Active partial eigenvalue assignment for friction-induced vibration using receptance method

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Abstract. Generally, a mechanical system always has symmetric system matrices. Nevertheless, when some non-conservative forces are included, such as friction and aerodynamic force, the symmetry of the stiffness matrix or damping matrix or both violated. Moreover, such an asymmetric system is prone to dynamic instability. Distinct from the eigenvalue assignment for symmetric systems to reassign their natural frequencies, the main purpose of eigenvalue assignment for asymmetric systems is to shift the unstable eigenvalues to the stable region. In this research, only the unstable eigenvalues and eigenvalues which are close to the imaginary axis of the complex eigenvalue plane are assigned due to their predominant influence on the response of the system. The remaining eigenvalues remain unchanged. The state-feedback control gains are obtained by solving the constrained linear least-squares problems in which the linear system matrices are deduced based on the receptance method and the constraint is derived from the unobservability condition. The numerical simulation results demonstrate that the proposed method is capable of partially assigning those targeted eigenvalues of the system for stabilisation.

1. Introduction

Numberless research has been studied extensively to actively control vibrational systems [1–3]. The dynamic response of a vibrating system can be modified through changing the natural frequencies and mode shapes which are also referred as the eigenvalues and eigenvectors of the system. Dating from the 1960s when Wonhan [4] demonstrated that if a system is controllable its eigenvalues can be placed to arbitrary position of the complex plane using state-feedback,eigenvalue assignment has been widely applied to vibration control problems.

The receptance-based method put forward by Ram and Mottershead [5] is to actively place required eigenvalues of a system, which has significant advantages over conventional matrix methods requiring no knowledge of the mass, damping and stiffness matrices of the system. The information of system matrices is typically obtained from the finite element method including modeling reductions and errors. In addition, only a small number of receptances need to be measured, thus no observer and or model reduction is required.

For symmetric systems whose damping and stiffness matrices are symmetric, the eigenvalue assignment is implemented for assigning appropriate imaginary part of the eigenvalues or shifting its natural frequencies to desirable values to avoid resonance [6]. Besides the eigenvalue placement, zeros of systems can be assigned to an antiresonance at a different coordinate to mitigate vibration [7].

Friction can induce undesirable dynamic characteristics in many mechanical systems. It is influenced by various factors, such as material properties, geometry of the sliding surfaces, sliding speed, temperature and normal load [8]. This, in turn, makes the friction-induced vibration complex and fugitive. For a system in which vibration is generated by friction, the symmetry of the stiffness matrix or damping matrix or both are violated. The friction-induced asymmetric system is prone to dynamic instability as a result of some of its eigenvalues are on the right-half-side of the complex plane. Ouyang [9, 10] developed a state-feedback control method and a hybrid control strategy in which passive structural modifications and an active state-feedback control are executed for an asymmetric system to relocate the unstable complex eigenvalues using the receptance method.

In practice, only a few of eigenvalues are undesirable. It is computational expensive, unnecessary and time consuming to assign all the eigenvalues of a system. Saad [11] proposed two projection methods for partial pole placement in first-order linear control systems. Datta et al. [12] considered partial pole assignment problems in the second-order form of the dynamic equations based on the orthogonality relations of the system matrices. Tehrani et al. [13] introduced a multi-input partial pole placement method and demonstrated experimentally using two test rigs. Tehrani and Ouyang [14] developed a partial pole assignment for asymmetric systems using the uncontrollability condition.

In this research, an active partial eigenvalue assignment method is developed for the friction-induced vibration whose damping and stiffness matrices are asymmetric. Only the first two pairs of the eigenvalues are assigned to the positions which are far from the imaginary axis of the complex plane. The control gains for assigning the only the first two pair of the eigenvalues and remain the rest unchanged are derived by solving the constrained linear least-squares problems in which the linear system matrices are deduced based on the receptance method and the constraint is derived from the unobservability condition. The numerical simulation results indicate that the partial assignment of the desired eigenvalues is achieved without spillover using the unobservability condition, thus stabilize the system.

2. Single-input state-feedback method for eigenvalue assignment

Asymmetric systems with asymmetric system matrices are prone to be unstable, thus, eigenvalue assignment is performed mainly for these systems to place unstable eigenvalues to stable regions using appropriate control gains.

