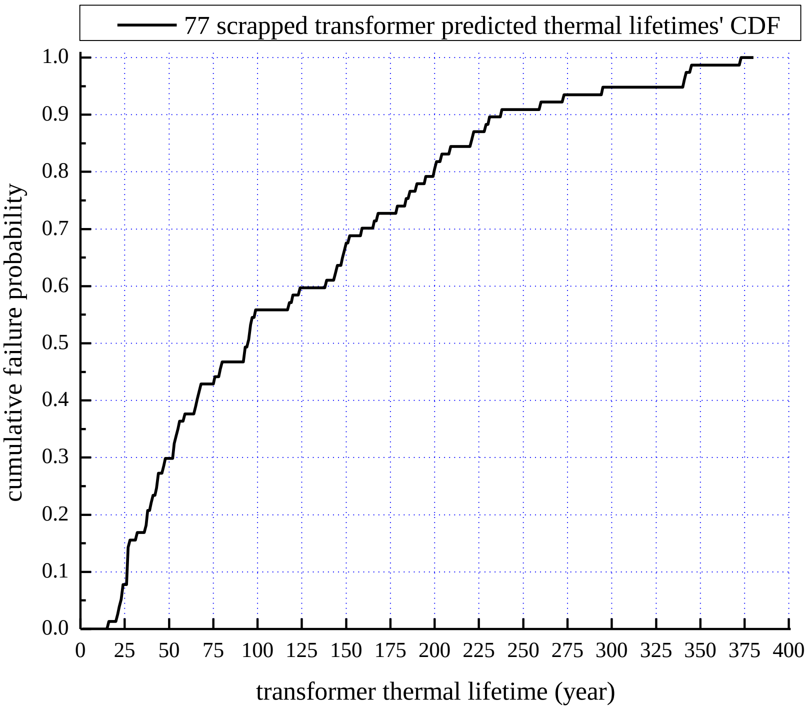


Figure 5-3 CDF of National Grid 77 Scrapped Transformers Predicted Thermal Lifetimes

Table 5-1 Ages of Reaching Typical CDFs Estimated from 77 Scrapped Transformers Predicted Thermal Lifetimes

CDF $F(t)$		Age t From 77 scrapped transformers thermal lifetime prediction
early onset of population significant unreliability	2.5%	21
median lifetime	50%	95
late onset of population significant unreliability	97.5%	342

According to Table 5-1, National Grid scrapped transformers' median thermal life is almost 100yrs and the late onset of the scrapped population unreliability is more than 300yrs. This is because a certain number of scrapped transformers were in good condition when scrapped. This is particularly true for transformers with low loading; these are estimated to have long thermal lifetimes, for instance more than 300yrs, if they continue to deteriorate at the previous extremely low ageing rate.

Meanwhile, the 77 scrapped transformers' thermal lifetimes can generate the scrapped population thermal hazard. In order to be consistent with the random hazard definition used in Chapter 3, scrapped transformer thermal hazard at age *t* is also derived as the ratio of the number of transformers whose thermal lifetimes end at age *t* and the number of transformers whose thermal lifetimes are greater or equal than age *t*. This is mathematically expressed as

$$h(t) = \frac{number\ of\ transformers\ with\ thermal\ lifetimes = age\ t}{number\ of\ transformers\ with\ thermal\ lifetimes \geq age\ t} \tag{5-5}$$

According to (5-5), 77 National Grid scrapped transformers' thermal hazard is shown in Figure 5-4.

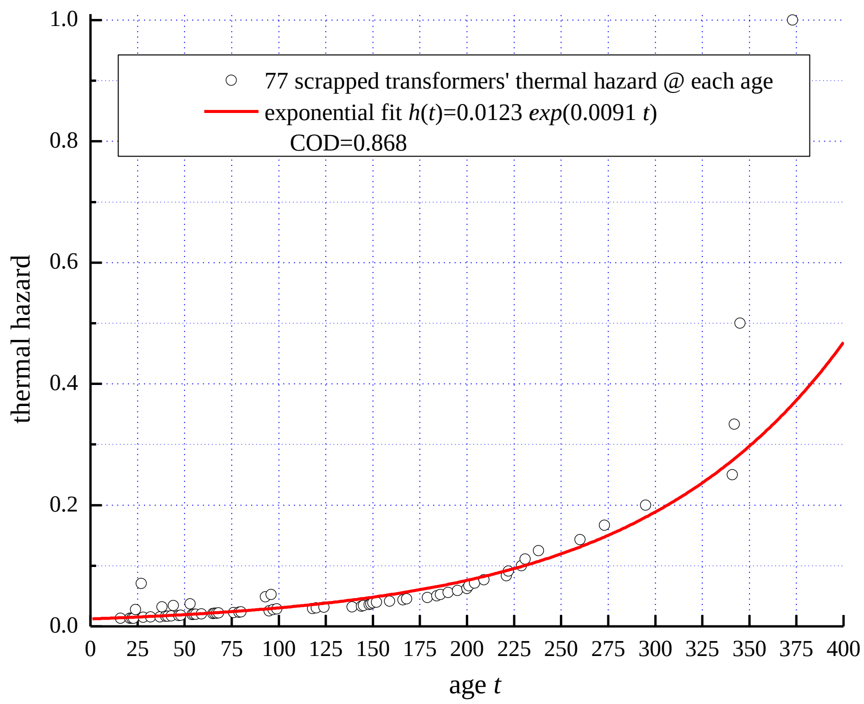


Figure 5-4 Thermal Hazard Curve of National Grid 77 Scrapped Transformers

By applying curve fitting on the 77 scrapped transformers' thermal hazard via the least square estimator (LSE), the best description is:

$$h(t) = 0.0123 \bullet exp(0.0091 \bullet t) \tag{5-6}$$

and the coefficient of determination (COD) is 0.868.

(5-6) indicates an exponentially increasing thermal hazard as shown in red in Figure 5-4. As discussed in Chapter 2, the exponential term of increasing hazard is the Gompertz Law [72] previously used to model the mortality of human beings, which is expressed as $a \bullet exp(bt)$. $a \bullet exp(bt)$ is also used to imply the increasing failure hazard at older ages in

Makecham's 1st Law [72], which is adopted by Hartford Steam Boiler (HSB) [62] to analyze its transformer lifetime data.

It can be noticed that the 1st unit, among the 77 scrapped transformers, approaches its thermal end-of-life at 16yrs, the scrapped population thermal hazard corresponding to the first failure at age 16 is derived as $\frac{1}{77} = 1.3\%$ according to (5-5), which exceeds 0.20% as the National Grid power transformer random failure rate. This high value of thermal hazard validates the reality that a certain number of poor-conditioned transformers were scrapped, and their estimated short thermal lifetimes result in a high thermal hazard at the early ages. It further suggests that 77 scrapped transformers may not be a sufficient sample to indicate the whole population thermal hazard.

5.2.3 Conclusion on the Above Analyses

It can be concluded based on the above calculations, that the scrapped transformers' ageing rates varies significantly and their thermal lifetimes distribute over a wide range. Some National Grid scrapped transformers were in a bad condition and thus deteriorated fast. However, some redundant units had long remaining thermal lifetimes, and others randomly failed with moderate ageing.

It is inferred that the 77 scrapped transformers may have experienced significantly different operation conditions or/and they were designed and manufactured differently. These scrapped samples may suggest possible design and operation discrepancies amongst the National Grid transformer population; however it is difficult at this stage to identify whether this group of scrapped transformers is a representative sample for the whole population. The cumulative failure probability derived from the 77 scrapped transformers, and clearly shown in Figure 5-3, indicates that up to age 40 more than 20% of the transformers would have reached their thermal end-of-life or would be at a high risk of ageing-related failure. This is obviously unrealistic, when compared with observed National Grid transformer failures.

The use of DP values from 77 scrapped transformers, to suggest population trends in ageing-related failure as applied to National Grid in-service transformers will be discussed in Section 5.3.

5.3 Analysis of Scrapped Transformers' Representativeness to National Grid In-Service Transformer Population

As concluded in 5.2, the thermal lifetimes of 77 scrapped transformers vary significantly and may not be an adequate sample, to represent the thermal ageing of the entire National Grid fleet of power transformers from which they are drawn. The characteristics of these scrapped transformers, in terms of their design specifications, operation conditions and installation sites, only constitutes a snapshot of the whole population operating on the National Grid network.

5.3.1 Transformer Thermal Lifetime Determinant Factors

Transformer thermal lifetime is actually determined using multiple variables, including transformer instantaneous load, ambient condition, transformer oil/winding design and cooling control. The estimation process of transformer thermal lifetime from these variables can be generally presented in Figure 5-5. This model is adopted by IEEE and IEC loading guide for oil-immersed power transformers.

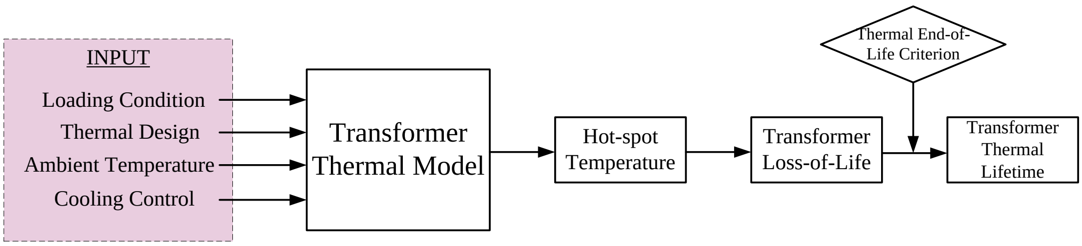


Figure 5-5 Transformer Thermal Lifetime Estimation via Transformer Thermal Model

Transformer thermal model is described using simple differential equations based on the fundamental loss of generation and the heat transfer principles [128, 129]. The oil viscosity change as a consequence of heat generated due to core and winding resistance loss and heat transfer is sometimes taken into account [128, 129]. Moreover, transformer losses are determined by transformer loading condition and ambient temperatures. Whilst, transformer heat transfer capability is affected by thermal design and cooling facilities.

The hottest temperature of a transformer winding, referred to as the transformer hot-spot temperature, is the output of the transformer thermal model. Transformer hot spot essentially determines the unit's loading capability, thermal ageing and the bubble generation risk under severe condition [130]. Transformer hot-spot temperature is generally derived as the sum of the transformer ambient temperature, top oil temperature rise over ambient and the hot-spot temperature rise over top oil temperature. Typically under a constant load (K) and unchanged ambient condition (θ_a), the steady-state hot-spot temperature (θ_h) can be obtained as

$$\theta_h = \left[\frac{1+K^2 \bullet R}{1+R}\right]^x \bullet \Delta\theta_{or} + K^y \bullet G_r \bullet HSF + \theta_a \qquad \text{(5-7) [97]}$$

in which K is the load factor in p.u. based on transformer rating,

θ_a is the ambient temperature,

x is the oil exponent, which determines the effect of total losses to the top oil temperature rise,

y is the winding exponent, which determines the effect of transformer loading on the winding temperature rise,

R, $\Delta\theta_{tor}$, G_r are thermal parameters, determined by a transformer temperature-rise test at rated load, and

HSF is the abbreviation for transformer hot-spot factor, which considers the temperature difference between the hot-spot and the top winding.

Details of the above parameters and their values, corresponding to the selected cooling mode, will be presented when the scrapped transformer thermal design characteristics are further discussed. Also it can be seen from (5-7) that the transformer loading condition, ambient temperature, thermal design and cooling control are the determinant factors for the transformer hot-spot temperature and thus the unit's loss-of-life and thermal end-of-life. The discrepancies of scrapped transformers' thermal lifetimes and their representativeness to the National Grid in-service population are also discussed.

5.3.2 Scrapped Transformer Representativeness Analysis

National Grid traditionally adopted 75/50 ℃ or 95/70 ℃ cooling control for transformers. There is however a lack of information to indicate the exact temperatures used in individual cooling control. It is assumed in this thesis that power transformers' cooler switching is controlled by the temperature of 75/50 ℃ and thus the effect from different cooler settings will not be considered in this thesis.

The reasons for the discrepancies in the scrapped transformer conditions is explained in this section, with reference to the large dispersion of the 77 scrapped transformers' thermal lifetimes, and why there is little indication to suggest National Grid transformer population ageing.

5.3.2.1 Variation of Transformer Loading

Transformer loading can be evaluated using: the level of loading and the profile of loading. Transformer load level concerns the equivalent burden when the transformer operates and the load profile indicates the transient load fluctuation upwards or downwards from a certain level.

- **Load Level**

According to the IEEE transformer loading guide, the transformer load level can be proposed as an equivalent constant load which converts the actual fluctuating load into a constant, within the same period of time, and with an equivalent loss generated during the transformer operation. The equivalent load is calculated as

$$L_{eq} = \sqrt{\frac{L_1^2 t_1 + L_2^2 t_2 + L_3^2 t_3 + \ldots + L_N^2 t_N}{t_1 + t_2 + t_3 + \ldots + t_N}} \quad \text{(5-8) [97]}$$

in which L_1, L_2, L_3, …, L_N are the various loads in MVA corresponding to the time interval t_1, t_2, t_3, …, t_N respectively, and N is the total number of time intervals.

Since National Grid power transformer loading in 2009 was effectively recorded, the scrapped transformers' loading condition can be briefly evaluated according to the loading records of the currently in-service transformers at the same site of those scrapped transformers. Due to the limited load record, 47 out of the 77 scrapped transformers' loading records, back to the years of scrapping, can be obtained. The calculation process is described in the following 3 steps.

Step 1: 2009 annual equivalent load L_{eq} of each of the 47 in-service transformers are calculated according to equation (5-8).

Step 2: each of the 2009 equivalent load L_{eq} is calculated backward to the year of transformer scrapping based on 1.1% annual growth of national demand given by National Grid [131]. This is formulated as

$$L'_{eq} = \frac{L_{eq}}{(1+1.1\%)^{(2009-Y_X)}} \quad \text{(5-9)}$$

where L'_{eq} is the equivalent load at the year of transformer scrapping and Y_X indicates the scrapping year of a particular transformer. The general assumption of individual transformer 1.1% annual load increase may not be realistic, because load on some transformers may be significantly different in 2009 to previous years. However, transformer loading in 2009 is the only information available that can be used for analysis.

Step 3: the transformer equivalent load at the year of scrapping L'_{eq} is expressed in per unit (p.u.) using the base of each transformer's rating, designated as the transformer load factor (K). The calculation is expressed as

$$K = \frac{L'_{eq}(MVA)}{Rating(MVA)} \tag{5-10}$$

The histogram of those 47 scrapped transformers' annual equivalent load factor (K) back to their scrapping year is shown in Figure 5-6.

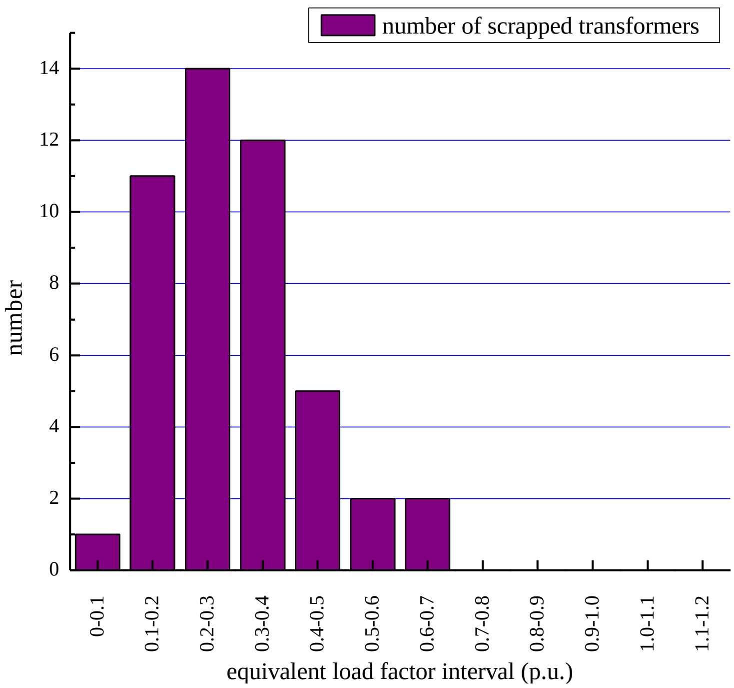
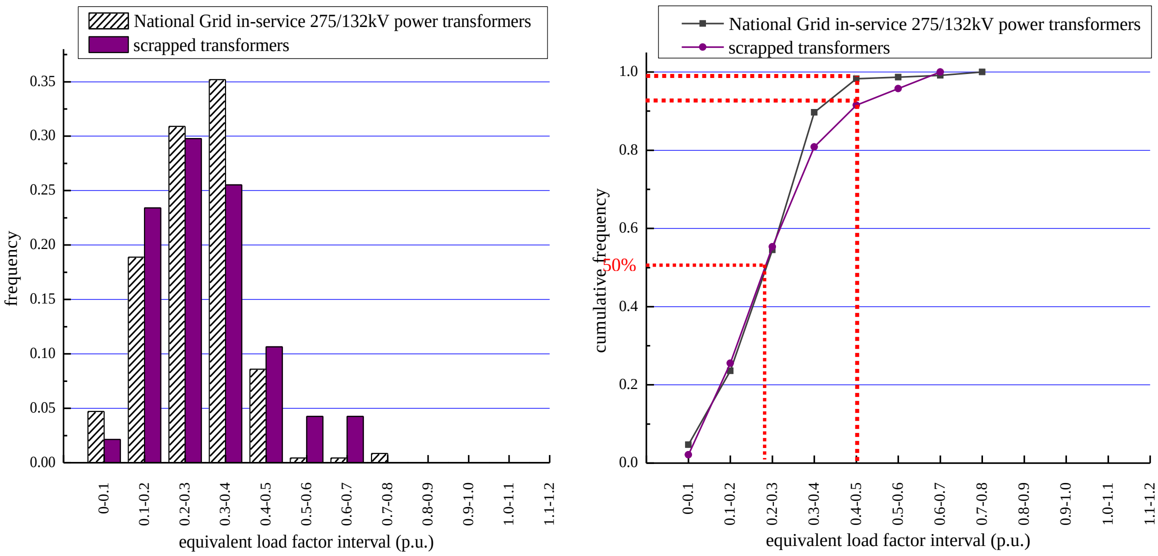


Figure 5-6 Histogram of 47 Scrapped Transformers' Equivalent Load Factor K backward to Transformer Scrapping Year

It can be concluded from Figure 5-6 that more than 90% of those 47 scrapped transmission transformers were on average loaded less than half of their power ratings. Besides transformer loads were centralized between 0.1p.u.-0.5p.u. and the average loading is about 0.25p.u[vi]. This is consistent with the *N*-1 planning and operation regulation conducted in the UK transmission network, according to which transmission network plant should not be loaded more than half of their ratings under the normal operation stress in concern of network stability when experiencing a through fault.

At present there are two hundred and seventy 275/132kV power transformers operating in National Grid's network. Two hundred and thirty three out of these two hundred and seventy transformers' 2009 real-time load are effectively recorded according to the data provided by National Grid [132]. The 2009 annual equivalent load factors of these 233 transformers are compared with the 47 scrapped transformers shown in Figure 5-7.



(a) Equivalent Load Frequency Comparison

(b) Equivalent Load Cumulative Frequency Comparison

Figure 5-7 National Grid In-Service 275/132kV Transformers 2009 Equivalent Loads vs. Scrapped Transformers Equivalent Loads @ Year of Scrapping

Figure 5-7(a) presents the frequencies of the annual equivalent load factor intervals among the in-service 275/132kV transformers and the scrapped transformers. Figure 5-7(b) simply plots the cumulative frequencies against the load factor intervals. It infers from Figure 5-7(a) that the equivalent load distribution of National Grid 275/132kV in-service transformers is similar to that of the scrapped transformers, except that on bands 0.5-0.7p.u., scrapped transformers have high frequency. The equivalent loads of

[vi] Great Britain national demand varies between about 18GW and 70GW with the average demand of 40GW. If assuming *N*-1 planning and operation regulation and some spare capacities, the total capacity of transmission transformers needs to be at least 140GW. The generic "average equivalent load factor" of a transformer is 40/140=0.29p.u., which is between 0.2p.u. and 0.3p.u..

275/132kV in-service transformers are also centralized between 0.1p.u. and 0.5p.u. According to Figure 5-7(b), more than 98% 275/132kV transformers are equivalently loaded no more than 0.5p.u. This is again consistent with the *N*-1 operation regulation conducted in the UK transmission network as indicated by 47 scrapped transformers. Besides it further reveals in Figure 5-7(b) that 50% of the 275/132kV population and also 50% of the scrapped transformers are on average lightly loaded, i.e. at no more than 0.3p.u.

According to the comparisons in Figure 5-7(a) and (b) the scrapped transformers' equivalent load factors reflect the real load level of the National Grid 275/132kV in-service transformer population. If the annual equivalent load factor *K* is the most dominant factor in the transformer thermal lifetime, 47 scrapped transformers would be a sufficient sample to directly indicate the National Grid in-service population thermal ageing. Hence the equivalent load of the scrapped transformers at the year of scrapping are displayed together with their predicted thermal lifetimes as shown in Figure 5-8.

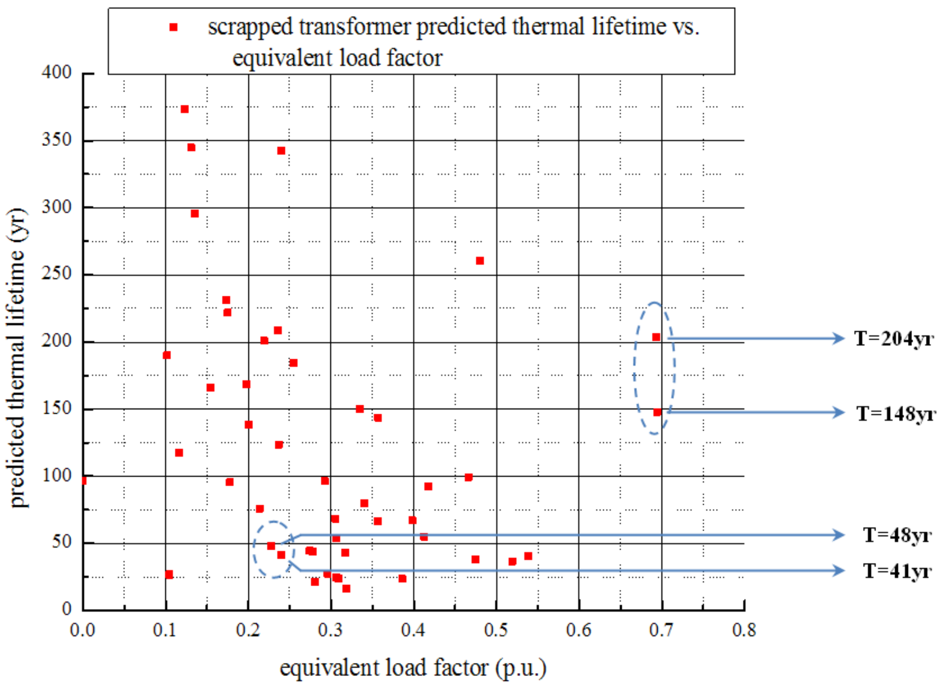


Figure 5-8 47 Scrapped Transformer Predicted Thermal Lifetimes against Annual Equivalent Load Factor

The correlation between the annual equivalent load (shown in *x* axis in Figure 5-8) and the predicted thermal lifetime (shown in *y* axis in Figure 5-8) of these 47 scrapped transformers is 24%[vii], which indicates a weak relationship between the transformer load level and thermal lifetime. It can be seen from Figure 5-8 that the transformer predicted thermal lifetimes vary significantly even though they are similarly loaded. In particular, as highlighted in Figure 5-8, two transformers, both loaded high i.e. 0.69 p.u., are estimated to reach their thermal end-of-life at 148yrs and 204yrs respectively. While two other transformers, both lightly loaded at 0.23p.u. are predicted to have thermal lifetimes less than 50yrs.

It is suggested from Figure 5-8 that a transformer's thermal lifetime is determined, not only by its load level, but also by its thermal design and the installation ambient temperature. However it should be borne in mind that 2009 transformer load records have included errors, when used to estimate the historical loading of scrapped transformers. These errors can be clearly stated as:

i) Loading of an in-service transformer does not represent the scrapped transformer's previous loading, typically when the substation was upgraded or changed in its layout.

ii) Current in-service transformer's loading (2009) may not accurately reflect the loading of the replaced unit. The 1.1% annual national demand growth predicted by National Grid in 2009 may be different from reality; for example unpredicted cold winter in 2009 and the figure varies per annum, for instance it was given as 1.3% in 2005 [133].

iii) One year loading record of a transformer does not adequately represent the whole loading history during its previous operation years, especially when the transformer was moved or used as a spare unit.

- **Load Profile**

In addition to the load level indicated by the transformer annual equivalent load factor *K*, the transformer load profile or the pattern of load is the other aspect of the transformer loading condition. Operation practice indicates that the transformer load varies

---

[vii] The correlation coefficient (correl) of array *x* and array *y* is calculated as $correl(x,y)=\frac{\sum(x-\bar{x})\bullet(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2\sum(y-\bar{y})^2}}$, where $\bar{x}$ and $\bar{y}$ are the arithmetic mean values of array *x* and *y* respectively.

cyclically with a duration of 24hrs [34, 96, 134]. Real-time variations in the transformer loading would affect the hot-spot temperature and thus the transformer ageing rate; even if the transformer's equivalent load remains constant.

According to the UK National Grid 7-year-statement (2008/2009), the Great Britain transmission system cyclic demand within 24hrs is represented by a national daily demand profile of one typical winter day (04/12/2008, the 1st Thursday of December, 2008) and one typical summer day (19/06/2008, the 3rd Thursday of June, 2008). These two daily demand curves are shown in Figure 5-9.

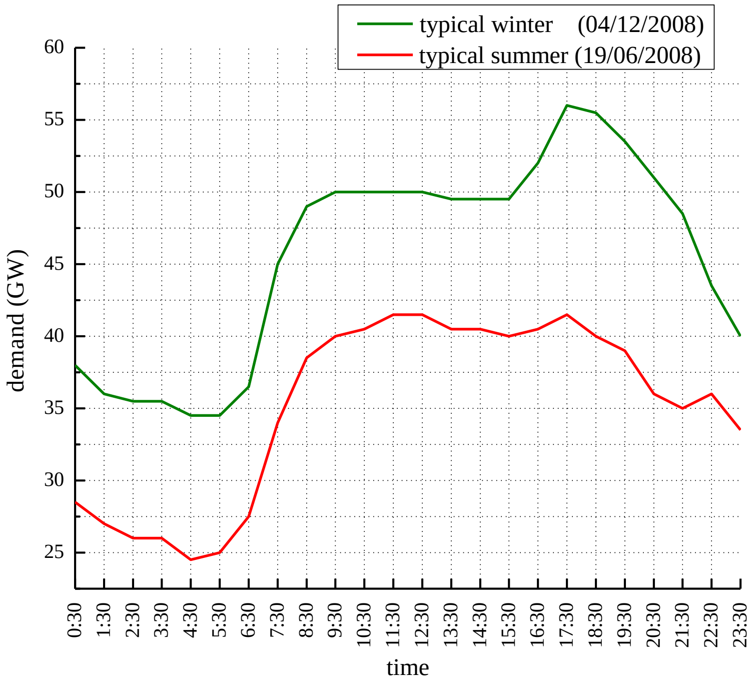
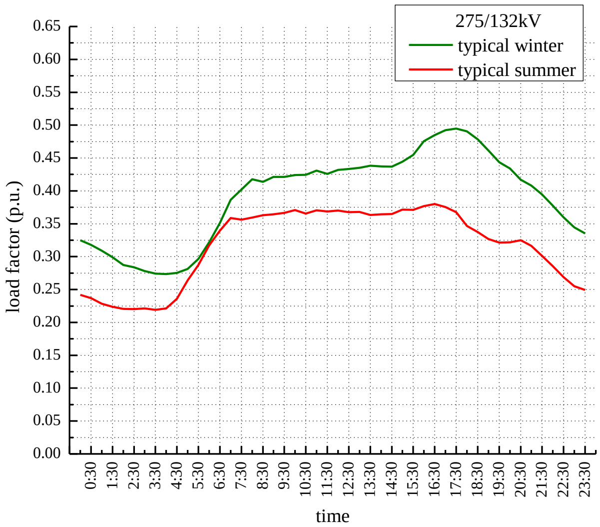


Figure 5-9 Great Britain National Demand Profile on Typical Winter Day and Typical Summer Day [131]

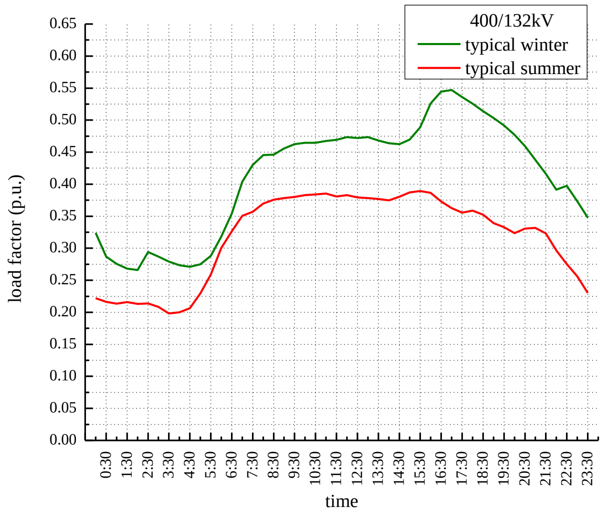
It can be seen from Figure 5-9 that the national demand varies within a typical day of winter and summer. In particular, significantly more electricity needs to be consumed during winter days due to significant heating demands and the early onset of evening lighting loads.

In general, a scrapped transformer's load profile can be represented by the records on the typical winter and summer day. In order to be consistent with the typical dates shown in Figure 5-9, the load records on the 1st Thursday of December and the 3rd Thursday of June of five different voltage level scrapped transformers are displayed in

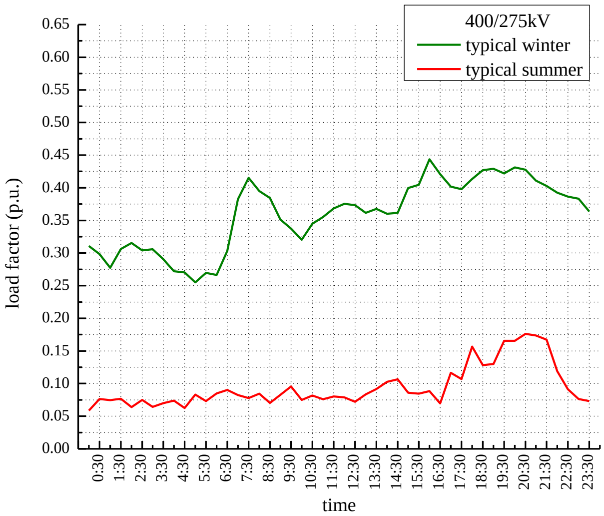
Figure 5-10. Transformer real-time load is recorded every 30min and is expressed in p.u. based on the transformer's MVA rating.



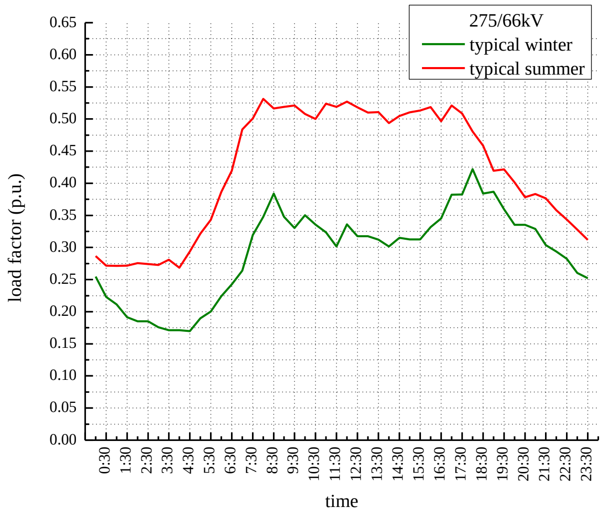
(a) 275/132kV



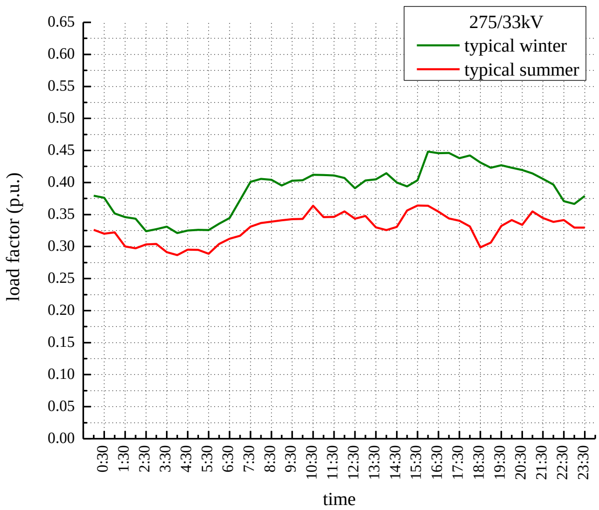
(b) 400/132kV



(c) 400/275kV



(d) 275/66kV



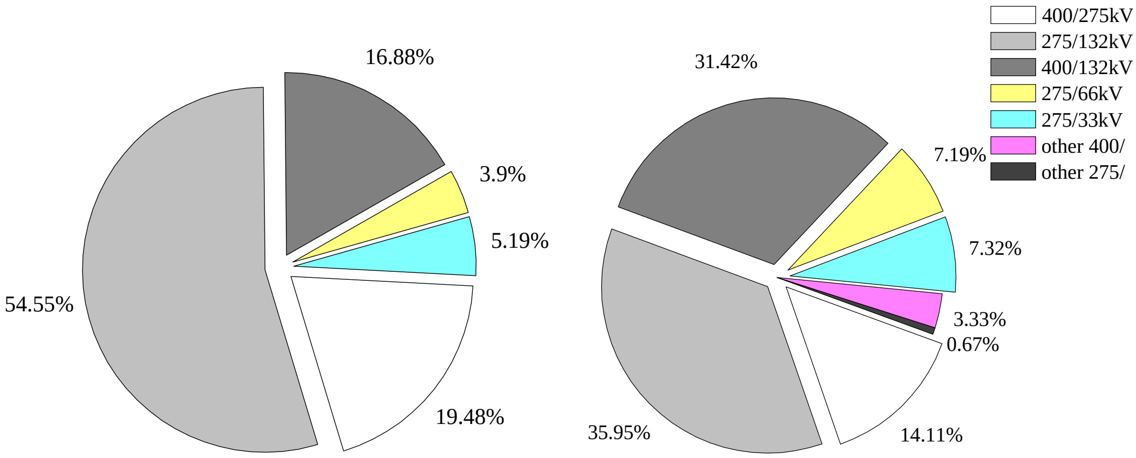
(e) 275/33kV

Figure 5-10 Daily Load Profiles of Transformers of Different Voltage Level on Typical Winter Day and Typical Summer Day [132]

It can generally be concluded from Figure 5-10 that transformer real-time load varies differently among transformers of different voltages. In particularly, the daily load profiles of 275/132kV and 400/132kV transformers are generally similar to the national daily demand profile; this is because these two groups of transformers deliver electricity from transmission level (400kV or 275kV) to distribution networks, which are below 132kV, whilst the customers receive energy from distribution network operators (DNOs) and national daily demand is obtained from the meter-readings at 132kV busbars. The load profiles of 275/66kV and 275/33kV transformers have their own patterns, which depend on the demand they supply; this is because these transformers connect to specific distribution networks that are often industrially focused. 400/275kV transformer is relatively even-loaded during the typical days and the load factor appears much less than 0.5p.u. This is because the operation of 400/275kV power transformers is to balance the power flow within transmission network in order to achieve high reliability transmission.

