

# $B_{s,d} \rightarrow \ell^+ \ell^-$ in the Standard Model with Reduced Theoretical Uncertainty

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We combine our new results for the  $\mathcal{O}(\alpha_{em})$  and  $\mathcal{O}(\alpha_s^2)$  corrections to  $B_{s,d} \rightarrow \ell^+ \ell^-$ , and present updated branching ratio predictions for these decays in the standard model. Inclusion of the new corrections removes major theoretical uncertainties of perturbative origin that have just begun to dominate over the parametric ones. For the recently observed muonic decay of the  $B_s$  meson, our calculation gives  $\overline{\mathcal{B}}(B_s \rightarrow \mu^+ \mu^-) = (3.65 \pm 0.23) \times 10^{-9}$ .

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Rare leptonic decays of the neutral  $B$  mesons are highly suppressed in the standard model (SM), and provide important constraints on models of new physics. In the SM, these flavor changing neutral current decays are generated first at one-loop level through W-box and Z-penguin diagrams. Their branching ratios undergo an additional helicity suppression by  $m_\ell^2/M_{B_q}^2$ , where  $m_\ell$  and  $M_{B_q}$  denote masses of the charged lepton and the  $B_q$  meson, respectively. This suppression can be lifted in models with extra Higgs doublets, such as the minimal supersymmetric standard model. Constraints on such models can be obtained even for the scalar masses reaching a few TeV, far above the current direct search limits (see e.g. Ref. [1]). However, one of the key factors in determining the constraints is the SM prediction accuracy. Improving this accuracy is the main purpose of the present work.

The average time-integrated branching ratios  $\overline{\mathcal{B}}_{q\ell} \equiv \overline{\mathcal{B}}[B_q \rightarrow \ell^+ \ell^-]$  ( $q = s, d$ ;  $\ell = e, \mu, \tau$ ) depend on details of  $B_q \overline{B}_q$  mixing [2]. A simple relation  $\overline{\mathcal{B}}_{q\ell} = \Gamma[B_q \rightarrow \ell^+ \ell^-]/\Gamma_H^q$  holds in the SM to a very good approximation, with  $\Gamma_H^q$  denoting the heavier mass-eigenstate total width. For  $\ell = \mu$ , the current experimental world averages read [3]

$$\overline{\mathcal{B}}_{s\mu} = (2.9 \pm 0.7) \times 10^{-9}, \quad \overline{\mathcal{B}}_{d\mu} = (3.6_{-1.4}^{+1.6}) \times 10^{-10}. \quad (1)$$

They have been obtained by combining the recent measurements of CMS [4] and LHCb [5]. In the  $\overline{\mathcal{B}}_{s\mu}$  case, reduction of uncertainties to a few percent level is expected in the forthcoming decade. To match such an accuracy, theoretical calculations must include the next-to-leading order (NLO) corrections of electroweak (EW) origin, as well as QCD corrections up to the next-to-next-to-leading order (NNLO). In the present paper, we combine our new calculations of the NLO EW [6] and NNLO QCD [7] corrections to the relevant coupling constant (Wilson coefficient)  $C_A$ , and present updated SM predictions for all the  $\overline{\mathcal{B}}_{q\ell}$  branching ratios.

A convenient framework for describing the considered processes is an effective theory derived from the SM by decoupling the top quark, the Higgs boson, and the heavy electroweak bosons  $W$  and  $Z$  (see, e.g., Ref. [8] for a pedagogical introduction). The effective weak interaction Lagrangian relevant for  $B_q \rightarrow \ell^+ \ell^-$  reads

$$\mathcal{L}_{\text{weak}} = N C_A(\mu_b) (\overline{b} \gamma_\alpha \gamma_5 q) (\overline{\ell} \gamma^\alpha \gamma_5 \ell) + \dots, \quad (2)$$

where  $C_A$  is the  $\overline{\text{MS}}$ -renormalized Wilson coefficient at the scale  $\mu_b \sim m_b$ . The ellipses stand for other, subleading weak interaction terms (operators) which we discuss below. The normalization constant  $N = V_{tb}^* V_{tq} G_F^2 M_W^2 / \pi^2$  is given in terms of the Fermi constant  $G_F$  (extracted from the muon decay), the  $W$ -boson on-shell mass  $M_W$ , and the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements  $V_{ij}$ .

