AN ALTERNATIVE PENSION REFORM FOR SPAIN BASED ON OPTIMISATION TECHNIQUES♦

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SUMMARY

The aim of this paper is to twofold: to design an alternative reform of the Spanish contributory retirement pension system based on optimal strategies to restore liquidity through changes in the key variables of the system (the contribution rate, retirement age and/or indexation of pensions) and at the same time to assess the reformed Spanish system with the focus on its liquidity in the long run. These optimal strategies, which we call Automatic Balancing Mechanisms (ABMs), calculate the optimal path of these variables over time and absorb fluctuations in longevity, fertility rates, life expectancy, salary growth or any other kind of uncertainty faced by the pension system.

JEL**: E62, H55, J11, J26.**

KEYWORDS: Financial equilibrium, Pay-as-you-go, Public pensions, Optimisation, Reform, Risk, Spain.

RESUMEN

El objetivo de este trabajo es doble. En primer lugar se diseña una reforma alternativa en el sistema español de pensiones contributivas de jubilación, basada en técnicas de optimización, para restablecer el equilibrio financiero minimizando los cambios en las principales variables del sistema como el tipo de cotización, la edad de jubilación y la revalorización de las pensiones.

A continuación se valora si la reforma actual del sistema de pensiones garantiza la liquidez del sistema en el largo plazo. La estrategia óptima diseñada, a la que llamamos mecanismo financiero de ajuste automático (MFA), identifica la trayectoria óptima de las variables y absorbe las fluctuaciones en la longevidad, las tasas de fecundidad, esperanza de vida, el crecimiento del salario o cualquier otro acontecimiento aleatorio que afecte al sistema de pensiones.

JEL**: E62, H55, J11, J26.**

Palabras clave: Equilibrio financiero, España, Pensiones públicas, Optimización, Reforma, Riesgo, Sistema de reparto.

1. Introduction

Public pension systems are usually financed on a Pay-As-You-Go (PAYG) basis where pensions for retirees are paid by the contributions of the working-age population. It is well understood that this method of financing requires a balance between the benefits paid to the pensioners and the contributions made by the active workers.

The decline in fertility rates, the increase in longevity and the current forecasts for the ageing of the baby-boom generation all point to a substantial increase in the age dependency ratio, and this will raise serious concerns for the sustainability of PAYG pension systems. This is a worldwide problem, and many European countries have already carried out some parametric reforms of their systems in the sense that promises of payment may be reasonably respected. For example in Italy the normal retirement age has increased from 60 to 65 for men and from 55 to 60 for women while the indexation of pension payments has been modified in order to reduce their rate of increase. Germany has also introduced penalties for early retirement[3]. As stated in Holzmann et al. (2003), countries such as Sweden, Poland, Italy and Latvia have undertaken some structural reforms by changing the formula to calculate their initial pension from a Defined Benefit (DB) to a Notional Defined-Contribution (NDC) pension system[4].

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[3] Detailed pension reforms for European countries can be found in Whiteford and Whitehouse (2006).
[4] According to Holzmann et al. (2012), many countries, including Egypt, China and Greece, are seriously considering the introduction of NDCs.

In the meantime, some countries, such as, Sweden, Germany and Japan, have decided to set up an automatic balance mechanism (ABM). This has been defined as a set of predetermined measures established by law to be applied immediately as required according to an indicator that reflects the financial health of the system (Vidal-Meliá et al. 2009 and 2010). Its purpose, through successive application, is to re-establish the financial equilibrium of PAYG pension systems without the repeated intervention of the legislator. Turner (2009) found that at least 12 countries have the indexation of benefits linked to life expectancy or some other kind of automatic adjustment[5].

Spain is not an exception to the pension crisis and concerns about the financial sustainability of the pension system have driven reforms enacted in recent years. In 2060, there will be fewer than two people of active-age per retiree compared to more than three currently while life expectancy at the retirement age will have increased from 16.5 to 20.8. There are many debates regarding the effectiveness of the Spanish reforms and lot of researchers, Vidal-Meliá (2014), De la Fuente and Doménech (2013), state that the 2011 reform that introduced parametric changes and the commitment to a sustainability factor by 2027 is not sufficient. According to Vidal-Meliá (2014) the 2011 reform preserves the structure of the pre-reform formula for calculating the initial retirement pension and reproduces most of its main flaws. Responding to the strong pressures from the European Commission, the Spanish Committee of Experts designed and introduced a new indexation on pensions (annual revaluation factor), to be implemented from the beginning of 2014, and sustainability factor[6] (intergenerational equity factor) for the pension system to be implemented in 2019 with the aim of reducing spending on pensions.

To our knowledge, the Spanish pension system reform does not have any mathematical basis other than trying to adapt the system to the demographic scenario, without any proof, based on the general view that indicators, such as the old-age dependency ratio, expenditure on pensions or life expectancy, will rise over time. With this in mind, the aim of this paper is to twofold. First, it seeks to design an alternative reform of the Spanish pension system, different to the ones proposed in Act 27/2011 and Act 23/2013, based on optimal strategies, that not only adapts the system to economic and demographic changes with a mathematical basis but also restores liquidity to

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[5] For more information on ABMs see, for instance, Turner (2007) (2009), Vidal-Meliá et al. (2009) and OECD (2012).

[6] For more information on the new indexation on pensions and sustainability factor see Act 23/2013.

the Spanish pension system through changes in the contribution rate, retirement age and/or indexation of pensions. Secondly, it assesses the reformed Spanish system (Act 27/2011) with the focus on its financial equilibrium in the long run. These optimal strategies, which we call Automatic Balancing Mechanisms[7] (ABMs), calculate the optimal path of these variables over time and absorb fluctuations in longevity, fertility rates, life expectancy, salary growth or any other kind of uncertainty faced by the pension system.

The paper is organised as follows. The next section presents some data on the Spanish pension system. The third section describes the main notation, definitions and the numerical optimisation method. The fourth section shows the practical application of this method to the Spanish pension system and suggests how an ABM should be designed, for both symmetric and asymmetric cases, to restore liquidity to the system. This section also provides a sensitivity analysis of the results. The fifth section focuses on the long run and provides an analysis over a 75-year horizon for the Spanish pension system. The final section concludes. Three appendices present the expressions to calculate the income from contributions and the expenditure on pensions; summarise the main automatic balancing mechanism, and; provide a detailed description of the formulae to calculate the monthly pension benefit before and after the Spanish pension reform.

2. Brief description of the Spanish pension system

This paper is focused on the reform of the contributory public social insurance programs run by the State and in particular to the old-age (also called retirement) contingency[8]. The retirement contingency is financed by contributions from employees and employers.

The key features of the reform are: a) raising the statutory retirement age from 65 to 67 over the period 2013-2027, b) lengthening the period used to calculate the base pension[9] from 15 to 25 years over the period 2013-2022, c) increasing the number of contributory years for a full pension from 35 to

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[7] See Appendix A for characteristics of some countries with ABMs in place.
[8] These programs award benefits to compensate for income no longer earned due to sickness, accident, family care responsibilities, unemployment, disability, old age or death.
[9] Also called regulating base. For more details of the calculation see Appendix C.

37[10] and d) changing the adjustment factors for anticipating the retirement age and the incentives for remaining in the labour market[11].

At the end of 2013, a reform was passed that establishes a new revaluation index and regulates the sustainability factor that complements the parametric changes. Under this reform, from 2014, pensions will be adjusted according to the performance of variables, such as revenue, expenditure and the number of pensions. This will replace the former system, in force since 1997, which linked pensions to the rate of change of the CPI. Moreover, from 2019, starting pensions will be automatically linked to the increase in life expectancy (the sustainability factor[12]).

Table 1 presents the main data on the retirement contingency during the last six years. Consequences of the reform are difficult to value at this first stage given that the reform will be applied gradually until 2027. It can be seen that the actual number of contributors has been decreasing while the number of pensioners increases. Thus the number of contributors funding a retirement pension falls from 3.87 to 2.98 over the last six years. At the same time the monthly benefit and the life expectancy at retirement age increase and this worsens the sustainability of the pension system.