Moreover, one should not forget that the typical winter/summer daily load profile of a transformer may not represent its real loading condition during a specific year, or one single transformer of a certain voltage level may not indicate the loading condition of other transformers of the same voltage. However the loading profiles of the above five, different voltage level, specific transformers reflects the transformer loading characteristics at different voltages. In other words, transformer loading condition is essentially related to its voltage level and depending on what the transformer supplies.

Amongst the 77 National Grid scrapped transformers, 275/132kV, 400/275kV and 400/132kV are the three majority groups; there are few 275/33kV and 275/66kV transformers. The distribution of different voltage groups within 77 scrapped transformers is shown in a pie chart in Figure 5-11 and is compared with the voltage group distribution of National Grid in-service transformers in 2009.



(c) among 77 Scrapped Transformers

(d) among National Grid In-Service Transformers

Figure 5-11 Voltage Distribution of Scrapped Transformers and National Grid In-Service Transformers

By comparing Figure 5-11(a) and Figure 5-11(b), it can be seen that a small number of 400/66kV, 400/33kV, 400/145kV, 400/26.25kV, 275/11kV, 275/26kV and 275/20.5kV transformers are currently operating on the National Grid transmission network, and no scrapped samples are available. The relative number of 400/275kV, 275/132kV, 400/132kV, 275/66kV and 275/33kV transformers within the scrapped samples and among the overall in-service transformers is of interest. In particular, because the 275kV network is older than 400kV network in the G.B., the number of 275/132kV transformers scrapped is high, and 275/132kV transformers are more than 50% of the scrapped transformers. However, within the in-service population 275/132kV transformers and 400/132kV transformers both take approximately 30% of the whole in-service transformers. Scrapped transformers should not be used to represent the overall in-service transformers from this point of view.

- **Conclusions on Scrapped Transformers' Loading Condition**

According to information about National Grid transformer 2009 real-time load and by analyzing scrapped transformers load level and load profile, it can be concluded that the loading conditions of 77 scrapped samples do not sufficiently indicate the loading conditions of other in-service transformers. Transformer load profile in particular, varies significantly between transformers of different voltage levels. The effect of transient load variations on transformer hot-spot temperature and transformer thermal lifetime is still under investigation. It is difficult to use the general loading of a few scrapped transformers to represent the loads of other transformers, even when they operate at the same voltage levels.

Moreover transformer loading is not the only determinant factor in evaluating the transformer thermal lifetime; transformer thermal design is also important.

5.3.2.2 Variation of Transformer Thermal Design

Transformer thermal design also determines the maximum temperature rises within the transformer and consequently is an important factor in evaluating the transformer thermal lifetime [36]. To obtain the exact temperature rises under continuous full load and to confirm that these temperatures are within acceptable limits, a temperature-rise test is usually undertaken within each individual transformer or each equivalently designed transformers.

The transformer temperature-rise test is performed at rated load. During the test, the transformer no-load losses (*NLL*), load losses (*LL*), oil temperature rises over ambient at the bottom and at the top of winding ($\Delta\theta_{bor}$ and $\Delta\theta_{tor}$) and the average winding temperature rise over ambient temperature ($\Delta\theta_{avwr}$) are all measured. Transformer thermal parameters R, $\Delta\theta_{or}$ and G_r at rated load are then obtained using the measured data. The parameter:

- R is the ratio of load losses (*LL*) to no-load losses (*NLL*):

$$R = \frac{LL}{NLL} \tag{5-11}$$

- $\Delta\theta_{tor}$ is the top oil temperature rise over ambient temperature in steady-state
- G_r is the gradient between the average winding temperature rise ($\Delta\theta_{avwr}$) and the average oil temperature rise ($\Delta\theta_{avor}$):

$$G_r = \Delta\theta_{avwr} - \Delta\theta_{avor} \tag{5-12}$$

  where the average oil temperature rise ($\Delta\theta_{avor}$) is calculated as the mean of the top oil temperature rise and the bottom oil temperature rise, i.e.:

$$\Delta\theta_{avor} = \frac{1}{2}\left(\Delta\theta_{tor} + \Delta\theta_{bor}\right) \tag{5-13}$$

These are the thermal parameters, used in (5-7) to calculate the transformer hot-spot temperature (θ_h) in the steady-state. Other parameters, x and y in (5-7), are related to transformer heat transfer, however they are normally empirically suggested, instead of being measured via the temperature-rise test. These parameters are not discussed in this thesis.

Values of the thermal parameters are also related to transformer cooling mode. Power transformers owned by National Grid are usually operating under dual cooling mode, referred to as ONAN/OFAF. The switching between ONAN and OFAF is controlled by the measurements recorded by the winding temperature indicators (WTIs) [43]. Under the ONAN mode, the transformer is cooled by naturally circulating the oil inside the main tank and by natural convection of the air surrounding the radiator. While under the OFAF cooling mode oil and air are forced to flow quickly over the tank and radiator by switching on extra pumps and fans. Thus the transformer is cooled more efficiently [130, 135].

The switching between these two cooling modes is controlled by an approximation of the winding hot-spot temperature, provided by data from the WTIs. National Grid ensures that for their power transformers, ONAN cooling remains if the estimated hot-spot temperature is no more than 75 ℃; extra fans and pumps will be switched on to kick start the OFAF cooling once the hot-spot temperature is estimated to exceed 75 ℃ and OFAF will remain in operation until the hot-spot temperature has been reduced below 50 ℃. The above WTIs control process is denoted as 75/50 ℃ control.

As indicated in [136], the temperature-rise test should be performed on transformers with multiple cooling schemes under all available cooling modes corresponding to the different MVA ratings. Therefore for National Grid power transformers equipped with ONAN/OFAF dual cooling mode, the base power rating for OFAF with fans and pumps switched on is the nominated rating as indicated in the transformer nameplate, which the base rating should be the half of the nominated power when natural cooling is used [136].

The IEC transformer loading guide provides typical values of thermal parameters R, $\Delta\theta_{or}$ and G_r under the rated load for both ONAN and OFAF cooling [97]. The values are listed in Table 5-2 as follows.

Table 5-2 Typical Values of Thermal Parameters Given by IEC Transformer Loading Guide [97]

	Cooling Mode	
Thermal parameter	ONAN	OFAF
Loss ratio *R*	6	6
Top oil temperature rise $\Delta\theta_{tor}$ (K)	52	56
Average winding-to-oil temperature gradient G_r (K)	20	17

Moreover the oil/winding temperatures measured via transformer temperature-rise test and the obtained values of thermal parameters can be shown in a thermal diagram. IEC defines typical values of transformer thermal parameters under ONAN and OFAF cooling mode; these are tabulated in Table 5-2 and displayed in Figure 5-12; assuming a constant ambient temperature of 20 ℃.

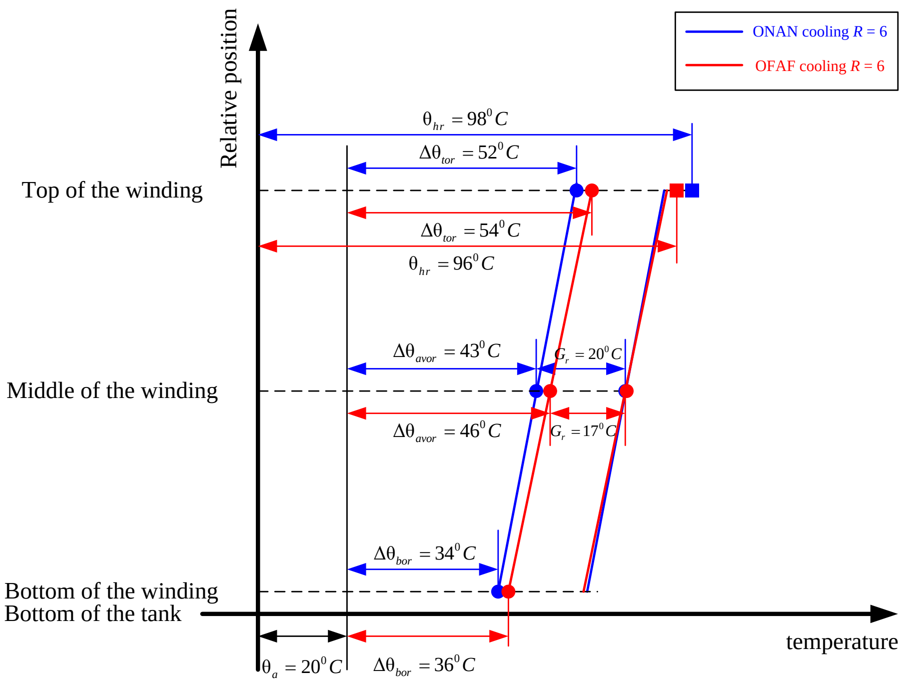


Figure 5-12 Transformer Thermal Diagram of IEC Typical Thermal Design under ONAN and OFAF Cooling Mode

As shown in Figure 5-12, the transformer thermal diagram is based on the assumptions that no matter what cooling mode is applied, the oil temperature within transformer main tank increases linearly from the bottom to the top of the tank, and the winding temperature rise is in parallel with the oil temperature rise, i.e. with a constant temperature difference [34]. The measured oil/winding temperatures are indicated by dots in Figure 5-12 under either ONAN or OFAF cooling mode. Lines in blue show the temperature rises under ONAN cooling and red lines represent the temperature rises under OFAF cooling.

It can be noticed from Figure 5-12 that the slope of each line indicates temperature difference between top oil and bottom oil or the temperature difference between top winding and bottom winding. The slope further infers the oil flow velocity from bottom to top, and the dissipation of the generated losses subject to the specific oil duct design. Additionally the temperature difference between the oil and winding under a certain cooling mode indicates the cooling efficiency, associated with transferring heat from conductors to the surrounding oil.

The measured oil/winding temperatures reveal the global temperature rise within the transformer main tank. However, the localized hottest temperature or the transformer hot-spot temperature cannot be easily measured. The transformer hot-spot is usually near the top of the winding due to the maximum leakage flux and the high surrounding oil temperature. The value is higher than the temperature at the top of winding owing to the additional losses and imperfections in the oil flow [34, 136]. The hot-spot extra temperature rise over top winding temperature can be considered by a dimensionless factor, denoted as the hot-spot factor (*HSF*). The hot-spot temperature from WTIs is given by measuring the top oil temperature and using *HSF* of 1.3 for either ONAN or OFAF cooling mode. Moreover based on *HSF* of 1.3 the IEC typical designed transformer hot-spot temperature at rated load ($\Delta\theta_{hr}$) reaches 98 ℃ and 96 ℃ under ONAN and OFAF cooling mode respectively, of which 98 ℃ is the maximum conductor temperature allowance for transformers with kraft paper insulation.

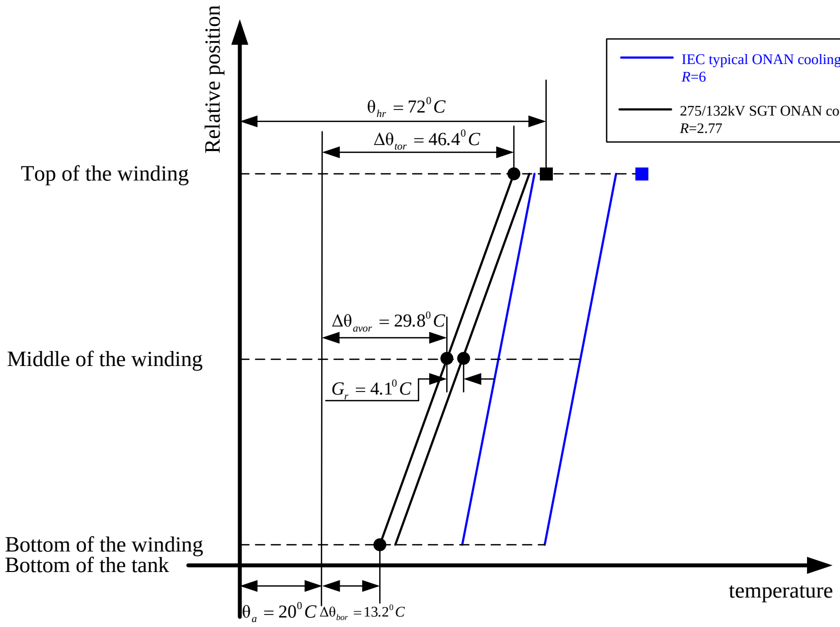
When analyzing the characteristics of the scrapped transformer, the transformer thermal parameters of loss ratio (R), top oil temperature rise ($\Delta\theta_{tor}$), average winding temperature rise and average oil temperature rise gradient (G_r) are discussed. Together they indicate the global heat transfer within the transformer main tank. Transformer hot-spot factor (*HSF*) is also discussed because it indicates the transformer localized hottest temperature. Moreover thermal design is revealed to be strongly related to the particular manufacturer of the transformer and the year of manufacturing. This information will also be presented.

- **Transformer Thermal Parameters**

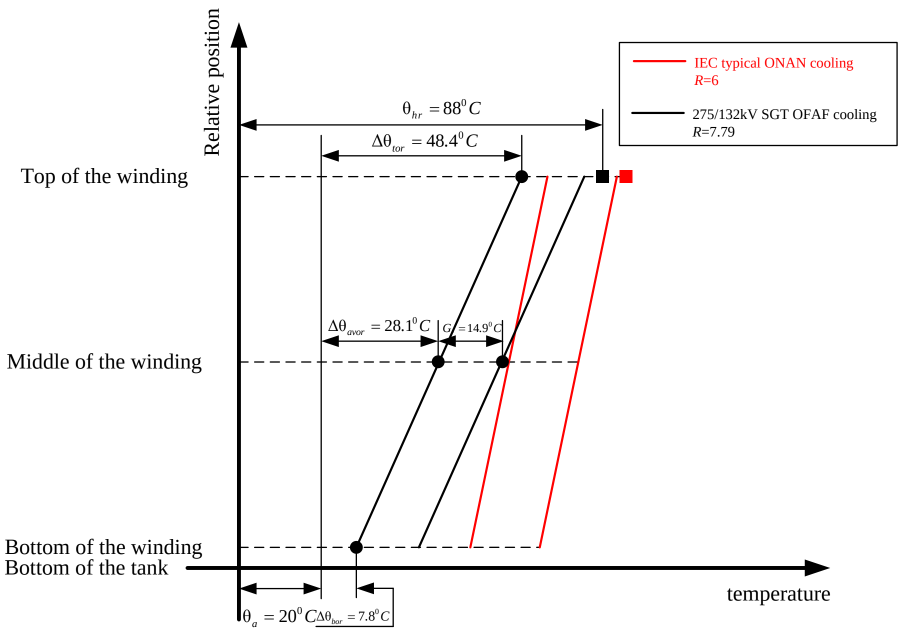
Amongst the 77 National Grid scrapped transformers, there are only thermal parameters for 16 transformers under both ONAN and OFAF cooling modes. These 16 are completely recorded, either in their factory testing reports or the reports of other equivalently designed sister transformers. In general, under each cooling mode the measurement of loss ratio (*R*), top and bottom oil temperature rise ($\Delta\theta_{tor}$ and $\Delta\theta_{bor}$), mean winding temperature rise ($\Delta\theta_{avwr}$) and the temperature rise gradient (G_r) are dispersedly distributed; the values are different to the IEC typical values shown in Figure 5-12. The large variation of each thermal parameter and its discrepancy to IEC typical designs are represented by thermal diagrams of three scrapped transformers at 275/132kV, 400/132kV and 400/275kV respectively. Each transformer's thermal diagrams under ONAN and OFAF cooling are drawn separately and they are displayed together with the IEC typical ONAN cooling design or the IEC typical OFAF cooling. These diagrams are shown in Figure 5-13 to Figure 5-15.

As indicated in Figure 5-13(a), under the ONAN cooling mode, the bottom oil temperature rise ($\Delta\theta_{bor}$), top oil temperature rise ($\Delta\theta_{tor}$), average oil temperature rise ($\Delta\theta_{avor}$) and the mean winding temperature rise ($\Delta\theta_{avwr}$) measured on this specific 275/132kV transformer are much less than the values suggested by IEC. The temperature difference between the top and bottom oil or between the top and bottom winding is also larger than suggested by IEC. However when ONAN is performed under the rated load, the generated loss is less than half that suggested from an IEC typical design, which infers a significantly slower oil flow within the transformer oil ducts. It is however difficult to compare oil and air natural convection efficiency between this specific transformer with ONAN cooling and the IEC typical ONAN design, because the loss ratio (*R*) and the winding-to-oil gradient (G_r) of this 275/132kV transformer are both significantly less than the IEC typical designed transformer.

When the OFAF cooling is performed as indicated in Figure 5-13(b), the generated loss appears a little higher, which is according to the loss ratio R=7.79, as compared to R=6 in an IEC typical design. The temperature gradient (G_r) is closer to the IEC typical OFAF design. This indicates that enhanced cooling is obtained within this transformer as compared to the IEC typical OFAF model; however the top-to-bottom temperature difference ($\Delta\theta_{tor}$ - $\Delta\theta_{bor}$) is still larger.



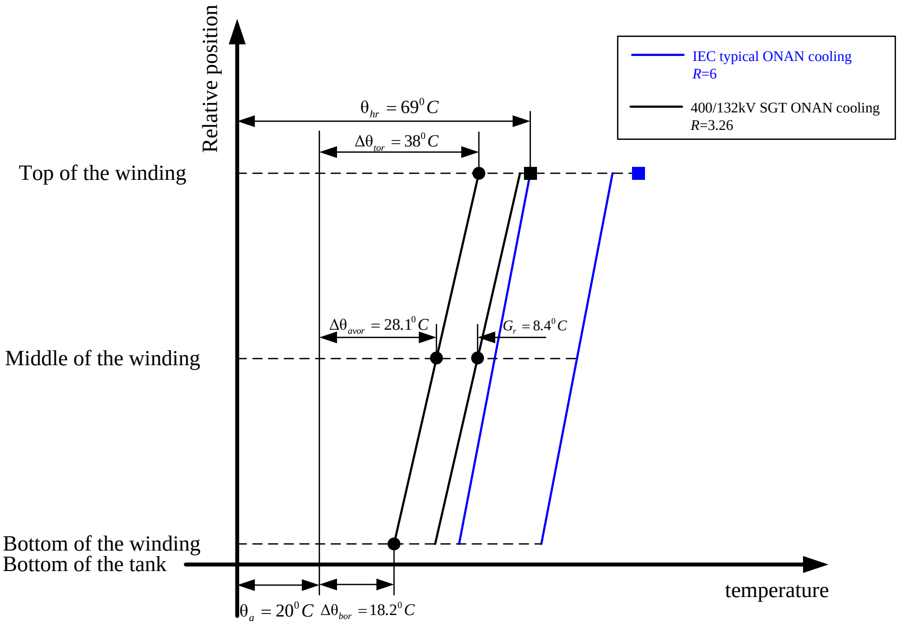
(a) SGTA ONAN Cooling vs. IEC ONAN Cooling



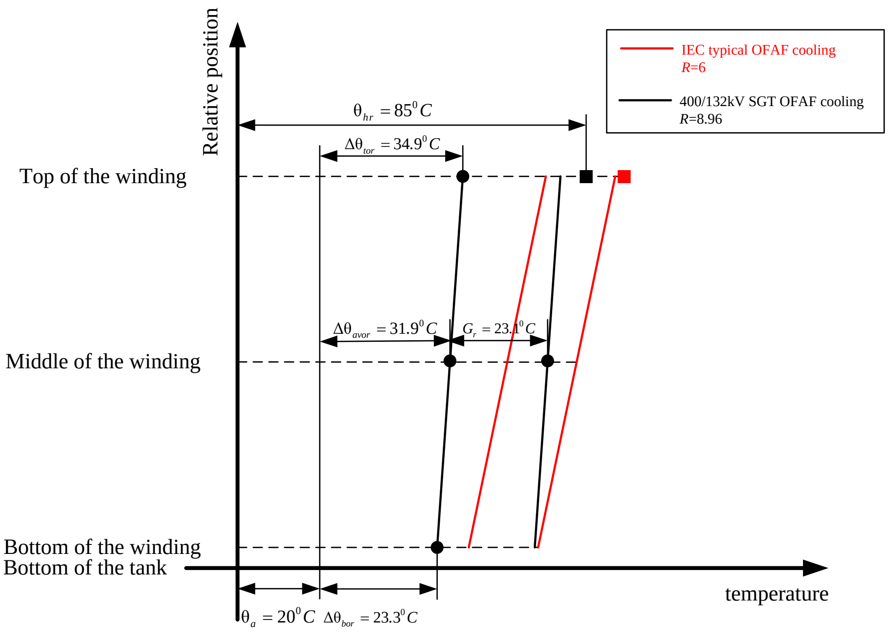
(b) SGTA OFAF Cooling vs. IEC OFAF Cooling

Figure 5-13 275/132kV SGTA ONAN/OFAF Dual Cooling vs. IEC Typical ONAN/OFAF Dual Cooling

This 275/132kV transformer was built around 1961-1963 and replaced in 1994 by a higher rated transformer. According to the investigation performed during scrapping, the specific design associated with this transformer showed signs of arcing and sparking in the main tank, and this resulted in the potential loose clamping problem [87]. The problem was further observed in other scrapped transformers designed and manufactured by the same company. However the above problems were identified as not directly related to transformer thermal design deficiencies. In fact transformers belonging to this specific design family are generally recognized as performing well, in terms of dielectric, thermal and mechanical behaviour.



(a) SGTB ONAN Cooling vs. IEC ONAN Cooling



(b) SGTB OFAF Cooling vs. IEC OFAF Cooling

Figure 5-14 400/132kV SGTB ONAN/OFAF Dual Cooling vs. IEC Typical ONAN/OFAF Dual Cooling

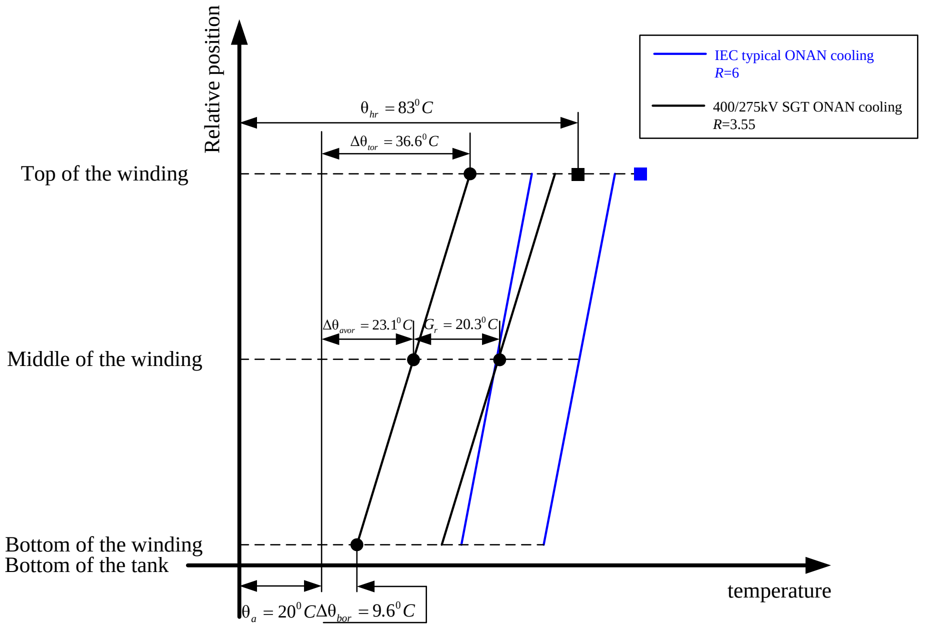
Again as shown in Figure 5-14, when operating under ONAN cooling, this 400/132kV transformer has relatively lower temperature rises ($\Delta\theta_{bor}$, $\Delta\theta_{tor}$, $\Delta\theta_{avor}$ and $\Delta\theta_{avwr}$) and lower temperature gradient (G_r) as compared to an IEC typical design. The oil flow rate within the transformer main tank is less than with the IEC typical design because the generated loss is much less (R=3.26), whilst the temperature difference between the top oil and bottom oil or between the top winding and bottom winding is comparable to the values associated with the IEC design. Similar to the above, the cooling efficiency of SGT B when operating under ONAN cooling is different to that observed with the IEC typically designed ONAN cooled transformer.

When OFAF cooling is switched on, the forced oil flows much faster than IEC typical designed OFAF cooling, which results in less temperature difference between the top oil and bottom oil or between the top winding and bottom winding even higher loss is generated (R=8.96). The cooling efficiency seems generally close to that of the typical IEC design.

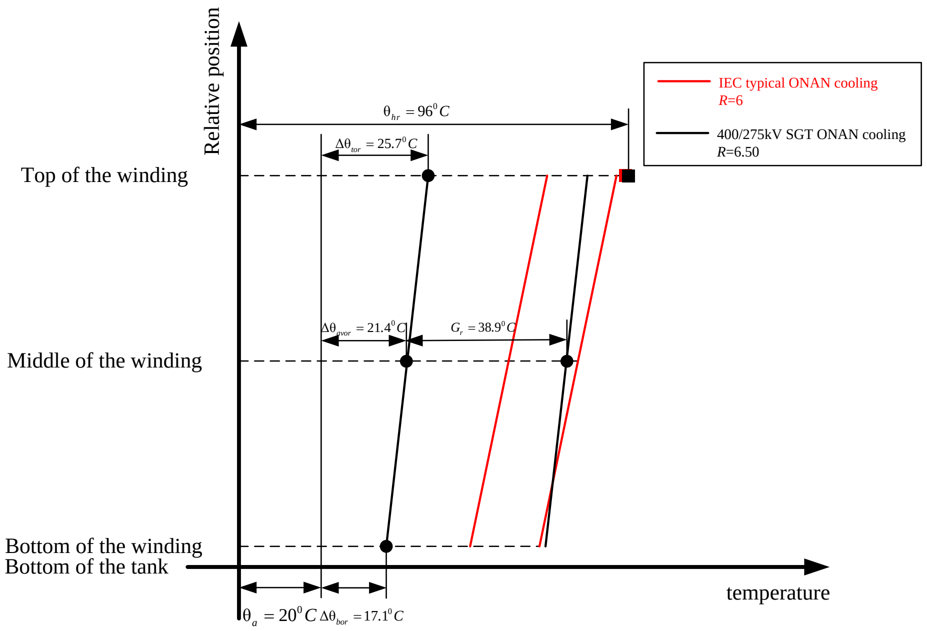
400/132kV transformers of this specific design were tested and studied by National Grid as they were considered defective in insulation mechanical strength during short circuit [87]. Meanwhile a large core and frame circulating current had also been observed when performing the temperature-rise test, which might lead to localized overheating and insulation thermal breakdown.

Figure 5-15(a) and Figure 5-15(b) show the thermal diagrams of a 400/275kV scrapped transformer. The oil flow rate within the transformer main tank is slower when ONAN cooling is performed and the ONAN cooling is less efficient than the IEC typically designed ONAN cooling. Although the oil flows much faster when the OFAF cooling is switched on, the ancillary fans and pumps do not work as efficiently as the IEC OFAF model designed to dissipate the heat from the winding to the oil.

Power transformers designed and built similarly to this 400/275kV transformer have been revealed by National Grid to suffer from inadequate mechanical strength during short circuits due to the wooden clamping beam over the windings rather than the conventional steel frame and the clamping screws arrangement [87]. However, no problems directly related to transformer thermal design have been observed.



(a) SGTC ONAN Cooling vs. IEC ONAN Cooling



(b) SGTC OFAF Cooling vs. IEC OFAF Cooling

Figure 5-15 400/275kV SGTC ONAN/OFAF Dual Cooling vs. IEC Typical ONAN/OFAF Dual Cooling

It can be concluded according to the sampled transformer thermal diagrams as shown in Figure 5-13, Figure 5-14 and Figure 5-15 that individual transformer thermal design, including loss ratio, temperature rises and temperature gradient, are different to an IEC typical design and besides the thermal design of one transformer could be significantly different to others. By reviewing the 16 scrapped transformers' temperature-rise test results, the following parameters were derived: arithmetic mean value, standard deviation and the range of transformer loss ratio (R), top oil temperature rise ($\Delta\theta_{tor}$) and the winding-to-oil temperature gradient (G_r). They are then presented in Table 5-3.

Table 5-3 Distribution of Scrapped Transformer Thermal Parameters

		ONAN		
		R	$\Delta\theta_{tor}$	G_r
16 Scrapped Data	Mean	3.10	39.89	13.71
	Standard deviation	0.93	6.56	6.51
	[min, max]	[1.76, 6.17]	[27.40, 57.50]	[4.10, 27.50]
		OFAF		
		R	$\Delta\theta_{tor}$	G_r
	Mean	8.74	36.09	26.63
	Standard deviation	3.80	7.35	6.66
	[min, max]	[4.06, 21.91]	[25.70, 52.00]	[14.90, 38.90]

According to Table 5-3, and noting that the information of transformer thermal design is fairly limited, the 16 transformers' temperature-rise test results roughly show that the value of individual thermal parameter R, $\Delta\theta_{tor}$ and G_r distribute over a wide range. This might be one of the reasons that causes the dispersed thermal lifetimes among the scrapped transformers. The variation of thermal parameters is actually determined by the transformer thermal design technique, the manufacturing process and the exact values of transformer thermal parameters. These can only be obtained via the temperature-rise test.

Except for the variations of transformer global temperature rises, the discrepancies of localized hot-spot temperature or the transformer hot-spot factor (*HSF*) needs to be further discussed. This is important, since the hot-spot is another critical factor in determining transformer thermal end-of-life.

- **Hot-spot Factor (*HSF*)**

The *HSF* is critical in determining the transformers thermal lifetime, since the ONAN/OFAF dual-cooling transformer thermal lifetime simulation model indicates that transformer thermal loss-of-life (*LoL*) increases exponentially with a linear increase in *HSF*. However, the value of *HSF* or the hot-spot temperature is not directly suggested from transformer temperature-rise test.

It is summarized in [137] that transformer *HSF* varies between windings and depends on the design, size and the short-circuit impedance of the specific transformer winding. The exact value of *HSF* may vary between 1.0 to 2.1 and needs to be investigated via direct measurement or calculation on the prototype transformer or windings [34]. IEC transformer loading guide suggests a value of *HSF*=1.3 is used for medium and large power transformers under either ONAN or OFAF cooling mode [34]. Further study, published by a CIGRÉ working group, shows that the variation of *HSF* distribute from 0.51 to 2.06 with a mean value of 1.29 based on measurement [138]. A *HSF*<1 is not realistic and this might be caused by the errors/mistakes during measurement; however, no clear explanation is given in [138]. Significant variations firstly indicates the uncertainties of *HSF*, although the mean *HSF* value of 1.29, validates the general assumption of *HSF*=1.3 adopted in IEC loading guide.

By knowing a scrapped transformer's lowest DP and its thermal parameters, and by simply using the new-installed transformer 2009 real-time load and a general ambient temperature record (i.e. Heathrow London 2007 real-time ambient temperature), the unit's *HSF* can be deduced according to the process described in Figure 5-16.

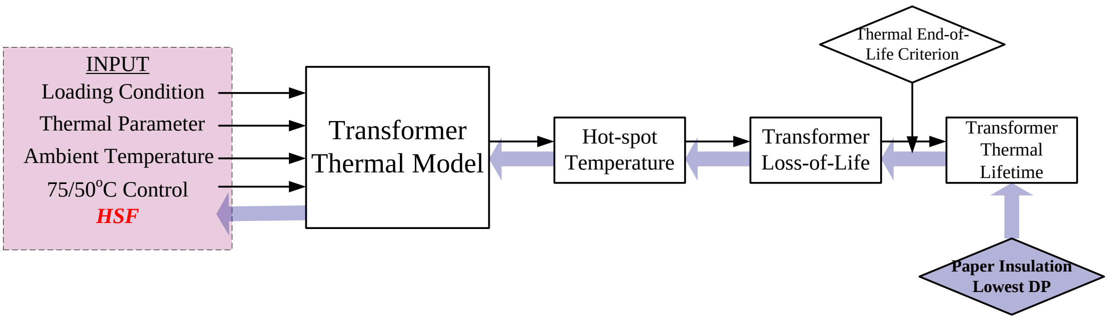


Figure 5-16 Process of Deriving Individual Scrapped Transformer *HSF*

Based on the information provided by 12 scrapped transformers', the following *HSF*s can be deduced and are presented in the inverted cumulative probability curve shown in Figure 5-17.

