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The Behaviour and Effects of Beam-End Buckling in Fire Using a Component-Based Method

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Abstract

A combination of beam-web shear buckling and flange buckling at the ends of steel beams is very commonly observed during full-scale fire tests. This can affect the behaviour of the steel beams, as well as on their adjacent connections, under fire conditions. This phenomenon has not previously been sufficiently investigated and cannot be simulated in high-temperature global frame analysis, which could potentially lead to unrealistic results being used in structural fire engineering design. In this research, a component-based beam-end buckling element has for the first time been created for Class 1 and 2 beams. The beam-end buckling element is composed of nonlinear springs, respectively representing the buckling of beam flange and web, also considering the interaction between these two buckling phenomena. Each spring is able to deal with loading-unloading-reloading force-deformation paths. A significant challenge is to enable the flange buckling spring to deal with post-buckling deformation reversal. The buckling element has been implemented into the structural fire engineering frame analysis software Vulcan, to be used adjacent to existing connection elements in frame modelling.

The buckling element has been verified against ABAQUS finite element modelling on isolated beams. It is shown that the newly created component-based buckling element is able to simulate the effects of beam-end shear buckling in the web and local buckling of the bottom-flange, with satisfactory accuracy. The influence of the buckling element on the bolt-row force distribution within the adjacent connection element has been investigated. Analyses using isolated beams indicate that the implementation of the buckling element considerably improves the prediction of connection force resultants. A general observation from numerical studies with and without the buckling element is that beam-end buckling seems to reduce the connection component forces generated at elevated temperatures.

Keywords: Post-Buckling Behaviour; Steel Beam; Component-Based Model; Connection; Fire.

Notation:

b	flange width
c	half flange width
d	beam web depth
D_B	spring deformation of Node B
D_{CREF}	spring deformation of the Reference Point
D_{CREF1}	spring deformation of the Reference Point at temperature T_1
D_{INTER}	spring deformation of the Intersection Point
D_{INTER1}	spring deformation of the Intersection Point at temperature T_1
D_{INTER2}	spring deformation of the Intersection Point at temperature T_2
D_n	spring deformation of Node P_n
D_{n-1}	spring deformation of Node P_{n-1}
D_x	real spring deformation in an arbitrary iteration
F_x	internal horizontal force of the buckling element
F_B	bottom spring force
F_{INTER1}	spring force of the Intersection Point at temperature T_1
F_{INTER2}	spring force of the Intersection Point at temperature T_2
F_R	spring yield strength
F_S	shear spring force
F_T	top spring force
F_{UB}	internal force of the unbuckled spring
F_y	internal vertical force of the buckling element
k_E	reduction factor for young's modulus at elevated temperatures
K_{IC}	initial elastic stiffness of the compression spring
K_{T1}	initial elastic stiffness of the compression spring at temperature T_1
K_{T2}	initial elastic stiffness of the compression spring at temperature T_2
k_y	reduction factor for yield stress at elevated temperatures
K_1	slope of the line segment between Point B and Point P_{n-1}
K_2	slope of the line segment between Point P_{n-1} and Point P_{n-2}
l	beam length
L_p	flange-buckling wavelength length
M	applied bending moment
M_n	moment resistance of Point P_n
M_{n-1}	moment resistance of Point P_{n-1}
M_P	plastic moment resistance of the buckling element
M_x	moment resistance in an arbitrary iteration
M_{xy}	in-plane elemental moment
t_f	thickness of the flange

t_w	thickness of the beam web
α_T	thermal elongation coefficient
Δ_T	top spring deformation
Δ_B	bottom spring/buckling spring deformation
Δ_{2y}	vertical displacement of Nodal 2
Δ_{1x}	horizontal displacement of Nodal 1
Δ_{1y}	vertical displacement of Nodal 1
Δ_{2x}	horizontal displacement of Nodal 2
θ	relative rotation of the two nodes of the buckling element
θ_1	rotation of Nodal 1
θ_2	rotation of Nodal 2
θ_B	relative rotation of the two nodes at the end of the plateau stage
θ_R	relative rotation of the two nodes at the end of the pre-buckling stage
θ_n	relative rotation of the two nodes at Point P_n
θ_{n-1}	relative rotation of the two nodes at Point P_{n-1}
$\sigma_{y,\theta}$	yield strength of steel at elevated temperatures

1. Introduction

Significant developments [1, 2] have been made in investigating the behaviour of steel structures under fire conditions in the last two decades. The Cardington full-scale fire tests [3, 4] demonstrated that the behaviour of a continuous composite structure can be completely different from the behaviour of isolated members seen in conventional standard fire testing. Structural behaviour in fire can be highly nonlinear and complex, and perhaps the most important goal of fire design is to prevent progressive collapse of the whole building. Therefore, designers are becoming increasingly aware of the importance of performance-based design, which treats the structure integrally, and attempts to sufficiently consider the interactions between different parts of a structure in analyses which contribute to the design process. Due to the high cost of full-scale fire testing and the complexity of the interactions involved, it has become essential to develop full-structure computational simulation tools in order to enable performance-based design. Detailed finite element modelling using software such as ABAQUS [5] and ANSYS [6] can provide sufficient accuracy, but the creation of structural models is time-consuming and the analysis can be computationally demanding; this approach is therefore undesirable for practical full-structure analysis, especially when detailed semi-rigid connection models have to be generated.

