

**Mechanism Sufficient Statistic  
in the Risk-Neutral Agency Problem**

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## **Abstract**

This paper analyses the efficient use of information in an agency relationship with moral hazard, when parties are risk-neutral. We show that, given an arbitrary information system, all relevant information from a mechanism-design point of view can be summarized by a binary statistic. We then show that this allows a complete ordering of information systems for the risk-neutral agency problem. These results are obtained under a weak convexity condition which does not rely on an exogenous ordering on signal sets. The condition is shown to be more general than existing requirements for justifying the first-order approach.

**KEYWORDS.** Moral hazard, principal-agent, limited liability, information systems.

**JEL classification:** D2, D8

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## 1. Introduction

This paper is concerned with the efficient use of information in the context of a moral hazard principal-agent framework when the participants are risk neutral and penalties are bounded.<sup>1</sup> Given an arbitrary information system, potentially with multidimensional observables, we examine how information should be aggregated optimally. The information system refers to the set of commonly observable variables correlated with the agent's hidden action and with respect to which the principal can design an incentive scheme.

Due to the risk-neutrality restriction we show that from a mechanism-design point of view all relevant information can be summarized by a binary statistic. Consequently, the principal can constraint mechanism without loss of generality to the set of bonus contracts. All the mechanism needs to specify are a fix payment, a bonus and the terms under which the agent will receive the bonus. We show that if the original signal satisfies the Monotone Likelihood Ratio Condition (hereafter MLRC, see MILGROM [1981]), the terms for the bonus are independent of the agent's action. We generalize our results by examining the case in which the sufficient binary statistics is state dependent. We conclude by showing that the

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<sup>1</sup>Otherwise the risk neutrality implies that the first-best solution would be possible with any information system, as is well known.

set up allows for a simple ranking criteria of information systems.

In the existing agency literature, there are two prevailing criteria for comparing information systems: HOLMSTRÖM's [1979] notion of informativeness and BLACKWELL's [1951, 1953] efficiency condition.<sup>2</sup> Although originally developed in a statistical or decision theoretic context, both criteria also proved useful to rank information systems in the agency framework. However, as emphasized by KIM [1995], there is an essential difference between a statistical or decision theoretic problem and an agency problem. In the first case the decision maker attempts to estimate some unobserved variable, while in the second the principal attempts to control an action. Based on this observation, Kim introduced a new criterion defined in terms of a mean-preserving spread (MPS) of the likelihood ratio distribution function.<sup>3</sup> He shows that for moral hazard situations with a risk-neutral principal the MPS criterion is less restrictive than the two aforementioned criteria and that it applies to a broader set of comparisons.

Based on Kim's work, we define the notion of *mechanism sufficiency* and distinguish it from the well known concept of statistical sufficiency. Heuristically a signal is said to be statistically sufficient for an other random variable if none

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<sup>2</sup>See also GJESDAL [1982] and GROSSMAN and HART [1983].

<sup>3</sup>This criterion is related to the Fisher measure of the quantity of information found in the statistical literature.

of the informational content is lost. However, in an agency problem the principal may not care about losing some statistically relevant information if it is not useful to correctly align the agent's incentive. As a result, we define a signal to be mechanism sufficient to an other random variable if none of the information relevant to the principal is lost. *The main purpose of this paper is to exemplify in a simple environment with risk neutral participants the distinction between these two concepts.* Though the risk neutrality assumption is strong, it is justified because it ensures that the difference between both notions of sufficiency is, in fact, maximal.<sup>4</sup>

From a more applied point of view, our results provide a strong theoretical justification for the widespread use of dichotomic monitoring schemes (of the “fail or pass” type) in the agency literature where the principal's monitoring effort is endogenized. Rather than assuming as a simplification that the signals observed by the principal are binary and that they satisfy the monotone likelihood ratio condition, we show that the efficient aggregation of information leads to a statistic with these kind of characteristics.<sup>5</sup> Our result is a generalization of PARK [1995], where it is shown in a similar model that if the first-best outcome can be attained

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<sup>4</sup>It is maximal in the sense that the informational content of any system can be contained in a binary statistic.

<sup>5</sup>For example BARON and BESANKO [1984], LAFFONT and TIROLE [1993] and MIRRLEES [1974].

the optimal contract is a bonus contract. One interpretation of our result is that bonus contract remain optimal even when first-best is not attainable.

The basic framework is described in section 2. In section 3 we introduce the notion of mechanism sufficiency. In section 4 our main result are derived under somewhat restrictive sufficient conditions. In section 5 we show that the same results can be obtained under much weaker conditions, without imposing any prior ordering requirements on signals. The implications for the ordering of information systems are drawn in section 6. Section 7 concludes.

## 2. The Model

The agent's action space is a real interval  $A = [\underline{a}, \bar{a}]$ . The action or effort level is observed by the agent only, but the principal obtains imperfect *ex post* information. The information system consists of a finite set  $S$  of possible observations with a family  $\{p(s, a), s \in S\}_{a \in A}$  of probability distributions. The signals  $s \in S$  may be multidimensional, with quantitative and (or) qualitative information. The characteristics of the information system are common knowledge. We assume the following:

ASSUMPTION (A1):  $p(s, a) > 0$  for every  $s \in S$  and every  $a \in A$ .