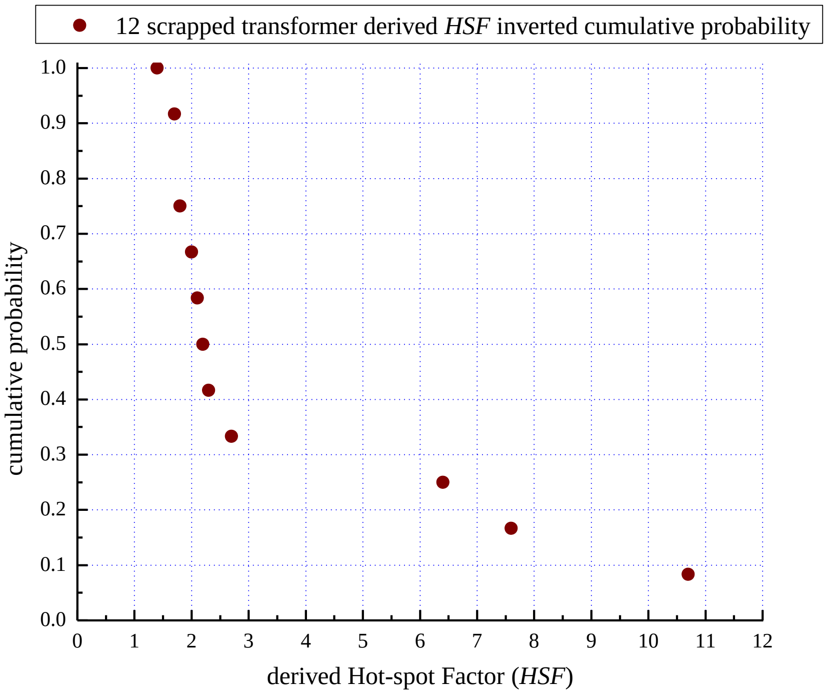


Figure 5-17 12 Scrapped Transformers Derived *HSF* Inverted Cumulative Probability

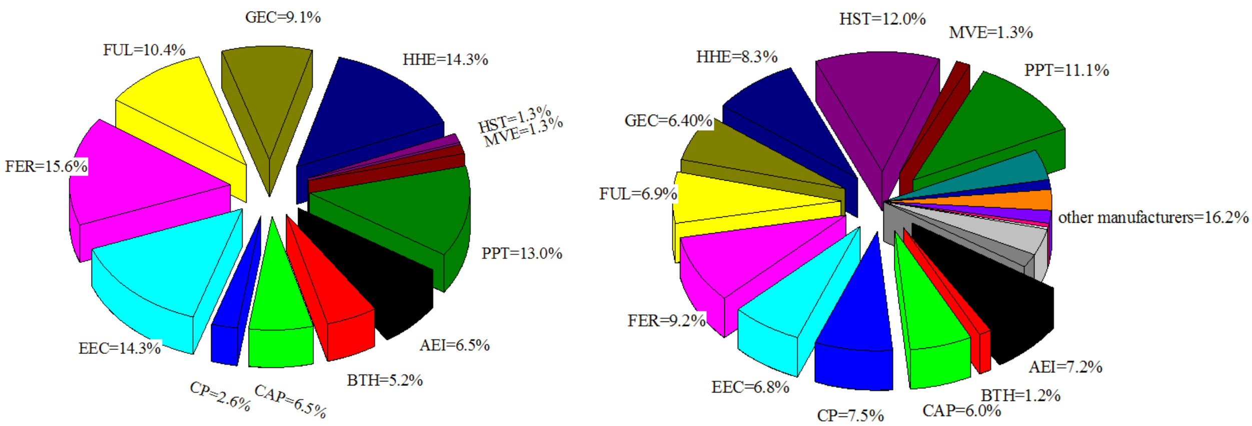
It can be seen from Figure 5-17 that the derived *HSF*s distribute in a wide range from 1.4 to 10.7, and except for three distinctly large *HSF*s the values distribute almost linearly between 1.4 and 2.7. However, this derived range of *HSF*s may not include all the possible *HSF*s for National Grid transformers, because the scrapped transformers do not constitute a sufficient sample and in many cases are badly designed units.

However according to the *HSF* distribution shown in Figure 5-17, the general assumption of *HSF*=1.3 is not realistic for the whole population, and especially for the National Grid scrapped transformers. In addition, using *HSF*=1.3 on the WTIs, may lead to false switching between the cooling modes, since the actual hot-spot temperature could have reached beyond 75 ℃, while the value is estimated to be much less by the WTIs and a presumed *HSF* of 1.3.

As indicated previously the variation of the transformers' thermal parameters and the *HSF*s are determined by the transformers' different design techniques and manufacturing processes. The manufacturers and the design groups of the scrapped transformers and the National Grid in-service population are therefore discussed in the following section.

- **Transformer Manufacturer**

The manufacturers involved in the 77 scrapped transformers and the National Grid in-service transformers are separately displayed in Figure 5-18(a) and (b). A transformer manufacturer is represented by a three letter abbreviation, and the full name of each manufacturer is given in Table 5-4. It should be noticed that some manufacturers might change their names. For example, the GEC Power Transformer (GEC) had merger English Electric Power Transformer (EEC) in 1970's and some of the EEC designs were changed to the Hackbridge and Hewittic designs (HHE) [87]. Figure 5-18 shows the companies' names when the transformers were manufactured.



(a) among 77 Scrapped Transformers

(b) among National Grid In-Service Transformers

Figure 5-18 Distribution of Manufacturers among Scrapped Transformers and among National Grid In-Service Transformers

Table 5-4 List of Transformer Manufacturers

Abbreviation	Company
AEI	Associated Electrical Industries Ltd – Transformer Division
BTH	British Thomson – Houston Co. Ltd
CAP	C. A. Parsons & Co. Ltd
CP	Crompton Parkinson Ltd (The B. E. T. Co. Ltd)
EEC	English Electric Power Transformer Ltd
FER	Ferranti Ltd
FUL	Fuller Electric Ltd
GEC	GEC Power Transformer Ltd
HHE	Hackbridge & Hewittic Elect. Co. Ltd
HST	Hawker Siddeley Power Transformers Ltd
MVE	Metropolitan – Vickers Electrical Company Ltd
PPT	Initially Bruce Peebles Ltd and later Parsons Peebles Ltd

It can be concluded by comparing the distribution of the manufacturers within the 77 scrapped transformers and within the in-service transformers that the scrapped transformers are not a sufficiently representative sample for the whole population. The reasons are: 16.2% in-service transformers are not sampled within the scrapped group, and good designed transformers (generally good condition in core, winding, oil and other auxiliary components) tend to be less scrapped and thus their manufacturers are less involved in the scrapped group; these transformers however may take a big portion in the in-service population.

Furthermore even transformers built by the same manufacturer may be designed and produced according to different schedules and in addition, designs change with time. Transformers are thus subdivided into different design groups or design families. A transformer manufacturer distinguishes its transformers' design families by assigning different codes according to transformer's voltage level, rating, impedance, core structure, winding type, tap-changer or occasionally manufacturing period. Examples are given for three typical transformer design families, produced by one transformer manufacturer, as shown in Table 5-5 [87].

Table 5-5 Examples of Different Design Families from One Transformer Manufacturer [87]

	G01	G06a	G06b
Rating (MVA)	750	240	240
Voltage (kV)	400/275/13	400/132/13	400/132/13
Built-up Period	1977	2000	2002
Impedance (p.u.)	0.2	0.2	0.2
Tap-changer	none	Neutral-end MR MI 802	ATL AN 315
Core Structure	five limb core	three limb core	three limb core
Winding Structure	interleaved series/ common windings	interleaved disc series winding, intershielded disc common winding	interleaved disc series winding, intershielded disc common winding
Known Problems or Comments	excessive hot-spot temperature possible thermal design problem	unusual tank design without bushing pockets, giving a large oil volume	conventional tank design with bushing pockets

According to the National Grid transformer database, more than 190 different design families can be classified within the current stock of in-service transformers, some of which are not sampled within the scrapped transformers. Besides, there is a lack of a mathematical model to clearly relate a transformer's thermal parameters or *HSF* with its design family; even when a specific design has been identified with a thermal problem (i.e. design family G01 as shown in Table 5-5). In fact most of the verified design problems are related to transformer random failures, instead of thermal degradation or the ageing-related failure.

- **Conclusions on Scrapped Transformers' Thermal Design**

According to the above analyses, applied to the scrapped transformers' thermal designs, it can be concluded that the discrepancy of the scrapped transformer thermal design is another factor in the variations in the thermal lifetime. Moreover the group of scrapped transformers is not an adequate sample to indicate the thermal performance of the National Grid in-service population because

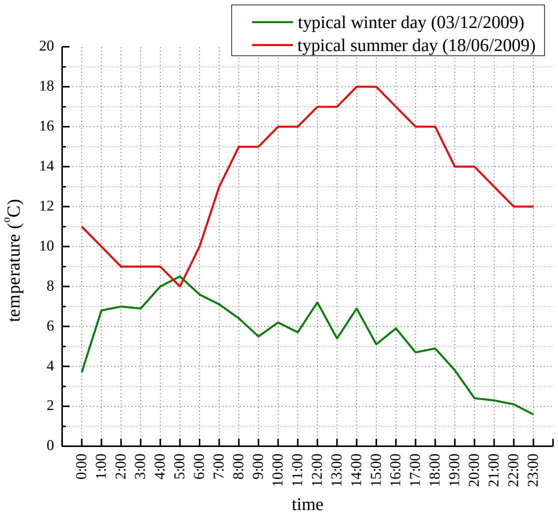
i) individual thermal parameters distribute in a dispersed manner and the exact values can be only obtained via a transformer temperature-rise test,
ii) scrapped transformer derived *HSF*s also distribute in a wide range and they may not identify the *HSF*s of the entire in-service population,
iii) a number of in-service transformers are not sampled within the scrapped group.

Besides the transformer installation site or the ambient environment would be another important factor in determining the unit's thermal lifetime.

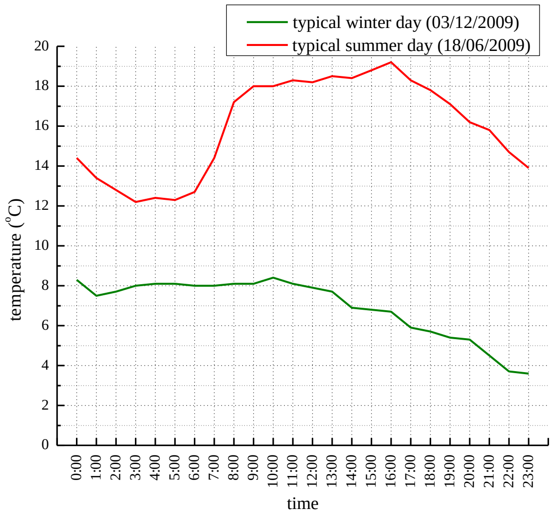
5.3.2.3 Variation of Transformer Ambient Condition

As indicated in equation (5-7) the air temperature of the transformer surrounding strongly influences the transformer temperature rise and loading capability [139]. According to the IEEE transformer loading guide, the ambient temperature accounts for the air temperature around the radiators or heat exchangers [96]. According to the IEC loading guide the natural condition of wind blow, sunshine and rainfall close to the transformer should be also taken into account, when considering the ambient condition [34]. However due to limited information, measured air temperature close to the transformer installation site is used as the transformer ambient temperature θ_a.

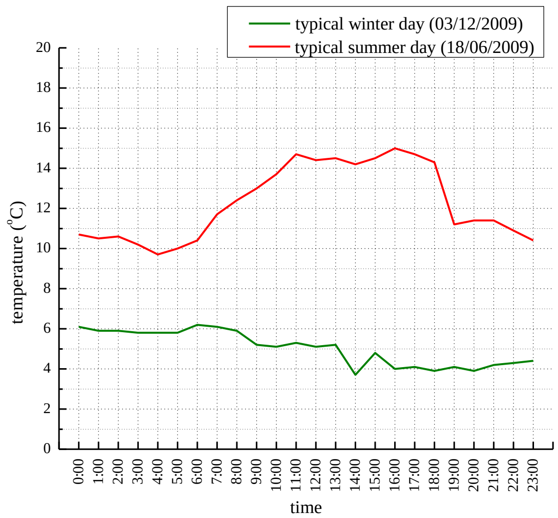
Areas of England and Wales, within which the installed National Grid power transformers are located, are customarily divided into four districts when recording the installation site of each transformer: Southwest England and Wales (SW), Southeast England (SE), Northwest England (NW) and Northeast England (NE). Figure 5-19(a) – Figure 5-19(d) plot the variation of hourly air temperatures measured at 4 weather stations from SW, SE, NW and NE respectively on a 2009 typical winter day (the 1st Thursday of December) and a typical summer day (the 3rd Thursday of June) according to the information from British Atmospheric Data Centre (BADC) [140]. These 4 weather stations are selected as they are close to the transformer installation sites and thus the recorded temperature indicates the air temperature of four specific transformers installed at Exeter, central London, Rochdale and Coventry respectively.



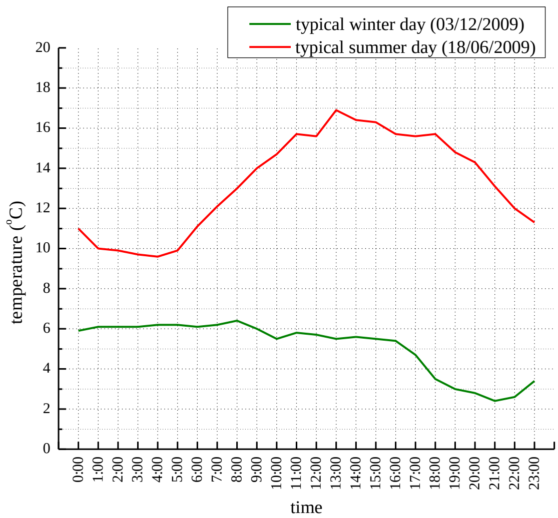
(a) Exeter Airport - SW



(b) London St. James's Park - SE



(c) Rochdale - NW

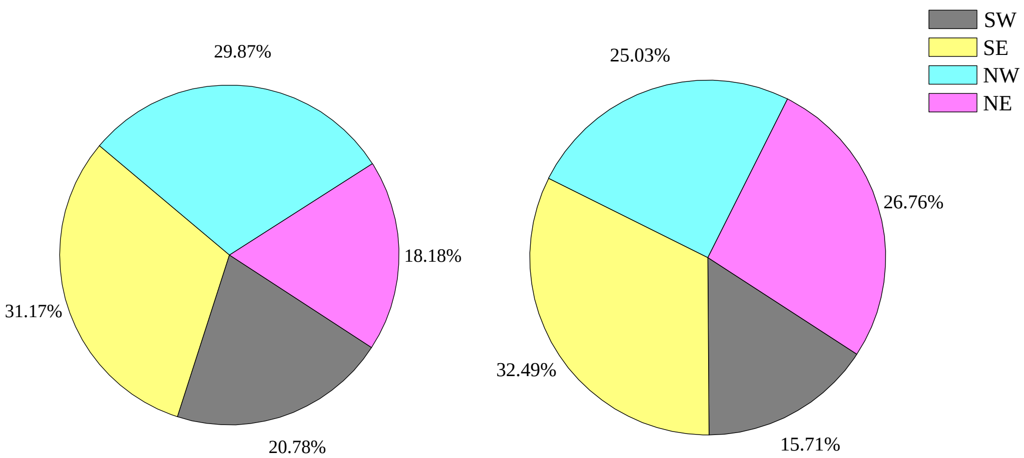


(d) Coventry Coundon - NE

Figure 5-19 Hourly Ambient Temperature Recorded at Different Weather Stations on 2009 Typical Winter and Summer Day [140]

It can be seen from Figure 5-19(a) - Figure 5-19(d) that the ambient air temperature varies differently in the four English regions. In fact the air temperatures are different, when using two weather stations in the same area. The discrepancies in scrapped transformer ambient temperature could be one of factors to determine the dispersed thermal lifetimes; however the effects of the real-time ambient temperature variation to the transformer thermal lifetime cannot be quantified since the mathematical model is unclear. Besides due to the small geographic coverage of England and Wales, the effects from ambient temperature variation is neglected in this thesis.

Hence in order to reveal the representativeness of the scrapped transformers to the whole population, the distribution of transformer installation sites among the scrapped transformers and among the in-service population are compared. They are displayed separately in Figure 5-20(a) and (b).



(a) among 77 Scrapped Transformers (b) among National Grid In-Service Transformers

Figure 5-20 Distribution of Transformer Installation Sites among Scrapped Transformers and among National Grid In-Service Transformers

As indicated in Figure 5-20(a) and (b), scrapped transformers previously installed in Southwest England (SW) or Northwest England (NW) are 5% more than those currently in service in these two areas, while the scrapped transformers located in Northeast England (NE) are 8% less than the transformers currently operating in NE. In particular transformers in Southeast England (SE) are more than 30% in both of the scrapped group and the in-service population. This is because many transformers are installed in this area, especially in London and along Thames River, in order to satisfy the large demand requirement.

According to the comparable pie charts of Figure 5-20(a) and (b), the in-service transformers in different meteorological areas are generally equally sampled by the scrapped group according to their similar portions of transformers. Hence, the 77 scrapped transformers represent the whole population from the view point of the general classification of England and Wales.

5.4 Summary

National Grid retired transformers are always scrapped and their DP values are measured along the main winding insulation. The lowest DP indicates the weakest part of the insulation paper and thus the lowest DP determines the transformer thermal end-of-life. National Grid scrapped transformers constitute a unique sample to indicate the ageing status of the in-service population.

The ageing status of 77 scrapped transformers are examined by calculating their average ageing rate and the average thermal lifetime according to the "1/DP increase against transformer service age" model. It concludes based on the above analyses that the scrapped transformers' ageing rates varies significantly and their thermal lifetimes also distribute over a wide range. It is therefore implied that the scrapped transformers may have operated and/or been designed differently. Next, it discusses the reasons for the scrapped transformers' thermal lifetimes and their representativeness with respect to existing National Grid in-service transformers. This is considered in terms of the ageing status and the effect of transformer loading, thermal design and installation ambient temperature.

The average equivalent loads of scrapped transformers are indicated between 0.1p.u. and 0.5p.u. and the distribution is similar to that of the 275/132kV in-service transformers. However transformer load level is not the only determinant factor to the unit's thermal lifetime. Transformer load profiles of these 77 scrapped units, differ significantly and consequently it is difficult to predict the loadings of other in-service transformers, especially transformers operating at other voltage levels.

Transformer thermal parameters also distribute in a dispersed manner and the exact values can only be obtained via the temperature-rise test implemented on the actual transformer or other similarly designed transformers. The hot-spot factors (*HSF*s) of the scrapped transformer are derived to distribute over a wide range. This section concludes the general assumption of *HSF*=1.3 may not be realistic for scrapped transformers or the in-service population. Scrapped transformers indicate only a small number of transformers which might have been defectively designed and may not constitute an adequate sample to suggest the thermal design of the whole population, especially when considering more recent well designed transformers.

By analyzing transformer ambient condition it further identifies that the National Grid in-service transformers in different meteorological areas are generally equally sampled by the scrapped units. Besides owing to the limited record of transformer ambient temperature and little knowledge of how real-time ambient variation affects the transformer thermal lifetime, a unified annual temperature is adopted in the following chapter for transformer thermal lifetime prediction.

It further concludes from the analysis undertaken in this chapter that transformer thermal lifetime is determined by multi-variables, such as instantaneous loading, thermal design parameters, winding *HSF* and ambient temperature. Information of these variables is usually incomplete in transformer operation practice. In order to predict an in-service transformer thermal lifetime, a simplified approach will be proposed in Chapter 6, using information available from scrapped transformers.

Chapter 6

National Grid In-Service Transformer Thermal Lifetime Estimation

6.1 Introduction

It is concluded in Chapter 5 that National Grid scrapped transformers had deteriorated at different rates and thus their thermal lifetimes vary over a wide range. This small group of scrapped transformers do not effectively represent the thermal ageing of the whole in-service population in respect of their loading conditions, thermal design characteristics, or installation ambient conditions. On the other hand however the scrapped transformers make up a unique sample to indicate National Grid in-service transformers ageing-related failure. Since an individual transformer thermal lifetime is determined by multi-variables, such as real-time loading, temperature-rise test results, winding *HSF* and ambient temperature, which are usually incomplete, a simplified approach is developed in order to predict the in-service thermal lifetime of National Grid transformers.

This chapter thus presents this simplified approach. Transformer winding hot-spot temperature compensation factor ($\mathbb{C}$) is first proposed, using the information available from scrapped transformers. Individual in-service transformer thermal lifetime is further predicted by MATLAB programming according to transformer thermal model described in the IEC Loading Guide 60076-7 [34]. The in-service transformer population thermal ageing trend, expressed in terms of a cumulative probability curve and a thermal hazard can be thereafter derived.

6.2 Simplified Approach of Transformer Thermal Lifetime Prediction via Transformer Winding Hot-Spot Temperature Compensation Factor

6.2.1 Introduction

Owing to incomplete thermal parameters, unpredictable *HSF*, limited real-time loading and inaccurate ambient temperatures, a simplified approach for individual transformer thermal lifetime calculations based on IEC transformer thermal model will be developed in this chapter. According to this approach an individual transformer thermal lifetime can be easily predicted by reforming its annual equivalent load to the national typical daily demand profiles, using the IEC typical thermal parameters and a universal ambient temperature record. The calculation process is illustrated in Figure 6-1 which is similar to that shown in Figure 5-5.

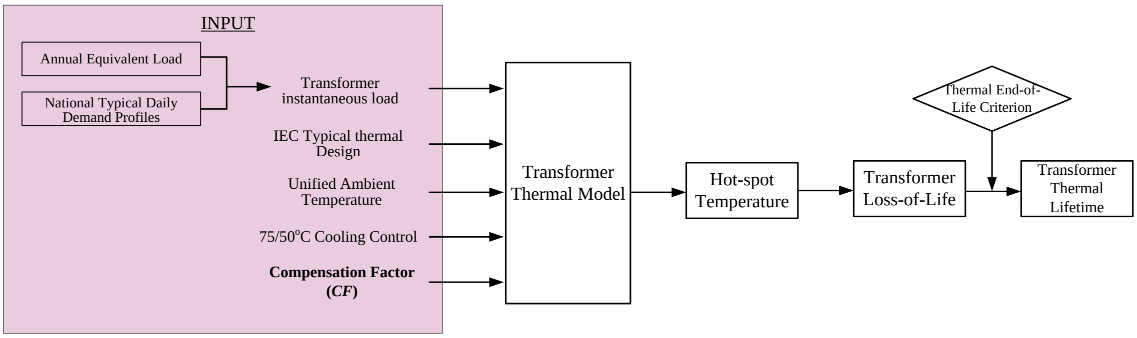


Figure 6-1 Illustration of Simplified Approach for Individual Thermal Lifetime Prediction

It is indicated in Figure 6-1, that a simplified approach to determine the transformer load profile is used to follow the national typical winter and summer daily demand curves by assuming its remaining equivalent load is identical to the real value. In addition, the transformer thermal parameters and ambient temperature are both unified. Particularly a dimensionless parameter, denominated as the compensation factor (*CF*) is proposed to compensate the discrepancies between the transformer real load and the national typical daily demand profile. The parameter also considers the differences between the real ambient condition and the universal ambient temperature, and the differences between a specific thermal design and the IEC typical design. The role of *CF* is highlighted in Figure 6-1.

CF is assigned to the position of the winding hot-spot factor (*HSF*). Hence the transformer steady-state hot-spot temperature (θ_h), under a constant load (K) and constant ambient temperature θ_a, according to the simplified approach is adjusted to be:

$$\theta_h = \left[\frac{1+K^2 \bullet R^*}{1+R^*}\right]^{x^*} \bullet \Delta\theta_{or}{}^* + K^{y^*} \bullet G_r{}^* \bullet CF + \theta_a \qquad (6\text{-}1)$$

where K is the load factor in p.u. based on transformer rating,

θ_a is the constant ambient temperature,

x^* and y^* are the oil and winding exponents suggested by IEC[viii]

R^*, $\Delta\theta_{tor}{}^*$, $G_r{}^*$ are IEC typical thermal design parameters, and

CF is the compensation factor.

CF can be explained as the substitution of the transformer *HSF*. It moreover includes all the uncertainties from the transformer loading record, thermal design and installation site ambient temperature. The value of *CF* is significantly different to *HSF* and any positive values are meaningful.

National Grid scrapped transformers with their thermal lifetime estimated using their lowest DP can be used to derive their compensation factors (*CF*) and the results are further used to predict the in-service transformers' thermal lifetimes according to the simplified approach. The inputs of this approach and the calculation process of the transformer hot-spot temperature (θ_h) and thermal lifetime (*EoL*) according to the IEC thermal model are discussed in 6.2.2 and 6.2.3 respectively. The derivation of the compensation factors (*CF*s) of the scrapped transformers are presented in 6.3.

6.2.2 Input Data of Simplified Approach

As indicated in Figure 6.1, except for the fixed 75/50 ℃ cooling control and the assigned *CF*, the transformer instantaneous load reformed from its annual equivalent load and the national typical daily demand profiles, IEC typical thermal design parameters and a universal ambient temperature record are the inputs to the transformer thermal model.

6.2.2.1 Transformer Instantaneous Load

In-service transformer hot-spot temperature (θ_h) and loss-of-life (*LoL*) calculations are undertaken using data from one year, which is subject to limited load information. The transformer is assumed to consume the equivalent amount of its useful thermal lifetime after one year operation until reaching its thermal end-of-life. A transformer would simply experience half winter days during the year, from January to March and from October to December and half summer days, from April to September. The instantaneous daily load is reformed to cyclically follow the national typical winter/summer daily demand profile, by assuming the annual equivalent load is

[viii] * is to distinguish IEC typical values from the real values of a specific transformer in the context.

identical to the real value. Moreover, transformer load, recorded in each hour and conformed to national typical daily demand profiles are converted into 1 minute sampling data using linear extrapolation. The above procedure to generate a transformer's per minute load is summarized in the following six steps.

Step 1: Calculate the transformer 2009 annual equivalent load as

$$L_{eq} = \sqrt{\frac{L_1{}^2 t_1 + L_2{}^2 t_2 + L_3{}^2 t_3 + ... + L_N{}^2 t_N}{t_1 + t_2 + t_3 + ... + t_N}} \tag{6-2} [96]$$

where L_1, L_2, L_3, …, L_N are the real-time loads in MVA recorded at t_1, t_2, t_3, …, t_N respectively, and N is the total number of recorded data in 2009.

Step 2: For a scrapped transformer in particular, convert the 2009 equivalent load L_{eq} to the equivalent value L'_{eq} at the year of scrapping based on 1.1% national demand annual growth factor [131]. L'_{eq} is expressed as

$$L'_{eq} = \frac{L_{eq}}{(1+1.1\%)^{(2009-Y_X)}} \tag{6-3}$$

in which Y_X indicates the scrapping year of a particular transformer.

Step 3: Convert the equivalent load at the year of scrapping L'_{eq} into per unit (p.u.) based on transformer rating as

$$K = \frac{L'_{eq}(MVA)}{Rating(MVA)} \tag{6-4}$$

where K is designated as transformer load factor in p.u.

The above 3 steps have been introduced in Chapter 5. Further steps generate transformer's load data per minute following national typical daily demand profiles and using the annual equivalent load factor K.

Step 4: Similarly to (6-2) calculate the national annual equivalent demand in GW based on the typical winter and summer national demand data, which can be further expressed as

$$D = \sqrt{\frac{\left(D_{w1}{}^2 t_{w1} + D_{w2}{}^2 t_{w2} + D_{w3}{}^2 t_{w3} + ... + D_{wN}{}^2 t_{wN}\right) + \left(D_{s1}{}^2 t_{s1} + D_{s2}{}^2 t_{s2} + D_{s3}{}^2 t_{s3} + ... + D_{sN}{}^2 t_{sN}\right)}{\left(t_{w1} + t_{w2} + t_{w3} + ... + t_{wN}\right) + \left(t_{s1} + t_{s2} + t_{s3} + ... + t_{sN}\right)}} \tag{6-5}$$

where D represents the national annual equivalent demand, D_{w1}, D_{w2}, D_{w3}, …, D_{wN} are the national demand in GW in winter at t_{w1}, t_{w2}, t_{w3}, …, t_{wN} respectively, D_{s1}, D_{s2},

D_{s3}, …, D_{sN} are the national demand in summer at t_{s1}, t_{s2}, t_{s3}, …, t_{sN} respectively and N is the number of recorded data samples obtained in winter or in summer.

Step 5: Scale the transformer's annual equivalent load factor K to the national annual equivalent demand D by factor α as:

$$\begin{aligned}\frac{K}{D} &= \frac{\sqrt{\left(K_{w1}{}^2+K_{w2}{}^2+K_{w3}{}^2+...+K_{wN}{}^2\right)+\left(K_{s1}{}^2+K_{s2}{}^2+K_{s3}{}^2+...+K_{sN}{}^2\right)}}{\sqrt{\left(D_{w1}{}^2+D_{w2}{}^2+D_{w3}{}^2+...+D_{wN}{}^2\right)+\left(D_{s1}{}^2+D_{s2}{}^2+D_{s3}{}^2+...+D_{sN}{}^2\right)}}\\ &= \frac{\sqrt{\alpha^2\bullet\left(D_{w1}{}^2+D_{w2}{}^2+D_{w3}{}^2+...+D_{wN}{}^2\right)+\alpha^2\bullet\left(D_{s1}{}^2+D_{s2}{}^2+D_{s3}{}^2+...+D_{sN}{}^2\right)}}{\sqrt{\left(D_{w1}{}^2+D_{w2}{}^2+D_{w3}{}^2+...+D_{wN}{}^2\right)+\left(D_{s1}{}^2+D_{s2}{}^2+D_{s3}{}^2+...+D_{sN}{}^2\right)}}\\ &= \alpha\\ &= \frac{K_{w1}}{D_{w1}}=\frac{K_{w2}}{D_{w2}}=...=\frac{K_{wN}}{D_{wN}}=\frac{K_{s1}}{D_{s1}}=\frac{K_{s2}}{D_{s2}}=...=\frac{K_{sN}}{D_{sN}}\end{aligned} \tag{6-6}$$

where K_{w1}, K_{w2}, K_{w3}, …, K_{wN} are the transformer load factor in winter at t_{w1}, t_{w2}, t_{w3}, …, t_{wN} respectively and K_{s1}, K_{s2}, K_{s3}, …, K_{sN} are the p.u. load factor corresponding to summer time t_{s1}, t_{s2}, t_{s3}, …, t_{sN} respectively.

Deduce the transformer's instantaneous load factor, K_{wN} at winter time t_{wN} as

$$K_{wN} = \alpha\bullet D_{wN} = \frac{K\bullet D_{wN}}{D} \tag{6-7}$$

and similarly K_{sN} in summer time t_{sN} as

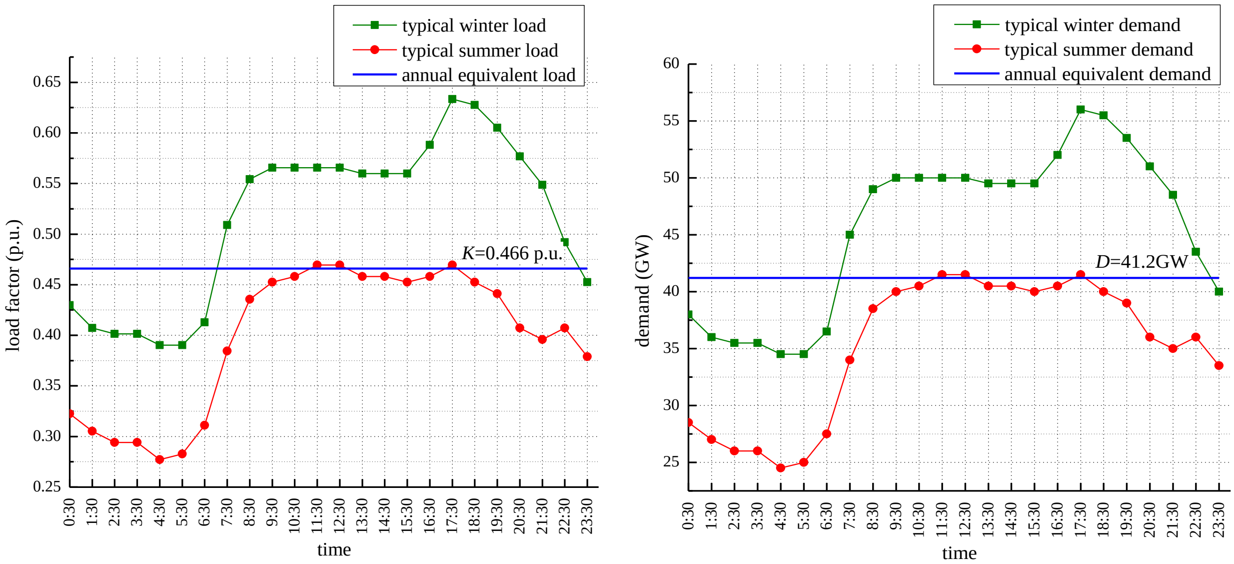
$$K_{sN} = \alpha\bullet D_{sN} = \frac{K\bullet D_{sN}}{D} \tag{6-8}$$

Particularly K_{w1}, K_{w2}, K_{w3}, …, K_{wN} and K_{s1}, K_{s2}, K_{s3}, …, K_{sN} are all obtained in intervals of 1 hour as national demand is given each hour.

Step 6: Convert the individual transformer load factor per hour into per minute using the linear extrapolation. For example the i^{th} minute's load between winter load $K_{wN\text{-}1}$ and K_{wN}, denoted as $K\begin{pmatrix} i \\ w \end{pmatrix}\begin{pmatrix} N-1 \\ N \end{pmatrix}$, can be calculated as

$$K\begin{pmatrix} i \\ w \end{pmatrix}\begin{pmatrix} N-1 \\ N \end{pmatrix} = \frac{K_{wN}-K_{wN-1}}{60}\times(i-1)+K_{wN-1} \tag{6-9}$$

Figure 6-2 gives an example of deriving a transformer typical winter and summer daily load in p.u. per minute according to the above six steps. The obtained load profiles are compared with national typical winter and summer daily demand curves in Figure 6-2.



(a) Transformer Typical Winter and Summer Daily Load Profile in p.u.

(b) Great Britain Typical Winter and Summer Daily Demand Profile

Figure 6-2 Example of Deriving Transformer Typical Winter and Summer Daily Load Profile Following National Typical Daily Demand Profile

It can be seen from Figure 6-2 (a) and (b) that the reformed transformer load pattern is identical to the national typical daily demand profile and the transformer's annual equivalent load. Again during the winter time, for one operation year, the typical winter load profile is assumed to be cyclically experienced, and during summer days, the typical summer load profile is cyclically experienced. The reformed load profile is the input to the transformer thermal model.

6.2.2.2 Unified Ambient Temperature

Since the exact ambient temperature seen by a transformer are seldom recorded, the real-time ambient temperature in the London Heathrow area in 2007 is used as the unified ambient input data to deduce each scrapped transformer's compensation factor (*CF*) and to predict the in-service transformer's thermal lifetime. Figure 6-3 shows the ambient temperature profile of London Heathrow in 2007 [141].

Similarly to the national demand data given by National Grid, Heathrow ambient temperature is also recorded in per hour interval. Hence the linear extrapolation is adopted to convert the hourly ambient temperatures into minute based ambient data for input to the transformer thermal model.