Table 1: Some data for the Spanish Pension System (old-age contingency)

Data \ Years	2008	2010	2012	2014
Number of contributors (in thousands)	19,129.81	17,667.47	16,845.86	16,553.79
Number of retirement pensions (in thousands)	4,936.84	5,123.88	5,330.20	5,558.96
Contributors/retirement pension ratio	3.87	3.45	3.16	2.98
Life expectancy at age 65	20	20.6	20.5	20.98
Monthly Average old-age pension	810.48	883.27	946.32	999.77

Own source based on www-seg-social.es and www.ine.es

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[10] Other features regarding the early retirement and voluntary work extension can be consulted in Act27/2011.

[11] For particularities or further explanation regarding the reform, please see the Appendix C and Act27/2011.

[12] Examination of the sustainability factor (see Act 23/2013) is beyond the scope of this paper whose main aim is to shed some light on how variables such as the contribution rate, retirement age and indexation on pensions should be modified to re-establish liquidity in the system in the long run.

According to Act27/2011, the aim of the reform is to meet the enormous challenge posed by an ageing population and to partially correct the imbalance between contributions made and pensions received. This paper assesses whether the main changes in the reform Act27/2011 are justified from a mathematical point of view with the aim to restore the liquidity into the system in the long run. The next section designs automatic balancing mechanisms to provide an alternative reform and assess the current reform of the Spanish pension system on a theoretical basis.

3. Liquidity ABMs

In this section two ABMs are built to analyse and restore the liquidity of the Spanish contributory retirement pension system, which is financed by the PAYG method. These ABMs are based on the work of Godínez-Olivares et al. (2015), in which the objective is to minimise changes over time in the main variables, such as the contribution rate, normal retirement age and indexation of pensions. This mechanism, which uses an intertemporal optimisation model, identifies and applies an optimal path for the contribution rate, normal retirement age and indexation of pensions to a PAYGO system. The first ABM does not include the accumulation or deficit of the buffer fund, while the second includes the accumulation or deficit of the fund over the time. The aim of both ABMs is to absorb fluctuations in longevity, fertility rates, life expectancy, salary growth or any other random events in a pension system. The main purposes of the ABM, through successive applications, are to re-establish the financial equilibrium of PAYG adapting the system to changes in socio-economic and demographic conditions; to create a credible institutional framework to increase the likelihood that promises of pension payments will be respected; and to minimise the use of the pension system as an electoral tool[13].

The functional objective is a logarithmic function which is set to minimise the percentage of the changes over time of the key variables. The logarithmic function is widely used in statistical forecasting (Nau (2015). Its great advantage is that small changes in the natural log of a variable are directly interpretable as percentage changes, to a very close approximation, and because changes in the natural logarithm are (almost) equal to percentage changes in the original series, it follows that the slope of a trend line fitted to logged data is equal to the average percentage growth in the original series. Godínez-Olivares et al. (2015) introduced the idea of minimising the rate

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[13] Nevertheless the system still is subject to a kind of political risk in the sense that the planning horizon of politicians is much shorter than the horizon of the pension system.

change of the decision variables involved in the projections of the contribution rate, normal retirement age and indexation of pensions.

The optimisation problem is defined by the objective function:

$$f(d_n, n) = min_{c_n, x_n^{(r)}, \lambda_n,} \left[\sum_{n=1}^{N} \left(\theta_1 log \left(\frac{c_{n+1}}{c_n} \right) + \varepsilon_1 \theta_2 log \left(\frac{x_{n+1}^{(r)}}{x_n^{(r)}} \right) + \right. \right.$$

$$\left. \left. \varepsilon_2 \theta_3 log \left(\frac{\lambda_{n+1}}{\lambda_n} \right) \right) \right] \tag{1}$$

where c_n, is the contribution rate; $x_n^{(r)}$, is the age of normal retirement; and, λ_n, is the indexation of pensions. $\theta_i, i = 1,2,3$ are the weights which measure the impact that occurs when the key variables are projected and $\theta_1 + \theta_2 + \theta_3 = 1$. These weights reflect the interest of the policy maker, since, a value of 0 in one parameter means that no change is allowed in this particular variable.

A metric problem is present due to the nature of the logarithmic function, as c_n, $x_n^{(r)}$ and λ_n have different units and constraints. To deal with this problem, $\varepsilon_i, i = 1,2$ have been introduced in the growth rates of the age of normal retirement and in the indexation of pensions.

$$\varepsilon_1 = \frac{log(c_\Delta)}{log(x_\Delta^{(r)})} \text{ and } \varepsilon_2 = \frac{log(c_\Delta)}{log(\lambda_{1\Delta})} 1_{\left\{ \frac{\lambda_{n+1}}{\lambda_n} > 0 \right\}} + \frac{log(c_\Delta)}{log(\lambda_{2\Delta})} 1_{\left\{ \frac{\lambda_{n+1}}{\lambda_n} \leq 0 \right\}}$$

where c_Δ, $x_\Delta^{(r)}$ and $\lambda_{1\Delta}$ are the maximum increase that are allowed (and $\lambda_{2\Delta}$ decrease for λ_n) per year in the contribution rate c_n, the age of normal retirement $x_n^{(r)}$ and the indexation of pensions λ_n; and $1_{\{j\epsilon\omega\}}$ is the indicator function[14].

The system's liquidity is measured in two ways; first, without the inclusion of the buffer fund, that is, only the income from contributions and the expenditure of pensions are taking into account; and secondly, the accumulation of the buffer fund over time and the actuarial projections of the income from contributions and the pensions to be paid are included in the liquidity of the system. The following subsections describe the constraints set in our optimisation model to build the ABMs.

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[14] The indicator function takes the value of 1 if the condition $j \epsilon \omega$ is satisfied and 0 in all other cases.

3.1 Liquidity ABM without buffer fund

In order built a realistic problem, the following constraints are imposed on the Liquidity ABM without buffer fund; for the contribution rate, c_n, the age of normal retirement, $x_n^{(r)}$, and the indexation of pensions, λ_n, upper (c_{max}, $x_{max}^{(r)}$, $\lambda_{max} \in \mathbb{R}$) and lower (c_{min}, $x_{min}^{(r)}$, $\lambda_{min} \in \mathbb{R}$) bounds are imposed. These bounds are set in order to avoid possible unrealistic increases in the key variables of the pension system. Another constraint to be considered is the smooth constraint. This constraint is necessary to prevent jumps in the contribution rate, age of retirement and indexation of pensions. Mathematically, the smooth constraint is set as:

$$c_{1\Delta} \leq \frac{c_{n+1}}{c_n} \leq c_{2\Delta};\ x_{1\Delta}^{(r)} \leq \frac{x_{n+1}^{(r)}}{x_n^{(r)}} \leq x_{2\Delta}^{(r)};\ \lambda_{1\Delta} \leq \frac{\lambda_{n+1}}{\lambda_n} \leq \lambda_{2\Delta}$$

where $c_{1\Delta}, c_{2\Delta}, x_{1\Delta}^{(r)}, x_{2\Delta}^{(r)}, \lambda_{1\Delta}, \lambda_{2\Delta} \in \mathbb{R}$.

Finally, the liquidity restriction is imposed as: $c_n W_n \left(n, g, x_n^{(r)} \right) - B_n \left(n, g, x_n^{(r)}, \lambda_n \right) \geq 0$. W_n is the total contribution base[15] paid at *n* and B_n is the total expenditure on pensions at *n* that depend on the growth of salaries, *g*, the retirement age $x_n^{(r)}$, and the indexation of pensions λ_n. The liquidity restriction helps to impose maximum levels of deficit or surplus into the system, and setting this restriction greater or equal to zero means that no deficit is allowed in the pension system.

