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Enhanced Bounded Integral Control of Input-to-State Stable Nonlinear Systems

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Abstract: The bounded integral controller (BIC) was recently proposed to replace the traditional integral controller (IC) for the regulation of any input-to-state stable (ISS) nonlinear system and guarantee closed-loop system stability with a bounded control output. In this paper, an enhanced version of the BIC is presented to provide a better approximation of the traditional IC in the entire bounded range of the control output and relax the assumption on the selection of the initial conditions of the original BIC. Using Lyapunov methods, it is analytically proven that the enhanced BIC maintains the zero-gain property and guarantees closed-loop stability of any nonlinear ISS plant with a given bound at the control output, without suffering from integrator windup issues. The plant dynamics and structure can be unkown as long as the plant is ISS. Hence, the proposed enhanced version of the BIC can replace the traditional IC in many applications where closed-loop stability cannot be proven, without changing the controller operation. A practical example is simulated to verify the performance of the proposed enhanced BIC compared to the original version and the traditional IC.

Keywords: Integral control, nonlinear systems, input-to-state stability, Lyapunov methods, bounded control.

1. INTRODUCTION

Integral control (IC) has been the core control method for regulating both linear and nonlinear systems. Especially under the presence of uncertainties and external disturbances, IC represents a key element of the closed-loop system to achieve robust regulation. In this case, the integrator dynamics are added to the dynamics of the original plant and the augmented closed-loop system that results from this action is investigated in terms of stability Khalil (2001); Tsinias (1989).

Closed-loop stability of a nonlinear plant with IC is not an easy task and has been an active area of research for many years. Local and semi-global stability results can be found in Isidori and Byrnes (1990); Huang and Rugh (1990); Mahmoud and Khalil (1996); Isidori (1997); Khalil (2000) especially for minimum-phase nonlinear plants with parameter perturbations. The results of Khalil (2000) were further extended in Jiang and Mareels (2001) without a priori knowledge of the bound of the uncertainties and also in Riachy et al. (2016) to use mollifiers instead of high-gain observers in the feedback loop. Conditional integrators have been also introduced in Singh and Khalil (2004); Li and Khalil (2012) to achieve asymptotic regulation of nonlinear systems and improve the transient response by applying an integral action inside a boundary layer. In Astolfi and Praly (2016), it is shown that robust output regulation can be accomplished using an integral action when a stabilizing feedback controller exists for the original plant. Hence, in most of the existing IC methods, several assumptions of the plant structure and dynamics are considered, i.e. minimum or non-minimum phase, relative degree, etc, including input-to-state stability (ISS) or integral ISS properties for the disturbance dynamics Jiang et al. (2004); Wu et al. (2011). The challenge of proving closed-loop stability increases when the dynamics of the system are unknown. For example, in Guay (2016), a proportional-integral controller with an extremum-seeking method is proposed to stabilize a nonlinear plant to an unknown optimal equilibrium.

In addition, since in many practical examples the IC is applied in plant systems with input constraints, either actuator or stability constraints (e.g. locally ISS systems), the controller output is often required to be bounded in a given range. Although the traditional IC can be combined with a saturation unit to maintain a given bound for the control output, the closed-loop system may still suffer from instability due to the integrator windup phenomenon. In these cases, several anti-windup techniques can be introduced in the control design and have been extensively studied in the literature, see for example Zaccarian and Teel (2002, 2004); Peng et al. (1996). The motivation for designing IC for nonlinear systems and guaranteeing closed-loop stability comes from a series of applications including motor control Choi and Lee (2009), unmanned aerial vehicles Nguyen and Damm (2012), power systems Zhong and Weiss (2011); Zhong et al. (2016), etc. A recent approach that introduces a saturating integrator to overcome the windup problem and guarantee closed-loop stability for a wide class of nonlinear systems is provided in Weiss and Natarajan (2016). However, most of the traditional anti-windup methods cannot guarantee a rigorous closed-loop stability and modern anti-windup techniques require knowledge of the plant dynamics, structure or several properties.