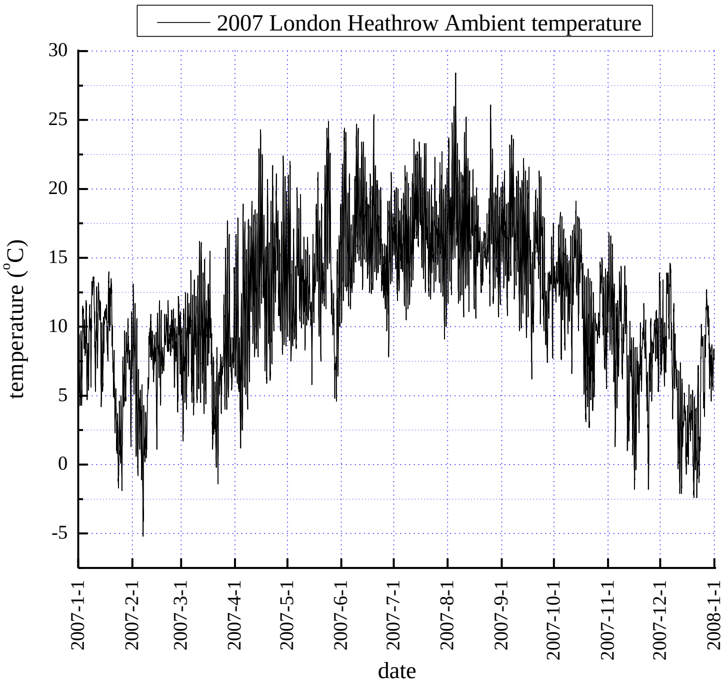


Figure 6-3 2007 London Heathrow Ambient Temperature Record [141]

6.2.2.3 IEC Transformer Typical Thermal Parameters

To avoid the reviewing of individual transformer's temperature-rise test results and to avoid the errors of temperature measurement, IEC typical thermal design parameters are assumed to be applicable to all the National Grid transformers. The typical values of transformer thermal parameters at rated load under ONAN and OFAF cooling modes are summarized in Table 6-1 as follows. Values of other empirical parameters, such as the oil exponent x, the winding exponent y, the oil response constant k_{11}, the winding-to-oil response constant k_{21} and k_{22}, the oil time constant τ_o and the winding time constant τ_w are also listed in Table 6-1.

Besides it is emphasized in Chapter 5 that when OFAF cooling is switched on, the base rating for the transformer load factor calculation is the nominated rating indicated on the transformer nameplate, whilst when ONAN is performing, the base rating changes to half the nominated MVA rating. Hence the instantaneous load factor, K_{w1}, K_{w2}, K_{w3}, …, K_{wN}, calculated according to (6-7) for instance, should be modified as $2 \cdot K_{w1}$, $2 \cdot K_{w2}$, $2 \cdot K_{w3}$, …, $2 \cdot K_{wN}$ under the ONAN cooling mode.

Table 6-1 Typical Values of Transformer Thermal Parameters Given by IEC Loading Guide 354:1991 and 60076-7 [34, 97]

	Cooling Mode	
Thermal parameter	ONAN	OFAF
Loss ratio R^*	6	6
Top-oil temperature rise $\Delta\theta_{tor}^*$ (K)	52	56
Average winding-to-oil temperature gradient G_r^* (K)	20	17
Oil exponent x^*	0.8	1.0
Winding exponent y^*	1.3	1.3
Oil response constant k_{11}^*	0.5	1.0
Winding-to-oil response constant k_{21}^*	2.0	1.3
Winding-to-oil response constant k_{22}^*	2.0	1.0
Oil time constant τ_o^* (min)	210	90
Winding time constant τ_w^* (min)	10	7

Moreover the switching between the ONAN and OFAF cooling modes is controlled by an approximation of the winding hot-spot temperature provided by the winding temperature indicators (WITs) using *HSF* of 1.3 for both cooling modes. The 75/50 ℃ control scheme is applied by National Grid, if the estimated hot-spot temperature (θ_h) is no more than 75 ℃, the ONAN cooling is used. Once θ_h is evaluated to have exceeded 75 ℃, extra fans and pumps will be switched on to kick start the OFAF cooling and OFAF remains unless the hot-spot temperature has been reduced to less than 50 ℃. Furthermore, although the mechanical response of fans and pumps is reported to be delayed in practice, which may essentially result in the overheating of transformer winding, they are assumed to switch ON/OFF immediately after OFAF cooling is activated.

6.2.3 IEC Process of Predicting Transformer Thermal Lifetime

The deduced 1 minute load factor, the unified ambient temperature, the IEC typical thermal parameters, the fixed 75/50℃ cooler control scheme and the winding compensation factor (*CF*) are the inputs to the IEC thermal model as described in Figure 6-1. The transformer hot-spot temperature (θ_h) is obtained in the interval of 1 minute.

The transformer thermal model is described by a series heat transfer differential equations. These equations are used to calculate the transformer hot-spot temperature (θ_h), the transformer loss-of-life (*LoL*) and thermal end-of-life (*EoL*) under arbitrarily time-varying load and ambient condition [34]. The calculation procedure is summarized as follows. The abbreviations are referred to in the list of symbols and abbreviations at the start of this thesis, and * indicates the values of the transformer thermal parameters according to IEC typical design.

Step 1: Calculate the initial steady-state temperatures at the beginning of calculation.

The initial top oil temperature θ_{toi} under the load factor K_i and ambient temperature θ_{ai} at the beginning of calculation is formulated as

$$\theta_{toi} = \left(\frac{1+K_i^{\,2}\bullet R^*}{1+R^*}\right)^{x^*}\bullet\Delta\theta_{tor}^{\ *}+\theta_{ai} \tag{6-10}$$

where K_i and θ_{ai} are the 1st recorded load factor and the ambient temperature respectively and are used to obtain the steady-state top oil temperature at the beginning of the calculation.

The initial hot-spot temperature rise over the top oil temperature $\Delta\theta_{hi}$ is calculated as the sum of two differential equations as

$$\Delta\theta_{hi} = \Delta\theta_{hi1} - \Delta\theta_{hi2} \tag{6-11}$$

in which

$$\Delta\theta_{hi1} = k_{21}^{\ *}\bullet K^{y^*}\bullet\left(G_r^{\ *}\bullet CF\right) \tag{6-12}$$

and

$$\Delta\theta_{hi2} = \left(k_{21}^{\ *}-1\right)\bullet K^{y^*}\bullet\left(G_r^{\ *}\bullet CF\right) \tag{6-13}$$

where, $G_r^{\ *}\bullet CF$ gives the value of transformer hot-spot to top oil gradient at the rated load, denoted as $\Delta\theta_{hr}^{\ *}$.

Hence, the initial hot-spot temperature θ_{hi} corresponding to the load factor K_i and the ambient temperature θ_{ai} is determined as

$$\theta_{hi} = \theta_{toi} + \Delta\theta_{hi} \tag{6-14}$$

ONAN cooling is always set as the initial status and thus the values of R^*, $\Delta\theta_{tor}{}^*$, $G_r{}^*$, x^*, y^* and $k_{21}{}^*$ are those expressed under ONAN cooling. Compare the initial hot-spot temperature θ_{hi} to 75 ℃, if $\theta_{hi} \geq$ 75 ℃, OFAF cooling is switched on and all the parameters consequently change to the values corresponding to OFAF cooling; otherwise ONAN cooling continues and accordingly the parameters remain. Transformer loss-of-life (LoL_i) is initially set 0.

Step 2: Calculate the top oil temperature rise over ambient $\Delta\theta_{to}(t)$ and the top oil temperature $\theta_{to}(t)$ at time interval t under the instantaneous load $K(t)$ and ambient temperature $\theta_a(t)$ recorded at t.

$$\Delta\theta_{to}(t) = \frac{1}{k_{11}{}^* \bullet \tau_0} \left\{ \left[\frac{1 + K(t)^2 \bullet R^*}{1 + R^*} \right]^{x^*} \bullet \Delta\theta_{tor}{}^* - \left[\theta_{to}(t-1) - \theta_a(t) \right] \right\} \tag{6-15}$$

and

$$\theta_{to}(t) = \Delta\theta_{to}(t) + \theta_a \tag{6-16}$$

Step 3: Calculate the hot-spot temperature rise over top oil $\Delta\theta_h(t)$ and then the hot-spot temperature $\theta_h(t)$ at time interval t.

$\Delta\theta_h(t)$ is again derived as the sum of two differential equations as

$$\Delta\theta_h(t) = \Delta\theta_{h1}(t) - \Delta\theta_{h2}(t) \tag{6-17}$$

where

$$\Delta\theta_{h1}(t) = \frac{1}{k_{22}{}^* \bullet \tau_w{}^*} \bullet \left[k_{21}{}^* \bullet \left(G_r{}^* \bullet CF \right) \bullet K(t)^{y^*} - \Delta\theta_{h1}(t-1) \right] \tag{6-18}$$

and

$$\Delta\theta_{h2}(t) = \frac{1}{\left(1/k_{22}{}^*\right) \bullet \tau_o{}^*} \bullet \left[\left(k_{21}{}^* - 1\right) \bullet \left(G_r{}^* \bullet CF \right) \bullet K(t)^{y^*} - \Delta\theta_{h2}(t-1) \right] \tag{6-19}$$

Thereafter the hot-spot temperature $\theta_h(t)$ is deduced as

$$\theta_h(t) = \Delta\theta_h(t) + \theta_{to}(t) \tag{6-20}$$

Step 4: Compare the estimated hot-spot temperature $\theta_h(t)$ at time t with WTIs 75/50 ℃ threshold temperatures to determine the cooling model and the corresponding thermal parameters for the calculation in the next time interval.

The algorithm can be described as: -

If ONAN cooling is performed at t-1 and $\theta_h(t) \geq 75$ ℃, OFAF will be switched on; otherwise if $\theta_h(t) < 75$ ℃, ONAN cooling will remain.

If OFAF cooling is conducted at t-1 and $\theta_h(t) \geq 50$ ℃, OFAF cooling will continue to perform; whilst if $\theta_h(t) < 50$ ℃, OFAF will be switched off and ONAN will be activated.

Step 5: Calculate the relative ageing rate factor $k_r(t)$ and the loss-of-life $LoL(t)$ within time interval *t*.

Transformer relative ageing rate factor $k_r(t)$ based on the estimated hot-spot temperature $\theta_h(t)$ is obtained according to the piecewise function discussed in Chapter 4. These equations are written here again as

$$k_r(t) = \begin{cases} \dfrac{A_{ox}}{A_0} \bullet exp\left[\dfrac{E_0}{\mathrm{R}\bullet(\theta_{h0}+273)} - \dfrac{E_{ox}}{\mathrm{R}\bullet[\theta_h(t)+273]}\right], & when\ \theta_h(t) \leq 60^0 C \\ \dfrac{A_{hy}}{A_0} \bullet exp\left[\dfrac{E_0}{\mathrm{R}\bullet(\theta_{h0}+273)} - \dfrac{E_{hy}}{\mathrm{R}\bullet[\theta_h(t)+273]}\right], & when\ 60^0 C < \theta_h(t) \leq 150^0 C \\ \dfrac{A_{py}}{A_0} \bullet exp\left[\dfrac{E_0}{\mathrm{R}\bullet(\theta_{h0}+273)} - \dfrac{E_{py}}{\mathrm{R}\bullet[\theta_h(t)+273]}\right], & when\ 150^0 C < \theta_h(t) \end{cases} \tag{6-21}$$

where the value of the relative ageing rate factors at time t is determined according to the deterioration process conducted within different ranges of the hot-spot temperatures: - when the hot-spot temperature is no more than 60 ℃, oxidation is the dominant degradation process; when the hot-spot temperature is between 60 ℃ and 150 ℃, hydrolysis is significant; when the hot-spot temperature exceeds 150 ℃, pyrolysis is predominant. A_{ox}, A_{hy} and A_{py} are the pre-exponential factor of oxidation, hydrolysis and pyrolysis respectively, E_{ox}, E_{hy} and E_{py} indicate the activation energy under these three deterioration processes, and A_0 and E_0 represent the pre-exponential factor and activation energy for the referenced condition. Values of these parameters are displayed in Table 4-1.

Furthermore transformer thermal loss-of-life after operating within time t, denoted as $LoL(t)$, is derived as

$$LoL(t) = k_r(t) \bullet t \tag{6-22}$$

where $k_r(t)$ is the relative ageing rate factor at time interval t.

Step 6: Obtain the total loss-of-life LoL_{total} consumed during the whole year operation.

Repeat **Step 2** to **Step 5** at each time interval of 1 minute until the end of the operation year, the total loss-of-life LoL_{total} is the sum of the LoLs obtained at each time interval, as

$$LoL_{total} = \sum LoL(t) \cdot t \tag{6-23}$$

Step 7: Predict transformer's thermal lifetime *EoL*.

A transformer is assumed to have experienced the identical loss-of-life during each year's operation until the end of its thermal lifetime. By using the normal insulation life of 150,000hrs based on the thermal end-of-life criterion of DP=200, the transformer's thermal lifetime in year is predicted as

$$EoL(yr) = \frac{150{,}000hr \times 1yr}{LoL_{total}(hr)} \tag{6-24}$$

6.3 National Grid Scrapped Transformers' Hot-Spot Compensation Factors (*CF*s) Derivation

Since a scrapped transformer's thermal lifetime can also be predicted by the lowest DP, its compensation factor (*CF*) can thus be inversely derived. It presents in this section the derivation of National Grid scrapped transformers' compensation factors and then the information is applied to the in-service population.

6.3.1 Scrapped Transformers' Compensation Factors Generation Procedure

A scrapped transformer compensation factor (*CF*) can be deduced by estimating its thermal lifetime according to the simplified approach equivalent to the value from its lowest DP. The calculation procedure is illustrated in Figure 6-4.

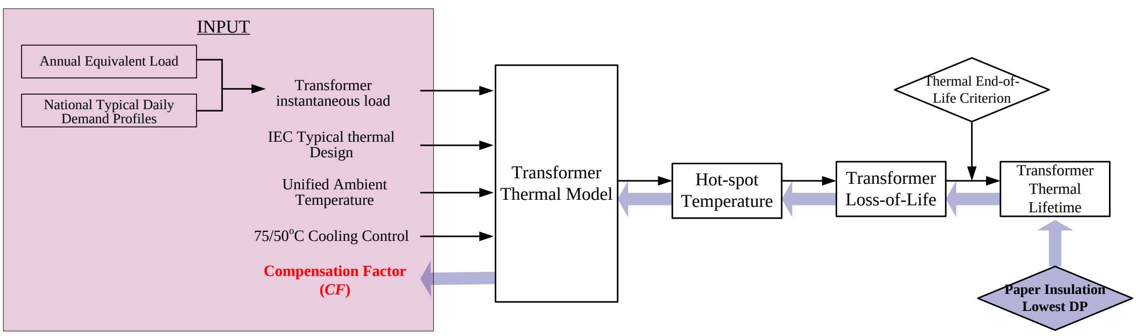
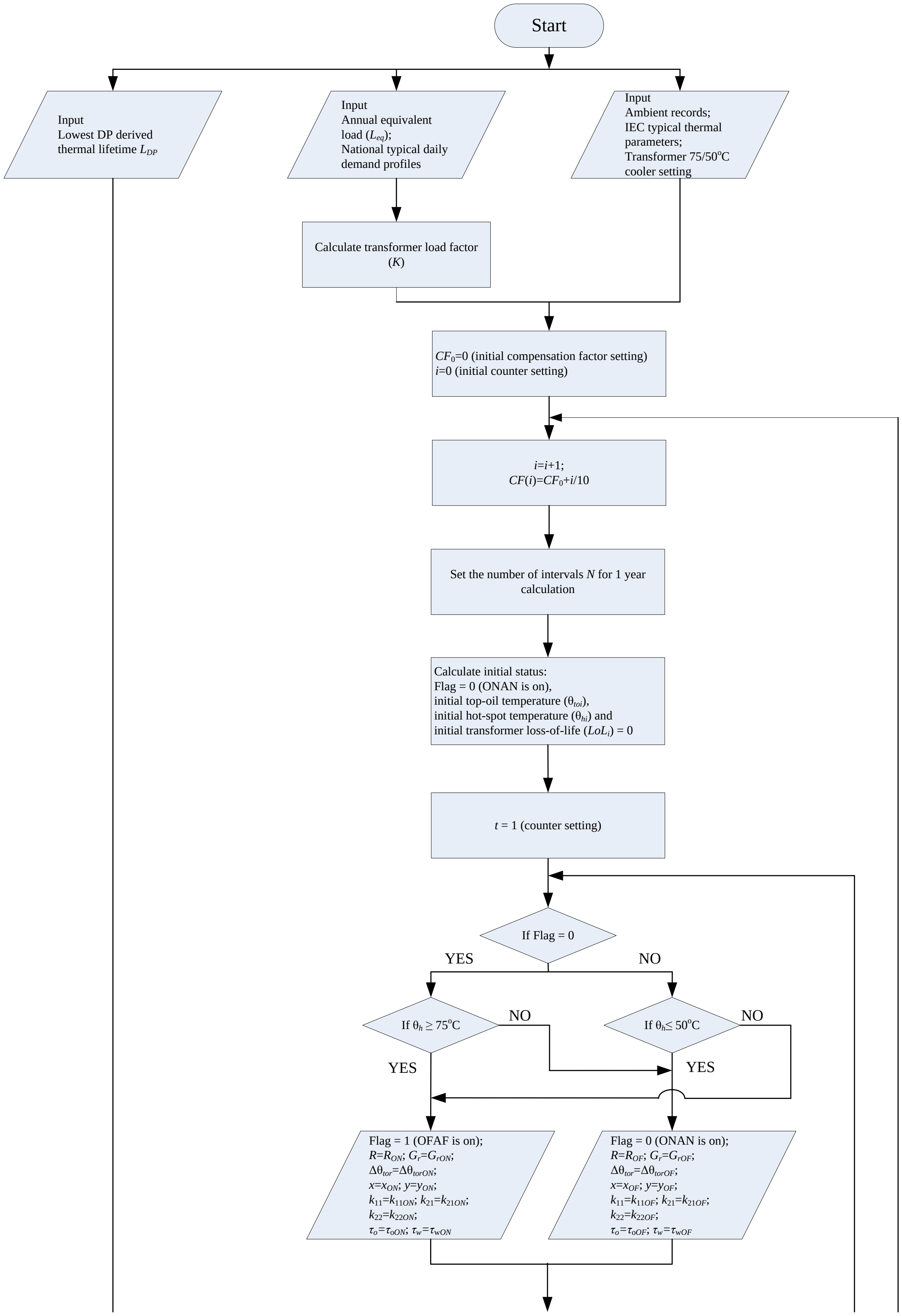


Figure 6-4 Process of Deriving Individual Scrapped Transformer *CF*

The derivation process can be further illustrated by the flowchart shown in Figure 6-5.



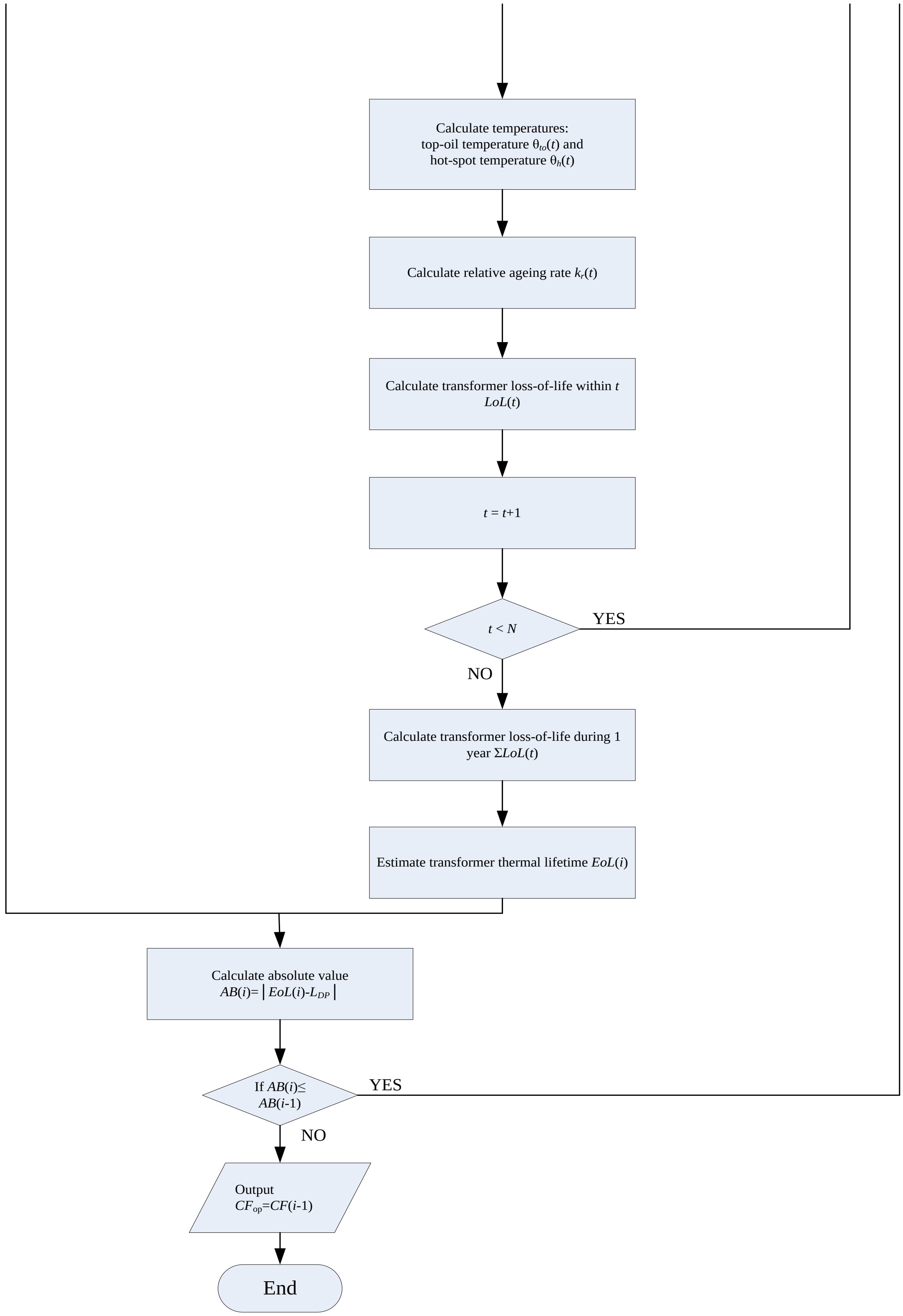


Figure 6-5 Scrapped Transformer Compensation Factor (*CF*) Derivation Flow Chart

According to Figure 6-5 *CF* is initially set as 0 and increases with the interval of 0.1; transformer hot-spot temperature (θ_h), loss-of-life (*LoL*) and thus thermal lifetime (*EoL*) corresponding to the load and ambient and under a certain *CF* are calculated using the IEC transformer thermal model and the optimal *CF* is the value under which the estimated transformer thermal lifetime is closest to that predicted by the lowest DP.

6.3.2 Scrapped Transformers' Compensation Factors (*CF*) against Transformer Annual Equivalent Load Factor (*K*)

Based on the provided load information, 44 out of 77 scrapped transformers' hot-spot temperature compensation factors (*CF*s) are derived. Scrapped transformer *CF* is revealed to be strongly correlated with the scrapped transformer annual equivalent load factor *K* backward to the year of scrapping. Figure 6-6 displays *CF* against transformer annual equivalent load factor *K*.

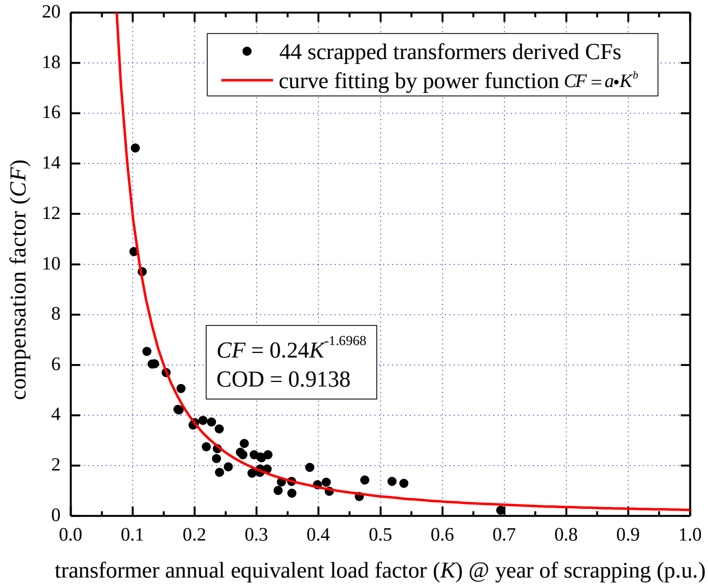


Figure 6-6 44 Scrapped Transformers Derived Compensation Factors (*CF*) vs. Annual Equivalent Load Factor *K* @ Year of Scrapping

Figure 6-6 indicates that the best fitting of 44 scrapped transformers' *CF*s is determined by a power function of the transformer annual equivalent load factor *K* at the year of transformer scrapping which is described as

$$CF = a \bullet K^b \tag{6-25}$$

where parameter *a* is derived to be 0.24 and *b* equals -1.6968, with the coefficient of determination (COD) as high as 0.9138.

It is thus suggested based on the above plot that transformer hot-spot compensation factor *CF* is strongly related to the annual equivalent load factor *K* and furthermore for an in-service transformer, *CF* can be obtained according to the unit's one year annual equivalent load and thus its thermal lifetime can be predicted via previously presented approach.

6.3.3 Curve Fitting Error Analysis

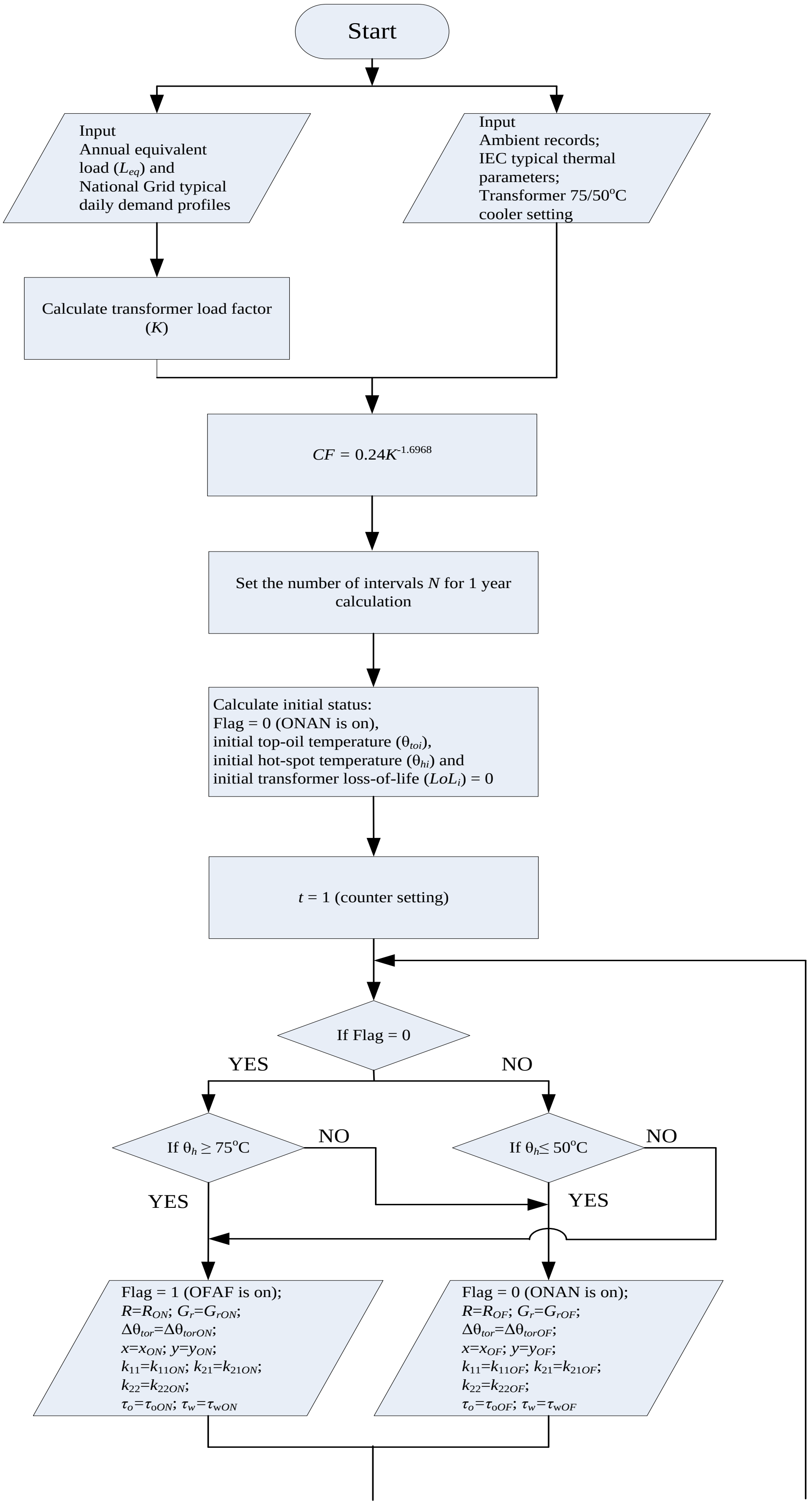
In order to extrapolate (6-25) to National Grid in-service transformers, the error of fitting *CF* into load factor *K* by (6-25) is analyzed in this section. (6-25) derived *CF*s of 44 scrapped transformers are used to calculate their thermal lifetimes according to the simplified approach. The calculation procedure is presented in Figure 6-7.

Transformer's re-calculated thermal lifetime is recorded as *EoL* and its lowest DP derived thermal lifetime is denoted as EoL_0. The error between *EoL* and EoL_0, denoted as Er_{EoL}, is calculated as

$$Er_{EoL} = \frac{EoL - EoL_0}{EoL_0} \tag{6-26}$$

Similarly the original estimated compensation factor in accord with transformer lowest DP determined thermal lifetime is denoted as CF_0 in order to distinguish the *CF* deduced by (6-25). The error between *CF* and CF_0, nominated as Er_{CF} is defined as

$$Er_{CF} = \frac{CF - CF_0}{CF_0} \tag{6-27}$$


Start
Input
Annual equivalent load (L_{eq}) and National Grid typical daily demand profiles
Input
Ambient records;
IEC typical thermal parameters;
Transformer 75/50^{o}C cooler setting
Calculate transformer load factor (K)
$CF = 0.24K^{-1.6968}$
Set the number of intervals N for 1 year calculation
Calculate initial status:
Flag = 0 (ONAN is on),
initial top-oil temperature (θ_{toi}),
initial hot-spot temperature (θ_{hi}) and
initial transformer loss-of-life (LoL_i) = 0
t = 1 (counter setting)
If Flag = 0
YES
NO
If $\theta_h \geq 75^o$C
NO
If $\theta_h \leq 50^o$C
NO
YES
YES
Flag = 1 (OFAF is on);
$R=R_{ON}$; $G_r=G_{rON}$;
$\Delta\theta_{tor}=\Delta\theta_{torON}$;
$x=x_{ON}$; $y=y_{ON}$;
$k_{11}=k_{11ON}$; $k_{21}=k_{21ON}$;
$k_{22}=k_{22ON}$;
$\tau_o=\tau_{oON}$; $\tau_w=\tau_{wON}$
Flag = 0 (ONAN is on);
$R=R_{OF}$; $G_r=G_{rOF}$;
$\Delta\theta_{tor}=\Delta\theta_{torOF}$;
$x=x_{OF}$; $y=y_{OF}$;
$k_{11}=k_{11OF}$; $k_{21}=k_{21OF}$;
$k_{22}=k_{22OF}$;
$\tau_o=\tau_{oOF}$; $\tau_w=\tau_{wOF}$

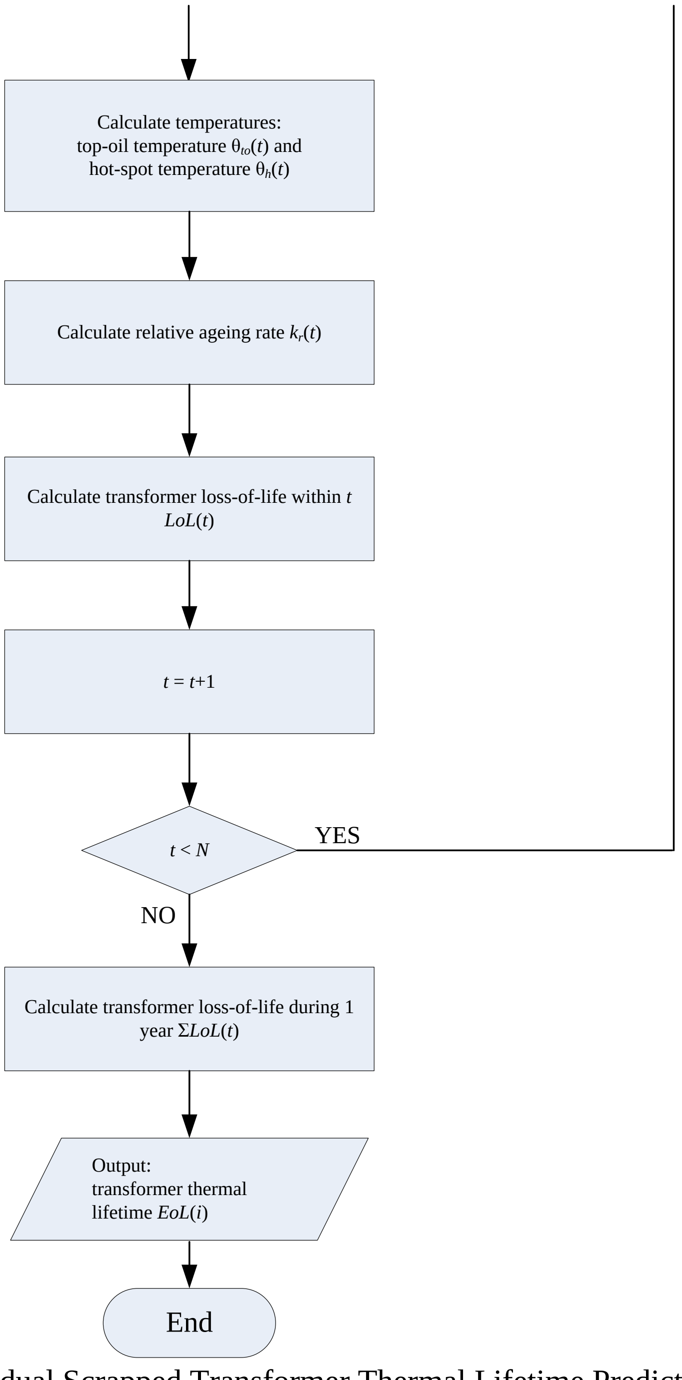


Figure 6-7 Individual Scrapped Transformer Thermal Lifetime Prediction Based on (6-25) Generated Compensation Factor

Er_{EoL} and Er_{CF} of 44 scrapped transformers are plotted together for comparison purpose in Figure 6-8.