ASSUMPTION (A2): For every  $s \in S$ ,  $p(s, a)$  is twice continuously differentiable with respect to  $a$ .

ASSUMPTION (A3): For every  $a \in A$ ,  $p_a(s, a) \neq 0$  for some  $s \in S$ .

(A1) eliminates information systems with moving supports, which lead to trivial solutions. (A2) is a regularity condition. Finally (A3) essentially states that signals are informative with respect to any action that may be required from the agent.

The agent is risk-neutral with utility  $t - C(a)$ , where  $t$  is the transfer from the principal to the agent and  $C(a)$  is the agent's cost of effort function, with  $C'(a) > 0$  and  $C''(a) \geq 0$ . The transfer to the agent is constrained by the limited liability condition  $t \geq -L$ , where the liability limit  $L \geq 0$  represents the maximum penalty than can be imposed on the agent. The agent's reservation utility is normalized to zero.

The *ex post* signal generated by the information system is observable by both parties. A contract or mechanism is a schedule  $t(s)$  specifying the transfer from the principal to the agent for each possible signal. The mechanism  $t(s)$  implements the action  $\hat{a}$  if



$$\hat{a} \in \arg \max_a \sum_{s \in S} p(s, a)t(s) - C(a), \quad (2.1)$$

$$\sum_{s \in S} p(s, \hat{a})t(s) - C(\hat{a}) \geq 0, \quad (2.2)$$

$$t(s) \geq -L, \text{ for every } s \in S. \quad (2.3)$$

That is, the mechanism provides the correct incentives for the action considered by the principal and it satisfies the agent's participation and limited liability constraints. For a required action  $\hat{a}$ , the principal's problem is to design the implementing mechanism so as to minimize the expected transfer

$$\sum_{s \in S} p(s, \hat{a})t(s). \quad (2.4)$$

### 3. Mechanism Sufficient Statistics

The signals generated by the information system can be aggregated. A real-valued statistic  $Y$  is a mapping from the set of signals  $S$  to the set of real numbers and may be interpreted as an aggregator of information. We write  $Y(S)$  for the image

of  $Y$  and  $y \in Y(S)$  for a possible realization of the variable. In what follows, different properties are defined with the qualification “for  $a \in B$ ”, where  $B$  is a subinterval of the action space  $A$ . When the qualification is omitted, this is understood to mean that the property holds with respect to the whole action space. A basic distinction is that between *statistical sufficiency* and *mechanism sufficiency*.

DEFINITION (Statistical Sufficiency): *The statistic  $Y$  is sufficient for  $a \in B$  if the conditional distribution of  $s \in S$  given  $Y$  is constant with respect to  $a \in B$ .*

This definition differs slightly from the one usually found in the agency literature. Its main advantage is that it does not rely on the agent’s action being interpreted as a random variable (see, for example, Holmström [1979]), which would not be appropriate in the present context.<sup>6</sup> Let  $\pi(s, y, a)$  denote the conditional probability of  $s$  given a realization  $y$  of some statistic  $Y$ . Then

$$\pi(s, y, a) = \begin{cases} 0 & \text{if } Y(s) \neq y, \\ \frac{p(s,a)}{\sum_{s:Y(s)=y} p(s,a)} & \text{if } Y(s) = y. \end{cases} \quad (3.1)$$

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<sup>6</sup>On the equivalence between this definition (due to Neyman) and the Bayesian definition, see for instance GOURIEROUX and MONTFORT [1995]. Note that assumption (A3) ensures that a sufficient statistics cannot be a degenerate random variable.

The statistic is sufficient for  $a \in B$  if the expression for the conditional probability, when  $Y(s) = y$ , is constant on  $B$ . This leads directly to the following characterization:

STATISTICAL SUFFICIENCY CRITERIA: *Let  $Y$  be a statistic with probability distribution  $g(y, a) \equiv \sum_{s:Y(s)=y} p(s, a)$ . The following statements are equivalent:*

- (i) *The statistic  $Y$  is sufficient for  $a \in B$ .*
- (ii) *If  $Y(s) = Y(s')$ ,  $\frac{p(s', a)}{p(s, a)}$  is constant on  $B$ .*
- (iii) *For every  $a \in B$ ,  $\frac{p_a(s, a)}{p(s, a)} = \frac{g_a(y, a)}{g(y, a)}$  if  $Y(s) = y$ .*

It is a well known result that a sufficient statistic includes all the relevant information for the principal's problem:

PROPOSITION 1 (HOLMSTRÖM [1979])<sup>7</sup>: *Let  $t(s)$  be a mechanism implementing  $\hat{a}$ . If  $Y$  is a sufficient statistic, there exists another mechanism  $\tau(s)$  that implements  $\hat{a}$  at the same expected cost for the principal and that satisfies  $\tau(s) = \varphi(Y(s))$ .*