Once  $C_A(\mu_b)$  is determined to sufficient accuracy, the branching ratio is easily expressed in terms of the lepton mass  $m_\ell$ , the  $B_q$ -meson mass  $M_{B_q}$ , and its decay constant  $f_{B_q}$ . The latter is defined by the QCD matrix element  $\langle 0 | \overline{b} \gamma^\alpha \gamma_5 q | B_q(p) \rangle = i p^\alpha f_{B_q}$ . One finds

$$\overline{\mathcal{B}}_{q\ell} = \frac{|N|^2 M_{B_q}^3 f_{B_q}^2}{8\pi \Gamma_H^q} \beta_{q\ell} r_{q\ell}^2 |C_A(\mu_b)|^2 + \mathcal{O}(\alpha_{em}), \quad (3)$$

where  $r_{q\ell} = 2m_\ell/M_{B_q}$  and  $\beta_{q\ell} = \sqrt{1 - r_{q\ell}^2}$ . Equation (3) holds at the leading order in flavor-changing weak interactions and in  $M_{B_q}^2/M_W^2$ , which is accurate up to permille-level corrections. In particular, operators like  $(\overline{b} \gamma_5 q)(\overline{\ell} \ell)$  from the Higgs boson exchanges give rise to  $\mathcal{O}(M_{B_q}^2/M_W^2)$  effects only. Thus, one neglects such operators in the SM. However, they often matter in beyond-SM theories.

As far as the  $\mathcal{O}(\alpha_{em})$  term in Eq. (3) is concerned, it requires more explanation because we are going to neglect it while including complete corrections of this order to  $C_A(\mu_b)$ . The first observation to make is that some of the  $\mathcal{O}(\alpha_{em})$  corrections to  $C_A(\mu_b)$  get enhanced by

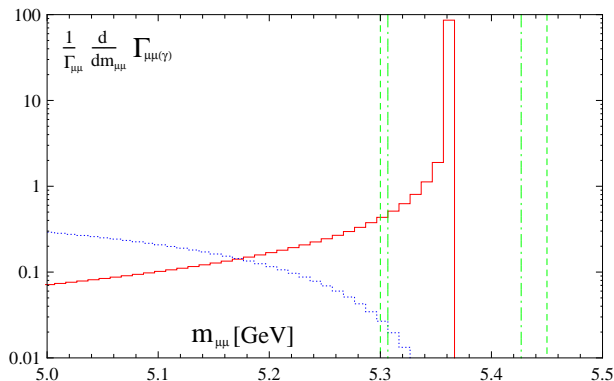


FIG. 1: Contributions to the dimuon invariant-mass spectrum in  $B_s \rightarrow \mu^+ \mu^- (n\gamma)$  with  $n = 0, 1, 2, \dots$  (see the text). Both of them are displayed in bins of 0.01 GeV width.

$1/\sin^2 \theta_W$ , powers of  $m_t^2/M_W^2$  or logarithms  $\ln^2 M_W^2/\mu_b^2$ , as explained in Ref. [6]. None of these enhancements is possible for the  $\mathcal{O}(\alpha_{em})$  term in Eq. (3) once  $\mu_b \sim m_b$ . This term is  $\mu_b$ -dependent and contains contributions from operators like  $(\bar{b}\gamma_\alpha\gamma_5 q)(\bar{\ell}\gamma^\alpha\ell)$  or  $(\bar{b}\gamma_\alpha P_L c)(\bar{c}\gamma^\alpha P_L s)$ , with photons connecting the quark and lepton lines. It depends on non-perturbative QCD in a way that is not described by  $f_{B_q}$  alone, and it must compensate the  $\mu_b$ -dependence of  $C_A(\mu_b)$ . Since we neglect this term, scale dependence serves as one of the uncertainty estimates. When  $\mu_b$  is varied from  $m_b/2$  to  $2m_b$ , our results for  $|C_A(\mu_b)|^2$  vary by about 0.3%, which corresponds to a typical size of  $\mathcal{O}(\alpha_{em})$  corrections that undergo no extra enhancement. On the other hand, the NLO EW corrections to  $|C_A(\mu_b)|^2$  often reach a few percent level [6].

