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Mod p Base Change transfer for $\mathbf{GL}_2$

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Abstract

We discuss Base Change functoriality for mod p eigenforms for $\mathbf{GL}_2$ over number fields. We carry out systematic computer experiments and collect data supporting its existence in cases of field extensions K/F where F is imaginary quadratic and K is CM quartic.

Contents

1   Introduction

Mod p eigenforms are Hecke eigenclasses in the characteristic p cohomology groups of arithmetic manifolds. Recently they have received a lot of attention. In [9], P. Scholze proved the existence of mod p Galois representations associated to mod p eigenforms for $\mathbf{GL}_n$ over CM fields. This can be viewed as a mod p version of the Reciprocity Principle of the Langlands Programme.

As for mod p versions of instances of the Functoriality Principle, a mod p version of the Jacquet-Langlands correspondence for mod p Bianchi eigenforms ($\mathbf{GL}_2$ over imaginary quadratic fields) has been formulated by F. Calegari and A. Venkatesh in [3]. Soon after this formulation, A. Page and the second author collected extensive numerical evidence for

the truth of this conjecture[1]. Recently in [11], D. Treumann and A. Venkatesh established a correspondence between mod p eigenforms for a semisimple group $\mathbf{G}$ and $\mathbf{G}^\sigma$ where σ is an automorphism of $\mathbf{G}$ of order p. This implies mod p Base Change transfer of mod p eigenforms for $\mathbf{SL}_n$ in p-power degree Galois extensions.

In this paper, we carry out systematic computations, using the computer programs developed in [10] and [7], and collect numerical data that strongly suggest the existence of Base Change transfer for mod p *Bianchi eigenforms* (that is, $\mathbf{GL}_2$ over imaginary quadratic fields) in *quadratic* extensions. In this case the results of [11] apply but only to mod 2 eigenforms. For a discussion of our data, see Section 2.2.

2 The Conjecture

Let F be a number field with signature (r, s) and ring of integers $\mathbb{Z}_F$. Let G denote the real Lie group $\mathrm{GL}_2(F \otimes \mathbb{R})$, $A \simeq \mathbb{R}_{>0}$ be embedded diagonally into G and K be a maximal compact subgroup of G. Then associated symmetric space is given by

$$D := G/AK \simeq \mathcal{H}_2^r \times \mathcal{H}_3^s \times \mathbb{R}_{>0}^{r+s-1}$$

where $\mathcal{H}_n$ denotes the real hyperbolic n-space.

Let $\hat{\mathbb{Z}}_F, A_F^f$ denote the rings of finite adèles of $\mathbb{Z}_F$ and of F, respectively. Fix an ideal $\mathfrak{N} \subseteq \mathbb{Z}_F$ and define the compact open subgroup

$$U_0(\mathfrak{N}) := \left\{ \gamma \in \mathrm{GL}_2(\hat{\mathbb{Z}}_F) : \gamma \equiv \begin{pmatrix} * & * \\ 0 & * \end{pmatrix} \bmod \mathfrak{N} \right\}.$$

Consider the adelic locally symmetric space

$$Y(\mathfrak{N}) = \mathrm{GL}_2(F) \backslash \left(\left(\mathrm{GL}_2(\mathbb{A}_F^f)/U_0(\mathfrak{N}) \right) \times D \right).$$

This space is a disjoint union

$$Y(\mathfrak{N}) = \bigcup_{j=1}^{h_F} \Gamma_j \backslash D$$

where Γ_j are arithmetic subgroups of $\mathrm{GL}_2(F)$ and h_F is the class number of F. When h_F equals 1, the quotient $Y(\mathfrak{N})$ is simply $\Gamma_0(\mathfrak{N}) \backslash D$.

We shall consider the cohomology groups

$$H^i(Y(\mathfrak{N}), \overline{\mathbb{F}}_p).$$

The groups $H^i(Y(\mathfrak{N}), \overline{\mathbb{F}}_p)$ come equipped with commutative Hecke algebras $\mathbb{T}_k^i(\mathfrak{N})$ (generated by Hecke operators $T_\mathfrak{q}$ associated to prime ideals $\mathfrak{q}$ of $\mathbb{Z}_F$ away from $p\mathfrak{N}$).

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[1]A written account is (still) under preparation.

A **mod p eigenform Ψ (over F) of level $\mathfrak{N}$ and degree i** is a ring homomorphism $\Psi : \mathbb{T}_k^i(\mathfrak{N}) \to \overline{\mathbb{F}}_p$.

It is well-known that the values of a mod p eigenform Ψ generate a finite extension $\mathbb{F}$ of $\mathbb{F}_p$. Let us say that two mod p eigenforms with levels $\mathfrak{N}, \mathfrak{M}$ are **equivalent** if their values agree on Hecke operators associated to prime ideals away from $p\mathfrak{N}\mathfrak{M}$. A conjecture of F. Calegari and M. Emerton (see [2]) predicts that any mod p eigenform should be equivalent to one with the same level and degree $r+s$. When F is imaginary quadratic, the conjecture holds as a result of low dimensionality. When F is totally real, the conjecture is known to be true under some hypotheses [8]. A weaker version of this conjecture would say that any mod p eigenform should be equivalent to one with the same level and degree in the interval $[r + s, \ldots, 2r + 3s - 1]$ (see [4, Section 3.1.1]).

A **complex eigenform f (over F) of level $\mathfrak{N}$ and degree i** is a complex valued character of the Hecke algebra associated to the complex cohomology groups

$$H^i(Y(\mathfrak{N}), \mathbb{C}).$$

It is well-known that the values of a complex eigenform are algebraic integers and they generate a finite extension $\mathbb{K}$ of $\mathbb{Q}$.

Given a complex eigenform f, one can fix an ideal $\mathfrak{p}$ of $\mathbb{K}$ over p and obtain a mod p eigenform Ψ_f (of the same level and degree) by declaring $\Psi_f(T_{\mathfrak{q}}) = f(T_{\mathfrak{q}}) \mod \mathfrak{p}$, for all $\mathfrak{q}$ coprime to $p\mathfrak{N}$. Let us say that a mod p eigenform Ψ *lifts to a complex one* if there is a complex eigenform f with the same level and degree such that $\Psi = \Psi_f$. Otherwise, we call Ψ simply *non-lifting*.

Let us call a complex eigenform f **trivial** if $f(T_{\mathfrak{q}}) = \mathbf{N}_{F/\mathbb{Q}}\mathfrak{q} + 1$ for all prime ideals $\mathfrak{q}$ away from $\mathfrak{N}$. Similarly, a mod p eigenform Ψ is **trivial** if $\Psi(T_{\mathfrak{q}}) = \mathbf{N}_{F/\mathbb{Q}}\mathfrak{q} + 1 \mod p$ for all prime ideals $\mathfrak{q}$ away from $p\mathfrak{N}$. Thanks to Eisenstein series associated to the cusps of $Y(\mathfrak{N})$, a trivial mod p eigenform lifts to a complex one.

2.1 Reciprocity and Base Change

Mod p eigenforms have intimate connections with arithmetic. The **mod p Reciprocity Conjecture**, roughly speaking, establishes a correspondence between mod p eigenforms and mod p Galois representations. We will be interested in the half of this correspondence that associates a mod p Galois representation to a mod p eigenform.

Conjecture 2.1. (mod p Reciprocity) *Let Ψ be a mod p eigenform for over F. Then there is a semisimple, continuous representation*

$$\rho(\Psi) : \mathrm{Gal}(\overline{F}/F) \to \mathrm{GL}_2(\overline{\mathbb{F}}_p)$$

such that

(i) $\rho(\Psi)$ is unramified outside $p\mathfrak{N}$,

(ii) $\mathrm{Tr}(\rho(\Psi)(\mathrm{Frob}_{\mathfrak{q}})) = \Psi(T_{\mathfrak{q}})$ for all primes $\mathfrak{q}$ away from $p\mathfrak{N}$.

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When F is CM, Conjecture 2.1 follows from results obtained by Scholze in [9].

