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Deriving The Standard Model From Superstring Theory*

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ABSTRACT

I outline a program to derive the Standard Model directly from superstring theory. I present a class of three generation superstring standard–like models in the free fermionic formulation. I discuss some phenomenological properties of these models. In particular these models suggest an explanation for the top quark mass hierarchy. A numerical estimate yielded $m_t \sim 175 - 180\ GeV$. The general texture of fermion mass matrices was obtained from analysis of nonrenormalizable terms up to order $N = 8$. I argue that the realistic features of these models are due to the underlying $Z_2 \times Z_2$ orbifold, with standard embedding, at the free fermionic point in toroidal compactification space.

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Superstring theory is the only known framework for the consistent unification of quantum gravity with the gauge interactions. The idea of gauge unification, and its big desert scenario, is supported by calculation of $\sin^2\theta_W$, m_b/m_τ and the recent observation of only three light left–handed neutrinos at LEP. Furthermore, LEP precision data, in general, support the validity of supersymmetric unified theories while they severely constrain theories that perturb the Standard Model very strongly, like Technicolor or composite models[1)]. Ultimately, the nature of the electroweak symmetry breaking sector will be decided by future accelerators.

Despite the success of the Standard Model and the qualitative support for unified gauge theories, point field theories are in general incomplete. First, there are still too many parameters and the choice of unifying group is arbitrary. Second, how does nature choose to have three generations, and the mass hierarchy among them. In particular the top–bottom mass hierarchy. Finally, gravity and point quantum field theories are incompatible. In the context of unified theories the solution to these problems must come from a more fundamental Planck scale theory. Furthermore, suppose that the Higgs sector of the Standard Model is strongly interacting or that the quark and leptons are composite. In this case we would have one point quantum field theory replacing the current one and we would continue our long march to the Planck scale, where gravitational interactions cannot be neglected. Eventually we will be faced with the need to reconcile between gravity and quantum mechanics. String theory does that in a very mild and natural way. The fundamental conceptual modification in string theory might be embodied in the modification of the uncertainty principle[2)]. In string theory the uncertainty principle receives an additional term that depends on the energy scale over M_{Pl}. Thus, as long as we are confined to low energies the additional term is small and point quantum field theories are a good approximation. However, when we get near the Planck the additional term gets large and point particle approximation breaks down.

Superstring theory provides us with a unique framework to study how the Planck scale may determine the parameters of the Standard Model. Keeping in mind the experimental observations that support the validity of the Standard Model up to a very large scale, it makes sense to ask whether it is possible to connect between superstring theory and the highly successful Standard Model. The aim of the work that I describe in this talk is to achieve precisely that. Using the free fermionic model building rules[3)] we ask whether it is possible to construct a "realistic" superstring standard–like model? A realistic model must posses the following properties. First, the gauge group is $SU(3) \times SU(2) \times U(1)^n \times hidden$. Second, the massless spectrum must have three chiral generations and a pair of Higgs doublets that can produce a realistic fermion mass spectrum. Finally, we require $N = 1$ space–time supersymmetry. This last requirement ensures vanishing of the cosmological constant. However, nonsupersymmetric models, and nonsupersymmetric models in which some electroweak doublets transform under a non–Abelian hidden gauge group, can be constructed. In these models, in general, the cosmological constant does not vanish.

Besides being formulated in four space–time dimensions, what is special about the free fermionic formulation? The heterotic string in ten space–time dimensions is more or less unique. However, when we compactify to four dimensions this uniqueness is lost. The free fermionic formulation is formulated at a highly symmetric point in the compactification space. It is an exact Conformal Field Theory and we can use the CFT calculational tools to calculate Yukawa couplings, etc. Finally, in free fermionic models one can naturally obtain three generations with standard $SO(10)$ embedding.

