Retinal Fundus Image Deconvolution

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Abstract

Blurring of images occurs in many fields, causing significant problems in retinal imaging. In any diabetic retinopathy screening programme, up to 10% of the images are ungradeable due to inadequate clarity or poor field definition. It is important to obtain as much information as possible of retinal vessels and other structures. Deblurring is a major technique which may be developed to restore the lost true image. We present some approaches to tackle this problem. As a key step, we have found a new way to make the Chan-Wong [4] method work.

Image Degradation

In order to derive a model for image deblurring, we consider the forward problem. We assume that the problem of blur and noise corruption of an image can be modelled as

$$z(x,y) = [h \circ u](x,y) + \eta(x,y) \qquad (1)$$

where we assume that the blur is linear special invariant and so can be modelled by the convolution operation of a point spread function (psf) and the true, uncorrupted image:

$$[h \circ u](x,y) = \int h(x-x', y-y')\, u(x',y')\, dx'\, dy'$$

and we have

- $z(x,y)$ is the received image
- $h(x,y)$ is the blur function which may be *known* or *unknown*
- $u(x,y)$ is the true image which we later wish to reconstruct from the received data
- $\eta(x,y)$ represents additive noise

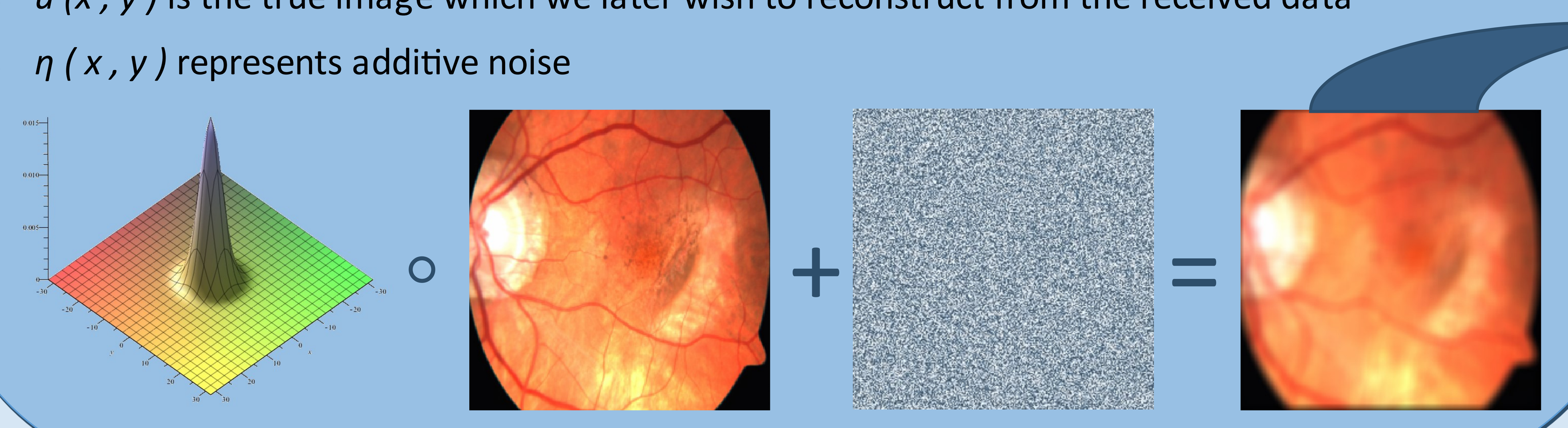


Image Deblurring

In image deblurring, our aim is to remove the blur and noise degradation from a given corrupted image in order to find the hidden true image.