The only other possible enhancement of QED corrections that one may worry about is related to soft photon bremsstrahlung. For definiteness, let us consider  $B_s \rightarrow \mu^+ \mu^- (n\gamma)$  with  $n = 0, 1, 2, \dots$ . The dimuon invariant-mass spectrum in this process is obtained by summing the two distributions shown in Fig. 1. The dotted (blue) curve corresponds to real photon emission from the quarks (Eq. (25) of Ref. [9]), while the tail of the solid (red) one is dominated by soft photon radiation from the muons (Eqs. (19)–(23) of Ref. [10]). The vertical dashed and dash-dotted (green) lines indicate the CMS [4] and LHCb [5] signal windows, respectively. In the displayed region below the windows (i.e. between 5 and 5.3 GeV), each of the two contributions integrates to around 5% of the total rate.

The determination of  $\overline{\mathcal{B}}_{s\mu}$  on the experimental side includes a correction due to photon bremsstrahlung from the muons. For this purpose, both CMS [4] and LHCb [5] apply PHOTOS [11]. Such an approach is practically equivalent to extrapolating along the solid curve in Fig. 1 down to zero. In the resulting quantity, all the soft QED logarithms cancel out, and we obtain  $\overline{\mathcal{B}}_{s\mu}$  as in Eq. (3), up to  $\mathcal{O}(\alpha_{em})$  terms that undergo no extra enhancement [10].

The direct emission, i.e. real photon emission from

Parameter	Value	Unit	Ref.
$G_F$	$1.166379 \times 10^{-5}$	$\text{GeV}^{-2}$	[13]
$\alpha_s^{(5)}(M_Z)$	0.1184 (7)	–	[13]
$\alpha_{em}^{(5)}(M_Z)$	1/127.944 (14)	–	[13]
$\Delta\alpha_{em,\text{hadr}}^{(5)}(M_Z)$	0.02772 (10)	–	[13]
$M_Z$	91.1876 (21)	GeV	[13]
$M_t$	173.1 (9)	GeV	[13]
$M_H$	125.9 (4)	GeV	[13]
$M_{B_s}$	5366.77 (24)	MeV	[13]
$M_{B_d}$	5279.58 (17)	MeV	[13]
$f_{B_s}$	227.7 (4.5)	MeV	[14]
$f_{B_d}$	190.5 (4.2)	MeV	[14]
$1/\Gamma_H^s$	1.615 (21)	ps	[15]
$2/(\Gamma_H^d + \Gamma_L^d)$	1.519 (7)	ps	[15]
$ V_{cb} $	0.0424 (9)	–	[16]
$ V_{tb}^* V_{ts}/V_{cb} $	0.980 (1)	–	[17, 18]
$ V_{tb}^* V_{td} $	0.0088 (3)	–	[17, 18]

TABLE I: Numerical inputs.

the quarks is infrared safe by itself because the decaying meson is electrically neutral. It is effectively treated as background on both the experimental and theoretical sides. On the experimental side, it is neglected in the signal window (being very small there, indeed), and not included in the extrapolation. On the theory side, it is just excluded from  $\overline{\mathcal{B}}_{s\mu}$  by definition. This contribution survives in the limit  $m_\mu \rightarrow 0$ , which explains its considerable size below the signal window in Fig. 1.