The optimisation function to minimise is as follows:

$$f(d_n, n) = min_{c_n, x_n^{(r)}, \lambda_n,} \left[\sum_{n=1}^{N} \left(\theta_1 log \left(\frac{c_{n+1}}{c_n} \right) + \varepsilon_1 \theta_2 log \left(\frac{x_{n+1}^{(r)}}{x_n^{(r)}} \right) + \varepsilon_2 \theta_3 log \left(\frac{\lambda_{n+1}}{\lambda_n} \right) \right) \right] \tag{2}$$

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[15] See Appendix B for details of calculations for the total contribution base and expenditure on pensions.

$$s.t. \begin{cases} c_{min} \leq c_n \leq c_{max}; x_{min}^{(r)} \leq x_n^{(r)} \leq x_{max}^{(r)}; \lambda_{min} \leq \lambda_n \leq \lambda_{max} \\[2mm] c_{1\Delta} \leq \dfrac{c_{n+1}}{c_n} \leq c_{2\Delta}; x_{1\Delta}^{(r)} \leq \dfrac{x_{n+1}^{(r)}}{x_n^{(r)}} \leq x_{2\Delta}^{(r)}; \lambda_{1\Delta} \leq \dfrac{\lambda_{n+1}}{\lambda_n} \leq \lambda_{2\Delta} \\[2mm] \theta_1 + \theta_2 + \theta_3 = 1 \\[2mm] \varepsilon_1 = \dfrac{\log(c_\Delta)}{\log(x_\Delta^{(r)})} \\[2mm] \varepsilon_2 = \dfrac{\log(c_\Delta)}{\log(\lambda_{1\Delta})} 1_{\left\{\frac{\lambda_{n+1}}{\lambda_n}>0\right\}} + \dfrac{\log(c_\Delta)}{\log(\lambda_{2\Delta})} 1_{\left\{\frac{\lambda_{n+1}}{\lambda_n}\leq 0\right\}} \\[2mm] c_n W_n\left(n, g, x_n^{(r)}\right) - B_n\left(n, g, x_n^{(r)}, \lambda_n\right) \geq 0 \end{cases}$$

3.2 Liquidity ABM with buffer fund

In this subsection a contingency fund is introduced into the minimisation function. It is important to highlight that four papers, Godínez-Olivares et al. (2015), Haberman and Zimbidis (2002), Pantelous and Zimbidis (2008), and Gannon, Legros and Touzé (2013), propose parametric reforms in the PAYG pension systems introducing the concept of a liquidity or contingency fund in order to absorb unexpected events that might affect their liquidity. Gannon, Legros and Touzé (2013) define this buffer fund as the inter-temporal budget balance of the pension system that brings promised future expenditures in line with expected future revenues. For example, the interest generated by the buffer fund in Spain covered the shortfall in contributions during 2010 (Vidal-Meliá (2014)).

The liquidity ABM with the buffer fund calculates the optimal path for the contribution rate, age of normal retirement and indexation of pensions building an optimisation problem that is solved by numerical analysis (Godínez-Olivares et al. (2015)) including the dynamics of the fund, F_n, that can be expressed as:

$$F_n = (1 + J_n)F_{n-1} + c_n W_n\left(n, g, x_n^{(r)}\right) - B_n\left(n, g, x_n^{(r)}, \lambda_n\right) \tag{3}$$

where J_n is the growth risk-free rate of the fund during year n; $c_n, x_n^{(r)}, \lambda_n$ are the projected variables during year n; W_n is the total contribution base paid at n; B_n is the total expenditure on pensions at n. The derived contingency fund is able to absorb fluctuations in longevity, fertility rates, salary growth or any other events.

In this ABM, the liquidity restriction is changed to, $F_n \geq 0$, that is, $(1 + J_n)F_{n-1} + c_n W_n \left(n, g, x_n^{(r)}\right) \geq B_n \left(n, g, x_n^{(r)}, \lambda_n\right)$. The other restrictions set for the ABM without buffer fund remains the same.

The optimisation problem to minimise in the ABM including the buffer fund is defined by:

$$f(d_n, n) = min_{c_n, x_n^{(r)}, \lambda_n,} \left[\sum_{n=1}^{N} \left(\theta_1 log \left(\frac{c_{n+1}}{c_n} \right) + \varepsilon_1 \theta_2 log \left(\frac{x_{n+1}^{(r)}}{x_n^{(r)}} \right) + \right.\right.$$

$$\left.\left. \varepsilon_2 \theta_3 log \left(\frac{\lambda_{n+1}}{\lambda_n} \right) \right) \right] \tag{4}$$

$$s.t. \begin{cases} c_{min} \leq c_n \leq c_{max}; x_{min}^{(r)} \leq x_n^{(r)} \leq x_{max}^{(r)}; \lambda_{min} \leq \lambda_n \leq \lambda_{max} \\[2mm] c_{1\Delta} \leq \dfrac{c_{n+1}}{c_n} \leq c_{2\Delta}; \; x_{1\Delta}^{(r)} \leq \dfrac{x_{n+1}^{(r)}}{x_n^{(r)}} \leq x_{2\Delta}^{(r)}; \lambda_{1\Delta} \leq \dfrac{\lambda_{n+1}}{\lambda_n} \leq \lambda_{2\Delta} \\[2mm] \theta_1 + \theta_2 + \theta_3 = 1 \\[2mm] \varepsilon_1 = \dfrac{\log(c_\Delta)}{\log(x_\Delta^{(r)})} \\[2mm] \varepsilon_2 = \dfrac{\log(c_\Delta)}{\log(\lambda_{1\Delta})} 1_{\left\{\frac{\lambda_{n+1}}{\lambda_n} > 0\right\}} + \dfrac{\log(c_\Delta)}{\log(\lambda_{2\Delta})} 1_{\left\{\frac{\lambda_{n+1}}{\lambda_n} \leq 0\right\}} \\[2mm] F_n \geq 0 \end{cases}$$

The next subsections describe two indicators to measure the liquidity of the system according to the inclusion or exclusion of the buffer fund.

3.3 Liquidity indicators

Two indicators emerge from our model to measure the liquidity of the system. The first one, which does not include the buffer fund, is defined by:

$$I_{NoBF} = \frac{c_n W_n \left(n, g, x_n^{(r)}\right)}{B_n \left(n, g, x_n^{(r)}, \lambda_n\right)} \tag{5}$$

The second includes the buffer fund as follows:

$$I_{BF} = \frac{(1+J_n)F_{n-1} + c_n W_n \left(n, g, x_n^{(r)}\right)}{B_n \left(n, g, x_n^{(r)}, \lambda_n\right)} \tag{6}$$

Both indicators measure the liquidity of the system, however, in the first one, only the income from contributions and the expenditure of pensions are included, whereas, in the second one, the accumulated buffer fund is added to the indicator.

3.4 Symmetric and asymmetric designs

Our ABM also allows us to design symmetric and asymmetric cases under both ABMs. Palmer (2012) states that under a *symmetric* ABM, any surplus that might arise would be automatically distributed. In the absence of a symmetric ABM, an undistributed surplus will be maintained. Under the *symmetric* design, the ABMs determine whether the contribution rate, the age of normal retirement and (indexation of pensions) are reduced (increased) when the system has a surplus or increased (decreased) in periods of deficit (Godínez-Olivares et al. 2015). Analytically, for the *asymmetric* case the change in the contribution rate and age of normal retirement are enforced to be strictly greater or equal to zero (strictly lower or equal to zero for the indexation of pensions). In other words, under the *symmetric* case, the change in the variables could be positive or negative.

The following section shows the application of the ABMs to the Spanish pension system.

4. ABMs for the Spanish pension system

This section presents different designs of ABMs for the Spanish pension system. First, the main data and assumptions are presented, secondly the results are discussed and finally this section provides a sensitivity analysis of the results.

