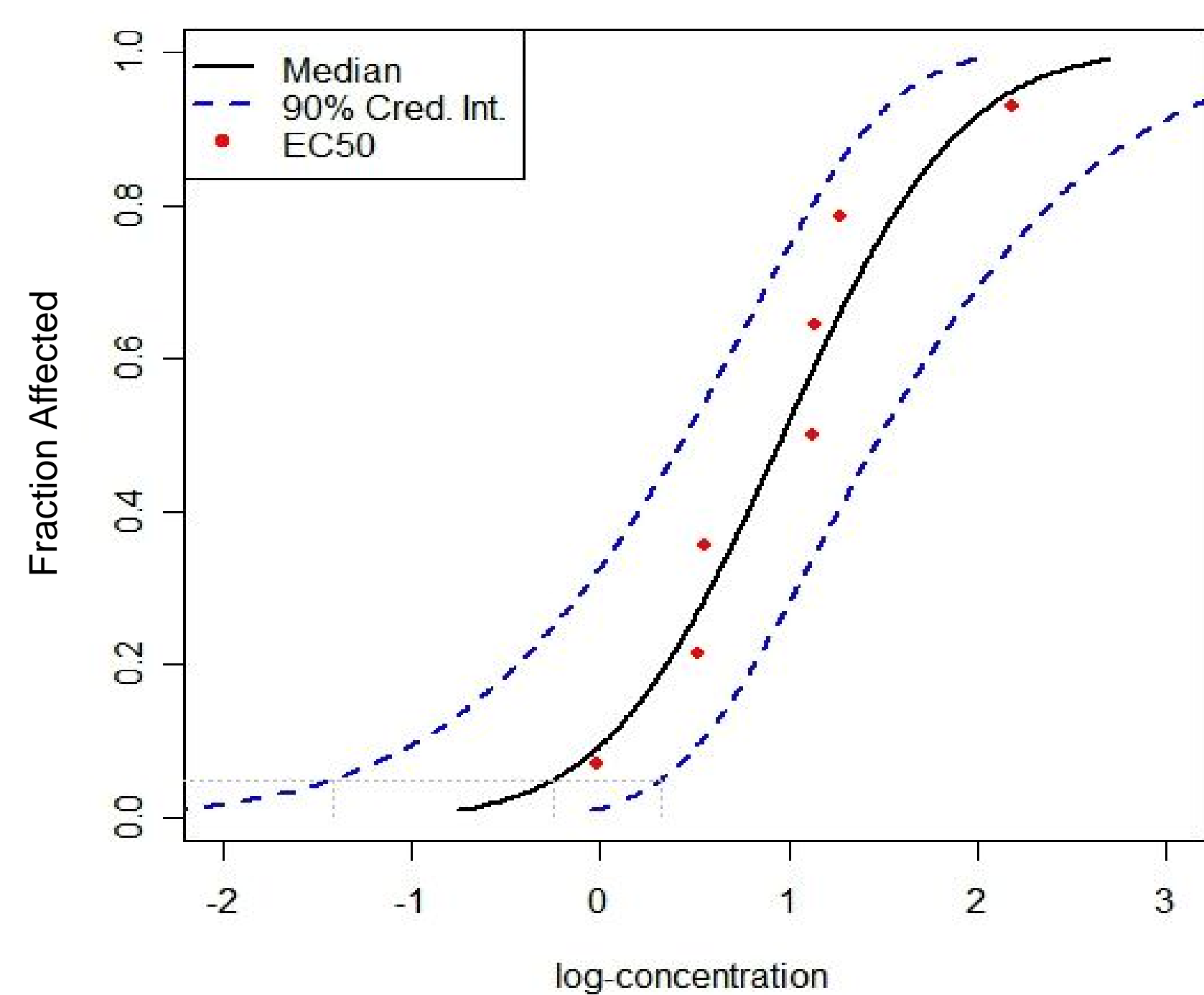


Graeme Hickey<sup>1\*</sup>, Peter Craig<sup>1</sup> & Andy Hart<sup>2</sup>

<sup>1</sup>Durham University, Dept. of Mathematical Sciences, Durham, UK

<sup>2</sup>The Food and Environment Research Agency, York, UK

\*Email: g.i.hickey@durham.ac.uk



- The hazardous concentration to  $p\%$  of species ( $HC_p$ ), defined up to a specific ecological community, is the concentration such that the probability a randomly selected species from the assemblage will have its toxicological endpoint violated is  $p\%$ .