We now consider the conjectural Base Change transfer for mod p eigenforms for $\mathbf{GL}_2$ over number fields. For compactness and flexibility, we will assume Conjecture 2.1 above and use it in our formulation. In the cases where we shall carry out our experiments, Conjecture 2.1 will hold via [9].

Conjecture 2.2. (mod p Base Change) *Let Ψ be a mod p eigenform for over F of level $\mathfrak{N}$. Let K/F be a finite extension. Then there is a mod p eigenform Φ for $\mathbf{G}_K$ of some level such that $\rho(\Psi)_{|G_K} \simeq \rho(\Phi)$.*

In general, we expect[2] to find Φ at level $\mathfrak{N}\mathbb{Z}_K$ where $\mathbb{Z}_K$ is the ring of the integers of K. Note that recent work of Treumann and Venkatesh [11] gives[3] the existence of Base Change transfer of mod p eigenforms for $\mathbf{SL}_n$ in Galois extensions K/F with p-power degree.

2.2 A summary of experiments and the results

In [7], the first author employed methods of P. Gunnells and D. Yasaki (as used in [6]) to develop computer programs that compute with $H^5(\Gamma, \mathbb{Z})$ as a Hecke module in the case of Γ_0-type congruence subgroups Γ of $\mathrm{GL}_2(\mathbb{Z}_K)$ for the CM quartic field $K = \mathbb{Q}(\zeta_{12})$. For this paper, the programs of [7] have been adapted to two other CM quartic fields K, namely $\mathbb{Q}(\zeta_8)$ and $\mathbb{Q}(t)$ where t is a root of $x^4 - x^3 + 2x^2 + x + 1$. The imaginary quadratic fields $F = \mathbb{Q}(\sqrt{-d})$ with $d = 1, 2, 3, 15$ lie inside at least one of these three CM quartic fields K, so we computed "interesting" (see below) mod p Bianchi eigenforms over F with $2 < p < 500$.

Within the bounds for the level we set ourselves (which were dictated by our computational limits in the CM quartic case), we found 34 mod p Bianchi eigenforms Ψ with $p \in \{3, 5, 7, 11, 13, 19, 29, 47, 67, 211\}$. In each case, our computations over K showed that there was a mod p eigenform Φ over K that seemed to be the Base Change transfer of Ψ. We also computed the relevant space of complex eigenforms over K and observed that none of these mod p eigenforms Φ actually lifted to complex eigenforms.

We also paid attention to the multiplicities of the mod p eigenforms. In char. p, the Hecke algebras are not semi-simple and thus it is interesting to keep an eye on the dimensions of the generalized eigenspaces. We observed that the multiplicity of Ψ and that of its Base Change transfer Φ matched except in three mod 3 cases. As Φ's do not lift to complex eigenforms, these exceptional jumps in the multiplicities do not come from congruences between torsion classes and automorphic classes. They arise from "extra" 3-torsion classes. In the first two instances (see 3.2.2, 3.2.3), we checked that the level group of Φ does not have any 3-torsion, while in the third instance (see 3.3.1) it does. Thus in the first two instances, the extra 3-torsion is "genuine" in the sense that it does

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[2]In classical Base Change, when $\mathfrak{N}$ is coprime to the discriminant of the extension K/F, the Base Change transfer appears at level $\mathfrak{N}\mathbb{Z}_K$.

[3]In that paper, the authors do not use a Galois theoretic formulation like we do.

not arise from the group torsion.

3   The Experiment

In this section, we discuss our computations regarding Conjecture 2.2. We operate within the set-up where, in the same notation, F is imaginary quadratic and K is a CM quartic. Thus we are testing quadratic Base Change transfer for mod p Bianchi eigenforms. Recall that results of Treumann and Venkatesh in [11] cover the case $p = 2$ in this set-up and as a consequence, we leave mod 2 eigenforms out of our experiments. Also note that the mod p reciprocity result of Scholze applies to mod p eigenform over both F and K.

Let us describe our experiment. We fix an imaginary quadratic field F. We start by, using modified versions of the algorithms developed in [10, 7], looking for a mod p eigenform Ψ over F of level $\mathfrak{N}$ and degree 2. These forms are known as **mod p Bianchi eigenforms** as the relevant modular group is the Bianchi group $\mathrm{GL}_2(\mathbb{Z}_F)$. Note that here the relevant degrees are 1 and 2. Moreover, any mod p Bianchi eigenform with degree 1 is equivalent to one with degree 2 and vice versa.

A remark is in order. If Ψ lifts to a complex one, then Conjecture 2.2 follows from the classical Base Change functoriality which is proven in quadratic extensions. So we need to focus on non-lifting Ψ. The obstruction to lift Ψ to a complex one arises from p-torsion in the integral cohomology $H^2(Y(\mathfrak{N}), \mathbb{Z})$. So in order to target non-lifting mod p Bianchi eigenforms, we compute, again using algorithms from [10], the torsion appearing in these integral cohomology groups. It follows that we should in particular focus on non-trivial mod p Bianchi eigenforms.

Next we fix a CM quartic field K, containing F. Here we expect, following the weak version of the Calegari-Emerton conjecture (see the Introduction), that all non-trivial mod p eigenforms appear in with degrees in the interval $[2, 5]$. Thanks to Poincaré duality, this reduces to degrees 4 and 5. The virtual cohomological dimension of $\mathrm{GL}_2(\mathbb{Z}_K)$ is 6 and the current methods (see [5] for a survey) allow one to go at most one degree below the virtual cohomological dimension to compute the Hecke action on the cohomology. Thus, adapting the programs of [7] to work with $\overline{\mathbb{F}}_p$ coefficients (see [1] in regards to mod 3 computations in the presence of 3-torsion in the level group), we look for a mod p eigenform Φ over K of degree 5 which is a Base Change transfer of Ψ, that is, $\rho(\Psi)_{|G_K} \simeq \rho(\Phi)$. We always set things so that $\mathfrak{N}$ is coprime to the discriminant of the extension K/F and look for Φ with level $\mathfrak{N}\mathbb{Z}_K$.

The Galois theoretic condition $\rho(\Psi)_{|G_K} \simeq \rho(\Phi)$ can be translated to the following conditions on the values of eigenforms: for every prime $\mathfrak{Q}$ of $\mathbb{Z}_F$ away from $p\mathfrak{N}$ and from

the discriminant of the extension K/F, we have

$$\Phi(T_{\mathfrak{q}}) = \begin{cases} \Psi(T_{\mathfrak{Q}}), & \text{if } \mathfrak{Q} \text{ splits in } K, \\ \Psi(T_{\mathfrak{Q}})^2 - 2\mathbf{N}_{K/F}\mathfrak{q}, & \text{if } \mathfrak{Q} \text{ is inert in } K, \end{cases}$$

for every prime $\mathfrak{q}$ of $\mathbb{Z}_K$ lying above $\mathfrak{Q}$.

Notation and Terminology

For an ideal $\mathfrak{N}$ of $\mathbb{Z}_F$, we define the *Hermite normal form label* of $\mathfrak{N}$ to be the triple $[N, r_1, r_2]$, where $N = \mathbf{N}_{F/\mathbb{Q}}(\mathfrak{N})$, and $\mathfrak{N} = (\frac{N}{r_1}, r_1 + r_2\omega)$.

By *multiplicity* of a mod p eigenform Ψ, we shall mean the dimension of the generalized Ψ-eigenspace $H^i(Y(\mathfrak{N}), \overline{\mathbb{F}}_p)[\Psi]$.

Recall that the field generated by the values of a mod p (reps. complex) eigenform is denoted by $\mathbb{F}$ (resp. $\mathbb{K}$).

The mod p eigenform whose existence is predicted by Conjecture 2.2 will be highlighted.

3.1 Examples with $F = \mathbb{Q}(\sqrt{-1})$

Let $F = \mathbb{Q}(\sqrt{-1})$ and $K = \mathbb{Q}(t)$, where $t = \zeta_{12}$ is a primitive twelfth root of unity. To ensure that our calculations are consistent, we fix an embedding $F \hookrightarrow K$ by setting $\sqrt{-1} = t^3$.