4.1 Main data and assumptions

Demographic data

According to the National Institute of Statistics (Instituto Nacional de Estadística (INE)[16]) population growth will progressively decrease over the next few decades. If current demographic trends continue, Spain will lose a more than million people (2.2%) in the next 15 years and more than 5.6 million (12.1%) in the next 50 years, continuing the negative trend that

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[16] Instituto Nacional de Estadística, from http://www.ine.es, accessed 1 June 2015.

began in 2012, hence the Spanish population will be reduced to 45.8 million in 2024 and to 40.9 million in 2064. The reduction is mainly due to the progressive increase in deaths and low fertility rates. Figure 1 shows the population structure for years 2014, 2024 and 2034. It can be seen that the largest group in 2014 is aged 35-39 while the most representative group in 2014 will be the one aged 45-49.

Figure 1 –Spanish population structure.
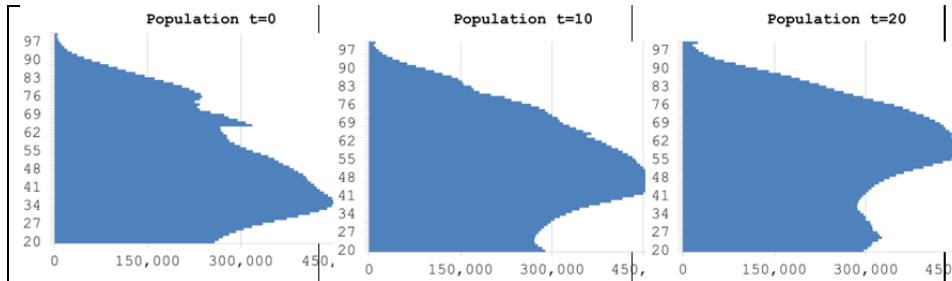
Own source based on www.ine.es

The increase in life expectancy also changes the structure of the population. In fact, life expectancy at birth will be around 84.0 years for men and 88.7 for women in 2030, 4 and 3 years more respectively from current values. Despite the decrease in the number of population and an increase in life expectancy, the number of deaths would continue to grow as a result of population ageing. In the period 2012-2032 more than six million deaths are expected, around 7% higher than those observed in the previous 15 years (1999-2012), which results in a decrease in the value of the old-age dependency ratio (Figure 2). Specifically, in 20 years there will be more than 11 million people over 64 years in Spain, that is an increase of 34%.

Figure 2 – Projected old-age dependency ratio.
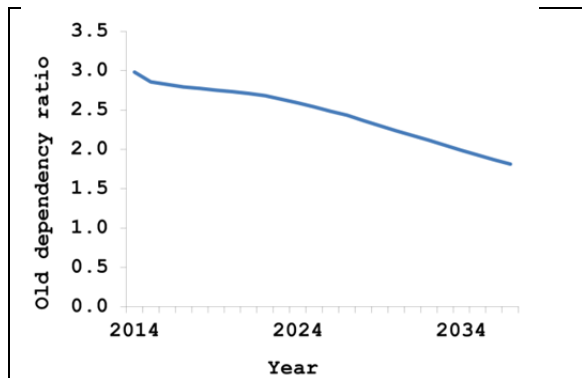
Own source based on www.ine.es

Other main data and assumptions

To set the minimisation problem, some assumptions are needed:

• The entry age into the labour market, x_e, is set in 20[17].

• At the beginning of the study, the decision variable contribution rate is assumed to be 18%, that is the weight that represents the expenditure on pensions over the total contributory expenditure.

• At the beginning of the study, the decision variable retirement age is assumed to be 65, that is the legal retirement age before the reform.

• At the beginning of the study, the decision variable indexation on pensions is assumed to be 1%. Although pensions are usually linked to the change of CPI, we want to see how this variable reacts with the economic and demographic scenario.

• The upper and lower bounds of the contribution rate, age of normal retirement and indexation of pensions are 23%, 69 and 2% and 18%, 65 and -2% respectively.[18]

• In order to have a smooth transition, it is also assumed that the change over time[19] in the contribution rate varies between 0.3% and 0.5%, the age of normal retirement between 1.5 and 3 months and the indexation of pensions between -2% and 2%.

• The buffer fund is assumed to increase at an annual rate[20] of 3% while the annual salary growth[21] is 2%.

• The salary structure is assumed to be the mean annual earnings in Spain of each age group [22].

• The average net replacement rate considered in our analysis is kept[23] at 75%. This value is higher than the replacement rate of 68.8% in the OECD-34. As a result of setting the replacement rate at 75%, the ratio of the

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[17] The entry age into the system does not have real implications in our model since we match the number of contributors and pensioners through the dependency ratio.

[18] Negative indexation would be a very unpopular and an unlikely measure to be taken in Spain. However some PAYG pension systems like Sweden has already adopted a negative indexation for some years to make its pension system solvent. For this reason, we would keep a more general setting in our model allowing for negative indexation with a minimum value of -2%.

[19] These values are in line with the most important reforms in the 34 OECD member countries between January 2009 and September 2013 (see OECD (2013)).

[20] Historical average of the Euribor in the last 15 years.

[21] We assume the expected growth of the GDP in the next 20 years according to The 2012 Ageing Report (2012).

[22] Eurostat Database, from http://ec.europa.eu/eurostat accessed 1 June 2015.

[23] Sensitivity analysis is provided regarding this variable.

average pension to the average salary in 2014 becomes 60% which is in line with real data[24].

4.2 Results

Figure 3 presents the optimal path for the contribution rate, age of normal retirement and the indexation of pensions when the buffer fund is included. This ABM, in contrast with the ABM without a buffer fund, accumulates the surpluses of the system. In both symmetric and asymmetric scenarios the optimal path of the decision variables is the same and the effect of the inclusion of the fund is seen, particularly, in the levels of the contribution rate, Figure 3a, and age of normal retirement, Figure 3b. The contribution rate stabilises at 20.5% and the age of normal retirement at 67.4. The indexation of pensions, Figure 3c, shows minor changes; with the inclusion of the buffer fund it stabilises at -0.75%. The dynamics of the fund are shown in Figure 3d. The accumulation period decreases over the last 5 years of the analysis, leaving the fund in levels near to zero as imposed in the restrictions.

Figure 3[25] – Liquidity ABM **<u>with buffer</u>** fund (Black line-Symmetric. Grey line - Asymmetric):

a)Contribution

b)Age

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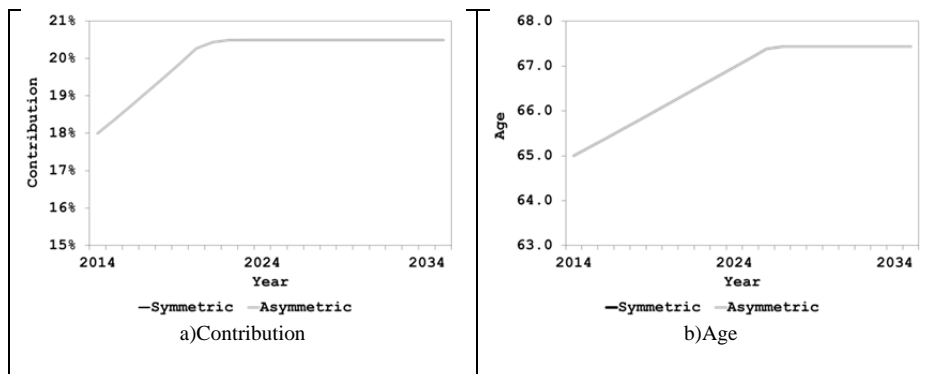
[24] According to the OCDE (2011) the Spanish pension reform on a full career worker reduces the replacement rate to 73.9% (on the OECD's standard assumptions of 2.5% price inflation and 2% real earnings growth).
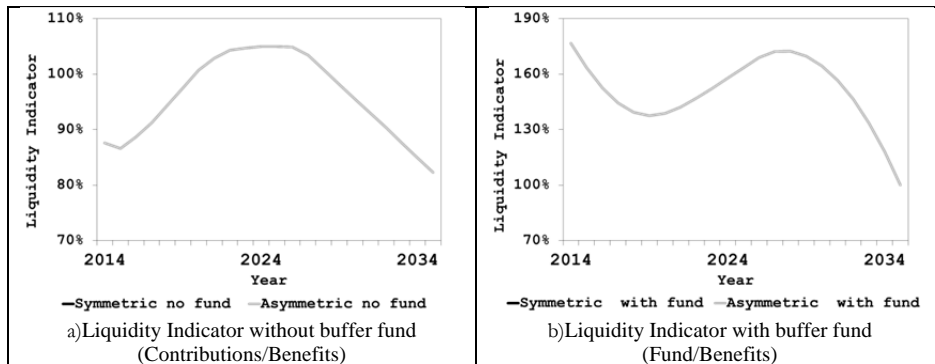
[25] As can be seen, the optimal path is exactly the same for both scenarios.