The present paper investigates nonlinear plant systems which may have unknown structure, dynamics and parameters. The only information is that the plant is ISS with respect to the control input and the main task is to regulate the plant and guarantee closed-loop stability. For simplicity, the investigation is restricted to single-input systems. For these type of systems, the bounded integral control (BIC) was recently proposed in Konstantopoulos et al. (2016) to achieve a desired regulation scenario and guarantee closed-loop stability. The BIC introduces a zero-gain property, i.e. the control output is bounded independently from the input signal; thus proving stability for any ISS plant. The main purpose was to replace the traditional IC in applications where the IC was used without a rigorous proof of stability. In this paper, an enhanced version of the BIC is proposed to overcome two main drawbacks of the original BIC: i) provide a better approximation of the traditional IC in the entire bounded range of the control output and ii) relax the assumption regarding the selection of the initial condition of the controller states. It is shown that a suitable modification of the BIC dynamics results in a more generic structure where the initial conditions of the BIC states can be chosen inside a wider area. Using Lyapunov methods, it is proven that the proposed enhanced BIC maintains that zero-gain property and the given bound for the control output without suffering from windup issues, leading to the closed-loop stability in the sense of boundedness for any ISS plant. This is achieved without additional saturation units and the continuous-time structure of the proposed controller allows the investigation of stability for the entire range of the plant input. Closed-loop stability in the sense of boundedness is proven using the generalized small-gain theorem Jiang et al. (1994) and represents the main task of the paper. For conditions of convergence to a desired equilibrium point, the reader is referred to the original BIC in Konstantopoulos et al. (2016). In order to verify the properties of the enhanced BIC, its performance is compared to the original BIC and the traditional IC with a saturation unit for a practical example of a dc/dc buck-boost converter system.

The rest of the paper is organized as follows. In Section 2, an overview of the original BIC is provided to explain the main concept and underline its limitations. In Section 3, the enhanced version of the BIC is proposed and analyzed. The zero-gain property and the main tasks that distinguish the proposed controller with the original one are explained. In Section 4, a practical example of a power converter is simulated to investigate the performance of the proposed approach and finally, the conclusions of the paper are presented in Section 5.

2. OVERVIEW OF THE BOUNDED INTEGRAL CONTROL

Consider the nonlinear plant system of the form

$$\dot{x} = f(x,u), \tag{1}$$

with x, u representing the state vector and input vector, respectively, $f : D \times D_u \to R^n$ being locally Lipschitz in x, u and D, D_u being open neighborhoods of the origin. To simplify the analysis, the investigation will be focused on single-input systems where the control task is the regulation of a scalar function $g(x)$ to zero, which includes the most common regulation scenario of one of the system states x_i to a reference value, i.e. $g(x) = x^{ref} - x_i$. The most widely used technique to achieve this task is to apply a traditional integral controller (IC) as

$$u = w \tag{2}$$

$$\dot{w} = k_I g(x), \tag{3}$$

with k_I being a positive integral gain, and is found in many engineering examples. However, the stability of the closed-loop system cannot be guaranteed in general, especially in the case where the dynamics of the plant are unknown. In addition, in several practical applications it is often required for the control output to be bounded in a given range, i.e. $u \in [-u_{max}, u_{max}]$, $u_{max} > 0$. In these cases, saturation units and additional anti-windup mechanisms are being placed, which may still require information of the plant dynamics or further complicate the stability analysis.

To this end, based solely on the assumption that the plant is ISS with respect to the input u and without any further knowledge on the plant, it has been shown in Konstantopoulos et al. (2016) that closed-loop stability can be guaranteed if the IC is replaced by the bounded integral controller (BIC) given in the form

$$u = w \tag{4}$$

$$\begin{bmatrix}\dot{w}\\ \dot{w}_q\end{bmatrix} = \begin{bmatrix} 0 & k_I g(x)\dfrac{w_q u_{max}^2}{u_{max}^2 - u_c^2} \\ -k_I g(x)\dfrac{w_q}{u_{max}^2 - u_c^2} & -k_q\left(\dfrac{w^2}{u_{max}^2} + w_q^2 - 1\right)\end{bmatrix}\begin{bmatrix} w \\ w_q\end{bmatrix}, \tag{5}$$

where $k_q > 0$ and $u_c \in (-u_{max}, u_{max})$ are constant gains. By considering the Lyapunov function candidate

$$W = \frac{w^2}{u_{max}^2} + {w_q}^2, \tag{6}$$

its derivative becomes

$$\dot{W} = -2\left(\frac{w^2}{u_{max}^2} + w_q^2 - 1\right) k_q w_q^2 \tag{7}$$

which implies that if the initial conditions of the BIC states w_0 and w_{q0} are defined on the ellipse

$$W_0 = \left\{ w, w_q \in R : \frac{w^2}{u_{max}^2} + {w_q}^2 = 1 \right\}, \tag{8}$$

then the trajectory of the BIC states response on the $w - w_q$ plane will be restricted on the ellipse W_0 for all future time, i.e. $u \in [-u_{max}, u_{max}]$ (see Fig. 1). This holds independently from the function $g(x)$, thus resulting in a zero-gain property of the controller, i.e. the controller states w and w_q are bounded independently from the input $g(x)$. In fact, this is a crucial property for guaranteeing closed-loop stability of the feedback interconnection of any ISS plant (1) with the BIC (4)-(5).

