

**1 Post-emplacment cooling and contraction of lava
2 flows: InSAR observations and a thermal model for
3 lava fields at Hekla volcano, Iceland**

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5 **Abstract.** Lava flows contract as they cool, causing progressive subsi-
6 dence of the flow surface. Here we study this process by measuring and mod-
7 elling the deformation of emplaced lava flows and the surrounding substrate.
8 The temporal trend of vertical lava movements was investigated using in-
9 terferometric analysis of synthetic aperture radar (InSAR) images from the
10 1991 and 2000 Hekla eruptions, covering periods of 23 and 12 years, respec-
11 tively. Data from six tracks from three satellites, including both ascending
12 and descending passes, were used to create 99 interferograms, from which
13 trends of accumulated subsidence and subsidence velocities were derived. Sub-
14 sidence rates are similar for both lava flows and decay approximately expo-
15 nentially from about 20 mm/year five years after emplacement to about 2
16 mm/year 15 years after emplacement. A one-dimensional, semi-analytical model
17 was fitted to the observed subsidence rates, with subsidence due to phase
18 change calculated analytically, and subsidence due to thermal contraction
19 calculated numerically using dilatometric constraints obtained experimentally.
20 The initial thicknesses of the 1991 and 2000 lava fields, D_{1991} and D_{2000} , scaled
21 thermal expansivity, $\gamma\alpha$, and thermal diffusivity, κ , are the crucial param-
22 eters influencing lava subsidence and subsidence rate. Inversion for these pa-
23 rameters reveal linear correlations between them. Best fitting results of in-
24 versions for D_{1991} range from 10 m to 27 m, for D_{2000} from 10 m to 30 m,
25 $\gamma\alpha = (9 - 24) \times 10^{-6} \text{ K}^{-1}$, and $\kappa = (1 - 7) \times 10^{-7} \text{ m}^2\text{s}^{-1}$.

1. Introduction

Lava flows often form ideal surfaces for interferometric analyses of satellite-acquired synthetic aperture radar images (InSAR measurements), given their high backscatter due to their roughness and their high coherence due to their stable surface and lack of vegetation. Post-emplacement lava subsidence has been observed at many volcanoes, e.g. at Etna volcano in Italy [Briole *et al.*, 1997; Stevens *et al.*, 2001a], Okmok volcano in Alaska [Patrick *et al.*, 2004; Lu *et al.*, 2005, 2010], Kilauea volcano in Hawaii [Peck, 1978; Hardee, 1980; Dietterich *et al.*, 2012], Santiaguito volcano, Guatemala [Ebmeier *et al.*, 2012], Parícutin volcano, Mexico [Chaussard, 2016], and in Iceland e.g. at Krafla volcano in North Iceland [Sigmundsson *et al.*, 1997] and Hekla volcano in South Iceland [Ofeigsson *et al.*, 2011]. In Iceland, however, the underlying processes have not been studied in detail. Hekla volcano, which has erupted five times in the last half century, has been targeted by frequent satellite acquisitions over the last few decades, providing excellent opportunities to study the processes governing lava compaction, as InSAR interferograms can map lava deformation over a wide area with a high spatial resolution. This study addresses cooling, thermal contraction and the associated vertical movement of two lava flows at Hekla volcano over 23 years with detailed observations and modelling. When interpreting ground deformation at active volcanoes, it is essential to separate the different sources contributing to it, including the thermal contraction of recent lava flows. Petrological, physical [e.g. Kattenhorn and Schaefer, 2008; Keszthelyi, 1994], and rheological response of cooling lava flows may bias the retrieval of parameters associated with deeper magmatic processes, such as inferred magma source location and strength. Constraints on the thermal contraction of

47 lava flows may thus enhance the resolution with which deep processes can be retrieved from
48 ground deformation studies, furthering the aspiration of acquiring a rheological description
49 of the state of magma during volcanic unrest.

50 InSAR techniques [*Rosen et al.*, 2000; *Massonnet and Feigl*, 1998] can map the change
51 in distance along the line of sight (LOS) from ground to satellite between the acquisition
52 times of SAR images by accurately comparing the phase of reflected radar waves in a set
53 of SAR images. The accuracy of inferred LOS change can be as good as a few millimeters
54 for patches of ground approximately 100 m² in size (e.g. *Massonnet and Feigl* [1998];
55 *Hooper et al.* [2012]), and thus it is a very useful technique to study post-emplacement
56 deformation of lava fields. Georeferenced data of lava flow outlines allow detailed cor-
57 relation of subsiding areas with the extent of lava flows. During the first phase of lava
58 contraction, decorrelation in interferograms may occur because of the repacking of surficial
59 clasts *Stevens et al.* [2001b], deformation of the viscous molten core of a lava flow [*Harris*
60 *and Rowland*, 2001] and creeping of the underlying substrate [*Stevens et al.*, 2001a, b]. A
61 pioneering study of lava cooling, solidification and accompanying degassing and its effect
62 on vertical movements was undertaken between 1963 and 1967 at Alae lava Lake, Hawaii
63 [*Peck*, 1978]. There, temperature and surface deformation measurements were carried
64 out until the maximum temperature in the lava lake fell below 100°C, which was reached
65 about four years after its emplacement. A recent study of the effects of solidification and
66 degassing on vertical lava movement at Okmok volcano, Alaska, suggests that exsolution
67 of gases may be an important contribution to volume changes during solidification near
68 solidus conditions [*Caricchi et al.*, 2014].

69 Hekla volcano is located in south Iceland (Figure 1) at the junction of the Eastern
70 Volcanic Zone (EVZ) and the South Iceland Seismic Zone (SISZ) (*Einarsson et al.* [1981],
71 *Sigmundsson et al.* [1995]). The volcanic edifice is a southwest-northeast striking volcanic
72 ridge, which rises 1488 meters above sea level. It is one of the most active volcanoes in
73 Iceland [*Thorarinsson*, 1950], with recent eruptions in 1970, 1980/1981, 1991 and 2000.
74 These eruptions began with an explosive phase (e.g. *Gudmundsson et al.* [1992]; *Gronvold*
75 *et al.* [1983]; *Höskuldsson et al.* [2007]), before transitioning into the effusion of lava flows,
76 which were emplaced onto the surroundings of the main volcanic edifice (Figure 1b). Two
77 of the most recent lava fields are studied here: one formed during the 17 January - 11
78 March 1991 eruption, and the other during the 26 February - 8 March 2000 eruption. Both
79 lavas were emplaced as 'a'ā flows and have similar basalt andesite chemical compositions
80 [*Moune et al.*, 2006].

81 The lava deformation at Hekla is superimposed on a much wider deformation field at
82 the volcano. Prior to, and after the eruption in 2000, a circular area of about 9 km radius
83 around the main edifice of the volcano showed steady uplift at rates of up to 5 mm/yr
84 [*Ofeigsson et al.*, 2011]. This is attributed to pressurisation of a source at a depth of
85 17 – 24 km [*Ofeigsson et al.*, 2011; *Geirsson et al.*, 2012]. Co-eruptive deflation in 2000
86 was of a similar magnitude as the precursory inflation between 1993 and 2000 [*Ofeigsson*
87 *et al.*, 2011]. Superimposed on the inter-eruptive uplift is a central region of subsidence,
88 extending beyond the lava fields and exceeding the lava deformation signals. This latter
89 observation may relate to viscous relaxation of the Earth due to loading of voluminous
90 lava flows [*Grapenthin et al.*, 2010], or may be a consequence of the shape of the deep
91 pressure source [*Geirsson et al.*, 2012]. The resulting inter-eruptive deformation between

1991 and 2000 of Hekla volcano forms a doughnut-shaped pattern of uplift surrounding
the main edifice [*Grapenthin et al.*, 2010; *Ofeigsson et al.*, 2011; *Grapenthin et al.*, 2010].
This wide-deformation signal is only visible in InSAR time series analysis when carried
out over a wider region than we focus on in this study.

2. Data analysis

2.1. Interferometric analysis

Interferometric analysis was carried out on SAR images of Hekla volcano acquired since
1993 by the European Remote Sensing (ERS), Envisat and COSMO-SkyMed (CSK) satel-
lites. Six tracks from these satellites enable us to investigate post-emplacement contraction
of lava flows for the last 20 years. An overview of the data and results are given in Table
1 and Figure 1. Interferograms were formed using the Doris software [*Kampes and Usai*,
1999]. Each satellite image was processed at full resolution corresponding to pixel sizes
of 3 m for CSK data, and 12.5 m for ERS and Envisat data. In our analysis we used an
intermediate TanDEM-X digital elevation model (DEM) formed from TerraSAR-X and
TanDEM-X (TDX) satellite SAR images acquired in 2011-2012, with a resolution of 25
m \times 12 m. Time series analysis was carried out on the resulting interferograms using
the StaMPS software [*Hooper*, 2008; *Hooper et al.*, 2012], which implements an InSAR
persistent scatterer (PS) approach [*Hooper et al.*, 2004]. In this approach, pixels are se-
lected that have a single bright scatterer that reflects the radar signal transmitted by the
satellite. No filtering is applied prior to the time series analysis. For each selected point,
the StaMPS approach evaluates LOS change due to deformation and atmospheric error.
DEM errors are estimated and removed. The PS approach works well on terrain such as
the ‘a’ lava fields at Hekla volcano because their textural variations and coarseness form

113 persistent scatters at the satellite wavelength. A visual inspection of the interferograms
114 was performed to exclude those with poor signal-to-noise ratio due to high atmospheric
115 disturbance and little coherence over the lava flows, resulting from snow and large satellite
116 baselines. Moreover, for the CSK time-series, no clear correlation between the phase and
117 topography were observed and no correction for atmospheric signal was performed. For
118 COSMO-SkyMed data the PS approach was considered because of the good signal-to-
119 noise ratio obtained on the lava field. For the ERS and Envisat data set few PS pixels
120 were selected by StaMPS, probably due to high satellite baselines, so a combined PS and
121 Small Baseline (SB) approach was used. In the SB approach, interferograms are selected
122 such their time interval is short and with a small difference in the satellite view, in order
123 to minimize decorrelation in interferograms. The combined PS and SB approach com-
124 bines advantages of both the approaches and can select more pixels than either of the
125 approaches alone, when the InSAR data contains pixels with a range of scattering char-
126 acteristics [Hooper *et al.*, 2012]. Phase unwrapping (inferring the appropriate number of
127 whole wavelengths to add to the observed modulo 2π phase) is carried as part of the time
128 series analysis. For one of the satellite tracks (CSK Track 2574) pre-unwrapping phase
129 filtering was applied to suppress unwrapping errors, but for other data good results were
130 achieved without this pre-filtering. The inferred average LOS changes for the different
131 time intervals are based on INSAR time series analysis shown in Appendix 1. Figure 2
132 shows average LOS changes at Hekla volcano for the tracks listed in Table 1 and shown
133 in Appendix 1. Each velocity image was referenced to its mean velocity in such a manner
134 that the average velocity is subtracted and the sum of the LOS change of all pixels is zero.
135 Each velocity map thus shows relative LOS changes. The central part of the volcano shows

136 LOS lengthening, partly overlapping with recent lava flows. Velocity maps overlapping
 137 in time and space from different satellite view angles allow an independent check of their
 138 noise level. Velocity maps of e.g. ERS Track 52 and ERS Track 324 for similar time
 139 spans (May 1995 - Jun 2000) were compared. Since both tracks are descending and have
 140 similar LOS unit vectors (compare Table 1), they should basically show similar results.
 141 The difference of the velocity maps shown in Figure 2a and b has a standard deviation of
 142 4 mm/yr for the whole area, much smaller than the signal studied. Averaging pixels over
 143 limited areas should result in significantly reduced uncertainty.

144 The 1991 and 2000 eruptions primarily generated lava flows on the southern and south-
 145 western flanks of Hekla volcano (Figure 1). Since their respective emplacement, the lava
 146 tongues formed in 1991 and in 2000 show continuous subsidence during the period of this
 147 study (Figure 2 and figures in Appendix 1 showing time series of LOS change). They were
 148 selected as structures of interest for the present study aiming to constrain cooling-induced
 149 phase change and contraction.

150 LOS changes give the projection of a three dimensional displacement vector onto the
 151 direction of line of LOS from ground to satellite. Since SAR satellites are side-looking, the
 152 line of sight forms an "incidence angle", θ , with the normal of the surface of the Earth.
 153 The incidence angles for the different satellite configurations considered here are listed in
 154 Table 1 as well as the direction of the unit vector from ground to satellite. In the absence
 155 of any horizontal movements, the relation of LOS change to vertical movement can be
 156 reduced to:

$$d = \frac{\overline{\text{LOS}}}{\cos \theta} \quad (1)$$

157 where d is the vertical movement and $\overline{\text{LOS}}$ is the length of the displacement vector along
158 the line of sight. In general there is, however, horizontal movement whose influence on
159 the LOS change depends on the view angle. When ascending and descending tracks are
160 available for the same region of interest and a similar time span, a linear combination of
161 the LOS changes can be applied to approximate vertical and east displacement compo-
162 nents. Adding together LOS changes from ascending and descending tracks results in a
163 linear combination that is mostly dependent on the vertical displacement (see LOS unit
164 vectors in Table 1), and subtracting them results in a linear combination mostly depen-
165 dent on the east displacement component. These linear combinations can be scaled to
166 reveal near-vertical and near-east displacement components [e.g. *Keiding et al.*, 2010]. In
167 our case (Figure 3), this decomposition of coupled LOS changes shows the lava tongues
168 selected as special target areas have a relatively well defined vertical displacement field.
169 For 1993-1999, the near-east displacement components next to the 1991 lava tongue are
170 similar compared to areas in its vicinity. For the 2000 lava tongue, an insignificant east
171 displacement extends beyond the lava outlines during the period 2011-2014.

172 In order to investigate the correlation between areas of LOS lengthening and lava field
173 extents, profiles striking West-East through both lava tongues were extracted (Figure 4).
174 For both profiles, LOS rates change abruptly from near constant values on either side of
175 the lava fields to higher negative values in the lava fields themselves, strengthening our
176 assumption LOS changes over the flows are a function of internal processes within them.
177 However, data to the West of both lava fields show a slight gradual change when approach-
178 ing the lava edge, which can be interpreted as minor deformation of the surrounding host
179 rock.