c)Indexation of pensions   d)Fund
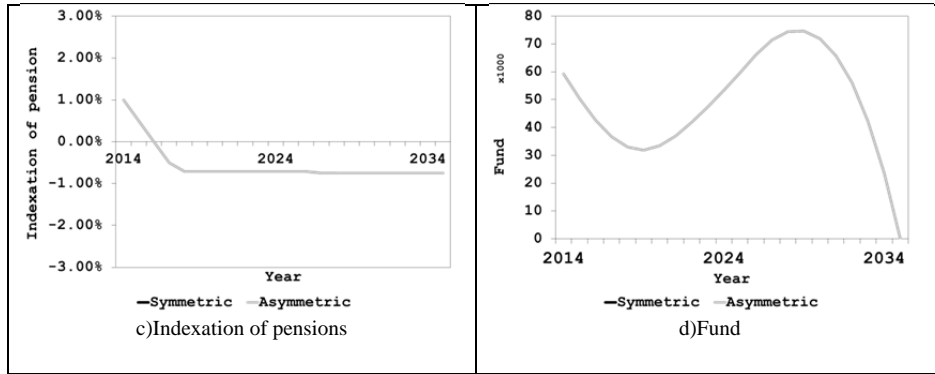
Own source

To illustrate how the inclusion of the buffer fund contributes to the liquidity of the system, two liquidity indicators for the ABM with buffer fund[26] are shown in Figure 4. The first one, Figure 4a, represent the ratio (equation 5) between the income from contributions and the expenditure of pensions, it is seen that the indicator, from year 2020 -2028, is above 1, so, the income from contribution does not cover the amount of pensions to be paid during 2014-2020 and 2029-2034. However, when the fund is included in the ratio (equation 6), Figure 4b, the ratio is always above 1. So, the inclusion of the buffer fund is contributing to the liquidity of the system.

Figure 4[27] Liquidity Indicators for the ABM **with buffer** fund (Black[28] line-Symmetric. Grey line- Asymmetric)

a)Liquidity Indicator without buffer fund
(Contributions/Benefits)

b)Liquidity Indicator with buffer fund
(Fund/Benefits)

Own source

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[26] For the ABM without buffer fund is not possible to calculate these indicators, because the optimisation problem does not take into account the accumulation of the surpluses.

[27] As can be seen, the optimal path is exactly the same for both scenarios.

[28] Given that the optimal path for the decision variables is the same, results under symmetric and asymmetric scenarios are identical.

If we apply the liquidity ABM without including the buffer fund, the projections of the contribution rate, age of normal retirement and indexation of pensions for Spain can be seen in Figure 5. In both scenarios, the symmetric and asymmetric, the contribution rate, Figure 5a, increases from 18% to 21.75%. The age of normal retirement, Figure 5b, increases to 68.8; 1.25% and 1.4 years more respectively compared to the ABM with the buffer fund. The indexation of pensions, Figure 5c, shows a small decrease compared to the previous ABM, in both the symmetric and asymmetric design, down to -1% by 2034. Figure 5d, shows that the liquidity indicator is always greater than 1, so, the expenditure of pensions is always covered by the income from contributions, as set in the liquidity restriction.

Figure 5 – Liquidity ABM **<u>without buffer fund</u>** (Black line-Symmetric. Grey line-Asymmetric).
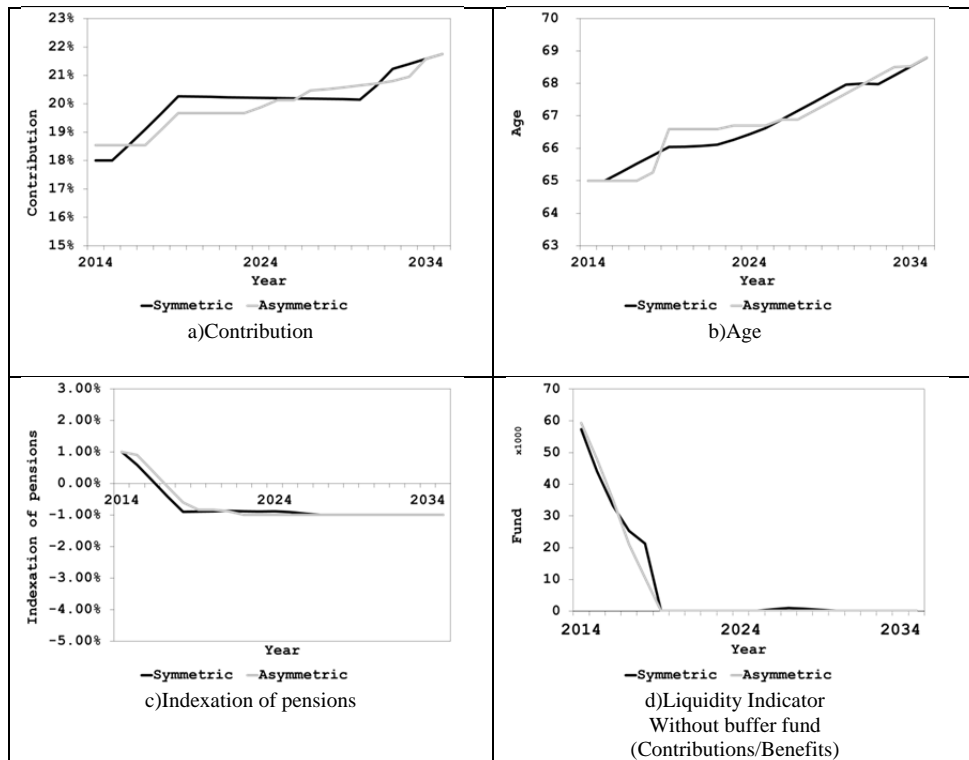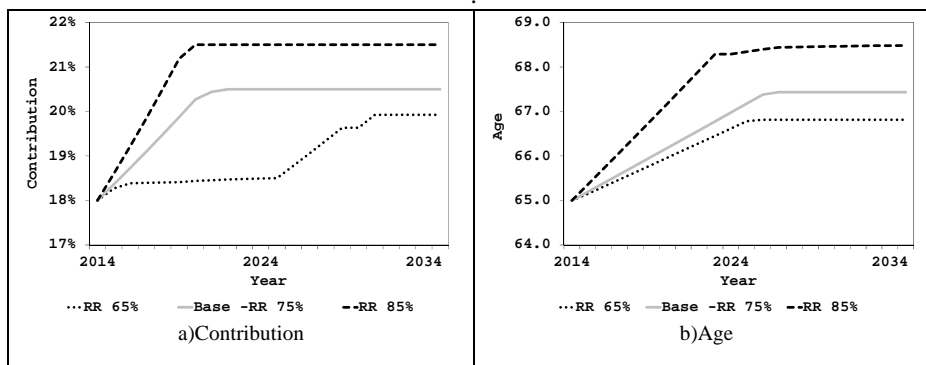