The software Vulcan [7] was developed by the Fire Engineering Research Group at the University of Sheffield. Vulcan allows engineers to conduct three-dimensional frame analysis and structural robustness assessments under fire conditions. A variety of element types (beam-column, slab, shear connector and connections) has already been implemented. The development of connection elements [8-10] in Vulcan has been based on a “component-based” representation instead of modelling the details of the connection using solid elements. In the component-based method a connection is considered as an assembly of nonlinear springs, each of which has its individual characteristics. This simplified model is able to represent the key behaviour of certain connection elements to an acceptable accuracy, but adds very few degrees of freedom to the structural model, which makes the computation considerably more efficient [8]. Recently, Khalaf et al. [11] have created a model for predicting the bond-slip between concrete and steel reinforcing bars at elevated temperatures in Vulcan.

It has been observed that both shear buckling of beam webs and beam bottom-flange buckling (Fig. 1), near the ends of steel beams, are very prevalent under fire conditions. These phenomena can affect particularly the internal forces in adjacent connections and the overall deflection of the beam, and may therefore influence the fire resistance of the assembled structure. However, there has not been sufficient research investigating the beam-end local buckling behaviour of Class 1 to 2 beams at high temperatures. On the one hand, there has been no theoretical model which can represent the plastic post-buckling behaviour of stocky (Classes 1 and 2) beams at elevated temperatures. On the other hand, although detailed modelling using commercial FEA packages such as ABAQUS or ANSYS can predict and follow the beam-end buckling phenomena, this is computationally very demanding and becomes unfeasible when global frame analysis is required in the context of practical performance-based structural fire engineering design. It has therefore become essential to develop a simplified model which can be integrated into global analysis, in order to simulate the beam-end buckling phenomena sufficiently accurately within an acceptable time-period, given that this includes both the creation of a model and the actual runtime. In this study, this has been achieved by developing a new buckling element and integrating it into Vulcan.

Previous work [12] conducted by the authors has led to the development of an analytical model which can consider the combination and interaction of flange buckling and beam-web shear buckling. Further parametric studies [12, 13] have indicated that this model is sufficiently accurate to reflect the most important aspects of the buckling zones in the vicinity of beam-to-column-face connections at elevated temperatures. In this study, a component-based model of the buckling zone has been created on the basis of this analytical model. Each nonlinear spring in the buckling element is able to deal with the reversal of spring deformation, to simulate strain reversal, which very often happens within a heated structure and must therefore be considered during modelling. The component-based buckling element has been implemented into Vulcan. It has been verified against ABAQUS modelling on isolated beams. The influence of the buckling element on the bolt-row force redistribution within the connection has then been investigated.

2. Creation of the component-based model

According to the analytical model, the force-deflection characteristics of the buckling element can be divided into three stages, described as pre-buckling, plateau and post-buckling. In the pre-buckling and plateau stages, the buckling element performs as an ordinary beam element. In the post-buckling stage, the deflection of the buckling zone is the sum of the deflection due to beam-web shear buckling and that caused by bottom-flange buckling. The bottom-flange buckling causes an additional rotation of the whole beam-end about its support (due mainly to local shortening of the bottom flange in buckling), as shown in Fig. 2 (a). It is assumed that the centre of rotation is at the top corner of the beam-end (Point A in Fig. 2 (a)), since the resistance of the bottom flange decreases after buckling. Beam-web shear buckling can cause transverse drift of the shear panel, as shown in Fig. 2 (b). Therefore, the combined effect of flange buckling and beam-web shear buckling on the overall vertical deflection of the beam is as illustrated in Fig. 2 (c).

The component-based buckling element is illustrated in Fig. 3. The flange-buckling element is composed of four nonlinear horizontal springs at the flange positions. Two springs, one to act in tension and one to act in compression, are located at each flange, representing its resistance. For the set of springs at either location, only one spring will be activated at any instant, depending on the sense of the spring force. The beam-web shear buckling is represented by the shear-buckling component (the vertical spring of the buckling element in Fig. 3). The length of the component-based buckling element is calculated according to Eq. (1) on the basis of elastic buckling theory [14], which has been modified to consider the effects of temperature and steel grade. In most fire tests only one shear-buckling wave has been observed, and this is usually aligned at around 45° to the horizontal. Therefore, the shear-buckling panel is not usually longer than the beam depth d, and the flange buckling wave lies between the two plastic hinges (Points B and C in Fig. 2 (b)) on the bottom flange. Hence, it has been assumed that the flange-buckling wavelength $\mathrm{L_p}$, calculated by Eq. (1), is limited not to be longer than the beam depth d.

$$\mathrm{L_p} = 2\beta\mathrm{c} = 2\times 0.713\sqrt{\frac{275}{\sigma_{\mathrm{y},\theta}}}\left(\mathrm{d}/\mathrm{b}\right)^{1/4}\left(\mathrm{t_f}/\mathrm{t_w}\right)^{3/4}\left(\mathrm{k_E}/(0.7\mathrm{k_y})\right)\times\mathrm{d}/2 \qquad (1)$$