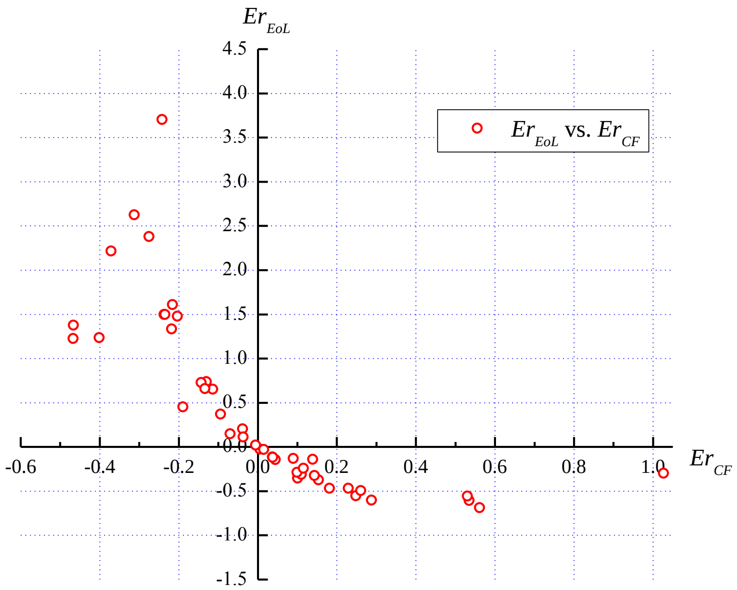


Figure 6-8 Error of transformer thermal *EoL* derived by (6-26) against Error of *CF* calculated by (6-27)

It is understandable that an underestimated hot-spot compensation factor would predict a longer thermal lifetime; while the overestimated *CF* would predict a shorter transformer thermal life. Therefore it is shown in Figure 6-8 that the plots of Er_{EoL} against Er_{CF} distribute in the quadrant-II and IV only. According to Figure 6-8, the Er_{CF}s derived from the above 44 scrapped transformers concentrate in the range of ±30%, while in an extreme case the value exceeds 100%. Er_{EoL}s of the 44 transformers distribute in a wide range; especially when the *CF* is deduced less than the original value CF_0. In an extreme case transformer re-calculated thermal lifetime *EoL* could be 3.7 times longer than the paper insulation lowest DP derived thermal lifetime EoL_0.

Moreover 44 transformers are sorted according to their annual equivalent load factor *K* from smallest to largest; their lowest DP derived EoL_0s, CF_0s, re-estimated *CF*s according to the fitted equation (6-25) and the re-calculated *EoL*s and errors Er_{CF}s and Er_{EoL}s in percentage are shown in Table 6-2.

Table 6-2 44 Scrapped Transformers Compensation Factor (CF_0) Corresponding to Lowest DP Predicted Thermal Lifetime (EoL_0), Re-Generated Compensation Factor (CF) by Fitted Equation, Re-Calculated Thermal Lifetime (EoL) and Their Errors (Er_{CF}s and Er_{EoL}s)

SGT	Annual equivalent load factor (p.u.)	Based on lowest DP		Based on (6-30)		Error%	
		CF_0	EoL_0 (yr)	CF	EoL (yr)	Er_{CF}%	Er_{EoL}%
1	0.102	10.50	190	11.56	123	10.1	-35.3
2	0.105	14.61	27	11.07	127	-24.2	370.4
3	0.116	9.70	118	9.33	142	-3.8	20.3
4	0.123	6.53	373	8.41	148	28.8	-60.3
5	0.131	6.03	345	7.52	154	24.8	-55.4
6	0.135	6.04	295	7.14	157	18.1	-46.8
7	0.154	5.69	166	5.72	162	0.5	-2.4
8	0.173	4.23	231	4.70	159	11.0	-31.2
9	0.175	4.20	222	4.62	158	9.9	-28.8
10	0.178	5.06	95	4.48	157	-11.4	65.3
11	0.198	3.60	168	3.76	144	4.4	-14.3
12	0.200	3.70	139	3.68	142	-0.5	2.2
13	0.214	3.79	76	3.30	132	-13.0	73.7
14	0.219	2.74	201	3.16	126	15.4	-37.3
15	0.227	3.72	48	2.96	119	-20.3	147.9
16	0.236	2.27	209	2.79	111	22.9	-46.9
17	0.237	2.67	124	2.77	110	3.7	-11.3
18	**0.240**	**3.45**	**41**	**2.71**	**107**	**-21.6**	**161.0**
19	**0.240**	**1.73**	**342**	**2.71**	**107**	**56.1**	**-68.7**
20	0.255	1.94	184	2.45	93	26.1	-49.5
21	0.274	2.52	44	2.16	76	-14.1	72.7
22	0.278	2.43	44	2.10	73	-13.4	65.9
23	0.280	2.87	21	2.08	71	-27.5	238.1
24	0.293	1.69	96	1.93	65	14.3	-32.3
25	0.296	2.42	27	1.89	63	-21.8	133.3
26	0.306	1.73	68	1.80	60	3.8	-11.8
27	0.306	1.86	54	1.79	60	-3.7	11.1
28	0.307	2.34	24	1.79	60	-23.7	150.0
29	0.308	2.31	24	1.77	60	-23.5	150.0
30	0.317	1.86	43	1.68	59	-9.4	37.2
31	0.319	2.43	16	1.67	58	-31.2	262.5
32	0.335	1.00	150	1.53	59	53.5	-60.7
33	0.340	1.34	79	1.49	60	11.5	-24.1
34	**0.357**	**1.36**	**66**	**1.38**	**64**	**1.5**	**-3.0**
35	**0.357**	**0.90**	**144**	**1.38**	**64**	**53.0**	**-55.6**
36	0.386	1.92	23	1.21	74	-37.1	221.7
37	0.398	1.23	67	1.14	77	-7.0	14.9
38	0.413	1.33	55	1.08	80	-18.9	45.5
39	0.417	0.97	93	1.06	81	9.0	-12.9
40	0.466	0.77	99	0.88	85	13.9	-14.1
41	0.475	1.42	38	0.85	85	-40.2	123.7
42	0.519	1.37	37	0.73	88	-46.6	137.8
43	0.538	1.29	40	0.69	89	-46.7	122.5
44	0.694	0.22	148	0.45	104	102.6	-29.7

It is highlighted in Table 6-2 a pair of transformers with the identical load factor of 0.240p.u. at their individual scrapping year. SGT18 and SGT19 are predicted via their lowest DP to have thermal lifetimes of 41yrs and 342yrs respectively, which determine their true *CF*s of 3.45 and 1.73. Their equivalent load factor however derives the same *CF* of 2.71 according to (6-25) and further generates the thermal lifetime of 107yrs. The re-calculated 107yrs thermal lifetime is actually 1.61 times longer ($Er_{EoL} = \frac{107-41}{41} = 1.61$) than SGT18 lowest DP derived thermal lifetime and is 0.69 shorter ($Er_{EoL} = \frac{107-342}{342} = -0.69$) than that of SGT19.

Also by comparing SGT34 and SGT35 as highlighted in Table 6-2, their identical load factor derived *CF* of 1.38 is fairly close to the original CF_0 of SGT34, which consequently generates a similar thermal lifetime as that determined by its lowest DP. However, the *CF* of 1.38 is 0.53 times higher ($Er_{CF} = \frac{1.38-0.90}{0.90} = 0.53$) than the original CF_0 of SGT35 (CF_0=0.90) which results in the thermal lifetime 0.56 times shorter ($Er_{EoL} = \frac{64-144}{144} = -0.56$) than that predicted by SGT35 lowest DP value.

It can be deduced either according to Figure 6-8 or Table 6-2 that the error of transformer re-calculated thermal lifetime (*EoL*) compared to the DP estimated thermal lifetime (EoL_0) is sensitive to the assigned compensation factor (*CF*), especially when the *CF* given by (6-25) is less than the CF_0 in accordance with the transformer lowest DP predicted thermal lifetime.

Since the error of re-calculated thermal lifetime (*EoL*) is considerably large, the uncertainties of *CF*s derived from a certain load factor *K* need to be taken into account when (6-25) is extrapolated to in-service transformer population.

6.3.4 Estimation of In-Service Transformer Compensation Factors ($\mathbb{C}$) by Using Scrapped Transformers Fitted Curve

In order to involve the error between (6-25) derived *CF* and the transformer real CF_0 in National Grid in-service transformers' thermal lifetime prediction, Er_{CF}s calculated according to (6-27) are plotted against *CF*s from (6-25) for analysis. Since Er_{CF} may be positive or negative according to the figures shown in Table 6-2, the absolute value of

Er_{CF} as $|Er_{CF}|$ is used to reduce the complexity in plotting. The dots in Figure 6-9 show $|Er_{CF}|$ against *CF*.

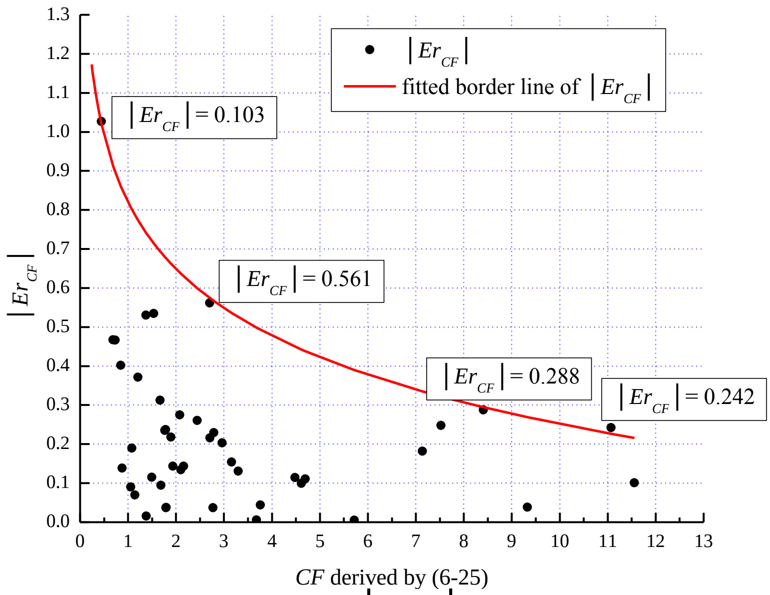


Figure 6-9 Absolute Error of *CF* $|Er_{CF}|$ against *CF* Derived by (6-25)

It indicates in a rough way in Figure 6-9 that $|Er_{CF}|$ is significant when the corresponding *CF* is small according to (6-25). In particular, the four outermost cases of $|Er_{CF}|$ determine a boundary line, assigned as “upper border line” of $|Er_{CF}|$. It is highlighted in Figure 6-9 by red that the best-fit curve can be expressed by a logarithms function, as

$$Er_{CF}{}^{U} = -0.247 \bullet ln(CF) + 0.8204 \tag{6-28}$$

in which $Er_{CF}{}^{U}$ indicates the upper border line of errors and *CF* is the transformer winding compensation factor estimated from the transformer annual equivalent load factor *K* according to (6-25).

Owing to the little knowledge to identify the exact error additionally to a specific *CF*, three assumptions are drawn prior to assign *CF* to an in-service transformer, which are:

- The errors of *CF* determined from 44 scrapped transformers indicate all the possible errors of the in-service transformers’ compensation factors.
- The range of errors is imposed to be within an envelope defined by an upper and lower border line, which are symmetric with error = 0.

- Error of a certain *CF* distributes normally with the mean value (μ) of 0 and 3 times of its standard derivation (σ) locates within the area defined by the "upper border line (Er_{CF}^{U})" and the "lower border line (Er_{CF}^{L})"[ix].

The lower border line Er_{CF}^{L} is symmetrical to the upper line. It is thus formulated as

$$Er_{CF}^{L} = -Er_{CF}^{U} = 0.247 \bullet ln(CF) - 0.8204 \tag{6-29}$$

The generation of the error bar Er_{CF} additionally to a specific *CF* based on the above 3 assumptions can be further graphically illustrated as shown in Figure 6-10.

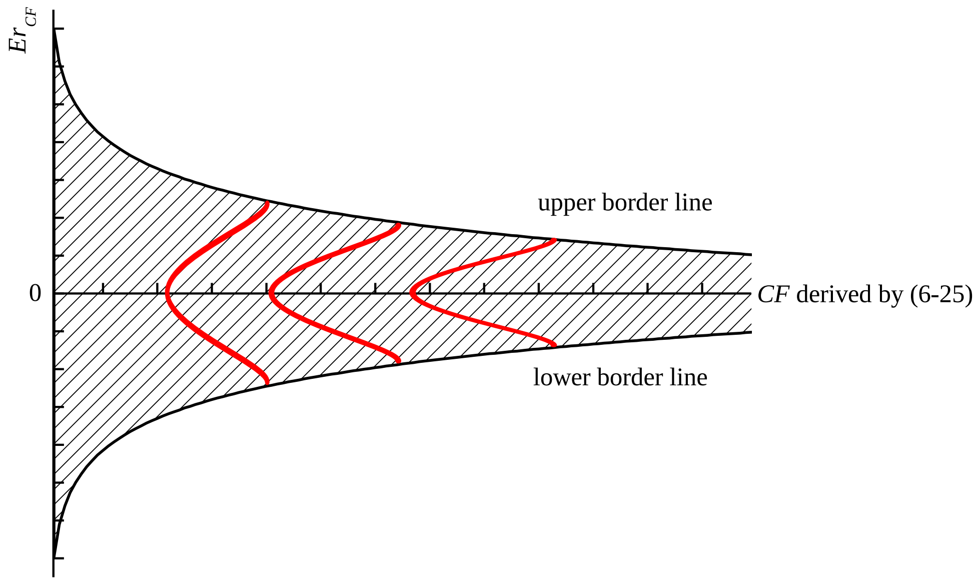


Figure 6-10 Illustration of *CF* Error Er_{CF} Generation

In particular, one should notice that according to (6-28), Er_{CF}^{U} would be derived to be negative when *CF* exceeded the critical value of 27.701 and the transformer was equivalently loaded less than 0.06p.u. The negative value of Er_{CF}^{U} essentially violates the definition of error "upper border line" which is positive by default. It is therefore stated here that National Grid in-service transformers with the equivalent load less than 0.06p.u. are out of consideration in the following thermal lifetime prediction. In fact, the transformer equivalent load of 0.06p.u. is a negligible load condition, under which the transformer has significantly little useful thermal life consumed during the year.

[ix] 3 times of standard deviation is selected as 99.86% cumulative probabilities of normal distribution are within the range of μ ± 3σ, which is explained in Chapter 2.

Er_{CF}, furthermore, is randomly generated within the area identified by Er_{CF}^{U} and Er_{CF}^{L} according to a normal distribution, of which the mean value (μ) is zero and 3 times standard deviation (σ) equals to Er_{CF}^{U}- Er_{CF}^{L}. The generation process for transformer *j*, for instance, can be represented as

$$Er_{CF}(j) = normrnd\left[\mu, \sigma(j)\right] \tag{6-30}$$

where *normrad* indicates the random generation process based on a normal distribution,

μ is the mean value of the normal distribution, μ=0, and

σ(*j*) is the standard derivation of the normal distribution accordingly, which for transformer *j*, is deduced as

$$\sigma(j) = \frac{Er_{CF}^{U}(j) - Er_{CF}^{L}(j)}{3} \tag{6-31}$$

Therefore by considering the possible error bar, the winding temperature compensation factor for this in-service transformer *j* with its annual equivalent load factor *K*(*j*) is estimated as

$$\begin{aligned} \mathbb{C}(j) &= CF(j) \Big/ \left(1 + Er_{CF}(j)\right) \\ &= 0.24 \bullet K(j)^{-1.6968} \Bigg/ \left(1 + normrnd\left[0, \frac{Er_{CF}^{U}(j) - Er_{CF}^{L}(j)}{3}\right]\right) \end{aligned} \tag{6-32}$$

where in particular, $\mathbb{C}$ represents the integrated compensation factor in order to distinguish from others like *CF* derived by (6-25) or CF_0; Er_{CF}^{U} and Er_{CF}^{L} are determined according to (6-28) and (6-29) respectively. Specifically a produced negative value of $\mathbb{C}$ would be invalid and should be generated again, as a negative compensation factor is conceptually against its definition.

(6-32) is thereafter adopted to calculate the winding compensation factors $\mathbb{C}$ of National Grid in-service transformers.

6.4 National Grid In-Service Transformer Thermal Lifetime Prediction

Individual National Grid in-service transformer's thermal lifetime can be predicted by the following the steps of

Step 1: Calculate the transformer 2009 annual equivalent load *K* according to (6-2).

Step 2: Generate the transformer hot-spot compensation factor $\mathbb{C}$ according to (6-32).

Step 3: Predict its thermal lifetime via the simplified approach according to the flow chart shown in Figure 6-7.

As the earliest transmission network established in the U.K., the 275/132kV electric infrastructures are most critically suffering the ageing-related failure. National Grid 275/132kV power transformers' thermal lifetimes are therefore analyzed in this section. The derived ageing trend can generally indicate the thermal ageing of National Grid in-service transformer population.

6.4.1 Calculation Procedure

Among National Grid 270 275/132kV in-service transformers, 231 units' 2009 load are well recorded and thus their thermal lifetimes can be estimated according to the proposed approach. Each of the 231 in-service transformers' compensation factors is estimated from (6-28) to (6-32). In order to adequately consider the possible uncertainties of each compensation factor $\mathbb{C}$, equation (6-30) of generating *CF* error is run 30 times for each transformer and thus the transformer thermal lifetimes are calculated 30 times corresponding to each generated compensation factor $\mathbb{C}$.

It needs to be figured out here that, the above calculation procedure for a certain transformer is in fact a one-dimension Monte Carlo simulation, in which the transformer thermal lifetime essentially relies on the repeated random sampling of compensation factor $\mathbb{C}$. A specific transformer's thermal lifetime can thus be simply expressed as

$$EoL = f(\mathbb{C}) \tag{6-33}$$

Supposing the tests of one transformer thermal lifetime *EoL* are independently undertaken *N* times, within which the transformer's real thermal lifetime is observed *n* times with the probability of $\mathbf{p}_t$, according to Bernoulli's law of large numbers, for any positive number ε,

$$\lim_{n\to\infty} \mathbf{P}\left(\left|\frac{n}{N} - \mathbf{p}_t\right| < \varepsilon\right) = 1$$ (6-34) [142]

which indicates that in a repetitive experiment (Bernoulli experiment [142]), the larger the number of trials undertaken, the higher the probability that the transformer's real thermal lifetime can be observed [142].

Furthermore according to Chebyshev's inequality and typically for a constant ε close to zero, (6-34) can be deduced to an inequality as

$$\mathbf{P}\left(\left|\frac{n}{N}-\mathbf{p}_t\right|<\varepsilon\right)>1-\frac{\sigma^2}{\varepsilon^2 \bullet N} \qquad \text{(6-35) [142]}$$

where σ is the standard deviation of observation $\frac{n}{N}$.

It can be seen from (6-35) that the value of **P**, or the probability of $\left|\frac{n}{N}-\mathbf{p}_t\right|<\varepsilon$, increases when the number of tests (N) increases. If $\mathbf{P}\left(\left|\frac{n}{N}-\mathbf{p}_t\right|<\varepsilon\right)$ is desired to reach a specific value $\mathbf{P}_0$, the number of tests is thus required to

$$1-\frac{\sigma^2}{\varepsilon^2 \bullet N} \geq \mathbf{P}_0 \qquad \text{(6-36)}$$

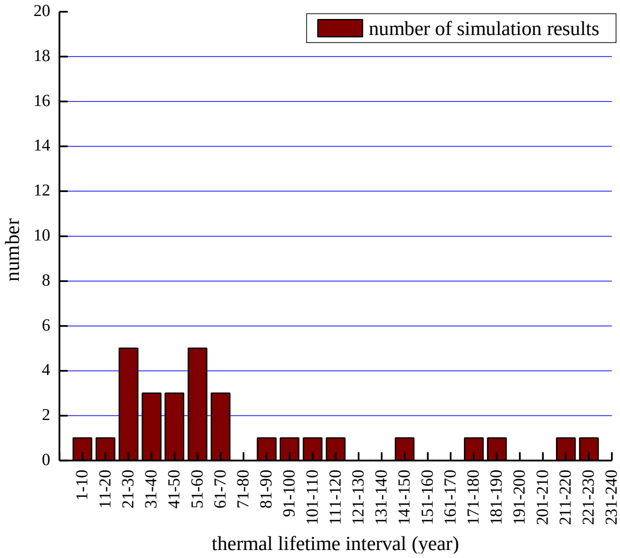
and thus

$$N \geq \frac{\sigma^2}{\varepsilon^2 \bullet (1-\mathbf{P}_0)} \qquad \text{(6-37)}$$

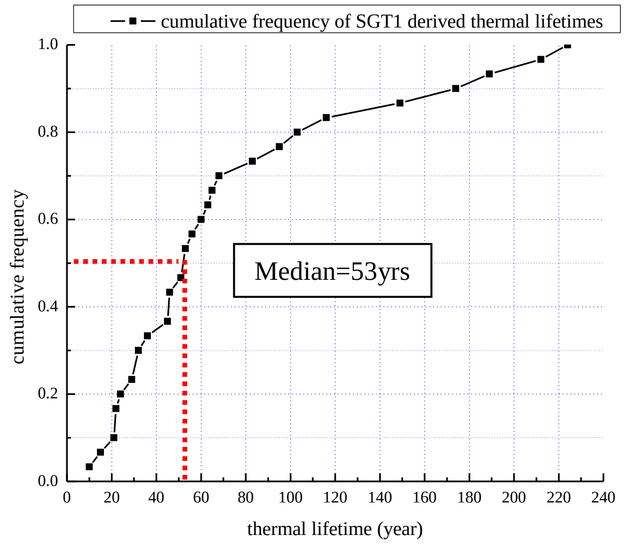
It hence indicates by (6-37) that the higher precision (reflected by larger $\mathbf{P}_0$) is desired, the more tests are needed to run; and particularly if the probability of observing the real thermal lifetime during tests is low (inferred by larger σ), the large iterations are required. For instance, suppose σ equals 3, ε is as small as 0.1, in order to achieve the accuracy of $\mathbf{P}_0$ = 0.90, the number of tests is requested no less than 9000, which is much more than 30 times of simulations carried out here for individual transformer thermal lifetime calculation.

In particular in the individual transformer thermal lifetime Monte Carlo simulation according to the simplified approach, the benchmark as the TRUE value of the probability of observing the unit's real thermal lifetime ($\mathbf{p}_t$) is difficult to know, which results in an uncertain value of σ and thus a large number of tests are tended to be run. In fact in engineering practice, 1000 iterations are usually sufficient undertaken; for problems of rare events, more iterations are required [19, 49]. However as introduced previously, in the transformer thermal lifetime simulation carried out in this chapter, the execution of 30 runs considers the uncertainties of the compensation factor (CF) and also reduces the computational complexity.

As two examples, the obtained 30 thermal lifetimes of two 275/132kV in-service transformers are presented in Figure 6-11 and Figure 6-12 respectively.

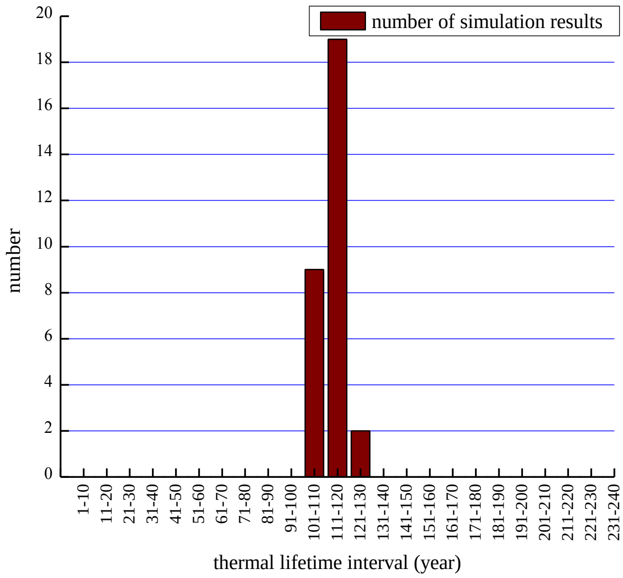


(a) SGT1 30 Runs of Thermal Lifetimes Histogram

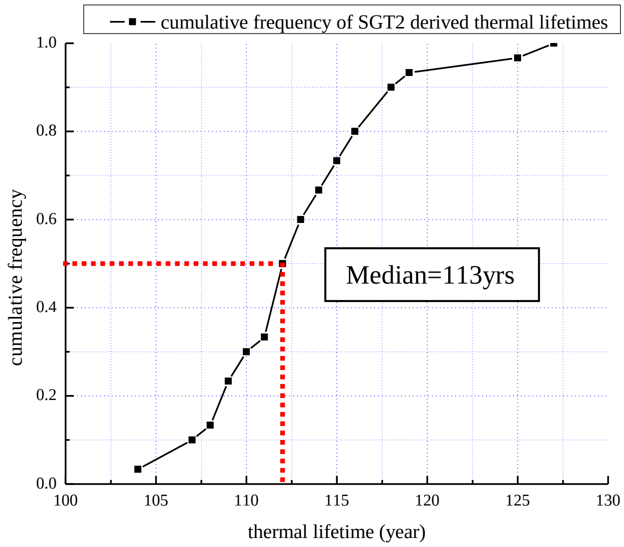


(b) SGT1 30 Runs of Thermal Lifetimes Cumulative Distribution Curve

Figure 6-11 National Grid 275/132kV In-Service SGT1 Estimated Thermal Lifetime Distribution



(a) SGT2 30 Runs of Thermal Lifetimes Histogram



(b) SGT2 30 Runs of Thermal Lifetimes Cumulative Distribution Curve

Figure 6-12 National Grid 275/132kV In-Service SGT2 Estimated Thermal Lifetime Distribution

SGT1 is equivalently loaded at 0.331p.u. in 2009, which derives its normal distributed compensation factor $\mathbb{C} \in N(\mu=1.571, \sigma=0.4725)$ according to (6-32); while SGT2's 2009 equivalent load is as small as 0.095p.u. and this generates $\mathbb{C} \in N(\mu=13.114, \sigma=0.1231)$. Due to the large σ assigned to SGT1, its 30 runs' thermal lifetimes are revealed to widely distribute from 10yrs to 230yrs. On the other hand, the 30 thermal lifetimes of SGT2 concentrate between 100yrs and 130yrs due to its small σ considered.

It can be generally summarized according to the above two samples of calculation that for an in-service transformer thermal lifetime prediction, the Monte Carlo simulation is run for 30 times,

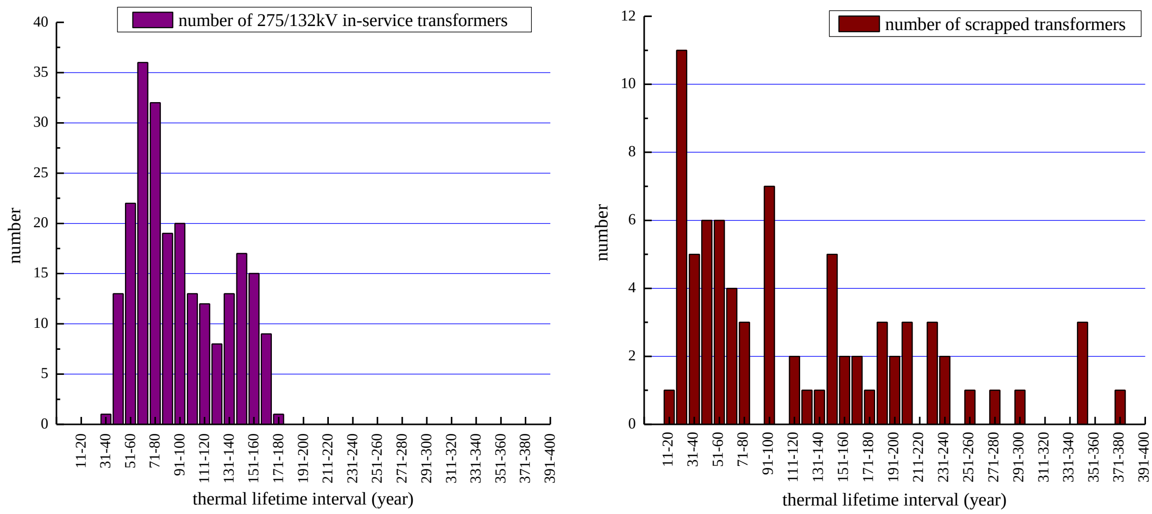
- the estimated 30 thermal lifetimes may distribute in a fairly wide range, especially for those equivalently loaded moderately or heavily, and
- the transformer's REAL thermal lifetime is difficult to determine by the limited 30 times of iteration.

Therefore the median value of the 30 thermal lifetimes is used to represent a transformer's thermal lifetime. The median figures out the average value among the 30 figures by simply separating the 15 lower lifetimes and other 15 higher lifetimes. Besides, the median value is not affected by the outliers (i.e. in Figure 6-11 (a) 2 simulation results of SGT1 ≥ 200yrs, which is far from the concentrated lifetimes ≤ 70yrs) as the arithmetic average can be [53]. The median lifetime can be easily determined as it corresponds to the 50% cumulative frequencies of the 30 results, and particularly for transformer lifetime prediction, the median is further rounded up or down to the nearest integer. It is thus indicated in Figure 6-11 (b) that the median value of 30 iterations as 53yrs is used as SGT1's thermal lifetime and in Figure 6-12 (b) the median life of 113yrs is SGT2's thermal lifetime.

Hence, for each of National Grid 275/132kV in-service transformers, Monte Carlo simulation is conducted 30 times according to 30 randomly generated compensation factors $\mathbb{C}$, and the median value of 30 lifetimes is calculated and rounded to simply represent the transformer's thermal lifetime *EoL*. Moreover National Grid 275/132kV in-service transformers' cumulative failure probability and thermal hazard curve can be derived.

6.4.2 National Grid 275/132kV Transformers' Thermal Lifetimes

The distribution of National Grid 275/132kV in-service transformers' predicted thermal lifetimes is displayed in Figure 6-13(a) as follows. The distribution of scrapped transformers' thermal lifetimes is also presented in the same scale in Figure 6-13(b) for comparison purpose.



(a) National Grid 275/132kV In-Service Transformer Thermal Lifetimes' Distribution

(b) National Grid Scrapped Transformer Predicted Thermal Lifetimes' Distribution

Figure 6-13 National Grid 275/132kV In-Service Transformer Thermal Lifetimes' Distribution vs. Scrapped Transformer Predicted Thermal lifetimes' Distribution

It can be seen from Figure 6-13(a) that the in-service transformers' thermal lifetimes intensively locate at 60-80yrs and 140-160yrs and the data vary from 39yrs to 173yrs. The range of in-service transformers' thermal lifetimes is much shorter than that between 16yrs and 373yrs derived from 77 scrapped transformers through their lowest DP values. The mean value of 275/132kV transformers' thermal lifetimes is predicted to be 96yrs with the standard deviation of 36yrs which are also less than the mean thermal lifetime 120yrs and standard deviation 91yrs derived from scrapped samples.

The less variation of National Grid in-service transformers' thermal lifetimes may be caused by firstly the arbitrarily normal distributed uncertainties assigned to a transformer's compensation factor ($\mathbb{C}$) and secondly the use of 30 runs' median value as the transformer's thermal lifetime, which effectively screens off the prolonged lifetimes that could be estimated within the 30 iterations.

The dual-peak-shape failure frequency of 275/132kV in-service population thermal lifetimes as shown in Figure 6-13 (a) suggests that a combined distribution model could be a better fit than a traditional distribution model. Curve fitting using a traditional distribution model is thereby not implemented.

National Grid 275/132kV in-service transformers' thermal lifetime cumulative probability curve is thereafter developed as shown in Figure 6-14. It is also compared with National Grid scrapped transformers' thermal lifetime cumulative probability.

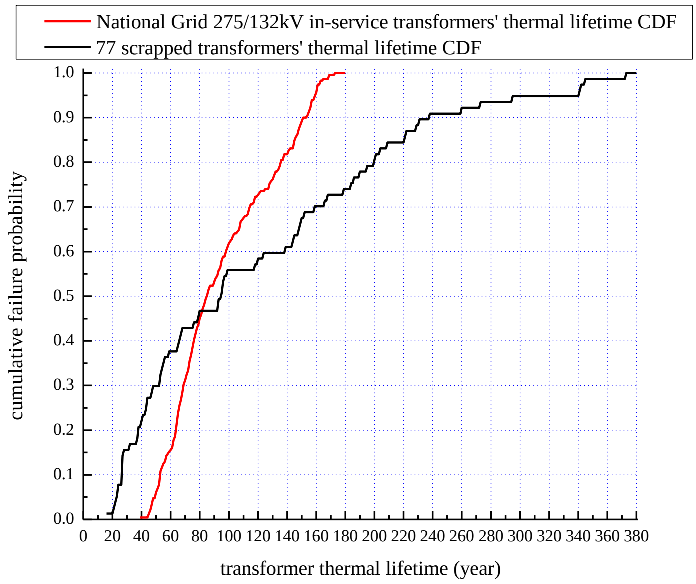


Figure 6-14 National Grid 275/132kV In-Service Transformers' Thermal Lifetimes CDF vs. Scrapped Transformers' Thermal Lifetimes CDF

The ages when reaching three typical cumulative probabilities; the early onset of population significant unreliability (corresponding to 2.5% of CDF), the median lifetime (50% of CDF) and the late onset of population significant unreliability (97.5% of CDF), determined from 275/132kV in-service population; are also compared with the ages estimated from the scrapped samples, see Table 6-3.

Table 6-3 Ages of Reaching Critical Thermal Life CDFs Estimated from National Grid 275/132kV In-service Population and from Scrapped Transformers

	Age *t*	
CDF *F*(*t*)	275/132kV In-Service Population	Scrapped Transformers
2.5%	46	21
50%	85	95
97.5%	161	342

Figure 6-14 and Table 6-3 show that the range of thermal lifetimes derived from 275/132kV in-service population is shorter than that for scrapped transformers. Particularly the population anticipated thermal lifetime when 50% approach their thermal end-of-life and the age of late onset of population unreliability are both smaller than those derived from scrapped samples.

The National Grid 275/132kV in-service population thermal hazard curve can also be generated based on the predicted thermal lifetimes. In order to be consistent with the National Grid power transformers' random failure hazard derived in Chapter 3 and the scrapped transformers' thermal hazard presented in Chapter 5, National Grid 275/132kV in-service population thermal hazard $h(t)$ as a function of age t is also deduced as the instantaneous observed thermal failure probability at a typical age, which is expressed as

$$h(t) = \frac{number\ of\ transformers\ with\ thermal\ lifetimes = age\ t}{number\ of\ transformers\ with\ thermal\ lifetimes \geq age\ t} \tag{6-38}$$

Figure 6-15 displays National Grid 275/132kV in-service population thermal hazard against age *t* together with the thermal hazard derived from scrapped transformers.