Looking at the first equation of the BIC (4), it can expressed as

$$\dot{w} = k_I g(x) \frac{w_q^2 u_{max}^2}{u_{max}^2 - u_c^2}. \tag{9}$$

Given a desired equilibrium point $(w_e, w_{qe}) = (u_e, w_{qe})$, corresponding to a point on the ellipse W_0, according to

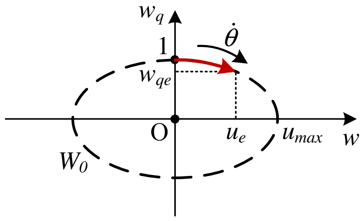


Figure 1. Original BIC states on $w - w_q$ plane

the analysis in Konstantopoulos et al. (2016), then near the desired equilibrium (9) becomes

$$\dot{w} \approx k_I g(x) \frac{u_{max}^2 - u_e^2}{u_{max}^2 - u_c^2}. \tag{10}$$

This indicates that if u_c is chosen equal to the steady-state value of the input u_e (or close to this value), then (10) results in $\dot{w} \approx k_I g(x)$ and the BIC approximates the traditional IC. However, this approximation holds only near the equilibrium point and not in the entire range of the input $[-u_{max}, u_{max}]$. Similarly, by selecting a different value for u_c, the BIC approximates the IC near a different operating point. In addition, it is clear from (9) that the integral gain of the BIC is $k_I \frac{w_q^2 u_{max}^2}{u_{max}^2 - u_c^2}$ and can change significantly while w and w_q operate on the desired ellipse W_0 because w_q changes.

Furthermore, the initial conditions of w and w_q should be defined on the ellipse W_0, i.e. they should satisfy $\frac{w_0^2}{u_{max}^2} + w_{q0}^2 = 1$ with a common choice of $w_0 = 0$ and $w_q = 1$. However, this can be a restrictive argument in a real environment where the system can be disturbed, leading to an inaccurate definition of the initial conditions.

Therefore, a better approximation of the traditional IC in the entire operating range of the input $[-u_{max}, u_{max}]$ and increased robustness of the controller in terms of the selection of the initial conditions are two important issues that need to be addressed. These should be accomplished without changing the zero-gain property of the BIC that guarantees the stability of any ISS nonlinear system with unknown dynamics and parameters. An enhanced version of the BIC that achieves these tasks is analyzed in the sequel.

3. ENHANCED VERSION OF THE BOUNDED INTEGRAL CONTROL

In order to overcome the limitations of the original BIC, the dynamics of the original controller (4) are modified to take the form

$$\begin{bmatrix} \dot{w} \\ \dot{w}_q \end{bmatrix} = \begin{bmatrix} -k\left(\frac{w^2}{u_{max}^2} + w_q^{2m} - 1\right) & k_I g(x) w_q^{2m-1} \\ -k_I g(x)\frac{w_q}{u_{max}^2} & -k\left(\frac{w^2}{u_{max}^2} + w_q^{2m} - 1\right) \end{bmatrix} \begin{bmatrix} w \\ w_q \end{bmatrix}, \tag{11}$$

where $m \geq 1 \in \mathbb{N}$ and k is a large positive constant gain.