In section 4 we derive the optimal mechanism implementing some required action. Considered as a statistic, a mechanism  $t(s)$  aggregates information. If a

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<sup>7</sup>The above definition of statistical sufficiency allows for a more general and simpler proof than in HOLMSTRÖM's [1979] article. In particular, the demonstration does not need to rely on the validity of the first-order approach.

mechanism were optimal only if it is a sufficient statistic, one would conclude that the principal must use all the statistically relevant information provided by the information system. By contrast, if the best mechanism is not a sufficient statistic, some of the statistically relevant information is redundant from a mechanism-design point of view. This observation suggests the following distinction:

DEFINITION (Mechanism Sufficiency): *A statistic  $X$  is mechanism sufficient for  $a \in B$  if the cost-minimizing mechanism implementing any  $a \in B$  can be written as  $t(s) = \varphi(X(s))$ .*

Though KIM [1995] did not explicitly formulate the terminology of a mechanism sufficient statistic, the concept follows easily from his work. The current formulation clarifies the distinction between the statistical definition of sufficiency and the mechanism design concept. The difference can arise because, from the principal’s perspective, losing statistically relevant information does not matter if it is not useful to align the agent’s incentive.

#### **4. Efficient Aggregation of Information**

The derivation of the optimal mechanism rests on the validity of the “first-order approach”; by this is meant the possibility of replacing the incentive compatibility

constraint with the first-order condition of the agent's optimization problem. It is well known that MLRC and CDFC are sufficient for the first-order approach to be valid.<sup>8</sup> We first impose these conditions and then show that similar results can be obtained under less restrictive conditions. For completeness, we recall some standard definitions where  $Y$  denotes a statistic with probability distribution  $g(y, a) \equiv \sum_{s:Y(s)=y} p(s, a)$ .

DEFINITION (MLRC): *The statistic  $Y$  is said to satisfy strict MLRC for  $a \in B$  if  $\frac{g_a(y, a)}{g(y, a)}$  is non-decreasing for all  $a \in B$  and strictly increasing for some  $a \in B$ .*

DEFINITION (CDFC): *The statistic  $Y$  satisfies CDFC if its cumulative probability distribution  $\sum_{y \leq y'} g(y, a)$  is convex in  $a$  for every  $y' \in Y(S)$ .*

The next proposition describes the least-cost mechanism for implementing an action  $a > \underline{a}$ .<sup>9</sup>

PROPOSITION 2: *Assume there exists a sufficient statistic  $Y$  satisfying the*

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<sup>8</sup>See ROGERSON [1985].

<sup>9</sup>To implement  $\underline{a}$  there is no need to provide incentives to the agent and  $t(s) = C(\underline{a})$  for every  $s \in S$  is optimal.

strict MLRC and CDFC. Let  $y^M = \max\{y \mid y \in Y(S)\}$ . Then

$$X(s) = \begin{cases} 1 & \text{if } Y(s) = y^M \\ 0 & \text{if } Y(s) \neq y^M \end{cases} \quad (4.1)$$

is a mechanism sufficient statistic for  $a \in A$ . Letting  $h(a) \equiv \sum_{s: X(s)=1} p(s, a)$ , the optimal mechanism implementing the action  $a \in (\underline{a}, \bar{a}]$  has the form

$$t(s) = \begin{cases} T + C'(a)/h'(a) & \text{if } X(s) = 1 \\ T & \text{if } X(s) = 0 \end{cases} \quad (4.2)$$

where

$$T = \max \left\{ -L, C(a) - \frac{C'(a)}{h'(a)/h(a)} \right\}, \quad (4.3)$$

and the expected cost to the principal is

$$C_P(a) = \max \left\{ C(a), \frac{C'(a)}{h'(a)/h(a)} - L \right\}. \quad (4.4)$$

PROOF: See the appendix.

The optimal mechanism is dichotomic. As in PARK (1995), it is a bonus

contract of the “pass or fail” type. It can be described by a non-contingent transfer  $T$  and a bonus equal to  $C'(a)/h'(a)$ , paid to the agent only if  $X = 1$  or equivalently  $Y = y^M$  is observed.<sup>10</sup> The value of the non-contingent transfer  $T$  depends on whether it is the limited liability or the participation constraint that is binding. If the limited liability constraint is binding, then  $T = -L$  and the cost to the principal is  $C_P(a) > C(a)$ , which means that the agent earns a rent. If it is not binding,  $T$  is set so as to satisfy the participation constraint.

Due to MLRC, the partition of the set of signals on which transfers are conditioned does not depend on the action that is to be implemented (nor does it depend on the agent’s liability limit or on his cost of effort function). Most of the information provided by the information system is irrelevant from the principal’s point of view, in the sense that the least cost mechanism would be feasible even if the principal were only able to observe the binary variable  $X$ . Due to the risk-neutrality assumption the result is extreme, but it demonstrates the importance to distinguish between statistical and mechanism sufficiency. In the remaining of the paper, we generalize the result and examine some of its implication for the ranking of information systems.

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<sup>10</sup>Obviously, the optimal scheme could also have been described by a non-contingent transfer and a penalty if  $X = 0$  is observed.