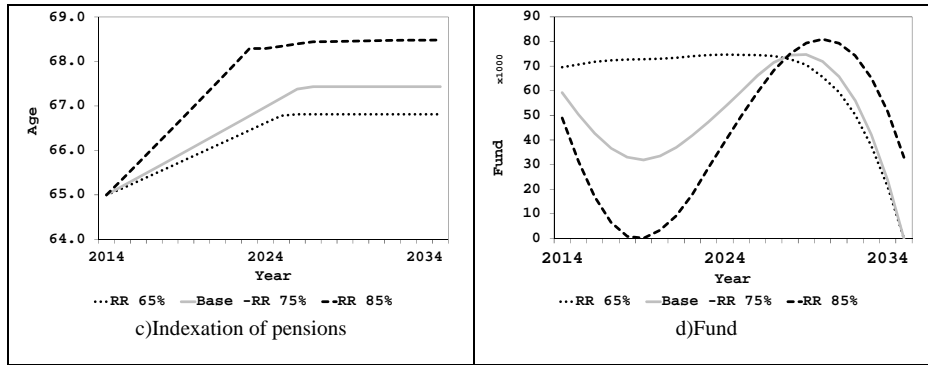Own source

4.3 Sensitivity analysis

This section presents, for the ABM with buffer fund under the asymmetric scenario, the results of increasing or decreasing the replacement rate (RR); increasing or decreasing the old-age dependency ratio; and, the results when only one variable is projected and the other two are fixed in the current values.

In Figure 6, two levels of replacement rates are analysed. First, we suppose a replacement rate of 65% (red line). Under this scenario, the contribution rate increases only by 1.9% to stabilise at 19.9% (0.51% less than in the base scenario of a replacement rate of 75%); the age of normal retirement increases to 66.8 (0.6 years less than in the base scenario) and the indexation of pensions decreases to -0.55%, that is 0.2% less compared with a replacement rate of 75%. When the replacement rate is set 85%, the contribution rate stabilises at 21.50%, the age of retirement at 68.5 and the indexation of pensions at -1.0% (that is, 1% and 1.1 years more, and 0.25% less indexation of pensions comparing with the scenario under the replacement rate of 75%).

Something interesting in this analysis is the dynamic of the buffer fund. For the replacement rate of 85%, the buffer fund has surpluses at the end of the analysis; however, as we are imposing an asymmetric scenario, the surpluses cannot be redistributed by the system.

Figure 6 – Liquidity ABM **with buffer fund** (Black dashed line - RR 85%. Grey line- RR 75% -base scenario-. Black points line - RR 65%)

a)Contribution

b)Age

c)Indexation of pensions

d)Fund

Own source

Figure 7 shows the results when different levels of the old-age dependency ratio (OD) are taken into account. First, a decrease of the ratio from 2.98 to 2.8 is analysed, then an increase to 3.2 is analysed. In the first case, the contribution rate and indexation of pension stabilises at the same level as in the base scenario, but the age of retirement stabilises at 67 instead at 67.5. That is, basically, because the age of retirement affect both, pensioners and contributors. In the second case, when the old-dependency ratio starts at 3.2, the contribution rate increase to 21.5%, the age to 67.6 and the indexation of pensions -0.85%. Again, the asymmetric design is leaving the fund with surpluses at the end of the study when the old-dependency ratio is 2.8 in the first year of analysis.

Figure 7 – Liquidity ABM **with buffer fund** (Black dashed line- OD 2.8. Grey line- OD 2.98 -base scenario-. Black points line - OD 3.2).

a)Contribution

b)Age

c)Indexation of pensions   d)Fund
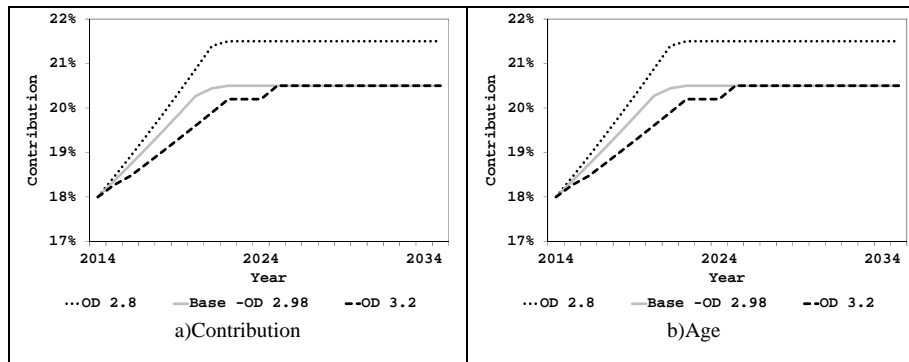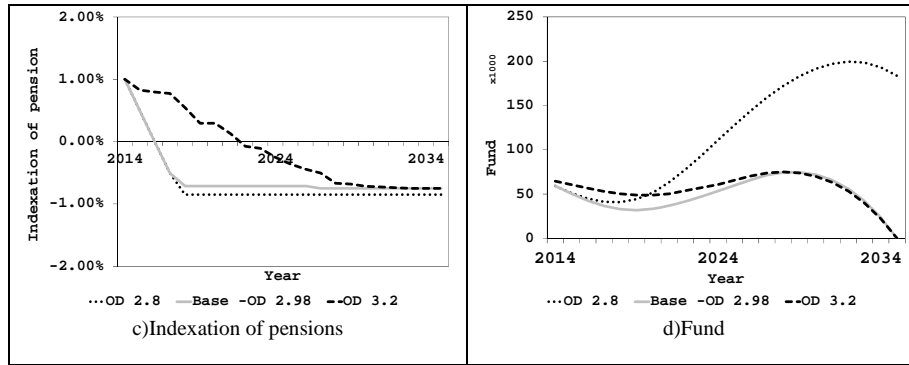
Own source

The ABMs proposed have the ability to modify only one variable, instead of modifying the three of them simultaneously. Therefore, we analyse the case when only one variable is studied. If we set the age of retirement and indexation of pensions at 65 and 1% respectively, the contribution rate would need to increase to 30%. If the age of retirement is the only player, having set the contribution rate and indexation of pensions at values 18% and 1% respectively, it would need to increase to 72.3. On the contrary, if the indexation of pensions is the only decision variable, the pension benefits would need to decrease by -10.3% every year.

5. Long run analysis of the Spanish pension system

In contrast, with the previous section, the model calculates the optimal path of the contribution rate, age of normal retirement, and indexation of pensions over a 75 year time horizon of. Figure 8a, presents an increase of 2.9% in the contribution rate comparing with the 20-year time horizon and the value stabilises at 23.4%. The age of retirement, Figure 8b, stabilises at 70.4, that is three years more than for the 20 years time horizon. The indexation of pensions, Figure 8c, now decreases to -1.5%, that is, double the number compared to the ABM over the 20 year horizon. Figure 8d, shows the dynamics of the buffer fund, which shows an accumulation period of almost 40 years after which it decreases reaching the value of zero at the end of the 75 years.