To analyze system (11), consider the Lyapunov function candidate

$$V_m = \frac{w^2}{u_{max}^2} + \frac{w_q^{2m}}{m}. \tag{12}$$

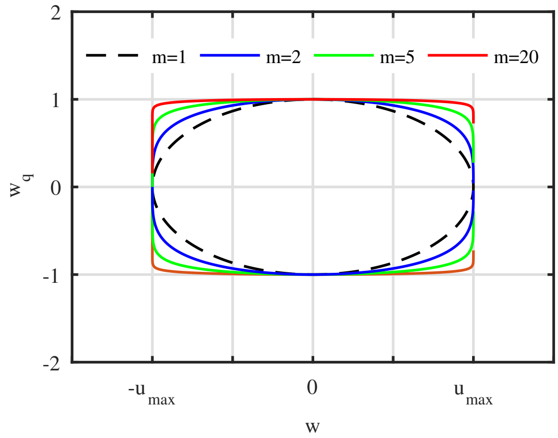


Figure 2. Enhanced BIC states on $w - w_q$ plane

Taking the time derivative of V_m and substituting from (11), it yields

$$\begin{aligned} \dot{V}_m &= \frac{2w\dot{w}}{u_{max}^2} + 2w_q^{2m-1}\dot{w}_q \\ &= -2k\left(\frac{w^2}{u_{max}^2} + w_q^{2m} - 1\right)\frac{w^2}{u_{max}^2} + 2k_I g(x)\frac{w w_q^{2m}}{u_{max}^2} \\ &\quad -2k_I g(x)\frac{w w_q^{2m}}{u_{max}^2} - 2k\left(\frac{w^2}{u_{max}^2} + w_q^{2m} - 1\right) w_q^{2m} \\ &= -2k\left(\frac{w^2}{u_{max}^2} + w_q^{2m} - 1\right)\left(\frac{w^2}{u_{max}^2} + w_q^{2m}\right), \end{aligned} \tag{13}$$

which shows that $\dot{V}_m$ negative outside of the closed curve

$$W_{m-1} = \left\{ w, w_q \in R : \frac{w^2}{u_{max}^2} + w_q^{2m} = 1 \right\} \tag{14}$$

positive inside, except from the origin where $\dot{V}_m$ is zero. Hence, W_{m-1} acts as an attractive closed curve since every trajectory $(w(t), w_q(t))$ starting from any point (w_0, w_{q0}) on the $w - w_q$ plane except from the origin will be attracted on W_{m-1}. Note that the larger the value of k, the faster the attraction of w and w_q on W_{m-1}. For $m = 1$, then the invariant set for w and w_q is defined by the union of the origin and the ellipse $\frac{w^2}{u_{max}^2} + w_q^2 = 1$, and the structure of (11) becomes similar to the original BIC in (5). The closed curve W_{m-1} for different values of $m \geq 1 \in \mathbb{N}$ is shown in Fig. 2.

It should be underlined that $\dot{V}_m < 0$ for any w and w_q outside W_{m-1} independently from the integral function $g(x)$. Note that for $m = 1$, then the BIC states w and w_q will be attracted on W_0 and the system (11) will operate similarly to the original BIC (4). Hence the proposed enhanced version represents a generalized structure of the original BIC.

Given the Lyapunov function candidate V_m defined in (12) and the fact that $\dot{V}_m < 0$ outside of W_{m-1}, then one can find a positive value $s > 0$ and a set $\Omega_m = \{V_m(w, w_q) \leq s\}$, outside of which $\dot{V}_m < 0$. By selecting $s = 1$ then

$$\Omega_m = \left\{ w, w_q \in R : \frac{w^2}{u_{max}^2} + \frac{w_q^{2m}}{m} \leq 1 \right\}. \tag{15}$$

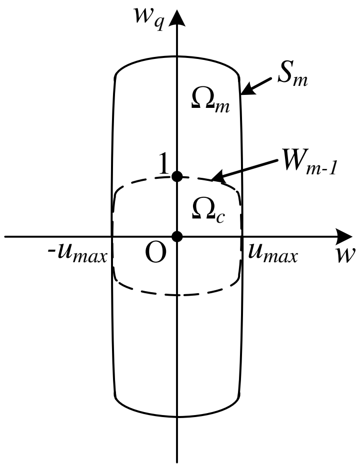


Figure 3. Maximum bound of the enhanced BIC states on $w - w_q$ plane

Note that the bound of Ω_m, i.e.