The dynamic equation of a closed-loop asymmetric system including state-feedback control gains with asymmetric damping and stiffness matrices is given in Laplace space as:

$$(\mathbf{M}s^2 + \mathbf{C}_{\mathrm{tol}}s + \mathbf{K}_{\mathrm{tol}})\mathbf{x}(s) = \mathbf{b}u(s) + \mathbf{p}(s) \tag{1}$$

$$u(s) = -(\mathbf{g} + s\mathbf{f})^{\mathrm{T}}\mathbf{x}(s) \tag{2}$$

where $\mathbf{M}, \mathbf{C}_{\mathrm{tol}}$ and $\mathbf{K}_{\mathrm{tol}} \in \mathbb{R}^{N\times N}$ are the mass, damping and stiffness matrices, respectively. Furthermore, $\mathbf{C}_{\mathrm{tol}} = \mathbf{C}_{\mathrm{s}} + \mathbf{C}_{\mathrm{as}}$ and $\mathbf{K}_{\mathrm{tol}} = \mathbf{K}_{\mathrm{s}} + \mathbf{K}_{\mathrm{as}}$. Moreover, $\mathbf{C}_{\mathrm{s}}$ and $\mathbf{K}_{\mathrm{s}}$ are the original symmetric matrices, $\mathbf{C}_{\mathrm{as}}$ and $\mathbf{K}_{\mathrm{as}}$ are the asymmetric terms generated by the non-conservative forces, such as friction and aerodynamic force. $\mathbf{b} \in \mathbb{R}^N$ is the vector indicating the position of the single control input $u(s)$. $\mathbf{g}$ and $\mathbf{f} \in \mathbb{R}^N$ are the displacement and velocity proportional feedback control gains, respectively. $\mathbf{x}(s)$ and $\mathbf{p}(s) \in \mathbb{R}^N$ are respectively the displacement vector and external force vector.

Using equation (1) and equation (2), eigenvalues of the asymmetric can be placed to any required positions on the complex plane if the system is controllable and all the details of $\mathbf{M}, \mathbf{C}_{\mathrm{tol}}, \mathbf{K}_{\mathrm{tol}}$ matrices are known. However, the detailed information of $\mathbf{M}, \mathbf{C}_{\mathrm{tol}}, \mathbf{K}_{\mathrm{tol}}$ always contains modeling errors. On the other hand, the receptance method which is based on the measured vibration data has no requirement of knowledge of system matrices. Thus, the recepatance method is adopted in this research.

The receptance is defined in the following formulation as:

$$\mathbf{H}_{\mathrm{s}}(s) = (\mathbf{M}s^2 + \mathbf{C}_{\mathrm{s}}s + \mathbf{K}_{\mathrm{s}})^{-1} \tag{3}$$

This equation provides the general definition of the receptance of symmetric systems. Similarly, the receptance for an asymmetric system is written in equation (4).

$$\mathbf{H}_{\mathrm{as}}(s) = [\mathbf{M}s^2 + (\mathbf{C}_{\mathrm{s}} + \mathbf{C}_{\mathrm{as}})s + (\mathbf{K}_{\mathrm{s}} + \mathbf{K}_{\mathrm{as}})]^{-1} \tag{4}$$

For assigning eigenvalues of asymmetric systems using receptance method, firstly, the closed-loop symmetric system is considered. Substitute equation (2) and equation (3) into equation (1) and omit the asymmetric terms $\mathbf{C}_{\mathrm{as}}$ and $\mathbf{K}_{\mathrm{as}}$, then equation (5) is arrived with further derivation.

$$[\mathbf{I} + \mathbf{H}_{\mathrm{s}}(s)\mathbf{b}(\mathbf{g} + s\mathbf{f})^{\mathrm{T}}]\mathbf{x}(s) = \mathbf{H}_{\mathrm{s}}(s)\mathbf{p}(s) \tag{5}$$