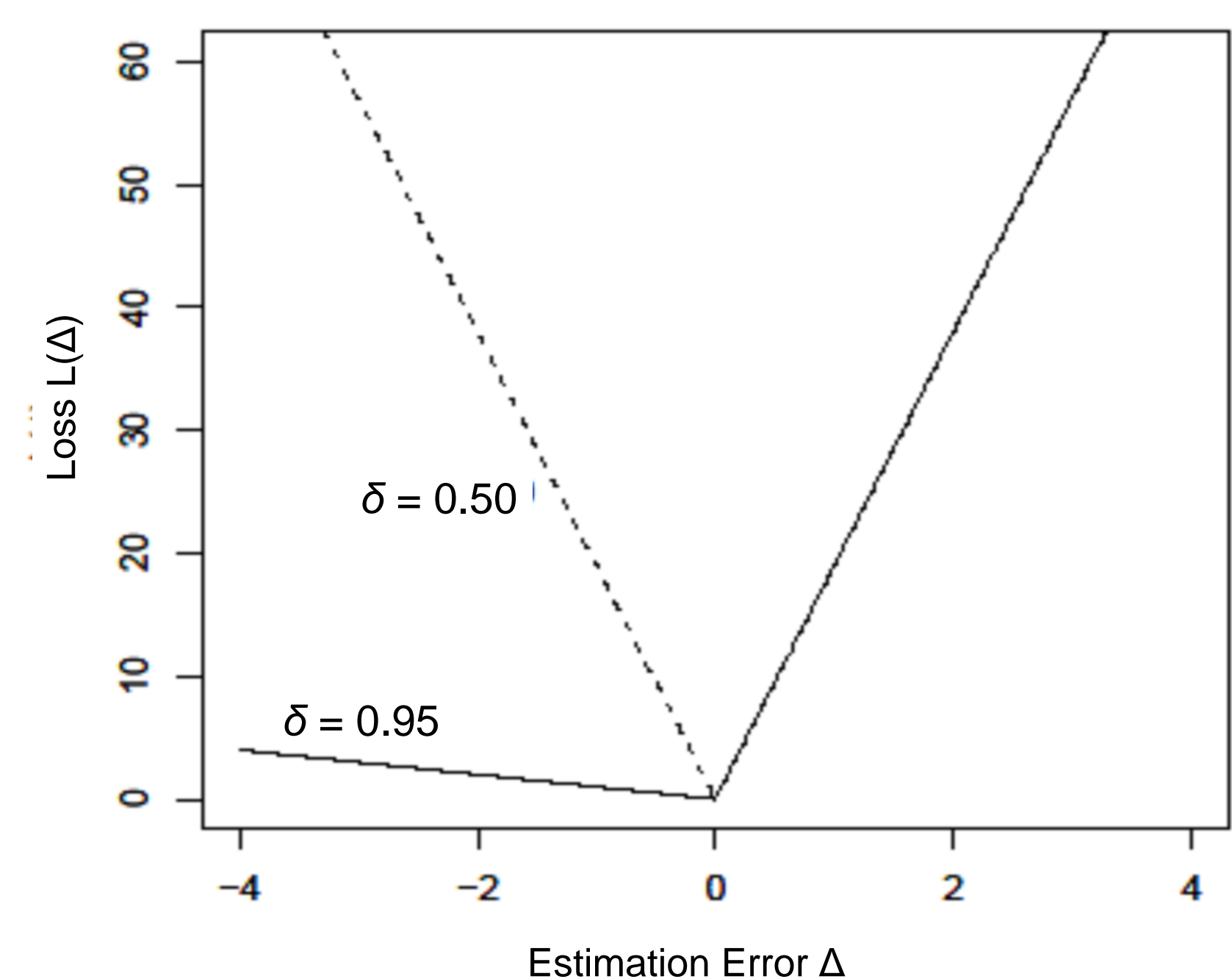
- The  $HC_5$  has become a standard benchmark safety limit which is presumed to have little adverse effect to species at the community level.