2.2. Temporal evolution of lava compaction

180 The subsidence time series was derived by selecting two sample areas well within each
181 of the lava tongues in order to avoid edge effects (Figure 1). These sample areas within
182 the lava tongues have well defined vertical displacements, and an east displacement that
183 does not correlate with the lava extent (Figure 3). Relative LOS change between the
184 sample areas and reference points outside the lava tongue is thus mostly due vertical
185 displacement. Furthermore, based on pre-emplacement DEMs, we infer the lava tongues
186 were emplaced on relatively flat ground and thus InSAR errors relating to significant
187 topographic relief will be minimal. For both lava tongues, the average LOS changes
188 of all pixels within circles of 100 m radius (centred at the orange and green stars in
189 Figure 1) were extracted from the InSAR time series (see Appendix 1). In order to infer
190 the internal deformation of the lava tongues, we subtracted the contribution from large
191 scale deformation near the lava tongues, but outside them, represented by mean values
192 from orange and green circles in Figure 1b., also of 100 m radius. In the case of the
193 ERS and Envisat data, the averaging over one circle included typically about 48 pixels.
194 For COSMO-SkyMed the number of points was about 1000 when no phase filtering was
195 applied prior to unwrapping (CSK Track 2575), but reduced to about 12 for the case when
196 filtering was applied prior to unwrapping (CSK Track 2574). However, in that case the
197 scatter between pixel values was less. Taking the difference between the average value on
198 a lava tongue and the corresponding average value outside of it reveals the lava-related
199 LOS change. Uncertainty on the lava-related LOS change inferred in this way was based
200 on the standard deviation of the selected pixels within the 100 m radius areas used. The

201 uncertainty (see error bars on Fig. 5) is comparable for the different satellites used,
202 although somewhat better for the CSK data.

203 Vertical deformation in the sample areas relative to their reference points was inferred
204 using Equation 1, assuming no horizontal deformation due to internal processes within
205 the lava tongues during the period of observation, in broad agreement with the inferred
206 near-vertical and near-east components of displacements (when LOS change over lava is
207 referenced to areas outside the lava). The LOS velocity field is thus scaled by $(\cos \theta)^{-1}$
208 to infer the vertical deformation. Figures 5a and 5b show the results for the lava fields
209 of 1991 and 2000, referencing each time series to its first acquisition. As can be seen
210 from Figures 5b and 2a-c, the region of the 2000 lava tongue shows slight subsidence
211 before 2000, likely due to compaction of the 1980-81 lava field underneath. This trend
212 was assumed to continue linearly after the eruption in 2000 and was subtracted from the
213 data. Considering the processes causing deformation of the lava fields are continuous,
214 the relative deformation data are transferred into deformation rates. This can be inferred
215 from linear regressions between groups of data points within each time series, with the
216 change between groups of points revealing deformation rates. Images from each summer
217 period were typically considered as one group. Years with less than two data points were
218 grouped together with a neighboring year. An example for ERS Track 52 is shown in
219 Figure 5c. Data and errors from linear regression for the six InSAR tracks are shown in
220 Figures 6a and 6b.

221 Clear evidence is found for decay of lava subsidence rate, ω , with time, t , since lava
222 emplacement. The observed variation in subsidence was initially fitted with an exponential
223 decay model:

$$\omega = \frac{d\delta y}{dt} = A \cdot e^{-t/b} \quad (2)$$

224 where δy is the vertical displacement, A is a constant and b is the exponential decay
 225 rate. For the lava tongue of 1991, the initial four years of data after emplacement were
 226 neglected since they show a different behavior. The best fitting parameters are $b = 4.7$
 227 yrs^{-1} and $A = 51.5$. For the lava tongue of 2000, best fitting parameters are $b = 6.0 \text{ yrs}^{-1}$
 228 and $A = 44.2$. The corresponding results are presented in Figures 6a and 6b.

229 By integrating equation 2, the accumulated deformation can be expressed as:

$$d = \delta y_0 + \delta y_{ts} - A \cdot b \cdot e^{-t/b} \quad (3)$$

230 $\delta y_{ts} + \delta y_0$ is the constant of integration and is different for each InSAR time series. It
 231 is split into two parts: δy_{ts} , which refers to the relative shifts of the single InSAR time
 232 series, and δy_0 , which is the same for all InSAR time series and refers to the accumulated
 233 deformation before the first observation. The value for δy_0 cannot be resolved from our
 234 observations and is set to zero in Figures 6c and 6d, which show the best fits of Equation
 235 3.

236 The lava subsidence rate follows approximately an exponential decay pattern for both
 237 lava fields. For the 1991 lava tongue, data before 1995 do not follow this exponential
 238 trend. Instead, the observations suggest lower subsidence rates during the first three to
 239 four years after the eruption. This is unlikely to result from InSAR related errors, such
 240 as atmospheric contributions, DEM errors or baselines, since six data points from two
 241 different satellite tracks show this behavior. This suggests that the lower initial subsidence
 242 rates are real and result from other contributing factors not considered here. A partial

243 explanation may be degassing-induced expansion while the lava fields still have a molten
244 core [e.g. *Caricchi et al.*, 2014], although it is uncertain how important this process is in
245 our case. The exponential model presented above is simple, fits well with the data, and
246 is therefore good to describe the temporal evolution. However, it does not explain the
247 physics of the process involved, limiting the interpretation of the meaning of the A and b
248 constants. A physics based model, considering the thermal evolution of the lava field as
249 presented below, can on the other hand provide insights into the physical process taking
250 place.

3. Thermal model

251 We used a thermal model to place constraints on the physical processes resulting from
252 thermal contraction following the emplacement of lava. This one-dimensional model of
253 lava cooling and contraction processes is subdivided into phases preceding and follow-
254 ing complete solidification. Before solidification, latent heat of crystallisation is released
255 between liquidus and solidus temperature conditions, and dissipated by conduction into
256 the air and bedrock. After solidification the evolution of the lava flow is governed by
257 conductive cooling. The problem then reduces to solving the heat diffusion equation and
258 superimposed thermal contraction. The temperature distribution at the time when the
259 lava fully solidifies is a parameter that needs quantification. During the initial phase of
260 solidification, the thermal distribution within a lava flow and the substrate can be calcu-
261 lated with formulas derived from *Carslaw and Jaeger* [1959, Chapter XI, §11.2] and *Crank*
262 [1984, Chapter 3.2]. Details of the method are presented in Appendix A, considering den-
263 sity changes and assumed constant initial temperature of the substrate. The predictions
264 for lava contraction of this conduction based thermal model are shown in Figures 6a and

265 6b. Deviation from the smooth decline in subsidence rates (peak lowering of subsidence
 266 rates) near the beginning of the time series is associated with period when lava becomes
 267 fully solidified. At this time the internal temperature profile within the lava readjusts
 268 and for a limited time period less heat leaves the lava field, and subsidence rates are
 269 temporarily lowered. The main features of the modeling approach are summarized in the
 270 following paragraphs, but a full treatment and derivation of the governing equations is
 271 found in Appendix B.

272 A lava flow of initial thickness, D , and of initial homogeneous crystallization temper-
 273 ature, T_l , is emplaced on a flat ground (Figure 7). Note that for simplicity, we assume
 274 that crystallization takes place at a given temperature instead of a temperature interval.
 275 The coordinate system is chosen with origin $y = 0$ at the bottom of the flow, and posi-
 276 tive values upwards. The temperature of the air, T_a , and of the substrate, T_b , at a large
 277 depth, $y \rightarrow -\infty$, are kept constant throughout the simulation. The lava flow solidifies
 278 from above and below, and upper and lower crusts develop. Under these assumptions,
 279 calculations for the lower and upper boundary are independent of each other as long as
 280 a liquid core exists. During this solidification process, the subsidence rate, u , of the lava
 281 flow due to thermal contraction, u_{th} , and due to density changes at the upper (u_{up}) and
 282 lower (u_{low}) crusts is given by

$$u = u_{up} + u_{th} + u_{low} \quad (4)$$

283 Before full solidification, the temperature distribution is described by equations A25
 284 - A28 in Appendix A. In order to estimate the contribution from thermal contraction,
 285 these equations are implemented in a finite difference scheme. After each time step, the

286 thermal contraction is calculated. The thermal contraction of a lava flow is a volumetric
 287 process, and one has to consider how the linear coefficient of thermal expansion (α) relates
 288 to the change in elevation. Contraction of the horizontal dimensions may be translated
 289 into lowering of the lava flow surface, in addition to the thermal contraction in the vertical
 290 dimension. In order to account for that possibility, we calculate the contraction in vertical
 291 dimension according to:

$$\Delta h = \gamma \alpha h \Delta T \quad (5)$$

292 where h is height and Δh change in height. If the coefficient γ equals 1, the basic linear
 293 thermal expansion formula is retrieved. If all the volumetric contraction of a cube element
 294 goes into lowering its height, then γ would equal 3. Intermediate value may also apply;
 295 when studying lava flow contraction *Chaussard* [2016] used a value of $\gamma = (1 + \nu)/(1 - \nu)$
 296 , where ν is the Poisson's ratio. For the typical value of $\nu = 0.25$, then $\gamma = 1.7$.

297 After complete solidification, temperature changes within the size of the finite differences
 298 are calculated via the heat diffusion equation. Between each two adjacent time steps,
 299 the thermal contraction within the lava field is given by equation 5. On this basis, the
 300 subsidence as well as the subsidence rate are calculated. Our modeling approach is simpler
 301 than taken in some of previous studies [e.g. *Patrick et al.*, 2004] as it is partly based on
 302 analytical equations and their implementation in a finite difference scheme.

303 The model is incorporated into an inversion procedure in order to find the best fitting
 304 values for thermal expansivity, thermal diffusivity and the lava thicknesses for each of the
 305 two study areas. Best fitting results are shown in Figures 6a,b and 12.

Equations A25 and A26 are dependent on time. Applying thermal contraction according to Equation 5 varies the size of the finite differences after each time step and introduces a small error when the temperature distribution is again calculated from Equations A25 and A26. However, compared to neglecting thermal contraction, the resulting deviation from the analytically imposed temperature distribution is presumably small.

4. Inversion

The thermal model is an approximation of the cooling and solidification processes. It accounts for conduction and constant values of the parameters throughout the whole simulation period. The parameters influencing contraction and contraction rate are the initial lava thickness, thermal expansivity and thermal diffusivity. Densities of the liquid and solid phases, latent heat and specific heat affect the temperature distribution and contraction during solidification. Density values for the liquid and solid phases were calculated using the software MELTS [Ghiorso *et al.*, 2002], based on the chemical composition of the 1991 lava reported by Gudmundsson *et al.* [1992] and of the 2000 lava reported by Moune *et al.* [2006]. At the liquidus temperature, $T = 1176^\circ\text{C}$, the density of the liquid phase at atmospheric pressure is $2.66 \times 10^3 \text{ kgm}^{-3}$, while the density becomes $2.80 \times 10^3 \text{ kgm}^{-3}$ when fully solidified at $T = 975^\circ\text{C}$.

Due to the partly analytical calculations of the model, a constant latent heat throughout the crystallization interval was used. Direct measurements of latent heat were carried out by Lange *et al.* [1994]. Their results show that latent heat varies from liquidus to solidus with a peak close to liquidus temperature. Constant values for latent heat of $L = 320 \times 10^3 \text{ Jkg}^{-1}$ and for specific heat of $c = 1200 \text{ Jkg}^{-1}\text{K}^{-1}$ were used [Turcotte and Schubert, 2002].

4.1. Inversion range of parameters in the thermal model

327 Inversions for best fitting models were carried out for four free model parameters: ther-
 328 mal expansivity, α , thermal diffusivity, κ and initial thicknesses for both lava fields, D_{1991}
 329 and D_{2000} . The two lavas show similar chemical compositions [Moune *et al.*, 2006], and
 330 it is therefore assumed that the thermal properties, α and κ , are the same for both lava
 331 tongues.

332 The initial thicknesses of the emplaced flows of 1991 and 2000 are constrained by values
 333 determined by comparison of differential digital elevation models (DEMs) by Pedersen
 334 *et al.* [2016]. For the lavas of 1991, a DEM from digital photogrammetry of aerial pho-
 335 tographs taken in 1979 was subtracted from the iDEM specified in Section 2.1. The differ-
 336 ential DEM for the 2000 lavas was created using an EMISAR DEM of 1998 [Magnússon,
 337 2003] and the TanDEM-X iDEM. The 1998 DEM was created from SAR survey using
 338 EMISAR, an airplane borne dual frequency (L- and C-band) fully polarimetric SAR sys-
 339 tem developed in Denmark for remote sensing applications [Christensen *et al.*, 1998]. Lava
 340 thicknesses estimated in this way give a range of 15 – 19 m for the 1991 tongue and a
 341 range of 12 – 14 m for the 2000 lava tongue [Pedersen *et al.*, 2016].

342 *A priori* constraints for parameter ranges of linear thermal expansivity and diffusivity
 343 are based on laboratory measurements and literature values for basaltic andesite and
 344 basalt as presented in Table 2. We carried out linear dilatometric measurements in the
 345 laboratory at the University of Liverpool using a Hyperion thermo-mechanical analyser
 346 TMA402 F1 from Netzsch with a length resolution of 0.125 nm. Four samples from the
 347 2000 Hekla lava tongue were selected to assess the range of thermal expansion coefficient
 348 for rocks with porosities (10.5 – 22.4 %) spanning the range dominant at this lava flow.

349 The dilatometric measurements were conducted on small (5 mm diameter by 6 mm high)
350 cylindrical cores in argon atmosphere, at an applied axial stress of 3 N rate and at a
351 heating rate of 2°C/min up to 1000°C; the sample was then cooled down at the same rate.
352 Prior to each measurement, a baseline measurement was first conducted on an alumina
353 ceramic standard (with identical geometry) to assess the thermal response of the sample
354 assembly. Then, a sample was measured with the same load and heating conditions,
355 and the linear thermal expansion of the sample was corrected to remove any thermal-
356 expansion effects from the sample assembly. The data first highlight the minor artifact
357 of thermal lag and equilibration at low temperatures up to 150°C (Figure 8). The data
358 shows a near constant expansion coefficient (α) in the range 150°C < T < 600 – 700°C
359 and suggest a value of $(8.4 \pm 0.6) \times 10^{-6} \text{K}^{-1}$ (see Figure 8). Above this temperature,
360 we note an increase in thermal expansivity due to phase change in the sample (likely
361 due to the presence of plagioclase in the mineralogical assemblage of these rocks). The
362 peak and subsequent decrease in expansion coefficient coincide with the onset of partial
363 melting at ca. 980°C, which mechanically results in softening of the sample (hence the
364 mild contraction due to relaxation of the applied stress). For the problem tackled here,
365 the measured value for the linear expansion coefficient was considered in the inversion.
366 A priori bounds on the γ coefficient in equation 5 were set to 0.5-3, accounting for the
367 possibility that all volumetric contraction would result in height contraction, but also that
368 the effective coefficient of linear expansion would be lower than the measured value.