Thus,

$$\mathbf{x}(s) = \frac{\mathrm{adj}(\mathbf{A}_{\mathrm{s}})}{\det(\mathbf{A}_{\mathrm{s}})}\mathbf{H}_{\mathrm{s}}(s)\mathbf{p}(s) = \hat{\mathbf{H}}_{\mathrm{s}}(s)\mathbf{p}(s) \tag{6}$$

where $\mathbf{A}_{\mathrm{s}} = \mathbf{I} + \mathbf{H}_{\mathrm{s}}(s)\mathbf{b}(\mathbf{g} + s\mathbf{f})^{\mathrm{T}}$. Moreover, $\hat{\mathbf{H}}_{\mathrm{s}}(s)$ is denoted as the closed-loop receptance of the symmetric system. Based on equation (6), it is obvious that eigenvalues of the closed-loop asymmetric system are roots of the denominator characteristic equation given in the following equation:

$$\det(\mathbf{A}_{\mathrm{s}}) = 0 \tag{7}$$

Taking the advantage of the matrix determinant lemma, the determine of matrix $\mathbf{A}_{\mathrm{s}}$ is derived as:

$$\det(\mathbf{A}_{\mathrm{s}}) = 1 + (\mathbf{g} + s\mathbf{f})^{\mathrm{T}}\mathbf{H}_{\mathrm{s}}(s)\mathbf{b} \tag{8}$$

Besides, the inverse of $\mathbf{A}_{\mathrm{s}}$ is derived using the Sherman-Morrison formula in equation (10).

$$\mathrm{inv}(\mathbf{A}_{\mathrm{s}}) = \mathbf{I} - \frac{\mathbf{H}_{\mathrm{s}}(s)\mathbf{b}(\mathbf{g} + s\mathbf{f})^{\mathrm{T}}}{1 + (\mathbf{g} + s\mathbf{f})^{\mathrm{T}}\mathbf{H}_{\mathrm{s}}(s)\mathbf{b}} \tag{9}$$

Subsequently, the receptance of the closed-loop symmetric system $\hat{\mathbf{H}}_{\mathrm{s}}$ is derived as:

$$\hat{\mathbf{H}}_{\mathrm{s}} = \mathrm{inv}(\mathbf{A}_{\mathrm{s}})\mathbf{H}_{\mathrm{s}}(s) = \mathbf{H}_{\mathrm{s}}(s) - \frac{\mathbf{H}_{\mathrm{s}}(s)\mathbf{b}(\mathbf{g} + s\mathbf{f})^{\mathrm{T}}\mathbf{H}_{\mathrm{s}}(s)}{1 + (\mathbf{g} + s\mathbf{f})^{\mathrm{T}}\mathbf{H}_{\mathrm{s}}(s)\mathbf{b}} \tag{10}$$

Similarly, the characteristic polynomial of the asymmetric closed-loop system is formulated as:

$$\det(\mathbf{A}_{\mathrm{as}}) = 0 \tag{11}$$

where $\mathbf{A}_{\mathrm{as}} = \mathbf{I} + \hat{\mathbf{H}}_{\mathrm{s}}(\mathbf{C}_{\mathrm{as}}s + \mathbf{K}_{\mathrm{as}})$.

As a consequence, substitute equation (10) into equation (11), thus the closed-loop eigenvalues can be assigned to the predetermined values using the measured open-loop receptance $\mathbf{H}_{\mathrm{s}}$ of the corresponding symmetric system.

3. Partial eigenvalue assignment using the unobservability condition

All of the eigenvalues of the asymmetric system can be assigned using the open-loop receptance of the corresponding symmetric system using equation (11). However, only some of the eigenvalues of the asymmetric system are on the right-half-side of the complex eigenvalue plane or close to the imaginary axis of the complex plane, there is no need to assign the other stable eigenvalues which are not the dominant eigenvalues. Hence, the effective method is to only relocate the eigenvalues which are unstable or have small negative real parts.

For the asymmetric system given in equation (1), the eigenvalue s_j associated with this equation is the root of the characteristic polynomial provided in the equation as:

$$\det[\mathbf{M}s^2 + \mathbf{C}_{\text{tol}}s + \mathbf{K}_{\text{tol}}] = 0 \tag{12}$$

The order of this polynomial is 2N and the roots of the above equation appear in complex conjugate pairs as $s_1, s_1^*, \ldots, s_N, s_N^*$. $(*)$ means the complex conjugate pair.

