

# Proton Stability, Gauge Coupling Unification and a Light $Z'$ in Heterotic-string Models

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## Abstract

We explore the phenomenological viability of a light  $Z'$  in heterotic-string models, whose existence has been motivated by proton stability arguments. A class of quasi-realistic string models that produce such a viable  $Z'$  are the Left-Right Symmetric (LRS) heterotic-string models in the free fermionic formulation. A key feature of these models is that the matter charges under  $U(1)_{Z'}$  do not admit an  $E_6$  embedding. The light  $Z'$  in the LRS heterotic-string models forbids baryon number violating operators, while allowing lepton number violating operators, hence suppressing proton decay yet allowing for sufficiently small neutrino masses via a seesaw mechanism. We show that the constraints imposed by the gauge coupling data and heterotic-string coupling unification nullify the viability of a light  $Z'$  in these models. We further argue that agreement with the gauge coupling data necessitates that the  $U(1)_{Z'}$  charges admit an  $E_6$  embedding. We discuss how viable string models with this property may be constructed.

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# 1 Introduction

The discovery of the Higgs boson at the LHC lends further credence to the hypothesis that the Standard Model (SM) provides a viable effective parametrisation of all subatomic interactions up to the GUT or heterotic–string unification scales. Support for this possibility stems from: the matter gauge charges; proton longevity; suppression of neutrino masses; and the logarithmic evolution of the SM parameters in its gauge and matter sectors. Preservation of the logarithmic running in the SM scalar sector entails that it must be augmented by a new symmetry. A concrete framework that fulfils the task is given by supersymmetry.

The supersymmetric extension of the SM introduces dimension four and five baryon and lepton number violating operators that mediate proton decay. This problem is particularly acute in the context of heterotic–string derived constructions, in which one cannot assume the existence of global or local discrete symmetries that simply forbid the undesired operators. Indeed, the issue has been examined in the past by a number of authors [1]. The avenues explored range from the existence of matter parity at special points in the moduli space of specific models, to the emergence of non–abelian custodial symmetries in some compactifications. However, a caveat to these arguments is that in addition to suppressing the proton decay mediating operators, one must also ensure that the mass terms of left–handed neutrinos are sufficiently suppressed. That is, while baryon number should be conserved to ensure proton longevity, lepton number must be broken to allow for suppression of left–handed neutrino masses. In heterotic–string constructions, due to the absence of higher–order representations of the Grand Unified Theory [2], one typically has to break lepton number by one unit, which generically results in both lepton and baryon number violation. An alternative solution to this conundrum is obtained if an additional  $U(1)$  gauge symmetry, beyond the SM gauge group, remains unbroken down to low scales. An additional abelian gauge symmetry, which is broken near the TeV scale, may also explain the suppression of the  $\mu$ –term in the supersymmetric potential [3].

The possibility of a low scale  $Z'$  arising from heterotic–string inspired models has a long history and continues to attract wide interest [4]. Surprisingly, however, keeping a  $Z'$  in explicit string derived constructions, unbroken down to the low scale, turns out to be notoriously difficult, as such an extra symmetry must satisfy a variety of phenomenological constraints. Obviously, to play a role in the suppression of proton decay mediating operators (PDMOs) implies that the SM matter states are charged under this symmetry. While forbidding baryon number violation, it should allow for lepton number violation, required for the suppression of neutrino masses. Furthermore, it should be family universal, otherwise there is a danger of generating Flavour Changing Neutral Currents (FCNC), or of generating the PDMOs via mixing. The additional symmetry should also allow for the fermion Yukawa couplings to electroweak Higgs doublets and must be anomaly free. Explicit string models that

do give rise to an extra  $U(1)$  symmetry with the required properties are the left–right symmetric models of [5, 6]. The existence of the required symmetry in explicit string constructions ensures that, in these examples, the extra  $U(1)$  is free of any gauge and gravitational anomalies. In [7] we constructed toy string–inspired models that are compatible with the charge assignments in the string derived models. In these models, the proton lifeguarding extra  $U(1)$  symmetry can, in principle, remain unbroken down to low scales.

An additional constraint that must be imposed on the extra gauge and matter states that arise in the  $Z'$  models, is compatibility with the gauge parameters,  $\sin^2 \theta_W(M_Z)$  and  $\alpha_3(M_Z)$ . The perturbative heterotic–string predicts that all the gauge couplings are unified at the string unification scale,  $M_S$ , which is of the order  $5 \cdot 10^{17} \text{GeV}$ . Nonperturbatively, the heterotic–string can be pushed to the GUT unification scale,  $M_{\text{GUT}}$ , of the order  $2 \cdot 10^{16} \text{GeV}$  [8]. In this paper we study the constraints that are imposed on the string inspired  $Z'$  models by gauge coupling unification and show that the gauge coupling data is not in agreement with the left–right symmetric heterotic–string models. The origin for the disagreement lies in the specific  $U(1)_{Z'}$  charges, which do not admit an  $E_6$  embedding. For comparison we also perform the analysis for  $U(1)_{Z'}$  charges that maintain the  $E_6$  embedding and show that, in this case, agreement with the data is achieved. We discuss how viable string derived models that preserve the  $E_6$  embedding may be constructed.

## 2 Additional $U(1)$ s in free fermionic models

In this section we review the structure of the free fermionic models. We focus on the extra  $U(1)$  symmetries that arise in the models and the charges of the matter states. We elaborate on the gauge symmetry breaking patterns induced by the Generalised GSO (GGSO) projections but concentrate here on the group theory structure and the matter charges. Further details of the free fermionic models and their construction are found in earlier literature [5, 9–16]. The free fermionic models correspond to  $Z_2 \times Z_2$  orbifold compactifications at special points in the moduli space [17]. It should be emphasized that our results are applicable to the wider range of orbifold models because they merely depend on the symmetry breaking patterns of the observable gauge symmetry.

Free fermionic heterotic–string models are constructed by specifying a consistent set of boundary condition basis vectors and the associated one–loop GGSO phases [9]. These basis vectors span a finite additive group,  $\Xi$ , where the physical states of a given sector,  $\alpha \in \Xi$ , are obtained by acting on the vacuum with bosonic and fermionic operators and by applying the GGSO projections. The  $U(1)$  charges, with respect to the unbroken Cartan generators of the four dimensional gauge group, are given by:

$$Q(f) = \frac{1}{2}\alpha(f) + F(f), \quad (2.1)$$

where  $\alpha(f)$  is the boundary condition of the complex world-sheet fermion  $f$  in the sector  $\alpha$ , and  $F_\alpha(f)$  is a fermion number operator counting each mode of  $f$  once ( $f^*$  minus once). For periodic fermions with  $\alpha(f) = 1$ , the vacuum is a spinor representing the Clifford algebra of the zero modes. For each periodic complex fermion,  $f$ , there are two degenerate vacua,  $|+\rangle$  and  $|-\rangle$ , annihilated by the zero modes,  $f_0$  and  $f_0^*$ , with fermion numbers  $F(f) = 0, -1$  respectively.