- Figure 1 (left).** An example Gaussian Species Sensitivity Distribution (SSD) with estimated median and 90% credible limits. Dashed-grey lines =  $HC_5$  (and 90% credible interval).

- Having adequately modelled the 'true' SSD, one needs to estimate the  $HC_5$ .

- A popular estimator is the Aldenberg & Jaworska (2000; *Ecotox. Environ. Saf.* 46: 1-18)  $\delta$ -estimator, where  $\delta$  is a measure of certainty. The data in Fig. 1 is the cadmium exposed soil organism data from Aldenberg & Jaworska.

- Proposals have emerged for one to use the lower one-sided  $\delta = 95\%$  underestimate confidence limit in order to have conservativeness.



- A loss function measures the 'cost' (not necessarily financial) of making an error in estimation.

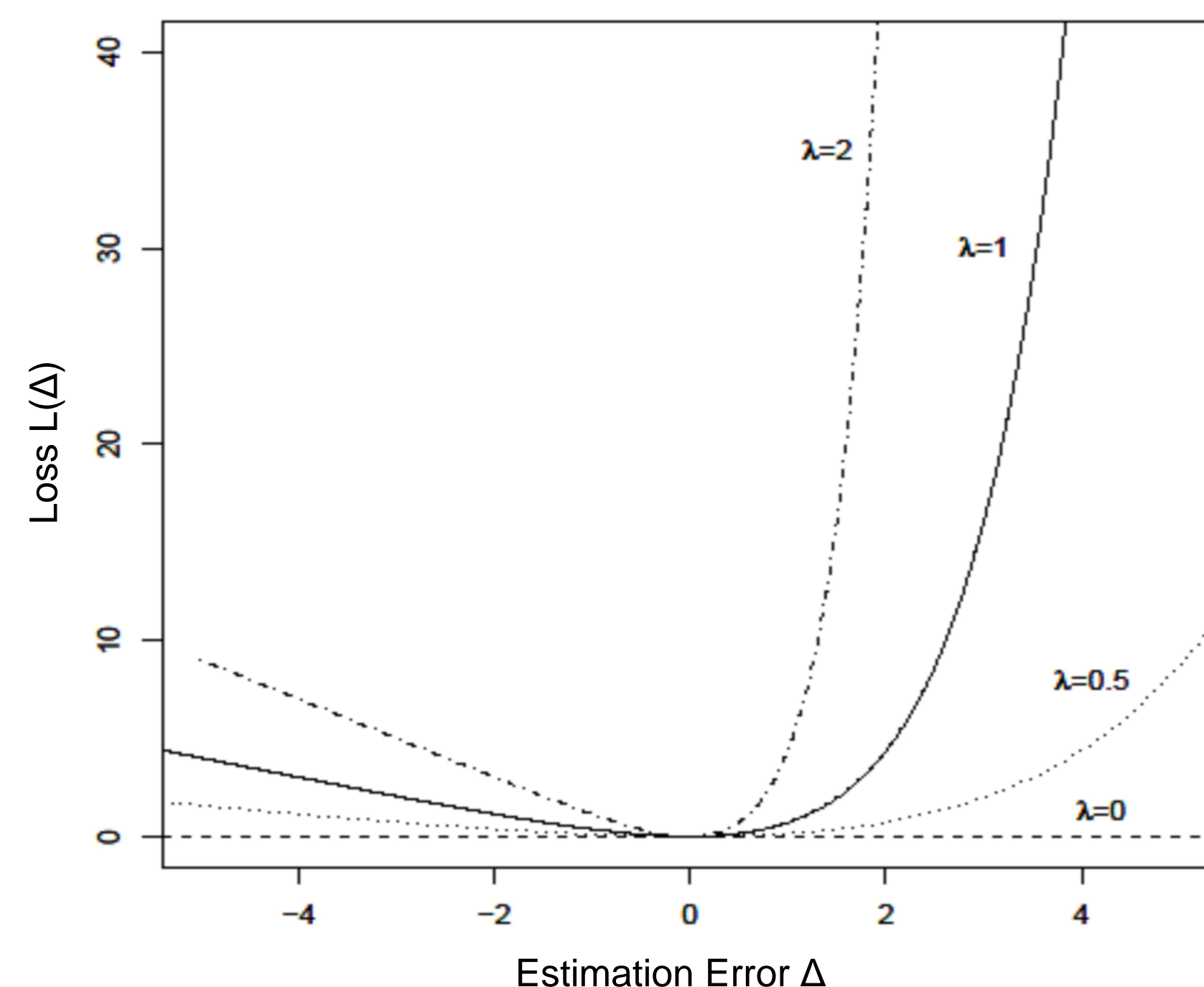
- The **precautionary principle** would imply that over-estimating the  $HC_p$  would be worse than under-estimating it. Therefore we would place a higher loss on overestimation.

