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Reliability Analysis of Complex Systems with Uncertainties by Monte Carlo Simulation Method

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Abstract: Survival signature has been presented recently to quantify the system reliability as an alternative to the system signature. It not only holds the merits of system signature, but has the advantage for system with more than one type of components. In real world, however, due to lack of knowledge or data, there always exit uncertainties and imprecisions in components parameters. This makes system reliability analysis by using survival signature-based analytical method intractable, which is a challenging problem in system reliability analysis area. In this paper, system reliability is computed by using a simulation based method and survival signature. The novel proposed method allow to estimate the reliability of realistic and large-scale systems including parameter uncertainty and imprecision. A numerical example is presented to show the applicability of the approach.

Keywords: Survival signature, Reliability analysis, Imprecision, Monte Carlo method, Complex System.

1. Introduction

A system is a collection of components, subsystems or assemblies arranged to a specific design in order to achieve desired functions with acceptable performance and reliability. System reliability analysis is essential to system safety and availability. In recent years, system signature has been recognized as an important tool to quantify reliability of systems and networks consist of independent and identically distributed (*iid*) components with random failure times (Samaniego 2007). System signature separates the structure of the system from the failure time distribution of the components. Some recent advances are presented by Eryilmaz (2010). However, when it comes to a system with more than one type of components, it requires computation of the probabilities of all possible different orderings of the order statistics of each component failure lifetime distributions, which is complex and tedious procedure not feasible for real work systems.

In order to overcome the limitations of the system signature, Coolen and Coolen-Maturi (2012) proposed the survival signature as improved concept, which does not rely any more on the restriction to one component type. Specifically, the characteristics of the components do not need to be independently and identically distributed. Recent developments have opened up a pathway to perform a reliability analysis using the concept of survival signature even for relatively complex systems. Aslett et al. (2014) presented the use of the survival signature for networked systems reliability quantification from a Bayesian perspective. Coolen and Coolen-Maturi (2015) have shown how the survival signature can be derived from the signatures of two subsystems in both series and parallel configuration, and they have developed a nonparametric-predictive inference scheme for system reliability using the survival signature.

In the real world, however, due to limited data and information available, it is difficult to characterize probabilistically the failure time of components. Since the reliability and performance of systems are directly affected by uncertainties, a quantitative assessment of uncertainty is widely recognized as an important task in practical engineering (Modarres 2006). Ferson et al. (2004) presented probability box, or p-box, to represent uncertain quantities. The defining characteristic of a p-box are the probability intervals that define upper and lower bounds on the cumulative probability over the domain of the uncertain quantity. Beer et al. (2013) used fuzzy set theory and imprecise probability theory to deal with the uncertainties. Considering imprecisions might lead to impractical computational costs especially for detailed models. Recently, Coolen et al. (2014) proposed non-parametric predictive inference for system reliability using the survival signature. Based on the above comcepts, Feng et al. (2015) developed an analytical method to calculate survival functions of systems with uncertain components parameters which belong to exponential distribution.

To deal with the imprecisions and uncertainties within components parameters, it is difficult to use analytical methods, especially for complex systems. Therefore, it can resort to simulation methods to quantify the system reliability. For instance, Marseguerra and Zio (2000) introduced optimization approach based on the combination of a generic algorithms and Monte Carlo simulation. Naess et al. (2009) presented the most advantage of Monte Carlo simulation method is that the failure criterision is usually relatively easy to check almost irrespective of the complexity of the system. The composite systems reliability evaluation based on Monte Carlo simulation and cross-entropy methods can be seen in Gonzalez et al. (2013). It can be seen that most of the current simulation methods evolve from Monte Carlo simulation, which is because this approach has its simplicity to implement at any levels of reliability analysis. However, Monte Carlo simulation method can require long computation time to produce satisfactory results. In order to speed up the simulation time for large systems and small failure probabilities, Au and Beck (2001) presented subset theory. The powerful software OpenCossan is able to deal with different representation of uncertainties, just refer to Patelli et al. (2012, 2014). Advanced line sampling was used to improve simulation efficiency in (De Angelis et al. 2015).

In this paper, an efficient simulation approach is proposed to estimate the reliability of system based on survival signature. The proposed methodology is very flexible and allows to consider different representation of the uncertainty. The applicability and efficiency of the proposed approach is demonstrated by solving an illustrative example.

This paper is organized as follows. Section 2 presents a brief introduction of the survival signature and the relating system survival signature. Survival signature-based simulation method for system reliability is presented in Section 3. The performance of the proposed method is illustrated by analyzing a complex system. Finally Section 4 closes the paper with conclusions.

2. Illustration of the Survival Signature

Suppose there is one system formed by m components. Let the state vector of components be $\underline{x} = (x_1, x_2, \cdots, x_m) \in \{0,1\}^m$ with $x_i = 1$ if the ith component is in working state and $x_i = 0$ if not. $\emptyset = \emptyset(\underline{x}) : \{0,1\}^m \rightarrow \{0,1\}$ defines the system structure function, i.e., the system status based on all possible $\underline{x}$. $\emptyset$ is 1 if the system functions for state vector $\underline{x}$ and 0 if not.