4.2. Inversion results

369 In order to find the contraction rate derived from the thermal model that best fits the
370 data presented in Figures 5 and 6, a non-linear inversion estimate of the free parameters

371 was carried out. The inversion was carried out in the programming language Python.
372 The Scientific Library for Python (SciPy) provides an implementation of inversion with
373 a limited memory Broyden-Fletcher-Goldfarb-Shanno (BFGS) scheme using simple box
374 constraints (L-BFGS-B) of the basin-hopping algorithm by *Byrd et al.* [1995]. It is based
375 on Monte-Carlo methods and the Metropolis algorithm. By allowing parameters to jump
376 within the parameter ranges with decreasing probability, the likelihood of finding the
377 globally best fitting parameters is maximized. A few hundred inversions were run starting
378 from randomly chosen sets of parameters within the *a priori* constrained intervals (Table
379 3). Initially, the inversion was carried out for latent heat L and specific heat c as well,
380 but they showed little influence on the inversion and thereafter their values were fixed to
381 $320 \times 10^3 \text{ Jm}^{-3}\text{K}^{-1}$ and 1200 Jkg^{-1} [*Turcotte and Schubert, 2002*], respectively, in order
382 to speed up calculations.

383 Inversions for each lava tongue separately revealed little difference for values of α and
384 κ . Since it is assumed that the two kinds of lava of 1991 and 2000 have similar thermal
385 properties, a joint inversion for the 1991 and 2000 lava tongues was carried out. Figure 9
386 shows the results of the inversions, which all have similar root-mean-square errors (RMSE)
387 of about 1.53 mm/yr. Free parameters are the thicknesses of both lava tongues, thermal
388 diffusivity and the linear coefficient of thermal expansion, α , multiplied by the γ coeffi-
389 cient to consider volumetric contraction. We refer to $\gamma\alpha$ as the effective vertical thermal
390 expansivity. The resolved values show strong correlation between pairs of parameters. For
391 a specific value of α , a narrow range of the corresponding thicknesses for both lava tongues
392 and the value for κ can be determined, and vice versa. Figure 9 further shows, that for
393 any specific thermal diffusivity and effective vertical thermal expansivity, the 2000 lava

394 tongue is similar in thickness as the 1991 lava tongue. This finding is only consistent with
395 the different a priori ranges for the thicknesses of the lava tongues, if both of them are
396 similar. Considering uncertainties, we therefore consider solutions with lava thickness in
397 the range of 11 – 19 m. We suggest the corresponding set of parameters is the preferred
398 solution to the problem, fitting the thermal model and the a priori bounds on the lava
399 thickness. This implies $\gamma\alpha$ has a value of $14 - 23 \times 10^{-6} \text{K}^{-1}$ and considering the laboratory
400 measured value of α , the γ coefficient has a value of 1.69 – 2.41. Accordingly, the lowering
401 of the lava surface is highly effective in accommodating the volumetric contraction of the
402 lava tongues. The range for the thermal diffusivity is $1.07 - 3.00 \times 10^{-7} \text{m}^2\text{s}^{-1}$. It is lower
403 than found by Peck [1978] for the Alae lava lake, but near the lower end of diffusivity
404 values found from thermal modeling of a lava field on Okmok volcano. In that case, the
405 range to tabulated parameters [Patrick *et al.*, 2004, Table 3] gives diffusivity values in the
406 range $3.5 - 8.6 \times 10^{-7} \text{m}^2\text{s}^{-1}$. Considering our values for thermal diffusivity in the context
407 of these other studies, we infer that the highest diffusivity value in our identified range is
408 the most likely (Fig. 12). We conclude the studied lava tongues at Hekla are not good
409 conductors of heat, and may have some internal insulation preventing heat to escape. We
410 suggest cracks or high porosity layers, isolated from air temperature, at depth in the lava
411 field hinder the flow of heat, and are the cause for the low effective heat diffusion. Our
412 simple thermal model can thus explain the observed deformation upon two conditions:
413 lowering of lava fields is effective in accommodating the full volumetric contraction of the
414 lava, and heat remains for long in the lava fields because of relatively low effective heat
415 diffusion.

5. Discussion

416 The deformation data presented here are based on InSAR observations. Application of
417 the presented method is therefore not only limited to easily accessible effusive eruption
418 sites, but also applies to remotely located volcanoes. Iceland in particular, with its overall
419 sparse and slowly growing vegetation, offers excellent preconditions for InSAR observa-
420 tions. Based on highly coherent data sets from six satellite tracks spanning time periods
421 of up to 23 years we find clear exponentially decaying rate of lava subsidence over long
422 time, excluding the initial few years after emplacement. A model of the thermal evolution
423 of a lava field and associated lava subsidence was implemented, considering both volume
424 contraction due to phase change and cooling. The model predicts subsidence and decaying
425 subsidence rates as observed, and allows for inversions for the best fitting values for lava
426 thickness, effective vertical thermal expansivity and thermal diffusivity. The parameter
427 values derived from the thermal model are "effective" and depend on the simplified model
428 assumptions.

429 The model assumes a constant temperature of the flow substrate at the time of lava
430 emplacement. This may be an issue for the 2000 lava flow as it was emplaced on top of the
431 lava flow of 1980-81, which shows ongoing subsidence in Figures 2a-c and 5b. The data
432 after the year 2000 were corrected for a linear trend that supposedly originates from the
433 lava flows of 1980-81. But the thermal model does not take into account a temperature
434 distribution originating from a still cooling and contracting lava of 1980-81. The influence
435 of this simplification was tested by incorporating a substrate below the 2000 lava field with
436 elevated temperatures due to the 1980-81 lava flow. We found it had a minor influence

437 on predicted subsidence and since the real temperature of the substrate is unknown, we
438 preferred the simple model of constant temperature for the flow substrate.

439 Inversions for thickness of both the lava tongues studied result in similar values (Figure
440 9). This is only in agreement with thickness measurements from differential DEMs of
441 the lava tongues if they are of similar thickness, about 14-15 m. For these thickness
442 values, a unique pair of effective thermal expansivity and thermal diffusivity was derived
443 (see Figure 9). The derived values maybe influenced by the model simplifications, and
444 effects such as viscous response of the Earth, clast repacking of underlying lava fields and
445 creeping of the flow core were not taken into account. A viscous relaxation of the lava
446 substrate has been invoked as an explanation for lava displacement, in particular when
447 subsidence extends outside lava fields. For example, *Briole et al.* [1997] found evidence
448 for such process at Etna volcano and a clear local deformation signal extending outside
449 a newly emplaced lava flow. We find no similar significant signal at Hekla. At Hekla, a
450 much wider viscoelastic relaxation signal due to lava loading has, however, been suggested
451 by *Grapenthin et al.* [2010]. Their model considers lava load emplaced on a few km thick
452 elastic uppermost crustal layer above a viscoelastic base. The predicted signal from this
453 process, when combined with a pressure increase at depth, could potentially explain the
454 volcano-wide deformation signal at Hekla (inflation signal with a central subsidence). For
455 our detailed study of lava tongues, its effect would mostly cancel out when interpreting
456 relative LOS changes with our approach over a limited area (the signal is about the same
457 at the lava tongues and just outside them). Additional cooling due to rainfall, wind and
458 radiation were also not incorporated specifically in our model. *Patrick et al.* [2004] found
459 effects from radiation during an initial period of lava cooling (about 200 days), but having

460 less significant effect afterwards. Even if rainfall affects cooling longer, an approach as
461 taken by *Shaw et al.* [1977] and applied by *Patrick et al.* [2004] would as well mostly cool
462 the lava in an initial period. The bulk material of the lava is dense, thus stopping the
463 water to percolate much inside the bulk. Since we keep the surface of the lava at 0°C,
464 part of the cooling effect of water is absorbed in this assumption. Furthermore, the initial
465 period after emplacement, which is affected by both radiation and direct effects of rainfall,
466 is not the main topic of our study due to timing of the InSAR acquisitions in our case.
467 These effects should, however, be considered in further studies of other lava fields, with
468 more dense InSAR coverage. These effects could increase cooling rates and there may
469 also be mechanically influenced subsidence, so that thermal contraction only accounts
470 for a fraction of the total subsidence. However, despite the simplicity of the model, the
471 inversion parameters are in the right order of magnitude, and the derived values should
472 be interpreted as effective values, with exact values influenced by the model assumptions.

473 Noting that *Peck* [1978] reports a maximum temperature of about 100°C after approx-
474 imately four years after emplacement of the solidifying Alae lava lake with an average
475 thickness of 14 m, it seems likely that lava may cool at very variable rate. This thickness
476 is similar as our inferred thickness for the Hekla lava tongues studies, but our inferred
477 maximum temperature in the flow core after four years is about 650°C. Based on this we
478 suggest the effective thermal conductivity within lava fields in general is highly variable,
479 and microcracks isolated from ambient air may act as insulators.

480 Investigation of contraction of lava flows, which are generally accessible for sampling, in-
481 vestigation, and importantly, visual inspection, is a step towards understanding of magma
482 body definition and evolution at depth. Volume variations at depth - whether due to crys-

483 tallisation, volatile exsolution, thermal contraction, etc. - may induce deformation on the
484 surface of the Earth, offering opportunities for observations and in return for the gen-
485 eration of inverse models that may define the physical-chemical evolution of magmatic
486 bodies. This study is therefore a step towards improving our understanding of magmatic
487 systems necessary for improved monitoring strategies in volcanic environments.

6. Conclusion

488 The combination of InSAR data over lava flows spanning time periods up to 23 years
489 after emplacement and an associated thermal model provide new insights and interpreta-
490 tions of cooling and contraction of lava flows. When line-of-sight (LOS) changes over lava
491 fields are corrected for LOS changes at reference areas outside the lava fields, the temporal
492 and spatial evolution of lava subsidence can be derived. At Hekla volcano, corresponding
493 subsidence rates decay from about 20 mm/year after five years to about 2 mm/year after
494 15 years after emplacement, with an approximately exponential decay. A simple thermal
495 model based on heat conduction can reproduce the observations. Despite the simplicity
496 of the model, all inversion parameters are in the right order of magnitude. *A priori* infor-
497 mation on lava thickness values allows us to select preferred values for effective vertical
498 thermal expansivity and thermal diffusivity, based on inversion results. Volumetric effects
499 of lava contraction due to cooling are absorbed into lowering of the lava fields, and we infer
500 a low effective thermal diffusivity. We suggest remaining deviation of inversion param-
501 eters from *a priori* parameter ranges is due to additional cooling effects beyond conductive
502 cooling. In general, a combined approach of InSAR observations and thermal modeling
503 is well suited to study lava fields at volcanoes worldwide, including remote locations.

Appendix A: Derivation of the governing equations for a cooling lava field

504 The cooling and contraction process can be subdivided into cooling before and after full
 505 solidification. Before the lava is fully solidified, there is need to consider latent heat at the
 506 phase transition. This mathematized problem of the development of a crust due to solid-
 507 liquid phase transition is commonly known as Stefan problem. The following derivation
 508 of equations are a variation and adaptation of [*Carslaw and Jaeger*, 1959, chapter XI] and
 509 [*Crank*, 1984, chapter 3.2].

510 The scheme of the model is shown in Figure 10. The lava flow is emplaced at a ho-
 511 mogeneous temperature before it starts cooling. The temperature of the uppermost layer
 512 (air) is kept constant at all times, thus releasing heat that is transferred by conduction
 513 to this element. Heat is also transferred to the flow substrate, which itself is kept at
 514 a constant temperature at all times at large depth. Each section of the model has its
 515 own thermal and mechanical properties. These are the density ρ , the specific heat c , the
 516 thermal conductivity k and the thermal diffusivity κ . As the lava flow cools, crusts evolve
 517 from the bottom and the top of the flow. Their moving and time dependent coordinates
 518 of the boundaries are $s_{low}(t)$ and $s_{up}(t)$.

519 Since the lava flow solidifies from the top and below, both upper and lower crust develop
 520 and each problem can initially be evaluated separately. The left-hand side of Figure 11
 521 schematically refers to the lower part of the lava flow and shows a slab of bedrock, crust
 522 and liquid lava, with parameters of interest for each region.

523 Similarly, the right-hand side of Figure 11 shows the equivalent problem for the upper
 524 crust. In order to make use of similarities, the coordinate system in this case was mirrored.
 525 In both cases, a crust develops and moves in positive y -direction upwards with $s(t)$, which

denotes the position of the surface of phase transition. Simplifying assumptions are a single temperature at which the lava solidifies, rather than a melting range between liquidus and solidus temperature. Furthermore, it is assumed that the lava is incompressible and that the crust has a higher density than liquid lava, i.e. $\rho_1 > \rho_2$.

The governing system of equations in this case becomes:

$$\frac{\partial T_0}{\partial t} = \kappa_0 \frac{\partial^2 T_0}{\partial y^2}, \quad \text{for } y < 0 \quad (\text{A1})$$

$$k_0 \frac{\partial T_0}{\partial y} - k_1 \frac{\partial T_1}{\partial y} = 0, \quad \text{for } y = 0 \quad (\text{A2})$$

$$\frac{\partial T_1}{\partial t} = \kappa_1 \frac{\partial^2 T_1}{\partial y^2}, \quad \text{for } 0 < y < s(t) \quad (\text{A3})$$

$$k_1 \frac{\partial T_1}{\partial y} - k_2 \frac{\partial T_2}{\partial y} = L\rho \frac{ds}{dt}, \quad \text{for } y = s(t) \quad (\text{A4})$$

$$\frac{\partial T_2}{\partial t} = \kappa_2 \frac{\partial^2 T_2}{\partial y^2} + \frac{\rho_1 - \rho_2}{\rho_2} \frac{ds}{dt} \frac{\partial T_2}{\partial y}, \quad \text{for } y > s(t) \quad (\text{A5})$$

T_0 , T_1 and T_2 refer to the temperatures of the bedrock, the crust and the liquid region of lava. The set of equations requires some explanations:

1. Equation A1 and A3 are the heat diffusion equations for the bedrock and the solidified lava.

2. Equation A2 is the boundary condition between these two regions.

3. Equation A4 is the boundary condition between liquid and solid phase of the lava.

When this boundary moves a distance ds , a quantity of heat $L\rho_2 ds$ per unit area must be removed by conduction [Carslaw and Jaeger, 1959].

4. Equation A5 is the heat diffusion equation in the liquid lava with an additional term taking into account the densification during solidification, which causes motion of the liquid above the crust. During solidification, the relative volume change per unit area is $\frac{\rho_1 - \rho_2}{\rho_2} ds$.