In the calculation procedure, the two nodal displacements of the component-based buckling element can be calculated by accumulating the nodal displacements from each iteration based on force equilibrium. The deformation of each spring can be related to the nodal displacements using the equations below:

$$\Delta_T = (\theta_2 - \theta_1)\ d/2 + (\Delta_{2x} - \Delta_{1x}) \tag{2}$$

$$\Delta_B = -(\theta_2 - \theta_1)\ d/2 + (\Delta_{2x} - \Delta_{1x}) \tag{3}$$

$$\Delta_S = \Delta_{2y} - \Delta_{1y} \tag{4}$$

According to each spring's deformation and stiffness, the spring force can be calculated. The elemental internal forces can be related to the individual spring force according to Eq. (5):

$$F_x = F_T + F_B \tag{5}$$

$$F_y = F_S \tag{6}$$

$$M_{xy} = (F_T + F_B)d/2 \tag{7}$$

As the element is two-dimensional at present, the out-of-plane elemental stiffnesses are all assumed to be infinitely large. The derivation of the individual spring stiffness and force according to its deformation in all the heating/loading stages will be introduced in Section 3.

3. Loading and unloading paths of the buckling element

During the course of a fire, the beam-end buckling zones can experience complex combinations of internal forces caused by high material nonlinearity and expansions due to temperature variation, interacting with end-restraint conditions. In the model, these forces will be resisted by the horizontal springs at the flanges. These springs can be subject to either compression or tension at different stages of loading/heating. For example, the bottom spring may be in compression during the initial heating phase, and in tension in the high-temperature catenary stage. Therefore, it is essential to establish a robust loading-unloading-reloading approach to deal with deformation reversal at both constant and transient temperatures. The vertical shear spring does not need a reversal path, as reversal of shear does not usually occur.

The Masing Rule [15] was initially created to model the dynamic force-deflection relationships of structural members under intensive cyclic seismic loading, when the members were loaded into the nonlinear range. It has been widely applied to deal with other engineering problems when the material is highly nonlinear and when residual strains are highly affected by the load-deformation history. Researchers [9, 16] have suggested that the Masing Rule could be used to model semi-rigid connections in heating and cooling. In this paper, the Masing Rule is incorporated into the characteristic curve of each flange spring of the buckling element, to enable modelling of the buckling panel under any possible loading-unloading-reloading sequence during either constant or transient heating. The Masing Rule has been modified for the post-buckling stage (after bottom-flange buckling occurs) to ensure that the hysteresis cycles are able to return to their initial points of unloading.

3.1. At constant temperature

Based on the Masing Rule, the component characteristics of a spring can be represented by the combination of a “skeleton” curve and a “hysteresis” curve. A schematic illustration of the Masing Rule is shown in Fig. 4. The hysteresis curve is the skeleton curve scaled by a factor of two and rotated by 180°.

If the skeleton curve is described as,

$$\delta = f(F) \tag{8}$$

Then the hysteresis curve can be described as,

$$(\delta_A - \delta) = 2f((F_A - F)/2) \tag{9}$$

where F_A is the force at which unloading starts and δ_A is the deformation at F_A.

The compression and tension springs at the same location (Fig. 3) can work in turn, depending on the sense of the spring force, to follow the complete loading-unloading-reloading path.

3.1.1 Compression Spring

In the post-buckling stage, previous research [12] has derived vertical force-deflection relationships for the buckling element, due to the combined effects of bottom flange buckling and shear buckling, based on yield-line theory. Deducting the effect of shear buckling from the total vertical deflection, the moment-rotation relationship of the buckling element due to flange buckling alone is as illustrated in Fig. 5. This relationship is based on the assumption that the beam is axially unrestrained, and therefore has no net axial force.

The force-deformation relationship of the compression spring at the buckling flange, including the three stages (pre-buckling, plateau and post-buckling), is shown in Fig. 6(a). The pre-buckling stage ends when the spring force reaches F_R, which is the axial force at which half of the I-section yields, as shown in Fig. 6(b). The value of F_R can be calculated using Eq. (10). The shape of the curve in the pre-buckling stage is based on EC3 Part 1-2 [17] for steel at elevated temperatures.

$$F_R = \sigma_{y,\theta} \times (b_f \times t_f + d \times t_f / 2) \tag{10}$$

The axial force remains constant until the initiation of plastic local buckling (Point B). Fig. 6(c) shows the yield line pattern in the post-buckling stage.

It is assumed that, in the post-buckling stage, the stiffness of the bottom spring representing the buckled flange is so low compared to that of the tension spring that the deformation of the non-buckling flange can be neglected. Therefore, the centre of rotation of the buckling element is assumed to be at the top flange, where the buckling element is connected to the connection element (Point A in Fig. 3). The compression spring deformation can be represented as:

$$\Delta_B = \theta d \tag{11}$$

For this axially unrestrained case, only the shear force and bending moment from the connected beam are transferred to the buckling element. Therefore, the force equilibrium within the buckling element gives:

$$F_{UB} + F_B = 0 \tag{12}$$

$$(F_{UB} - F_B) \times 0.5d = M \tag{13}$$

where Δ_B is the axial deformation of the buckled flange, M is the moment about the centre-line of the I-section, and F_{UB} and F_B are the forces in the unbuckled (top for this case) and buckled (bottom for this case) springs, respectively.