Three generation models in the free fermionic construction have been obtained by using two constructions: the first were the NAHE based models [13]; and the second class of models are those constructed by the classification method of [15]. The important distinction between the two cases is that the latter has only been applied for symmetric orbifolds, whereas, in the former, most of the constructions utilise asymmetric boundary conditions.

In NAHE based models [5, 10–12, 14] the first set of five basis vectors,  $\{\mathbf{1}, S, b_1, b_2, b_3\}$ , are fixed;  $b_1, b_2$  and  $b_3$  correspond to the three twisted sectors of the  $Z_2 \times Z_2$  orbifold and  $S$  is the spacetime supersymmetry generator. The gauge symmetry at the level of the NAHE set is  $SO(10) \times SO(6)^3 \times E_8$  with  $N = 1$  spacetime supersymmetry. The second stage of the construction consists of adding three additional basis vectors to the NAHE set. The additional vectors reduce the number of generations to three and simultaneously break the four dimensional group. The  $SO(10)$  symmetry is broken to one of its maximal subgroups:  $SU(5) \times U(1)$  (FSU5) [10];  $SU(3) \times SU(2) \times U(1)^2$  (SLM) [11];  $SO(6) \times SO(4)$  (PS) [12];  $SU(3) \times U(1) \times SU(2)^2$  (LRS) [5]; and  $SU(4) \times SU(2) \times U(1)$  (SU421) [14].

An important distinction between the last two cases and the first three is in regard to the anomalous  $U(1)_A$  symmetry that arises in these models [5, 14, 18]. The Cartan subalgebra of the observable rank eight gauge group is generated by eight complex fermions, denoted by  $\{\bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}\}$ , where  $\bar{\psi}^{1,\dots,5}$  are the Cartan generators of the  $SO(10)$  group and  $\bar{\eta}^{1,2,3}$  generate three  $U(1)$  symmetries, denoted by  $U(1)_{1,2,3}$ . In the FSU5, PS and SLM cases the  $U(1)_{1,2,3}$ , as well as their linear combination,

$$U(1)_\zeta = U(1)_1 + U(1)_2 + U(1)_3, \quad (2.2)$$

are anomalous, whereas in the LRS and SU421 models they are anomaly free. The distinction can be seen to arise from the symmetry breaking patterns induced in the two cases from the underlying  $N = 4$  toroidal model in four dimensions. Starting from the  $E_8 \times E_8$ , in the first case the symmetry is broken to  $SO(16) \times SO(16)$  by the choice of GGSO projection phases in the fermionic models, or equivalently by a Wilson line in the corresponding orbifold models. The basis vectors  $b_1$  and  $b_2$  break the symmetry further to  $SO(10) \times U(1)^3 \times SO(16)$ . Alternatively, we can implement the  $b_1$  and  $b_2$  twists in the  $E_8 \times E_8$  vacuum, which break the gauge symmetry to  $E_6 \times U(1)^2 \times E_8$ . The Wilson line breaking then reduces the symmetry to  $SO(10) \times U(1)_\zeta \times U(1)^2 \times SO(16)$ . It is then clear that the  $U(1)_\zeta$  becomes anomalous because of the  $E_6$  symmetry breaking to  $SO(10) \times U(1)_\zeta$  and the projection of some states from the spectrum by the GGSO projections [18]. On the other hand, the LRS

and SU421 heterotic–vacua arise from an  $N = 4$  vacuum with  $E_7 \times E_7 \times SO(16)$  gauge symmetry [5, 14]. In this case, one of the  $E_7$  factors produces the observable gauge symmetry and the second is hidden. The important point here is that these models circumvent the  $E_6$  embedding. Hence, in these cases, the  $U(1)_\zeta$  does not have an  $E_6$  embedding and therefore remains anomaly free.

The case of the symmetric orbifolds studied in [15] only allows for models with an  $E_6$  embedding of  $U(1)_\zeta$ . Thus, in these models  $U(1)_\zeta$  is, generically, anomalous. There is, however, a class of models in which it is anomaly free. This is the case in the self–dual models under the spinor–vector duality of [19]. In these models the number of  $SO(10)$  spinorial **16** representations and the number of vectorial **10** representations, arising from the twisted sectors is identical, although the  $E_6$  symmetry is broken. This situation occurs when the spinorial and vectorial representations are obtained from different fixed points of the  $Z_2 \times Z_2$  toroidal orbifold. A self–dual, three generation model with unbroken  $SO(10)$  symmetry is given in ref. [15], however, a viable model, of this type, with broken  $SO(10)$  symmetry has not been constructed to date.

Alternatively, we may construct  $U(1)_\zeta \subset E_6$  as an anomaly free combination by following a different symmetry breaking pattern to the  $E_6 \rightarrow SO(10) \times U(1)$  discussed above. Originally, the  $E_6 \rightarrow SO(10) \times U(1)$  breaking is achieved by projecting the vector bosons that arise in the spinorial **128** representation of  $SO(16)$  and enhance the  $SO(16)$  symmetry to  $E_8$ . We may construct models in which these vector bosons are not projected and, thus, the  $E_6$  symmetry is broken to a different subgroup. Examples of such models include the three generation  $SU(6) \times SU(2)$  models of [20]. In this case, the  $U(1)_\zeta$  is anomaly free by virtue of its embedding in the enhanced symmetry.

### 3 Gauge coupling analysis

In this section we present a comparative analysis of the two classes mentioned above. It will be instructive to specify a model in each class:

- Model I: This model was first presented in [7]. In this case the extra  $U(1)_\zeta$  does not admit an  $E_6$  embedding, *i.e.*  $SO(10) \times U(1)_\zeta \not\subset E_6$ .
- Model II: This model preserves the  $E_6$  embedding of the  $U(1)_\zeta$  and is akin to  $Z'$  models arising in string inspired  $E_6$  models [4].

Before proceeding with the gauge coupling analysis, it is instructive to detail the symmetry breaking patterns applicable to both models. The SM gauge group will be embedded, for our analysis, in  $SO(10)$ . As previously mentioned, this is broken to the LRS gauge group via the addition of basis vectors,  $\alpha, \beta$ , and  $\gamma$  at the string scale,  $M_S$ . The  $SU(2)_R$  is then broken at some intermediate scale,  $M_R$ . An anomaly

free  $U(1)$  combination that remains is the  $U(1)_{Z'}$  which is required to survive to low energies to preserve proton longevity [6, 7].