Filtering Approach

Given the discrete form $\boldsymbol{z} = h\,\boldsymbol{u} + \boldsymbol{\eta}$ of the forward problem, we attempt to recover the image by inverting the blur function h:

$$h = V\,\mathrm{diag}(s_i)\,W^T \quad \Longrightarrow \quad h^{-1}\boldsymbol{z} = \boldsymbol{u} + \sum s_i^{-1}(\boldsymbol{v}_i^T\boldsymbol{\eta})\,\boldsymbol{w}_i$$

In order to avoid division by small singular values, we employ a regularisation filter function, thereby solving

$$\boldsymbol{u} = \sum \omega_\alpha(s_i^2)\, s_i^{-1}(\boldsymbol{v}_i^T\boldsymbol{z})\,\boldsymbol{w}_i, \quad e.g.\ \omega_\alpha(s^2) = s^2/(s^2+\alpha).$$

Variational Approach

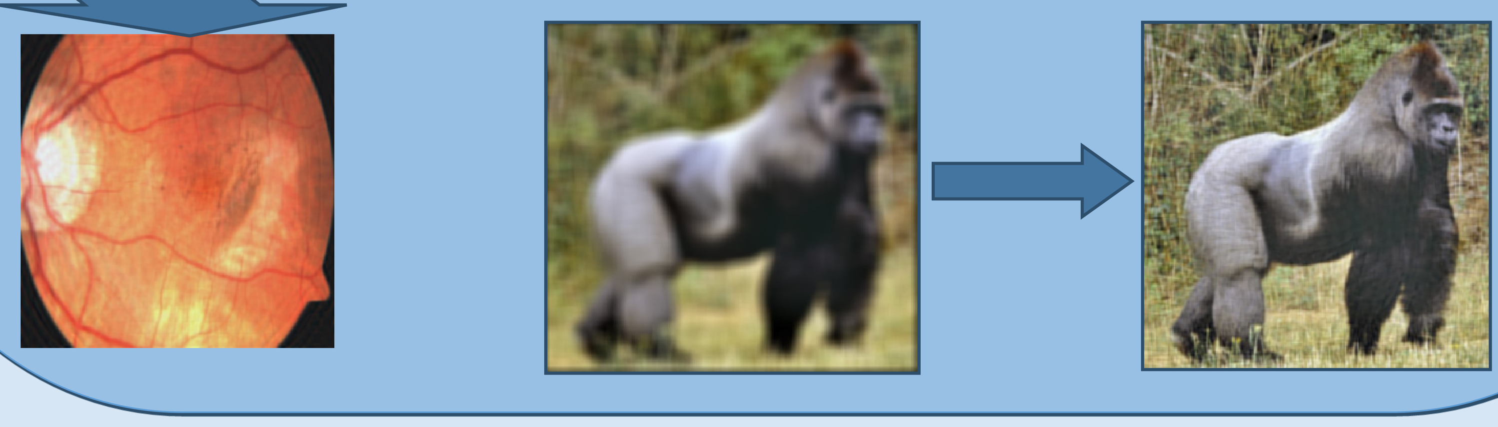
In order to better cope with noise, we take a variational approach to recovering the image. We aim to minimise the below energy functional, composed of data fitting and smoothing, with respect to the image u. Using this, we can recover a good approximation of the true image.

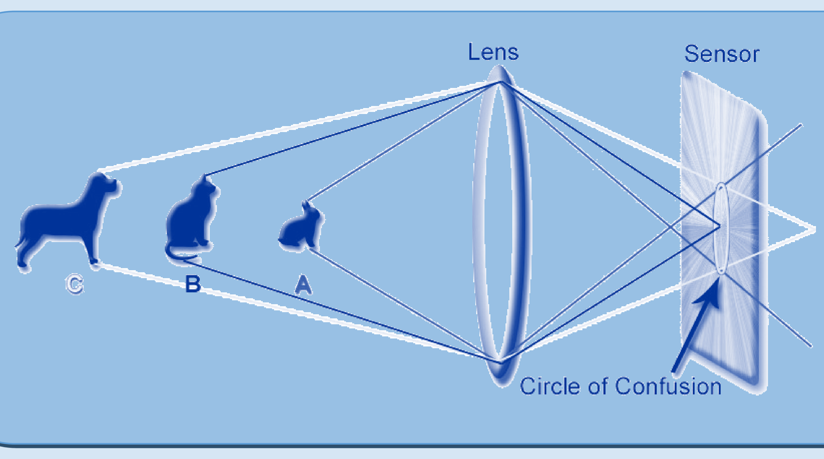
$$F(u) = \int_\Omega \underbrace{([h\circ u](\boldsymbol{x}) - z(\boldsymbol{x}))^2}_{\text{Fitting term}} + \alpha\,\underbrace{\Psi(|\nabla u(\boldsymbol{x})|^2)}_{\text{Regularisation term}}\, d\boldsymbol{x} \qquad (2)$$