We begin by searching for non-trivial and non-lifting mod p Bianchi eigenforms Ψ. The capacity of the computers at our disposal forced us *restrict our search* levels $\mathfrak{N}$ of norm up to 200 (up to Galois conjugacy) and to primes $2 < p < 500$.

In total we found 3 non-lifting non-trivial mod p Bianchi eigenforms. We list these below, together with values $\Psi(T_{\mathfrak{Q}})$ for a number of ideas $\mathfrak{Q} = [N, r_1, r_2]$ of $\mathbb{Z}_F$. The columns labeled "m_F" and "m_K" denote the multiplicities of Ψ and its Base Change transfer to K respectively.

$\mathfrak{N}$	p	m_F	m_K	$[2,1,1]$	$[9,0,3]$	$[13,8,1]$	$[13,5,1]$	$[5,2,1]$	$[5,3,1]$	$[37,31,1]$	$[37,6,1]$	$[49,0,7]$
$[97,22,1]$	5	1	1	2	4	1	1	$*$	$*$	3	3	1
$[157,28,1]$	3	1	1	2	$*$	1	1	0	0	0	1	2
$[178,55,1]$	7	1	1	$*$	3	2	2	4	2	3	0	$*$

Non-trivial non-lifting mod p Bianchi eigenforms over $\mathbb{Q}(\sqrt{-1})$

Now we look for the Base Change transfer to GL_2 over K of each of the above mod p Bianchi eigenforms. We shall compute with the prime ideals $\mathfrak{q}$ in $\mathbb{Z}_K$ of norm at most 50. We label these according to the following convention:

3.1.1 $\mathfrak{N} = [97, 22, 1], p = 5$

The mod 5 Bianchi eigenform has the following values:

$\mathfrak{q}$	Generator	$\mathbf{N}_{K/\mathbb{Q}}(\mathfrak{q})$	Prime in $\mathbb{Z}_F$ below $\mathfrak{q}$	Splitting behaviour in K/F
$\mathfrak{q}_4$	$-t^2+t+1$	4	$[2,1,1]$	Inert
$\mathfrak{q}_9$	$-t^2-1$	9	$[9,0,3]$	Ramifies
$\mathfrak{q}_{13,1}$	t^3-t^2+t+1	13	$[13,5,1]$	Splits
$\mathfrak{q}_{13,2}$	$-t^3-t^2+2$	13	$[13,5,1]$	Splits
$\mathfrak{q}_{13,3}$	t^3-t^2+2	13	$[13,8,1]$	Splits
$\mathfrak{q}_{13,4}$	t^3+t^2+1	13	$[13,8,1]$	Splits
$\mathfrak{q}_{25,1}$	t^3+2	25	$[5,3,1]$	Inert
$\mathfrak{q}_{25,2}$	$-t^3+2$	25	$[5,2,1]$	Inert
$\mathfrak{q}_{37,1}$	$-t^3-t-2$	37	$[37,6,1]$	Splits
$\mathfrak{q}_{37,2}$	$2t^3+t^2+1$	37	$[37,31,1]$	Splits
$\mathfrak{q}_{37,3}$	$-2t^3+t^2+1$	37	$[37,6,1]$	Splits
$\mathfrak{q}_{37,4}$	$2t^3-t^2+2$	37	$[37,31,1]$	Splits
$\mathfrak{q}_{49,1}$	t^2+2	49	$[49,0,7]$	Splits
$\mathfrak{q}_{49,2}$	$-t^2+3$	49	$[49,0,7]$	Splits

$[2,1,1]$	$[9,0,3]$	$[13,8,1]$	$[13,5,1]$	$[5,2,1]$	$[5,3,1]$	$[37,31,1]$	$[37,6,1]$	$[49,0,7]$
2	4	1	1	$*$	$*$	3	3	1

Let $\mathfrak{n}$ be the ideal generated by the element $-4t^3+9$. Then $H^5(Y(\mathfrak{N}),\overline{\mathbb{F}}_5)$ is 8-dimensional and affords the following eigenforms.

	$[\mathbb{F}:\mathbb{F}_5]$	Multiplicity
φ_1	1	7
φ_2	1	1

The mod 5 eigenforms φ_1 and φ_2 admit the following values:

	$\mathfrak{q}_4$	$\mathfrak{q}_9$	$\mathfrak{q}_{13,1}$	$\mathfrak{q}_{13,2}$	$\mathfrak{q}_{13,3}$	$\mathfrak{q}_{13,4}$	$\mathfrak{q}_{25,1}$	$\mathfrak{q}_{25,2}$	$\mathfrak{q}_{37,1}$	$\mathfrak{q}_{37,2}$	$\mathfrak{q}_{37,3}$	$\mathfrak{q}_{37,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$
φ_1	0	0	4	4	4	4	$*$	$*$	3	3	3	3	0	0
φ_2	0	4	1	1	1	1	$*$	$*$	3	3	3	3	1	1

The complex cohomology group $H^5(\Gamma_0(\mathfrak{n}),\mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.1.2 $\mathfrak{N}=[157,28,1], p=3$

The mod 3 Bianchi eigenform has the following values:

$[2,1,1]$	$[9,0,3]$	$[13,8,1]$	$[13,5,1]$	$[5,2,1]$	$[5,3,1]$	$[37,31,1]$	$[37,6,1]$	$[49,0,7]$
2	$*$	1	1	0	0	0	1	2

Let $\mathfrak{n}$ be the ideal generated by the element $-6t^3-11$. Then $H^5(Y(\mathfrak{N}),\overline{\mathbb{F}}_3)$ is 9-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_3]$	Multiplicity
φ_1	1	8
φ_2	1	1

The mod 3 eigenforms φ_1 and φ_2 admit the following values:

	$\mathfrak{q}_4$	$\mathfrak{q}_9$	$\mathfrak{q}_{13,1}$	$\mathfrak{q}_{13,2}$	$\mathfrak{q}_{13,3}$	$\mathfrak{q}_{13,4}$	$\mathfrak{q}_{25,1}$	$\mathfrak{q}_{25,2}$	$\mathfrak{q}_{37,1}$	$\mathfrak{q}_{37,2}$	$\mathfrak{q}_{37,3}$	$\mathfrak{q}_{37,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$
φ_1	2	*	2	2	2	2	2	2	2	2	2	2	2	2
φ_2	0	*	1	1	1	1	2	2	0	1	0	1	2	2

The complex cohomology group $H^5(\Gamma_0(\mathfrak{n}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.1.3 $\mathfrak{N} = [178, 55, 1], p = 7$

The mod 7 Bianchi eigenform has the following eigenvalues:

$[2,1,1]$	$[9,0,3]$	$[13,8,1]$	$[13,5,1]$	$[5,2,1]$	$[5,3,1]$	$[37,31,1]$	$[37,6,1]$	$[49,0,7]$
*	3	2	2	4	2	3	0	*

Let $\mathfrak{n}$ be the ideal generated by the element $-3t^3 + 13$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_7)$ is 10-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_7]$	Multiplicity
φ_1	1	7
φ_2	1	2
φ_3	1	1

The mod 7 eigenforms $\varphi_1, \varphi_2, \varphi_3$ admit the following values:

	$\mathfrak{q}_4$	$\mathfrak{q}_9$	$\mathfrak{q}_{13,1}$	$\mathfrak{q}_{13,2}$	$\mathfrak{q}_{13,3}$	$\mathfrak{q}_{13,4}$	$\mathfrak{q}_{25,1}$	$\mathfrak{q}_{25,2}$	$\mathfrak{q}_{37,1}$	$\mathfrak{q}_{37,2}$	$\mathfrak{q}_{37,3}$	$\mathfrak{q}_{37,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$
φ_1	*	3	0	0	0	0	5	5	3	3	3	3	*	*
φ_2	*	6	1	1	3	3	2	1	6	2	6	2	*	*
φ_3	*	3	2	2	2	2	6	1	3	0	3	0	*	*

The complex cohomology group $H^5(\Gamma_0(\mathfrak{n}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.2 Examples with $F = \mathbb{Q}(\sqrt{-2})$

Let $F = \mathbb{Q}(\sqrt{-2})$ and $K = \mathbb{Q}(t)$, where $t = \zeta_8$ is a primitive eighth root of unity. To ensure that our calculations are consistent, we fix an embedding $F \hookrightarrow K$ by setting $\sqrt{-2} = t^3 + t$.