$$S_m = \left\{ w, w_q \in R : \frac{w^2}{u_{max}^2} + \frac{w_q^{2m}}{m} = 1 \right\} \tag{16}$$

has two common points $(u_{max}, 0)$ and $(-u_{max}, 0)$ with W_{m-1} for any $m \geq 2$ and the two sets coincide for $m = 1$. For $m \geq 2$, this can be proven by combining (14) and (16), from which it yields that

$$\frac{w_q^{2m}}{m} - w_q^{2m} = 0.$$

Since $m \geq 2$, then $w_q = 0$ which corresponds to points $(u_{max}, 0)$ or $(-u_{max}, 0)$ on the $w - w_q$ plane. By defining $\Omega_c = \left\{ w, w_q \in R : \frac{w^2}{u_{max}^2} + w_q^{2m} \leq 1 \right\}$ then $\Omega_c \subseteq \Omega_m$ as shown in Fig. 3. Since $\dot{V}_m < 0$ outside the closed set Ω_m, then for any set of initial conditions w_0, w_{q0} defined in Ω_m, every trajectory starting from point (w_0, w_{q0}) on the $w - w_q$ plane will remain in Ω_m for all $t \geq 0$. This means that $u = w \in [-u_{max}, u_{max}]$, $\forall t \geq 0$. Since the control output u is proven to remain inside the required bounded range for any set of initial conditions $(w_0, w_{q0}) \in \Omega_m$, opposed to the original BIC (4) where w_0 and w_{q0} must be specifically defined on the ellipse W_0, the enhanced BIC shows increased robustness in the selection of the initial conditions compared to the original structure.

It is therefore proven that for $(w_0, w_{q0}) \in \Omega_m$, then $u \in [-u_{max}, u_{max}]$ independently from the function $g(x)$ that needs to be regulated to zero according to the analysis in (13). Hence the enhanced BIC maintains the zero-gain property of the original BIC, i.e. the controller states w and w_q are bounded independently from the controller input $g(x)$. As a result, considering the plant (1) satisfying the ISS property with respect to u and with a gain $\gamma_{plant} > 0$, and taking into account that the enhanced BIC (2), (11) is also ISS with zero gain ($\gamma_{control} = 0$), then according to the generalized small-gain theorem Jiang et al. (1994), the closed-loop system represented by the feedback interconnection of the plant and the proposed controller is stable in the sense of boundedness. It is underlined that this holds independently from the dynamics of the plant, i.e. function $f(x, u)$, number of states, system parameters or structure, leading to the closed-loop stability in the sense of boundedness for any unknown nonlinear plant (1) satisfying the ISS property.

As already mentioned, for a sufficiently large value of the controller parameter k, the BIC states will be attracted quickly to the curve W_{m-1} and remain very close to it for all future time. For any set of initial conditions $(w_0, w_{q0}) \in \Omega_m$ with $w_{q0} > 0$, the controller states will remain very close to the upper curve of W_{m-1}, i.e. above the horizontal axis, since as soon as they try to approach the horizontal axis, then $w_q \to 0$ and from (11) there is $w \to \pm u_{max}$, which means that the states slow down and approach one of the points $(u_{max}, 0)$ or $(-u_{max}, 0)$ independently from $g(x)$. This provides and inherent anti-windup strategy for the enhanced BIC because the integration smoothly slows down near the upper and lower limit of the control output.

Additionally, since w and w_q remain close to W_{m-1} for a large k, the controller state dynamics can be approximated by

$$\dot{w} \approx k_I g(x) w_q^{2m} \tag{17}$$

$$\dot{w}_q \approx -k_I g(x) \frac{w_q}{u_{max}^2}, \tag{18}$$

since the diagonal terms of the 2×2 matrix in (11) become very close to zero. According to Fig. 2, for a high value of m, the controller state w_q remains close to 1 almost in the entire area of the control output $u = w \in [-u_{max}, u_{max}]$. In this case, from (18), the rate of change of w_q increases since the gain $k_I \frac{w_q}{u_{max}^2}$ is larger for a wider range of values of the control output. Hence, w_q will change faster and the enhanced BIC will provide a better approximation of the traditional IC almost in the entire range of $u = w \in [-u_{max}, u_{max}]$. As a result, the enhanced BIC can replace the traditional IC to regulate a function $g(x)$ to zero and i) guarantee a bounded control output $u \in [-u_{max}, u_{max}]$ without additional saturation units, ii) approximate the IC almost in the entire range of the control output, iii) guarantee nonlinear stability in the sense of boundedness for any nonlinear plant in the form of (1) that is ISS with respect to u, and iv) will not suffer from integrator windup. The fact that the proposed enhanced BIC provides a better approximation of the IC in the entire range and inherits an anti-windup property is verified in the simulation results that follow.