In our analysis we vary the unification scale in the range  $2 \cdot 10^{16} - 5 \cdot 10^{17}$  GeV. The lower scale is the natural MSSM unification scale [21],  $M_X$ , whereas the higher scale corresponds to the heterotic-string unification scale [22],  $M_S$ . This factor of 20 discrepancy was discussed in [23] and it was concluded that intermediate matter thresholds contributed enough to overcome the difference, allowing coupling unification in a wide class of realistic free-fermionic string models [24]. From the spectra of our models, we will see that it is natural to include intermediate matter thresholds to achieve string unification. It has also been demonstrated that nonperturbative effects arising in heterotic M-theory [25] can push the unification scale down to the MSSM unification scale [8]. Our aim here is to study, qualitatively, the question of gauge coupling unification in the LRS heterotic-string models. In particular, to demonstrate that a low scale  $Z'$  in these models is incompatible with the gauge coupling data at the electroweak scale. The novel feature of the LRS models is the  $U(1)_{Z'}$  charge assignments. These admit an  $E_8$  embedding and therefore similar charge assignments also arise in heterotic M-theory and so we take the unification scale to vary between  $M_X$  and  $M_S$  to allow for the possible nonperturbative effects. We contrast the analysis in the LRS heterotic-string models with the models that admit the  $E_6$  embedding of the  $U(1)_{Z'}$  charges. In both models there are four intermediate scales between  $M_S$  and  $M_Z$ , corresponding to:

$M_R$ : *SU(2)<sub>R</sub> breaking scale.* The neutral components of  $\mathcal{H}_R + \bar{\mathcal{H}}_R$  acquire a VEV to break the  $SU(2)_R$  symmetry and leave the  $U(1)_{Z'}$  unbroken.

$M_D$ : *Colour triplet scale.* The additional colour triplets in our model acquire a mass at this scale. This will also resolve the discrepancy between the MSSM unification scale and string scale unification.

$M_{Z'}$ : *U(1)<sub>Z'</sub> breaking scale.* The  $U(1)_{Z'}$  is broken at this scale by singlets acquiring VEVs. The anomaly cancelling doublets also acquire mass at this scale and only the MSSM spectrum survives to lower scales.

$M_{\text{SUSY}}$ : *Supersymmetry breaking scale.* The current bounds from the LHC will be included here to get a phenomenologically viable supersymmetry scale. Only the SM states remain down to the  $M_Z$ -scale, at which the gauge data is extracted. Threshold corrections for the top quark and Higgs boson are included in the analysis.

In addition, due to the extra abelian gauge symmetry acting as our proton protector,  $M_{Z'}$  should be sufficiently low in order for adequate suppression of induced PDMOs [6, 7]. By starting from the string scale and evolving the couplings down to  $M_Z$ , our analysis may test whether the predictions of these models are in accordance with low-energy experimental data.

## Low-energy inputs

For our analysis, we take the following values for the masses and couplings [26]:

$$\begin{aligned} M_Z &= 91.1876 \pm 0.0021 \text{ GeV} & \sin^2 \theta_W (M_Z)|_{\overline{\text{MS}}} &= 0.23116 \pm 0.00012 \\ \alpha^{-1} \equiv \alpha_{\text{e.m.}}^{-1} (M_Z) &= 127.944 \pm 0.014 & \alpha_3 (M_Z) &= 0.1184 \pm 0.0007. \end{aligned} \quad (3.1)$$

We also include the top quark mass of  $M_t \sim 173.5$  GeV [26] and the Higgs boson mass of  $M_H \sim 125$  GeV [27] in our analysis.

## Renormalization Group Equations

For the analyses of both models, we follow [23]. String unification implies that the SM gauge couplings are unified at the heterotic-string scale. The one-loop renormalization group equations (RGEs) for the couplings are given by

$$\frac{4\pi}{\alpha_i(\mu)} = k_i \frac{4\pi}{\alpha_{\text{string}}} + \beta_i \log \frac{M_{\text{string}}^2}{\mu^2} + \Delta_i^{(\text{total})}, \quad (3.2)$$

where  $\beta_i$  are the one-loop beta-function coefficients, and  $\Delta_i^{(\text{total})}$  represents possible corrections from the additional gauge or matter states. By solving the one-loop RGEs we obtain expressions for  $\sin^2 \theta_W (M_Z)$  and  $\alpha_3 (M_Z)$ . In each model, we initially assume the MSSM spectrum between the string scale,  $M_S$ , and the  $Z$  scale,  $M_Z$ , and treat all perturbations as effective correction terms. At the string unification scale we have

$$\alpha_S \equiv \alpha_3(M_S) = \alpha_2(M_S) = k_1 \alpha_Y(M_S), \quad (3.3)$$

where  $k_1 = 5/3$  is the canonical  $SO(10)$  normalisation. Thus, the expression for  $\sin^2 \theta_W (M_Z)|_{\overline{\text{MS}}}$  takes the general form [23]

$$\sin^2 \theta_W (M_Z)|_{\overline{\text{MS}}} = \Delta_{\text{MSSM}}^{\sin^2 \theta_W} + \Delta_{\text{I.M.}}^{\sin^2 \theta_W} + \Delta_{\text{L.S.}}^{\sin^2 \theta_W} + \Delta_{\text{I.G.}}^{\sin^2 \theta_W} + \Delta_{\text{T.C.}}^{\sin^2 \theta_W} \quad (3.4)$$

with  $\alpha_3 (M_Z)|_{\overline{\text{MS}}}$  taking similar form with corresponding  $\Delta^{\alpha_3}$  corrections. Here  $\Delta_{\text{MSSM}}$  represents the one-loop contributions from the spectrum of the MSSM between the unification scale and the  $Z$  scale. The following three  $\Delta$  terms correspond to corrections from the intermediate matter thresholds, the light SUSY thresholds, and the intermediate vector bosons corresponding to the  $SU(2)_R$  symmetry breaking. The last term,

$$\Delta_{\text{T.C.}}^{\sin^2 \theta_W} = \Delta_{\text{H.S.}}^{\sin^2 \theta_W} + \Delta_{\text{Yuk.}}^{\sin^2 \theta_W} + \Delta_{\text{2-loop}}^{\sin^2 \theta_W} + \Delta_{\text{Conv.}}^{\sin^2 \theta_W}, \quad (3.5)$$

includes the corrections due to heavy string thresholds, and those arising from Yukawa couplings, two-loops and scheme conversion. These corrections are small and are neglected for this demonstrative analysis.