A straightforward generalization consists in showing that MLRC is not needed. In the following result, we show that the essence of a “pass or fail” mechanism is that the agent is penalized (at least in the sense of not getting the “bonus”) whenever a more favorable signal could have been observed<sup>11</sup>.

PROPOSITION 3: *Let  $Y$  be a sufficient statistic and assume there exists a realization  $y^M$  that is more favorable than any  $y \neq y^M$ . Define the statistic*

$$X(s) = \begin{cases} 1 & \text{if } Y(s) = y^M \\ 0 & \text{if } Y(s) \neq y^M \end{cases} \quad (4.5)$$

*If  $X$  satisfies CDFC, it is mechanism-sufficient for  $a \in A$  and the optimal mechanism is as in proposition 2.*

PROOF: The result follows directly from the proof of proposition 2 by using the definition of a more favorable signal and equation (7.14). ■

MLRC and CDFC with respect to a sufficient statistic, as in proposition 2, imply the conditions in proposition 3. That is, they imply the existence of a realization that is more favorable than any other and they also imply that the

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<sup>11</sup>Following MILGROM [1981], we say  $y$  is more favorable than  $y'$  for  $a \in B$  if  $\frac{g_a(y,a)}{g(y,a)} \geq \frac{g_a(y',a)}{g(y',a)}$ , for all  $a \in B$  with a strict inequality for some  $a \in B$ .



binary statistic defined in (4.5) satisfies CDFC. The conditions of proposition 3 are of course much weaker, as an example at the end of the section will show. In either case the mechanism-sufficient statistic satisfies MLRC. Observe also that under the conditions of proposition 3 any  $a \in A$  can be implemented.

In the next section, we show that similar results can be obtained under even weaker conditions, but with different mechanism-sufficient statistics depending on the action to be implemented.

#### AN EXAMPLE

Consider a situation where a signal  $s \in S$  corresponds to the realization of  $n$  independently distributed Bernoulli variables. That is,  $s = (s_1, \dots, s_n)$  where  $s_i \in \{0, 1\}$  with probability  $\Pr[s_i = 1 | a] = q_i(a)$ . Letting  $q'_i(a) > 0$  and  $q''_i(a) < 0$ , the event  $s_i = 1$  can be interpreted as “pass” with respect to the  $i$ -th check in a test involving  $n$  checks, the event  $s_i = 0$  as “fail” with respect to the  $i$ -th check.

#### CASE 1

Suppose that the  $s_i$ 's are identically distributed so that  $q_i(a) = q(a)$  for  $i = 1, \dots, n$ . Define the statistic  $Y(s_1, \dots, s_n) = \sum_{i=1}^n s_i$ . Such a statistic counts the total number of “pass” marks and its possible realizations are  $\{0, 1, \dots, n\}$ , as compared to the  $2^n$  possible signals in  $S$ .  $Y$  has the binomial distribution

$\mathcal{B}(n, q(a))$  and is well known to be a sufficient statistic for the probability  $q(a)$ . Because the latter is monotonic in  $a$ ,  $Y$  is also sufficient for  $a$ . It is easily verified that  $Y$  satisfies the strict MLRC; if there is enough concavity in the function  $q(a)$ , it will also satisfy CDFC. Proposition 2 will then hold and the statistic

$$X(s) = \begin{cases} 1 & \text{if } s = (1, \dots, 1) \\ 0 & \text{if } s \neq (1, \dots, 1) \end{cases} \quad (4.6)$$

will be mechanism-sufficient for  $a \in A$ . Equivalently, the mechanism-sufficient statistic could have been defined by  $X = 1$  if  $Y = n$  and  $X = 0$  if  $Y < n$ .

## CASE 2

Suppose now that the  $s_i$ 's are not identically distributed, so that  $q_i(a)$  and  $q_j(a)$  are different functions for  $i \neq j$ . This implies that in general the image of a sufficient statistic will usually have the same cardinality as  $S$ . Furthermore, the information system does not satisfy MLRC in that two arbitrary signals in  $S$  are not necessarily comparable. However, there is a partial ranking between signals in the sense that  $s'$  is more favorable than  $s$  if  $s' \geq s$ ,  $s' \neq s$ . In particular,  $s = (1, \dots, 1)$  is more favorable than any other signal. If there is enough concavity in the functions  $q_i(a)$ , the conditions for proposition 3 will therefore hold and the statistic  $X$ , as defined in (4.6), will be mechanism-sufficient for  $a \in A$ .

In both cases the agent is penalized (or does not get his “bonus”) if he fails in at least one of the  $n$  checks and the sanction is the same irrespective of the number of “fails”. In other words, the benchmark for “pass” in the optimal dichotomic scheme is “pass” with respect to all of the  $n$  checks. This should not be interpreted as implying that the agent is punished more often than he is rewarded: if the  $q_i$ ’s are close to one, the probability of not receiving the bonus may be small. The point is not to penalize often but to penalize whenever a more favorable signal could have been observed, as emphasized in the previous section; in the example, this occurs whenever the agent gets a “fail” mark in any of the  $n$  checks.