Figure 8 – Liquidity ABM **with buffer** fund (Grey line Asymmetric) in the long run

a)Contribution

b)Age

c)Indexation of pensions

d)Fund
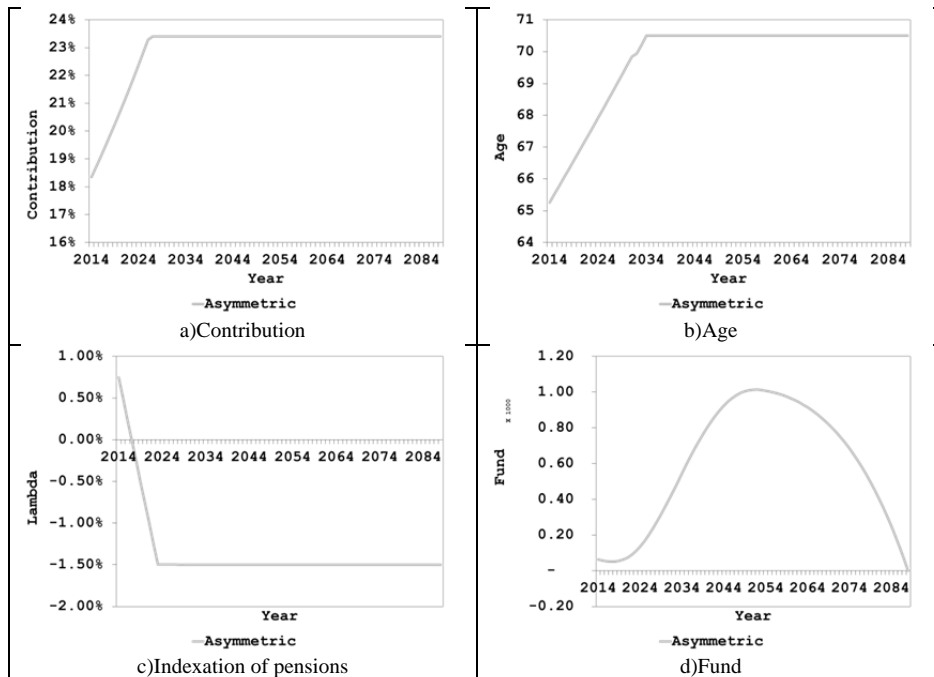
Own source

Figure 9 shows the liquidity indicators defined in equation 5 (Figure 9a), and equation 6 (Figure 9b). The liquidity indicator without the buffer fund, shows a deficit in the system from 2045 onwards, in contrast to the indicator with the buffer fund, which does not present any deficit in the 75 years.

Figure 9. Liquidity Indicators for the ABM **with buffer** fund (Grey line-Asymmetric) in the long run

a) Liquidity Indicator without buffer fund (Contributions/Benefits)

b) Liquidity Indicator with buffer fund (Fund/Benefits)
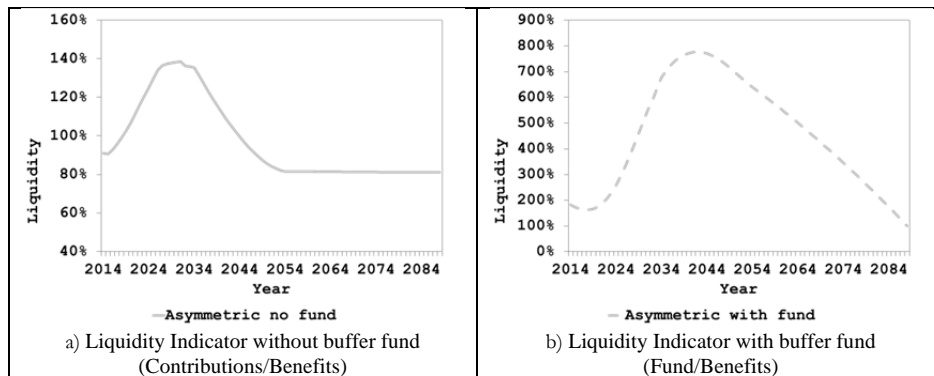
Own source

Finally, if only one variable is projected at a time, the contribution rate needs to increase to 47% when the age of retirement and indexation of pensions are fixed to 65 and 1% respectively; the age of retirement needs to increase from 65 to 79.9 if the contribution rate and indexation of pensions are fixed to 18% and 1% respectively; and the indexation of pension needs to decrease to -9.2% when the contribution rate and age of normal retirement are fixed to 18% and 65.

6. Conclusions

Restoring the liquidity of a PAYG pension system is on the agenda for most governments and Spain is no exception. Even after the 2011 reform, the pension system's sustainability in the long run has been widely questioned. This paper contributes to the debate on the policies needed to strengthen the pension system and proposes alternative reforms based on optimisation techniques to re-establish the liquidity of the Spanish PAYG pension system without the repeated intervention of the legislator.

The aim of these alternative reforms, also called automatic balancing mechanisms (ABMs), is to guide the system back onto the road to long-term liquidity and at the same time to automate the measures to be taken, isolating them from the political arena, avoiding any delay and lack of time perspective.

The ABMs can be designed in different ways considering or not in our analysis the amount of the accumulated buffer fund. We show that for the Spanish pension system the contribution rate stabilises at 20.5% and the age of normal retirement and indexation of pensions at 67.5 and -0.75% respectively after 20 years. As expected, if the amount of the buffer fund is not included in our analysis, the values for the contribution rate, age of normal retirement and indexation of pensions are higher (lower for the indexation of pensions) and stabilise at 21.75%, 68.8 and -1% respectively. The ABMs proposed have the ability to modify only one variable, instead of modifying the three of them simultaneously. Therefore, if the contribution rate is the only player into the system, it would need to increase to 30%, whereas if the age of retirement is the only variable it would need to increase to 72.3, questioning then the measures taken by Act27/2011. On the contrary, if the indexation of pensions is set as the only decision variable, the pension benefits must decrease by -10.3% every year.

Following some of the ideas of the current Swedish ABM we design a symmetric ABM that determines whether the contribution rate, the age of normal retirement (and indexation of pensions) are reduced (increased) when the system has a surplus or increased (decreased) in periods of deficit. Although there are some differences in the values of the variables under the symmetric and asymmetric design during the first years, at the end of the analysis both ABMs reach similar values for the contribution rate, retirement age and indexation of pensions.

Some countries like Japan or the USA are concerned about the sustainability of their system in a 95 or 75-time horizon. The results for the Spanish pension system if a 75-year time horizon is considered are quite alarming. If the three decision variables are modified simultaneously, the contribution rate stabilises at 23.4, whereas the retirement age needs to increase to 70.4 and the indexation on pension decreases -1.5% every year.

Our paper questions the adequacy of the current reforms and considers that the sustainability of the Spanish pension system is not guaranteed in a long run, unless the structure of population changes dramatically with an increase in the number of contributors and a decrease in the number of pensioners.

Appendix A: Expressions for income from contributions and expenditure on pensions

The total contribution base for year 1 is modelled as a function of the individuals' average wage, wage(x) at age x, the number of people alive, $l_{\{x,1\}}$, (uniformly distributed over the year), at age x at the first year of study (that is, at time 1) and the entry age into the labour market, x_e (for this particular example the entry age is 20). Thus, at time n=1, the total contribution base, W_1, is modelled as:

$$W_1 = \sum_{x=x_e}^{x_1^{(r)}-1} l_{\{x,1\}} wage(x)$$

For *n>1*, the total contribution base, W_n, depends additionally on variables such as the growth of salaries, g and the normal retirement age, $x_n^{(r)}$. Where the floor function is $\left\lfloor x_n^{(r)} \right\rfloor$; i.e. it maps a real number to the largest previous integer number, and $mod \left\lfloor x_n^{(r)} \right\rfloor$ is the modulus operation that finds the

remainder of the division $x_n^{(r)} \Big/ \left\lfloor x_n^{(r)} \right\rfloor$. Therefore, the expression of the total

contribution base for *n* greater than 1 is:

$$W_n = \left(\sum_{x=x_e}^{\left\lfloor x_n^{(r)} \right\rfloor - 1} l_{\{x,n\}} wage(x)(1+g)^n \right)$$
$$+ \left(x_n^{(r)} \, mod \left\lfloor x_n^{(r)} \right\rfloor \right) l_{\left\{ \left\lfloor x_n^{(r)} \right\rfloor, n \right\}} wage\left(\left\lfloor x_n^{(r)} \right\rfloor \right)(1+g)^n$$