The right eigenvalue problem associated with equation (12) is denoted as:

$$s_j^2\mathbf{M}\mathbf{u}_j + s_j\mathbf{C}_{\text{tol}}\mathbf{u}_j + \mathbf{K}_{\text{tol}}\mathbf{u}_j = \mathbf{0}, \qquad \forall j = 1, \ldots, N \tag{13}$$

where $s_j \in \mathbb{C}$ is the jth eigenvalue and $\mathbf{u}_j \in \mathbb{C}$ is the jth right eigenvector. The corresponding left eigenvalue problem can be represented in the similar formula as:

$$s_j^2\mathbf{v}_j^{\mathrm{T}}\mathbf{M} + s_j\mathbf{v}_j^{\mathrm{T}}\mathbf{C}_{\text{tol}} + \mathbf{v}_j^{\mathrm{T}}\mathbf{K}_{\text{tol}} = \mathbf{0}, \qquad \forall j = 1, \ldots, N \tag{14}$$

where $\mathbf{v}_j \in \mathbb{C}$ is the jth left eigenvector. Besides, $(\bullet)^{\mathrm{T}}$ denotes the transpose of a vector. Since the system focused in this research is asymmetric, right and left eigenvectors of the system are different and they can be calculated using the first-order state-space method [15].

Equation (1) can be transformed in the state space and after the Laplace transform it is provided in the first-order formula as:

$$s\mathbf{A}\mathbf{X}(s) + \mathbf{B}\mathbf{X}(s) = \mathbf{0} \tag{15}$$

where $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{\mathrm{N}\times\mathrm{N}}$ are the system matrices of the first-order system and $\mathbf{X}(s)$ is the state vector given by:

$$\mathbf{A} = \begin{pmatrix} \mathbf{O} & \mathbf{M} \\ \mathbf{M} & \mathbf{C} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -\mathbf{M} & \mathbf{O} \\ \mathbf{O} & \mathbf{K} \end{pmatrix}, \quad \text{and} \quad \mathbf{X}(s) = \begin{pmatrix} s\mathbf{x}(s) \\ \mathbf{x}(s) \end{pmatrix}$$

In addition, $\mathbf{O} \in \mathbb{R}^{\mathrm{N}\times\mathrm{N}}$ is the null matrix.

Consequently, the right eigenvalue problem of the first-order system is defined as:

$$s_j\mathbf{A}\mathbf{z}_j + \mathbf{B}\mathbf{z}_j = \mathbf{0}, \qquad \forall j = 1, \ldots, N \tag{16}$$

where s_j and $\mathbf{z}_j$ are respectively the jth eigenvalue and jth right eigenvector of the first-order system defined in equation (15). $\mathbf{z}_j$ is related to the jth eigenvector of second-order system given in equation (1) as follows [16]:

$$\mathbf{z}_j = \begin{pmatrix} s_j\mathbf{u}_j \\ \mathbf{u}_j \end{pmatrix} \tag{17}$$

The left eigenvalue problem of the first-order system is given as:

$$s_j\mathbf{y}_j^{\mathrm{T}}\mathbf{A} + \mathbf{y}_j^{\mathrm{T}}\mathbf{B} = \mathbf{0}, \qquad \forall j = 1, \ldots, N \tag{18}$$

where s_j and $\mathbf{y}_j$ are respectively the jth eigenvalue and jth left eigenvector of the first-order system defined in equation (15).

Similarly, the $\mathbf{y}_j$ is related to the second-order system by [16]:

$$\mathbf{y}_j = \begin{pmatrix} s_j \mathbf{v}_j \\ \mathbf{v}_j \end{pmatrix} \tag{19}$$

The right and left eigenvectors of the first-order system satisfy an orthogonality relationship denoted in equation (20) for distinct eigenvalues. This imply that the system given in equation (15) has set of biorthogonal eigenvectors with respect to the system matrices.

$$\mathbf{y}_j^{\mathrm{T}} \mathbf{A} \mathbf{z}_{\mathrm{k}} = 0 \quad \text{and} \quad \mathbf{y}_j^{\mathrm{T}} \mathbf{B} \mathbf{z}_{\mathrm{k}} = 0, \quad \forall j \neq k \tag{20}$$