## 5. A Generalization

So far the information system has been assumed to exhibit a “most favorable” signal (possibly aggregated) that serves as the “pass” benchmark in the optimal dichotomic mechanism, irrespective of the action to be implemented. We now relax the condition that there exists such a signal.

Let  $Y$  be a minimal sufficient statistic with probability distribution  $g(y, a)$ , where a statistic is said to be minimal sufficient if its image has the smallest

cardinality among all sufficient statistics. Define<sup>12</sup>

$$Y^M(S) = \left\{ y \in Y(S) \mid \text{for every } y' \neq y, \frac{g_a(y, a)}{g(y, a)} > \frac{g_a(y', a)}{g(y', a)} \text{ for some } a \in A \right\} \quad (5.1)$$

In some sense, the set  $Y^M(S)$  replaces the most favorable signal: it is formed by taking the realizations of the statistic that may constitute a “most favorable” signal at least *locally*, with respect to a subinterval of the action-space. The subset of elementary signals that generate such realizations is

$$S^M = \{s \in S \mid Y(s) \in Y^M(S)\} \quad (5.2)$$

Assumption (A3) ensures that  $Y^M(S)$  and therefore  $S^M$  are not empty. The following convexity assumption is introduced:

ASSUMPTION (A4): For every  $s \in S^M$  and  $a \in A$ ,  $p_{aa}(s, a) \leq 0$  if  $p_a(s, a) > 0$ .

We will use (A4) in lieu of MLRC and CDFC. In the standard model MLRC generates an exogenous ordering and CDFC imposes a convexity structure on this ordering. By contrast, we do not impose any ordering. In fact, it is the purpose

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<sup>12</sup>We restrict our attention to minimal sufficient statistics to ensure that  $Y^M(S) \neq \emptyset$ . This observation follows immediately from the definition since, in the case of a minimal sufficient statistic for any  $y_1, y_2 \in Y(S)$ ,  $\frac{g_a(y_1, a)}{g(y_1, a)} \neq \frac{g_a(y_2, a)}{g(y_2, a)}$  for some  $a \in A$ .

of this section to show that signals are endogenously ordered according to their usefulness to the principal. In particular, the ordering might depend on the action to be implemented. As a result, we have to rely on a local convexity hypothesis. The requirement in (A4) can be interpreted as a decreasing return hypothesis for favorable information.

To further motivate this assumption, we show that (A4) is implied by the conditions for proposition 3. Recall that the latter proposition holds under both an ordering and a convexity condition: for a sufficient statistic  $Y$ , it was assumed that there existed a “most favorable” realization  $y^M$  and that the probability distribution of the statistic satisfied  $g_{aa}(y^M, a) \leq 0$  for every  $a \in A$ . Because  $y^M$  is a globally most favorable signal, it is clear that  $s \in S^M$  if and only if<sup>13</sup>  $Y(s) = y^M$ . Also, because  $Y$  is a sufficient statistic, it is easily verified that  $g_{aa}(y^M, a) \leq 0$  implies  $p_{aa}(s, a) \leq 0$  for  $s \in S^M$ . Thus, the convexity condition of proposition 3 (as well as the more restrictive MLRC and CDFC conditions of proposition 2) imply the statement in the assumption.

We now show that assumptions (A1) to (A4) are sufficient for any action to be implementable and for the least-cost mechanism to be dichotomic, as in the

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<sup>13</sup>By restricting the requirement in (A4) to the set  $S^M$ , we keep the hypothesis to the smallest essential set. We also note that otherwise this equivalence would not hold and as a result MLRC and CDFC would not imply (A4).

previous sections:

PROPOSITION 4: *The statistic*

$$Z_a(s) = \begin{cases} 1 & \text{if } s \in \arg \max_{s \in S^M} \frac{p_a(s,a)}{p(s,a)}, \\ 0 & \text{otherwise.} \end{cases} \quad (5.3)$$

*is mechanism sufficient for action*  $a \in A$ .

PROOF: See the appendix.

The difference with respect to the previous results is that the “ordering” of signals for the purpose of determining the “pass” and “fail” events now derives endogenously from the cost-minimizing process and will in general depend on the action required from the agent. An interesting question concerns the relationship between the “ordering” induced by the mechanism sufficient statistic  $Z_a$  and the ordering usually defined on the set of signals by the “more favorable than” relation (or equivalently by MLRC). We have:

PROPOSITION 5: *For every*  $a \in A$ , *the statistic*  $Z_a$  *defined in (5.3) satisfies the strict MLRC and CDFC in some neighborhood of action*  $a$ .

PROOF: See the appendix.

In other words, the mechanism-sufficient statistic satisfies at least a *local* MLRC, in the sense that  $Z_a = 1$  represents a most favorable signal at least with respect to actions in some neighborhood containing the action to be implemented. To conclude the section, we provide an example where there is a switching of the most favorable signal depending on the action to be implemented.