4. EXAMPLE

In order to evaluate the performance of the enhanced BIC, the same practical example investigated in Konstantopoulos et al. (2016) is tested again with the same parameters. The system represents a dc/dc buck-boost converter with dynamics

$$L\frac{di}{dt} = -(1-u)\,v + uE \tag{19}$$

$$C\frac{dv}{dt} = (1-u)i - \frac{v}{R}, \tag{20}$$

where L is the converter inductance, C is the output capacitor, E is a constant input voltage (uncontrollable) and R is the load resistor, as shown in Fig. 4. The system states are the inductor current i and the output voltage v, while the control input is defined as u and represents the

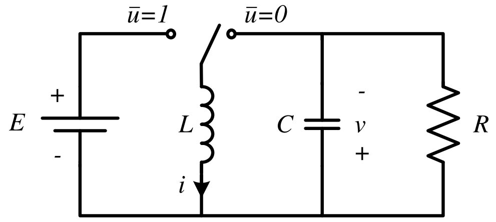
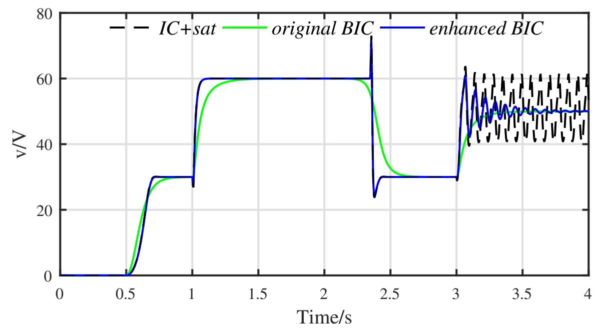


Figure 4. Schematic of a dc/dc buck-boost converter

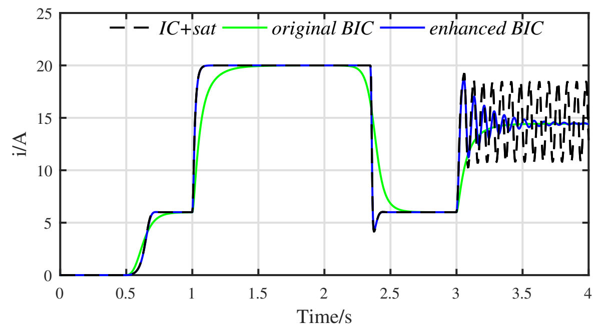
duty ratio of the converter restricted in the range $[0, 1]$. The continuous-time model (19)-(20) is obtained using average analysis on the actual switching converter model which is controlled by the switching element $\bar{u} = \{0, 1\}$. By suitably controlling the duty ratio, the main task is to regulate the output voltage v to a reference value v^{ref}. The plant is obviously a nonlinear system, due to the multiplication of the control input with the system states, and for any bounded control input u in the range $[0, 1-\gamma]$, for $0 < \gamma \leq 1$, the system states remain bounded. However, when $u = 1$, it is clear from (19) that the inductor current will escape to infinity leading to instability. This is why in practice the duty ratio u is desired to be limited in a lower range, e.g. in $[0, 0.8]$, and is often achieved with additional saturation units.

In order to regulate the output voltage v to a reference value v^{ref}, the traditional IC can be used with integral function $k_I g(x) = k_I(v^{ref} - v)$. Since the duty ratio should be limited in the range $[0, 0.8]$, a saturation unit can be added at the output of the IC. Similarly, the original and the enhanced BIC can be applied to achieve the same regulation. Given that $u \in [0, 0.8]$ should hold true, then the control input can be defined as $u = w^2$ and the controller state w is limited in the range $[-\sqrt{0.8}, \sqrt{0.8}]$, i.e. $u_{max} = \sqrt{0.8}$ for the BIC. In order to have a direct comparison, the IC with the saturation unit is applied with $u = w^2$ and dynamics $\dot{w} = k_I g(x)$, where $k_I = 0.22$. For the original BIC, the rest of the parameters are chosen as $k_q = 1000$ and $u_c = 0.6$ and for the enhanced BIC, there is $k = 1000$ and $m = 100$. The plant parameters are $L = 10mH$, $C = 30\mu F$, $R = 15\Omega$ and $E = 15V$. The simulation has been performed using Matlab/Simulink.

