Advanced Blur Removal Methods with Applications to Retinal Imaging for Ophthalmology

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Abstract

Blurring of images occurs in many fields, causing significant problems in retinal imaging. In any diabetic retinopathy screening programme, up to 10% of the images are ungradable due to inadequate clarity or poor field definition. It is important to obtain as much information as possible of retinal vessels and other structures.

Deblurring is a major technique that may be developed to restore the lost true image. We present non-blind and blind approaches to tackle this problem, including an approach for multi-channel images. We propose a new solution algorithm for removing unknown blur (blind case) and show results for retinal images.

1 Introduction

Image processing techniques, such as image reconstruction which includes removing image noise from a given image (denoising) [12], reconstructing an image from an given blurred image (deblurring) [6], reconstructing the missing or damaged portion of an image (inpainting) [3], emphasizing the boundaries of an image by different filters or segmenting a image into subregions (segmentation) [5], have been widely used in many areas. Despite significant development in photographic techniques and technology, blur is still a major cause for image quality degradation in clinical settings. While new segmentation models can cope with noise, they become inflective for blurred images. This is due to many factors such as motion of the camera or more commonly in the case of retinal images the target scene, defocusing of the lens system, imperfections in the electronic, photographic, transmission medium, or obstructions.

An observed blurred image can be written as a convolution of the true image with a Linear Shift-Invariant (LSI) blur, known as the Point Spread Function (PSF) or spatial invariant/variant unknown kernel K [9].

There are three main deconvolution problems: (1) blind deconvolution, which includes the cases when both kernel and image are unknown, (2) semi-blind restoration, in which the kernel is assumed to belong to a class of parametric functions, or (3) non-blind deconvolution where only the image is unknown. All three types are important not only in many scientific applications such as astronomical imaging, medical imaging, and remote sensing, but also for consumer photography.

Deconvolution in the case of known blur, assuming the linear degradation model, has been investigated widely in the last few decades giving rise to a variety of solutions [2, 10, 13]. In non-blind deconvolution, the point spread function is assumed known even though this information is not available in most of the real applications. In cases when the blur is not known the problem becomes harder and much more challenging. Blind deconvolution, which is our main concern below, was first introduced by [6] and a lot of work has been carried out so far to improve the model [1, 7, 11] and an excellent tutorial has been provided by [9].

This paper is organised as follows: in Section 2, we present the formulation for non-blind deblurring and the splitting idea with some results. In Section 3 we show the formulation of the blind method with results, including applications in retinal imaging. In the Section 4, we give the conclusion.

2 Non-Blind Deblurring

The idea of minimising an energy functional of the form

$$\min_u \left\{ ||k*u-z||_2^2 + \alpha \int_\Omega L(u)d\Omega \right\} \tag{1}$$

where the first term is the least squares term which aims to keep the restored image as close to the true image as possible and the second term is a regularisation term which aims to restore edges lost in the reconstruction, was proposed by [12]. A number of functions are commonly selected for $L(u)$, such as $L(u) = u^2$ (Tikhonov [14]) or $L(u) = |\nabla u|^2$. The Total Variation (TV) regularisation term given by $L(u) = |\nabla u|$ has been widely used due to its effectiveness with preserving edges. Minimising equation (1) we derive the Euler Lagrange equation

$$K^T Ku - K^T z - \alpha \nabla \cdot \left(\frac{\nabla u}{|\nabla u|} \right) = 0,$$

where K is an $n^2 \times n^2$ dense matrix of structured blocks. To cope with $\nabla u = 0$, we modify $|\nabla u|$ by $|\nabla u|_\beta = \sqrt{(\nabla_u)^2 + \beta}$ for $\beta > 0$. We therefore aim to solve

$$K^T Ku - K^T z - \alpha \nabla \cdot \left(\frac{\nabla u}{|\nabla u|_\beta} \right) = 0. \tag{2}$$

To solve this non-linear partial differential equation (PDE) we use iterative methods. Time marching is effective but typically proves to be slow to obtain good results. Instead, we use a Conjugate Gradient method aided by preconditioners which aim to increase the stability and speed of the system, such as the Product Preconditioner [15] or the Cosine Transform Preconditioner [4].