AN EXAMPLE

Let  $A = [0, 1]$  and  $S = \{s_1, s_2, s_3\}$ , with  $p(s_i, a) = \gamma_i \exp[\alpha_i a - \frac{1}{2}\beta_i a^2]$  for  $i = 1, 2$  and  $p(s_3, a) = 1 - p(s_1, a) - p(s_2, a)$ . Let  $\gamma_i > 0$  and  $\alpha_i^2 < \beta_i < \alpha_i < 1$  so that  $p_a(s_i, a) > 0$  and  $p_{aa}(s_i, a) < 0$  for  $i = 1, 2$ ;  $p(s, a) > 0$  for every  $s \in S$  is ensured by choosing the  $\gamma_i$ 's small enough. For  $i = 1, 2$

$$\frac{p_a(s_i, a)}{p(s_i, a)} = \alpha_i - \beta_i a > 0 \quad \text{for all } a \in [0, 1]. \quad (5.4)$$

Assume that  $\alpha_1 > \alpha_2$  and  $\alpha_1 - \beta_1 < \alpha_2 - \beta_2$  and let  $\hat{a}$  solve

$$\alpha_1 - \beta_1 \hat{a} = \alpha_2 - \beta_2 \hat{a}. \quad (5.5)$$

Then the statistic

$$X_0(s) = \begin{cases} 1 & \text{if } s = s_1 \\ 0 & \text{if } s \in \{s_2, s_3\} \end{cases} \quad (5.6)$$

is mechanism-sufficient for  $a \in [0, \hat{a}]$ , while the statistic

$$X_1(s) = \begin{cases} 1 & \text{if } s = s_2 \\ 0 & \text{if } s \in \{s_1, s_3\} \end{cases} \quad (5.7)$$



is mechanism-sufficient for  $a \in [\hat{a}, 1]$ .

The intuition is that the probability of observing  $X_0 = 1$  rather than  $X_0 = 0$  is more sensitive to changes in  $a$  for  $a < \hat{a}$  than the probability of observing  $X_1 = 1$  rather than  $X_1 = 0$ . Therefore,  $X_0$  corresponds to a more efficient partition of the set of signals when the action to be implemented is less than  $\hat{a}$ . The converse is true for actions greater than  $\hat{a}$ . The principal is indifferent between  $X_0$  and  $X_1$  if she wants to implement  $a = \hat{a}$ . Consider now the statistic<sup>14</sup>

$$Z_a(s) = \begin{cases} 1 & \text{if } s \in \arg \max_{s \in S} p_a(s, a)/p(s, a), \\ 0 & \text{otherwise.} \end{cases} \quad (5.8)$$

It should be obvious that  $Z_a = X_0$  if  $a < \hat{a}$  while  $Z_a = X_1$  if  $a > \hat{a}$ . For  $a = \hat{a}$ , we have  $Z_{\hat{a}} = 1$  if  $s \in \{s_1, s_2\}$  and  $Z_{\hat{a}}$  is mechanism-sufficient only at  $\hat{a}$ . To implement  $\hat{a}$ , the principal is therefore indifferent between  $X_0$ ,  $X_1$  and  $Z_{\hat{a}}$ .

Given these results, it is now feasible to obtain the principal's cost function of

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<sup>14</sup>Notice that  $S^M = \{s_1, s_2\}$ . However if we assumed  $\alpha_1 - \beta_1 = \alpha_2 - \beta_2$  so that the two lines in the figure intersect at the point  $a = 1$ , then  $S^M = \{s_1\}$ . We see in that case why it would not be important to impose restrictions on  $s_2$  since  $X_0$  is mechanism sufficient over the entire range.

implementing an action:

$$C_P(a) = \max \left\{ C(a), \frac{C'(a)}{\max \{ \alpha_1 - \beta_1 a, \alpha_2 - \beta_2 a \}} - L \right\}. \quad (5.9)$$

If  $C(0) = 0$  and  $C'(0) > 0$ , this function is discontinuous at  $a = 0$  if  $L < C'(0)/\alpha_1$ .

We note that for a sufficiently large liability limit,  $L \geq C'(1)/(\alpha_2 - \beta_2) - C(1)$ ,

the principal's cost is the same as under full information for every  $a \in [0, 1]$ .

## 6. Comparison of Information Systems

Let  $(S_1, p_1)$  and  $(S_2, p_2)$  be two information systems ( $S$  refers to the signal set and  $p$  to the family of distribution functions). Proposition 4 leads directly to a simple criterion for comparing information systems.

**DEFINITION:** *Information system  $(S_2, p_2)$  is more efficient than  $(S_1, p_1)$  at  $a$  if, for every liability limit  $L$  and cost of effort function  $C$ , the cost to the principal for implementing  $a$  is not greater with  $(S_2, p_2)$  than with  $(S_1, p_1)$ , and is strictly less for some  $L$  and  $C$ .*

From the proof of proposition 4, we easily obtain the following result:

**COROLLARY 1:** *Let  $Z_a^i \in \{0, 1\}$  with probability distribution  $h_i(a)$  be the*

*mechanism-sufficient statistic for implementing action  $a \in A$  using the information system  $(S_i, p_i)$ . Then information system  $(S_2, p_2)$  is more efficient than  $(S_1, p_1)$  at  $a \neq \underline{a}$  if and only if*

$$\frac{h'_2(a)}{h_2(a)} > \frac{h'_1(a)}{h_1(a)}. \quad (6.1)$$

In a context where the agent is risk averse, KIM [1995] introduced a sufficient condition for the (partial) ordering of information systems and showed that his criterion was implied by Blackwell's efficiency condition and that it nested Holmström's concept of informativeness. In contrast to Kim, and because the agent is assumed here to be risk-neutral, our condition is necessary and sufficient for an information system to be more efficient than another; it therefore provides a complete ordering of information systems (though the ordering is only valid for implementing action  $a \in A$ ).