We begin by searching for non-trivial and non-lifting mod p Bianchi eigenforms Ψ. The capacity of the computers at our disposal forced us *restrict our search* to levels $\mathfrak{N}$ of norm up to 75 (up to Galois conjugacy) and to primes $2 < p < 500$

In total we found 5 non-lifting non-lifting Bianchi eigenforms. We list these below, together with values $\Psi(T_\mathfrak{Q})$ for a number of ideas $\mathfrak{Q} = [N, r_1, r_2]$ of $\mathbb{Z}_F$. The columns labeled "m_F" and "m_K" denote the multiplicities of Ψ and its Base Change transfer to K respectively.

Level	p	m_F	m_K	$[2,0,1]$	$[3,1,1]$	$[3,2,1]$	$[17,10,1]$	$[17,7,1]$	$[25,0,5]$	$[41,30,1]$	$[41,11,1]$	$[49,0,7]$
$[33,19,1]$	3	1	1	2	$*$	$*$	2	0	2	0	0	0
$[38,6,1]$	3	1	2	$*$	$*$	$*$	1	0	1	0	2	2
$[66,52,1]$	3	2	3	$*$	$*$	$*$	2	0	2	0	0	0
$[67,47,1]$	3	1	1	1	$*$	$*$	0	0	1	1	1	1
$[73,12,1]$	19	1	1	9	16	5	9	14	13	4	10	3

Non-trivial non-lifting mod p Bianchi eigenforms over $\mathbb{Q}(\sqrt{-2})$

We shall compute with the prime ideals $\mathfrak{q}$ in $\mathbb{Z}_K$ of norm at most 50. We label these according to the following convention:

$\mathfrak{q}$	Generator	$\mathbf{N}_{K/\mathbb{Q}}\mathfrak{q}$	Prime in $\mathbb{Z}_F$ below $\mathfrak{q}$	Splitting behaviour in K/F
$\mathfrak{q}_2$	$t-1$	2	$[2,0,1]$	Ramifies
$\mathfrak{q}_{9,1}$	t^2+t-1	9	$[3,1,1]$	Inert
$\mathfrak{q}_{9,2}$	$-t^3-t^2-1$	9	$[3,2,1]$	Inert
$\mathfrak{q}_{17,1}$	$t+2$	17	$[17,10,1]$	Splits
$\mathfrak{q}_{17,2}$	t^3+2	17	$[17,10,1]$	Splits
$\mathfrak{q}_{17,3}$	$-t^3+2$	17	$[17,7,1]$	Splits
$\mathfrak{q}_{17,4}$	$-t+2$	17	$[17,7,1]$	Splits
$\mathfrak{q}_{25,1}$	t^2+2	25	$[25,0,5]$	Splits
$\mathfrak{q}_{25,2}$	$-t^2+2$	25	$[25,0,5]$	Splits
$\mathfrak{q}_{41,1}$	$-t^3+t^2-t+2$	41	$[41,30,1]$	Splits
$\mathfrak{q}_{41,2}$	$-2t^3+t^2-t+1$	41	$[41,11,1]$	Splits
$\mathfrak{q}_{41,3}$	$-t^3-t^2-t+2$	41	$[41,30,1]$	Splits
$\mathfrak{q}_{41,4}$	t^3+t^2+t+2	41	$[41,11,1]$	Splits
$\mathfrak{q}_{49,1}$	$-t^3-2t^2+2$	49	$[49,0,7]$	Splits
$\mathfrak{q}_{49,2}$	$2t^2-t+2$	49	$[49,0,7]$	Splits

3.2.1 $\quad \mathfrak{N}=[33,19,1], p=3$

The mod 3 Bianchi eigenform has the following values:

$[2,0,1]$	$[3,1,1]$	$[3,2,1]$	$[17,10,1]$	$[17,7,1]$	$[25,0,5]$	$[41,30,1]$	$[41,11,1]$	$[49,0,7]$
2	$*$	$*$	2	0	2	0	0	0

Let $\mathfrak{n}$ be the ideal generated by the element $2t^3+2t+5$. Then $H^5(Y(\mathfrak{N}),\overline{\mathbb{F}}_3)$ is 10-dimensional and affords the following eigenforms

	$[\mathbb{F}:\mathbb{F}_3]$	Multiplicity
φ_1	1	7
φ_2	1	2
φ_3	1	1

The mod 3 eigenforms φ_1, φ_2 and φ_3 admit the following values.

	$\mathfrak{q}_2$	$\mathfrak{q}_{9,1}$	$\mathfrak{q}_{9,2}$	$\mathfrak{q}_{17,1}$	$\mathfrak{q}_{17,2}$	$\mathfrak{q}_{17,3}$	$\mathfrak{q}_{17,4}$	$\mathfrak{q}_{25,1}$	$\mathfrak{q}_{25,2}$	$\mathfrak{q}_{41,1}$	$\mathfrak{q}_{41,2}$	$\mathfrak{q}_{41,3}$	$\mathfrak{q}_{41,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$
φ_1	0	$*$	$*$	0	0	0	0	2	2	0	0	0	0	2	2
φ_2	2	$*$	$*$	1	1	0	0	2	2	0	2	0	2	1	1
φ_3	2	$*$	$*$	2	2	0	0	2	2	0	0	0	0	0	0

The complex cohomology group $H^5(Y(\mathfrak{N}),\mathbb{C})$ is 8-dimensional and affords the following complex eigenforms.

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	$[\mathbb{K}:\mathbb{Q}]$	Multiplicity
ϕ_1	1	7
ϕ_2	1	2

The complex eigenforms ϕ_1 and ϕ_2 admit the following values:

	$\mathfrak{q}_2$	$\mathfrak{q}_{9,1}$	$\mathfrak{q}_{9,2}$	$\mathfrak{q}_{17,1}$	$\mathfrak{q}_{17,2}$	$\mathfrak{q}_{17,3}$	$\mathfrak{q}_{17,4}$	$\mathfrak{q}_{25,1}$	$\mathfrak{q}_{25,2}$	$\mathfrak{q}_{41,1}$	$\mathfrak{q}_{41,2}$	$\mathfrak{q}_{41,3}$	$\mathfrak{q}_{41,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$
ϕ_1	3	*	10	18	18	18	18	26	26	42	42	42	42	50	50
ϕ_2	2	*	6	-2	-2	-6	-6	2	2	-6	2	-6	2	10	10

3.2.2 $\mathfrak{N} = [38, 6, 1], p = 3$

The non-lifting mod 3 Bianchi eigenform has the following values:

$[2,0,1]$	$[3,1,1]$	$[3,2,1]$	$[17,10,1]$	$[17,7,1]$	$[25,0,5]$	$[41,30,1]$	$[41,11,1]$	$[49,0,7]$
*	*	*	1	0	1	0	2	2

Let $\mathfrak{n}$ be the ideal generated by the element $-t^3 - t - 6$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_3)$ is 15-dimensional and affords the following eigenforms:

	$[\mathbb{F}:\mathbb{F}_3]$	Multiplicity
φ_1	1	13
φ_2	1	2

The mod 3 eigenforms φ_1 and φ_2 admit the following values:

	$\mathfrak{q}_2$	$\mathfrak{q}_{9,1}$	$\mathfrak{q}_{9,2}$	$\mathfrak{q}_{17,1}$	$\mathfrak{q}_{17,2}$	$\mathfrak{q}_{17,3}$	$\mathfrak{q}_{17,4}$	$\mathfrak{q}_{25,1}$	$\mathfrak{q}_{25,2}$	$\mathfrak{q}_{41,1}$	$\mathfrak{q}_{41,2}$	$\mathfrak{q}_{41,3}$	$\mathfrak{q}_{41,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$
φ_1	*	*	*	0	0	0	0	2	2	0	0	0	0	2	2
φ_2	*	*	*	1	1	0	0	1	1	0	2	0	2	2	2

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 11-dimensional and affords only trivial complex eigenforms.