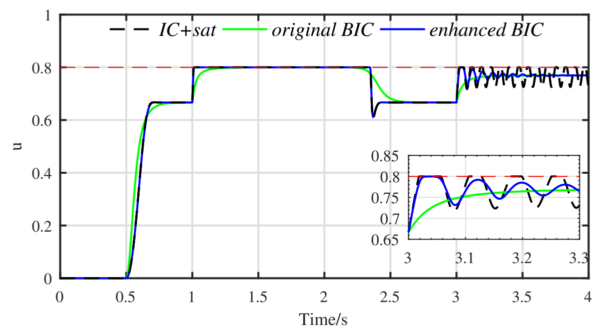
Starting from zero initial conditions for the plant states, the reference output voltage changes to $v^{ref} = 30\,\mathrm{V}$ at $t = 0.5\,\mathrm{s}$. As it is demonstrated in Fig. 5(a), the output voltage of the converter is regulated at the desired value for all three cases including the IC with saturation and both versions of the BIC. The inductor current response is shown in Fig. 5(b). At $t = 1\,\mathrm{s}$, the reference voltage increases to $70\,\mathrm{V}$ which requires a high duty ratio for the converter leading the control input u to its maximum value of 0.8 (Fig. 5(c)). The IC with the saturation unit forces the control input to saturate to the upper limit while both versions of the BIC smoothly converge to the upper limit. In this case, the output voltage is regulated to a lower value. After 1 s, the reference voltage v^{ref} returns to $30\,\mathrm{V}$, and all three controllers regulate the output voltage to the desired value after a short transient. From Fig. 5, it is clear that the response of the enhanced BIC almost overlaps with the response of the IC with a saturation unit for the first 3 s, opposed to the original BIC. The difference is clear when v^{ref} changes to $50\,\mathrm{V}$ at the time



(a) output voltage v



(b) inductor current i



(c) duty-ratio u

Figure 5. Simulation results of the buck-boost converter comparing the IC with a saturation unit, with the original and the enhanced BIC

instant $t = 3\,\mathrm{s}$, where the IC with saturation suffers from integrator windup, while the enhanced BIC can regulate the output voltage to the desired value. This is better observed in the response of the input u around $t = 3\,\mathrm{s}$ in Fig. 5(c), where the enhanced BIC does not saturate but slows down near the upper limit and does not introduce an integrator windup. The theoretical analysis of the BIC states w and w_q presented in the paper is verified in Fig. 6, showing the controller state trajectory on the $w - w_q$ plane, where it is clear that w and w_q take values on W_0 and W_{99}, for the cases of the original and the enhanced BIC, respectively.

Therefore, it becomes clear that the enhanced BIC can guarantee the crucial properties of the original BIC, i.e. zero-gain property, given bound for the control output, no windup issues, and additionally provides a better approximation of the traditional IC almost in the entire range of the control output. Hence, the enhanced BIC can easily replace the traditional IC with a saturation unit in many engineering applications and offer an inherent anti-

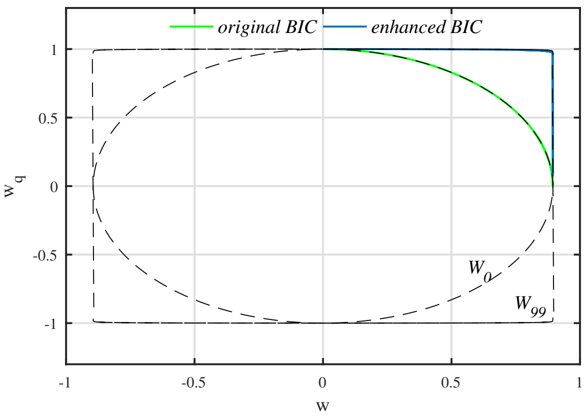


Figure 6. Controller states response on the $w - w_q$ plane for the original and the enhanced BIC

windup property without changing the control operation in the entire bounded range, maintaining a continuous-time structure that allows the investigation of the closed-loop stability.

5. CONCLUSIONS

An enhanced version of the recently proposed bounded integral controller was presented in this paper. It was analytically proven that the enhanced BIC maintains the crucial zero-gain property which guarantees the stability of the closed-loop system for any ISS plant, even with unknown dynamics and structure. Additionally, the enhanced BIC relaxes the assumption of the initial conditions of the controller states being defined on a specific ellipse, introduced by the original BIC, resulting in a larger set of possible initial conditions. It was also shown that the enhanced BIC provides a better approximation of the traditional IC in the entire range of the control output, providing a more suitable solution for replacing the IC. The theoretical contribution of the proposed method was verified using a practical example of a dc/dc buck-boost converter.

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