The relationship between condition (6.1) and Kim's criterion is as follows. For  $i = 1, 2$ , define the statistics  $Z_a^i(s) = p_a^i(s, a)/p^i(s, a)$  and denote their distribution functions by  $F_a^i$ . According to Kim's criterion, information system 2 is more efficient for action  $a$  than information system 1 if  $F_a^2$  is a mean-preserving spread

of  $F_a^1$ . Now, it is easily verified that this condition implies

$$\frac{h_2'(a)}{h_2(a)} \geq \frac{h_1'(a)}{h_1(a)}. \quad (6.2)$$

Thus, in a risk-neutral environment, Kim's criterion is sufficient for system 2 to be at least as efficient as system 1 (in the weak sense), but it is neither necessary nor in fact sufficient for system 2 to be (strictly) more efficient than system 1.

Obviously, if information systems are to be compared on the basis of the cost of implementing any  $a \in (\underline{a}, \bar{a}]$ , the ordering induced by condition (6.1) can only be a partial one. On this basis,  $(S_2, p_2)$  will be more efficient than  $(S_1, p_1)$  if and only if  $h_2'(a)/h_2(a) \geq h_1'(a)/h_1(a)$  for every  $a$ , with strict inequality for some  $a$ .

## 7. Concluding Remarks

This paper has introduced the notion of a mechanism sufficient statistic and shown that it must be distinguished from the standard concept of a sufficient statistic. To substantiate the difference, we have analyzed the efficient use of information in simple agency relationship with risk-neutral parties, under a fairly general characterization of the information system available to the principal. With risk-neutral parties the difference between both concepts of sufficiency is shown to be maximal:

the optimal incentive scheme is dichotomic and therefore a binary statistic is all that is needed to summarize the signals generated by the information system.

It follows that, to compare different information systems, it is sufficient to examine the binary mechanism-sufficient statistics from the different systems with respect to the action that is to be implemented. This, we show, generates an endogenous ordering of signals according to their usefulness to the principal. It is important to note that we did not impose the standard requirements of MLRC and CDFC. Instead, we introduced a weaker requirement that can be interpreted as a hypothesis of decreasing return for favourable information.<sup>15</sup> Specifically we showed that our hypothesis is implied by MLRC and CDFC, but that the reverse is not true. Owing to the risk-neutrality hypothesis, our criterion for comparing information systems was shown to be less demanding than Kim's MPS criterion. Like the MPS criterion, we used the likelihood ratio distribution function to define the ranking. However, because of the risk neutrality, only one point on that distribution function matters. As a result, from the principal's perspective each distribution function can be characterized by a scalar and the resulting ordering of information structures is complete. With respect to this ordering, we proved

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<sup>15</sup>In this respect, albeit in a simpler framework because of the risk-neutrality assumption, our paper is related to a recent article by SINCLAIR-DESGAGNÉ [1994] on the validity of the first-order approach in multi-signal agency problems.

the interesting corollary that the mechanism sufficient statistic satisfies a local version of the standard MLRC ordering.

There are numerous interesting extensions of the present paper. One possibility would be to make use of the simplicity of these results to derive a general characterization of the principal's decision problem in choosing between different information systems, i.e., in trading-off the cost and efficiency of different systems. A second possibility would be to go back to the standard model with risk averse participants and attempt to reexamine the difference between statistical and mechanism sufficiency.

## APPENDIX

PROOF OF PROPOSITION 2: Using the result from proposition 1, we can rewrite the principal's problem of implementing action  $a \in A$  at minimal cost as:

$$\min_{\{w(y)|y \in Y\}} \sum_{y \in Y} g(y, a)w(y) \tag{7.1}$$

$$a \in \arg \max_{\tilde{a}} \sum_{y \in Y} g(y, \tilde{a})w(y) - C(\tilde{a}), \tag{7.2}$$

$$\sum_{y \in Y} g(y, a)w(y) - C(a) \geq 0, \tag{7.3}$$

$$w(y) \geq -L, \quad \forall y \in Y. \tag{7.4}$$

By assumption, because of MLRC and CDFC the first order approach is valid.