For  $\sin^2 \theta_W (M_Z)$  we obtain

$$\begin{aligned}
\Delta_{\text{MSSM}}^{\sin^2 \theta_W} &= \frac{1}{1+k_1} \left[ 1 - \frac{\alpha}{2\pi} (11 - k_1) \log \frac{M_S}{M_Z} \right]; \\
\Delta_{\text{I.M.}}^{\sin^2 \theta_W} &= \frac{1}{2\pi} \sum_i \frac{k_1 \alpha}{(1+k_1)} \left( \beta_{2_i} - \beta_{1_i} \right) \log \frac{M_S}{M_i}; \\
\Delta_{\text{L.S.}}^{\sin^2 \theta_W} &= \frac{1}{2\pi} \frac{k_1 \alpha}{(1+k_1)} \left( \beta_{1_{\text{L.S.}}} - \beta_{2_{\text{L.S.}}} \right) \log \frac{M_{\text{SUSY}}}{M_Z},
\end{aligned} \tag{3.6}$$

where  $\alpha = \alpha_{\text{e.m.}}(M_Z)$  and  $M_i$  are the intermediate gauge and matter scales discussed earlier. Similarly for  $\alpha_3(M_Z)$ , we have:

$$\begin{aligned}
\Delta_{\text{MSSM}}^{\alpha_3} &= \frac{1}{1+k_1} \left[ \frac{1}{\alpha} - \frac{1}{2\pi} (15 + 3k_1) \log \frac{M_S}{M_Z} \right]; \\
\Delta_{\text{I.M.}}^{\alpha_3} &= \frac{1}{2\pi} \frac{1}{(1+k_1)} \sum_i \left[ (1+k_1) \beta_{3_i} - (\beta_{2_i} + k_1 \beta_{1_i}) \right] \log \frac{M_S}{M_i}; \\
\Delta_{\text{L.S.}}^{\alpha_3} &= -\frac{1}{2\pi} \frac{1}{(1+k_1)} \left[ (1+k_1) \beta_{3_{\text{L.S.}}} - (\beta_{2_{\text{L.S.}}} + k_1 \beta_{1_{\text{L.S.}}}) \right] \log \frac{M_{\text{SUSY}}}{M_Z}.
\end{aligned} \tag{3.7}$$

A subtle issue in the analysis of gauge coupling unification in string models is the normalisation of the  $U(1)$  generators. In GUTs the normalisation of abelian generators is fixed by their embedding in non-abelian groups. However, in string theory the non-abelian symmetry is not manifest, and the proper normalisation of the  $U(1)$  currents is obscured. The  $U(1)$  normalisation in string models that utilise a world-sheet conformal field theory construction is fixed by their contribution to the conformal dimensions of physical states. The procedure for fixing the normalisation was outlined in [23, 28] and we repeat it here for completeness.

In the free fermionic heterotic-string models, the Kač-Moody level of non-abelian group factors is always one. In general, a given  $U(1)$  current,  $U$ , in the Cartan subalgebra of the four dimensional gauge group, is a combination of the simple world-sheet currents  $U(1)_f \equiv f^* f$ , corresponding to individual world-sheet fermions,  $f$ .  $U$  then takes the form  $U = \sum_f a_f U(1)_f$ , where the  $a_f$  are model dependent coefficients. Each  $U(1)_f$  is normalised to one, so that  $\langle U(1)_f, U(1)_f \rangle = 1$ , and each of the linear combinations must also be normalised to one. The proper normalisation coefficient for the linear combination  $U$  is given by  $N = (\sum_f a_f^2)^{-\frac{1}{2}}$ , and the properly normalised  $U(1)$  current is, thus, given by  $\hat{U}(1) = N \cdot U$ .

In general, the Kač-Moody level,  $k$ , of a  $U(1)$  generator can be deduced from the operator product expansion between two of the  $U(1)$  currents, and is given by

$$k = 2N^{-2} = 2 \sum_f a_f^2. \tag{3.8}$$



The result is generalised to  $k = \sum_i a_i^2 k_i$  when the  $U(1)$  is a combination of several  $U(1)$ s with different normalisations. This procedure is used to determine the Kač–Moody level,  $k_1$ , of the weak–hypercharge generator, as well as that of any other  $U(1)$  combination in the effective low–energy field theory.

In the LRS heterotic–string models, the  $SO(10)$  symmetry is broken to  $SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R$ , where the combinations of world–sheet currents

$$\frac{1}{3} (\bar{\psi}_1^* \bar{\psi}_1 + \bar{\psi}_2^* \bar{\psi}_2 + \bar{\psi}_3^* \bar{\psi}_3) \quad (3.9)$$

and

$$\frac{1}{2} (\bar{\psi}_4^* \bar{\psi}_4 + \bar{\psi}_5^* \bar{\psi}_5) \quad (3.10)$$

generate  $U(1)_C$  and  $T_{3_R}$ , respectively, where the latter is the diagonal generator of  $SU(2)_R$ . The weak–hypercharge is then given by

$$U(1)_Y = T_{3_R} + \frac{1}{3} U(1)_C. \quad (3.11)$$

The symmetry of  $SU(2)_R$  is incorporated in the analysis at the  $M_R$  scale, where above this scale the multiplets are in representations of the LRS gauge group and below the  $M_R$  scale they are in SM representations. The weak–hypercharge coupling relation is given by

$$\frac{1}{\alpha_1(M_R)} = \frac{1}{\alpha_{2R}(M_R)} + \frac{k_C}{9} \frac{1}{\alpha_{\hat{C}}(M_R)} = \frac{1}{\alpha_{2R}(M_R)} + \frac{2}{3} \frac{1}{\alpha_{\hat{C}}(M_R)}. \quad (3.12)$$

Here we have used (3.8) to find that the Kač–Moody level of  $U(1)_C$  is  $k_C = 6$ . Again using (3.8) we find that  $k_1 = \frac{5}{3}$  as expected. This reproduces the expected result at the unification scale

$$\sin^2 \theta_W (M_S) = \frac{1}{1 + k_1} \equiv \frac{3}{8}. \quad (3.13)$$

### 3.1 Coupling unification in LRS heterotic–string models

This model is an example of a three generation, free fermionic model that yields an unbroken, anomaly free  $U(1)$  symmetry. Heterotic–string models with this property break the  $SO(10)$  symmetry to the left–right symmetric subgroup [5] and are therefore supersymmetric and completely free of gauge and gravitational anomalies. The  $U(1)_\zeta$  symmetry in the string models is an anomaly free, family universal symmetry that forbids the dimension four, five and six PDMOs, while allowing for the SM fermion mass terms. A combination of  $U(1)_\zeta$ ,  $U(1)_{B-L}$  and  $U(1)_{T_{3_R}}$  remains unbroken down to low energies and forbids baryon number violation while allowing for lepton number violation. Hence, it allows for the generation of small left–handed neutrino masses via

a seesaw mechanism, specifically an extended seesaw with the singlets,  $\phi$  [5,7]. Proton decay mediating operators are only generated when the  $U(1)_{Z'}$  is broken. Thus, the scale of the  $U(1)_{Z'}$  breaking is constrained by proton lifetime limits and can be within reach of the contemporary experiments. A field theory model demonstrating these properties was presented in [7].