From the last point, a velocity u of the moving liquid can be deduced:

$$u = \left(1 - \frac{\rho_1}{\rho_2}\right) \frac{ds}{dt} \quad (\text{A6})$$

This set of equations is valid for both the lower and the upper crust, but the initial and boundary conditions are different. For the lower crust, the heat transfer to the bedrock (initially homogeneously at temperature T_b) has to be taken into account, which is fulfilled by the following boundary conditions:

$$T_1 = T_0 \quad , \text{ for } y = 0, t \geq 0 \quad (\text{A7})$$

$$T_2 \rightarrow T_l \quad , \text{ for } y \rightarrow \infty, t \geq 0 \quad (\text{A8})$$

$$T_0 \rightarrow T_b \quad , \text{ for } y \rightarrow -\infty, t \geq 0 \quad (\text{A9})$$

$$T_1 = T_2 = T_m \quad , \text{ for } y = s(t), t \geq 0 \quad (\text{A10})$$

⁵⁴² T_m is the temperature of the melting point and T_l the temperature of emplacement of the
⁵⁴³ lava flow. A solution to Equation A1 is

$$T_0 = A + B \cdot \operatorname{erf} \frac{y}{2\sqrt{\kappa_0 t}} \quad (\text{A11})$$

⁵⁴⁴ And for Equation A3:

$$T_1 = C + D \cdot \operatorname{erf} \frac{y}{2\sqrt{\kappa_1 t}} \quad (\text{A12})$$

⁵⁴⁵ At the phase boundary $y = s(t)$, T_1 must be equal to the melting point T_m , and this must
⁵⁴⁶ hold at all times t . From Equation A12 then follows that $s(t)$ must be proportional to
⁵⁴⁷ \sqrt{t} :

$$s(t) = 2\lambda\sqrt{\kappa_1 t} \quad (\text{A13})$$

548 λ is a dimensionless coordinate determined from Equation A4. A solution to Equation
 549 A5 is:

$$T_2 = T_l - D \cdot \operatorname{erfc} \left(\frac{y}{2\sqrt{\kappa_2 t}} + \lambda \frac{\rho_1 - \rho_2}{\rho_2} \sqrt{\frac{\kappa_1}{\kappa_2}} \right) \quad (\text{A14})$$

550 Applying these boundary conditions to the lower crust and calculating the constants
 551 A, B, C, D reveals:

$$T_0(t) = T_b + \frac{k_1 \sqrt{\kappa_0} T_m}{k_0 \sqrt{\kappa_1} \operatorname{erf}(\lambda_{low}) + k_1 \sqrt{\kappa_0}} \cdot \left(1 + \operatorname{erf} \frac{y}{2\sqrt{\kappa_0 t}} \right) \quad (\text{A15})$$

$$T_1(t) = T_b + \frac{T_m - T_b}{k_1 \sqrt{\kappa_0} + k_0 \sqrt{\kappa_1} \operatorname{erf}(\lambda_{low})} \cdot \left(k_1 \sqrt{\kappa_0} + k_0 \sqrt{\kappa_1} \operatorname{erf} \frac{y}{2\sqrt{\kappa_1 t}} \right) \quad (\text{A16})$$

$$T_2(t) = T_l - \frac{T_l - T_m}{\operatorname{erfc} \left(\lambda_{low} \sqrt{\frac{\kappa_1 \rho_1}{\kappa_2 \rho_2}} \right)} \cdot \operatorname{erfc} \left(\frac{y}{2\sqrt{\kappa_2 t}} + \lambda_{low} \frac{\rho_1 - \rho_2}{\rho_2} \sqrt{\frac{\kappa_1}{\kappa_2}} \right) \quad (\text{A17})$$

$$\frac{k_0 \sqrt{\kappa_1} e^{-\lambda_{low}^2}}{k_1 \sqrt{\kappa_0} + k_0 \sqrt{\kappa_1} \operatorname{erf}(\lambda_{low})} - \frac{k_2 \sqrt{\kappa_1} (T_l - T_m) e^{-\lambda_{low}^2 \frac{\kappa_1 \rho_1^2}{\kappa_2 \rho_2^2}}}{k_1 \sqrt{\kappa_2} T_m \operatorname{erfc} \left(\lambda_{low} \sqrt{\frac{\kappa_1 \rho_1}{\kappa_2 \rho_2}} \right)} = \frac{\lambda_{low} L \sqrt{\pi} \rho_2}{c_1 T_m \rho_1} \quad (\text{A18})$$

For the lower boundary, the region $y < 0$ is initially liquid and at constant temperature T_l , while the surface at $y = 0$ is maintained at air temperature T_a . The boundary conditions are in this case:

$$T_1 = T_a \quad , \text{ for } y = 0, t \geq 0 \quad (\text{A19})$$

$$T_2 \rightarrow T_l \quad , \text{ for } y \rightarrow \infty, t \geq 0 \quad (\text{A20})$$

$$T_1 = T_2 = T_m \quad , \text{ for } y = s(t), t \geq 0 \quad (\text{A21})$$

552 Proceeding in a similar way the upper crust, its solution in the coordinate system of
 553 the right-hand side of Figure 11 is:

$$T_1(t) = T_a + \frac{T_m - T_a}{\operatorname{erf}(\lambda_{up})} \operatorname{erf} \frac{y}{2\sqrt{\kappa_1 t}} \quad (\text{A22})$$

$$T_2(t) = T_l - \frac{T_l - T_m}{\operatorname{erfc}\left(\lambda_{up}\sqrt{\frac{\kappa_1}{\kappa_2}\frac{\rho_1}{\rho_2}}\right)} \operatorname{erfc}\left(\frac{y}{2\sqrt{\kappa_2 t}} + \frac{\lambda_{up}(\rho_1 - \rho_2)\sqrt{\kappa_1}}{\rho_2\sqrt{\kappa_2}}\right) \quad (\text{A23})$$

$$\frac{e^{-\lambda^2}}{\operatorname{erf}(\lambda_{up})} - \frac{(T_l - T_m)k_2\sqrt{\kappa_1}e^{-\lambda_{up}^2\rho_1^2\kappa_1/\rho_2^2\kappa_2}}{(T_m - T_a)k_1\sqrt{\kappa_2}\operatorname{erfc}(\lambda_{up}\rho_1\sqrt{\kappa_1}/\rho_2\sqrt{\kappa_2})} = \frac{\lambda_{up}L\sqrt{\pi}}{c_1(T_m - T_a)} \quad (\text{A24})$$

Note that λ_{low} for the lower crust is calculated from Equation A18, and λ_{up} for the upper crust from Equation A24. When combining these two solutions, the problem arises that the boundary conditions, $\lim_{y \rightarrow \infty} T_2 = T_l$, for the liquid phase are not valid any more. For simplicity, it is therefore assumed that the lava field is emplaced homogeneously at the melting point. Combining these solutions means flipping the upper crust along the x-axis and moving it by D in positive y-direction, i.e. replacing y in the solution for the upper crust by $D - y$.

In our case the lava flows are emplaced on older lava, thus the thermal properties of lava and substrate are assumed to be the same. In this situation, the temperature distribution of Equations A16 and A22 simplifies to:

$$T_{low}(t) = T_b + \frac{T_l - T_b}{\operatorname{erf}(\lambda_{low}) + 1} \cdot \left(1 + \operatorname{erf}\frac{y}{2\sqrt{\kappa t}}\right), \quad \text{for } y \leq s_{low}(t) \quad (\text{A25})$$

$$T_{up}(t) = T_a + \frac{T_l - T_a}{\operatorname{erf}\lambda_{up}} \operatorname{erf}\frac{D - y}{2\sqrt{\kappa t}}, \quad \text{for } y \geq s_{up}(t) \quad (\text{A26})$$

L denotes latent heat, κ thermal diffusivity and c the specific heat of the lava. $s_{low}(t)$ and $s_{up}(t)$ are the positions of the solidification boundaries as in Equation A13. λ_{up} and λ_{low} are constants for the upper and lower crusts, which can be derived implicitly from Equations A18 and A24. In the case of uniform thermal properties these equations simplify to:

$$\frac{e^{-\lambda_{low}^2}}{1 + \operatorname{erf}(\lambda_{low})} = \frac{\lambda_{low} L \sqrt{\pi}}{c \cdot (T_l - T_b)} \frac{\rho_{liq}}{\rho_{sol}} \quad (\text{A27})$$

$$\frac{e^{-\lambda_{up}^2}}{\operatorname{erf}(\lambda_{up})} = \frac{\lambda_{up} L \sqrt{\pi}}{c \cdot (T_l - T_a)} \quad (\text{A28})$$

569 ρ_{liq} and ρ_{sol} denote densities of the liquid and solid phase. The total contraction is a
 570 combination of thermal contraction and contraction related to phase change of crystalliz-
 571 ing minerals. The latter is associated with density change from liquid to solid state of the
 572 lava.

573 Because of the density change during solidification, which occurs in either crust, the lava
 574 contracts at a velocity $u_{low} + u_{up}$, which can be derived from Equation A6. Additionally,
 575 the lava cools and thermal contraction occurs, so that the total vertical velocity u_{tot} of
 576 the lava surface is

$$u_{tot} = u_{low} + u_{up} + u_{th} \quad (\text{A29})$$

577 with u_{th} denoting vertical velocity related to thermal contraction. The resulting volume
 578 change due to thermal contraction is calculated from equation 5 in main text.

579 One must keep track of the evolving thicknesses of the crusts to determine the time
 580 they meet. After this time of solidification, the two separated problems for the lower and
 581 upper crust must be merged together, as schematically shown in Figure 10.

$$\nabla^2 T = \frac{1}{\kappa} \frac{\partial T}{\partial t} \quad (\text{A30})$$

582 The output of equations A25 - A28 form a unique temperature distribution dependent on
 583 thermal parameters (see Figure 12). Following the solidification process, heat is conducted

584 away from the lava core according to eq. A30, under the assumption that k , κ and ρ are
585 constants and that there is no internal heat generation.

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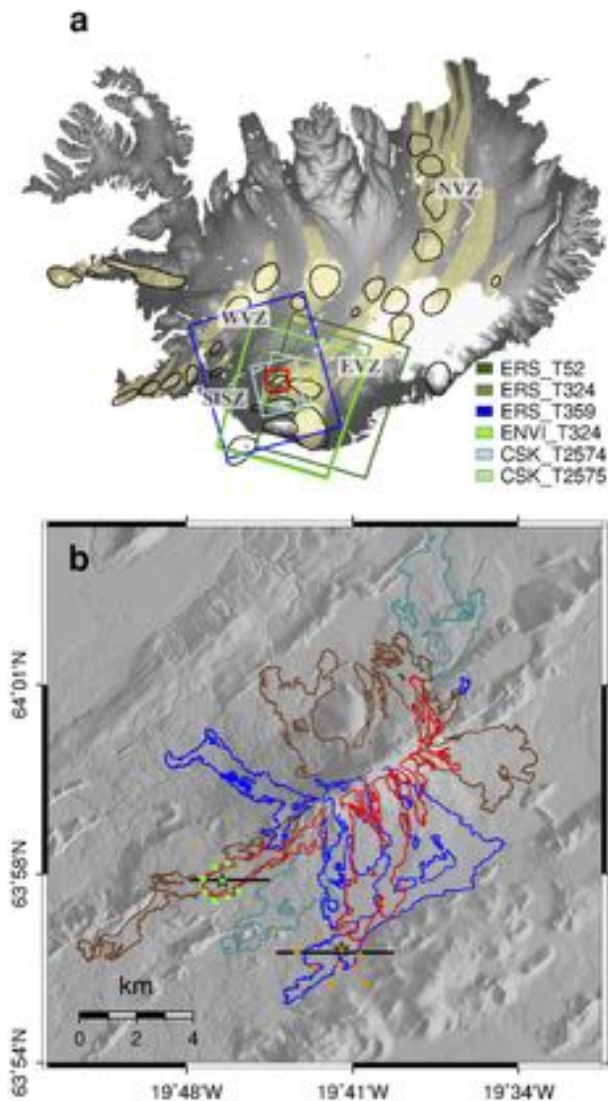


Figure 1. (a) Shaded relief map of Iceland with glaciers in white, fissure swarms in yellow and outlines of central volcanoes in black. SAR images from satellite tracks in ascending configuration are shown as blue rectangles and in descending configuration as green rectangles. A red rectangular shows the zoom region of (b) covering Hekla volcano. (b) Shaded relief map of Hekla volcano (center of map) with outlines of recent lava flows (1970: turquoise, 1980/1981: brown, 1991: blue, 2000: red).

Dots encompass two lava tongues which are the focus of this study. Orange circles show the reference areas for part of the 1991 lava field, with the orange star marking the sample area on the lava field. Green dots show the reference areas for the 2000 lava field, with the green star marking the sample area on the lava field. Radii are 100 m for all areas. Black lines mark profiles crossing the 1991 and 2000 lava fields shown in Figure 4.

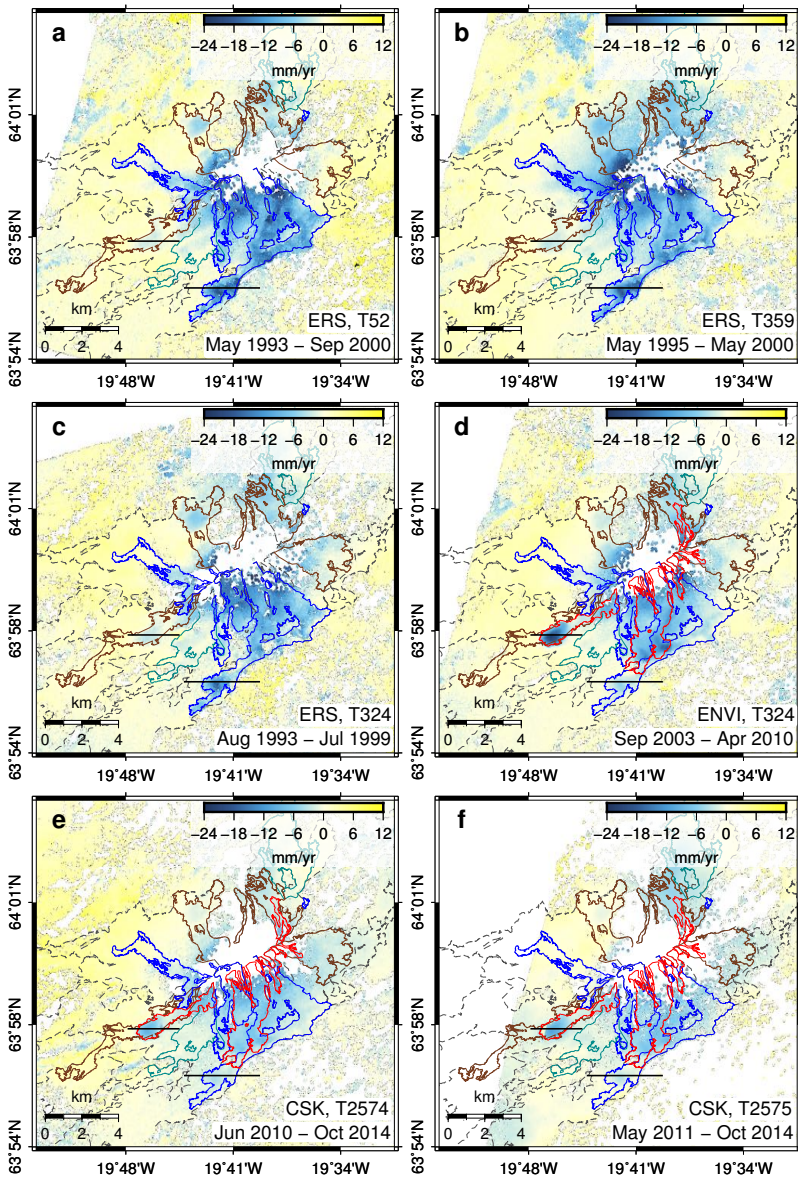


Figure 2. Average LOS velocities calculated from time series shown in Appendix 1. The panels display a zoom of the LOS velocity fields, with Hekla in the center. White corresponds to areas without data, including areas where no data were acquired and incoherent areas. Blue corresponds to LOS lengthening, orange to LOS shortening. Outlines of recent lava flows (1970: turquoise, 1980/1981: brown, 1991: blue, 2000: red). **(a)** Descending ERS Track 52 from 25/05/1993 to 29/09/2000, **(b)** Descending ERS Track 324 from 06/05/1995 to 31/05/2000, **(c)** Ascending ERS Track 359 from 24/08/1993 to 23/07/1999, **(d)** Descending Envisat Track 324 from 03/09/2003 to 14/04/2010, **(e)** Ascending CSK Track 2574 from 18/06/2010 to 11/10/2014, **(f)** Descending CSK Track 2575 from 19/05/2011 to 16/10/2014.