We resolve the optimization problem (7.1) by substituting for (7.2) its first-order condition. We define the Lagrangian:

$$\begin{aligned} \mathcal{L}(L, a; \mu, \nu, w(y), \lambda(y)_{y \in Y}) = & \sum_{y \in Y} g(y, a)w(y) \\ & - \mu \left( \sum_{y \in Y} g_a(y, a)w(y) - C'(a) \right) \\ & + \nu \left( \sum_{y \in Y} g(y, a)w(y) - C(a) \right) + \sum_{y \in Y} \lambda(y)(w(y) + L) \end{aligned} \quad (7.5)$$

where  $\mu$  is the multiplier of the agent's first-order condition of the incentive compatibility constraint,  $\nu$  the multiplier of the participation constraint and the  $\lambda(y)$ 's are the multipliers of the limited liability constraints. We consider two situations.

1.  $\nu > 0$ . In this case by complementary slackness, we know that the participation constraint is binding. Thus:

$$\mathcal{L}(L, a; \mu, \nu, w(y), \lambda(y)_{y \in Y}) = C(a). \quad (7.6)$$

2.  $\nu = 0$ . We partition the sample space  $Y$ . We define  $Y'$  to be the set of  $y$ 's where the constraints  $w(y) + L$  are not binding, i.e.  $Y' = \{y \in Y | \lambda(y) = 0\}$ .

The partition implies:

$$\forall y \in Y', \quad \frac{g(y, a)}{g_a(y, a)} = \mu \quad (7.7)$$

$$\forall y \notin Y', \quad w(y) = -L \quad (7.8)$$

(7.7) results from the first-order condition of the Lagrangian by setting  $\nu = 0$  and  $\lambda(y) = 0$ . Define  $\tau(y) = w(y) + L$ . We note that  $\forall y \notin Y'$ ,  $\tau(y) = 0$ , thus from the first-order condition of the agent's problem:

$$\sum_{y \in Y} g_a(y, a) w(y) = \sum_{y \in Y'} g_a(y, a) \tau(y) \quad (7.9)$$

$$= C'(a) \quad (7.10)$$

Using this equality and complementary slackness, we can solve for the La-



grangian:

$$\mathcal{L}(L, a; \mu, \nu, s(y), \lambda(y)_{y \in Y}) = \sum_{y \in Y} g(y, a)w(y) \quad (7.11)$$

$$= \sum_{y \in Y'} g(y, a)\tau(y) - L \quad (7.12)$$

$$= \mu \sum_{y \in Y'} g_a(y, a)\tau(y) - L \quad (7.13)$$

$$= \frac{g(y', a)}{g_a(y', a)}C'(a) - L \quad (7.14)$$

where  $y'$  is an element of  $Y'$ . Given the resulting expected cost and the strict MLRC hypothesis, we conclude immediately  $y' = y^M$  i.e.  $Y' = \max \{y|y \in Y(S)\}$ .

Whether  $\nu > 0$  or  $\nu = 0$  depends on whether there is enough information content in the outcome of the random experiment to extract the entire rent. When  $\nu = 0$ , then  $T = -L$  and when  $\nu > 0$ , then  $T = -\frac{g(y^M, a)}{g_a(y^M, a)}C'(a) + C(a)$ . The result of the proposition follows immediately by change of notation. ■

PROOF OF PROPOSITION 4: We rewrite the principal's problem:

$$\min_{\{t(s)|s \in S\}} \sum_{s \in S} p(s, a)t(s) \quad (7.15)$$

$$a \in \arg \max_{\tilde{a}} \sum_{s \in S} p(s, \tilde{a})t(s) - C(\tilde{a}) \quad (7.16)$$

$$\sum_{s \in S} p(s, a)t(s) - C(a) \geq 0 \quad (7.17)$$

$$t(s) \geq -L, \quad \forall s \in S. \quad (7.18)$$

We follow standard practice. We assume that the first-order approach is valid, solve for the optimal solution under this hypothesis and conclude by showing that (A4) guarantees that the resulting mechanism is globally incentive compatible. If we follow the same steps as in the proof of proposition 2, adjusting for the change in notation, we obtain:

$$\mathcal{L}(L, a; \mu, \nu, t(s), \lambda(s)_{s \in S}) = \frac{p(s', a)}{p_a(s', a)} C'(a) - L \quad (7.19)$$

with  $s' \in S'$ . Given the expected cost structure, we have  $S' \ni \arg \max_{s \in S^M} \frac{p_a(s, a)}{p(s, a)}$ . To conclude, we show that (A4) guarantees that the second order condition of the agent's problem is satisfied. We proceed by contradiction.

1. We note that if the first-order condition is at a point  $a \in A$  then it is a local maximum. Indeed, for the first order condition to be satisfied, we must have  $p_a(a, s) > 0$  which implies  $p_{aa}(a, s) \leq 0$ .

2. Suppose  $a \in A$  is not the global maximum. Then there exists  $\hat{a}$  which yields a higher profit. This in turns implies there exists a local minimum  $\tilde{a}$  between those two points. For a local minimum, we must have  $p_a(s, a) > 0$  and  $p_{aa}(s, a) > 0$  which by (A4) yields a contradiction. ■

PROOF OF PROPOSITION 5 : Let  $h'(a) > 0$  for some  $a \in A$ , then  $h'(a) > 0$  for some neighbourhood and by (A4)  $h''(a) \leq 0$  in that same neighborhood. ■

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