Premultiplying equation (16) by $\mathbf{y}_j^{\mathrm{T}}$, the following equation can be obtained.

$$s_j \mathbf{y}_j^{\mathrm{T}} \mathbf{A} \mathbf{z}_j + \mathbf{y}_j^{\mathrm{T}} \mathbf{B} \mathbf{z}_j = \mathbf{0} \tag{21}$$

Hence, the eigenvectors can be normalized so that:

$$\mathbf{y}_j^{\mathrm{T}} \mathbf{A} \mathbf{z}_j = 1 \tag{22}$$

Substitute equation (17) and equation (19) into equation (22), the normalized equation is formed as:

$$\mathbf{v}_j^{\mathrm{T}} [2 s_j \mathbf{M} + \mathbf{C}] \mathbf{u}_j = 1 \tag{23}$$

Furthermore, the first-order dynamical equation of the asymmetric closed-loop system is given as:

$$s\mathbf{A}\mathrm{X}(s) + \mathbf{B}\mathrm{X}(s) = \begin{pmatrix} \mathbf{0} \\ \mathbf{b} \end{pmatrix} u(s) = \begin{pmatrix} \mathbf{0} \\ -(\mathbf{g} + s\mathbf{f})^{\mathrm{T}} \mathbf{x}(s) \end{pmatrix} \tag{24}$$

Therefore, the right eigenvalue problem of the closed-loop system is given as

$$\lambda_j \mathbf{A} \bar{\mathbf{z}}_j + \mathbf{B} \bar{\mathbf{z}}_j = \begin{pmatrix} \mathbf{0} \\ -(\mathbf{g} + \lambda_j \mathbf{f})^{\mathrm{T}} \end{pmatrix} \bar{\mathbf{z}}_j \tag{25}$$

where λ_j is the jth required closed-loop eigenvalue and $\bar{\mathbf{z}}_j$ is the closed-loop right eigenvalue of the closed-loop system in first-order. Premultiply equation (25) by the closed-loop left eigenvalue $\bar{\mathbf{y}}_j^{\mathrm{T}}$ of the closed-loop system in first-order with further derivation, the following equation is arrived.

$$\begin{pmatrix} \lambda_j \bar{\mathbf{v}}_j^{\mathrm{T}} & \bar{\mathbf{v}}_j^{\mathrm{T}} \end{pmatrix} (\lambda_j \mathbf{A} + \mathbf{B}) \begin{pmatrix} \lambda_j \bar{\mathbf{u}}_j \\ \bar{\mathbf{u}}_j \end{pmatrix} = -\bar{\mathbf{v}}_j^{\mathrm{T}} \mathbf{b} (\mathbf{g} + \lambda_j \mathbf{f})^{\mathrm{T}} \bar{\mathbf{u}}_j \tag{26}$$

According to equation (26), it is obvious that if certain required eigenvalues λ_j and the corresponding closed-loop right and left eigenvectors $\bar{\mathbf{u}}_j, \bar{\mathbf{v}}_j$ of the closed-loop system in second-order formulation make the right-hand side of equation (26) vanish, then equation (26) degenerated to equation (21), thus this particular required closed-loop eigenvalue λ_j remains the same as the open-loop eigenvalue s_j. Simultaneously, the closed-loop right and left eigenvectors $\bar{\mathbf{u}}_j, \bar{\mathbf{v}}_j$ keep the same as the right and left eigenvectors $\mathbf{u}_j, \mathbf{v}_j$ of the open-loop system. This indicates the eigenvalues can be partially assigned using particular control gains.

Furthermore, the right-hand side of equation (26) can be vanished either when

$$\bar{\mathbf{v}}_j^{\mathrm{T}} \mathbf{b} = \mathbf{0} \quad (\bar{\mathbf{v}}_j = \mathbf{v}_j) \tag{27}$$

or

$$\bar{\mathbf{u}}_j^{\mathrm{T}}(\mathbf{g}+\lambda_j\mathbf{f})=\mathbf{0} \quad (\lambda_j=s_j \quad \text{and} \quad \bar{\mathbf{u}}_j=\mathbf{u}_j) \tag{28}$$

equation (27) is the uncontrollable condition for jth mode. Likewise, the jth mode is said to be unobservable when equation (28) satisfied. This means

$$[\mathbf{u}_j^{\mathrm{T}} \quad s_j\mathbf{u}_j^{\mathrm{T}}]\begin{pmatrix}\mathbf{g}\\ \mathbf{f}\end{pmatrix}=\mathbf{0} \tag{29}$$

Hence, the control gains should be in the null space of $\mathbf{U}_{\mathrm{o}}=[\mathbf{u}_j^{\mathrm{T}} \quad s_j\mathbf{u}_j^{\mathrm{T}}]$. In the subsequent study, the unobservability condition is applied to select appropriate control gains.