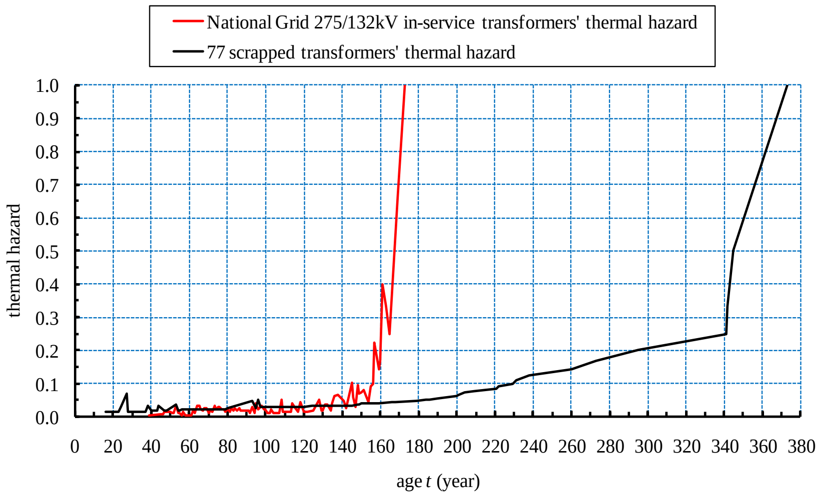


Figure 6-15 National Grid 275/132kV In-Service Transformers' Thermal Hazard vs. Scrapped Transformers' Thermal Hazard

It reveals in Figure 6-15 that National Grid 275/132kV in-service population's thermal hazard increases with transformer service age; in particular the hazard increases much faster when approaching older ages. Again the range of in-service transformers' thermal lifetimes is much shorter than for scrapped transformers. 275/132kV in-service population thermal hazard curve is further zoomed in between hazard of 0 and 0.2 as shown in Figure 6-16.

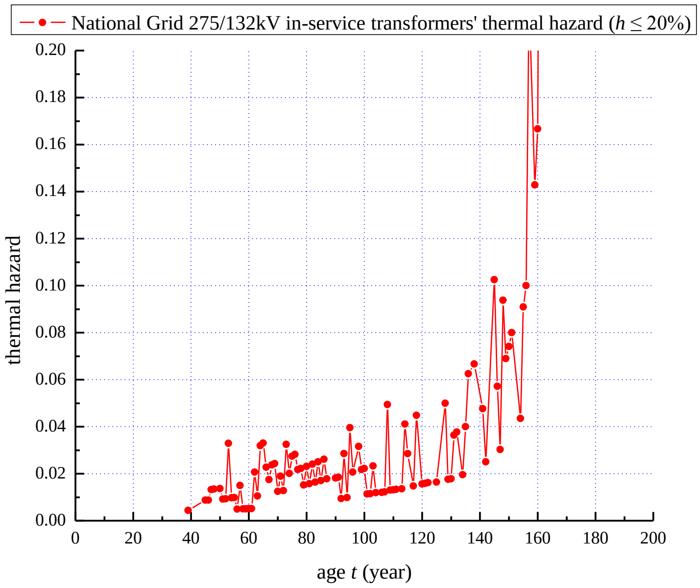


Figure 6-16 National Grid 275/132kV In-Service Transformers' Thermal Hazard Curve (hazard ≤ 20%)

It can be seen from Figure 6-16 that the in-service population thermal hazard will fluctuate during successive years, due to the different number of transformers predicted to approach their thermal end-of-life. However, 275/132kV in-service transformers' thermal hazard can be closely fitted into the exponential form $a \bullet exp(bt)$; i.e. the ageing-related failure hazard model proposed by Hartford Steam Boiler (HSB) [24, 62]. The exponentially fitted hazard curve against age is shown in Figure 6-17 together with the originally derived thermal hazard.

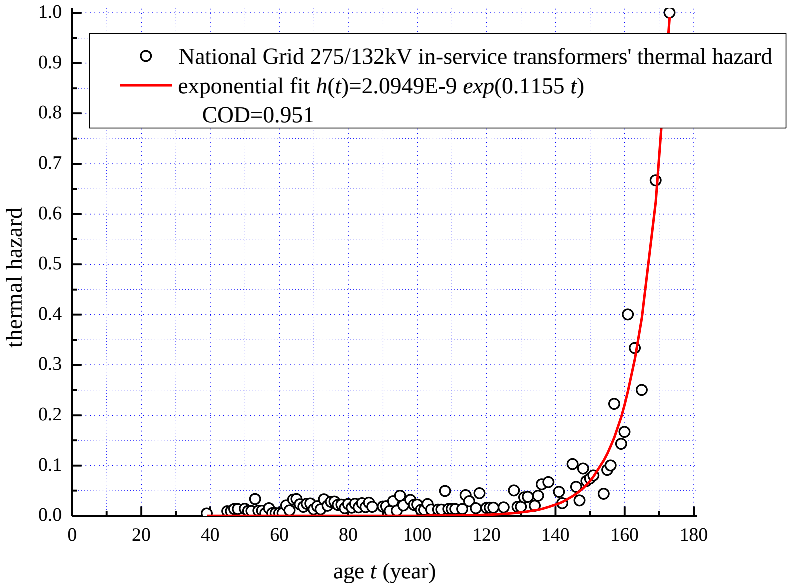


Figure 6-17 Exponential Fitting on National Grid 275/132kV In-Service Transformers' Thermal Hazard

As indicated in Figure 6-17 the best fit is determined as

$$h(t) = 2.0949\text{E}-9 \bullet exp(0.1155 \bullet t) \tag{6-39}$$

with the coefficient of determination (COD) equal to 0.951.

This derived exponential term of the 275/132kV in-service transformer hazard curve generally indicates the thermal ageing trend which the National Grid transformer population is experiencing. However uncertainties need to be considered, and these are discussed in the following section.

6.4.3 National Grid In-Service Transformer Population Thermal Ageing Trend

As explained in 6.4.1, errors are unavoidably generated when calculating the individual thermal lifetime according to the simplified approach. A transformer's thermal lifetime may not be confidently predicted by assigning an arbitrarily distributed error to the load deduced compensation factor $\mathbb{C}$; this is achieved by running a Monte Carlo simulation 30 times, or by using the median value of the 30 runs as the transformer thermal lifetime. Error bars need to be considered when predicting an individual transformer's thermal lifetime, see Section 6.3.4.

Moreover, 275/132kV in-service transformers thermal hazard curve may not accurately represent the thermal ageing trend of the whole in-service population, since transformers from other voltage levels, i.e. 400/275kV, 400/132kV, 275/66kV, 275/33kV and smaller transformers, are approximately 2/3 of the 2009 National Grid in-service population. As discussed in Chapter 5, loadings of these transformers are different from that of 275/132kV transformers. In addition, because the relationship between the transformer equivalent load and the compensation factor $\mathbb{C}$ is deduced from 44 scrapped samples, mostly 275/132kV and a few 400/132kV transformers, (6-32) may not be applicable to transformers of other voltage groups. Hence, the National Grid transformer population may have been experiencing an increasing thermal ageing trend as indicated by 275/132kV transformers' thermal hazard curve; however significant uncertainties exist.

6.5 Summary

In this Chapter, a simplified approach for transformer thermal lifetime prediction based on the IEC transformer thermal model is proposed to overcome the deficient input data in the classical calculation approach. The idea of a transformer hot-spot compensation factor ($\mathbb{C}$) is established, based on the transformer 2009 annual equivalent load and other universal inputs such as national typical winter/summer daily demand profiles, IEC thermal design parameters and UK Heathrow area 2007 ambient temperature records.

A transformer's hot-spot compensation factor ($\mathbb{C}$) is revealed to strongly relate to its annual equivalent load factor according to a study on 44 scrapped transformers. By extrapolating the relation and by reasonably considering the errors, the in-service transformers' compensation factors can be effectively evaluated according to their loading conditions.

The simplified approach is used to predict National Grid 275/132kV in-service transformers' thermal lifetimes via a one-dimension Monte Carlo simulation. The results generally suggest a shorter range of thermal lifetimes than derived from 77 scrapped transformers through their lowest DP values. Also the mean value of 275/132kV transformers' thermal lifetimes and the standard deviation are less than suggested from the scrapped samples. Uncertainties of individual thermal lifetime however need to be borne in mind owing to the errors from assigning the compensation factor and inadequate number of Monte Carlo simulation.

The thermal ageing trend of the National Grid in-service population can be generally matched to the 275/132kV in-service transformers' thermal hazard $h(t)$ against age t. However, error bars should be taken into account, especially because the ageing characteristics of transformers operating at other voltages are not represented by 275/132kV transformers.

Chapter 7

National Grid Transformer Population Actual Failure Hazard Derivation

7.1 Introduction

Analysis on National Grid historical failed and in-service transformers (2009), summarizes that UK National Grid transformers have experienced the normal operating stage. The normal operating stage, inferred by a low and stable hazard rate, corresponds to a random failure mechanism. Under the random failure mechanism, a transformer fails due to randomly occurring short-circuit faults, lightning, over-voltage or other system transients. Random failure is the predominant failure mechanism during the early operation ages of a transformer.

As a transformer ages, the unit is more prone to fail since the insulation withstand strength degrades. A transformer fails due to the ageing failure mechanism, according to which the hazard rate increases with age as indicated in the wear-out stage of the bathtub curve. Transformer thermal hazard is used as the baseline for the transformer population ageing-related failure hazard.

In order to develop the National Grid transformer population actual failure hazard curve, the hazard rate under random failure mechanism and thermal hazard due to an ageing failure mechanism need to be combined. This chapter presents a mathematical model for linking the transformer random failure hazard with the thermal hazard.

7.2 Derivation of National Grid Transformer Population Actual Failure Hazard

7.2.1 National Grid Transformers' Random Failure Hazard

The National Grid power transformers' random failure hazard rate, for the normal operating stage, denoted as h_0, is determined to be:

$$h_0 = 0.20\% \tag{7-1}$$

up to the age of 36 according to the study on historical failures and in-service transformers, see 3.6. h_0 is shown in the common logarithm scale (log_{10}) against age t in Figure 7-1 as follows.

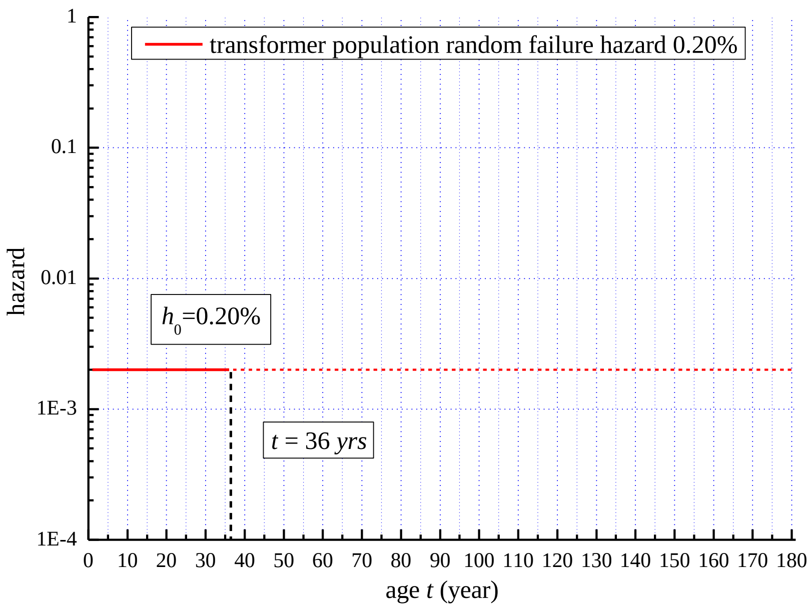


Figure 7-1 Illustration of National Grid Transformer Population Random Failure Hazard

When the insulation strength is sufficiently high, a transformer fails randomly, this is mainly due to exceptional system transients. The random failure hazard consequently represents the frequency of transients in the network within a year. In a well-maintained transmission network in a country with mild climate condition, the occurrences of exceptional events are well controlled under a certain level and the natural events, such as lightning, are rare. The random failure hazard does not rise over time [2]. For National Grid transformer population h_0 = 0.20% is thereby assumed to continue. The dash line in Figure 7-1 indicates the constant random failure hazard beyond 36yrs.

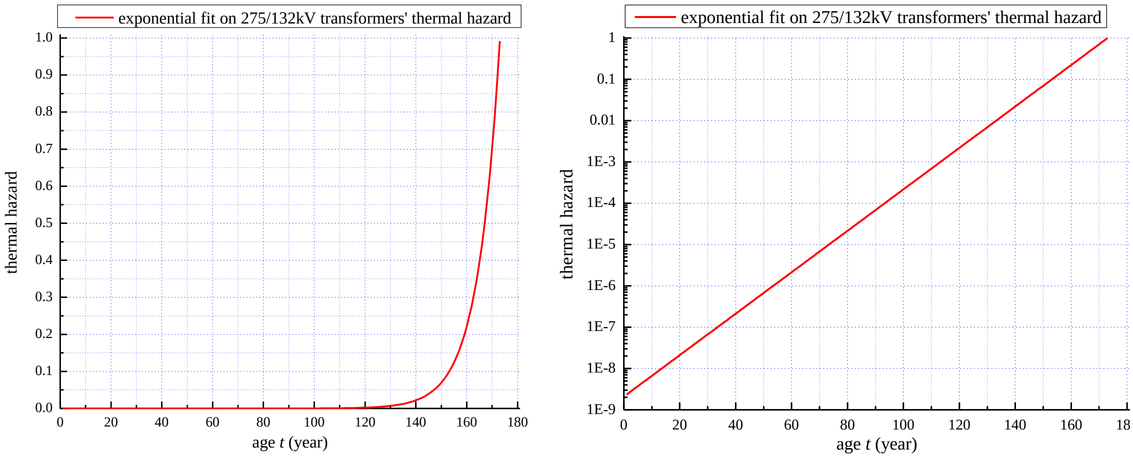
7.2.2 National Grid Transformers' Thermal Hazard

National Grid 275/132kV in-service transformers' thermal hazard is derived according to a simplified approach presented in Chapter 6. Accordingly the thermal hazard of the National Grid transformer population can be presented by adding reasonable error bars to the 275/132kV transformers' thermal hazard curve.

However, to get rid of the uncertainties of the whole population thermal hazard curve, the exponential fit on 275/132kV transformers' thermal hazard curve is adopted to indicate the population thermal hazard against age. As presented in Chapter 6, the best exponential fit of 275/132kV transformers' thermal hazard $h_t(t)$ is obtained as

$$h_t(t) = 2.0949\text{E}-9 \bullet exp(0.1155 \bullet t) \quad (7\text{-}2)$$

$h_t(t)$ is shown in Figure 7-2(a), for clarity $h_t(t)$ is also shown in the common logarithm scale (log_{10}) against age t in Figure 7-2(b).



(a) Population Thermal Hazard Curve

(b) Population Thermal Hazard Curve in log_{10} Scale

Figure 7-2 Illustration of National Grid Transformer Population Thermal Hazard

Transformers' thermal hazard curve, refers to the ageing-related failure risk of the whole population.

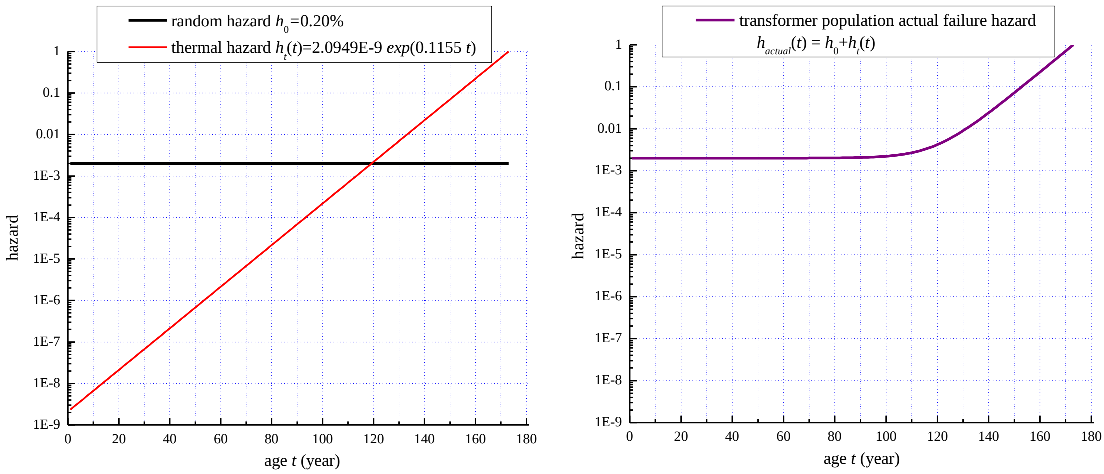
7.2.3 Linking Population Thermal Hazard with Random Failure Hazard

The principle of transformer random failure mechanism is different from that of the ageing failure mechanism: the former depends on the frequency of transmission system transients that are seen by the transformer and the latter is coincident with the transformer insulation ageing process. Hence they are independently affecting a transformer's actual failure hazard; the random failure mechanism is predominant during early operation ages, whilst the ageing failure mechanism dominates when the units approaching old ages.

Therefore the transformer population actual failure hazard h_{actual} can be obtained by linking the random failure hazard with the thermal hazard along the time scale, which is mathematically expressed as

$$\begin{aligned} h_{actual} &= h_0 + h_t(t) \\ &= 0.20\% + 2.0949\text{E}-9 \bullet exp(0.1155 \bullet t) \end{aligned} \tag{7-3}$$

According to (7-3) the combination of random failure hazard h_0 and thermal hazard $h_t(t)$ can be graphically illustrated as the superposition of Figure 7-1 and Figure 7-2, the derivation process is shown in Figure 7-3(a) and the deduced National Grid transformer population actual hazard $h_{actual}(t)$ against age t is indicated in Figure 7-3(b), in the common logarithm scale (log_{10}) against age t.



(a) Derivation of Transformer Population Actual Failure Hazard

(b) Transformer Population Actual Failure Hazard

Figure 7-3 Derivation of Transformer Population Actual Failure Hazard

It needs to be noticed that a hazard model that independently considers the random failure period and the wear out period was illustrated by CIGRÉ in [6] for SF_6 circuit breakers' failure analysis. Moreover the combined hazard, as the sum of a constant value and an exponentially increase hazard $h_0 + a \bullet exp(b \bullet t)$, is equivalent to the transformer failure risk model developed by Hartford Steam Boiler (HSB) based on Makeham's 1st Law [62, 63]. The mathematical principles beneath the combined hazard expression were illustrated in Chapter 2.

7.3 Transformer Population Critical Hazard Rate and Knee Point Age of Failure

In addition from an asset manager's perspective, a critical hazard $h_{critical}$ corresponding to a critical age is of special concern, because $h_{critical}$ essentially suggests a dramatic increase of population actual hazards as transformers reach "old" age. A utility cannot be tolerant of large numbers of transformer failures because of the detrimental impact on operational security and network reliability. Hence by quoting [143], a critical age t, designated as the knee point age t_{knee}, is figured out in the population actual failure hazard curve where the increase of age can result in an increase of the population actual failure hazard. The parameter t_{knee} in the population actual failure hazard curve implies the start of a dangerous period with too many failures, which the system operator cannot accept. Figure 7-4 generally presents a hypothesized $h_{critical}$ in the common logarithm scale (log_{10}), and the corresponding knee point age t_{knee} at transformer population actual failure hazard curve for illustration purpose.

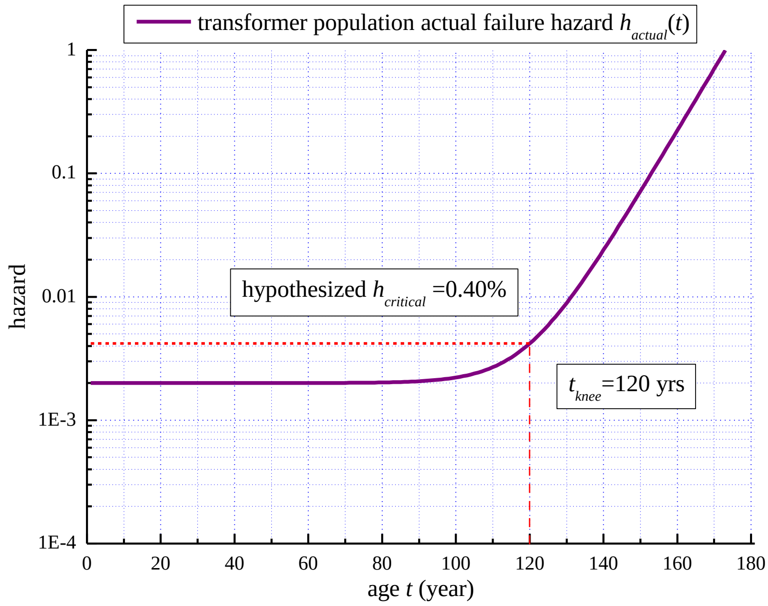


Figure 7-4 Illustration of Transformer Population critical hazard $h_{critical}$ and knee point age t_{knee}

The critical hazard value $h_{critical}$ and the knee point age t_{knee} are essentially determined by the transformer ageing status and the network reliability requirement. In the hypothesized case as shown in Figure 7-4 for example, if the critical hazard $h_{critical}$ is determined as 0.40% by asset managers, the age of 120 yrs would suggest the onset of a dangerous period with a remarkable increasing number of transformer failures, since when the proactive actions need to be carried out. t_{knee} may shift to older ages, if more newly installed transformers are operating in the network. Thus higher transmission reliability and security are ensured. Besides the critical hazard $h_{critical}$ and the knee point age t_{knee}, the replacement decision for transformer may be different among utilities. This is influenced by the number of transformer failures that can be tolerated on a specific network.

7.4 Summary

The random failure mechanism is predominant before the onset of transformer ageing-related failure and the effect of random failures works constantly within a well-maintained network. However, it is always difficult to design and maintain a reliable network in mountainous countries or under the severe climatic condition. Ageing failure mechanism, especially thermal ageing, dominates at older ages and consequently the transformer population thermal hazard is derived as an exponential term. The actual failure hazard curve is proposed as the superposition of the random failure hazard and the thermal hazard with respect to the service age.

Based on the deduced actual failure hazard curve, asset managers refer to a knee point age t_{knee} when the actual hazard reaches a critical value. This identifies when a large number of transformers owned by the utility would have approached their end-of-life and accordingly the actual failure hazard would increase dramatically. Preventative replacement therefore needs to be implemented effectively in order to maintain network security.

Chapter 8 Conclusions and Further Work

8.1 Conclusions

8.1.1 General

An electric power system requires a cost-efficient strategy for ageing infrastructure replacement to satisfy the increasing requirement for system reliability. UK National Grid in particular, owns a significant number of power transformers which were installed during the 1960's and 1970's. These transformers are approaching or have exceeded their 40 years designed end-of-life. It is thus of great importance to predict the transformer population failure trend based on present knowledge and information.

Intensive studies on UK National Grid power transformers' lifetimes were carried out in this thesis using:

- statistical analyses applied to National Grid transformer lifetime data, and
- a physical ageing model, based on using the reduction in insulation paper mechanical strength to estimate the transformer thermal lifetime.

The objective of the above studies was to predict the National Grid transformer population failure hazard curve and use it to help asset managers make proper replacement decisions. The main conclusions are summarized in the following section.

8.1.2 Summary of Main Findings

Manufacturer predicted product design end-of-life does not tell a product's real lifetime as the operation condition varies among different units. Product failure can be expressed in a hazard curve, which implies the conditional failure probability at a specific age. This is achieved by knowing a certain number of products have survived till the end of the previous annual age period. Literature from engineering applications, revealed that statistical approaches have been widely used since the 1940's in generating population failure hazard curve. The commonly used method was to calculate the average lifetime ($\bar{t}$) and the standard deviation (σ) of failed products. This method was recognized to be not suitable for a population with relatively long operation life and with very limited number of failures, i.e. power transformers.

By considering the information from failed products and also the surviving units, the procedure of lifetime data statistical analysis was systematically developed as, firstly collecting lifetime data, secondly selecting proper distribution model(s), thirdly fitting lifetime data into the presumed distribution model(s) and determining the best fitted parameters, and finally implementing a "Goodness-of-Fit" test on the presumed model(s). If a hypothesised model is identified adequate according to the "Goodness-of-Fit" test, the population future failure can be expressed as the hazard curve related to age, based on the presumed models and determined parameter(s).

The transformer lifetimes were classified as a group of "multiply right-time censored" data, because transformers were commissioned in different years, some transformers were manually taken out of service before failure for system security reason, or some are still in service. Traditional distributions, for example the normal distribution, Weibull distribution and etc, are widely used to fit the population lifetime data. Besides, hazard models from vital statistics are often used, and utilities always define empirical models developed according to their operational experience and expert opinions. It therefore emphasized in engineering applications, that the curve-fitting obtained hazard rate should be consistent with the utility's hazard model expression. The least-square estimator (LSE), maximum likelihood estimator (MLE) and Bayesian approach were concluded as 3 commonly used techniques to derive the values of parameters under the presumed distribution model(s). In particular, the key advantage of LSE as mathematically simple-to-implement results in its popularity in practice. Based on the LSE algorithm, Nelson developed a graphic plotting method, including the hazard plotting approach and the CDF plotting approach. Li further modified Nelson's method and applied his simplified method to analyze Canada BC Hydro 500kV reactors' lifetime data. The "Goodness-of-Fit" test is usually not carried out in engineering practice, especially when utilities' empirical hazard model is used. Approaches suggested from literatures were then implemented using the lifetime data from National Grid transformers.

National Grid has recorded the installation of its transformer assets since 1952. Until the end of 2008, 52 transformers failed, 62 were manually retired and 751 transformers are still in service. The service ages of these 865 (52+62+751) transformers constitute the data base for statistical analysis. National Grid transformer lifetime data are heavily censored as only 52 transformers failed. The normal distribution, lognormal distribution, 2-parameter Weibull distribution and extreme value distributions were the selected models used to describe the lifetime data, because they are commonly used models in the field of engineering. The least-square estimator (LSE), including Nelson's hazard plotting approach and CDF plotting approach, and Li's simplified approach, were used to fit the data into the presumed models. The maximum-likelihood estimator (MLE) was also used and the results were compared with those from LSE. The Kolmogorov-Smirnov test (K-S test) is implemented to examine the "goodness-of-fit" of the above distribution models.

By using Nelson's hazard plotting approach, the National Grid transformer population mean life ($\bar{t}$) was derived to be 73 yrs under the normal distribution and the standard deviation (σ) was 27.8 yrs; $\bar{t}$ was 12932 yrs with σ of 157030 yrs under the lognormal distribution; $\bar{t}$ was 387 yrs and σ was 353.1 yrs under the 2-parameter Weibull distribution; and $\bar{t}$ was 53 yrs and σ was 14.0 yrs under the extreme value distribution. By curve fitting, the values of mean lives and standard deviations were much longer than the mean life and the standard deviation obtained from the failed transformers ($\bar{t}$ =20 yrs and σ=11.6 yrs). It is verified that the population mean life can be estimated to be reasonably longer than from the failure samples, by considering the information from the surviving transformers. In particular, under the lognormal distribution, both the population mean life ($\bar{t}$=12932 yrs) and the standard deviation (σ=157030 yrs) were very large, and typically σ was larger than $\bar{t}$. The large values were purely derived from a mathematical process and the large standard deviation indicates the high dispersion of the lifetime data.

According to the K-S test, the normal distribution and extreme value distribution were rejected; while the lognormal distribution and Weibull distribution were identified as adequate models to describe the data. However, the lognormal distribution and Weibull distribution with constant hazard are inconsistent with the engineers' prior knowledge that transformers will approach the wear-out stage with increasing hazard. Moreover, the predicted population mean life, $\bar{t}$=12932 yrs, under the lognormal distribution or $\bar{t}$ =387 yrs under the Weibull distribution is unconvincing to asset managers. It was concluded that, when statistically analyzing the product lifetime data, information about engineering prior knowledge has to be suspended.

In fact, National Grid power transformers' operational experience has not shown an adequate number of failures, especially failures at older ages (only one failure was recorded during age 40 and 50). The limited number of failures prevents any meaningful statistical analysis to be conducted. Statistical curve fitting results would be distorted by the lifetime data of poor quality. Particularly, the last failure at age 48 was caused by a random failure mechanism; generally historical transformer failures indicate a random failure mode. The high value of hazard at age 48 does not indicate the onset of an increasing hazard.

In the CDF plotting approach, the Kaplan-Meier method, Herd-Johnson method and the actuarial method were used to calculate the reliability function and the cumulative distribution function. The values of the reliability function $R(i)$ and the cumulative distribution function $F(i)$ of failure i are similarly derived by these three methods. The population mean life ($\bar{t}$), standard deviation (σ) and the hazard curve obtained by using the CDF plotting approach, are similar to those derived by hazard plotting.

Li's modified method was also implemented to compare with Nelson's graphic plotting method. It was concluded that Li's method is simple to implement and restriction need to be born in mind especially when dealing with heavily censored lifetime data.

It was indicated by using MLE to fit the National Grid transformer lifetime data, that MLE can fit the later failures better than the earlier failures; it is also advantageous in dealing with heavily censored data.

Statistical analysis on product lifetime data, for example on transformer lifetime data, is straightforward; however the deduced failure trend might be biased against the prior knowledge due to the limitations of

- using transformer service age as the only variable considered in the analysis, as it is not the dominant factor in determining a transformer failure,
- operational experience has not yet suggested any appropriate distribution models to best describe transformer lifetime data, and
- the number of failures is limited up to the present and these failures were mainly caused by random system transient events; i.e. this data should not be used to indicate future ageing-related failures.

According to the study described in this thesis, National Grid transformer historical failures until age 36 had an average hazard rate of 0.20%. A significantly high hazard rate was obtained at age 48, due to the limited number of exposed transformers at that old age and further investigation implied a random failure mechanism for this failed transformer. It was further emphasized that, UK National Grid has little experience of operating transformers at old ages, and except for one transformer that failed in 2007, caused by severe solid insulation ageing, most other failures resulted from the random failure mechanism. Observed random failures up to the present should not be used to predict ageing-related failures in the future. Consequently 0.20% was concluded as the random failure hazard rate of the National Grid transformers. This low and stable value of hazard indicates that the normal operating stage is at least age 36, which benefited from a proper commissioning scheme, maintenance strategy and the mild weather in most part of England and Wales.

Due to the intrinsic restrictions of statistics, a physical model of transformer ageing is used to predict National Grid transformer future failures under the ageing failure mechanism. A transformer's end-of-life is eventually determined by the useful lifetime of its insulation paper subject to the unit's operation condition, due to the irreplaceability of the paper insulation. Literatures in regard to transformer insulation paper ageing were reviewed. According to literature, cellulose ageing is implied by the paper mechanical property deterioration, caused by the effects from heat, water, oxygen and low molecular weight acids (LMA). A simple 6 ℃ rule is widely used to express the effect of heat or temperature, as the cellulose paper ageing rate doubles when the temperature increases by 6 ℃. The presence of water in paper can shorten and weaken the fibre molecules and the effect is strongly related to the acids dissolved. Oxygen dissolved in transformer oil can attack the carbon atoms in the cellulose molecule and the released acids and water will initiate and then catalyze the cellulose hydrolysis process. The content of the low molecular weight acids (LMA) determines the rate of cellulose hydrolysis, which is the dominant degradation process at normal operating temperatures.

Transformer "***thermal ageing***" was used to describe the transformer paper insulation mechanical property degradation under the effects from heat, water, oxygen and acids. Transformer ageing failure mechanism discussed in this study was the failure mechanism related to thermal ageing. Based on the definition, transformer "***thermal***

end-of-life" was used to indicate the end of paper insulation useful life, i.e. when the mechanical strength reduces to an unacceptable level and the transformer is not fit for operation. "***Thermal lifetime***" was to represent the transformer operational period, i.e. when the paper insulation reaches a critical physical condition. The cellulose tensile strength (TS) and the paper degree of polymerisation (DP) are both measurable properties and they are used to assign the critical physical condition, as the "***thermal end-of-life criterion***". The absolute DP value was concluded as a better indicator of thermal ageing, and a DP=200, is a widely adopted value used as the thermal end-of-life criterion.

The development of models describing transformer insulation paper DP reduction was reviewed. A commonly used model that assumes 1/DP linearly increases with transformer service age was used for individual thermal lifetime estimation. Accordingly in this model, DP of 1000 was identified to represent the new insulation and DP of 200 indicated the end of thermal life.

UK National Grid has been carrying out post-mortem analysis on its retired transformers since about 1993. The group of retired transformers constitutes a unique sample to indicate the ageing status of the in-service transformer population. The DP values of a scrapped transformer's winding insulation paper are usually measured; the lowest value of DP indicates the weakest part of paper insulation, and this determines the transformer's thermal end-of-life. The model of 1/DP linearly increasing with transformer service age was applied to the National Grid scrapped transformers. According to the model, scrapped transformers were examined by calculating individual ageing rate and thermal end-of-life. The results yielded a large variation of National Grid scrapped transformers' ageing rates and also their thermal lifetimes. It suggested that these transformers might have been loaded differently, designed/manufactured differently, or/and experienced different ambient conditions and this small group of scrapped transformers did not effectively represent the thermal ageing of the National Grid in-service transformer population.

The scrapped transformers were further assessed in respect of their loading, thermal design and installation site ambient temperature. This was used to explain the reasons for the large variations in their thermal lifetime and also to identify their inability to represent the in-service population.

The level and profile of transformer loading were analyzed to evaluate the loading condition of scrapped transformers. It was concluded that the average equivalent loads of scrapped transformers are between 0.1p.u. and 0.5p.u.. Although the distribution is similar to that of the 275/132kV in-service transformers, the level of transformer loading is not the only determinant factor in a unit's thermal lifetime. Transformer load profiles of scrapped units differ significantly and they are powerless to suggest the load of other in-service transformers, especially transformers at different voltage levels.