Now consider a system with $K \geq 2$ types of m components, with m_k indicating the number of components of each type and $\sum_{k=1}^{K} m_k = m$. It is assumed that the failure times of the same component type are independently and identically distributed or exchangeable. The survival signature becomes $\emptyset(l_1, l_2, \ldots, l_k)$, with $l_k = 0,1,\ldots,m_k$ for $k = 1,2,\ldots,K$. There are $\binom{m_k}{l_k}$ state vectors $\underline{x}^k$ with precisely l_k components x_i^k equal to 1, so with $\sum_{i=1}^{m} x_i^k = l_k$. Let $S_{l_1,l_2,\ldots,l_K}$ denote the set of all state vectors for the whole system for which $\sum_{i=1}^{m_k} x_i^k = l_k$, $k = 1,2,\ldots,K$. Assume that the random failure times of components of the different types are fully independent, and in addition the components are exchangeable within the same component types, the survival signature can be written as:

$$\emptyset(l_1, \ldots, l_k) = \left[\prod_{k=1}^{K} \binom{m_k}{l_k}^{-1}\right] \times \sum_{\underline{x} \in S_{l_1,\ldots,l_k}} \emptyset(\underline{x}) \quad (1)$$

$C_k(t) \in \{0,1,\dots,m_k\}$ denotes the number of k components working at time t. Assume that the components of the same type have a known CDF, $F_k(t)$ for type k. Moreover, the failure times of different component types are assumed independent, then:

$$P\left(\cap_{k=1}^{K}\{C_k(t)=l_k\}\right)=\prod_{k=1}^{K}P(C_k(t)=l_k)=\prod_{k=1}^{K}\binom{m_k}{l_k}[F_k(t)]^{m_k-l_k}[1-F_k(t)]^{l_k} \quad (2)$$

Hence, the survival function of the system with K types of components becomes

$$P(T_s>t)=\sum_{l_1=0}^{m_1}\cdots\sum_{l_k=0}^{m_k}\emptyset(l_1,\dots,l_K)P\left(\cap_{k=1}^{K}\{C_k(t)=l_k\}\right) \quad (3)$$

It is obvious from the above equations that the survival signature can separate the structure of the system from the failure time distribution of its components, which is the main advantage of the system signature. What is more, the survival signature only need to be calculated once for any system, which is similar to the system signature for systems with only single type of components. It is easily seen that survival signature is closely related with system signature. For a special case of a system with only one type of components, the survival signature and the Samaniego's signature (Samaniego 2007) is directly linked to each other through a simple equation, however, the latter cannot be easily generalized for systems with multiple types of components (Coolen et al. 2012).

This implies that all attractive properties of the system signature also hold for the method using the survival signature, also the survival signature is easy to apply for systems with multiple types of components, and one could argue it is much easier to interpret than the system signature.

3. Survival Signature-based Simulation Method

In this section, survival signature-based simulation methods are used for reliability analysis on systems without and with parameter uncertainties and imprecisions, respectively. The definitions and calculation steps can be illustrated with reference to the complex system shown in Fig. 1.

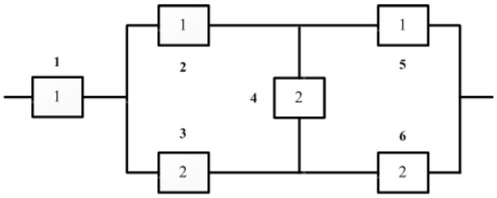

Figure 1. Complex system with two types of components

It can be seen from the above figure that the complex system has no series section or parallel section which can enable simplification and it comprises of six components, which belonging to two types. Results of survival signature of the system can be seen in Table 1.

Table 1. Survival signature of the complex system.

l_1	l_2	$\emptyset(l_1,l_2)$
0	0	0
0	1	0
0	2	0
0	3	0
1	0	0
1	1	0
1	2	1/9
1	3	1/3
2	0	0
2	1	0
2	2	4/9
2	3	2/3
3	0	1
3	1	1
3	2	1
3	3	1

3.1 Reliability analysis on system without imprecision

Suppose components failure times of type 1 and type 2 obey Weibull distribution and gamma distribution, respectively. Their parameters can be seen in Table 2.

Table 2. Parameters of components in the complex system.

Component Type	Distribution Type	Parameters (α,β)
1	Weibull	(1.2, 2.3)
2	Gamma	(0.8, 1.3)

It can use simulation method to conduct the complex system reliability assessment because there are no approximations or assumptions in the simulation approach. The survival signature gives the probability that the system functions knowing the number of components for each type (i.e. l_1 and l_2) that are working. This system is equivalent to a system composed by two components that can be in four states (state 0, state 1, state 2 and state 3). Therefore, the survival signature can be interpreted as the ``production capability" of the system. For instance, if all the components are working (or they are all in state 3), the system output is 1. If only one component of type 1 and two components of type 2 are working (or component 1 in state 1 while component 2 in state 2), the system output is $1/9$ and so on.