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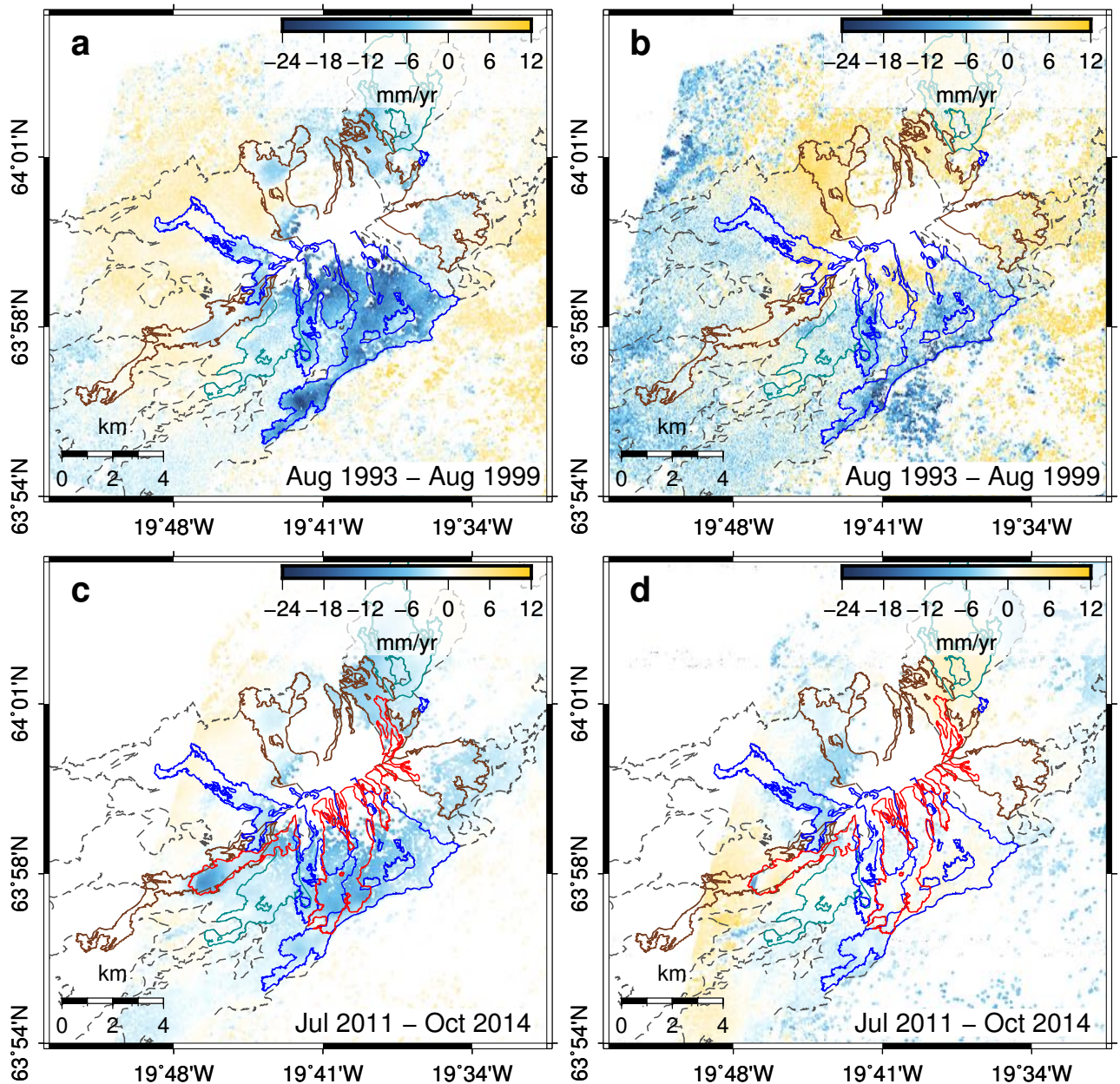


Figure 3. Linear combinations of LOS changes in ascending and descending ERS and CSK tracks. (a) Sum of LOS velocity fields from descending ERS Track 52 and ascending ERS Track T359, normalized to reveal near-vertical displacement. (b) The corresponding difference, normalized to reveal approximate east displacement. (c) Sum of LOS velocity fields from of ascending CSK Track 2574 and descending CSK Track T2575 normalized to reveal near-vertical displacement. (d) The corresponding difference, normalized reveal approximate east displacement.

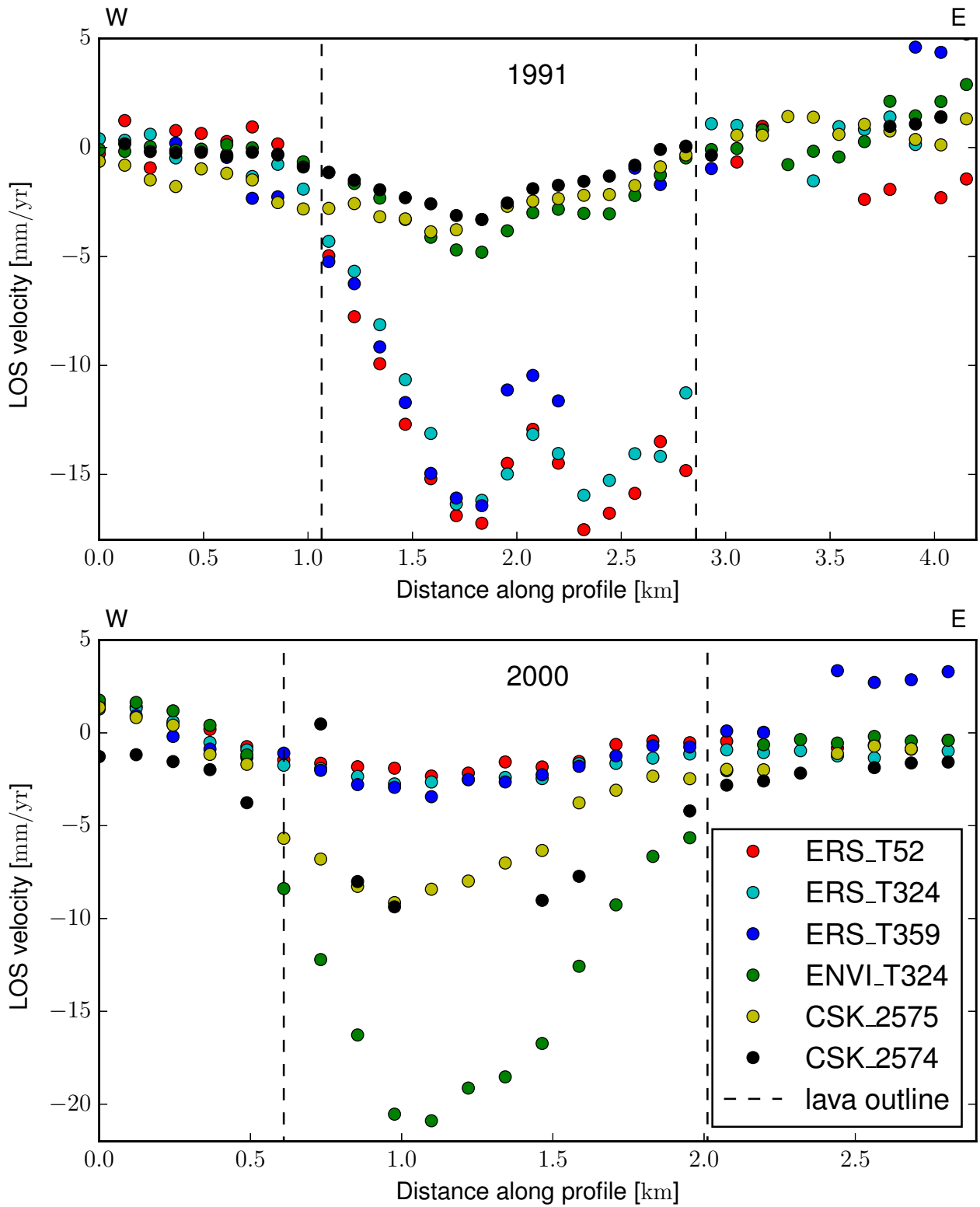


Figure 4. LOS velocity profiles across lava tongues. Distance along profiles (see black lines in Figure 1) are from west to east. The upper figure shows the profile through the 1991 lava tongue. The lower figure shows the profile through the 2000 lava tongue. Vertical dashed lines

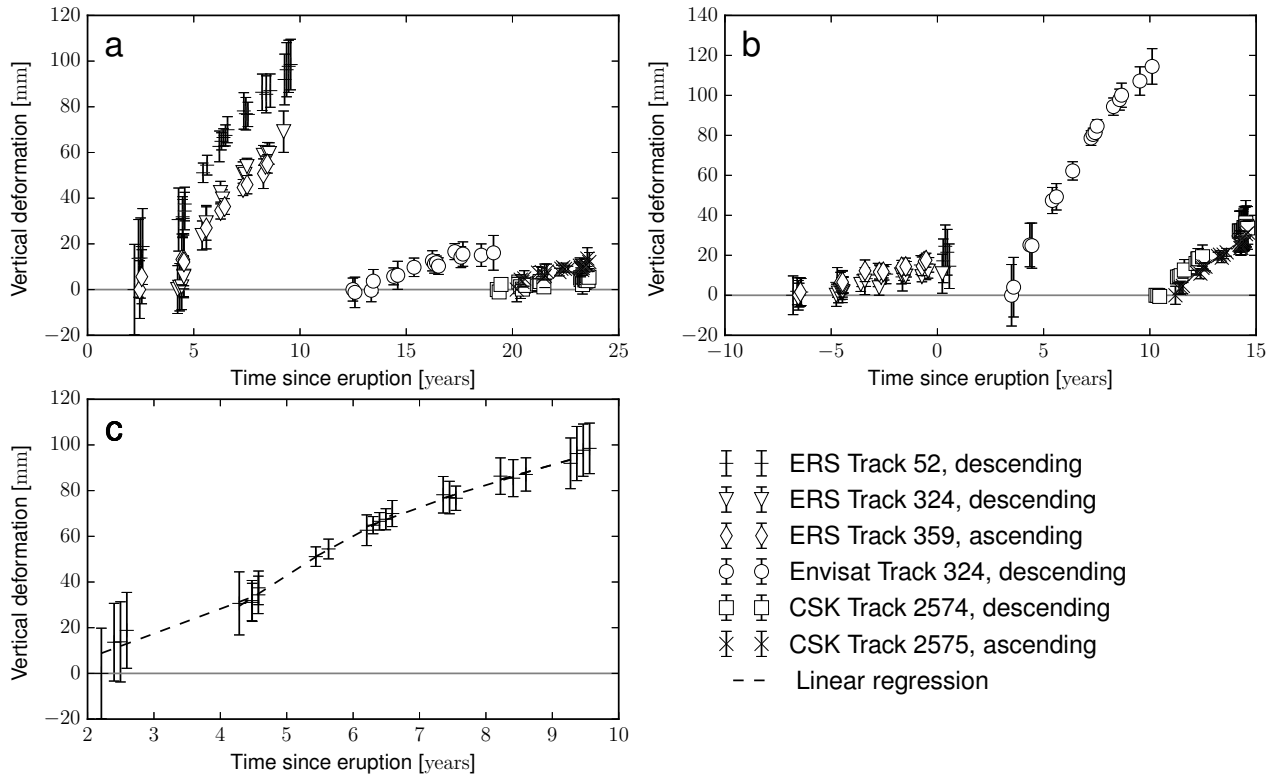


Figure 5. Inferred vertical change (subsidence is positive) and linear regression. (a) Inferred vertical deformation of the sample area of the 1991 Hekla lava tongue relative to surrounding area. (b) Inferred vertical deformation of the sample area of the 2000 Hekla lava tongue relative to surrounding area. A linear trend before the eruption ($t = 0$ years) is noticeable. (c) Example of linear regression for ERS Track 52. The slopes of the individual regressions yield the vertical velocities for each time span.