Several properties are required from a heterotic string theory[4)]: conformal invariance, modular invariance and world–sheet supersymmetry. In the free fermionic formulation of the heterotic string all the degrees of freedom that are needed to cancel the conformal anomaly are represented in terms of free fermions propagating on the string world–sheet. Under parallel transport around a noncontractible loop the fermionic states pick up a phase. A model in this construction is defined by a set of basis vectors of boundary conditions for all world–sheet fermions. The basis vectors are constrained by modular invariance and world–sheet supersymmetry and span a finite additive group Ξ. For every vector in the additive group correspond a sector in the string Hilbert space. The physical spectrum is obtained by applying the generalized GSO projections. The low energy effective field theory is obtained by S–matrix elements between external states. The Yukawa couplings and higher order terms in the superpotential are obtained by calculating correlators between vertex operators[5)]. For a correlator to be nonvanishing all the symmetries of the model must be conserved. Thus, the boundary condition vectors completely determine the phenomenology of the models.

The first five vectors in the basis that generate the standard–like models consist of the NAHE* set, $\{\mathbf{1}, S, b_1, b_2, b_3\}$, and are common to all the realistic free fermionic models. The gauge group after the NAHE set is $SO(10) \times SO(6)^3 \times E_8$ with $N = 1$ space–time supersymmetry, and 48 spinorial 16 of $SO(10)$, sixteen from each sector b_1, b_2 and b_3. The NAHE set divides the internal world–sheet fermions in the following way: $\bar{\phi}^{1,\cdots,8}$ generate the hidden E_8 gauge group, $\bar{\psi}^{1,\cdots,5}$ generate the $SO(10)$ gauge group, and $\{\bar{y}^{3,\cdots,6}, \bar{\eta}^1\}$, $\{\bar{y}^1, \bar{y}^2, \bar{\omega}^5, \bar{\omega}^6, \bar{\eta}^2\}$, $\{\bar{\omega}^{1,\cdots,4}, \bar{\eta}^3\}$ generate the three horizontal $SO(6)^3$ symmetries. The left–moving $\{y, \omega\}$ states are divided to $\{y^{3,\cdots,6}\}$, $\{y^1, y^2, \omega^5, \omega^6\}$, $\{\omega^{1,\cdots,4}\}$ and χ^{12}, χ^{34}, χ^{56} generate the left–moving $N = 2$ world–sheet supersymmetry.

The internal fermionic states $\{y, \omega | \bar{y}, \bar{\omega}\}$ correspond to the six left–moving and six right–moving compactified dimensions in a geometric formulation. This correspondence is illustrated by adding the vector with periodic boundary conditions for the set $\{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^{1,2,3}\}$ to the NAHE set[7)]. This extends the gauge group to $E_6 \times U(1)^2 \times E_8 \times SO(4)^3$ with $N = 1$ space–time supersymmetry and twenty four chiral 27 of E_6. The same model is generated in the orbifold language[8)] by moding out an $SO(12)$ lattice by a $Z_2 \times Z_2$ discrete symmetry with standard embedding. The $SO(12)$ lattice is obtained for special values of the metric and antisymmetric tensor and at the self dual point in compactification space. The metric is the Cartan matrix of $SO(12)$ and the antisymmetric tensor is given by $b_{ij} = g_{ij}$ for $i > j$.

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* This set was first constructed[6)] by Nanopoulos, Antoniadis, Hagelin and Ellis (NAHE) in the construction of the flipped $SU(5)$. *nahe*=pretty, in Hebrew.

The sectors b_1, b_2 and b_3 correspond to the three twisted sectors in the orbifold models and the Neveu–Schwarz sector corresponds to the untwisted sector. In the construction of the standard–like models beyond the NAHE set, the assignment of boundary conditions to the set of internal fermions $\{y,\omega|\bar{y},\bar{\omega}\}$ determines many of the properties of the low energy spectrum, such as the number of generations, the presence of Higgs doublets, Yukawa couplings, etc.