3.2.3 $\mathfrak{N} = [66, 52, 1], p = 3$

The non-lifting Bianchi eigenform has the following eigenvalues:

$[2, 0, 1]$	$[3, 1, 1]$	$[3, 2, 1]$	$[17, 10, 1]$	$[17, 7, 1]$	$[25, 0, 5]$	$[41, 30, 1]$	$[41, 11, 1]$	$[49, 0, 7]$
*	*	*	2	0	2	0	0	0

Let $\mathfrak{n}$ be the ideal generated by the element $5t^3 + 5t - 4$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_3)$ is a 32-dimensional and affords the following eigenforms:

	$[\mathbb{F} : \mathbb{F}_3]$	Multiplicity
φ_1	1	23
φ_2	1	6
φ_3	1	3

The mod 3 eigenforms φ_1, φ_2 and φ_3 admit the following eigenvalues:

	$\mathfrak{q}_2$	$\mathfrak{q}_{9,1}$	$\mathfrak{q}_{9,2}$	$\mathfrak{q}_{17,1}$	$\mathfrak{q}_{17,2}$	$\mathfrak{q}_{17,3}$	$\mathfrak{q}_{17,4}$	$\mathfrak{q}_{25,1}$	$\mathfrak{q}_{25,2}$	$\mathfrak{q}_{41,1}$	$\mathfrak{q}_{41,2}$	$\mathfrak{q}_{41,3}$	$\mathfrak{q}_{41,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$
φ_1	*	*	*	0	0	0	0	2	2	0	0	0	0	2	2
φ_2	*	*	*	1	1	0	0	2	2	0	2	0	2	1	1
φ_3	*	*	*	2	2	0	0	2	2	0	0	0	0	0	0

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 26-dimensional and affords the following complex eigenforms:

	$[\mathbb{K} : \mathbb{Q}]$	Multiplicity
ϕ_1	1	23
ϕ_2	1	3

The complex eigenforms ϕ_1 and ϕ_2 admit the following eigenvalues:

	$\mathfrak{q}_2$	$\mathfrak{q}_{9,1}$	$\mathfrak{q}_{9,2}$	$\mathfrak{q}_{17,1}$	$\mathfrak{q}_{17,2}$	$\mathfrak{q}_{17,3}$	$\mathfrak{q}_{17,4}$	$\mathfrak{q}_{25,1}$	$\mathfrak{q}_{25,2}$	$\mathfrak{q}_{41,1}$	$\mathfrak{q}_{41,2}$	$\mathfrak{q}_{41,3}$	$\mathfrak{q}_{41,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$
ϕ_1	*	*	10	18	18	18	18	26	26	42	42	42	42	50	50
ϕ_2	*	*	6	-2	-2	-6	-6	2	2	-6	2	-6	2	10	10

3.2.4 $\mathfrak{N} = [67, 47, 1], p = 3$

The non-lifting Bianchi eigensystem has the following eigenvalues:

$[2,0,1]$	$[3,1,1]$	$[3,2,1]$	$[17,10,1]$	$[17,7,1]$	$[25,0,5]$	$[41,30,1]$	$[41,11,1]$	$[49,0,7]$
1	*	*	0	0	1	1	1	1

Let $\mathfrak{n}$ be the ideal generated by the element $3t^3 + 3t + 7$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_3)$ is a 7-dimensional and affords the following mod 3 eigenforms:

	$[\mathbb{F} : \mathbb{F}_3]$	Multiplicity
φ_1	1	5
φ_2	1	1
φ_3	1	1

The mod 3 eigenforms φ_1, φ_2 and φ_3 admit the following eigenvalues:

	q_2	$q_{9,1}$	$q_{9,2}$	$q_{17,1}$	$q_{17,2}$	$q_{17,3}$	$q_{17,4}$	$q_{25,1}$	$q_{25,2}$	$q_{41,1}$	$q_{41,2}$	$q_{41,3}$	$q_{41,4}$	$q_{49,1}$	$q_{49,2}$
φ_1	0	*	*	0	0	0	0	2	2	0	0	0	0	2	2
φ_2	1	*	*	0	0	0	0	1	1	1	1	1	1	1	1
φ_3	1	*	*	1	1	2	2	2	2	0	1	0	1	1	1

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 4-dimensional and affords the following complex eigenforms:

	$[\mathbb{K} : \mathbb{Q}]$	Multiplicity
ϕ_1	1	3
ϕ_2	1	1

The complex eigenforms ϕ_1 and ϕ_2 admit the following eigenvalues:

	q_2	$q_{9,1}$	$q_{9,2}$	$q_{17,1}$	$q_{17,2}$	$q_{17,3}$	$q_{17,4}$	$q_{25,1}$	$q_{25,2}$	$q_{41,1}$	$q_{41,2}$	$q_{41,3}$	$q_{41,4}$	$q_{49,1}$	$q_{49,2}$
ϕ_1	3	10	10	18	18	18	18	26	26	42	42	42	42	50	50
ϕ_2	-2	5	5	-2	-2	8	8	-4	-4	-3	-8	-3	-8	-5	-5

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3.2.5 $\mathfrak{N} = [73, 12, 1], p = 19$

The non-lifting Bianchi eigenform has the following eigenvalues:

$[2,0,1]$	$[3,1,1]$	$[3,2,1]$	$[17,10,1]$	$[17,7,1]$	$[25,0,5]$	$[41,30,1]$	$[41,11,1]$	$[49,0,7]$
9	16	5	9	14	13	4	10	3

Let $\mathfrak{n}$ be the ideal generated by the element $-6t^3 - 6t + 1$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_{19})$ is an 8-dimensional and affords the following eigenforms:

	$[\mathbb{F} : \mathbb{F}_{19}]$	Multiplicity
φ_1	1	7
φ_2	1	1

The mod 19 eigenforms φ_1 and φ_2 admit the following eigenvalues:

	$\mathfrak{q}_2$	$\mathfrak{q}_{9,1}$	$\mathfrak{q}_{9,2}$	$\mathfrak{q}_{17,1}$	$\mathfrak{q}_{17,2}$	$\mathfrak{q}_{17,3}$	$\mathfrak{q}_{17,4}$	$\mathfrak{q}_{25,1}$	$\mathfrak{q}_{25,2}$	$\mathfrak{q}_{41,1}$	$\mathfrak{q}_{41,2}$	$\mathfrak{q}_{41,3}$	$\mathfrak{q}_{41,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$
φ_1	3	10	10	18	18	18	18	7	7	4	4	4	4	12	12
φ_2	9	3	0	9	9	14	14	13	13	4	10	4	10	3	3

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.3   Examples with $F = \mathbb{Q}(\sqrt{-3})$

Let $F = \mathbb{Q}(\sqrt{-3})$ and $K = \mathbb{Q}(t)$, where $t = \zeta_{12}$ is a primitive twelfth root of unity. To ensure that our calculations are consistent, we fix an embedding $F \hookrightarrow K$ by setting $\sqrt{-3} = 2t^2 - 1$.

We begin by searching for non-trivial and non-lifting mod p Bianchi eigenforms Ψ. The capacity of the computers at our disposal forced us *restrict our search* to levels $\mathfrak{N}$ of norm up to 200 (up to Galois conjugacy) and to primes $2 < p < 500$.

In total we found 1 such eigenform. We list it below, together with values $\Psi(T_{\mathfrak{Q}})$ for a number of ideas $\mathfrak{Q} = [N, r_1, r_2]$ of $\mathbb{Z}_F$. The columns labeled "m_F" and "m_K" denote the multiplicities of Ψ and its Base Change transfer to K respectively.