In this context, one may wonder whether the helicity suppression factor  $r_{q\ell}^2$  in Eq. (3) can be relaxed at higher orders in QED. For the two-body decay it is not possible in the SM because a generic non-local interaction of  $B_q$  with massless leptons contains vector or axial-vector lepton currents contracted with the lepton momenta, which means that it vanishes on shell. On the other hand, contributions with (real or virtual) photons coupled to the quarks may survive in the  $m_\ell \rightarrow 0$  limit, but they are phase-space suppressed in the signal window (cf. the dotted line in Fig. 1). In the  $\overline{\mathcal{B}}_{s\mu}$  case, the phase-space suppression is at least as effective as the helicity suppression, given the applied window sizes in both experiments.

We are now ready to numerically evaluate the branching ratios in Eq. (3). Our inputs are collected in Table I. The  $\overline{\text{MS}}$ -renormalized coupling constants  $\alpha_s^{(5)}(M_Z)$  and  $\alpha_{em}^{(5)}(M_Z)$  are defined in the SM with decoupled top quark. Hadronic contributions to the evolution of  $\alpha_{em}$  are given by  $\Delta\alpha_{em,\text{hadr}}^{(5)}$ . This quantity is used to evaluate the  $W$ -boson pole mass according to the fit formula in Eqs. (6) and (9) of Ref. [12], which gives  $M_W = 80.358 (8) \text{ GeV}$ , consistently with the direct measurement  $M_W = 80.385 (15) \text{ GeV}$  [13]. All the masses in Table I are interpreted as the on-shell ones. In the top-quark case, this is equivalent to assuming that the

so-called color reconnection effects are included in the uncertainty. Converting  $M_t$  to the  $\overline{\text{MS}}$ -renormalized mass with respect to QCD (but still on shell with respect to EW interactions), we get  $m_t \equiv m_t(m_t) = 163.5$  GeV.

The decay constants  $f_{B_q}$  are adopted from the most recent update of the  $N_f = (2+1)$  FLAG compilation [14] which averages the  $N_f = 2+1$  results of Refs. [19–21]. More recent calculations with  $N_f = 2+1+1$  [22] and  $N_f = 2$  [23] are consistent with these averages. As far as the lifetimes are concerned, using the explicit result for  $\tau_H^s \equiv 1/\Gamma_H^s$  from Ref. [15] allows to avoid considering correlations between the decay width difference and the average lifetime. In the case of  $B_d$ , we can safely set  $1/\Gamma_H^d \simeq 2/(\Gamma_H^d + \Gamma_L^d) \equiv \tau_{\text{av}}^d$  given the tiny SM expectation for  $(\Gamma_L^d - \Gamma_H^d)/(\Gamma_L^d + \Gamma_H^d) \equiv \Delta\Gamma^d/(2\Gamma_{\text{av}}^d) = 0.0021$  (4) [24].

The CKM matrix element  $|V_{cb}|$  is treated in a special manner, as it is now responsible for the largest parametric uncertainty in  $\overline{\mathcal{B}}_{s\mu}$ . One should be aware of a long-lasting tension between its determinations from the inclusive and exclusive semileptonic decays [14]. Here, we adopt the recent inclusive fit from Ref. [16]. It is the first one where both the semileptonic data and the precise quark mass determinations from flavor-conserving processes have been taken into account. Once  $|V_{cb}|$  is fixed, we evaluate  $|V_{tb}^* V_{ts}|$  using the accurately known ratio  $|V_{tb}^* V_{ts}/V_{cb}|$ .