## Spectrum

Field	$SU(3)_C$	$\times SU(2)_L$	$\times SU(2)_R$	$U(1)_C$	$U(1)_\zeta$	$\beta_3$	$\beta_{2L}$	$\beta_Y$
$Q_L^i$	3	2	1	$+\frac{1}{2}$	$-\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{1}{6}$
$Q_R^i$	$\bar{3}$	1	2	$-\frac{1}{2}$	$+\frac{1}{2}$	1	0	$\frac{5}{3}$
$L_L^i$	1	2	1	$-\frac{3}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
$L_R^i$	1	1	2	$+\frac{3}{2}$	$+\frac{1}{2}$	0	0	1
$H_0$	1	2	2	0	0	0	1	1
$H_L^{ij}$	1	2	1	$+\frac{3}{2}$	$+\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
$H_L'^{ij}$	1	2	1	$-\frac{3}{2}$	$+\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
$H_R^{ij}$	1	1	2	$-\frac{3}{2}$	$-\frac{1}{2}$	0	0	1
$H_R'^{ij}$	1	1	2	$+\frac{3}{2}$	$-\frac{1}{2}$	0	0	1
$D^n$	3	1	1	+1	0	$\frac{1}{2}$	0	$\frac{1}{3}$
$\bar{D}^n$	$\bar{3}$	1	1	-1	0	$\frac{1}{2}$	0	$\frac{1}{3}$
$\mathcal{H}_R$	1	1	2	$+\frac{3}{2}$	$-\frac{1}{2}$	0	$\frac{3}{5}$	1
$\bar{\mathcal{H}}_R$	1	1	2	$-\frac{3}{2}$	$+\frac{1}{2}$	0	$\frac{3}{5}$	1
$S^i$	1	1	1	0	-1	0	0	0
$\bar{S}^i$	1	1	1	0	+1	0	0	0
$\phi^a$	1	1	1	0	0	0	0	0

Table 1: *High scale spectrum and  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_C \times U(1)_E$  quantum numbers, with  $i = 1, 2, 3$  for the three light generations,  $j = 1, 2$  for the number of doublets required by anomaly cancellation,  $n = 1, \dots, k$ , and  $a = 1, \dots, p$ . The  $\beta_i$  show the contributions for each state, relevant for the RGE analysis later.*

The spectrum of our model above the left–right symmetry breaking scale is summarised in Table 1. The spectrum below the intermediate symmetry breaking scale is shown in Table 2. The spectra above and below the  $SU(2)_R$  breaking scale are

both free of all gauge and gravitational anomalies. Hence, the  $U(1)_{Z'}$  combination given in equation (3.14) is viable to low energies.

Field	$SU(3)_C$	$\times SU(2)_L$	$T_{3R}$	$U(1)_Y$	$U(1)_{Z'}$	$\beta_3$	$\beta_{2L}$	$\beta_Y$
$Q_L^i$	3	2	0	$+\frac{1}{6}$	$-\frac{2}{5}$	1	$\frac{3}{2}$	$\frac{1}{6}$
$u_L^{c\ i}$	$\bar{3}$	1	$-\frac{1}{2}$	$-\frac{2}{3}$	$+\frac{3}{5}$	$\frac{1}{2}$	0	$\frac{4}{3}$
$d_L^{c\ i}$	$\bar{3}$	1	$+\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{1}{5}$	$\frac{1}{2}$	0	$\frac{1}{3}$
$L_L^i$	1	2	0	$-\frac{1}{2}$	$-\frac{4}{5}$	0	$\frac{1}{2}$	$\frac{1}{2}$
$e_L^{c\ i}$	1	1	$-\frac{1}{2}$	+1	$+\frac{3}{5}$	0	0	1
$\nu_L^{c\ i}$	1	1	$+\frac{1}{2}$	0	+1	0	0	0
$H^u$	1	2	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{5}$	0	$\frac{1}{2}$	$\frac{1}{2}$
$H^d$	1	2	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{5}$	0	$\frac{1}{2}$	$\frac{1}{2}$
$H_L^i$	1	2	0	$+\frac{1}{2}$	$+\frac{4}{5}$	0	$\frac{3}{2}$	$\frac{3}{2}$
$H'_L{}^i$	1	2	0	$-\frac{1}{2}$	$+\frac{1}{5}$	0	$\frac{3}{2}$	$\frac{3}{2}$
$E_R^i$	1	1	$-\frac{1}{2}$	-1	$-\frac{3}{5}$	0	0	1
$N_R^i$	1	1	$+\frac{1}{2}$	0	-1	0	0	0
$E'_R{}^i$	1	1	$+\frac{1}{2}$	+1	$-\frac{2}{5}$	0	0	1
$N'_R{}^i$	1	1	$-\frac{1}{2}$	0	0	0	0	0
$D^n$	3	1	0	$+\frac{1}{3}$	$+\frac{1}{5}$	$\frac{1}{2}$	0	$\frac{1}{3}$
$\bar{D}^n$	$\bar{3}$	1	0	$-\frac{1}{3}$	$-\frac{1}{5}$	$\frac{1}{2}$	0	$\frac{1}{3}$
$S^i$	1	1	0	0	-1	0	0	0
$\bar{S}^i$	1	1	0	0	+1	0	0	0
$\phi^a$	1	1	0	0	0	0	0	0

Table 2: *Low scale matter spectrum and  $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Z'}$  quantum numbers with  $\beta_i$  contributions.*

The heavy Higgs',  $\mathcal{H}_R^k + \bar{\mathcal{H}}_R^k$  that break the  $SU(2)_R \times U(1)_C \rightarrow U(1)_Y$ , along a flat direction, leave the orthogonal combination

$$U(1)_{Z'} = \frac{1}{5}U_C - \frac{2}{5}T_{3R} + U_\zeta \quad (3.14)$$

unbroken. Here, the index  $k$  allows for the possibility that the heavy Higgs sector contains more than two fields, as is typically the case in the string constructions.

Further discussion of this model, including a trilinear level superpotential, can be found in [7]. Here we notice that the incomplete representations added to the MSSM may cause problems with gauge coupling unification. The induced gauge anomalies in the  $SU(2)_{L/R}^2 \times U(1)_\zeta$  diagrams require the addition of  $H_L^{ij}, H_L'^{ij}, H_R^{ij}, H_R'^{ij}$ , which differ from the  $E_6$  case. The addition of triplets may help subdue any adverse effects and will also give scope for the inclusion of intermediate matter scales.