The information from transformer design includes the transformer thermal parameters, i.e. the ratio of load losses to no-load losses R, the top oil temperature rise over the ambient temperature $\Delta\theta_{tor}$, the gradient between the average winding temperature rise and the average oil temperature rise G_r, and the transformer winding hot-spot factor (*HSF*). *HSF* describes the hot-spot temperature rise over the top winding temperature. Transformer thermal parameters are assumed to distribute in a dispersed manner and the exact values are eventually determined by the transformer design technique and the manufacturing process. The values of transformer thermal parameters can only be obtained via the temperature-rise test implemented on the actual transformer or other similarly designed transformers. The scrapped transformer hot-spot factors (*HSF*s) were also derived, they distribute over a wide range, and the values were not consistent with the general assumption of *HSF*=1.3. *HSF*s of the in-service transformers would not be precisely known. Moreover, by comparing the manufacturers involved in the scrapped transformers and the in-service transformers, the scrapped transformers only indicate that a small number of transformers might have been defectively designed. This small number of scrapped transformers does not constitute an adequate sample to suggest the thermal design of the in-service population, especially when considering more recent well designed transformers.

By customarily dividing England and Wales into the Southwest England and Wales (SW), Southeast England (SE), Northwest England (NW) and Northeast England (NE), the National Grid in-service transformers in different meteorological areas are generally equally sampled by the scrapped units. However, because limited information of transformer ambient temperatures were provided and little knowledge of how real-time ambient variation affects the transformer thermal lifetime is understood, a typical annual temperature profile would be a reasonable approximation to be used for individual in-service transformer thermal lifetime estimation. The National Grid scrapped transformers provided an inadequate number of samples to indicate the ageing status of the National Grid in-service population.

Transformer thermal lifetime is determined by multi-variables, such as instantaneous loading, thermal design parameters, winding *HSF* and ambient temperature. Information of these variables is usually incomplete in transformer operation practice. A simplified approach was therefore proposed to predict individual transformer thermal lifetime based on an IEC thermal model. The idea of transformer hot-spot compensation factor ($\mathbb{C}$) was established to replace the uncertain transformer winding hot-spot factor (*HSF*). Accordingly the inputs of the traditional IEC thermal model were simplified to the transformer 2009 annual equivalent load, national typical daily demand profiles, IEC typical thermal design parameters, UK Heathrow area 2007 yearly ambient temperature and the assigned transformer compensation factor ($\mathbb{C}$). A study was undertaken to estimate the compensation factors ($\mathbb{C}$) of National Grid scrapped transformers. A scrapped transformer's compensation factor $\mathbb{C}$ was revealed to strongly relate to its annual equivalent load and this relationship was extrapolated to other in-service transformers. Hence an in-service transformer's compensation factor $\mathbb{C}$ was designated firstly from its equivalent load and secondly by considering the normal-distributed errors.

This proposed approach was used to predict National Grid 275/132kV in-service transformers' thermal lifetimes. A one-dimension Monte Carlo simulation was used in each transformer's calculation to adequately consider the possible uncertainties of each transformer's compensation factor ($\mathbb{C}$). The median value of 30 simulated thermal lifetimes was adopted to represent a transformer's thermal lifetime. The National Grid 275/132kV in-service transformers' deduced lifetimes were further illustrated by the population cumulative failure curve and the hazard curve. The curves generally suggested a shorter lifetime range for the 275/132kV transformer population and in particular, the population median life (50% of CDF) was extended to 85 yrs compared to the 55 yrs, previously expected by National Grid, and the late onset of the population significant unreliability (97.5% of CDF) was also prolonged to 161 yrs compared to the 85 yrs used by National Grid. Moreover, the hazard curve of 275/132kV transformers could be closely fitted by an exponential formula $h(t) = 2.0949E-9 \bullet exp(0.1155 \bullet t)$, that is a function of age *t*. The thermal ageing trend of the National Grid in-service population can be generally matched to the 275/132kV in-service transformers' thermal hazard $h(t)$. However, error bars should be taken into account, since the ageing characteristics of transformers operating at other voltages are not represented by 275/132kV transformers.

Transformer random failure mechanism and ageing failure mechanism were realized in this study as two different mechanisms, because they affect actual transformer failures independently. It was concluded that, UK National Grid transformers have experienced the normal operating stage with the random failure hazard rate of 0.20%, and the population ageing failure rate can be represented by the transformers thermal hazard $h(t) = 2.0949\text{E}-9 \bullet exp(0.1155 \bullet t)$. Random failure mechanism dominates transformer failure at the early ages, and the random failures continue when transformers are used within a well-maintained network. When operating at older ages, the ageing failure mechanism is dominant. Transformer population actual hazard curve was thereafter proposed as a superposition of the random failure hazard and the thermal hazard with respect to the service age.

The derived transformer population actual failure hazard curve could be used by asset managers to identify the critical hazard rate and the corresponding knee point age, which suggests the onset of a dangerous period where a significant number of transformers would have approached their actual end-of-life. Preventive replacement needs to be effectively implemented in order to maintain secure and reliable operation of the network.

Based on the study described in this thesis, it should be pointed out that, if the critical hazard of transformer failure is determined as 0.40% by National Grid, the age of 120 yrs would suggest the knee point age, at which a remarkable number of transformer failures would start to occur. This is when proactive actions need to be carried out. However, the value of critical hazard, the knee point age and the replacement decision for transformers may be different amongst utilities. This is essentially influenced by the number of transformer failures that can be tolerated on a specific network.

8.2 Further Work

The thesis described the analysis of UK National Grid transformer failures and presented the general modelling of transformer population end-of-life. According to the model, the random failure hazard is determined by statistics, the thermal hazard is obtained via the insulation paper ageing model, and the population actual hazard is produced as the superposition of the above two hazards. The restrictions of statistical approaches and the complexities of individual thermal lifetime simulation however lead to the errors when finally developing the actual failure hazard of the transformer population. Further work needs to be carried out to ultimately determine the National Grid transformer population failure trend.

This generic model of transformer population end-of-life could be improved by: - firstly reassessing the transformer failure statistically when more lifetime data are available, secondly verifying the random hazard rate at early ages, thirdly more accurately estimating the thermal lifetimes of individual transformers, and finally by considering the effects from other ageing failure mechanisms, other than thermal ageing.

- **Reassessment of Transformer Lifetime Data via Statistical Approaches**

In order to improve the transformer end-of-life model, electric utilities, i.e. UK National Grid, would be expected to allow more transformers to fail and risk operating transformers at an older age. Once more failures are provided, especially when a “knee point” is observed, indicating the onset of the increasing-hazard period, the statistical analyses on lifetime data as presented in Chapter 3, could be conducted again. The results would be more meaningful compared to those based on inadequate lifetime data.

However, it could be argued by asset managers that an increased number of transformer failures and the potentially higher risk of operating old transformers would threaten the network reliability, which in fact departs from the objective of this study. The dilemma caused for the statistical analysis should be discussed, related to the utility’s operation policy.

Moreover, in terms of mathematical definition on “adequate lifetime data”, a method related to the binomial distribution could be used. For example, in order to examine the 86 exposed transformers as an inadequate number at age 48, the binomial distribution could be used to calculate the probability of observing one failure out of 86 exposed transformers at that age. A low value of the probability would suggest the limited number of exposed transformers at that age.

- **Verification of Population Random Hazard Rate**

If more failure data could be provided, the transformer population random failure hazard would be derived again based on the larger samples.

However, when dealing with the National Grid transformers' limited lifetime data, it had been conservatively assumed that the average hazard rate of 0.20% up to age 36 indicates the transformer population random failure hazard and this constant hazard would predominate until the onset of population ageing failure. The age 36 was selected as it is the age corresponding to the "early onset of transformer population significant unreliability" in National Grid technical guidance. Intensive mathematical discussions need to be undertaken to examine the accuracy of this specific age 36 and the random failure hazard rate 0.20%:-

i. how long will the random failure period or transformer normal operating stage last, and
ii. to what extent, this 0.20% would be regarded as sufficiently low and stable, and can be used as the random hazard rate of National Grid power transformers.

A Bayesian analysis could be implemented to examine the reliability of the average hazard 0.20%. The conditional probability of observing the hazard rate of 0.20%, by given the number of failures at each age, could be calculated. A large value of the conditional probability at a certain age would indicate a high reliability of the hazard rate 0.20% at that particular age.

In a previous study, the frequency of electric system transient events was assigned constant by assuming the system could be well maintained. Further studies on the increase of transient event frequency caused by the network layout change and the effects from the increased frequency of the transient events, would help in deriving a more accurate random hazard rate. Besides, transformer random failures related to the impact of global/regional climate change are expected to be of greater concern. For example, transformer random hazard rate might increase due to more frequent lightning or/and stronger wind etc.

Furthermore, failures of transformer subsystems, i.e. cooling system, bushing, on-load tap-changer (OLTC), oil preservation and expansion system, protection and monitoring system are related to the randomly-occurred system transient events. These subsystems should also be taken into account in order to build up an integrated model of transformer random failure.

- **Reduction of Uncertainties on Individual Thermal lifetime and Population Thermal Hazard Curve**

The major incentive for the simplified approach to transformer thermal lifetime prediction was to avoid the problem caused by incomplete information about the input variables. For example, the temperature-rise test results might not have been fully recorded in the test report, the instantaneous loading data might not be provided, the transformer winding temperature hot-spot factor (*HSF*) was only empirically suggested, and/or the ambient temperature was difficult to obtain. The individual transformer thermal lifetime could be more accurately predicted if the transformer's loading history, ambient temperature records and the temperature-rise test results were all available. Meanwhile, the studies on the effects of transient load variation and the real-time ambient temperature variation to the transformer thermal lifetime, the correlation between transformer load variation and real-time temperature, and the transformer winding temperature-rise modelling would also cope with the problem of incomplete input data.

In particular, for a better understanding of transformer temperature-rise and especially for obtaining the exact value of transformer hot-spot factor (*HSF*), more transformers would be scrapped and their operation histories need to be provided, because the small size of scrapped transformers up to the present does not imply effective information related to transformer ageing and they are not adequate samples to suggest the ageing trend of the in-service population. Although the large scale of replacement would cause intensive capital expenditure, more information from failed/replaced transformers however, would be essential to reduce the uncertainties in dealing with individual transformer lifetime estimation.

Furthermore, the mechanical response of fans and pumps is reported to be severely delayed in practice in the ONAN and OFAF cooling mode switching, which may essentially result in the overheating of transformer winding. Transformer thermal lifetime would be more accurately predicted if the delayed time was considered in the thermal model.

As mentioned in Chapter 1, the long-term monitoring of transformer defects and the tracking of operation condition would also contribute a better understanding of transformer ageing and thus is another solution to cope with the problem of limited input data in transformer lifetime simulation. A mathematical model on transformer condition propagation and a combination of monitoring data and thermal lifetime are worth developing. The study on transformer condition long-term monitoring would also improve the mathematical modelling of the transformer ageing rate factor, which essentially varies related to the development of the oxygen, water and acidity content dissolved in the insulation paper.

- **Consideration of other Ageing Failure Mechanisms**

It was clearly stated in this thesis that transformer ageing-related failure is indicated by transformer main winding insulation paper mechanical strength deterioration or briefly transformer paper thermal ageing, because the mathematical model of describing mechanical strength reduction is only well understood. It is therefore worthy in the future to consider other failure modes related to ageing failure mechanism, for example transformer electromagnetic ability degradation, current carrying ability degradation and dielectric property deterioration.

Hence, furthermore, the ideal of “transformer functional life” would be developed, determined by the effects from all the failure modes related to transformer ageing failure mechanism.

Ultimately the successful outcome of the research will identify at any specific ages the National Grid transformer failure hazard with a clearly defined error bar, on which the asset manager’s transformer replacement decision relies.

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Appendix-I

Commonly Used Curve Fitting Techniques

To estimate the values of parameters under a presumed distribution model, the least square estimator (LSE), the maximum likelihood estimator (MLE) and the Bayesian method are commonly used. In this appendix these techniques are explained by fitting product lifetime data into a 2-parameter Weibull distribution.

A. Least Square Estimator (LSE)

A least square estimator (LSE) is used to determine the parameters of a presumed distribution model that best fits the lifetime data. The principle is to minimise the sum of squares of the vertical distances from the observed points to the fitted regression line; the values of parameters corresponding to that minimum verify the most suitable model of fitting.

LSE is always associated with the graphic plotting approaches of lifetime data curve fitting, in which the cumulative distribution function (CDF) $F(t)$ or the cumulative hazard function $H(t)$ is linearly related to the expression of age t as

$$ln t = \frac{1}{\beta} ln\left\{-ln\left[1-F(t)\right]\right\} + ln\alpha \quad \text{(I-1)}$$

or

$$ln(t) = \frac{1}{\beta} \bullet ln\left[H(t)\right] + ln\alpha \quad \text{(I-2)}$$

under the 2-parameter Weibull distribution for instance, where typically α and β are parameters. According to (I-1) or (I-2), the analysis is simplified as to mathematically determine a straight line best fit to the data points.

Supposing there are M failed products with the age of failure of t_1, t_2, t_3, …, t_M respectively, the fitting process of LSE is illustrated by fitting this group of complete data under the presumed 2-parameter Weibull distribution.

As indicated in (I-1), $ln t$ is linear related t $ln\left\{-ln\left[1-F(t)\right]\right\}$. Let $Y = ln t$, $X = ln\left\{-ln\left[1-F(t)\right]\right\}$, $A = \frac{1}{\beta}$ and $B = ln\alpha$, (I-1) can be substituted as

$$Y = AX + B \quad \text{(I-3)}$$

in which again Y is linear related to X.

Calculate Y_i and X_i at each observed failure t_i (i=1, 2, 3, …, M) and plot the corresponding (X_1, Y_1), (X_2, Y_2), …, (X_M, Y_M) in linear scale as shown in the red dots in Figure I-1. The blue straight line $Y = AX + B$ indicates the best fit.

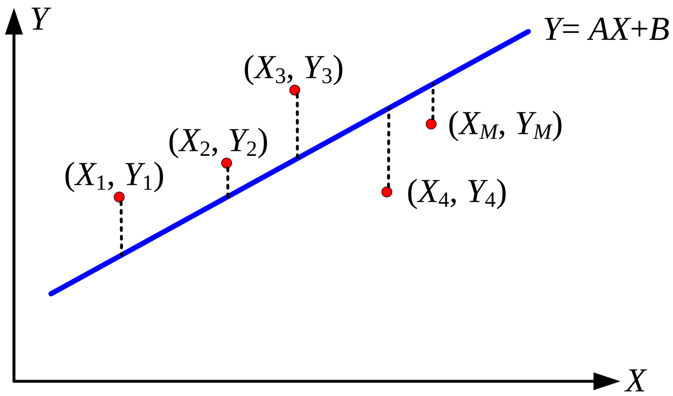

Figure I-1 Least Square Estimator Illustration

In order to determine the proper values of A and B, the distance between each point and the straight line, also called residual of each point is calculated as

$$\begin{aligned} d_1 &= Y_1 - (AX_1 + B) \\ d_2 &= Y_2 - (AX_2 + B) \\ &\cdots \\ d_M &= Y_M - (AX_M + B) \end{aligned} \tag{I-4}$$

and the sum of squares of the residuals S is further obtained as

$$S = \sum_{i=1}^{M} (d_i)^2 \tag{I-5}$$

S is then minimized by letting $\frac{\partial S}{\partial A} = 0$ and $\frac{\partial S}{\partial B} = 0$. The corresponding A and B denoted as $\hat{A}$ and $\hat{B}$ respectively, indicate the best fitting. Therefore parameter α and β of the best fit, denoted as $\hat{\alpha}$ and $\hat{\beta}$, are obtained as

$$\hat{\alpha} = exp(\hat{B}) \tag{I-6}$$

and

$$\hat{\beta} = \frac{1}{\hat{A}} \tag{I-7}$$

In order to evaluate the goodness of LSE fitting, the coefficient of determination (COD) is always calculated as the ratio of the "regression sum of squares" SS_{reg} and the "total sum of squares" TSS, as

$$\mathrm{COD} = \frac{SS_{reg}}{TSS} = \frac{\sum_{i=1}^{M}\left(\hat{Y}_i - \bar{Y}\right)^2}{\sum_{i=1}^{M}\left(Y_i - \hat{Y}_i\right)^2} \tag{I-8}$$

where $Y_i, i = 1,2,3,\ldots,M$ is the plotted point, $\bar{Y}$ is the mean value of plotted points and $\hat{Y}_i, i = 1,2,3,\ldots,M$ is the fitted point. In particular, $\sum_{i=1}^{M}\left(\hat{Y}_i - \bar{Y}\right)^2$ calculates the portion that is explained by the fitted model and $\sum_{i=1}^{M}\left(Y_i - \hat{Y}_i\right)^2$ considers the total variation between data points and their fitted values. COD thereby presents the ratio of dispersion that is illustrated by the fitted model; value closer to 1 indicates a better fit.

The hazard function under this 2-parameter Weibull model is further determined as

$$h(t) = \left(\frac{\hat{\beta}}{\hat{\alpha}}\right)\left(\frac{t}{\hat{\alpha}}\right)^{\hat{\beta}-1} \tag{I-9}$$

According to the above processes, it can be seen that LSE is straightforward to apply; however it is mathematically improper when the failure samples are large or the data is heavily censored.

B. Maximum Likelihood Estimator (MLE)

The fundamental idea behind maximum likelihood estimator (MLE) is based on the presumed distribution model and observed data, to estimate the most likely values of parameters by maximizing the likelihood function.

The likelihood function is structured according to the principle that for the independent observed complete failure data, the likelihood of obtaining such failures is proportional to the probability density function (PDF) of each failure; and for the multiply censored data, the likelihood is proportional to the PDF of each failed unit and the reliability function of each survival.

- **Complete Data**

Taking 2-parameter Weibull distribution for example, to analyze M products fail at t_1, t_2, t_3, … , t_M by MLE, the likelihood function for this complete data is calculated as

$$L = \prod_{i=1}^{M} f(t_i) \tag{I-10}$$

where $f(t_i)$ (i=1, 2, 3, …, M) is the probability density function (PDF) at each failure under Weibull distribution assumption. Particularly a specific $f(t_i)$ can be rewritten as

$$f(t_i) = \frac{1}{\alpha}\left(\frac{t_i}{\alpha}\right)^{\beta-1} exp\left[-\left(\frac{t_i}{\alpha}\right)^{\beta}\right] \tag{I-11}$$

Logarithm of the likelihood function is always applied instead of (I-10) in practice since the logarithm of (I-10) simplifies the product into superposition. The natural logarithm of (I-10) is expressed as

$$ln\,L = \sum_{i=1}^{M} ln\left[f(t_i)\right] \tag{I-12}$$

- **For Incomplete Data**

If the sample size is M+C in which M products are failed units and C are survival products. The likelihood function should be modified by considering the reliability of those survivals, as

$$L = \prod_{i=1}^{M} f(t_i) \times \prod_{i=1}^{C} R(t'_i) \tag{I-13}$$

where $R(t'_i)$ (i=1, 2, 3, …, C) are the reliability function of those censored data and t'_i indicates the service life of the i^{th} censored product. Again under Weibull distribution $R(t'_i)$ is expressed as

$$R(t'_i) = exp\left[-\left(\frac{t'_i}{\alpha}\right)^{\beta}\right] \tag{I-14}$$

The natural logarithm of likelihood function (I-13) is written as

$$ln\,L = \sum_{i=1}^{M} ln\left[f(t_i)\right] + \sum_{i=1}^{C} ln\left[R(t'_i)\right] \tag{I-15}$$

One can obtain the values of α and β (denoted by $\hat{\alpha}$ and $\hat{\beta}$) by optimizing the likelihood function (I-13) or the log likelihood function (I-15) via letting $\frac{\partial L}{\partial \alpha} = 0$ and $\frac{\partial L}{\partial \beta} = 0$, or

$\frac{\partial ln(L)}{\partial \alpha}=0$ and $\frac{\partial ln(L)}{\partial \beta}=0$. For large sample size the parameters are iteratively solved by numerical techniques.

$\hat{\alpha}$ and $\hat{\beta}$ are designated as the maximum likelihood estimates of Weibull parameters. Particularly for time censored data with large failures involved, the asymptotic distribution of ($\hat{\alpha}$, $\hat{\beta}$) is proved close to a normal distribution, with the mean value equivalent to the true value (α_0 , β_0) under the presumed Weibull distribution. One of the attractive properties of MLE is that it not only provides the most probable parameter values under the presumed distribution model but also estimates the asymptotic confidence limits of parameters and the confidence limit for the corresponding reliability function.

In order to obtain the asymptotic confidence limits of Weibull parameters α and β, the **Fisher Information Matrix** is defined as the negative second partial derivatives of (I-10) for complete data or (I-13) for incomplete data, as

$$\mathbf{F}=\begin{bmatrix} -\frac{\partial^2 L}{\partial \alpha^2} & -\frac{\partial^2 L}{\partial \alpha \partial \beta} \\ -\frac{\partial^2 L}{\partial \beta \partial \alpha} & -\frac{\partial^2 L}{\partial \beta^2} \end{bmatrix} \tag{I-16}$$

Substituting α and β by $\hat{\alpha}$ and $\hat{\beta}$, (I-16) is called the **Local Fisher Information Matrix**, denoted by $\hat{\mathbf{F}}$. The maximum likelihood asymptotic covariance matrix $\mathbf{var}\left(\hat{\alpha},\hat{\beta}\right)$ is then developed as

$$\mathbf{var}\left(\hat{\alpha},\hat{\beta}\right)=\begin{bmatrix} var(\hat{\alpha}) & cov\left(\hat{\alpha},\hat{\beta}\right) \\ cov\left(\hat{\alpha},\hat{\beta}\right) & var\left(\hat{\beta}\right) \end{bmatrix}=\hat{\mathbf{F}}^{-1} \tag{I-17}$$

where $var(\hat{\alpha})$ and $var\left(\hat{\beta}\right)$ respectively indicate the maximum likelihood estimated variance of determined parameter $\hat{\alpha}$ and $\hat{\beta}$, and $cov\left(\hat{\alpha},\hat{\beta}\right)$ estimates the covariance of $\hat{\alpha}$ and $\hat{\beta}$. The square root of (I-17) provides the asymptotic standard deviation of the maximum likelihood estimate of Weibull parameters.

Based on the general assumption that ($\hat{\alpha}$, $\hat{\beta}$) is normal distributed, the two-sided approximation of parameters ($\hat{\alpha}$, $\hat{\beta}$) with *p*% confidence limit are given as

$$
\begin{aligned}
\underset{\sim}{\alpha} &= \frac{\hat{\alpha}}{exp\left\{\dfrac{K_p \bullet \sqrt{var(\hat{\alpha})}}{\hat{\alpha}}\right\}}, \qquad \tilde{\alpha} = \hat{\alpha} \bullet exp\left\{\frac{K_p \bullet \sqrt{var(\hat{\alpha})}}{\hat{\alpha}}\right\} \\
\underset{\sim}{\beta} &= \frac{\hat{\beta}}{exp\left\{\dfrac{K_p \bullet \sqrt{var(\hat{\beta})}}{\hat{\beta}}\right\}}, \qquad \tilde{\beta} = \hat{\beta} \bullet exp\left\{\frac{K_p \bullet \sqrt{var(\hat{\beta})}}{\hat{\beta}}\right\}
\end{aligned}
\tag{I-18}
$$

in which $\underset{\sim}{\alpha}$, $\underset{\sim}{\beta}$ are the lower limit of $\hat{\alpha}$ and $\hat{\beta}$, and $\tilde{\alpha}$, $\tilde{\beta}$ are their upper limits, K_p is the standard normal distribution percentile that indicates the age by which $\frac{100+p}{2}\%$ of the population has failed and *p* is the significance level.

When the sample size is sufficiently large, parameters obtained by MLE are much close to their true values and moreover (I-18) estimates the confidence limits of parameter true values based on the normal distribution assumption. This idea is also presented by the asymptotic theory which identifies MLE is asymptotically efficient and typically for large sample size, MLE provides the most precise estimates for parameters. When dealing with incomplete data in addition, especially when the data are heavily censored, MLE produces better fit than LSE. However if the sample size is small, the results from MLE could be badly biased.

C. Bayesian Parameter Estimation

In the Bayesian estimation variable matrix, **t** indicates the observed lifetime data and **Θ** is the matrix of distribution parameters. Bayesian estimation concerns that by observing the historical failure censored data **t**, can more information be drawn to derive parameter **Θ**?

It is stated in Bayesian theory that the conditional probability $f(\mathbf{\Theta}|\mathbf{t})$ of random variable **Θ** by given the condition of *T*=**t** can be calculated as dividing the joint probability $f(\mathbf{t},\mathbf{\Theta})$ by the probability function $f(\mathbf{t})$, as

$$f(\mathbf{\Theta}|\mathbf{t}) = \frac{f(\mathbf{t},\mathbf{\Theta})}{f(\mathbf{t})} \tag{I-19}$$

in which **t** and **Θ** are both matrixes of random variables.

Besides the joint probability function $f(\mathbf{t},\mathbf{\Theta})$ can be expressed as

$$f(\mathbf{t},\mathbf{\Theta}) = f(\mathbf{t}|\mathbf{\Theta})\bullet f(\mathbf{\Theta}) \tag{I-20}$$

where again $f(\mathbf{t}|\mathbf{\Theta})$ is the conditional probability of variable **t** by given the value of parameters **Θ**, $f(\mathbf{\Theta})$ is the prior distribution which contains all the possible values of **Θ**. $f(\mathbf{\Theta})$ models the uncertainties of **Θ** before observing outcomes of lifetime **t**. By integrating (I-20) into (I-19), $f(\mathbf{\Theta}|\mathbf{t})$ is further expressed as

$$f(\mathbf{\Theta}|\mathbf{t}) = \frac{f(\mathbf{t}|\mathbf{\Theta})\bullet f(\mathbf{\Theta})}{f(\mathbf{t})} \tag{I-21}$$

where similarly $f(\mathbf{t})$ is called prior predictive distribution. $f(\mathbf{\Theta}|\mathbf{t})$ is thereafter called posterior distribution function.

In application practice the non-informative distribution is always assigned to the prior distribution $f(\mathbf{\Theta})$, such as $f(\mathbf{\Theta}) \equiv 1$. The information included in $f(\mathbf{\Theta})$ is thus very limited due to little prior knowledge provided.

The prior predictive distribution $f(\mathbf{t})$ can be derived as

$$f(\mathbf{t}) = \int_{\mathbf{\Theta}} f(\mathbf{t}|\mathbf{\Theta})\bullet f(\mathbf{\Theta})\, d\mathbf{\Theta} \tag{I-22}$$

$f(\mathbf{t})$ hereby contains the information of variable matrix **t** within all the possible values of **Θ**. (I-22) is often used for statistical models comparison purpose, and it is always assigned to be a constant in engineering practice. Hence the posterior distribution $f(\mathbf{\Theta}|\mathbf{t})$ can be deduced as proportional to the joint probability $f(\mathbf{t},\mathbf{\Theta})$ as

$$f(\mathbf{\Theta}|\mathbf{t}) \propto f(\mathbf{t}|\mathbf{\Theta})\bullet f(\mathbf{\Theta}) \tag{I-23}$$

By considering the constant value of $f(\mathbf{\Theta})$, (I-23) is further simplified as

$$f(\mathbf{\Theta}|\mathbf{t}) \propto f(\mathbf{t}|\mathbf{\Theta}) \tag{I-24}$$

According to (I-24) the posterior distribution $f(\mathbf{\Theta}|\mathbf{t})$ is essentially transferred to a likelihood function with unknown parameter matrix **Θ**. The further steps of solving the likelihood function are consequently the same as those presented in MLE analysis.

For example, in fitting product lifetime data under 2-paramter Weibull distribution via the Bayesian estimator, the posterior distribution function $f(\mathbf{\Theta}|\mathbf{t})$ for M failed products and C survival units is expressed as

$$f\left(\alpha,\beta\left|t_{i(i=1,2,3,\ldots,M)},t'_{i(i=1,2,3,\ldots,C)}\right.\right)\propto f\left(t_{i(i=1,2,3,\ldots,M)},t'_{i(i=1,2,3,\ldots,C)}\left|\alpha,\beta\right.\right) \tag{I-25}$$

in which α and β are Weibull parameters to be estimated, $t_{i(i=1,2,\ldots,M)}$ indicates the lifetimes of those failed units and $t'_{i(i=1,2,\ldots,C)}$ are the censoring time of those survived products.

Furthermore the right hand side of (I-25) can be extended to the product of PDF of failed units and the reliability function of the survivals, as shown in (I-13) in the likelihood function of MLE analysis,

$$f\left(\alpha,\beta\left|t_{i(i=1,2,3,\ldots,M)},t'_{i(i=1,2,3,\ldots,C)}\right.\right)\propto f\left(t_{i(i=1,2,3,\ldots,M)},t'_{i(i=1,2,3,\ldots,C)}\left|\alpha,\beta\right.\right)=\prod_{i=1}^{M}f(t_i)\times\prod_{i=1}^{C}R(t'_i) \tag{I-26}$$

in which again $f(t_i)$ (i=1, 2, 3, …, M) is the probability density function (PDF) at each failure and $R(t'_i)$ (i=1, 2, 3, …, C) are the reliability function of those censored data.

As elaborated in MLE, by computing the maximal value of the likelihood function $f\left(t_{i(i=1,2,3,\ldots,M)},t'_{i(i=1,2,3,\ldots,C)}\left|\alpha,\beta\right.\right)$ given the lifetime data t_i and t'_i, values of parameters α and β (denoted by $\hat{\alpha}$ and $\hat{\beta}$) are determined. This similar process reveals that Bayesian analysis and MLE are closely related. However it should be kept in mind that the simple proportional relation shown in (I-25) is derived based on the very general prior information included in the analysis. Product lifetime data analysis via Bayesian approach incorporates the observed data with the use's prior knowledge. Once the prior judgement is updated, a more precise prior distribution $f(\mathbf{\Theta})$ should be considered instead of using the previous non-informative prior distribution $f(\mathbf{\Theta})\equiv 1$.

Bayesian analysis can further provide the credit bound for single-parameter models and the credit region for multiple-parameter models. Unlike the parameter confidence limits obtained in MLE, which concern the confidence intervals for a constant value of determined parameters, the parameter credit bound/region in Bayesian analysis explains the likelihood with which the posterior distribution $f(\boldsymbol{\Theta}|\mathbf{t})$ locates within a pre-assigned interval. Alternatively it tells the credibility of the determined parameters within a certain bound. In consequence the credibility that parameter $\boldsymbol{\Theta}$ would be within the interval of ($\mathbf{R}_1$, $\mathbf{R}_2$) is derived as

$$p\left(\mathbf{R}_1 \le \boldsymbol{\Theta} \le \mathbf{R}_2\right) = \int_{\mathbf{R}_1}^{\mathbf{R}_2} f\left(\boldsymbol{\Theta} \middle| t_{i(i=1,2,3,...,M)}, t'_{i(i=1,2,3,...,C)}\right) d\boldsymbol{\Theta} \tag{I-27}$$

Meanwhile, for a certain significant level γ ($\gamma \in [0,1]$), the credit bound ($\mathbf{R}_1$, $\mathbf{R}_2$) within which 100(1- γ)% probability that $\boldsymbol{\Theta}$ can be obtained is calculated as

$$\int_{\mathbf{R}_1}^{\mathbf{R}_2} f\left(\boldsymbol{\Theta} \middle| t_{i(i=1,2,3,...,M)}, t'_{i(i=1,2,3,...,C)}\right) d\boldsymbol{\Theta} = 1-\gamma \tag{I-28}$$

Moreover according to the equal tail interval assumption, credibility outside the bound ($\mathbf{R}_1$, $\mathbf{R}_2$), as (0, $\mathbf{R}_1$) or ($\mathbf{R}_2$, 1) is equal to $\frac{\gamma}{2}$. This is mathematically expressed as

$$\int_{0}^{\mathbf{R}_1} f\left(\boldsymbol{\Theta} \middle| t_{i(i=1,2,3,...,M)}, t'_{i(i=1,2,3,...,C)}\right) d\boldsymbol{\Theta} = \frac{\gamma}{2} = \int_{\mathbf{R}_2}^{1} f\left(\boldsymbol{\Theta} \middle| t_{i(i=1,2,3,...,M)}, t'_{i(i=1,2,3,...,C)}\right) d\boldsymbol{\Theta} \tag{I-29}$$

According to (I-28) and (I-29), once the value of significant level γ is assigned the credit bounds of the posterior distribution $f(\boldsymbol{\Theta}|\mathbf{t})$ can be determined.

According to the above calculation process, the Bayesian analysis yields the values of parameters equivalent to those derived by MLE; however they should be interpreted differently. The confidence limits of parameters used in MLE essentially tell the repeatability of the observation: if the user would like to resample data from the population, 95% of the confidence limits, for instance, would contain the true values of the parameters. This in fact verifies the large sample size required in the test as explained for MLE. By contraries in Bayesian analysis, the credible bound/region indicates the probability of 100(1- γ)% that the bound/region contains the true value(s) of parameter(s). The advantage in the Bayesian analysis is that the prior judgement of parameter $\boldsymbol{\Theta}$ is involved in the prior distribution $f(\boldsymbol{\Theta})$, such that the large size of samples is unnecessary.

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Appendix-II

Kolmogorov-Smirnov (K-S) Test

Because the distribution models are subjectively selected when fitting product lifetime data, a statistical test is always carried out to verify the goodness of fit. The Kolmogorov-Smirnov test or briefly K-S test is one of the most commonly used tests. The procedures of applying K-S test on complete data and right-time censored data are described separately in this appendix.

- **Complete Data:** M failed products, with failure ages from low to high: t_1, t_2, t_3, …, t_M.

Step 1: Order the observed CDF (denoted by $F^0(t_i), i=1,2,3,...,M$) according to failure ages from low to high t_1, t_2, t_3, …, t_M.

Step 2: Based on the hypothesized distribution model and curve fitting obtained parameters, calculate the theoretical CDF (denoted by $F_0(t_i), i=1,2,3,...,M$) corresponding to each of observe failure ages t_1, t_2, t_3, …, t_M.

Step 3: Compare the observed CDF $F^0(t_i)$ with the theoretical CDF $F_0(t_i)$ at each failure age t_1, t_2, t_3, …, t_M. The maximum absolute difference between $F^0(t_i)$ and $F_0(t_i)$ among M complete data, denoted as D_M, is expressed as

$$D_M = max\left|F^0(t_i) - F_0(t_i)\right|, \quad i = 1,2,3,...,M \qquad \text{(II-1)}$$

D_M is used to evaluate the goodness of the fitting under the hypothesized distribution model with the obtained parameters.

Step 4: Compare D_M with the critical value D_M^γ. If $D_M \le D_M^\gamma$, the hypothesized distribution is acceptable and thus the data are properly fitted; otherwise the hypothesized model should be rejected.

The criterion relies on the theory that under a proper distribution model, the observed failure CDF ($F^0(t_i), i=1,2,3,...,M$) at age t_i is asymptotically normally distributed with the mean value of the hypothesized model theoretical CDF ($F_0(t_i), i=1,2,3,...,M$) at age t_i.