The dynamics of *B(n)* for *n>1* could be written as:

$$B_n = \left(1 - \left(x_n^{(r)} \, mod \left\lfloor x_n^{(r)} \right\rfloor \right) l_{\left\{ \left\lfloor x_n^{(r)} \right\rfloor, n \right\}} \right) P_{\left\lfloor x_n^{(r)} \right\rfloor, n} + \sum_{x=\left\lceil x_n^{(r)} \right\rceil}^{w} P_{x,n} l_{\{x,n\}}$$

where $P_{x,n}$ is $P_{x,n} = P_{x-1,n-1}(1 + \lambda_{n-1})$, $\left\lceil x_n^{(r)} \right\rceil$ is the ceiling function, $l_{\{x,n\}}$, the number of people alive at age x in time n; *w* the last age to which a person can survive, and λ is the indexation of pensions that is dynamic over time and the expenditure on pensions at year *n=1* is equal to:
$B_1 = \sum_{x=x_1^{(r)}}^{w} P_{x,1} l_{\{x,1\}}$,

where $P_{x_1^{(r)},1} = \alpha \dfrac{\sum_{x=x_e}^{x_1^{(r)}-1} l_{\{x,1\}} wage(x)}{\sum_{x=x_e}^{x_1^{(r)}-1} l_{\{x,1\}}}$

Appendix B: Main automatic balancing mechanisms

The aim of the automatic balancing mechanisms is to reduce the growth of the liabilities in a pension system and adapt it to the new economic and demographic scenario. According to D'Addio and Whitehouse (2012) three main automatic mechanisms can be considered for changing pension values. First, adjustments can be made in benefit levels to reflect changes in life expectancy; second, adjustments can be made through revalorization of earlier years' contribution bases and third, adjustments may occur through the indexation of pension payments.

This non-exhaustive[29] appendix shows some of the main countries that have an ABM in place in their public pension systems. The list is sorted alphabetically.

Table 2: Characteristics of some countries with ABMs in place.

Country	Type	ABM
Canada (DB PAYG)	Third type and increase of contribution rate	If the plan is not sustainable a (semi-automatic) mechanism will increase the contribution rate by the amount necessary to cover 50% of the deficit and the benefits will be frozen for three years.
Finland (DB PAYG)	Third type	Life expectancy coefficient adjusts the amount of pensions in payment.
Germany (DB PAYG)	Third type	Revaluation of pensions includes a factor that takes into account the system's rate of dependence.
Japan (DB PAYG)	Second and third type	A modified indexation, that takes into account improvement in life expectancy and population changes, is applied to both the revaluation of the contribution bases and pension in payment
Latvia (DC PAYG)	First type	Conversion from NDC capital to an annuity taking into account projected cohort life tables adjusted annually.
Poland (DC PAYG)	First type	Conversion from NDC capital to an annuity taking into account the average life expectancy at retirement age.
Portugal (DB PAYG)	Third type	Indexation of benefits taking into account improvements in life expectancy.
Sweden (DC PAYG)	First and second type	If the system is insolvent the growth in pension liability (i.e. pension in payment and contributors' notional capital) is reduced

Source: Own source based on Vidal-Meliá et al (2009) and Godínez-Olivares et al (2015)

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[29] For a deep understanding see Vidal-Meliá et al (2009), Fall and Bloch (2014), Turner (2007) and (2009), OECD (2012) and (2013).

Appendix C: The Spanish pension reform

In Spain, in order to be eligible to receive a retirement pension it is necessary to contribute for at least 15 years, including at least two of them in the last 15 years prior your retirement. The table 3 shows the main differences in the expression to calculate the amount of the monthly initial pension, pre-reform (P_{x_r}) and post-reform (P'_{x_r}) where:

Wage$_i$: is the contribution base of one individual in month i.
x_r : is the retirement age
S_1 : is the monthly contribution base in month 1, that is, the month before retirement.
S_i : is the monthly contribution base in month *i*.
$\%_{x_r,C}$: is the percentage to be applied to the base [30] pension before the reform according to the retirement age, x_r , and number of contributory years, C.
$(\%_{x_r,C})'$: is the percentage to be applied to the base pension after the reform according to the retirement age, x_r , and number of contributory years, C.
$\%_C$: is the percentage to be applied to the base pension before the reform according to the total number of years of contributions (*C*).
$(\%_C)'$: is the percentage to be applied to the base pension after the reform according to the total number of years of contributions (*C*).

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[30] The table shows a very simplified expression of this value without considering the particularities of different groups.

Table 3: Expressions to calculate the pension benefits before and after the reform[31]

Pre-Reform	Post-Reform

Pre-Reform

$$P_{x_r} = \%_C \cdot \%_{x_r,C} \cdot \frac{\overbrace{\sum_{i=1}^{24} wage_i + \sum_{i=25}^{180} wage_i \frac{RPI_{25}}{RPI_i}}^{Base\ Pension}}{210}$$

With

$$\%_{x_r,C} = \begin{cases} 100\% - (65-x_r)\cdot 8\%, & 60 \leq x_r <65\ and\ 15 \leq C <30 \\ 100\% - (65-x_r)\cdot 7.5\%, & 60 \leq x_r <65\ and\ 30 \leq C <35 \\ 100\% - (65-x_r)\cdot 7\%, & 60 \leq x_r <65\ and\ 35 \leq C \leq 37 \\ 100\% - (65-x_r)\cdot 6.5\%, & 60 \leq x_r <65\ and\ 38 \leq C \leq 39 \\ 100\% - (65-x_r)\cdot 6\%, & 60 \leq x_r <65\ and\ C \geq 40 \\ 100\% - (x_r-65)\cdot 2\%, & x_r \geq 65\ and\ 30 \leq C <40 \\ 100\% - (x_r-65)\cdot 3\%, & x_r \geq 65\ and\ C \geq 40 \end{cases}$$

And

$$\%_C = \begin{cases} 0, & C<15 \\ 50\% + (C-15)\cdot 3\%, & 15 \leq C \leq 25 \\ 80\% + (C-25)\cdot 2\%, & 25 < C \leq 35 \\ 100\%, & C >35 \end{cases}$$

Post-Reform

$$P'_{x_r} = (\%_C)' \cdot (\%_{x_r,C})' \cdot \frac{\overbrace{\sum_{i=1}^{24} wage_i + \sum_{i=25}^{300} wage_i \frac{RPI_{25}}{RPI_i}}^{Base\ Pension}}{350}$$

With

$$(\%_{x_r,C})' = \begin{cases} \text{Worker voluntarily gives up her job} \\ 100\% - (67-x_r)\cdot 8\%, & 65 \leq x_r <67\ and\ 35 \leq C <38.5 \\ 100\% - (65-x_r)\cdot 7.5\%, & 63 \leq x_r <65\ and\ 38.5 \leq C <41.5 \\ 100\% - (65-x_r)\cdot 7\%, & 63 \leq x_r <65\ and\ 41.5 \leq C \leq 44.5 \\ 100\% - (65-x_r)\cdot 6.5\%, & 63 \leq x_r <65\ and\ C \geq 44.5 \\ \text{Worker voluntarily prolongs her working life} \\ 100\% - (x_r-67)\cdot 2\%, & x_r \geq 67\ and\ 15 \leq C <25 \\ 100\% - (x_r-67)\cdot 2.75\%, & x_r \geq 67\ and\ 25 \leq C <37 \\ 100\% - (x_r-67)\cdot 4\%, & x_r \geq 67\ and\ C \geq 37 \\ 100\% - (x_r-65)\cdot 4\%, & x_r \geq 65\ and\ C \geq 38.5 \end{cases}$$

And

$$(\%_C)' = \begin{cases} 0, & C<15 \\ 50\% + 2.28\% (C-15), & 15 \leq C <35.67 \\ 80\% + 2\% (C-35.67), & 35.67 \leq C <37 \\ 100\%, & C >37 \end{cases}$$

Source: Based on Act 27/2011

[1] The expression of factor corresponding to the retirement age after the reform $(\%_{x_r,C})'$ does not contain the case when the worker loses her job through no fault of her own.

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The resulting pension should be higher than the minimum pension per year and is capped if its value exceeded the amount of the maximum pension.

It is worth highlighting that before the reform, one individual who contributes 35 years and retires at the age 65 has a full amount of pension equal to the base pension. Once that the reform is totally implemented in 2027 the individual should retire at the age 65 after 38.5 years of contributions or at the age of 67 after 37 contributory year to have full pension.

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