- In the Bayesian paradigm we can use available toxicity data (e.g.  $EC_{50}$ s) to update prior knowledge for the distribution of the  $HC_p$  — known as the *posterior* distribution.

- A popular choice of estimator in decision theoretic statistics is the **Bayes rule** — the decision which minimises the statistically expected loss with respect to the posterior distribution of the true  $HC_p$ .

- Figure 2 (left).** The Generalised Absolute Loss function. Aldenberg & Jaworska's class of estimators corresponds to Bayes rules under this loss function class. The symmetric 'V' shape corresponds to the median ( $\delta = 0.50$ ) estimator, and the union of the solid lines corresponds to the lower one-sided ( $\delta = 0.95$ ) underestimate confidence limit.  $\Delta$  = the estimation error ( $\Delta > 0$  : overestimation,  $\Delta < 0$  : underestimation).  $L(\Delta)$  = cost of estimation error (arbitrary scale). GAL is parameterised by  $C_2 / C_1$  — the cost of overestimation relative to underestimation.

- Clearly the asymmetrical sub-class ( $\delta > 0.50$ ) is appealing from a conservative perspective. However, is the linearity reflective of a risk managers true cost-benefit portfolio? Furthermore, is punishing overestimation 19 times more than underestimation reasonable?



- A risk manager can choose any suitable loss function which represents their requirements.

- An alternative loss function is the modified-LINEAR Exponential (LINEX) function. This is a non-linear asymmetrical loss function parameterised by conservatism control parameter  $\lambda$ . As  $\lambda > 0$  increases, so does the level of conservatism.

- Figure 3 (left).** Example of standard-LINEX loss functions for some different values of  $\lambda$ .

- A modified version of LINEX allows for the risk manager to specify loss on a scale which is independent of the SSD variability.

- Reducing estimators to decision-theoretic interpretations *potentially* allows for more transparency in estimation methods.

- An algorithm for obtaining a suitable value of  $\lambda$  is suggested in Hickey *et al.* (2009; *Ecotox. Environ. Saf.* 72: 293-300).

Loss Function	Parameterisation	$HC_5$ Estimate ( $\mu\text{g Cd / mg}$ )
GAL (A&J $\delta = 0.95$ )	$C_2 = 19C_1$	0.038
GAL (A&J $\delta = 0.50$ )	$C_2 = C_1$	0.568
GAL (A&J $\delta = 0.05$ )	$19C_2 = C_1$	2.112
Modified-LINEX	$\lambda = 0.5$	0.633
Modified-LINEX	$\lambda = 1$	0.542
Modified-LINEX	$\lambda = 3$	0.235
Modified-LINEX	$\lambda = 5$	0.055

- It is important for a risk manager to understand the level of conservatism in their estimates, otherwise they might reject or allow a substance inappropriately.

- Table 1 (left).**  $HC_5$  estimates for the classical cadmium toxicity dataset (discussed in Aldenberg and Jaworska 2000) for two different loss function classes: GAL and modified-LINEX, and different parameterisations.

- There is a wide range in estimates — reinforcing the need to suitably estimate the  $HC_5$ .

- An introduction to loss functions for estimating  $HC_p$ s is discussed in Hickey *et al.* (2009).

- Overall conclusion:** some current  $HC_p$  estimation methods are arbitrary, but loss functions can potentially help risk managers to control the degree of conservatism in more appropriate ways.