Thus, for partial eigenvalue assignment, the closed-loop eigenvalues are composed of the assigned eigenvalues $\lambda_1,\lambda_1^*,\ldots,\lambda_k,\lambda_k^*$ and the unchanged eigenvalues $s_1,s_1^*,\ldots,s_{N-k},s_{N-k}^*$. For the unchanged eigenvalues, equation (28) has to be satisfied. On the other hand, for the newly assigned eigenvalues, the control gains can be obtained using equation (11). Since the number of the assigned eigenvalues is less than the number of the unknown terms of control gains, the equation for solving the control gains is an under-determined equation that allows an infinite number of solutions of control gains. Therefore, the control gains for assigning required eigenvalues and remaining unchanged eigenvalues can be obtained solving the constrained linear least-square problems in which the constraint condition is that the control gains has to be in the null space of $\mathbf{U}_{\mathrm{o}}=[\mathbf{u}_j^{\mathrm{T}} \quad s_j\mathbf{u}_j^{\mathrm{T}}]$.

4. Numerical simulation

In this section, a simplistic model is provided in figure (1). This model is modified from the model proposed by Ouyang [9] by adding a contact damping c_0. Ouyang originally used the model in which the damping and stiffness matrices are asymmetric for investigating the friction-induced vibration.

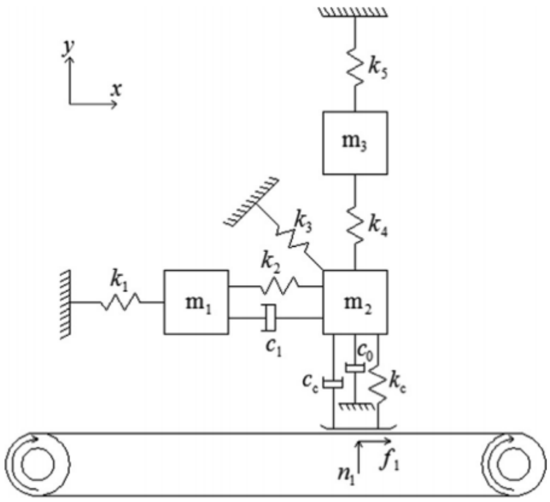

Figure 1: Four-degrees-of-freedom model for FIV

In this model, the single-point masses m_1, m_3 can move only in one direction and the single-point mass m_2 can move in both the horizontal x direction and the vertical y direction. These masses are supported by linear springs and linear dampers. Moreover, the oblique linear spring k_3 is verified to cause the coupling of x and y directions in Hoffmann's work [17]. The friction f_1 acting on m_2 abides by the Column law. Thus, $f_1=\mu n_1$, where n_1 is the normal force acting on m_2. In addition, the friction coefficient is set as a constant value deliberately to avoid the stick-slip motion.

The equation of motion of this model with state-feedback control gains is the same as equation (1). The details of the system matrices are given as:

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_3 & 0 & 0 \\ 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix}, \ \mathbf{C}_{\mathrm{s}} = \begin{bmatrix} c_1 & 0 & -c_1 & 0 \\ 0 & 0 & 0 & 0 \\ -c_1 & 0 & c_1 & 0 \\ 0 & 0 & 0 & c_0 + c_c \end{bmatrix}$$