The standard–like models are constructed by adding three additional vectors to the NAHE set[9,10,11,12)]. One example is presented in the table, where only the boundary conditions of the "compactified space" are shown. In the gauge sector $\alpha,\beta\{\bar{\psi}^{1,\cdots,5},\bar{\eta}^{1,2,3},\bar{\phi}^{1,\cdots,8}\} = \{1^3,0^5,1^4,0^4\}$ and $\gamma\{\bar{\psi}^{1,\cdots,5},\bar{\eta}^{1,2,3},\bar{\phi}^{1,\cdots,8}\} = \{\frac{1}{2}^9,0,1^2,\frac{1}{2}^3,0\}$ break the symmetry to $SU(3)\times SU(2)\times U(1)_{B-L}\times U(1)_{T_{3_R}}\times SU(5)_h\times SU(3)_h\times U(1)^2$. Three additional vectors are needed to reduce the number of generations to one generation from each sector b_1, b_2 and b_3. Each generation has horizontal symmetries that constrain the allowed interactions. Each generation has two gauged $U(1)$ symmetries, $U(1)_{R_j}$ and $U(1)_{R_{j+3}}$. For every right–moving $U(1)$ symmetry there is a corresponding left–moving global $U(1)$ symmetry, $U(1)_{L_j}$ and $U(1)_{L_{j+3}}$. Finally, each generation has two Ising model operators that are obtained by pairing a left–moving real fermion with a right–moving real fermion.

Table 1. A three generations $SU(3)\times SU(2)\times U(1)^2$ model[11)].

	y^3y^6, $y^4\bar{y}^4$, $y^5\bar{y}^5$, $\bar{y}^3\bar{y}^6$	$y^1\omega^6$, $y^2\bar{y}^2$, $\omega^5\bar{\omega}^5$, $\bar{y}^1\bar{\omega}^6$	$\omega^1\omega^3$, $\omega^2\bar{\omega}^2$, $\omega^4\bar{\omega}^4$, $\bar{\omega}^1\bar{\omega}^3$
α	1, 1, 1, 0	1, 1, 1, 0	1, 1, 1, 0
β	0, 1, 0, 1	0, 1, 0, 1	1, 0, 0, 0
γ	0, 0, 1, 1	1, 0, 0, 0	0, 1, 0, 1

Higgs doublets in the standard–like models are obtained from two distinct sectors. The first type are obtained from the Neveu–Schwarz sector, which produces three pairs of electroweak doublets. Each pair can couple at tree level only to the states from the sector b_j. There is a stringy doublet–triplet splitting mechanism that projects out the color triplets and leaves the electroweak doublets in the spectrum. Thus, the superstring standard–like models resolve the GUT hierarchy problem. The second type of Higgs doublets are obtained from the vector combination $b_1 + b_2 + \alpha + \beta$. The states in this sector are obtained by acting on the vacuum with a single fermionic oscillator and transform only under the observable sector.

The cubic level Yukawa couplings for the quarks and leptons are determined by the boundary conditions in the vector γ according to the following rule[12)]

$$\Delta_j = |\gamma(U(1)_{\ell_{j+3}}) - \gamma(U(1)_{r_{j+3}})| = 0,1 \qquad (j=1,2,3) \tag{3a}$$

$$\Delta_j = 0 \rightarrow d_j Q_j h_j + e_j L_j h_j; \;\; \Delta_j = 1 \rightarrow u_j Q_j \bar{h}_j + N_j L_j \bar{h}_j, \tag{3b,c}$$

where $\gamma(U(1)_{R_{j+3}})$, $\gamma(U(1)_{\ell_{j+3}})$ are the boundary conditions of the world–sheet fermionic currents that generate the $U(1)_{R_{j+3}}$, $U(1)_{\ell_{j+3}}$ symmetries.

The superstring standard–like models contain an anomalous $U(1)$ gauge symmetry. The anomalous $U(1)$ generates a Fayet–Iliopoulos term by the VEV of the dilaton field that breaks supersymmetry and destabilizes the vacuum[13)]. Supersymmetry is restored by giving VEVs to standard model singlets in the massless spectrum of the superstring models. However, as the charge of these singlets must have $Q_A < 0$ to cancel the anomalous $U(1)$ D–term equation, in many models a phenomenologically realistic solution does not exist. In fact a very restricted class of standard–like models with $\Delta_j = 1$ for $j = 1,2,3$, were found to admit a solution to the F and D flatness constraints. Consequently, the only models that were found to admit a solution are models which have tree level Yukawa couplings only for $+\frac{2}{3}$ charged quarks.