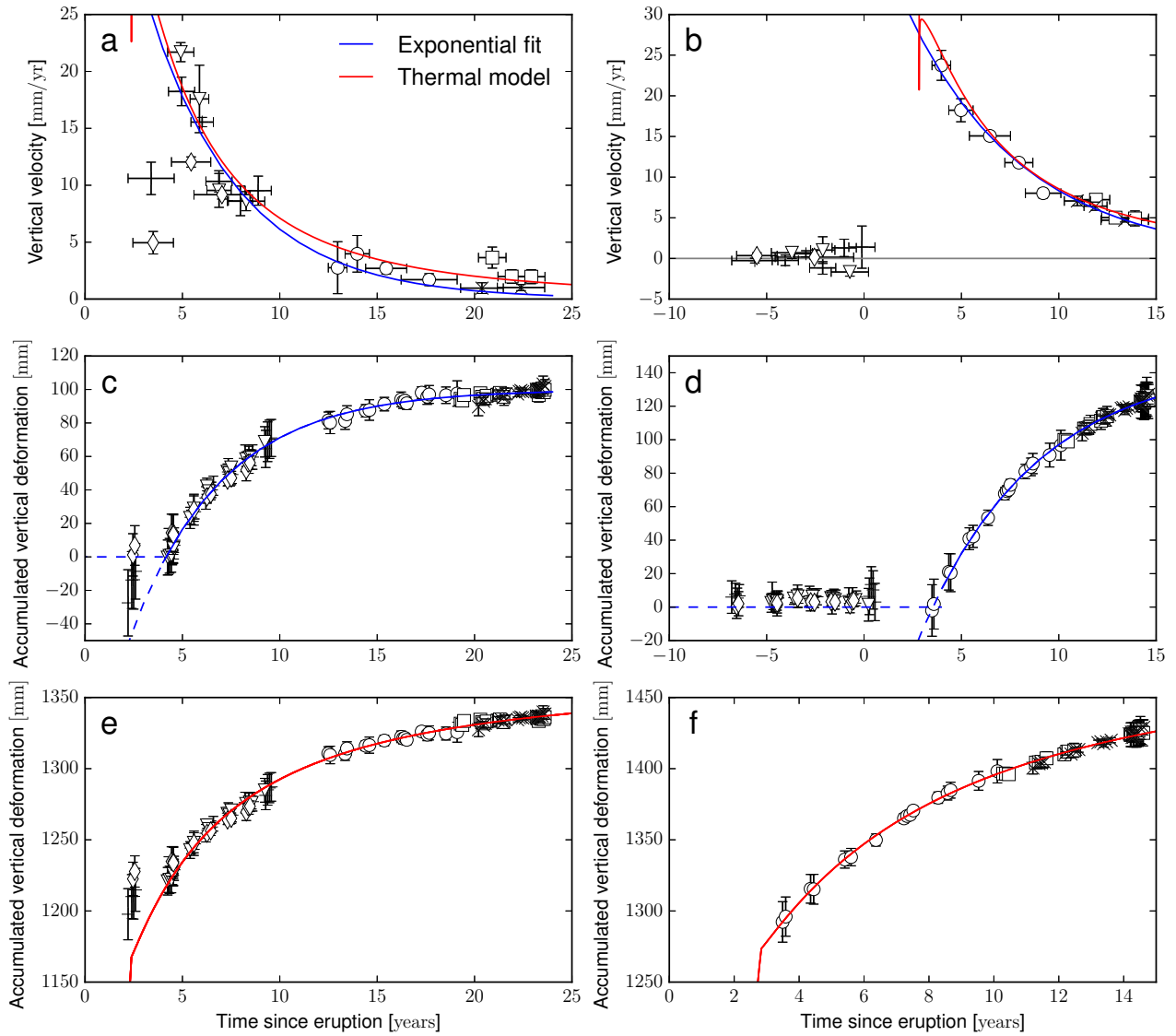


Figure 6. Inferred vertical velocity and deformation (subsidence is positive) (a) Vertical surface velocity of 1991 lava tongue relative to surrounding area and best fitting models. (b) Vertical surface velocity of 2000 lava tongue relative to surrounding area and best fitting models. (c) Exponential fit for accumulated vertical deformation of 1991 lava tongue. (d) Exponential fit for accumulated vertical deformation of 2000 lava tongue. (e) Best fitting thermal model for accumulated vertical deformation of 2000 lava tongue. (f) Best fitting thermal model for accumulated vertical deformation of 1991 lava tongue.

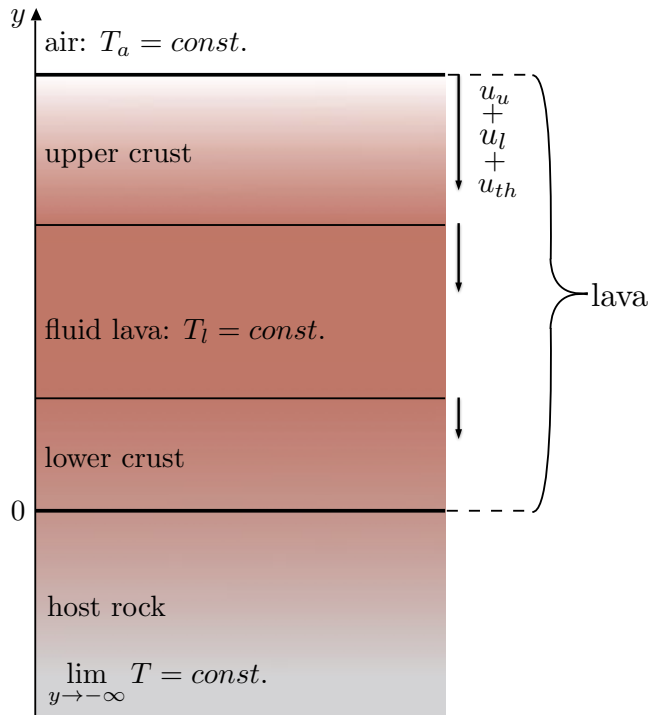


Figure 7. Scheme of the thermal model. A lava flow is emplaced at temperature T_l and heat is conducted away into the air and bed rock. The upper surface of the flow moves at a velocity $u_{up} + u_{low} + u_{th}$, due to density changes during phase transition and cooling.

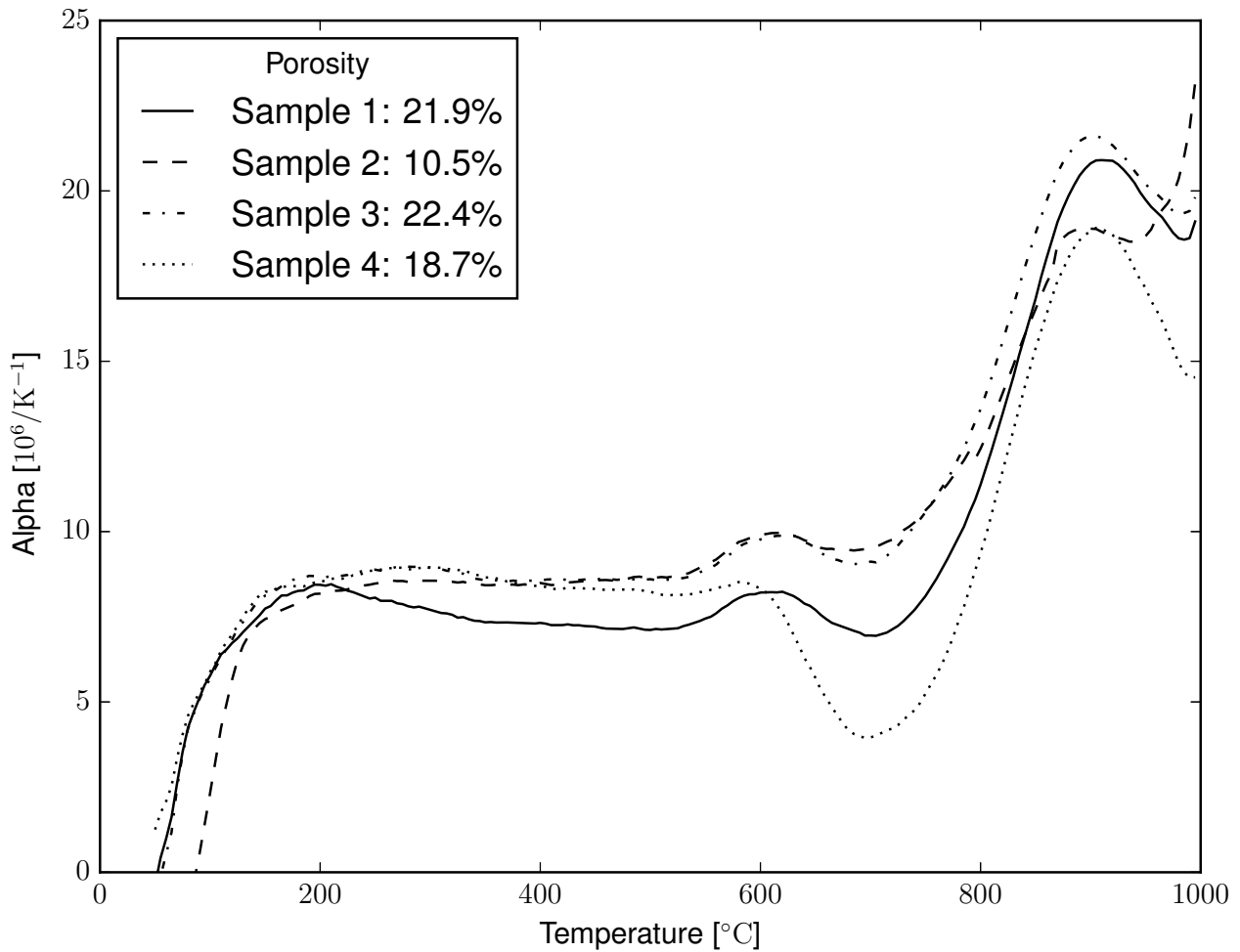


Figure 8. Expansion coefficient of 2000 Hekla lava tongue as a function of temperature. Data below $\approx 150^\circ\text{C}$ highlight the initial thermal adjustment of the furnace and sample assembly. The data shows that the expansion coefficient is constant between 200 and 600 – 700 $^\circ\text{C}$ for all samples. Above this temperature we note an increase in thermal expansivity and a peak due to partial melting at circa 980°C , which mechanically results in softening of the sample.

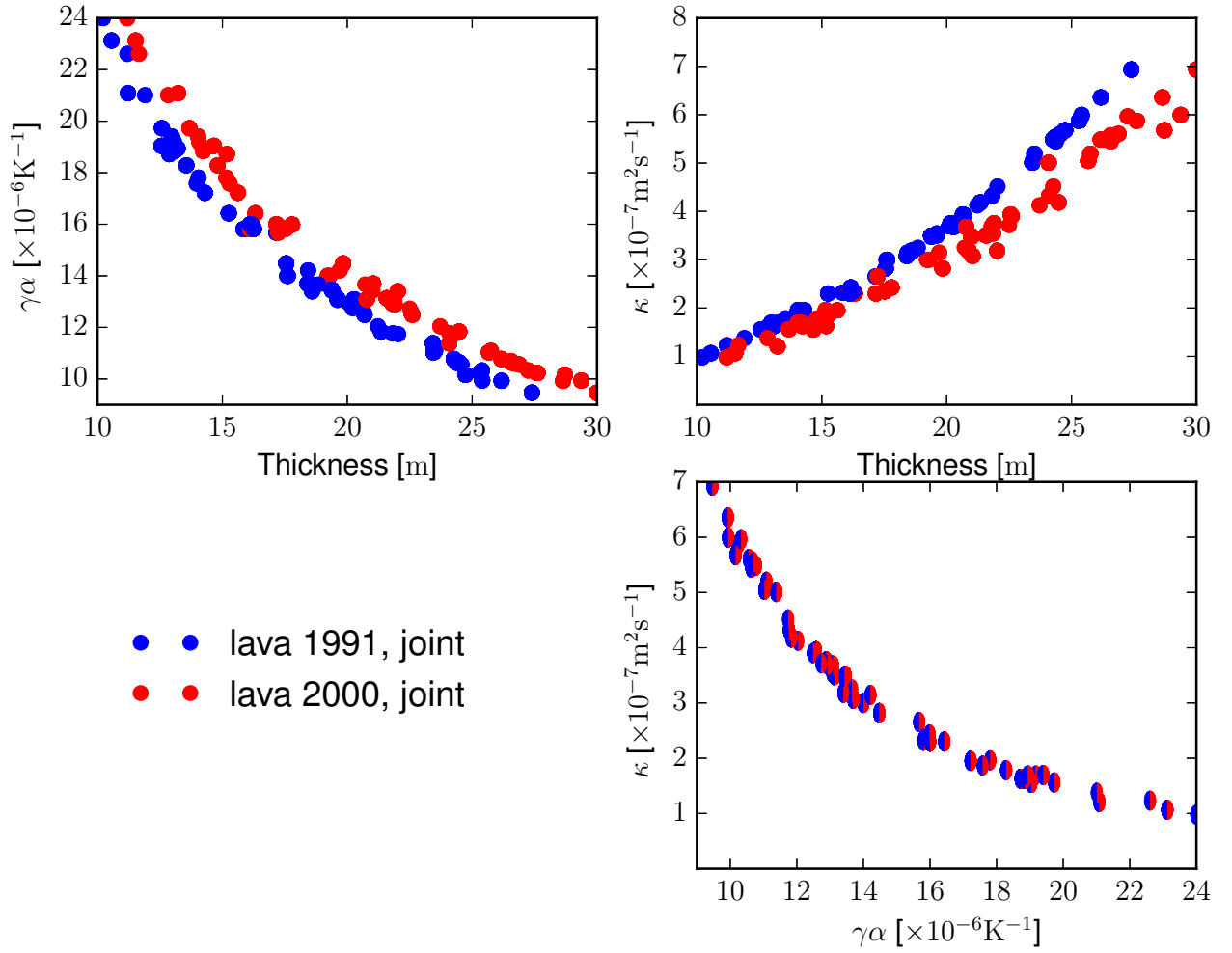


Figure 9. Cross-correlation between pairs of parameters of the joint inversion. Each point represents a single solution of the inversion. All the points on the plots result in a similar residual between models and observations and lead to root-mean-square errors of about 1.53 mm/yr. Blue dots refer to the 1991 lava tongue, and red dots to the 2000 lava tongue. The inversion process seeks to minimize the standard deviation between the thermal model and measured subsidence rates (see Figures 6a and b). Parameters are initial thicknesses of the 1991 and 2000 lava tongues, effective vertical thermal expansivity, $\gamma\alpha$, and thermal diffusivity, κ . Latent heat L and specific heat c were kept constant at $L = 320 \times 10^3 \text{ Jkg}^{-1}$ and $c = 1200 \text{ Jkg}^{-1}\text{K}^{-1}$, respectively.

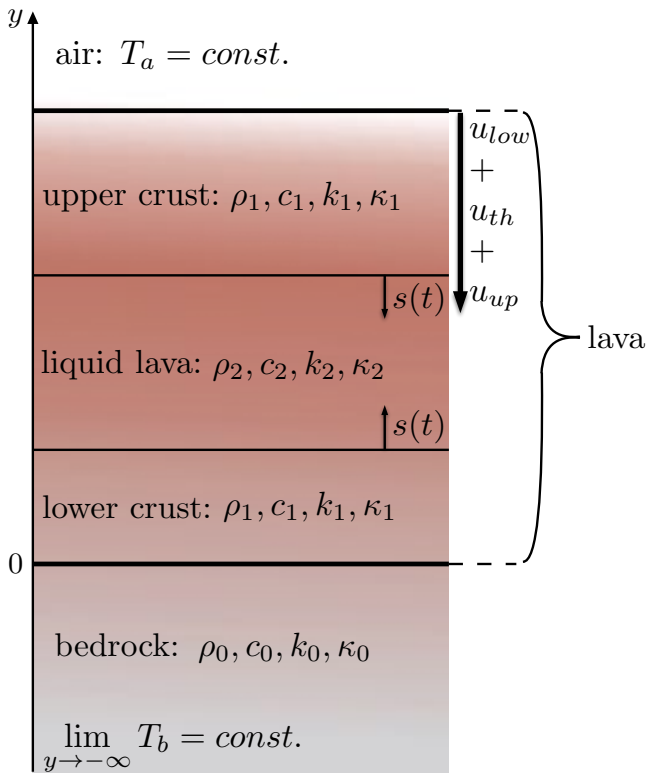


Figure 10. Scheme of the model. Indices 0 refer to bedrock, 1 to solidified lava and 2 to fluid lava. Thus, the bedrock may have different properties than the solidified lava. For explanation see text.