2.1 Splitting Deblurring and Denoising

We modify the functional by replacing the restored image variable in the least squares term with a new variable, distinct from that used in the regularisation term, and add a further term to minimise the difference between them. Our modified functional is given by

$$f(u,v) = \frac{1}{2}||k*v-z||_2^2 + \alpha \int_\Omega |\nabla u| d\Omega + \frac{\gamma}{2}||u-v||_2^2. \tag{3}$$

Minimisation of equation (3) with respect to u and v yields the Euler Lagrange equations

$$K^T K v - K^T z - \gamma(u - v) = 0 \tag{4}$$

$$\gamma(u - v) - \alpha \nabla \cdot \left(\frac{\nabla u}{|\nabla \tilde{u}|_\beta} \right) = 0 \tag{5}$$

which we solve for v and u respectively [8]. We may use fixed point and the 2-dimensional fast Fourier transform (fft) to solve equation (4) for v. In order to solve equation (5) for u we use Time Marching or Conjugate Gradient. We make an initial estimate u of the true image u_{true}, which we typically take to be the received image z and repeatedly solve equations (4) and (5) for u and v respectively until u and v are sufficiently close. Our algorithm is given as

Algorithm 1 Non-blind Deblurring

procedure NonBlindSplitting($z, k, \alpha, \gamma, tol$)
  $u \leftarrow z$
  repeat
    Solve $(K^T K + \gamma I)v = K^T z + \gamma u$ for v
    Solve $\gamma(u - v) - \alpha \nabla \cdot \left(\frac{\nabla u}{|\nabla \tilde{u}|_\beta} \right) = 0$ for u
  until $||u - v||_2^2 \leq tol$
end procedure

2.2 Experimental Results

We present experimental results of this method in the non-blind case using the satellite and retinal image examples corrupted by motion and Gaussian blur in figures 1—3.

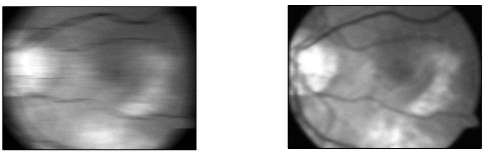

Figure 1: Retina image with motion blur. The PSNR increases from 22.847 in the received image to 27.145 in the restored image. The CPU time to obtain the restored image is 1.48.

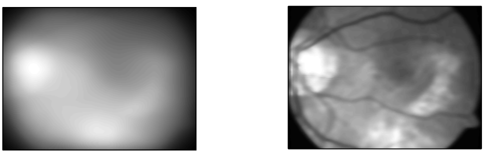

Figure 2: Retina image with strong Gaussian blur. The PSNR increases from 17.862 in the received image to 27.003 in the restored image. The CPU time to obtain the restored image is 1.63.

Figure 3: Satellite image with motion blur. The PSNR increases from 142.688 in the received image to 147.510 in the restored image.

3 Blind Deblurring

A model for Blind Deblurring, given by minimising the functional

$$f(u,k)=\frac{1}{2}||k*u-z||_2^2+\alpha_1\int_\Omega|\nabla u|d\Omega+\alpha_2\int_\Omega|\nabla k|d\Omega. \tag{6}$$

with respect to u and k and solving via an alternate minimisation scheme was proposed by [6]. The model was shown to give excellent results for model problems, but not for testing general images where convergence is an issue. We modify this functional, splitting the restored image and kernel terms as follows:

$$f(u,v,k,h)=\frac{1}{2}||h*v-z||_2^2+\alpha_1\int_{\Omega_1}|\nabla u|d\Omega_1+\alpha_2\int_{\Omega_2}|\nabla k|d\Omega_2+\frac{\gamma_1}{2}||u-v||_2^2+\frac{\gamma_2}{2}||k-h||_2^2.$$