### Renormalization group analysis

The properly normalised  $\beta$ -function coefficients are shown in Tables 1 and 2. The numerical output of equation (3.6) and (3.7) is generated subject to the variation of the scales and is displayed in Figure 1. The intermediate scales are varied to find phenomenologically viable areas of the parameter space. The scales and ranges of  $\sin^2 \theta_W(M_Z)$  and  $\alpha_3(M_Z)$  were first restricted to the experimentally allowed regions and then also allowed to take values outside this range. The hierarchy of scales was constrained to be

$$M_S \gtrsim M_R > M_D \gtrsim M_{Z'} \gtrsim M_{SUSY} > M_Z. \quad (3.15)$$

To this end, we restricted the allowed range of  $\sin^2 \theta_W(M_Z)$  and  $\alpha_3(M_Z)$  to five sigma deviations from the central values shown in eq (3.1). The RGEs were run in Mathematica. Restricting the output to the experimentally constrained interval produced no phenomenologically viable results. Allowing the values of  $\sin^2 \theta_W(M_Z)$  and  $\alpha_3(M_Z)$  to run freely and restricting the relevant mass scales to (in GeV)

$$\begin{aligned} 2 \cdot 10^{16} \leq M_S \leq 5 \cdot 10^{17}; & & 10^5 \leq M_D \leq 10^{12}; \\ 10^9 \leq M_R \leq 5 \cdot 10^{17}; & & 10^3 \leq M_{Z'}, M_{SUSY} \leq 10^{10}, \end{aligned} \quad (3.16)$$

also produced no phenomenologically viable results, as shown in Figure 1.

### Contrasting analysis with $E_6$ embedding of $U(1)_\zeta$

To further elucidate the constraints on the LRS heterotic-string models arising from coupling unification, we contrast the outcome with the corresponding results when the  $U(1)_\zeta$  charges are embedded in  $E_6$  representations. For models that allow the  $E_6$  embedding of the  $U(1)_{Z'}$  charges, the spectrum consists of three generations of  $\mathbf{27}$ s that decompose under  $SO(10)$  as:

$$\mathbf{27}^i \rightarrow \mathbf{16}_{\frac{1}{2}}^i + \mathbf{10}_{-1}^i + \mathbf{1}_2^i. \quad (3.17)$$

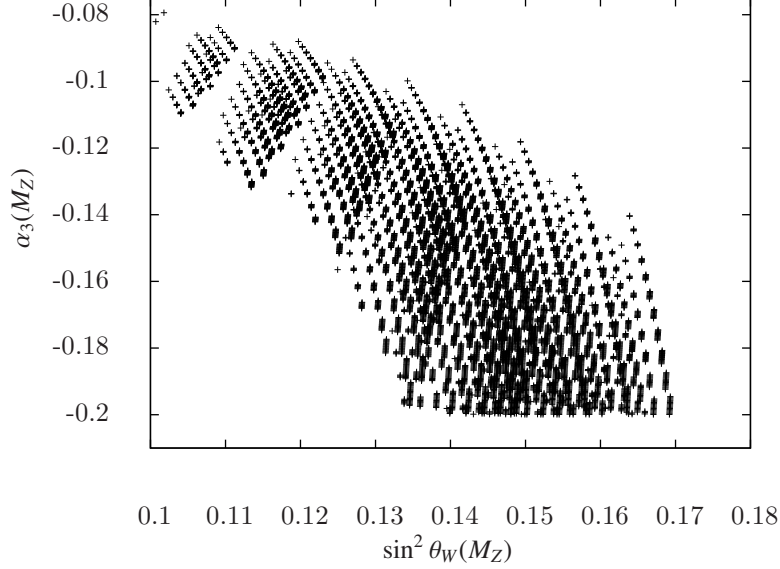


Figure 1: Freely running  $\sin^2 \theta_W(M_Z)$  and  $\alpha_3(M_Z)$ :  $\sin^2 \theta_W(M_Z)$  vs.  $\alpha_3(M_Z)$  with  $0.05 \lesssim \alpha_{\text{string}} \lesssim 0.1$ .

Under  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_C \times U(1)_\zeta$ , this results in a similar spectrum to the LRS model. The **16** decomposes exactly as for the LRS model,

$$\begin{aligned}
 Q_L^i &\sim \left(3, 2, 1, +\frac{1}{2}, +\frac{1}{2}\right); & L_L^i &\sim \left(1, 2, 1, -\frac{3}{2}, +\frac{1}{2}\right); \\
 Q_R^i &\sim \left(\bar{3}, 1, 2, -\frac{1}{2}, +\frac{1}{2}\right); & L_R^i &\sim \left(1, 1, 2, +\frac{3}{2}, +\frac{1}{2}\right),
 \end{aligned} \tag{3.18}$$

with the proviso that the charges under  $U(1)_\zeta$  take the same sign. The **10** decomposes as

$$H^i \sim (1, 2, 2, 0, -1); \quad D^i \sim (3, 1, 1, +1, -1); \quad \bar{D}^i \sim (\bar{3}, 1, 1, -1, -1). \tag{3.19}$$

The remaining singlets are neutral under the SM gauge group and are used to break the  $U(1)_{Z'}$ . In addition to the complete  $SO(10)$  representations above, the  $E_6$  spectrum includes a bidoublet,

$$H_0 \sim (1, 2, 2, 0, -1), \tag{3.20}$$

that facilitates gauge coupling unification. The model also contains the pair of heavy Higgs right-handed doublets,

$$\mathcal{H}_R + \bar{\mathcal{H}}_R = \left(1, 1, 2, \frac{3}{2}, \frac{1}{2}\right) + \left(1, 1, 2, -\frac{3}{2}, -\frac{1}{2}\right), \tag{3.21}$$

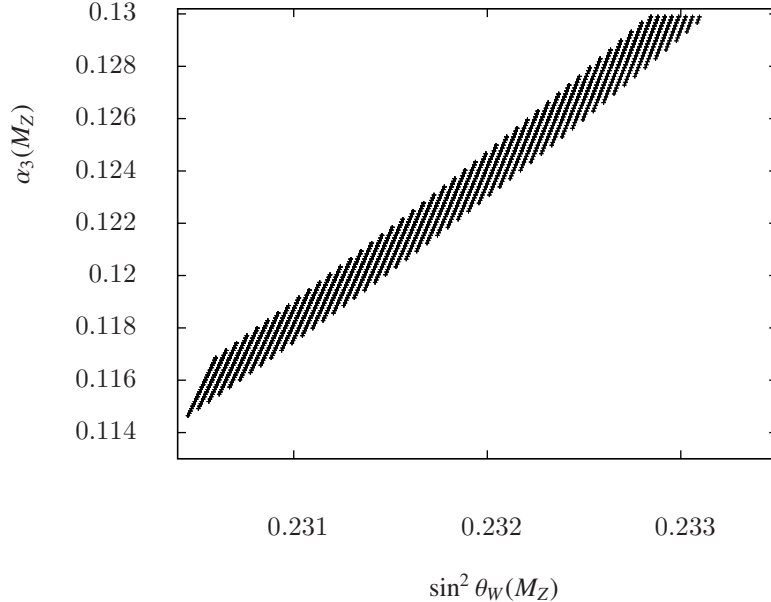


Figure 2: Freely running  $\sin^2 \theta_W(M_Z)$  and  $\alpha_3(M_Z)$ :  $\sin^2 \theta_W(M_Z)$  vs.  $\alpha_3(M_Z)$  with  $0.05 \lesssim \alpha_{\text{string}} \lesssim 0.1$  for Model II.