$$\mathbf{K}_{\mathrm{s}} = \begin{bmatrix} k_1+k_2 & 0 & -k_2 & 0 \\ 0 & k_4+k_5 & 0 & -k_4 \\ -k_2 & 0 & k_2+0.5k_3 & -0.5k_3 \\ 0 & -k_4 & -0.5k_3 & k_4+0.5k_3+k_c \end{bmatrix}, \ \mathbf{E} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where $m_{\mathrm{i}} = 1$ kg $(i = 1, 2, 3)$, $c_{\mathrm{i}} = 0.5$ Ns/m (i $= 0, 1$) $k_{\mathrm{i}} = 100$ N/m (i $= 1, 2, 3, 4, 5$), and $k_{\mathrm{c}} = 2k_1$. Moreover, the asymmetric damping and stiffness matrices are obtained as $\mathbf{C}_{\mathrm{as}} = \mu c_{\mathrm{c}}\mathbf{E}$ and $\mathbf{K}_{\mathrm{as}} = \mu k_{\mathrm{c}}\mathbf{E}$, where $\mathbf{E}$ is the matrix indicating the location of the friction-affected degrees-of-freedom. Besides, the friction coefficient μ is set to be 0.5. The vector $\mathbf{b}$ indicating the position of the control input is set as $\mathbf{b} = (0,\ 0,\ 1,\ 1)^{\mathrm{T}}$.

The eigenvalues of this open-loop system without control input are provided in the table 1. For the open-loop system, the first pair of its eigenvalues has positive real parts. This leads the system to be unstable with large vibration amplitudes. Moreover, the second pair of the eigenvalues is quite closed to the imaginary axis of the complex plane. Therefore, this pair of eigenvalues is also considered to be vital to the stability of the system. The third and the fourth pair of the open-loop eigenvalues have large negative real parts. Thus, they have little influence on the response of the system. As a consequence, there is no necessity to reassign all of the open-loop eigenvalues and only the first and the second pair of the open-loop system need to be relocated to the left-hand-side half of the complex plane with large negative real parts. Consequently, the required closed-loop are set as the ones presented in table 1. Only the first two pairs of the eigenvalues are assigned to new positions with large negative real parts, the rest two pairs of the eigenvalues remain unchanged with the ones of the open-loop system.

Table 1: Eigenvalues of the open-loop and the closed-loop system

Eigenvalues	1st pair	2nd pair	3rd pair	4th pair
Open-loop eigenvalues s	$0.005 \pm 8.946i$	$-0.066 \pm 12.134i$	$-0.527 \pm 16.823i$	$-0.212 \pm 19.732i$
Closed-loop eigenvalues λ	$-0.200 \pm 9.000i$	$-0.400 \pm 12.000i$	$-0.527 \pm 16.823i$	$-0.212 \pm 19.732i$

Based on equation (11), for the model displayed in figure (1), the displacement and velocity proportional control gains $\mathbf{g}, \mathbf{f}$ to assign the required eigenvalues satisfies:

$$1 + (s\mu c_{\mathrm{c}} + \mu k_{\mathrm{c}})h_{43}(s) + \mathbf{t}(s)^{\mathrm{T}}(\mathbf{g} + s\mathbf{f}) = 0 \tag{30}$$

in which $\mathbf{t}(s) = \mathbf{H}_{\mathrm{s}}(s)\mathbf{b} + (s\mu c_{\mathrm{c}} + \mu k_{\mathrm{c}})[h_{43}(s)\mathbf{H}_{\mathrm{s}}(s)\mathbf{b} - \mathbf{e}_4^{\mathrm{T}}\mathbf{H}_{\mathrm{s}}(s)\mathbf{b}\mathbf{H}_{\mathrm{s}}(s)\mathbf{e}_3]$. $\mathbf{e}_3 = \{0, 0, 1, 0\}^{\mathrm{T}}$ and $\mathbf{e}_4 = \{0, 0, 0, 1\}^{\mathrm{T}}$ are the vectors indicating the friction-affected degree-of-freedom.