A theoretical model has previously been developed by the authors [12], which can be used to determine the moment-rotation relationship (Eq. (14)) of the buckling element:

$$M = f(\theta) \tag{14}$$

Substituting Eqs. (11)-(13) into Eq. (14), the axial force-deformation relationship of the buckled spring is:

$$F_B = 0.5 \times f(\Delta_B) / d^2 \qquad (15)$$

The curved descending part of the moment-rotation relationship of the buckling element has been simplified to a multi-linear relationship. The rotation θ at any given bending moment can be found through linear interpolation within each linear increment (Fig. 5). The corresponding spring force is derived based on Eq. (15) as:

$$F_x = F_R + K_1 \times (D_{n\text{-}1} - D_B) + K_2 \times (D_x - D_{n\text{-}1}) \qquad (16)$$

where

$$K_1 = (M_P - M_{n\text{-}1}) / (d^2 \times (\theta_B - \theta_{n\text{-}1})) \qquad (17)$$

$$K_2 = (M_{n\text{-}1} - M_n) / (d^2 \times (\theta_{n\text{-}1} - \theta_n)) \qquad (18)$$

Fig. 7 (a)-(c) illustrate the various possibilities for re-loading curves, when unloading initiates at the different stages (pre-buckling, plateau and post-buckling) for the compression spring. The initiation point of unloading in a convergent time step is defined as the “Intersection Point” for the following time step. The loading curve prior to the plateau stage (at Point A in Fig. 7) is composed of an initial linear part followed by a nonlinear part. It is assumed that the heights of the linear and nonlinear parts are identical. When unloading starts from the plateau stage, since the hysteresis curve (the thick line) is the skeleton curve (the thin line) scaled by a factor of two and rotated by 180° following the Masing Rule, the linear part of the unloading curve finishes exactly at the point where it meets the horizontal axis; their intersection is defined as the “Reference Point”. The Intersection Point and the Reference Point update at every convergent time step. In order to simplify the calculation, it is assumed that the linear part of the unloading path always stops at the Reference Point; the linear path is followed by a curved part, which is the nonlinear part of the initial loading curve scaled by a factor two and rotated 180°. In other words, the unloading path will stop its linearity when it hits the X axis, and be followed by a nonlinear curve for which the tension spring is activated. The force resistance of the tension spring is F_R. At the end of the unloading curve, the tensile force in this spring is equal to the magnitude of the compressive resistance F_R of the compression spring.

Once reloading occurs, if the compressive deformation of the compression spring is larger than the recorded position of the Intersection Point, the load path will follow the initial loading curve.

The coordinate of the Reference Point is determined by Eq. (19).

$$D_{CREF} = D_C - F_C / K_{IC} \tag{19}$$

where D_{CREF} is the coordinate of the Reference Point, K_{IC} is the initial elastic stiffness of the compression spring. The compressive deformation D_C and force F_C are absolute values (always positive).

If the total spring deformation at the end of an arbitrary iteration is smaller than that of the pre-existing Intersection Point, deformation reversal will occur, following the thick line between the Intersection Point and Reference Point (the existing unloading path). The slope of the unloading path is equal to the initial elastic stiffness K_{IC} at the relevant temperature. The compression force on the unloading curve is:

$$F_C = F_{INTER} + (D_{INTER} - D_C)K_{IC} \tag{20}$$

For an arbitrary iteration, the spring force can be calculated based on the spring deformation of the previous iteration. In the post-buckling stage, the spring stiffness on the loading path is negative. This negative stiffness leads to the situation that one force corresponds to two possible deformations (one on the loading path, one on the unloading path). In order to avoid this numerical singularity when using the static solver during the calculation, the following approach has been proposed. Point C is assumed to be the start of an arbitrary iteration (Fig. 7(c)). For the loading path, a 'zero' stiffness (instead of a negative stiffness) is assumed to define the start of the next iteration. The unloading path remains unchanged. The loading and unloading paths become the dashed lines starting from Point C. When the internal force is larger than the external, the iteration will follow the unloading path. Otherwise, the loading path is adopted, in which case Point D_1 is assumed to be the end of this iteration. The position of D_1 depends on the size of the iteration step. In the next iteration Point D_2, which has the same deformation as that of Point D_1, will be used as the starting point, but the spring force is calculated based on the descending post-buckling curve. At the end of each iteration, the difference between the internal and external forces is checked; this iterative process stops when a balance between the external and internal forces is found.

The deformation of the compression spring D_C can be calculated according to the differential displacement of the two nodes of the buckling element. When the spring deformation is on the unloading path above the Reference Point (Point RP in Fig. 7(a, b, c)), the spring will be under tension. The compression spring is deactivated; the tension spring at the same location will be activated instead.

The model has been developed on the basis of the assumption that there is no restraint to thermal expansion. The model is also valid for restrained cases, since the buckling criterion (the bottom spring experiences a certain amount of compressive squash) is calculated from a yield line mechanism, which is not affected by the restraint conditions. The only difference between the restrained and unrestrained cases is that the bottom spring force is larger in the former case than in the latter, and the model is capable of adjusting the spring force level to achieve equilibrium.

3.1.2 Tension Spring

The characteristics of the tension spring for the initial loading stage are similar to those of the compression spring, but lack the post-buckling phase (Fig. 8). The force-deformation relationship in the pre-buckling stage in Fig. 8 is an illustration based on the steel material properties at temperatures above 400°C. The characteristics in the pre-buckling stage can be updated if a different material is used.