$\mathfrak{N}$	p	m_F	m_K	$[4,0,2]$	$[3,1,1]$	$[13,3,1]$	$[13,9,1]$	$[25,0,5]$	$[37,26,1]$	$[37,10,1]$	$[7,4,1]$	$[7,2,1]$
$[133,102,1]$	3	1	2	1	*	1	1	2	1	2	*	0

Non-trivial non-lifting mod p Bianchi eigenforms over $\mathbb{Q}(\sqrt{-3})$

We shall compute with the prime ideals $\mathfrak{q}$ in $\mathbb{Z}_K$ of norm at most 50. We label these according to the following convention:

$\mathfrak{q}$	Generator	$\mathbf{N}_{K/\mathbb{Q}}\mathfrak{q}$	Prime in $\mathbb{Z}_F$ below $\mathfrak{q}$	Splitting behaviour
$\mathfrak{q}_4$	$-t^2 + t + 1$	4	$[4,0,2]$	Ramifies
$\mathfrak{q}_9$	$-t^2 - 1$	9	$[3,1,1]$	Inert
$\mathfrak{q}_{13,1}$	$t^3 - t^2 + t + 1$	13	$[13,9,1]$	Splits
$\mathfrak{q}_{13,2}$	$-t^3 - t^2 + 2$	13	$[13,3,1]$	Splits
$\mathfrak{q}_{13,3}$	$t^3 - t^2 + 2$	13	$[13,3,1]$	Splits
$\mathfrak{q}_{13,4}$	$t^3 + t^2 + 1$	13	$[13,9,1]$	Splits
$\mathfrak{q}_{25,1}$	$t^3 + 2$	25	$[25,0,5]$	Splits
$\mathfrak{q}_{25,2}$	$-t^3 + 2$	25	$[25,0,5]$	Splits
$\mathfrak{q}_{37,1}$	$-t^3 - t - 2$	37	$[37,10,1]$	Splits
$\mathfrak{q}_{37,2}$	$2t^3 + t^2 + 1$	37	$[37,26,1]$	Splits
$\mathfrak{q}_{37,3}$	$-2t^3 + t^2 + 1$	37	$[37,26,1]$	Splits
$\mathfrak{q}_{37,4}$	$2t^3 - t^2 + 2$	37	$[37,10,1]$	Splits
$\mathfrak{q}_{49,1}$	$t^2 + 2$	49	$[7,2,1]$	Inert
$\mathfrak{q}_{49,2}$	$-t^2 + 3$	49	$[7,2,1]$	Inert

3.3.1 $\mathfrak{N} = [133, 102, 1], p = 3$

The mod 3 Bianchi eigenform has the following values:

$[4,0,2]$	$[3,1,1]$	$[13,3,1]$	$[13,9,1]$	$[25,0,5]$	$[37,26,1]$	$[37,10,1]$	$[7,4,1]$	$[7,2,1]$
1	*	1	1	2	1	2	*	0

Let $\mathfrak{n}$ be the ideal generated by the element $4t^2 + 9$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_3)$ is 13-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_3]$	Multiplicity
φ_1	1	10
φ_2	1	2
φ_3	1	1

The mod 3 eigenforms φ_1, φ_2 and φ_3 admit the following values:

	$\mathfrak{q}_4$	$\mathfrak{q}_9$	$\mathfrak{q}_{13,1}$	$\mathfrak{q}_{13,2}$	$\mathfrak{q}_{13,3}$	$\mathfrak{q}_{13,4}$	$\mathfrak{q}_{25,1}$	$\mathfrak{q}_{25,2}$	$\mathfrak{q}_{37,1}$	$\mathfrak{q}_{37,2}$	$\mathfrak{q}_{37,3}$	$\mathfrak{q}_{37,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$
φ_1	2	*	2	2	2	2	2	2	2	2	2	2	2	*
φ_2	1	*	1	1	1	1	2	2	2	1	1	2	1	*
φ_3	0	*	1	2	2	1	0	0	2	1	1	2	1	*

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

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3.4 Examples with $F = \mathbb{Q}(\sqrt{-15})$

Let $F = \mathbb{Q}(\sqrt{-15})$ and $K = \mathbb{Q}(t)$, where t is a root of the polynomial $x^4 - x^3 + 2x^2 + x + 1$. To ensure that our calculations are consistent, we fix an embedding $F \hookrightarrow K$ by setting $\sqrt{-15} = 2t^3 - 2t^2 + 6t + 1$.

We begin by searching for non-trivial and non-lifting mod p Bianchi eigenforms Ψ. The capacity of the computers at our disposal forced us *restrict our search* to levels $\mathfrak{N}$ of norm up to 100 (up to Galois conjugacy) and to primes $2 < p < 500$.

We found the following 25 examples of non-lifting non-trivial mod p Bianchi forms. We list these below, together with values $\Psi(T_\mathfrak{Q})$ for a number of ideas $\mathfrak{Q} = [N, r_1, r_2]$ of $\mathbb{Z}_F$. The columns labeled "m_F" and "m_K" denote the multiplicities of Ψ and its Base Change transfer to K respectively.

Level	p	m_F	m_K	$[19,8,1]$	$[19,10,1]$	$[31,17,1]$	$[31,13,1]$	$[49,0,7]$	$[61,15,1]$	$[61,45,1]$
$[17,5,1]$	3	1	1	0	2	0	1	2	0	2
$[34,5,1]$	3	2	2	0	2	0	1	2	0	2
$[46,39,1]$	7	1	1	13	0	0	2	*	3	4
$[47,9,1]$	7	1	1	1	4	6	1	*	0	4
$[51,22,1]$	3	3	3	0	2	0	1	2	0	2
$[53,20,1]$	7	1	1	3	1	2	6	*	6	4
$[53,20,1]$	47	1	1	5	42	23	29	12	34	3
$[61,45,1]$	11	1	1	9	1	7	2	10	5	*
$[62,17,1]$	3	1	1	2	1	*	2	2	2	0
$[64,24,2]$	3	1	1	1	0	1	1	1	2	1
$[68,39,1]$	3	3	3	0	2	0	1	2	0	2
$[76,67,1]$	5	1	1	2	*	3	3	0	4	3
$[79,35,1]$	29	1	1	19	1	13	21	12	11	27
$[80,24,2]$	5	1	1	0	1	1	0	1	1	3
$[83,51,1]$	7	1	1	1	2	5	5	*	3	6
$[85,62,1]$	3	2	2	2	0	1	0	2	2	0
$[85,62,1]$	11	1	1	6	0	1	6	6	10	0
$[85,62,1]$	13	1	1	1	5	0	4	11	9	5
$[92,52,1]$	7	2	2	0	3	2	0	*	4	3
$[92,52,1]$	7	1	1	0	1	1	1	*	1	5
$[93,79,1]$	7	1	1	5	0	*	5	*	4	1
$[94,9,1]$	7	2	2	1	4	6	1	*	0	4
$[94,9,1]$	211	1	1	99	92	41	201	88	15	185
$[94,37,1]$	7	2	2	4	1	1	6	*	4	0
$[94,37,1]$	67	1	1	3	20	0	59	18	29	18

Non-trivial non-lifting mod p Bianchi eigenforms over $\mathbb{Q}(\sqrt{-15})$

We shall compute with the prime ideals $\mathfrak{q}$ in $\mathbb{Z}_K$ of norm at most 70. We label these according to the following convention:

$\mathfrak{q}$	Generator	$\mathbf{N}_{K/\mathbb{Q}}\mathfrak{q}$	Prime in $\mathbb{Z}_F$ below $\mathfrak{q}$	Splitting behaviour
$\mathfrak{q}_{19,1}$	$-t^3 + t^2 - 2t + 1$	19	$[19, 10, 1]$	Splits
$\mathfrak{q}_{19,2}$	$\frac{1}{2}(-t^3 + 2t^2 - 4t - 3)$	19	$[19, 8, 1]$	Splits
$\mathfrak{q}_{19,3}$	$\frac{1}{2}(-t^3 - 2t - 5)$	19	$[19, 8, 1]$	Splits
$\mathfrak{q}_{19,4}$	$t - 2$	19	$[19, 10, 1]$	Splits
$\mathfrak{q}_{31,1}$	$-2t^3 + 2t^2 - 4t - 1$	31	$[31, 13, 1]$	Splits
$\mathfrak{q}_{31,2}$	$\frac{1}{2}(t^3 + 2t^2 - 2t + 3)$	31	$[31, 17, 1]$	Splits
$\mathfrak{q}_{31,3}$	$\frac{1}{2}(t^3 + 2t^2 - 2t + 5)$	31	$[31, 17, 1]$	Splits
$\mathfrak{q}_{31,4}$	$\frac{1}{2}(3t^3 - 2t^2 + 2t + 3)$	31	$[31, 13, 1]$	Splits
$\mathfrak{q}_{49,1}$	$t^3 - 2t^2 + 2t + 2$	49	$[49, 0, 7]$	Splits
$\mathfrak{q}_{49,2}$	$-t^3 + 2t^2 - 2t + 2$	49	$[49, 0, 7]$	Splits
$\mathfrak{q}_{61,1}$	$t^3 - 3t^2 + 4t - 1$	61	$[61, 15, 1]$	Splits
$\mathfrak{q}_{61,2}$	$\frac{1}{2}(-3t^3 + 4t^2 - 2t - 3)$	61	$[61, 45, 1]$	Splits
$\mathfrak{q}_{61,3}$	$\frac{1}{2}(-t^3 - 4t - 5)$	61	$[61, 15, 1]$	Splits
$\mathfrak{q}_{61,4}$	$\frac{1}{2}(3t^3 - 4t^2 + 8t - 3)$	61	$[61, 45, 1]$	Splits