Consequently, the control gains can be arrived using the following equation:

$$\mathbf{G}_1 \begin{pmatrix} \mathbf{g} \\ \mathbf{f} \end{pmatrix} = -\mathbf{d}_1 \tag{31}$$

where the jth $(j = 1, \ldots, 4)$ row of $\mathbf{G}_1$ and $\mathbf{d}_1$ are:

$$\mathrm{G}_{1j} = [\mathbf{t}_j^{\mathrm{T}} \quad s_j \mathbf{t}_j^{\mathrm{T}}]|_{s_j=\lambda_j} \tag{32}$$

$$\mathrm{d}_{1j} = 1 + (s_j c_{\mathrm{c}} + \mu k_{\mathrm{c}}) \mathrm{H}_{\mathrm{s}}(4,3)|_{s_j=\lambda_j} \tag{33}$$

Additionally, λ_j is the jth required eigenvalue. Besides, (λ_1, λ_2) and (λ_3, λ_4) are the first two pairs of eigenvalues to be assigned.

Furthermore, according to equation (29), the constraint is denoted as:

$$\mathbf{G}_2 \begin{pmatrix} \mathbf{g} \\ \mathbf{f} \end{pmatrix} = \mathbf{0} \tag{34}$$

where the ith $(i = 1, \ldots, 4)$ row of $\mathbf{G}_2$ is given in:

$$\mathrm{G}_{2i} = [\mathbf{u}_j^{\mathrm{T}} \quad s_j \mathbf{u}_j^{\mathrm{T}}]|_{s_j=\lambda_j} \quad j = i + 4 \tag{35}$$

where $\mathbf{u}_j$ is the jth right eigenvector of the open-loop system. (λ_5, λ_6) and (λ_7, λ_8) are the two pairs of the eigenvalues which are unchanged from the ones of the open-loop system.

As a consequence, the constrained linear least-squares problem is defined as:

$$\begin{aligned} &\min_{\mathbf{g},\ \mathbf{f}} \quad \frac{1}{2} ||\mathbf{G}_1 \{\mathbf{g},\ \mathbf{f}\}^{\mathrm{T}} - \mathbf{d}_1||_2^2 \\ &\text{s.t.} \quad \mathbf{G}_2 \{\mathbf{g},\ \mathbf{f}\}^{\mathrm{T}} = \mathbf{0} \end{aligned} \tag{36}$$

The resultant control gains are $\mathbf{g} = \{2.0731, \ -7.2787, \ 1.9299, \ -3.1739\}^{\mathrm{T}}$ and $\mathbf{f} = \{-0.0163, \ 1.6155, \ 0.2994, \ 0.7796\}^{\mathrm{T}}$. Thus, for the closed-loop system, the eigenvalue problem is designated as:

$$\det(\lambda \mathbf{A}_{\mathrm{c}} + \mathbf{B}_{\mathrm{c}}) = 0 \tag{37}$$

where $\mathbf{A}_{\mathrm{c}}$ and $\mathbf{B}_{\mathrm{c}}$ are the system matrices of the first-order closed-loop system with state feedback control gains. The details of $\mathbf{A}_{\mathrm{c}}$ and $\mathbf{B}_{\mathrm{c}}$ are given:

$$\mathbf{A}_{\mathrm{c}} = \begin{bmatrix} \mathbf{O} & \mathbf{M} \\ \mathbf{M} & \mathbf{C} + \mathbf{b}\mathbf{f}^{\mathrm{T}} \end{bmatrix}, \ \mathbf{B}_{\mathrm{c}} = \begin{bmatrix} -\mathbf{M} & \mathbf{O} \\ \mathbf{O} & \mathbf{K} + \mathbf{b}\mathbf{g}^{\mathrm{T}} \end{bmatrix}$$

Using this control gains $\mathbf{g}$ and $\mathbf{f}$ the closed-loop eigenvalues can be calculated based on equation (37). The resultant closed-loop eigenvalues are exactly the ones listed in table 1.

5. Conclusion

In this study, the partial pole assignment method is developed for the asymmetric system using unobservability condition. The eigenvalue placement of the asymmetric system is concentrated on placing the unstable eigenvalues or the ones closed to the imaginary axis of the complex plane. The receptance of the corresponding symmetric system is adopted to obtain the state feedback control gains. The receptance method requires no knowledge of the mass, damping and stiffness matrices. The constraint conditions are derived from the unobservability condition. Therefore, the control gains can be achieved by solving the constrained linear least-squares problem. Using the obtained control gains only the first two pairs of the open-loop eigenvalues are reassigned to the stable region with the rest eigenvalues remain unchanged. This numerical simulation result demonstrates the effectiveness of the proposed method.

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