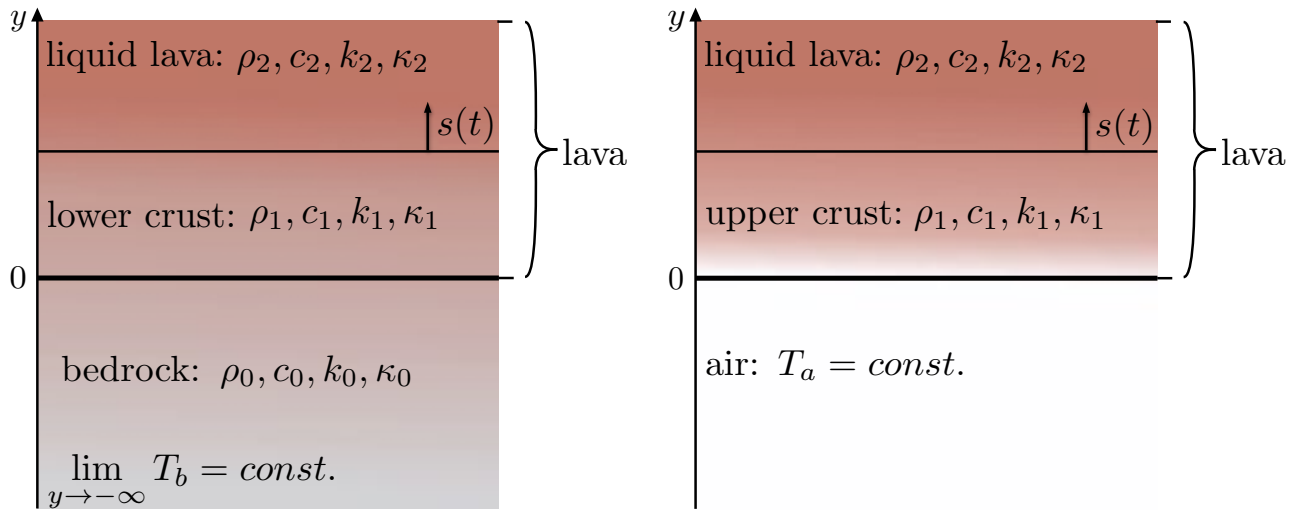


Figure 11. Solving the model as two separate problems for the lower (left-hand side) and upper (right-hand side) crust. Afterwards, the problem is merged back into the original problem by mirroring the coordinate system for the upper crust along the x-axis.

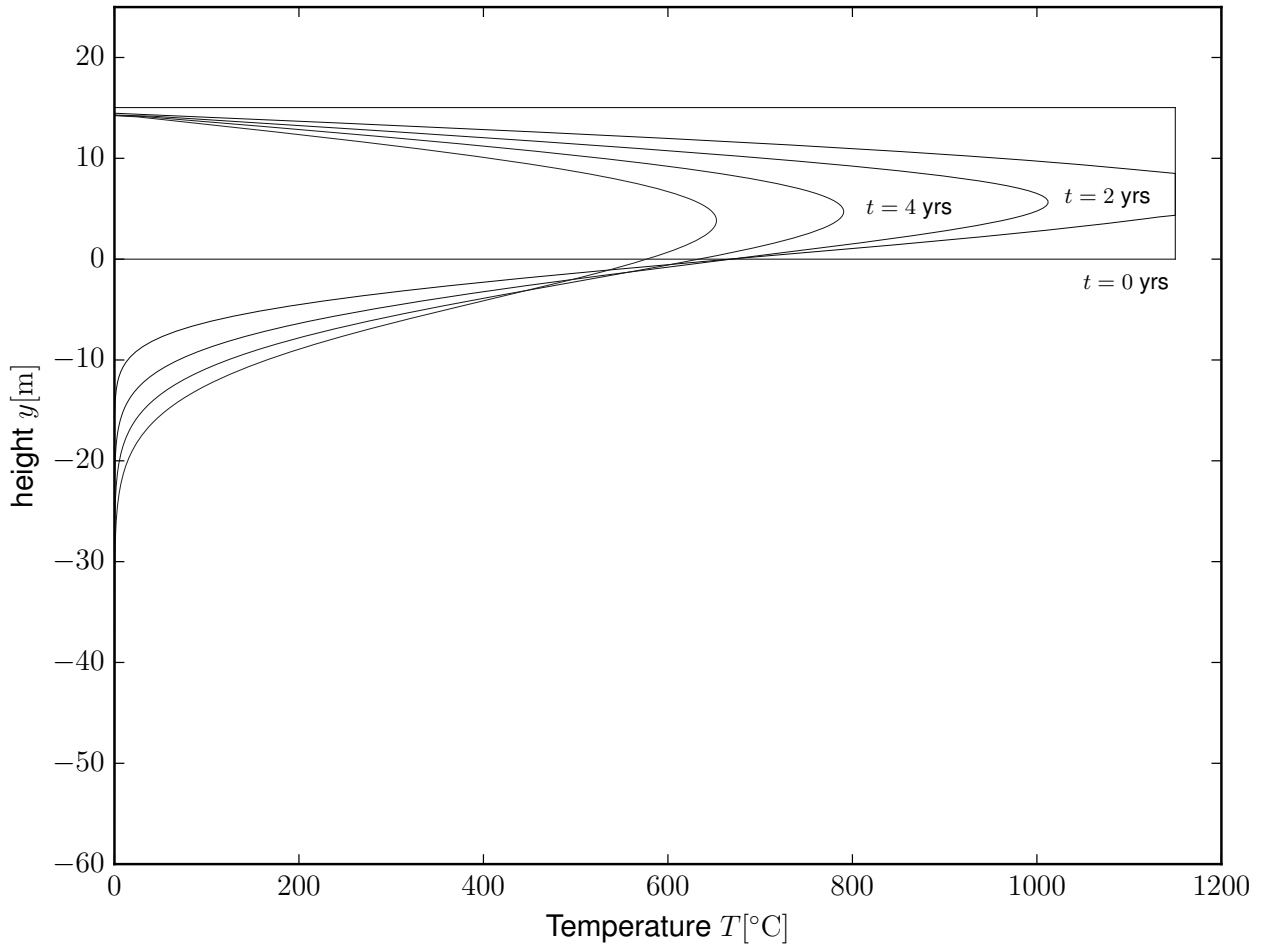


Figure 12. Time evolution of the temperature distribution. The rectangle represents the initial distribution at $t = 0$ years with a lava thickness of 15.00 m. A scaled thermal expansivity value of $\gamma \cdot \alpha = 1.66 \times 10^{-5} \text{K}^{-1}$ and a thermal diffusivity value of $\kappa = 3.00 \times 10^{-7} \text{m}^2 \text{s}^{-1}$ has been used (compare Section 4.2). The temperature distribution is plotted every year of simulation time. The upper and lower crusts meet at $t = 1.67$ yrs, the time of full solidification. Note the decreasing thickness of the lava (the upper intersection of the curves with the y -axis).

Table 1. Satellite data and time span of acquisitions.

Satellite	Track	Time span	Geometry	Incidence angle	LOS unit vector (Up/N/E)	Figure
ERS	52	05/1993 - 09/2000	descending	24.7°	(0.92/-0.13/0.35)	S1
	324	05/1995 - 05/2000	descending	22.3°	(0.93/-0.14/0.35)	S2
	359	08/1993 - 07/1999	ascending	22.0°	(0.94/-0.12/-0.31)	S3
Envisat	324	09.2003 - 04/2010	descending	22.3°	(0.93/-0.14/0.35)	S4
CSK	2574	06/2010 - 10/2014	ascending	32.2°	(0.85/-0.17/-0.5)	S5
	2575	05/2011 - 10/2014	descending	23.9°	(0.91/-0.14/0.38)	S6

Table 2. Table of parameters

Parameter name	Symbol	Value* [SI]	Literature	comment
Density of solid lava	ρ_{sol}	1660 - 2780 2799	[Peck, 1978] MELTS	Alae, Hawaii Hekla (1991, 2000)
Density of liquid lava	ρ_{liq}	(2790 \pm 30), 2740 2654 2663	[Peck, 1978] MELTS MELTS	Alae, Hawaii Hekla lava (1991) Hekla lava (2000)
Thermal expansivity	α	8.3 $\times 10^{-6}$ (3 \pm 1) $\times 10^{-6}$	Laboratory [Peck, 1978]	Hekla lava (1991) Alae, Hawaii
Thermal diffusivity	κ	5 \cdots 6 $\times 10^{-7}$	[Peck, 1978]	Alae, Hawaii
Latent heat of fusion	L	(335 \pm 42) $\times 10^3$	[Peck, 1978]	Alae, Hawaii
Specific heat	c	0.75 \cdots 1.23 $\times 10^3$	[Peck, 1978]	Alae, Hawaii

* All numerical values are given in corresponding SI-units.

Table 3. *A priori* values of parameters used in the inversion

Parameter name	Symbol	<i>A priori</i> range
Initial thickness (1991)	D_{1991}	8 \cdots 30 m
Initial thickness (2000)	D_{2000}	8 \cdots 30 m
Thermal expansion	α	8.3 $\times 10^{-6}$ K $^{-1}$
Scaling coefficient	γ	0.25 \cdots 3
Thermal diffusivity	κ	0.5 \cdots 9 $\times 10^{-7}$ m 2 s $^{-1}$
Latent heat of fusion	L	320 $\times 10^3$ Jkg $^{-1}$
Specific heat	c	1.2 $\times 10^3$ Jkg $^{-1}$ K $^{-1}$

**1 Supporting Information for ”Post-emplacement
2 cooling and contraction of lava flows: InSAR
3 observations and a thermal model for lava fields at
4 Hekla volcano, Iceland”**

5 DOI: 10.1002/

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7 **Additional Supporting Information: InSAR time series**

- 8 1. Figure 1: ERS Track 52
- 9 2. Figure 2: ERS Track 324
- 10 3. Figure 3: ERS Track 359
- 11 4. Figure 4: Envisat Track 324
- 12 5. Figure 5: CSK Track 2574
- 13 6. Figure 6: CSK Track 2575

14 **Introduction**

15 All InSAR time series shown in this supporting information were processed using the
16 StaMPS processing software *Hooper et al.* [2012]. The data points result from a merged
17 processing approach, combining both permanent scatterer and small baseline approaches
18 in the interferometric analysis. Furthermore, ramps in the interferograms were removed.
19 Table 1 shows the satellites and the time span of the data used. Figures S1 through S6
20 show the corresponding time series.

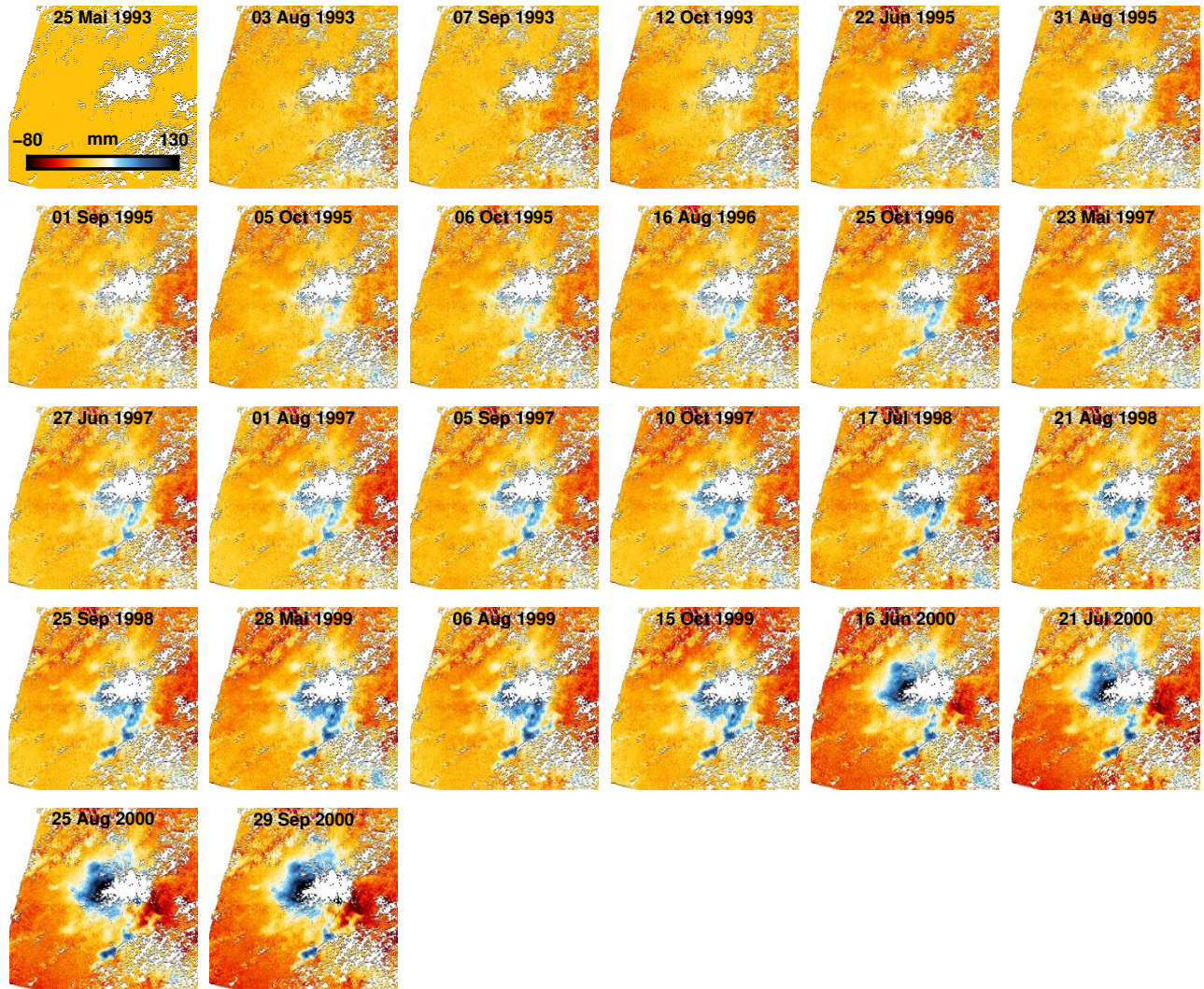


Figure S1. ERS time series spanning 1993 to 2000. Accumulated LOS unwrapped phase change, Track 52, descending.