where $\boldsymbol{x} = (x_1, x_2)$ and α is a small, non-negative parameter which measures the trade-off between data fitting and smoothness. We solve the following PDE resulting from the minimisation of (2).

$$h^\dagger \circ (h\circ u - z) - \alpha\,\nabla\cdot(\Psi(|\nabla u|^2)\nabla u) = 0$$

where $h^\dagger$ is the adjoint $h^\dagger(\boldsymbol{x}) = h(-\boldsymbol{x})$.





What if we don't know the cause of degradation?

In the likely event that we do not know the cause of the blur, we may still be able to obtain some a priori information from the image. It may be clear that the blur was caused by motion of the camera or incorrect focus, in which case we may use **semi-blind** deblurring [1] using a parametric representation of the blur function.

The problem of deblurring an image with no knowledge of or assumptions about the blur function is known as **blind deblurring**. To solve this problem, we may attempt to identify the blur using sophisticated **blur identification** methods [2,3] or by **simultaneous recovery** of the image and point spread function [4].

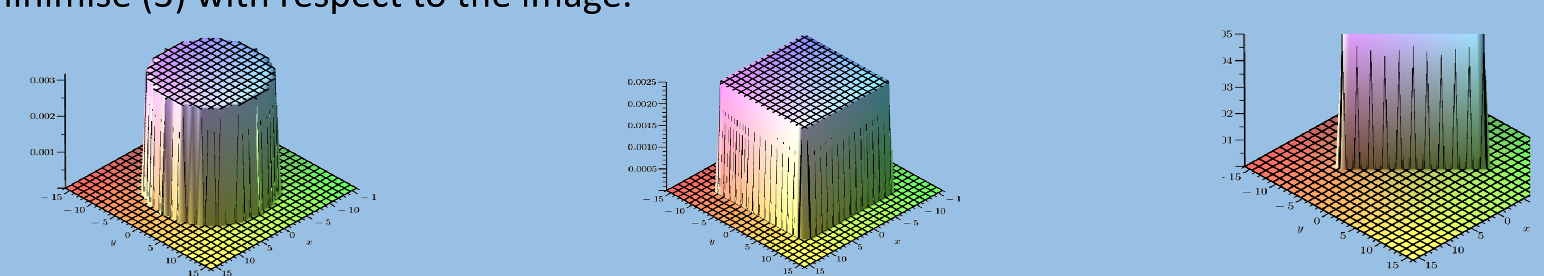
Semi-Blind Image Deconvolution

Chan and Wong [4] proposed *alternate minimisation* of the blur function and image. Incorporating the ideas of [5], we reformulate this in an implicitly constrained way as the minimisation of the energy functional:

$$f(\psi,\sigma) = \int (h_\sigma(\boldsymbol{x}) \circ \tau_a(\psi(\boldsymbol{x})) - z(\boldsymbol{x}))^2 + \vartheta(\psi - \zeta^*)^2 + \alpha\,\Psi(|\nabla\tau_a(\psi(\boldsymbol{x}))|^2)\, d\boldsymbol{x}. \qquad (3)$$