3.2. During transient heating

At elevated temperatures, the material stress-strain characteristic is temperature-dependent. The essential assumption for deformation reversal at increasing temperatures is that the permanent deformation of a spring is unaffected by change of temperature, and so the Reference Point of the unloading curve does not change between two adjacent temperature steps. The new unloading path will still be linear, following the initial slope of the force-deformation relationship at the new temperature, and so the Intersection Point (at which unloading initiates) relocates. Taking the compression spring at plateau stage as an example, Fig. 9 shows the loading and unloading procedure when the spring's temperature increases.

The loading and uploading paths at the initial temperature T_1 are shown in Fig. 9 (a). The deformation at the Reference Point can be calculated using,

$$D_{CREF1} = D_C - F_C / K_{T1} \tag{21}$$

When the temperature increases to T_2 the Reference Point remains identical, while a new Intersection Point can be found using the new initial elastic stiffness K_{T2}. The spring deformation of the new Intersection Point is,

$$D_{INTER2} = D_{CREF1} + F_{INTER2} / K_{T2} \tag{22}$$

The deformation of the compression spring remains identical between adjacent iterations within the same temperature step. Therefore, when the temperature increases to T_2, the Intersection Point falls onto the unloading path of the new force-deformation relationship where K_{T2} is lower than K_{T1}, as shown in Fig. 9 (b). This sudden jump disturbs the force equilibrium, and so in the following iterations the spring deformation is adjusted until a balance between the internal and external forces is found. The spring will follow the loading path if its deformation is larger than that of the Intersection Point ($D_C > D_{INTER2}$), where D_C and D_{INTER2} are absolute values (always positive) of the spring deformation. Otherwise, the spring follows the unloading path. The spring force is calculated using Eq. (23):

$$F_C = F_{INTER2} + (D_C - D_{INTER2})K_{T2} \tag{23}$$

When the spring follows the unloading path above the Reference Point, it will be subject to tension. The characteristics of the tension spring follow the same rules described in Section 3.1.1. A flowchart of the procedure for modelling the compression spring is shown in Fig. 10.

Although the shear spring is a necessary component of the buckling element, it has been found to have little influence on the analytical results. The main benefit of having a non-rigid shear spring is to allow the simulation of the transverse drift of the buckling panel, whereas the transverse drift is negligible here as it does not influence the bolt-row force distributions, which is the main purpose of this research. Moreover, the shear spring starts to function from the post-buckling stage and its behaviour is related to the buckled flange spring, not to the applied shear force, thus it is not possible develop a shear force-deformation relationship for the shear spring. Therefore, the shear spring has been assumed to be rigid in the proposed model.

The elemental 12 x 12 stiffness matrix uses six degrees of freedom at each node and it is assembled by spring stiffness, as presented in Eq. (24). The elemental stiffness matrix can be implemented in the overall stiffness matrix for further calculations.

$$K = \begin{bmatrix} (K_T+K_B) & 0 & 0 & 0 & \frac{(K_T+K_B)d}{2} & 0 & -(K_T+K_B) & 0 & 0 & 0 & -\frac{(K_T+K_B)d}{2} & 0 \\ 0 & \infty & 0 & 0 & 0 & 0 & 0 & 0 & -\infty & 0 & 0 & 0 \\ 0 & 0 & K_S & 0 & 0 & 0 & 0 & 0 & -K_S & 0 & 0 & 0 \\ 0 & 0 & 0 & \infty & 0 & 0 & 0 & 0 & 0 & -\infty & 0 & 0 \\ \frac{(K_T+K_B)d}{2} & 0 & 0 & 0 & \frac{(K_T+K_B)d^2}{2} & 0 & -\frac{(K_T+K_B)d}{2} & 0 & 0 & -\frac{(K_T+K_B)d^2}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \infty & 0 & 0 & 0 & 0 & 0 & -\infty \\ -(K_T+K_B) & 0 & 0 & 0 & -\frac{(K_T+K_B)d}{2} & 0 & (K_T+K_B) & 0 & 0 & 0 & \frac{(K_T+K_B)d}{2} & 0 \\ 0 & -\infty & 0 & 0 & 0 & 0 & 0 & \infty & 0 & 0 & 0 & 0 \\ 0 & 0 & -K_S & 0 & 0 & 0 & 0 & 0 & K_S & 0 & 0 & 0 \\ 0 & 0 & 0 & -\infty & 0 & 0 & 0 & 0 & 0 & \infty & 0 & 0 \\ -\frac{(K_T+K_B)d}{2} & 0 & 0 & 0 & -\frac{(K_T+K_B)d^2}{2} & 0 & \frac{(K_T+K_B)d}{2} & 0 & 0 & 0 & -\frac{(K_T+K_B)d^2}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\infty & 0 & 0 & 0 & 0 & 0 & \infty \end{bmatrix} \tag{24}$$

4. Results

4.1 Verification of the Vulcan models

In order to verify the newly created component-based element in Vulcan, example beams were modelled using both Vulcan and ABAQUS. A sketch of the Vulcan model using buckling elements is shown in Fig. 11 (a). The beam element in Vulcan is actually a line element, and the end zones of the beam are simulated by the new component-based buckling elements.

In order to allow reasonable comparison, the ABAQUS models also consist of three parts: two beam ends modelled by shell elements and the rest of the beam simulated using wire elements. The images of ABAQUS models are shown in Fig. 12 (a). Example beams without the buckling elements at the beam ends were also built up in Vulcan (Fig. 11 (b)) and in ABAQUS (Fig. 12 (b)) to investigate the effects of the buckling elements by comparing results from the models with and without the buckling elements.