3.4.1 $\quad \mathfrak{N} = [17, 5, 1], p = 3$

The mod 3 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
0	2	0	1	2	0	2

Let $\mathfrak{n}$ be the ideal generated by the element $\frac{1}{2}(5t^3 - 6t^2 + 10t - 3)$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_3)$ is 4-dimensional and affords the following eigenforms

	$[\mathbb{F} : \mathbb{F}_3]$	Multiplicity
φ_1	1	3
φ_2	1	1

The mod 3 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
φ_2	0	2	2	0	0	1	1	0	2	2	0	2	0	2

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 3-dimensional and affords only trivial complex eigenforms.

3.4.2 $\mathfrak{N} = [34, 5, 1], p = 3$

The mod 3 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
0	2	0	1	2	0	2

Let $\mathfrak{n}$ be the ideal generated by the element $\frac{1}{2}(3t^3 - 10t^2 + 8t - 9)$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_3)$ is 9-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_3]$	Multiplicity
φ_1	1	7
φ_2	1	2

The mod 3 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
φ_2	0	2	2	0	0	1	1	0	2	2	0	2	0	2

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.4.3 $\mathfrak{N} = [46, 39, 1], p = 7$

The mod 7 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
3	0	0	2	*	3	4

Let $\mathfrak{n}$ be the ideal generated by the element $t^3 - t^2 + 3t - 6$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_7)$ is 8-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_7]$	Multiplicity
φ_1	1	7
φ_2	1	1

The mod 7 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	6	6	6	6	4	4	4	4	*	*	6	6	6	6
φ_2	3	0	0	3	0	2	2	0	*	*	3	4	3	4

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.4.4 $\mathfrak{N} = [47, 9, 1], p = 7$

The mod 7 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
1	4	6	1	$*$	0	4

Let $\mathfrak{n}$ be the ideal generated by the element $-5t^3 + 6t^2 - 6t - 4$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_7)$ is 4-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_7]$	Multiplicity
φ_1	1	3
φ_2	1	1

The mod 7 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	6	6	6	6	4	4	4	4	$*$	$*$	6	6	6	6
φ_2	1	4	4	1	6	1	1	6	$*$	$*$	0	4	0	4

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 3-dimensional and affords only trivial complex eigenforms.

3.4.5 $\mathfrak{N} = [51, 22, 1], p = 3$

The mod 3 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
0	2	0	1	2	0	2

Let $\mathfrak{n}$ be the ideal generated by the element $2t^3 - 2t^2 + 6t - 5$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_3)$ is 10-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_3]$	Multiplicity
φ_1	1	7
φ_2	1	3

The mod 3 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
φ_2	0	2	2	0	0	1	1	0	2	2	0	2	0	2

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.4.6 $\mathfrak{N} = [53, 20, 1], p = 7$

The mod 7 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
3	1	2	6	*	6	4

Let $\mathfrak{n}$ be the ideal generated by the element $\frac{1}{2}(7t^3 - 10t^2 + 14t - 9)$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_7)$ is 4-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_7]$	Multiplicity
φ_1	1	3
φ_2	1	1

The mod 7 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	6	6	6	6	4	4	4	4	*	*	6	6	6	6
φ_2	3	1	1	3	2	6	6	2	*	*	6	4	6	4

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 3-dimensional and affords only trivial complex eigenforms.

3.4.7 $\mathfrak{N} = [53, 20, 1], p = 47$

The mod 47 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
5	42	23	29	12	34	3

Let $\mathfrak{n}$ be the ideal generated by the element $\frac{1}{2}(7t^3 - 10t^2 + 14t - 9)$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_{47})$ is 4-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_{47}]$	Multiplicity
φ_1	1	3
φ_2	1	1

The mod 47 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	20	20	20	20	32	32	32	32	3	3	15	15	15	15
φ_2	5	42	42	5	23	29	29	23	12	12	34	3	34	3

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 3-dimensional and affords only trivial complex eigenforms.

3.4.8 $\mathfrak{N} = [61, 45, 1], p = 11$

The mod 11 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
9	1	7	2	10	5	*

Let $\mathfrak{n}$ be the ideal generated by the element $-4t^3 + 4t^2 - 12t - 1$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_{11})$ is 8-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_{11}]$	Multiplicity
φ_1	1	7
φ_2	1	1

The mod 11 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	9	9	9	9	10	10	10	10	6	6	7	*	7	*
φ_2	9	1	1	9	7	2	2	7	10	10	5	*	5	*

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.4.9 $\mathfrak{N} = [62, 17, 1], p = 3$

The mod 3 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
2	1	*	2	2	2	0

Let $\mathfrak{n}$ be the ideal generated by the element $\frac{1}{2}(-3t^3 + 6t^2 - 20t + 5)$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_3)$ is 18-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_3]$	Multiplicity
φ_1	1	17
φ_2	1	1

The mod 3 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	2	2	2	2	*	2	2	*	2	2	2	2	2	2
φ_2	2	1	1	2	*	2	2	*	2	2	2	0	2	0

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 17-dimensional and affords only trivial complex eigenforms.

3.4.10 $\mathfrak{N} = [64, 24, 2], p = 3$

The mod 3 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
1	0	1	1	1	2	1

Let $\mathfrak{n}$ be the ideal generated by the element $4t^3 - 6t^2 + 4t + 6$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_3)$ is 24-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_3]$	Multiplicity
φ_1	1	23
φ_2	1	1

The mod 3 eigenforms φ_1 and φ_2 admit the following values.:

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
φ_2	1	0	0	1	1	1	1	1	1	1	2	1	2	1

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 23-dimensional and affords only trivial complex eigenforms.

3.4.11 $\mathfrak{N} = [68, 39, 1], p = 3$

The mod 3 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
0	2	0	1	2	0	2

Let $\mathfrak{n}$ be the ideal generated by the element $-2t^3 - 3t^2 + 3t - 7$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_3)$ is 14-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_3]$	Multiplicity
φ_1	1	11
φ_2	1	3

The mod 3 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
φ_2	0	2	2	0	0	1	1	0	2	2	0	2	0	2

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 11-dimensional and affords only trivial complex eigenforms.

3.4.12 $\mathfrak{N} = [76, 67, 1], p = 5$

The mod 5 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
2	$*$	3	3	0	4	3

Let $\mathfrak{n}$ be the ideal generated by the element $\frac{1}{2}(-9t^3 + 16t^2 - 14t - 11)$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_5)$ is 24-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_5]$	Multiplicity
φ_1	1	23
φ_2	1	1

The mod 5 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	0	$*$	$*$	0	2	2	2	2	0	0	2	2	2	2
φ_2	2	$*$	$*$	2	3	3	3	3	0	0	4	3	4	3

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 23-dimensional and affords only trivial complex eigenforms.