Apart from the parameters listed in Table I, our results depend on two renormalization scales  $\mu_0 \sim M_t$  and  $\mu_b \sim m_b$  used in the calculation of the Wilson coefficient  $C_A$ . This dependence is very weak thanks to our new calculations of the NLO EW and NNLO QCD corrections. Since this issue is discussed at length in the parallel articles [6, 7], we just fix here these scales to  $\mu_0 = 160$  GeV and  $\mu_b = 5$  GeV. Our results for the Wilson coefficient  $C_A$  are then functions of the first seven parameters in Table I. Allowing only the top-quark mass and the strong coupling constant to deviate from their central values, we find the following fits for  $C_A$

$$\begin{aligned} C_A(\mu_b) &= 0.4802 R_t^{1.52} R_\alpha^{-0.09} - 0.0112 R_t^{0.89} R_\alpha^{-0.09} \\ &= 0.4690 R_t^{1.53} R_\alpha^{-0.09}, \end{aligned} \quad (4)$$

$$\begin{aligned} C_A(\mu_b) &= 0.4802 \tilde{R}_t^{1.50} R_\alpha^{0.015} - 0.0112 \tilde{R}_t^{0.86} R_\alpha^{-0.031} \\ &= 0.4690 \tilde{R}_t^{1.51} R_\alpha^{0.016}, \end{aligned} \quad (5)$$

where  $R_\alpha = \alpha_s(M_Z)/0.1184$ ,  $R_t = M_t/(173.1 \text{ GeV})$  and  $\tilde{R}_t = m_t/(163.5 \text{ GeV})$ . The fits are accurate to better than 0.1% in  $C_A$  for  $\alpha_s(M_Z) \in [0.11, 0.13]$ ,  $M_t \in [170, 175] \text{ GeV}$ , and  $m_t \in [160, 165] \text{ GeV}$ .

In the first lines of Eqs. (4) and (5),  $C_A$  is given as a sum of two terms. The first one corresponds to the leading order EW but NNLO QCD matching calculation [7]. The second one accounts for the NLO EW matching corrections [6] at the scale  $\mu_0$ , as well as for the logarithmically enhanced QED corrections that originate from the renormalization group evolution between  $\mu_0$  and  $\mu_b$  [25, 26].

Inserting Eq. (4) into Eq. (3), we obtain for  $\overline{\mathcal{B}}_{s\mu}$

$$\overline{\mathcal{B}}_{s\mu} \times 10^9 = (3.65 \pm 0.06) R_{t\alpha} R_s = 3.65 \pm 0.23, \quad (6)$$

where  $R_{t\alpha} = R_t^{3.06} R_\alpha^{-0.18} = \tilde{R}_t^{3.02} R_\alpha^{0.032}$  and

$$R_s = \left( \frac{f_{B_s} [\text{MeV}]}{227.7} \right)^2 \left( \frac{|V_{cb}|}{0.0424} \right)^2 \left( \frac{|V_{tb}^* V_{ts}/V_{cb}|}{0.980} \right)^2 \frac{\tau_H^s [\text{ps}]}{1.615}.$$

Correlations between  $f_{B_s}$  and  $\alpha_s$  have been ignored above. Uncertainties due to parameters that do not occur in the quantities  $R_\alpha$ ,  $R_t$  and  $R_s$  have been absorbed into the residual error in the middle term of Eq. (6). This residual error is actually dominated by a non-parametric uncertainty, which we set to 1.5% of the branching ratio. Such an estimate of the non-parametric uncertainty is supposed to include:

- (i) Effects of the neglected  $\mathcal{O}(\alpha_{em})$  term in Eq. (3). They account for the fact that  $|C_A(\mu_b)|^2$  changes by around 0.3% when  $\mu_b$  is varied between  $m_b/2$  and  $2m_b$ . Such a dependence on  $\mu_b$  must cancel order-by-order in perturbation theory.
- (ii) Higher-order  $\mathcal{O}(\alpha_s^3, \alpha_{em}^2, \alpha_s \alpha_{em})$  matching corrections to  $C_A$  at the electroweak scale  $\mu_0$ . Such corrections must remove the residual  $\mu_0$ -dependence of  $C_A(\mu_b)$ . When  $\mu_0$  is varied between  $m_t/2$  and  $2m_t$ , the variation of  $|C_A(\mu_b)|^2$  due to EW and QCD interactions amounts to around 0.2% in each case [6, 7]. Effects of similar size in the branching ratio are observed in Ref. [6] when comparing several EW renormalization schemes.
- (iii) Higher-order  $\mathcal{O}(M_{B_q}^2/M_W^2)$  power corrections.
- (iv) Uncertainties due to evaluation of  $m_t$  from the experimentally determined  $M_t$  using a three-loop relation. Note that half of the three-loop correction shifts  $m_t$  by about 200 MeV, which affects  $\overline{\mathcal{B}}_{s\mu}$  by around 0.3%. Non-perturbative uncertainties at this point (renormalons, color reconnection) are expected to be of the same order of magnitude.
- (v) Tiny  $\mathcal{O}(\Delta\Gamma^q/\Gamma^q)$  corrections due to deviations from the relation  $\overline{\mathcal{B}}_{q\ell} = \Gamma[B_q \rightarrow \ell^+ \ell^-]/\Gamma_H^q$ , i.e. due to decays of the lighter mass eigenstate in the  $B_q \overline{B}_q$  system. At the leading order in  $\alpha_{em}$  and  $M_{B_q}^2/M_W^2$ , such corrections are non-vanishing only because of CP-violation in the absorptive part of the  $B_q \overline{B}_q$  mixing matrix. Apart from being suppressed by  $\Delta\Gamma^q/\Gamma^q$ , they vanish in the limit  $m_c \rightarrow m_u$ , and receive additional CKM suppression in the  $B_s$  case. Beyond the leading order in  $\alpha_{em}$  or  $M_{B_q}^2/M_W^2$ , the lighter eigenstate can decay to leptons also in the CP-conserving limit of the SM.

All the other  $\overline{\mathcal{B}}_{q\ell}$  branching ratios are calculated along

	$f_{B_q}$	CKM	$\tau_H^q$	$M_t$	$\alpha_s$	other param.	non-param.	$\Sigma$
$\overline{\mathcal{B}}_{s\ell}$	4.0%	4.3%	1.3%	1.6%	0.1%	< 0.1%	1.5%	6.4%
$\overline{\mathcal{B}}_{d\ell}$	4.5%	6.9%	0.5%	1.6%	0.1%	< 0.1%	1.5%	8.5%

TABLE II: Relative uncertainties from various sources in  $\overline{\mathcal{B}}_{s\ell}$  and  $\overline{\mathcal{B}}_{d\ell}$ . In the last column they are added in quadrature.

the same lines. We find

$$\begin{aligned}
\overline{\mathcal{B}}_{se} \times 10^{14} &= (8.54 \pm 0.13) R_{t\alpha} R_s = 8.54 \pm 0.55, \\
\overline{\mathcal{B}}_{s\tau} \times 10^7 &= (7.73 \pm 0.12) R_{t\alpha} R_s = 7.73 \pm 0.49, \\
\overline{\mathcal{B}}_{de} \times 10^{15} &= (2.48 \pm 0.04) R_{t\alpha} R_d = 2.48 \pm 0.21, \\
\overline{\mathcal{B}}_{d\mu} \times 10^{10} &= (1.06 \pm 0.02) R_{t\alpha} R_d = 1.06 \pm 0.09, \\
\overline{\mathcal{B}}_{d\tau} \times 10^8 &= (2.22 \pm 0.04) R_{t\alpha} R_d = 2.22 \pm 0.19, \quad (7)
\end{aligned}$$

with

$$R_d = \left( \frac{f_{B_d} [\text{MeV}]}{190.5} \right)^2 \left( \frac{|V_{tb}^* V_{td}|}{0.0088} \right)^2 \frac{\tau_d^{\text{av}} [\text{ps}]}{1.519}.$$

A summary of the error budgets for  $\overline{\mathcal{B}}_{s\ell}$  and  $\overline{\mathcal{B}}_{d\ell}$  is presented in Table II. It is clear that the main parametric uncertainties come from  $f_{B_q}$  and the CKM angles.