For the shell elements in the ABAQUS models, the four-noded shell element (S4R), which is capable of simulating buckling behaviour with reasonable accuracy, was adopted. A mesh sensitivity analysis was conducted, which indicated that elements of size 15mm x 15mm provided an optimum between accuracy and computing efficiency. For the wire element a mesh size of 250mm was adopted after a mesh sensitivity analysis. Material properties, including the thermal expansion coefficient of steel given by EC3 [17], were used. The models were subject to full axial restraint at both ends, and were restrained from out-of-plane deflection so that no overall buckling across the weak axis could occur. Two beams, spanning 6m and 9m, were modelled. The beam section was UB356x171x51 for the 6m beams, and UB457x191x98 for the 9m beams. In order to achieve different combinations of axial force and bending moment the 6m beams were loaded with uniformly distributed load of intensity 26.8N/mm (load ratio = 0.4), 33.5N/mm (load ratio = 0.5) and 40.2N/mm (load ratio = 0.6). The 9m beams were loaded with uniformly distributed load of intensity 29.6N/mm (load ratio = 0.4), 37.0N/mm (load ratio = 0.5) and 44.4N/mm (load ratio = 0.6). The beams were uniformly heated beyond 700°C.

The force-deformation relationships of the springs for the 6m beam with the buckling element subject to a load ratio of 0.5 are shown in Fig. 13. In Fig. 13 (a) the dashed curve represents the deformation-temperature relationship of the bottom spring. The thin solid curve shows the force-deformation response of this compression spring. It has been shown that, the compression force in the bottom spring initially increases linearly with the load increase. After the load is fully applied to the beam, temperature starts to elevate until around 220°C, while the bottom-spring goes into plateau stage. When temperature is above 220°C, the bottom spring starts to buckle. After the bottom flange buckles (when the deformation is around 6mm), the compressive force decreases with increase of compressive deformation as temperature further elevates. The thick solid curve is the fully-yielded force-deformation relationship of this spring, for which the decrease in the axial force would only be due to the degradation of the material as temperature rises (according to EC3 [17]). The difference between these curves illustrates the reduction in strength due to buckling. Deformation reversal occurs when the spring deformation is 33.5 mm when the temperature is 694°C. The deformation of a compression spring can be calculated as the summation of (1) nodal displacement at the intersection points (Points A and B in Fig. 11) of the beam and buckling elements and (2) spring deformation within the buckling element caused by the nodal rotation at the intersection. There is always an equation relating the beam elongation due to elevated temperature and the beam elongation needed for geometrical deformation. The latter is the summation of (1) nodal displacement at both beam ends and (2) beam-end movement due to mid-span deflection at the intersection point between the beam and buckling elements. Prior to the bottom-spring deformation reversal, the rate of beam elongation caused by thermal expansion is relatively large. Points A and B tend to be compressed, in common with the bottom spring in the buckling element. The deformation reversal of the bottom-spring means that, when large mid-span deflection has been produced in the beam at very high temperatures, the rate of beam elongation caused by thermal expansion is smaller than the beam-end movement due to mid-span deflection. Therefore, the intersection nodes will be stretched towards the beam mid-span. The bottom-flange tends to be stretched rather than being compressed, due to the combined effect of nodal movement and rotation when temperature rises further. During this reversal, the compression spring follows the unloading curve introduced in Section 3. The spring force and its compressive deformation reduce together. After the spring force has changed to tension, the compression spring is disabled and the tension spring becomes active.The response of this spring is represented by the dotted curve. The deformation (33.5mm) of the compression spring before it enters tension, representing the permanent deformation of the bottom flange, will be taken forward by the tension spring. The springs will eventually fail in tension if a failure strain is defined for them. Fig. 13 (b) illustrates the force/temperature-deformation relationships of the top spring. It can be seen that this is under tension when the beam is initially loaded. When the temperature is elevated to around 200°C, the top spring is under compression due to thermal expansion. The stiffness of the top spring remains identical below 200°C. When the temperature is above 200°C, the top spring experiences tension force again, due to large nodal rotation at the intersection point between the buckling element and the beam element. An amplified figure of the top-spring force/temperature and deformation relationship below 600°C is shown in Fig. 13 (b).

Comparisons of the results from the ABAQUS and Vulcan models, in terms of beam mid-span deflection, axial net force and beam-end moment, against temperature, are shown in Figs. 14 - 19. Figs. 14 - 15 show the temperature-deflection relationships for two beams (those modelled in ABAQUS, as described in Section 4.1) under the same load ratios (0.4, 0.5 and 0.6). Figs. 16 - 17 show the temperature-axial force relationships. Figs. 18 - 19 show the temperature-moment relationships. The thick solid lines represent the results from the Vulcan models with buckling elements, whilst the thick dotted lines are for the equivalent ABAQUS models (with shell elements at the beam ends). The thin lines show results from the Vulcan and ABAQUS models without buckling/shell elements.

It can be seen from Figs. 14 - 19 that the results from Vulcan and ABAQUS compare well for beams with and without the buckling elements, for all load ratios.

Figs. 14 - 15 show that the use of the buckling element in Vulcan can improve the accuracy of prediction of mid-span deflection. Models with the buckling elements show the greater deflections, due to the additional beam-end rotations caused by bottom-flange buckling.