The survival signature-based method used to simulate the expected output of the system over the time is derived from the approach proposed in Zio et al. (2006). The major steps of the method are as follows:

Step 1: sampling the transition times of the first component type, hence a sequence of transition times t_1, t_2, t_3 and t_4 are obtained.

Step 2: Repeating the procedure of step 1 for the component type 2, which will obtain 4 additional transition times.

Step 3: Reordering all the transition times of $(t_1, t_2, \ldots, t_8)$.

Step 4: Computing the probability that the system functions for each time interval, which based on survival signature.

Step 5: Repeating the steps 1 to 4 for n system histories and averaging the obtained results.

Therefore, the probability of the system to survival over the time t is obtained. The survival signature-based simulation method follows the productivity idea, which gives each run an expected survival function. This simulation method fully takes survival signature into account. Therefore, this approach is more efficient for inference on the system survival function.

In order to verify the simulation method, the results obtained by the survival signature-based simulation method are compared with the analytical solution and shown in Fig. 2.

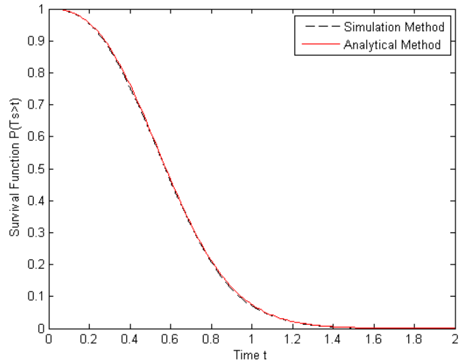

Figure 2. Survival functions of the complex system calculated by survival signature-based simulation method and analytical method

It can be derived from the above figure that the survival function got by survival signature-based simulation method is agree with the result got through analytical solution.

3.2 Reliability analysis on system with imprecision

The Probabilistic uncertainty and imprecision in components parameters are challenging phenomena in reliability analysis of complex systems. They require appropriate mathematical modelling and quantification to obtain realistic results when calculating the reliability of systems (Beer et al. 2013). The imprecisions in the component parameters will lead to lower and upper bounds of survival function of the systems, how to deal with the imprecisions within parameters is a practical problem in engineering.

Continue using the complex system in Fig.1, the distribution types and imprecise parameters of components are shown in Table 3.

Table 2. Parameters of components in the complex system considering imprecision.

Component Type	Distribution Type	Parameters (α, β)
1	Weibull	([1.2,1.8], [2.3,2.9])
2	Gamma	([0.8,1.6], [1.3,2.1])

In some instances analytical method will not be an appropriate means to analysis a system.

For example, the system may be too complex or exits uncertainties within components, all of which will prevent the use of a general analytical method codes. In these situations, the system reliability performance can be simulated using survival signature-based Monte Carlo method. This method not only has the advantage of survival signature to handle complex systems reliability problems, but can recur to Monte Carlo simulation to deal with the uncertainties within the systems.

Double loop sampling involves two layers of sampling: the outer loop is called the parameter loop since it concerns sampling different values for the set of distribution parameters for all of the uncertain quantities; while the inner loop goes by the name of probability loop because it involves sampling from precise probability distribution functions. As a matter of fact, double loop sampling implicates sampling from an analytical distribution whose parameters have been generated by sampling.

To solve the parameter epistemic imprecision within components, it is just need to add an optimization loop around the survival signature-based simulation methods cited in Section 3.1 to estimate the bounds. In other words, it can be done by adding a simple Monte Carlo loop and sampling the values of components parameters from uniform distributions.

Fig. 3 presents the survival function bounds calculated by the double loop sampling simulation method which based on the survival signature.

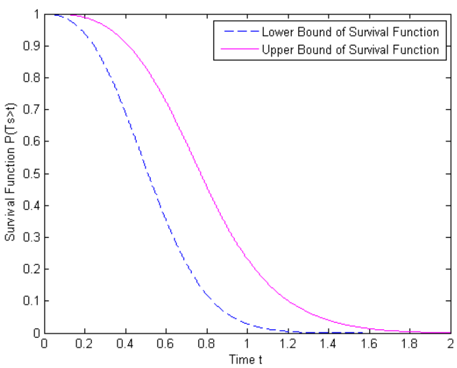

Figure 3. Lower and upper bounds of survival functions of the complex system calculated by survival signature-based simulation method

4. Conclusions

The survival signature has been shown to be a practical method for conducting reliability analysis of complex systems with multiple component types. A survival signature-based simulation method has been proposed for system reliability estimation. The proposed approach is extremely efficient since the system model only needs to be analysed once in order to get the survival signature. The proposed simulation method gives out a stepped survival function for every run. In order to get the survival function of the system, it is necessary to average the stepped survival functions after simulation. The proposed approach is generally applicable. It allows to solve and analyse problems considering different representation of the uncertainty.

In principle, the simulation method proposed in this paper has the ability to analyse system reliability by using only component failure time simulation and the survival signature, which is of great value for many systems in the real world. The feasibility of the proposed method has been illustrated analysing complex system. Future research direction may focus on repairable systems.

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