that break the intermediate  $SU(2)_R$  symmetry. We run the RGEs in exactly the same way as shown for the LRS model, constraining the mass scales to the hierarchy

$$M_S \gtrsim M_R \gtrsim M_D = M_{Z'} \gtrsim M_{\text{SUSY}} \gg M_Z. \quad (3.22)$$

In this model we find that unification does occur, as found in previous literature. We note that the phenomenologically viable results (see Figure 2) required  $M_S \sim M_X \sim 2 \cdot 10^{16}$  GeV as expected. The intermediate scales were found to be (in GeV)

$$1 \cdot 10^{13} \leq M_R \leq 1 \cdot 10^{16}; \quad 1 \cdot 10^3 \leq M_D \leq 1 \cdot 10^8; \quad 1 \cdot 10^3 \leq M_{\text{SUSY}} \leq 1 \cdot 10^6, \quad (3.23)$$

with  $M_{Z'}$  between  $1 - 10^5$  TeV. In this case we have taken the mass of the vector-like doublets,  $M_{Z'}$ , and triplets,  $M_D$  to be degenerate, which is the case in  $E_6$  inspired models, as they are generated by the same singlet VEV. String models afford more flexibility that we do not make use of in our analysis here. Fine-tuning the  $M_{\text{SUSY}}$  allows for  $M_{Z'}$  to be in agreement with current experimental bounds.

The contrast between the two cases can be elucidated further by examining more closely the contributions of the intermediate gauge and matter thresholds to  $\sin^2 \theta_W(M_Z)$  and  $\alpha_3(M_Z)$ . Using the general expressions in equations (3.6) and (3.7) we find that, in the case of the spectrum and charge assignments in the LRS heterotic-string model, shown in Tables 1 and 2, the threshold corrections from intermediate

gauge and matter scales are given by

$$\begin{aligned}\delta(\sin^2\theta_W(M_Z))_{\text{i.t.}} &= \frac{1}{2\pi} \frac{k_1\alpha}{1+k_1} \left( \frac{12}{5} \log \frac{M_S}{M_R} - \frac{24}{5} \log \frac{M_S}{M_{Z'}} - \frac{2n_D}{5} \log \frac{M_S}{M_D} \right), \\ \delta(\alpha_3(M_Z))_{\text{i.t.}} &= \frac{1}{2\pi} \left( \frac{3}{2} \log \frac{M_S}{M_R} - 9 \log \frac{M_S}{M_{Z'}} + \frac{3n_D}{4} \log \frac{M_S}{M_D} \right).\end{aligned}\tag{3.24}$$

In the case of models that admit an  $E_6$  embedding of the charges, the same threshold corrections are given by

$$\begin{aligned}\delta(\sin^2\theta_W(M_Z))_{\text{i.t.}} &= \frac{1}{2\pi} \frac{k_1\alpha}{1+k_1} \left( \frac{12}{5} \log \frac{M_S}{M_R} + \frac{6}{5} \log \frac{M_S}{M_H} - \frac{6}{5} \log \frac{M_S}{M_D} \right), \\ \delta(\alpha_3(M_Z))_{\text{i.t.}} &= \frac{1}{2\pi} \left( \frac{3}{2} \log \frac{M_S}{M_R} - \frac{9}{4} \log \frac{M_S}{M_H} + \frac{9}{4} \log \frac{M_S}{M_D} \right).\end{aligned}\tag{3.25}$$

If we take  $M_S$  to coincide with the MSSM unification scale and with  $M_R$  as well, then the first lines in equations (3.6) and (3.7), which only contain the MSSM contributions, are in good agreement with the observable data. The corrections arising from the intermediate gauge and matter thresholds in equations (3.24) and (3.25) then have to cancel. We see from equation (3.24) that the corrections from the intermediate doublet and triplet thresholds contribute with equal sign in  $\sin^2\theta_W(M_Z)$ . For  $\alpha_3(M_Z)$ , the corrections from these thresholds contribute with opposite sign, but the contribution of the doublets outweigh the contribution of the triplets. We may compensate for the negative contribution from the extra doublets by lowering the  $SU(2)_R$  breaking scale. Requiring that  $m_{\nu\tau} \lesssim 1\text{eV}$  necessitates that  $M_R \geq 10^9\text{GeV}$ . Keeping the extra triplets at the GUT scale, and the  $Z'$  scale at  $10^{12}\text{GeV}$  then yields rough agreement with  $\sin^2\theta_W(M_Z)$  but gross disagreement with  $\alpha_3(M_Z)$ . Lowering the triplet scale improves the agreement with  $\alpha_3(M_Z)$  but conflicts with the data for  $\sin^2\theta_W(M_Z)$ . We therefore conclude that a low scale  $Z'$  in the LRS heterotic-string models is incompatible with the gauge data at the  $Z$ -boson scale. In contrast, from equation (3.25) we see that the corresponding corrections cancel each other, provided that  $M_H = M_{Z'} = M_D$ . This is the case as both are generated by the  $Z'$  breaking VEV. This cancellation is, of course, the well known cancellation that occurs when the representations fall into  $SU(5)$  multiplets. Allowing  $M_R$  to be at  $10^{15}\text{GeV}$  then compensates for the SUSY threshold at  $1\text{TeV}$ , enabling accommodations of the low-energy data, as illustrated in Figure 2.