Figs. 16 - 17 show that the net compression force at the beam-end decreases when the bottom flange buckles. The beam-end bottom-flange buckling can relieve the axial compression force caused by restraint to thermal expansion, and therefore transfers less compression force onto the adjacent connection element. Beams with the buckling elements initiate their catenary tension phase sooner than those without buckling elements.

Figs. 18 - 19 show the development of the beam-end major-axis moment as temperature increases. It can be seen that, for beams without the buckling element, the beam-end moment increases from 100ºC to 400ºC. This is because both the restraint to thermal expansion and the stiffness reduction of the beam induce an increase in the curvature at its ends, while the steel strength remains unchanged up to 400ºC. The end-moment starts to decrease when the temperature reaches 400ºC, due to the progressive reduction of steel strength above that temperature. For beams with the buckling elements the rotational stiffness of the steel beam-ends is reduced due to the occurrence of bottom flange buckling, resulting in less moment being transferred to the adjacent connections. On the other hand, as the applied load increases the beam-end curvatures increase, causing an increase in the beam-end moment. Therefore, the variation of beam-end moment depends on which of these two is more dominant. It can be observed from Figs. 18 - 19 that in most cases the beam-end moment decreases between 100ºC and 400ºC after bottom-flange buckling has occurred at around 100ºC. However, for the 6m beam subject to a high load ratio of 0.6, and the 9m beams under load ratios of 0.5 and 0.6, the beam-end moment tends to increase slightly between 300ºC and 400ºC.

It has been indicated in the Figs. 13 - 18 that the newly created buckling element is capable of accurately modelling the combined effects of beam-end shear buckling and bottom-flange buckling of steel beams at elevated temperatures. The good match between the Vulcan and ABAQUS modeling results confirms that the buckling element is able to account for the major structural effects of the net axial compression due to restraint to thermal expansion, as mentioned in Section 3.1.

4.2 Verification of the ABAQUS models

The differences between the Vulcan models with and without the buckling elements and between the ABAQUS models with and without shell elements, as shown in Section 4.1, have indicated the importance of considering the beam-end buckling phenomena. This is true only if such differences are not caused by the adoption of different element types, especially for the ABAQUS models. This section examines the sensitivity of the ABAQUS modelling results to the element types adopted. Two ABAQUS models, one using wire elements to model the entire beam and one using shell elements which are restrained against buckling, were used to model the beam ends, retaining the wire elements for the rest of the beam, and results were compared as shown in Fig. 12.

The same mesh size, element type and temperature curve used for the ABAQUS models described in Section 4.1 were used. The beam was of section UB356x171x51 and 3m length; this is short enough to avoid bottom-flange buckling. The beam web was fully restrained against out-of-plane deformation, and therefore no beam-web shear buckling was allowed. The two ABAQUS models resulted in indistinguishable deflection and axial force, as shown in Fig. 20. This confirms that the differences in behaviour between models with and without shell elements at beam-ends are entirely due to beam-end buckling, and are not caused by the use of different element types.

4.3 Illustrative examples of beams with buckling and connection elements

The component-based buckling element has been verified, and the influence of the buckling element on the behaviour of a beam has been demonstrated in Section 4.1. In this section the buckling element is used together with the existing component-based connection element of Vulcan to model isolated beams. Models with and without the buckling elements are compared. The influence of the buckling elements on the beam deflection, and on the internal force distribution among the bolt rows of the adjacent connections, are investigated.

The models are of the same dimensions previously used for the 6m and 9m beams described in Section 4.1. End-plate connections, designed to be moment resistant in accordance with BS EN 1993-1-8 [18] and its accompanying National Annex [19], are used. The connection details are shown in Fig. 21. Grade 8.8 M20 bolts and 15mm thick endplates are used. One purpose of this research is to investigate the influence of the buckling element on the force distribution within the bolt rows of the adjacent connection. To focus on this, the stress area of all bolts is assumed to be 500mm^2 (instead of the usual nominal value 245mm^2 for M20 bolts) to avoid bolt fracture.

Figs. 22 - 23 show comparisons of the force distribution between connection bolt-rows, for beams of different spans (6m and 9m) subject to various load ratios (0.4, 0.5 and 0.6) with and without the buckling element. The component-based connection element is composed of six horizontal springs (two compression springs, representing the top and bottom flanges and four tension springs, representing the bolt rows). The bolt-row springs can only transfer tension force [10]. The general trend is that, in the initial ambient-temperature loading stage, the top three tension bolt-row springs and the bottom flange spring are mobilized to resist the beam-end rotation caused by the external load. After heating starts, the beam starts to expand and the connections are subjected to a combination of compression and bending. At this stage all the four tension bolt rows are progressively deactivated due to the compression caused by the restraint to thermal expansion. Once all the tension bolt rows are deactivated, the top compression spring starts to work. The deflection of the beam increases dramatically when its temperature increases further, and the four tension bolt rows are progressively re-activated; the top compression spring is switched off. At around 700°C the beams start to develop catenary tension, and the bending action is reduced. Eventually all the four tension bolt rows are again activated, and both compression springs are deactivated.