This result suggests an explanation for the top quark mass hierarchy relative to the lighter quarks and leptons. At the cubic level only the top quark gets a mass term and the mass terms for the lighter quarks and leptons are obtained from nonrenormalizable terms. To study this scenario we have to examine the nonrenormalizable contributions to the doublet Higgs mass matrix and to the fermion mass matrices[14,15)].

At the cubic level there are two pairs of electroweak doublets. At the non-renormalizable level one additional pair receives a superheavy mass and one pair remains light to give masses to the fermions at the electroweak scale. Requiring F–flatness imposes that the light Higgs representations are $\bar{h}_1$ or $\bar{h}_2$ and h_{45}.

The nonrenormalizable fermion mass terms of order N are of the form $cgf_if_jh\phi^{N-3}$ or $cgf_if_j\bar{h}\phi^{N-3}$, where c is a calculable coefficient, g is the gauge coupling at the unification scale, f_i, f_j are the fermions from the sectors b_1, b_2 and b_3, h and $\bar{h}$ are the light Higgs doublets, and ϕ^{N-3} is a string of standard model singlets that get a VEV and produce a suppression factor $(\langle\phi\rangle/M)^{N-3}$ relative to the cubic level terms. Several scales contribute to the generalized VEVs. The leading one is the scale of VEVs that are used to cancel the anomalous D–term equation. The next scale is generated by Hidden sector condensates. Finally, there is a scale which is related to the breaking of $U(1)_{Z'}$, $\Lambda_{Z'}$. Examination of the higher order nonrenormalizable terms reveals that $\Lambda_{Z'}$ has to be suppressed relative to the other two scales.

At the cubic level only the top quark gets a nonvanishing mass term. Therefore only the top quark mass is characterized by the electroweak scale. The remaining quarks and leptons obtain their mass terms from nonrenormalizable terms. The cubic and nonrenormalizable terms in the superpotential are obtained by calculating correlators between the vertex operators. The top quark Yukawa coupling is generically given by

$$g\sqrt{2} \tag{1}$$

where g is the gauge coupling at the unification scale. In the model of Ref. [11], bottom quark and tau lepton mass terms are obtained at the quartic order,

$$W_4 = \{d^c_{L_1}Q_1h'_{45}\Phi_1 + e^c_{L_1}L_1h'_{45}\Phi_1 + d^c_{L_2}Q_2h'_{45}\bar{\Phi}_2 + e^c_{L_2}L_2h'_{45}\bar{\Phi}_2\}. \tag{2}$$

The VEVs of Φ are obtained from the cancelation of the anomalous D–term equation. The coefficient of the quartic order mass terms were calculated by calculating the quartic order correlators and the one dimensional integral was evaluated numerically. Thus after inserting the VEV of $\bar{\Phi}_2$ the effective bottom quark and tau lepton Yukawa couplings are given by[11),

$$\lambda_b = \lambda_\tau = 0.35 g^3. \tag{3}$$

They are suppressed relative to the top Yukawa by

$$\frac{\lambda_b}{\lambda_t} = \frac{0.35 g^3}{g\sqrt{2}} \sim \frac{1}{8}. \tag{4}$$

To evaluate the top quark mass, the three Yukawa couplings are run to the low energy scale by using the MSSM RGEs. The bottom mass is then used to calculate $\tan\beta$ and the top quark mass is found to be[11),

$$m_t \sim 175 - 180 GeV. \tag{5}$$

The fact that the top Yukawa is found near a fixed point suggests that this is in fact a good prediction of the superstring standard–like models. By varying $\lambda_t \sim 0.5 - 1.5$ at the unification scale, it is found that λ_t is always $O(1)$ at the electroweak scale.