According to this theory a limiting equation needs to be satisfied in order to accept a hypothesized distribution, which is

$$\lim_{M\to\infty} P\left(\sqrt{M}D_M \le y\right) = 1-\gamma \tag{II-2}$$

in which $P\left(\sqrt{M}D_M \le y\right)$ indicates the probability of the cumulative absolute differences ($\sqrt{M}D_M$) no more than a critical value y, γ is the significance level and also presents the probability that the hypothesized model would be rejected by the test.

Let $y_{1-\gamma}$ denote the value of a specific y under which (II-2) is satisfied corresponding to a certain significant level γ and the critical D_M^{γ} is defined as

$$D_M{}^{\gamma} = \frac{y_{1-\gamma}}{\sqrt{M}} \tag{II-3}$$

for a certain amount of failure data (M in this specific case).

Values of $y_{1-\gamma}$ are listed in Table II-1.

Table II-1 Values of $y_{1-\gamma}$ under Various Significant Level γ

γ	0.20	0.10	0.05	0.01
$y_{1-\gamma}$	1.07	1.22	1.36	1.63

Particularly for $M \le 50$, the values of D_M^{γ} are directly given as listed in Table II-2.

Table II-2 Values of D_M^{γ} under Significant Level γ and Sample Size M

M \ γ	0.20	0.10	0.05	0.01
5	0.45	0.51	0.56	0.67
10	0.32	0.37	0.41	0.49
15	0.27	0.30	0.34	0.40
20	0.23	0.26	0.29	0.36
25	0.21	0.24	0.27	0.32
30	0.19	0.22	0.24	0.29
35	0.18	0.20	0.23	0.27
40	0.17	0.19	0.21	0.25
45	0.16	0.18	0.20	0.24
50	0.15	0.17	0.19	0.23

- **Right-Time Censored Data:** M failed products, with failure ages from low to high: t_1, t_2, t_3, …, t_M; C censored products, with censoring ages from low to high: t'_1, t'_2, t'_3, …, t'_C. Let $r = \frac{M}{M+C}$, which hereby indicates the portion of failures out of the total number of products under analysis.

Step 1: Order the observed CDF (denoted by $F^0(t_i), i = 1,2,3,...,M$) at each age of failure according to failure ages from low to high t_1, t_2, t_3, …, t_M. The mathematical expression of $F^0(t_i)$ can be referred to Kaplan-Meier method, Herd-Johnson method or actuarial method as discussed in Chapter 3.

Step 2: The same as Step 2 for complete data. Based on the hypothesized distribution model and curve fitting obtained parameters, calculate the theoretical CDF (denoted by $F_0(t_i), i = 1,2,3,...,M$) at each of observe failure ages t_1, t_2, t_3, …, t_M.

Step 3: Compare the observed CDF $F^0(t_i)$ with the theoretical CDF $F_0(t_i)$. The maximum absolute difference between $F^0(t_i)$ and $F_0(t_i)$ among M failures, denoted as $D_M(r)$, is calculated as

$$D_M(r) = max\left|F^0(t_i) - F_0(t_i)\right|, \quad i = 1,2,3,...,M \ \ failure \tag{II-4}$$

in which, similarly to D_M for the complete data, $D_M(r)$ is the measurement of goodness of fit for the right-time censored data with failure portion r.

Step 4: Compare $D_M(r)$ with the critical value $D_M^{\gamma}(r)$. (II-2) in the complete data is modified by substituting D_M by $D_M(r)$ and D_M^{γ} by $D_M^{\gamma}(r)$ respectively, as

$$\lim_{M\to\infty} P\left[\sqrt{M} D_M(r) \le y(r)\right] = 1 - \gamma \tag{II-5}$$

Use $y_{1-\gamma}(r)$ to represent the value of $y(r)$ when (II-5) is satisfied under a certain significant level γ, the critical $D_M^{\gamma}(r)$ is defined as

$$D_M^{\gamma}(r) = \frac{y_{1-\gamma}(r)}{\sqrt{M}} \tag{II-6}$$

clearly according to which the value of $y_{1-\gamma}(r)$ varies depending on the variation of r and γ, and therefore cannot be suggested from Table II-1. Anna Colosi from University of

North Carolina at Chapel Hill, U.S. developed the table of critical values $y_{1-\gamma}(r)$ for censored data. Table II-3 shows a part of her results.

Table II-3 Values of $y_{1-\gamma}(r)$ under Significant Level γ and Failure Portion r for Censored Data

γ \ r	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.100	0.1953	0.2753	0.3360	0.3867	0.4308	0.4703	0.5062	0.5392	0.5699	0.5985
0.050	0.2233	0.3147	0.3839	0.4417	0.4920	0.5569	0.5577	0.6152	0.6500	0.6825
0.025	0.2488	0.3505	0.4276	0.4918	0.5477	0.5975	0.6428	0.6844	0.7230	0.7589
0.010	0.2796	0.3938	0.4803	0.5523	0.6149	0.6707	0.7214	0.7679	0.8110	0.8512
0.005	0.3011	0.4240	0.5171	0.5946	0.6619	0.7219	0.7764	0.8264	0.8726	0.9157
0.001	0.3466	0.4880	0.5950	0.6840	0.7613	0.8303	0.8927	0.9500	1.0029	1.0523
γ \ r	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	
0.100	0.8155	0.9597	1.0616	1.1334	1.1813	1.2094	1.2216	1.2238	1.2238	
0.050	0.9268	1.0868	1.1975	1.2731	1.3211	1.3471	1.3568	1.3581	1.3581	
0.025	1.0285	1.2024	1.3209	1.3997	1.4476	1.4717	1.4794	1.4802	1.4802	
0.010	1.1505	1.3419	1.4696	1.5520	1.5996	1.6214	1.6272	1.6276	1.6276	
0.005	1.2361	1.4394	1.5735	1.6582	1.7056	1.7258	1.7306	1.7308	1.7308	
0.001	1.4171	1.6456	1.7931	1.8828	1.9292	1.9464	1.9494	1.9495	1.9495	

The critical value of $D_M^{\gamma}(r)$ is thereafter calculated according to (II-6). Only if $D_N(r) \le D_N^{\gamma}(r)$ is satisfied, the hypothesized distribution is accepted as a proper model.

The beauty of K-S goodness-of-fit test is that it is a non-parametric test and it is distribution free: the test does not rely on the underlying cumulative distribution function being tested. The test is easily to implement which is another attractive feature for its widely application in engineering practice.

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Appendix-III

Tables for UK National Grid Power Transformer Lifetime Data Statistical Analysis

Table III-1 National Grid Transformer Lifetime Data Analysis Using Hazard Plotting Approach

age t	Censored/ Failed	Reversed Rank k_{rr}	Hazard $h(t)$	Cumulative hazard $H(t)$	age t	Censored/ Failed	Reversed Rank k_{rr}	Hazard $h(t)$	Cumulative hazard $H(t)$
1	0	865			39	0	432		
1	0	864			39	0	431		
1	0	863			39	0	430		
1	0	862			39	1	429	0.002331	0.080994
1	1	861	0.001161	0.001161	39	1	428	0.002336	0.083331
1	1	860	0.001163	0.002324	39	0	427		
1	0	859			39	0	426		
1	0	858			39	0	425		
1	0	857			39	0	424		
1	0	856			40	0	423		
2	0	855			40	0	422		
2	0	854			40	0	421		
2	0	853			40	0	420		
2	0	852			40	0	419		
2	0	851			40	0	418		
2	0	850			40	0	417		
2	0	849			40	0	416		
2	1	848	0.001179	0.003503	40	0	415		
2	0	847			40	0	414		
2	0	846			40	0	413		
2	0	845			40	0	412		
2	0	844			40	0	411		
2	0	843			40	0	410		
2	0	842			40	0	409		
3	0	841			40	0	408		
3	0	840			40	0	407		
3	0	839			40	0	406		
3	1	838	0.001193	0.004697	40	0	405		
3	0	837			40	0	404		
3	0	836			40	0	403		
4	0	835			40	0	402		
4	0	834			40	0	401		
4	0	833			40	0	400		
4	0	832			40	0	399		
4	0	831			41	0	398		
4	0	830			41	0	397		
4	0	829			41	0	396		
4	0	828			41	0	395		
4	0	827			41	0	394		
4	1	826	0.001211	0.005907	41	0	393		
4	1	825	0.001212	0.007120	41	0	392		
4	0	824			41	0	391		
4	0	823			41	0	390		
4	0	822			41	0	389		
4	0	821			41	0	388		
4	0	820			41	0	387		
4	0	819			41	0	386		
4	0	818			41	0	385		
4	0	817			41	0	384		
5	0	816			41	0	383		
5	0	815			41	0	382		
5	0	814			41	0	381		
5	0	813			41	0	380		

5	0	812			41	0	379		
5	0	811			41	0	378		
5	0	810			41	0	377		
5	0	809			41	0	376		
5	0	808			41	0	375		
5	0	807			41	0	374		
5	0	806			41	0	373		
5	0	805			41	0	372		
5	0	804			41	0	371		
5	0	803			41	0	370		
5	0	802			41	0	369		
5	0	801			41	0	368		
5	0	800			41	0	367		
5	0	799			41	0	366		
5	0	798			41	0	365		
5	0	797			41	0	364		
5	0	796			41	0	363		
5	0	795			41	0	362		
5	0	794			41	0	361		
6	0	793			41	0	360		
6	0	792			41	0	359		
6	0	791			41	0	358		
6	0	790			41	0	357		
6	0	789			41	0	356		
6	0	788			41	0	355		
6	1	787	0.001271	0.008390	41	0	354		
6	1	786	0.001272	0.009662	41	0	353		
6	0	785			41	0	352		
6	0	784			41	0	351		
6	0	783			41	0	350		
6	0	782			41	0	349		
6	0	781			41	0	348		
7	0	780			41	0	347		
7	0	779			41	0	346		
7	0	778			41	0	345		
7	0	777			41	0	344		
7	0	776			41	0	343		
7	0	775			41	0	342		
7	1	774	0.001292	0.010954	41	0	341		
7	0	773			42	0	340		
7	0	772			42	0	339		
7	0	771			42	0	338		
7	0	770			42	0	337		
7	0	769			42	0	336		
8	0	768			42	0	335		
8	0	767			42	0	334		
8	0	766			42	0	333		
8	1	765	0.001307	0.012262	42	0	332		
8	1	764	0.001309	0.013571	42	0	331		
8	0	763			42	0	330		
8	0	762			42	0	329		
9	0	761			42	0	328		
9	0	760			42	0	327		
9	0	759			42	0	326		
9	1	758	0.001319	0.014890	42	0	325		
9	0	757			42	0	324		
9	0	756			42	0	323		
9	0	755			42	0	322		
10	0	754			42	0	321		
10	0	753			42	0	320		
10	0	752			42	0	319		
10	0	751			42	0	318		
10	0	750			42	0	317		
10	0	749			42	0	316		
10	0	748			42	0	315		
10	0	747			42	0	314		
10	0	746			42	0	313		
10	1	745	0.001342	0.016232	42	0	312		
10	1	744	0.001344	0.017576	42	0	311		

10	1	743	0.001346	0.018922	42	0	310		
10	0	742			42	0	309		
10	0	741			42	0	308		
10	0	740			42	0	307		
10	0	739			42	0	306		
10	0	738			42	0	305		
10	0	737			42	0	304		
10	0	736			42	0	303		
10	0	735			42	0	302		
10	0	734			42	0	301		
11	0	733			42	0	300		
11	0	732			42	0	299		
11	0	731			42	0	298		
11	0	730			42	0	297		
11	0	729			42	0	296		
11	0	728			42	0	295		
11	0	727			42	0	294		
11	0	726			42	0	293		
11	0	725			42	0	292		
11	0	724			42	0	291		
11	0	723			42	0	290		
11	0	722			42	0	289		
11	0	721			42	0	288		
11	0	720			42	0	287		
11	0	719			42	0	286		
11	0	718			42	0	285		
12	0	717			42	0	284		
12	0	716			42	0	283		
12	0	715			42	0	282		
12	0	714			42	0	281		
12	0	713			42	0	280		
12	0	712			42	0	279		
12	0	711			42	0	278		
12	0	710			42	0	277		
12	0	709			43	0	276		
12	0	708			43	0	275		
13	0	707			43	0	274		
13	0	706			43	0	273		
13	0	705			43	0	272		
13	0	704			43	0	271		
13	1	703	0.001422	0.020345	43	0	270		
13	0	702			43	0	269		
13	0	701			43	0	268		
13	0	700			43	0	267		
14	0	699			43	0	266		
14	0	698			43	0	265		
14	0	697			43	0	264		
14	0	696			43	0	263		
14	1	695	0.001439	0.021783	43	0	262		
14	1	694	0.001441	0.023224	43	0	261		
14	0	693			43	0	260		
14	0	692			43	0	259		
14	0	691			43	0	258		
14	0	690			43	0	257		
15	0	689			43	0	256		
15	0	688			43	0	255		
15	0	687			43	0	254		
15	0	686			43	0	253		
15	0	685			43	0	252		
15	0	684			43	0	251		
15	0	683			43	0	250		
15	0	682			43	0	249		
15	0	681			43	0	248		
15	1	680	0.001471	0.024695	43	0	247		
15	1	679	0.001473	0.026168	43	0	246		
15	0	678			43	0	245		
15	0	677			43	0	244		
15	0	676			43	0	243		
15	0	675			43	0	242		

15	0	674			43	0	241		
15	0	673			43	0	240		
15	0	672			43	0	239		
15	0	671			43	0	238		
15	0	670			43	0	237		
16	0	669			43	0	236		
16	0	668			43	0	235		
16	0	667			43	0	234		
16	0	666			43	0	233		
16	0	665			43	0	232		
16	0	664			43	0	231		
16	0	663			43	0	230		
16	1	662	0.001511	0.027678	43	0	229		
16	0	661			43	0	228		
16	0	660			43	0	227		
16	0	659			43	0	226		
16	0	658			43	0	225		
16	0	657			43	0	224		
16	0	656			43	0	223		
16	0	655			43	0	222		
17	0	654			43	0	221		
17	0	653			44	0	220		
17	0	652			44	0	219		
17	0	651			44	0	218		
17	0	650			44	0	217		
17	0	649			44	0	216		
17	0	648			44	0	215		
17	0	647			44	0	214		
17	0	646			44	0	213		
17	0	645			44	0	212		
17	0	644			44	0	211		
17	0	643			44	0	210		
17	0	642			44	0	209		
17	0	641			44	0	208		
17	0	640			44	0	207		
17	0	639			44	0	206		
17	0	638			44	0	205		
17	0	637			44	0	204		
17	0	636			44	0	203		
17	0	635			44	0	202		
17	0	634			44	0	201		
18	0	633			44	0	200		
18	0	632			44	0	199		
18	0	631			44	0	198		
18	0	630			44	0	197		
18	0	629			44	0	196		
18	0	628			44	0	195		
18	0	627			44	0	194		
18	0	626			44	0	193		
18	0	625			44	0	192		
18	0	624			44	0	191		
18	0	623			44	0	190		
19	0	622			44	0	189		
19	0	621			44	0	188		
19	0	620			44	0	187		
19	0	619			44	0	186		
19	0	618			44	0	185		
19	0	617			44	0	184		
19	0	616			44	0	183		
19	0	615			44	0	182		
19	1	614	0.001629	0.029307	44	0	181		
19	1	613	0.001631	0.030938	44	0	180		
19	0	612			44	0	179		
19	0	611			44	0	178		
19	0	610			44	0	177		
19	0	609			44	0	176		
19	0	608			44	0	175		
19	0	607			44	0	174		
19	0	606			44	0	173		

20	0	605			44	0	172		
20	0	604			44	0	171		
20	0	603			44	0	170		
20	1	602	0.001661	0.032599	44	0	169		
20	0	601			44	0	168		
20	0	600			45	0	167		
21	0	599			45	0	166		
21	0	598			45	0	165		
21	1	597	0.001675	0.034274	45	0	164		
21	0	596			45	0	163		
22	0	595			45	0	162		
22	0	594			45	0	161		
22	1	593	0.001686	0.035961	45	0	160		
22	1	592	0.001689	0.037650	45	0	159		
22	1	591	0.001692	0.039342	45	0	158		
22	0	590			45	0	157		
23	0	589			45	0	156		
23	0	588			45	0	155		
23	0	587			45	0	154		
23	0	586			45	0	153		
23	0	585			45	0	152		
23	1	584	0.001712	0.041054	45	0	151		
23	0	583			45	0	150		
23	0	582			45	0	149		
23	0	581			45	0	148		
23	0	580			45	0	147		
24	0	579			45	0	146		
24	0	578			45	0	145		
24	0	577			45	0	144		
24	1	576	0.001736	0.042790	45	0	143		
24	1	575	0.001739	0.044530	45	0	142		
24	0	574			45	0	141		
24	0	573			45	0	140		
25	0	572			45	0	139		
25	1	571	0.001751	0.046281	45	0	138		
25	0	570			45	0	137		
26	0	569			45	0	136		
26	0	568			45	0	135		
26	1	567	0.001764	0.048045	45	0	134		
26	1	566	0.001767	0.049811	45	0	133		
26	0	565			45	0	132		
27	0	564			45	0	131		
27	0	563			45	0	130		
27	1	562	0.001779	0.051591	46	0	129		
27	1	561	0.001783	0.053373	46	0	128		
27	1	560	0.001786	0.055159	46	0	127		
27	0	559			46	0	126		
28	0	558			46	0	125		
28	1	557	0.001795	0.056954	46	0	124		
28	1	556	0.001799	0.058753	46	0	123		
29	0	555			46	0	122		
29	0	554			46	0	121		
29	0	553			46	0	120		
29	1	552	0.001812	0.060564	46	0	119		
29	1	551	0.001815	0.062379	46	0	118		
29	0	550			46	0	117		
29	0	549			46	0	116		
29	0	548			46	0	115		
30	0	547			46	0	114		
30	0	546			46	0	113		
30	0	545			46	0	112		
30	0	544			46	0	111		
30	1	543	0.001842	0.064221	47	0	110		
30	0	542			47	0	109		
30	0	541			47	0	108		
30	0	540			47	0	107		
30	0	539			47	0	106		
31	0	538			47	0	105		
31	0	537			47	0	104		

31	0	536			47	0	103		
31	0	535			47	0	102		
31	0	534			47	0	101		
31	1	533	0.001876	0.066097	47	0	100		
31	0	532			47	0	99		
31	0	531			47	0	98		
31	0	530			47	0	97		
31	0	529			47	0	96		
31	0	528			47	0	95		
32	0	527			47	0	94		
32	0	526			47	0	93		
32	0	525			47	0	92		
32	0	524			47	0	91		
32	0	523			47	0	90		
32	0	522			47	0	89		
32	0	521			47	0	88		
32	0	520			47	0	87		
32	0	519			48	0	86		
32	0	518			48	0	85		
32	0	517			48	0	84		
32	0	516			48	0	83		
32	0	515			48	0	82		
32	0	514			48	0	81		
32	0	513			48	0	80		
32	0	512			48	0	79		
32	0	511			48	0	78		
33	0	510			48	0	77		
33	0	509			48	0	76		
33	0	508			48	1	75	0.013333	0.096664
33	0	507			48	0	74		
33	0	506			48	0	73		
33	0	505			48	0	72		
33	0	504			48	0	71		
33	0	503			48	0	70		
33	0	502			48	0	69		
33	0	501			48	0	68		
33	0	500			48	0	67		
33	0	499			48	0	66		
33	0	498			48	0	65		
34	0	497			48	0	64		
34	0	496			49	0	63		
34	0	495			49	0	62		
34	0	494			49	0	61		
34	0	493			49	0	60		
34	1	492	0.002033	0.068130	49	0	59		
34	1	491	0.002037	0.070166	49	0	58		
34	0	490			49	0	57		
34	0	489			49	0	56		
34	0	488			49	0	55		
34	0	487			49	0	54		
35	0	486			49	0	53		
35	0	485			49	0	52		
35	0	484			49	0	51		
35	0	483			49	0	50		
35	1	482	0.002075	0.072241	49	0	49		
35	1	481	0.002079	0.074320	49	0	48		
35	0	480			50	0	47		
35	0	479			50	0	46		
35	0	478			50	0	45		
35	0	477			50	0	44		
36	0	476			50	0	43		
36	0	475			50	0	42		
36	0	474			50	0	41		
36	0	473			50	0	40		
36	0	472			50	0	39		
36	0	471			50	0	38		
36	0	470			50	0	37		
36	0	469			50	0	36		
36	0	468			51	0	35		

37	0	467			51	0	34		
37	0	466			51	0	33		
37	0	465			51	0	32		
37	0	464			51	0	31		
37	0	463			51	0	30		
37	0	462			51	0	29		
37	1	461	0.002169	0.076489	51	0	28		
37	1	460	0.002174	0.078663	51	0	27		
37	0	459			52	0	26		
37	0	458			52	0	25		
37	0	457			52	0	24		
37	0	456			52	0	23		
37	0	455			52	0	22		
37	0	454			52	0	21		
38	0	453			52	0	20		
38	0	452			53	0	19		
38	0	451			53	0	18		
38	0	450			53	0	17		
38	0	449			53	0	16		
38	0	448			53	0	15		
38	0	447			53	0	14		
38	0	446			54	0	13		
38	0	445			54	0	12		
38	0	444			54	0	11		
38	0	443			54	0	10		
38	0	442			54	0	9		
38	0	441			55	0	8		
38	0	440			55	0	7		
38	0	439			55	0	6		
38	0	438			55	0	5		
38	0	437			55	0	4		
38	0	436			55	0	3		
38	0	435			55	0	2		
39	0	434			57	0	1		
39	0	433							

Table III-2 National Grid Transformer Lifetime Data Analysis Using Probability Plotting involving Kaplan-Meier Method

age t	Censored/ Failed	Reversed Rank k_{rr}	Reliability $R(i)=\frac{k_{rr}(i)-1}{k_{rr}(i)}\bullet R(i-1)$			Cumulative Failure Probability $F(i)$=1-$R(i)$
			Conditional $\frac{k_{rr}(i)-1}{k_{rr}(i)}$	Previous $R(i$-1)	Current $R(i)$	
1	0	865			1	
1	0	864				
1	0	863				
1	0	862				
1	1	861	0.998839	1	0.998839	0.001161
1	1	860	0.998837	0.998839	0.997677	0.002323
1	0	859				
1	0	858				
1	0	857				
1	0	856				
2	0	855				
2	0	854				
2	0	853				
2	0	852				
2	0	851				
2	0	850				
2	0	849				
2	1	848	0.998821	0.997677	0.996501	0.003499
2	0	847				
2	0	846				
2	0	845				
2	0	844				
2	0	843				
2	0	842				
3	0	841				
3	0	840				
3	0	839				
3	1	838	0.998807	0.996501	0.995311	0.004689
3	0	837				
3	0	836				
4	0	835				
4	0	834				
4	0	833				
4	0	832				
4	0	831				
4	0	830				
4	0	829				
4	0	828				
4	0	827				
4	1	826	0.998789	0.995311	0.994106	0.005894
4	1	825	0.998788	0.994106	0.992902	0.007098
4	0	824				
4	0	823				
4	0	822				
4	0	821				
4	0	820				
4	0	819				
4	0	818				
4	0	817				
5	0	816				
5	0	815				
5	0	814				
5	0	813				
5	0	812				
5	0	811				
5	0	810				

5	0	809				
5	0	808				
5	0	807				
5	0	806				
5	0	805				
5	0	804				
5	0	803				
5	0	802				
5	0	801				
5	0	800				
5	0	799				
5	0	798				
5	0	797				
5	0	796				
5	0	795				
5	0	794				
6	0	793				
6	0	792				
6	0	791				
6	0	790				
6	0	789				
6	0	788				
6	1	787	0.998729	0.992902	0.991640	0.008360
6	1	786	0.998728	0.991640	0.990378	0.009622
6	0	785				
6	0	784				
6	0	783				
6	0	782				
6	0	781				
7	0	780				
7	0	779				
7	0	778				
7	0	777				
7	0	776				
7	0	775				
7	1	774	0.998708	0.990378	0.989099	0.010901
7	0	773				
7	0	772				
7	0	771				
7	0	770				
7	0	769				
8	0	768				
8	0	767				
8	0	766				
8	1	765	0.998693	0.989099	0.987806	0.012194
8	1	764	0.998691	0.987806	0.986513	0.013487
8	0	763				
8	0	762				
9	0	761				
9	0	760				
9	0	759				
9	1	758	0.998681	0.986513	0.985211	0.014789
9	0	757				
9	0	756				
9	0	755				
10	0	754				
10	0	753				
10	0	752				
10	0	751				
10	0	750				
10	0	749				
10	0	748				
10	0	747				
10	0	746				
10	1	745	0.998658	0.985211	0.983889	0.016111
10	1	744	0.998656	0.983889	0.982566	0.017434
10	1	743	0.998654	0.982566	0.981244	0.018756
10	0	742				
10	0	741				

10	0	740				
10	0	739				
10	0	738				
10	0	737				
10	0	736				
10	0	735				
10	0	734				
11	0	733				
11	0	732				
11	0	731				
11	0	730				
11	0	729				
11	0	728				
11	0	727				
11	0	726				
11	0	725				
11	0	724				
11	0	723				
11	0	722				
11	0	721				
11	0	720				
11	0	719				
11	0	718				
12	0	717				
12	0	716				
12	0	715				
12	0	714				
12	0	713				
12	0	712				
12	0	711				
12	0	710				
12	0	709				
12	0	708				
13	0	707				
13	0	706				
13	0	705				
13	0	704				
13	1	703	0.998578	0.981244	0.979848	0.020152
13	0	702				
13	0	701				
13	0	700				
14	0	699				
14	0	698				
14	0	697				
14	0	696				
14	1	695	0.998561	0.979848	0.978438	0.021562
14	1	694	0.998559	0.978438	0.977029	0.022971
14	0	693				
14	0	692				
14	0	691				
14	0	690				
15	0	689				
15	0	688				
15	0	687				
15	0	686				
15	0	685				
15	0	684				
15	0	683				
15	0	682				
15	0	681				
15	1	680	0.998529	0.977029	0.975592	0.024408
15	1	679	0.998527	0.975592	0.974155	0.025845
15	0	678				
15	0	677				
15	0	676				
15	0	675				
15	0	674				
15	0	673				
15	0	672				

15	0	671				
15	0	670				
16	0	669				
16	0	668				
16	0	667				
16	0	666				
16	0	665				
16	0	664				
16	0	663				
16	1	662	0.998489	0.974155	0.972683	0.027317
16	0	661				
16	0	660				
16	0	659				
16	0	658				
16	0	657				
16	0	656				
16	0	655				
17	0	654				
17	0	653				
17	0	652				
17	0	651				
17	0	650				
17	0	649				
17	0	648				
17	0	647				
17	0	646				
17	0	645				
17	0	644				
17	0	643				
17	0	642				
17	0	641				
17	0	640				
17	0	639				
17	0	638				
17	0	637				
17	0	636				
17	0	635				
17	0	634				
18	0	633				
18	0	632				
18	0	631				
18	0	630				
18	0	629				
18	0	628				
18	0	627				
18	0	626				
18	0	625				
18	0	624				
18	0	623				
19	0	622				
19	0	621				
19	0	620				
19	0	619				
19	0	618				
19	0	617				
19	0	616				
19	0	615				
19	1	614	0.998371	0.972683	0.971099	0.028901
19	1	613	0.998369	0.971099	0.969515	0.030485
19	0	612				
19	0	611				
19	0	610				
19	0	609				
19	0	608				
19	0	607				
19	0	606				
20	0	605				
20	0	604				
20	0	603				

20	1	602	0.998339	0.969515	0.967905	0.032095
20	0	601				
20	0	600				
21	0	599				
21	0	598				
21	1	597	0.998325	0.967905	0.966283	0.033717
21	0	596				
22	0	595				
22	0	594				
22	1	593	0.998314	0.966283	0.964654	0.035346
22	1	592	0.998311	0.964654	0.963024	0.036976
22	1	591	0.998308	0.963024	0.961395	0.038605
22	0	590				
23	0	589				
23	0	588				
23	0	587				
23	0	586				
23	0	585				
23	1	584	0.998288	0.961395	0.959749	0.040251
23	0	583				
23	0	582				
23	0	581				
23	0	580				
24	0	579				
24	0	578				
24	0	577				
24	1	576	0.998264	0.959749	0.958082	0.041918
24	1	575	0.998261	0.958082	0.956416	0.043584
24	0	574				
24	0	573				
25	0	572				
25	1	571	0.998249	0.956416	0.954741	0.045259
25	0	570				
26	0	569				
26	0	568				
26	1	567	0.998236	0.954741	0.953057	0.046943
26	1	566	0.998233	0.953057	0.951373	0.048627
26	0	565				
27	0	564				
27	0	563				
27	1	562	0.998221	0.951373	0.949681	0.050319
27	1	561	0.998217	0.949681	0.947988	0.052012
27	1	560	0.998214	0.947988	0.946295	0.053705
27	0	559				
28	0	558				
28	1	557	0.998205	0.946295	0.944596	0.055404
28	1	556	0.998201	0.944596	0.942897	0.057103
29	0	555				
29	0	554				
29	0	553				
29	1	552	0.998188	0.942897	0.941189	0.058811
29	1	551	0.998185	0.941189	0.939481	0.060519
29	0	550				
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30	0	547				
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30	0	544				
30	1	543	0.998158	0.939481	0.937751	0.062249
30	0	542				
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31	1	533	0.998124	0.937751	0.935991	0.064009
31	0	532				
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34	0	493				
34	1	492	0.997967	0.935991	0.934089	0.065911
34	1	491	0.997963	0.934089	0.932186	0.067814
34	0	490				
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35	0	483				
35	1	482	0.997925	0.932186	0.930252	0.069748
35	1	481	0.997921	0.930252	0.928318	0.071682
35	0	480				
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37	1	461	0.997831	0.928318	0.926305	0.073695
37	1	460	0.997826	0.926305	0.924291	0.075709
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39	1	429	0.997669	0.924291	0.922137	0.077863
39	1	428	0.997664	0.922137	0.919982	0.080018
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48	1	75	0.986667	0.919982	0.907716	0.092284
48	0	74				
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55	0	3				
55	0	2				
57	0	1				

Table III-3 National Grid Transformer Lifetime Data Analysis Using Probability Plotting involving Actuarial Method

						Reliability		CDF
Failure Age t	Number of Exposure $N_E(t)$	Number of Failure $n_F(t)$	Number of Censoring $n_C(t)$	Number of Surviving $N_S(t)$	$R(t\text{-}1)$	$1-\frac{n_F(t)}{N_E(t)-0.5n_C(t)}$	$R(t)$	$F(t)$
1	865	2	8	855	1	0.997677	0.997677	0.002323
2	855	1	13	841	0.997677	0.998821	0.996501	0.003499
3	841	1	5	835	0.996501	0.998807	0.995313	0.004687
4	835	2	17	816	0.995313	0.997580	0.992904	0.007096
5	816	0	23	793		1		
6	793	2	11	780	0.992904	0.997460	0.990383	0.009617
7	780	1	11	768	0.990383	0.998709	0.989104	0.010896
8	768	2	5	761	0.989104	0.997387	0.986520	0.013480
9	761	1	6	754	0.986520	0.998681	0.985218	0.014782
10	754	3	18	733	0.985218	0.995973	0.981251	0.018749
11	733	0	16	717		1		
12	717	0	10	707		1		
13	707	1	7	699	0.981251	0.998579	0.979856	0.020144
14	699	2	8	689	0.979856	0.997122	0.977036	0.022964
15	689	2	18	669	0.977036	0.997059	0.974163	0.025837
16	669	1	14	654	0.974163	0.998489	0.972691	0.027309
17	654	0	21	633		1		
18	633	0	11	622		1		
19	622	2	15	605	0.972691	0.996745	0.969525	0.030475
20	605	1	5	599	0.969525	0.998340	0.967916	0.032084
21	599	1	3	595	0.967916	0.998326	0.966296	0.033704
22	595	3	3	589	0.966296	0.994945	0.961412	0.038588
23	589	1	9	579	0.961412	0.998289	0.959767	0.040233
24	579	2	5	572	0.959767	0.996531	0.956437	0.043563
25	572	1	2	569	0.956437	0.998249	0.954762	0.045238
26	569	2	3	564	0.954762	0.996476	0.951398	0.048602
27	564	3	3	558	0.951398	0.994667	0.946324	0.053676
28	558	2	1	555	0.946324	0.996413	0.942929	0.057071
29	555	2	6	547	0.942929	0.996377	0.939512	0.060488
30	547	1	8	538	0.939512	0.998158	0.937782	0.062218
31	538	1	10	527	0.937782	0.998124	0.936023	0.063977
32	527	0	17	510		1		
33	510	0	13	497		1		
34	497	2	9	486	0.936023	0.995939	0.932221	0.067779
35	486	2	8	476	0.932221	0.995851	0.928353	0.071647
36	476	0	9	467		1		
37	467	2	12	453	0.928353	0.995662	0.924326	0.075674
38	453	0	19	434		1		
39	434	2	9	423	0.924326	0.995343	0.920022	0.079978
40	423	0	25	398		1		
41	398	0	58	340		1		
42	340	0	64	276		1		
43	276	0	56	220		1		
44	220	0	53	167		1		
45	167	0	38	129		1		
46	129	0	19	110		1		
47	110	0	24	86		1		
48	86	1	22	63	0.920022	0.986667	0.907755	0.092245
49	63	0	16	47		1		
50	47	0	12	35		1		
51	35	0	9	26		1		
52	26	0	7	19		1		
53	19	0	6	13		1		
54	13	0	5	8		1		
55	8	0	7	1		1		
56	1	0	0	1		1		
57	1	0	1	0		1		

Appendix-IV

List of Publications

1. P. Jarman, Z.D. Wang, Q. Zhong, and M.T. Ishak, "End-of-Life Modelling for Power Transformers in Aged Power System Networks," in *CIGRE 2009. 6th Southern Africa. Regional Conference* Cape Town, Southern Africa, 2009.

2. M.T. Ishak, Q. Zhong, and Z.D. Wang, "Impact of Loading Profiles, WTI Settings and Hotspot Factors on Loss-of-Life of Dual Cooling Mode Transformers," in *16th International Symposium on High Voltage Engineering*, Johannesburg, 2009.

3. P. Jarman, R. Hooton, L. Walker, Q. Zhong, M.T. Ishak, and Z.D. Wang, "Transformer Life Prediction Using Data from Units Removed from Service and Thermal Modelling," in *CIGRE Session 2010* Paris, France, 2010.

4. Q. Zhong, Z.D. Wang, and P.A. Crossley, "Power Transformer End-of-Life Modelling: Linking Statistical Approach with Physical Ageing Process," in *CIRED Workshop* Lyon, France, 2010.