In Figs. 22 - 23 results for models with and without the buckling element are distinguished by line thickness (thick lines for models with buckling element; thin ones for those without). Colours and marker shapes are used to distinguish different bolt rows. When the tension springs (representing bolt rows) are activated at high temperatures, the thick lines are all below the thin lines. In other words, the tension forces at the bolt rows of a connection are lower when buckling is allowed, compared to the equivalent case without buckling elements. This is reasonable, given the lower rotational stiffness at the beam-end in the presence of local buckling. Therefore, the adjacent connection rotates less, resulting in lower forces in each bolt row. Without buckling elements (thin lines), the forces in the upper bolt rows reach their maximum values at lower temperature. This is because, in this case, the peak spring force corresponds to the yielding of a bolt row, and the upper bolt rows yield earlier than the lower ones as temperature rises. After the upper bolt rows have yielded, more load is distributed to the lower bolt rows, accelerating the yielding of those bolt rows. The decrease in the spring forces after their peak values are due to the reduction of yield strength as temperature increases. On the other hand, in the cases with buckling elements, the tension forces in all four bolt-rows reach their peaks at the same temperature. The inclusion of the buckling element allows the consideration of the reduction of bending moment at beam-ends at the post-buckling stage. This causes a reversal of the beam-end rotation, resulting in a decrease in the bolt force.

5. Conclusion

In this study a component-based beam-end buckling element, which considers beam-web shear buckling and bottom-flange buckling within the beam-end buckling zone, has for the first time been created for beams of Classes 1 and 2. The component-based model is able to consider the post-buckling descending force-deflection relationship of its bottom spring, which simulates the bottom-flange buckling behaviour. Each spring in the buckling element is able to deal with deformation reversal, which commonly happens at high temperatures. The buckling element has been implemented into the global frame analysis software Vulcan. The buckling element has been verified against ABAQUS models on isolated beams. After implementing the buckling element the Vulcan models agree considerably better with the ABAQUS models, compared to the Vulcan models without the buckling elements. The influence of the buckling element on the adjacent connection has also been investigated. The results indicate that, by including the buckling element, the net axial compression force and moment transferred from beam to connection have been reduced. Hence, the stresses within the connection bolt rows are reduced when the beam-end buckling is taken into consideration. In general, by ignoring buckling near to the beam-to-column connections in the structural fire engineering design process, the results will tend to be conservative in terms of the connection details specified.

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List of Figure Captions

Fig. 1 Flange buckling and beam-web shear buckling in combination [4]

Fig. 2 The effects of (a) bottom-flange buckling, (b) shear buckling and (c) total deflection on beam vertical deflection

Fig. 3 Component-based column-face connection and beam-end buckling elements

Fig. 4 A schematic illustration of application of the Masing Rule

Fig. 5 Moment-rotation relationship of the buckling element

Fig. 6 Post-buckling behaviour: (a) schematic characteristics of the compression spring on the buckling flange; (b)yield zone for the bottom flange ; (c) yield line pattern

Fig. 7 New unloading-loading curves at different stages: (a) pre-buckling stage; (b) plateau; (c) post-buckling stage

Fig. 8 Schematic characteristics of the tension spring

Fig. 9 Loading and unloading procedure when temperature increases: (a) at temperature T_1; (b) at temperature T_2 ($T_2 > T_1$)

Fig. 10 Flowchart of the developed procedure for modelling the compression spring

Fig. 11 Comparison of Vulcan models: (a) beam with buckling elements; (b) beam with beam elements only

Fig. 12 Comparison of ABAQUS models: (a) beam with shell elements; (b) beam with line elements only

Fig. 13 Force-deformation relationship of the springs in the buckling element: (a) bottom spring; (b) top spring

Fig. 14 The development of mid-span deflection as temperature rises for 6m beams: (a) for load ratio = 0.4; (b) for load ratio = 0.5; (c) for load ratio = 0.6

Fig. 15 The development of mid-span deflection as temperature rises for 9m beams: (a) for load ratio = 0.4; (b) for load ratio = 0.5; (c) for load ratio = 0.6

Fig. 16 Comparisons of axial net force against temperature for 6m beams: (a) for load ratio = 0.4; (b) for load ratio = 0.5; (c) for load ratio = 0.6

Fig. 17 Comparisons of axial net force against temperature for 9m beams: (a) for load ratio = 0.4; (b) for load ratio = 0.5; (c) for load ratio = 0.6

Fig. 18 Comparisons of beam-end bending moment against temperature for 6m beams: (a) for load ratio = 0.4; (b) for load ratio = 0.5; (c) for load ratio = 0.6

Fig. 19 Comparisons of beam-end bending moment against temperature for 9m beams: (a) for load ratio = 0.4; (b) for load ratio = 0.5; (c) for load ratio = 0.6

Fig. 20 Comparison of the ABAQUS results

Fig. 21 Details of the analysed endplate connections: (a) for 6m beams; (b) for 9m beams

Fig. 22 Bolt row force distribution for 6m beams: (a) for load ratio = 0.4; (b) tension zone only for load ratio = 0.4; (c) for load ratio = 0.5; (d) tension zone only for load ratio = 0.5; (e) for load ratio = 0.6; (e) tension zone only for load ratio = 0.6

Fig. 23 Bolt row force distribution for 9m beams: (a) for load ratio = 0.4; (b) tension zone only for load ratio = 0.4; (c) for load ratio = 0.5; (d) tension zone only for load ratio = 0.5; (e) for load ratio = 0.6; (e) tension zone only for load ratio = 0.6