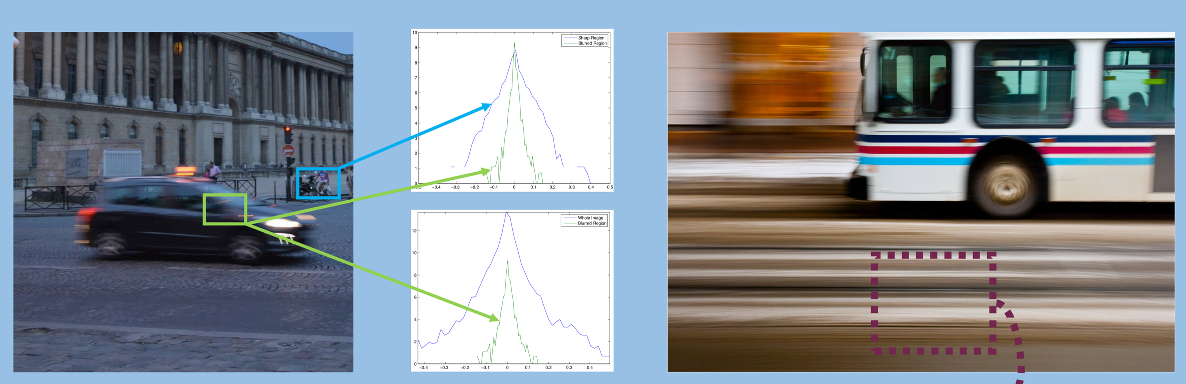
where $\tau_a : \boldsymbol{R} \to \boldsymbol{R}_a$ is a function whose range is a subset of $\boldsymbol{R}$ determined by $\boldsymbol{a}$. This can be used to constrain intensity values to improve the quality of the result. Smoothness is considered for both the blur function h and the image $\tau(\psi)$ and α is a regularisation parameter.

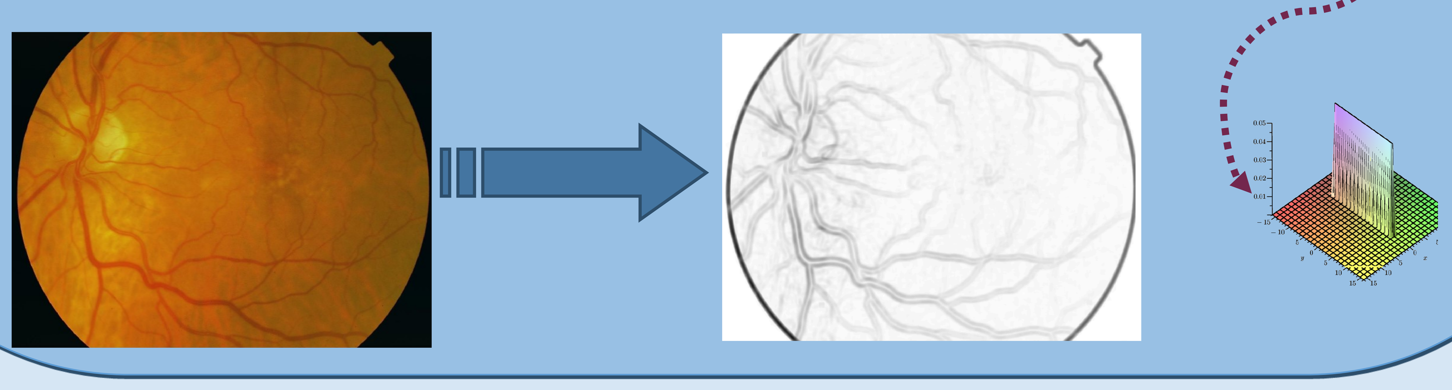
Solving this problem, we simultaneously reconstruct the blur function and the image. In order to minimise this functional, we make initial estimates of the image and point spread function and, fixing the image, we minimise (3) with respect to the psf argument. Then, keeping the psf fixed, we minimise (3) with respect to the image.



Blur Identification

Much recent research has been targeted at identifying blur in an image by considering its statistics. Such methods typically attempt to distinguish blurred and sharp regions while others aim to identify the psf [4,5]. For example, log-gradient histograms show certain trends in blurred/ sharp regions.