To get rid of such uncertainties, one may take advantage [27] of their cancellation in ratios like

$$\kappa_{q\ell} \equiv \frac{\overline{\mathcal{B}}_{q\ell} \Gamma_H^q \Delta M_{B_q}^{-1}}{(G_F M_W m_\ell)^2 \beta_{q\ell}} \stackrel{\text{SM}}{\simeq} \frac{3 |C_A(\mu_b)|^2}{\pi^3 C_{LL}(\mu_b) B_{B_q}(\mu_b)}, \quad (8)$$

where  $\Delta M_{B_q}$  is the mass difference in the  $B_q \bar{B}_q$  system, and  $C_{LL}$  enters through the  $\Delta B = 2$  term in  $\mathcal{L}_{\text{weak}}$ , namely  $-\frac{1}{4} N V_{tb}^* V_{tq} C_{LL} (\bar{b} \gamma_\alpha P_L q) (\bar{b} \gamma^\alpha P_L q)$ . The bag parameters  $\hat{B}_{B_q}$  are defined by the QCD matrix elements  $\langle \bar{B}_q | (\bar{b} \gamma_\alpha P_L q) (\bar{b} \gamma^\alpha P_L q) | B_q \rangle = \frac{2}{3} f_{B_q}^2 B_{B_q} M_{B_q}^2$ .

Following FLAG [14], we take  $\hat{B}_{B_s} = 1.33(6)$  and  $\hat{B}_{B_d} = 1.27(10)$  [28]. For the Wilson coefficient  $C_{LL}$ , including the NLO QCD [29] and NLO EW [30] corrections, we find  $\hat{C}_{LL} \equiv C_{LL}(\mu_b) B_{B_q}(\mu_b) / \hat{B}_{B_q} = 1.27 R_t^{1.51}$  for  $\alpha_s^{(5)}(M_Z) = 0.1184$  and  $\mu_b = 5 \text{ GeV}$ . The r.h.s. of

Eq. (8) gives then  $\kappa_{s\ell} = 0.0126(7)$  and  $\kappa_{d\ell} = 0.0132(12)$ . It follows that the overall theory uncertainties in  $\kappa_{q\ell}$  and  $\overline{\mathcal{B}}_{q\ell}$  are quite similar at present. The l.h.s. of Eq. (8) together with Eq. (1) give  $\kappa_{s\mu}^{\text{exp}} = 0.0104(25)$  and  $\kappa_{d\mu}^{\text{exp}} = 0.047(20)$ , which is consistent with the SM predictions.

To conclude, we have presented updated SM predictions for all the  $\overline{\mathcal{B}}_{q\ell}$  branching ratios. Thanks to our new results on the NLO EW [6] and NNLO QCD [7] matching corrections, a significant reduction of the non-parametric uncertainties has been achieved. Such uncertainties are now estimated at the level of around 1.5% of the branching ratios, compared to around 8% prior to our calculations. As far as the parametric ones are concerned, their reduction will depend on progress in the lattice determinations of  $f_{B_q}$  and  $B_{B_q}$  in the cases of  $\overline{\mathcal{B}}_{q\ell}$  and  $\kappa_{q\ell}$ , respectively. For  $\overline{\mathcal{B}}_{q\ell}$ , the CKM uncertainties are now equally important, with  $|V_{cb}|$  being one of the main limiting factors in the precise determination of  $\overline{\mathcal{B}}_{s\ell}$ .

The increased theory accuracy is essential in interpreting the experimental findings in terms of the SM or new physics. This will be particularly important after the LHCb upgrade (see e.g. Ref. [31]), when the experimental accuracy in  $\overline{\mathcal{B}}_{s\mu}$  is expected to reach the same level as the current theoretical one. Even if no deviation from the SM is found, the role of  $B_q \rightarrow \ell^+ \ell^-$  in constraining new physics will become significantly stronger.

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