Minimising with respect to u, v, k and h respectively, we obtain the following Euler Lagrange equations

$$\gamma_1(v(x,y)-u(x,y))+h(-x,-y)*(h(x,y)*v(x,y)-z(x,y))=0 \tag{7}$$

$$\gamma_1(u(x,y)-v(x,y))-\alpha_1\nabla\cdot\left(\frac{\nabla u(x,y)}{|\nabla u(x,y)|_\beta}\right)=0 \tag{8}$$

$$\gamma_2(h(x,y)-k(x,y))+v(-x,-y)*(v(x,y)*h(x,y)-z(x,y))=0 \tag{9}$$

$$\gamma_2(k(x,y)-h(x,y))-\alpha_2\nabla\cdot\left(\frac{\nabla k(x,y)}{|\nabla k(x,y)|_\beta}\right)=0 \tag{10}$$

An overall algorithm is given in Algorithm 2.

3.1 Experimental Results

We present experimental results of the blind restoration using this model of the satellite and retina images corrupted by motion blur in figures (4) and (5).

4 Conclusions

A new splitting method algorithm is proposed for blind deconvolution restoration. Test results in retinal images are encouraging and show that it is potentially useful in medical imaging.

Algorithm 2 Blind Deblurring

function BLINDSPLITTING($z, k_{initial}, \alpha_1, \alpha_2, \gamma_1, \gamma_2, tol, maxit$)
  $u \leftarrow z$
  $k \leftarrow k_{initial}$
  for $i \leftarrow 1$ **to** $maxit$ **do**
    repeat
      Solve $u(-x,-y)uk - u(-x,-y)z + \gamma(k-h) = 0$ for k
      Solve $\gamma(h-k) - \alpha\nabla \cdot \left(\frac{\nabla h}{|\nabla \tilde{h}|}\right) = 0$ for h
    until $||h-k||_2^2 \leq tol$
    Impose: $h(x,y) \leftarrow 0$ if $h(x,y) < 0$
    Impose: $h(x,y) \leftarrow (h(x,y) + h(-x,-y))/2 \; \forall x,y \in \Omega$
    Impose: $h \leftarrow h / \int_\Omega h(x,y)dxdy$
    repeat
      Solve $k(-x,-y)kv - k(-x,-y)z + \gamma(v-u) = 0$ for v
      Solve $\gamma(u-v) - \alpha\nabla \cdot \left(\frac{\nabla u}{|\nabla \tilde{u}|}\right) = 0$ for u
    until $||u-v||_2^2 \leq tol$
    Impose: $u(x,y) \leftarrow 0$ if $u(x,y) < 0$
  end for
end function

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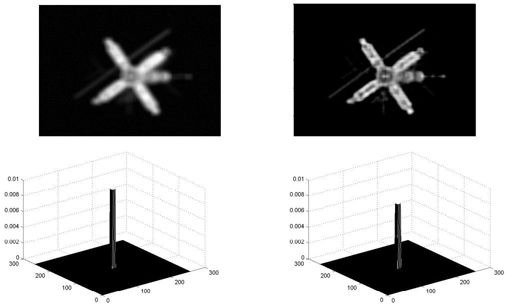

Figure 4: Satellite image corrupted with motion blur. The CPU time to obtain the restored image is 16.71. On the top left we show the received image, on the top-right is the restored image, bottom left is the initial estimate of the kernel and bottom right is the restored kernel.

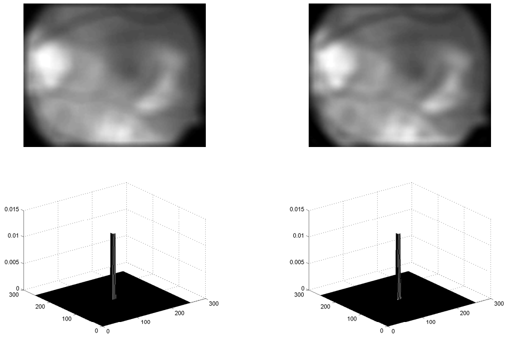

Figure 5: Retina image corrupted with motion blur. The CPU time to obtain the restored image is 18.77. On the top left we show the received image, on the top-right is the restored image, bottom left is the initial estimate of the kernel and bottom right is the restored kernel.

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