An analysis of fermion mass terms up to order $N = 8$ revealed the general texture of fermion mass matrices in these models. The sectors b_1 and b_2 produce the two heavy generations and the sector b_3 produces the lightest generation. This is due to the horizontal $U(1)$ charges and because the Higgs pair h_3 and $\bar{h}_3$ necessarily get a Planck scale mass[14). The mixing between the generations is obtained from exchange of states from the sectors $b_j + 2\gamma$. The general texture of the fermion mass matrices in the superstring standard–like models is of the following form,

$$M_U \sim \begin{pmatrix} \epsilon, a, b \\ \tilde{a}, A, c \\ \tilde{b}, \tilde{c}, \lambda_t \end{pmatrix} ; \; M_D \sim \begin{pmatrix} \epsilon, d, e \\ \tilde{d}, B, f \\ \tilde{e}, \tilde{f}, C \end{pmatrix} ; \; M_E \sim \begin{pmatrix} \epsilon, g, h \\ \tilde{g}, D, i \\ \tilde{h}, \tilde{i}, E \end{pmatrix},$$

where $\epsilon \sim (\Lambda_{Z'}/M)^2$. The diagonal terms in capital letters represent leading terms that are suppressed by singlet VEVs, and $\lambda_t = O(1)$. The mixing terms are generated by hidden sector states from the sectors $b_j + 2\gamma$ and are represented by small letters. They are proportional to $(\langle TT \rangle / M^2)$. In Ref. [15] it was shown that if the states from the sectors $b_j + 2\gamma$ obtain VEVs in the application of the DSW mechanism, then a Cabibbo angle of the correct order of magnitude can be obtained in the superstring standard–like models. In later work the analysis was extended to show that reasonable values for the entire CKM matrix parameters can be obtained for appropriate flat F and D solutions. Texture zeroes in the fermion mass matrices are obtained if the VEVs of some states from the sectors $b_j + 2\gamma$ vanish. These texture zeroes are protected by the symmetries of the string models to all order of nonrenormalizable terms[15)].

Next, I turn to the problem of gauge coupling unification in the superstring standard–like models[16)]. While LEP results indicate that the gauge coupling in the minimal supersymmetric standard model unify at $10^{16} GeV$, superstring theory predicts that the unification scale is at $10^{18} GeV$. The superstring standard–like models may resolve this problem due to the existence of color triplets and electroweak doublets from exotic sectors that arise from the additional vectors α, β and γ. These exotic states carry fractional charges and do not fit into standard $SO(10)$ representations. Therefore, they contribute less to the evolution of the $U(1)_Y$ beta function than standard $SO(10)$ multiplets. The standard–like models predict $\sin^2\theta_W = 3/8$ at the unification scale due to the embedding of the weak hypercharge in $SO(10)$. In Ref. [16], I showed that provided that the additional exotic color triplets and electroweak doublets exist at the appropriate scales, the scale of gauge coupling unification is pushed to $10^{18} GeV$, with the correct value of $\sin^2\theta_W$ at low energies.

Next, I comment on the problem of supersymmetry breaking. In Ref. [17] we address the following question: Given a supersymmetric string vacuum at the Planck scale, is it possible to obtain hierarchical supersymmetry breaking in the observable sector? A supersymmetric string vacuum is obtained by finding solutions to the cubic level F and D constraints. We take a gauge coupling in agreement with gauge coupling unification, thus taking a fixed value for the dilaton VEV.

We then investigate the role of nonrenormalizable terms and strong hidden sector dynamics. The hidden sector contains two non–Abelian hidden gauge groups, $SU(5) \times SU(3)$, with matter in vector–like representations. The hidden $SU(3)$ group is broken near the Planck scale. We analyze the dynamics of the hidden $SU(5)$ group. The hidden $SU(5)$ matter mass matrix is nonsingular for specific F and D flat solutions. We find that, in some scenarios, the matter and gaugino condensates can brake supersymmetry in a hierarchically small scale.

To conclude, the superstring standard–like models contain in their massless spectrum all the necessary states to obtain realistic phenomenology. They resolve the problems of proton decay through dimension four and five operators that are endemic to other superstring and GUT models. The existence of only three generations with standard $SO(10)$ embedding is understood to arise naturally from $Z_2 \times Z_2$ twisting at the free fermionic point in compactification space. Better understanding of the correspondence with other superstring formulations will provide further insight into the realistic properties of these models. Finally, the free fermionic standard-like models provide a highly constrained and phenomenologically realistic laboratory to study how the Planck scale may determine the parameters of the Standard Model.

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