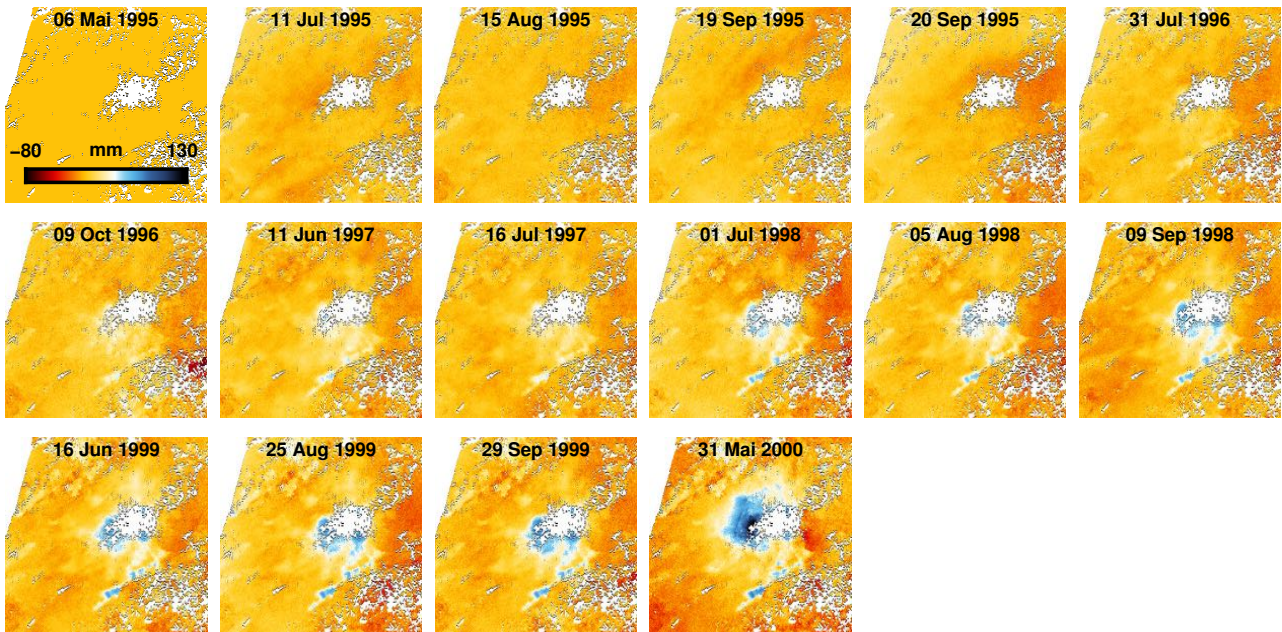


Figure S2. ERS time series spanning 1995 to 2000. Accumulated LOS unwrapped phase change, Track 324, descending.

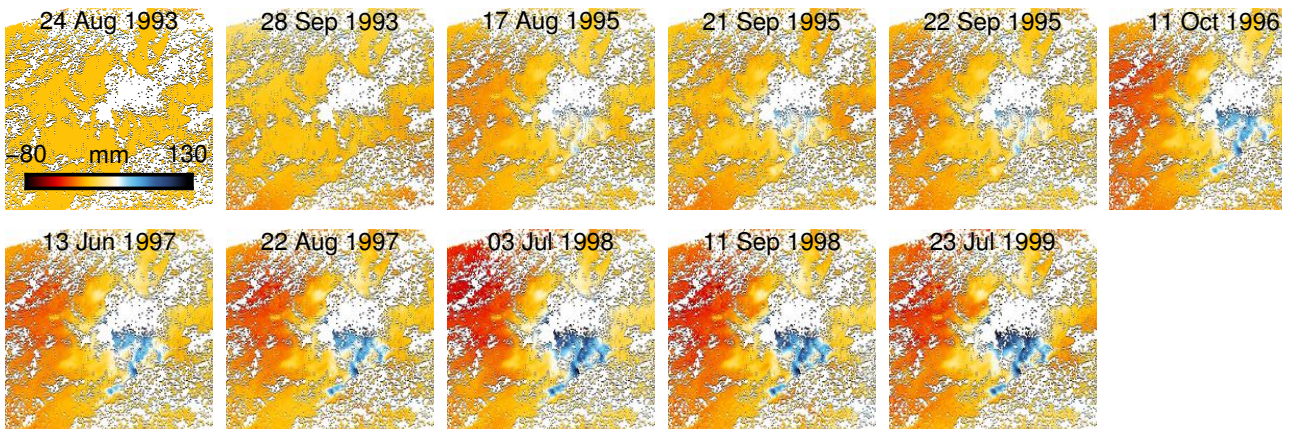


Figure S3. ERS time series spanning 1993 to 1999. Accumulated LOS unwrapped phase change, Track 359, ascending.

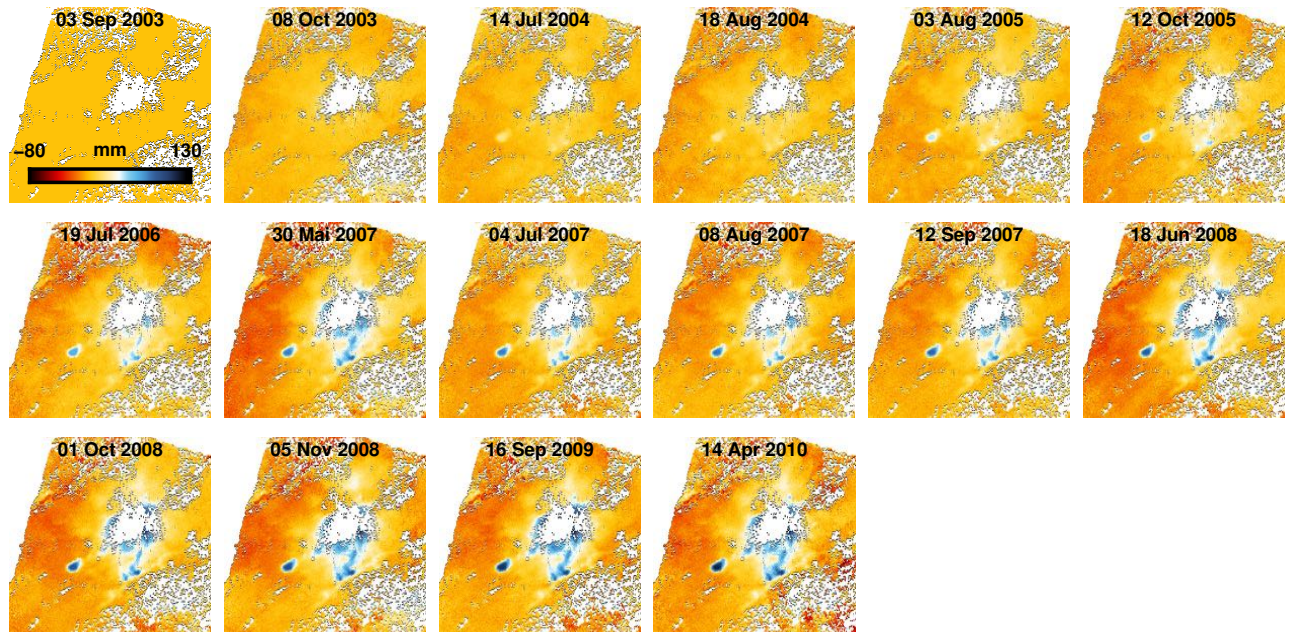


Figure S4. Envisat time series spanning 2003 to 2010. Accumulated LOS unwrapped phase change, Track 324, descending.

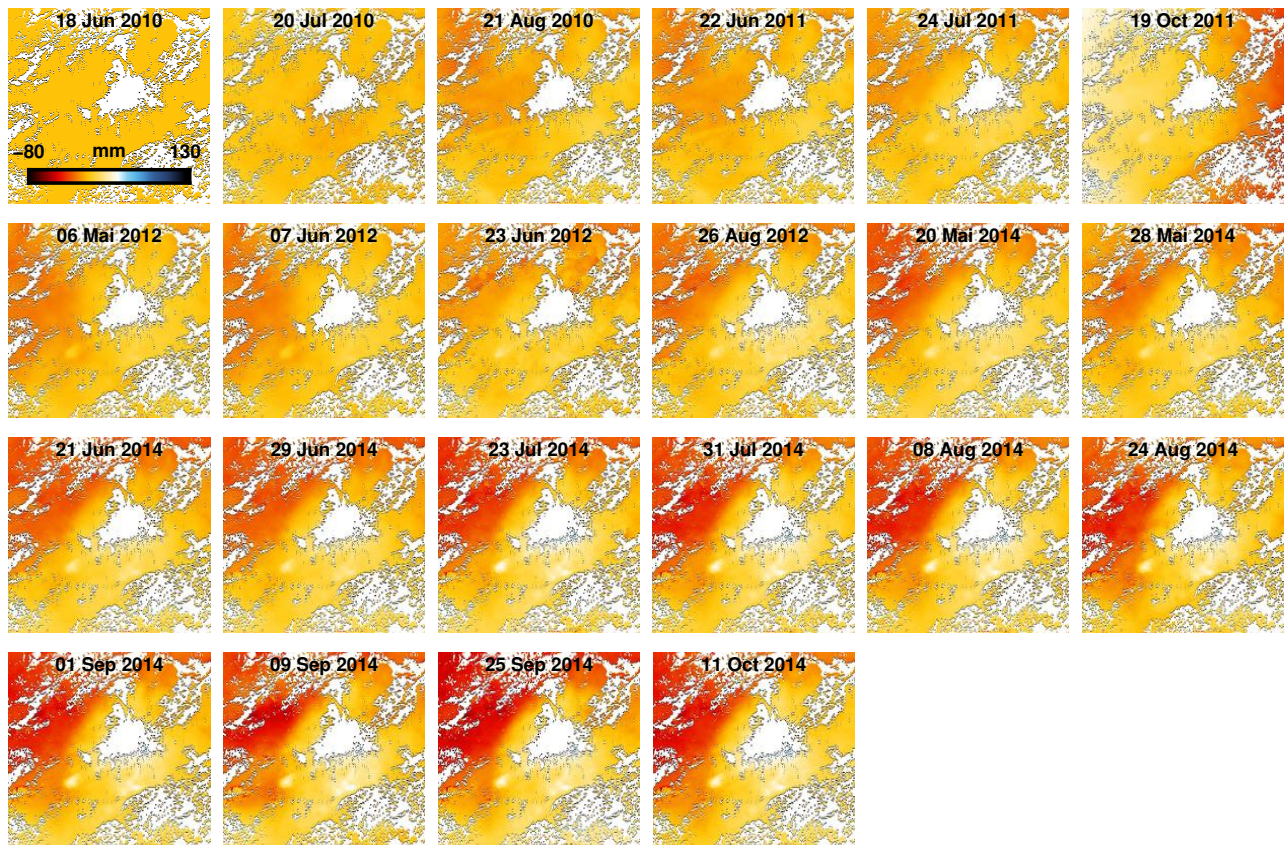


Figure S5. CSK time series spanning 2010 to 2014. Accumulated LOS unwrapped phase change, Track 2574, ascending.

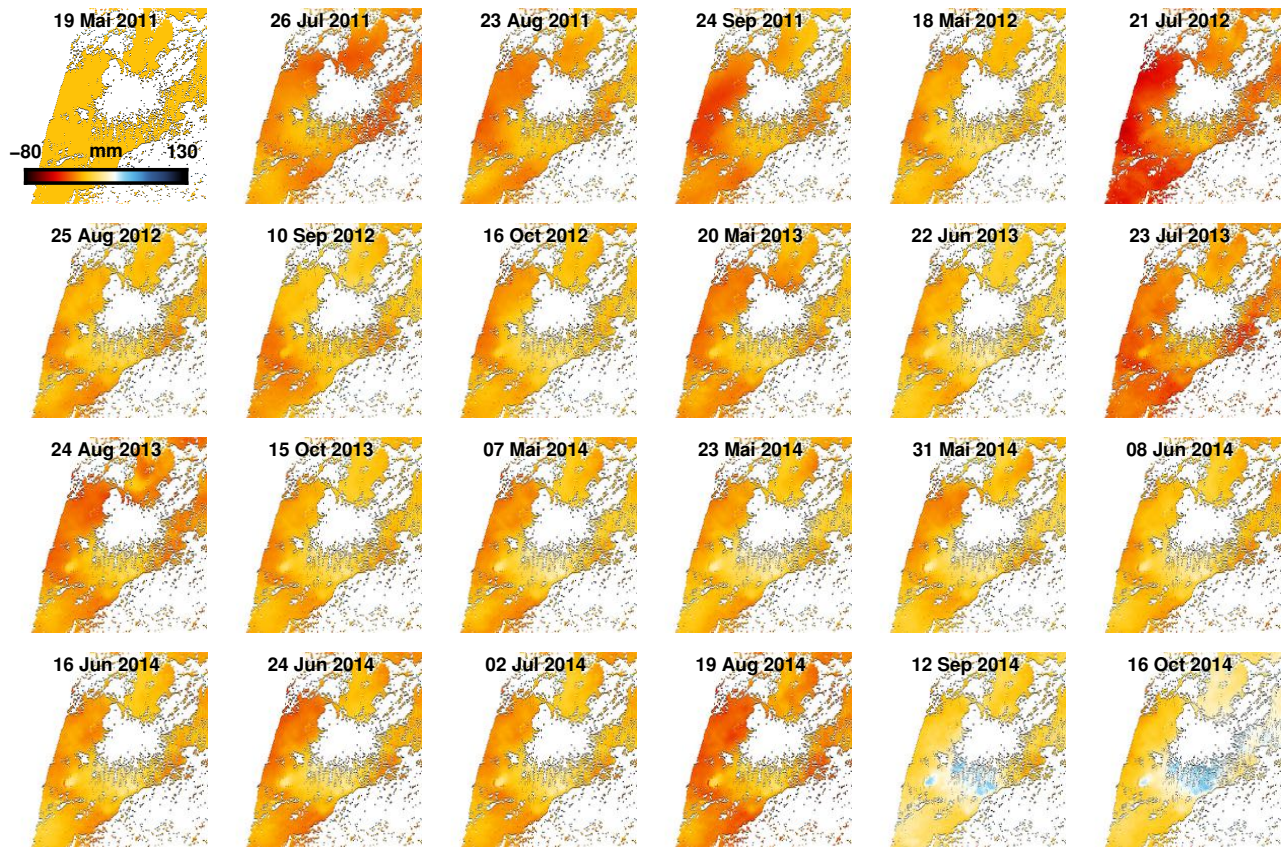


Figure S6. CSK time series spanning 2011 to 2014. Accumulated LOS unwrapped phase change, Track 2575, ascending.