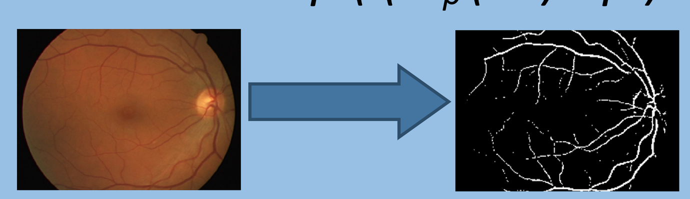




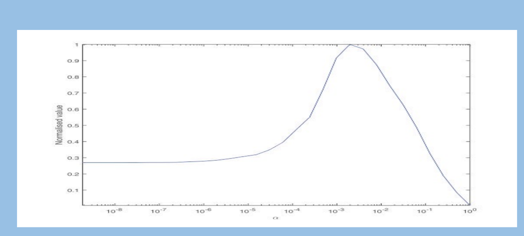
Vessel Segmentation

We aim to select the regularisation parameter automatically. This typically requires some information of the true image. We aim to make use of an expert graded vessel map which may be compared with the restored image.

$$V(u) = \frac{(\kappa - \delta)\circ C_p(u)}{1 + \exp((C_p(u) - \mu)\,\varepsilon^{-1})}$$

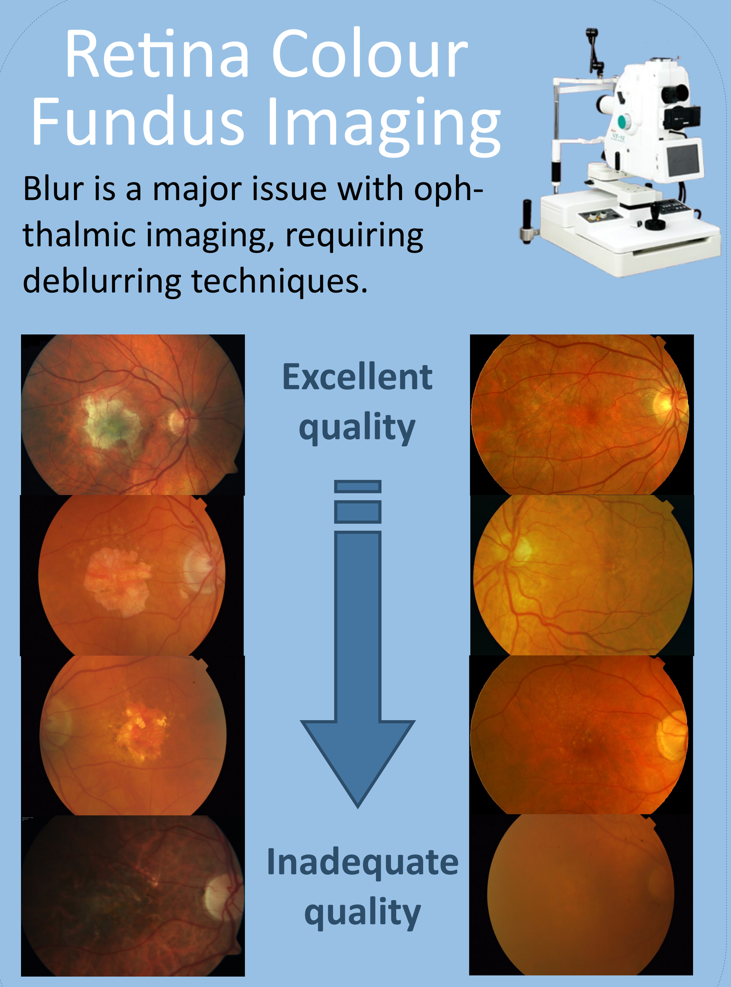


Using this comparison, we may select the degree of regularisation without human intervention.



Retina Colour Fundus Imaging

Blur is a major issue with ophthalmic imaging, requiring deblurring techniques.



References

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