3.4.13 $\mathfrak{N} = [79, 35, 1], p = 29$

The mod 29 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
19	1	13	21	12	11	27

Let $\mathfrak{n}$ be the ideal generated by the element $\frac{1}{2}(13t^3 - 18t^2 + 22t - 1)$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_{29})$ is 8-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_{29}]$	Multiplicity
φ_1	1	7
φ_2	1	1

The mod 29 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	20	20	20	20	3	3	3	3	21	21	4	4	4	4
φ_2	19	1	1	19	13	21	21	13	12	12	11	27	11	27

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.4.14 $\mathfrak{N} = [80, 24, 2], p = 5$

The mod 5 Bianchi eigenform has the following values:

$[19,8,1]$	$[19,10,1]$	$[31,17,1]$	$[31,13,1]$	$[49,0,7]$	$[61,15,1]$	$[61,45,1]$
0	1	1	0	1	1	3

Let $\mathfrak{n}$ be the ideal generated by the element $2t^3 - 4t^2 + 2t + 8$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_5)$ is 32-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_5]$	Multiplicity
φ_1	1	31
φ_2	1	1

The mod 5 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	0	0	0	0	2	2	2	2	0	0	2	2	2	2
φ_2	0	1	1	0	1	0	0	1	1	1	1	3	1	3

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 31-dimensional and affords only trivial complex eigenforms.

3.4.15 $\mathfrak{N} = [83, 51, 1], p = 7$

The mod 7 Bianchi eigenform has the following values:

$[19,8,1]$	$[19,10,1]$	$[31,17,1]$	$[31,13,1]$	$[49,0,7]$	$[61,15,1]$	$[61,45,1]$
1	2	5	5	*	3	6

Let $\mathfrak{n}$ be the ideal generated by the element $-5t^3 + 2t^2 - 2t - 8$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_7)$ is 4-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_7]$	Multiplicity
φ_1	1	3
φ_2	1	1

The mod 7 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	6	6	6	6	4	4	4	4	*	*	6	6	6	6
φ_2	1	2	2	1	5	5	5	5	*	*	3	6	3	6

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 3-dimensional and affords only trivial complex eigenforms.

3.4.16 $\mathfrak{N} = [85, 62, 1], p = 3$

The mod 3 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
2	0	1	0	2	2	0

Let $\mathfrak{n}$ be the ideal generated by the element $\frac{1}{2}(-7t^3 + 6t^2 + 2t - 15)$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_3)$ is 9-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_3]$	Multiplicity
φ_1	1	7
φ_2	1	2

The mod 3 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
φ_2	2	0	0	2	1	0	0	1	2	2	2	0	2	0

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.4.17 $\mathfrak{N} = [85, 62, 1], p = 11$

The mod 11 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
6	0	1	6	6	10	0

Let $\mathfrak{n}$ be the ideal generated by the element $\frac{1}{2}(-7t^3 + 6t^2 + 2t - 15)$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_{11})$ is 8-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_{11}]$	Multiplicity
φ_1	1	7
φ_2	1	1

The mod 11 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	9	9	9	9	10	10	10	10	6	6	7	7	7	7
φ_2	6	0	0	6	1	6	6	1	6	6	10	7	10	7

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.4.18 $\mathfrak{N} = [85, 62, 1], p = 13$

The mod 13 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
1	5	0	4	11	9	5

Let $\mathfrak{n}$ be the ideal generated by the element $\frac{1}{2}(-7t^3 + 6t^2 + 2t - 15)$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_{13})$ is 8-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_{13}]$	Multiplicity
φ_1	1	7
φ_2	1	1

The mod 13 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	7	7	7	7	6	6	6	6	11	11	10	10	10	10
φ_2	1	5	5	1	0	4	4	0	11	11	9	5	9	5

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.4.19 $\mathfrak{N} = [92, 52, 1], p = 7$

The two mod 7 Bianchi eigenforms have the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
0	3	2	0	$*$	4	3
0	1	1	1	$*$	1	5

Let $\mathfrak{n}$ be the ideal generated by the element $7t^3 - 9t^2 + 9t + 5$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_7)$ is 14-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_7]$	Multiplicity
φ_1	1	11
φ_2	1	2
φ_3	1	1

The mod 7 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	6	6	6	6	4	4	4	4	$*$	$*$	6	6	6	6
φ_2	0	3	3	0	2	0	0	2	$*$	$*$	4	3	4	3
φ_3	0	1	1	0	1	1	1	1	$*$	$*$	1	5	1	5

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 11-dimensional and affords only trivial complex eigenforms.

3.4.20 $\mathfrak{N} = [93, 79, 1], p = 7$

The mod 7 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
5	0	*	5	*	4	1

Let $\mathfrak{n}$ be the ideal generated by the element $-t^3 + 8t^2 - 8t + 6$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_7)$ is 16-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_7]$	Multiplicity
φ_1	1	15
φ_2	1	1

The mod 7 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	6	6	6	6	*	4	4	*	*	*	6	6	6	6
φ_2	5	0	0	5	*	5	5	*	*	*	4	1	4	1

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 15-dimensional and affords only trivial complex eigenforms.

3.4.21 $\mathfrak{N} = [94, 9, 1], p = 7$

The mod 7 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
1	4	6	1	*	0	4

Let $\mathfrak{n}$ be the ideal generated by the element $-t^3 + t^2 - 3t - 10$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_7)$ is 9-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_7]$	Multiplicity
φ_1	1	7
φ_2	1	2

The mod 7 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	6	6	6	6	4	4	4	4	*	*	6	6	6	6
φ_2	1	4	4	1	6	1	1	6	*	*	0	4	0	4

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.4.22 $\mathfrak{N} = [94, 9, 1], p = 211$

The mod 211 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
99	92	41	201	88	15	185

Let $\mathfrak{n}$ be the ideal generated by the element $-t^3 + t^2 - 3t - 10$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_{211})$ is 8-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_{211}]$	Multiplicity
φ_1	1	7
φ_2	1	1

The mod 211 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	20	20	20	20	32	32	32	32	50	50	62	62	62	62
φ_2	99	92	92	99	41	201	201	41	88	88	15	185	15	185

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.4.23 $\mathfrak{N} = [94, 37, 1], p = 7$

The mod 7 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
4	1	1	6	*	4	0

Let $\mathfrak{n}$ be the ideal generated by the element $\frac{1}{2}(13t^3 - 6t^2 + 16t + 17)$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_7)$ is 9-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_7]$	Multiplicity
φ_1	1	7
φ_2	1	2

The mod 7 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	6	6	6	6	4	4	4	4	*	*	6	6	6	6
φ_2	4	1	1	4	1	6	6	1	*	*	4	0	4	0

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

3.4.24 $\mathfrak{N} = [94, 37, 1], p = 67$

The mod 67 Bianchi eigenform has the following values:

$[19, 8, 1]$	$[19, 10, 1]$	$[31, 17, 1]$	$[31, 13, 1]$	$[49, 0, 7]$	$[61, 15, 1]$	$[61, 45, 1]$
3	20	0	59	18	29	18

Let $\mathfrak{n}$ be the ideal generated by the element $\frac{1}{2}(13t^3 - 6t^2 + 16t + 17)$. Then $H^5(Y(\mathfrak{N}), \overline{\mathbb{F}}_{67})$ is 8-dimensional and affords the following eigenforms.

	$[\mathbb{F} : \mathbb{F}_{67}]$	Multiplicity
φ_1	1	7
φ_2	1	1

The mod 67 eigenforms φ_1 and φ_2 admit the following values.

	$\mathfrak{q}_{19,1}$	$\mathfrak{q}_{19,2}$	$\mathfrak{q}_{19,3}$	$\mathfrak{q}_{19,4}$	$\mathfrak{q}_{31,1}$	$\mathfrak{q}_{31,2}$	$\mathfrak{q}_{31,3}$	$\mathfrak{q}_{31,4}$	$\mathfrak{q}_{49,1}$	$\mathfrak{q}_{49,2}$	$\mathfrak{q}_{61,1}$	$\mathfrak{q}_{61,2}$	$\mathfrak{q}_{61,3}$	$\mathfrak{q}_{61,4}$
φ_1	20	20	20	20	32	32	32	32	50	50	62	62	62	62
φ_2	3	20	20	3	0	59	59	0	18	18	29	18	29	18

The complex cohomology group $H^5(Y(\mathfrak{N}), \mathbb{C})$ is 7-dimensional and affords only trivial complex eigenforms.

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