## 4 String models with $E_6$ embedding

The low scale  $Z'$  in the string models is, in essence, a combination of the Cartan generators,  $U(1)_{1,2,3}$ , that are generated by the right-moving complex world-sheet fermions  $\bar{\eta}^{1,2,3}$ , together with a  $U(1)$  symmetry, embedded in the  $SO(10)$  GUT, and is

orthogonal to the weak hypercharge. Whether, or not, the symmetry is anomaly free depends on the specific symmetry breaking pattern induced by the GGSO projections. As we discussed above, in the FSU5, PS and SLM the symmetry is anomalous, whereas in the LRS models it is anomaly free. The difference stems from the fact that in the former cases the combination for  $U(1)_\zeta$  admits the  $E_6$  embedding but in the latter it does not. On the other hand, as we have seen in Section 3, the  $E_6$  embedding allows for compatibility with the low scale gauge coupling data. The  $Z'$  in the LRS models, which do not admit the  $E_6$  embedding, is constrained to be heavier than at least  $10^{12}\text{GeV}$ . Gauge coupling data, therefore, seems to indicate that the  $E_6$  embedding of the charges is necessary. We emphasize that the indication is that the charges must admit an  $E_6$  embedding and *not* that the  $E_6$  symmetry is actually realised. An illustration of this phenomenon is the existence of self-dual models under the spinor–vector duality without  $E_6$  enhancement [19]. The question then arises as to how one constructs heterotic–string models with anomaly free  $U(1)_\zeta$ , which admit an  $E_6$  embedding. Here we discuss how viable heterotic–string models with  $E_6$  embedding of the  $U(1)_{Z'}$  charges may be obtained. The main constraint being that the extra  $U(1)$  symmetry has to be anomaly free. For this purpose, we first give a general overview as to how the gauge symmetry is generated in the string models.

The vector bosons that generate the four dimensional gauge group in the string models arise from two principal sectors: the untwisted sector and the sector  $x = \{\bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}\}$ . In the  $x$ –sector the complex right–moving world–sheet fermions, that generate the Cartan subalgebra of the observable gauge group, are all periodic. At the level of the  $E_8 \times E_8$  heterotic–string in ten dimensions, the vector bosons of the observable  $E_8$  are obtained from the untwisted sector and from the  $x$ –sector. Under the decomposition  $E_8 \rightarrow SO(16)$ , the adjoint representation decomposes as  $\mathbf{248} \rightarrow \mathbf{120} + \mathbf{128}$ , where the adjoint  $\mathbf{120}$  representation is obtained from the untwisted sector and the spinorial  $\mathbf{128}$  representation is obtained from the  $x$ –sector. The set  $\{\mathbf{1}, S, x, \zeta\}$  produces a model with  $N = 4$  spacetime supersymmetry in four dimensions. The gauge symmetry arising in this model, at a generic point in the compactified space, is either  $E_8 \times E_8$  or  $SO(16) \times SO(16)$  depending on the GGSO phase  $c(\frac{x}{\zeta}) = \pm 1$ .

Adding the basis vectors  $b_1$  and  $b_2$  reduces the spacetime supersymmetry to  $N = 1$ . The observable gauge symmetry reduces from  $E_8$  to  $E_6 \times U(1)^2$  or  $SO(16) \rightarrow SO(10) \times U(1)^3$ . Additional vectors reduce the gauge symmetry further. Aside from the model of [20], all the quasi–realistic free fermionic models follow the second symmetry breaking pattern. That is, in all these models, the vector bosons arising from the  $x$ –sector are projected out.

We consider, then, the symmetry breaking pattern induced by the following



boundary condition assignments in two separate basis vectors

$$1. \quad b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5}\} = \left\{\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}\right\} \Rightarrow SU(5) \times U(1), \quad (4.1)$$

$$2. \quad b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5}\} = \{1 \ 1 \ 1 \ 0 \ 0\} \Rightarrow SO(6) \times SO(4). \quad (4.2)$$

The assignment in equation (4.1) reduces the untwisted  $SO(10)$  gauge symmetry to  $SU(5) \times U(1)$ , however the assignment in eq. (4.2) reduces it to  $SO(6) \times SO(4)$ . Thus, the inclusion of equations (4.1) and (4.2) in two separate boundary condition basis vectors reduces the  $SO(10)$  gauge symmetry to  $SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L$ , where  $2U(1)_C = 3U(1)_{B-L}$  and  $U(1)_L = 2U(1)_{T_{3R}}$ . For appropriate choices of the GGSO projection coefficients, the vector bosons arising from the  $x$ -sector enhance the  $SU(3) \times SU(2) \times U(1)^2 \times U(1)_\zeta$  arising from the untwisted sector to  $SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_{\zeta'}$ , where

$$U(1)_4 = U(1)_C + 3U(1)_L - 3U(1)_\zeta; \quad (4.3)$$

$$U(1)_2 = U(1)_C + U(1)_L + U(1)_\zeta; \quad (4.4)$$

$$U(1)_{\zeta'} = -3U(1)_C + 3U(1)_L + U(1)_\zeta. \quad (4.5)$$

$U(1)_4$  and  $U(1)_2$  are embedded in  $SU(4)_C$  and  $SU(2)_R$ , respectively, and  $U(1)_\zeta$  is given by equation (2.2). The matter representations charged under this group arise from the sectors  $b_j$  and are complemented by states from  $b_j + x$  to form the ordinary representations of the Pati–Salam model. The difference, as compared to the Pati–Salam string models of [12], is that  $U(1)_{\zeta'}$  is anomaly free. The reason is that all the states of the **27** representation of  $E_6$  are retained in the spectrum, whereas in the Pati–Salam models of [12] the corresponding states are projected out. The symmetry breaking of the Pati–Salam  $SU(4)_C \times SU(2)_R$  group is induced by the VEV of the heavy Higgs in the  $(\bar{4}, 1, 2)_{-\frac{1}{2}} \oplus (4, 1, 2)_{+\frac{1}{2}}$  representation of  $SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_{\zeta'}$ . In addition to the weak–hypercharge, this VEV leaves the unbroken combination

$$U(1)_{Z'} = \frac{1}{2}U(1)_{B-L} - \frac{2}{3}U(1)_{T_{3R}} + \frac{5}{3}U(1)_{\zeta'}, \quad (4.6)$$

which is anomaly free and admits the  $E_6$  embedding of the charges.

## 5 Conclusions

In this paper we examined the gauge coupling unification constraints imposed on a low scale  $Z'$  arising in LRS heterotic–string derived models. The existence of a low–scale  $Z'$  in these models guarantees that PDMOs are sufficiently suppressed. However, we have shown that the hypothesis of a low scale  $Z'$  in these models is incompatible with the gauge coupling data at the electroweak scale. We contrasted this result

with the corresponding result in string models that admit an  $E_6$  embedding of the  $U(1)$  charges. In the latter case the possibility of a low scale  $Z'$  is viable. We further discussed how heterotic–string models that admit the  $E_6$  embedding may be obtained in the free fermionic formulation, though an explicit three generation viable model is yet to be constructed. Similarly, a more complete analysis of the phenomenological realisation of this  $U(1)$  symmetry in heterotic–string models is warranted and will be reported in future publications. We also remark that other  $U(1)$  symmetries that have been proposed in the literature to suppress proton decay mediating operators [4, 29] have also been invalidated due to neutrino masses and other constraints [6]. The enigma of the proton lifetime in heterotic–string unification continues to serve as an important guide in the search for viable string vacua.

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