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Renormalisation Invariance and the Soft β-Functions

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We demonstrate that the soft supersymmetry–breaking terms in a $N = 1$ theory can be linked by simple renormalisation group invariant relations which are valid to all orders of perturbation theory. In the special case of finite $N = 1$ theories, the soft terms preserve finiteness to all orders.

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Recently there has been remarkable progress in the understanding of the soft supersymmetry-breaking β-functions [1]–[3]. For a $N = 1$ supersymmetric gauge theory with superpotential

$$W(\Phi) = \frac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{ij}\Phi_i\Phi_j, \tag{1}$$

we take the soft breaking Lagrangian L_{SB} as follows:

$$\begin{aligned} L_{SB}(\Phi, W) = &- \left\{ \int d^2\theta\eta \left(\frac{1}{6}h^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}b^{ij}\Phi_i\Phi_j + \frac{1}{2}MW_A{}^\alpha W_{A\alpha} \right) + \text{h.c.} \right\} \\ &- \int d^4\theta\bar{\eta}\eta\bar{\Phi}^j(m^2)^i{}_j(e^{2gV})_i{}^k\Phi_k. \end{aligned} \tag{2}$$

Here $\eta = \theta^2$ is the spurion external field and M is the gaugino mass. Use of the spurion formalism in this context was pioneered by Yamada [4]; in [2], [3] it was shown that β_h, β_b and β_M are given by the following simple expressions:

$$\beta_h^{ijk} = \gamma^i{}_l h^{ljk} + \gamma^j{}_l h^{ilk} + \gamma^k{}_l h^{ijl} - 2\gamma_1^i{}_l Y^{ljk} - 2\gamma_1^j{}_l Y^{ilk} - 2\gamma_1^k{}_l Y^{ijl} \tag{3a}$$

$$\beta_b^{ij} = \gamma^i{}_l b^{lj} + \gamma^j{}_l b^{il} - 2\gamma_1^i{}_l\mu^{lj} - 2\gamma_1^j{}_l\mu^{il} \tag{3b}$$

$$\beta_M = 2\mathcal{O}\left(\frac{\beta_g}{g}\right) \tag{3c}$$

where

$$\mathcal{O} = \left(Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial Y^{lmn}} \right), \tag{4}$$

and

$$(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j. \tag{5}$$

$\gamma^i{}_j(g, Y, Y^*)$ is the anomalous dimension of the chiral multiplet. These relations are valid in DRED (supersymmetric dimensional regularisation with minimal subtraction). A straightforward application of the spurion formalism leads to the result

$$(\beta_{m^2})^i{}_j = \Delta\gamma^i{}_j, \tag{6}$$

where

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2MM^*g^2\frac{\partial}{\partial g^2} + \tilde{Y}_{lmn}\frac{\partial}{\partial Y_{lmn}} + \tilde{Y}^{lmn}\frac{\partial}{\partial Y^{lmn}}, \tag{7}$$

$Y_{lmn} = (Y^{lmn})^*$, and

$$\tilde{Y}^{ijk} = (m^2)^i{}_l Y^{ljk} + (m^2)^j{}_l Y^{ilk} + (m^2)^k{}_l Y^{ijl}. \tag{8}$$

This result, however, is not valid in DRED because the ϵ-scalars associated with DRED acquire a mass through radiative corrections [5]. Moreover there is no scheme perturbatively related to DRED such that Eq. (6) is valid. It is, however, possible to define a scheme, DRED$'$, closely related to DRED, such that β_{m^2} is independent of the ϵ-scalar mass [6]. Let us hypothesise that in DRED$'$ the correct result for β_{m^2} is

$$(\beta_{m^2})^i{}_j = \left[\Delta + X(g, Y, Y^*, h, h^*, m, M)\frac{\partial}{\partial g}\right]\gamma^i{}_j, \tag{9}$$

where the X term represents in some way the contribution of the ϵ-scalar mass renormalisation. From the explicit calculation of [5], [6] we know that in DRED$'$ the leading contribution to X is given by

$$X = -2Sg^3(16\pi^2)^{-1} \tag{10}$$

where

$$S\delta_{AB} = (m^2)^k{}_l(R_A R_B)^l{}_k - MM^*C(G)\delta_{AB} \tag{11}$$

(see [5] for our group theory conventions). In this paper we will show that the existence of the X-term is a necessary consequence of a quite different hypothesis: namely, the existence of a set of renormalisation group invariant relations expressing Y as a function of g; and h, b and m^2 as functions of M, μ and g. These relations amount to a generalisation to the softly-broken case of the coupling constant reduction program [7]. Remarkably, implementation of this program will enable us to verify Eq. (10).

In what follows we will specialise for simplicity to the case of a single real Yukawa coupling Y and a single superfield Φ transforming according to a representation R of the gauge group, which we assume admits both a cubic and a quadratic invariant, and we will take all the soft terms to be real as well. Let us suppose that there exists a RG invariant trajectory $Y(g)$. It follows immediately that

$$\beta_Y = 3\gamma Y = Y'\beta_g, \quad \text{where} \quad Y' = \frac{dY}{dg}. \tag{12}$$

If we seek a perturbative solution to Eq. (12) of the form

$$Y = ag + bg^3 + cg^5 + \cdots \tag{13}$$

then we have[8]

$$a^2 = 4C(R) + 2Q/3 \quad \text{and} \quad b = 0 \tag{14}$$

where $\beta_g = Qg^3(16\pi^2)^{-1} + \cdots$. The special case $Q = a^2 - 4C(R) = 0$ corresponds to a one–loop finite theory; it is possible to construct $Y(g)$ then so that $\beta_g = \gamma = 0$ to all orders [9]. The extension of the finite case to include the soft–breaking terms has been considered recently by Kazakov [10]; our results generalise his to the case of a non-trivial solution to Eq. (12).

It was shown in [8] that, given Eq. (14), the following relations among the soft parameters are RG invariant through two loops:

$$h = -MY, \tag{15a}$$

$$m^2 = \frac{1}{3}(1 - \frac{1}{16\pi^2}\frac{2}{3}g^2Q)M^2, \tag{15b}$$

$$b = -\frac{2}{3}M\mu. \tag{15c}$$

We will now proceed to extend these relations to all orders in g.

For real Y, we have

$$\mathcal{O} = \frac{1}{2}\left(Mg\frac{\partial}{\partial g} - h\frac{\partial}{\partial Y}\right), \tag{16}$$

inspection of which suggests immediately the possibility

$$h = -MgY' \tag{17}$$

since then we have simply

$$\mathcal{O} = \frac{1}{2}Mg\frac{d}{dg}. \tag{18}$$

In particular, for the finite case such that $\gamma\left(g, Y(g)\right) = 0$ for any g, then also $\gamma_1 = \mathcal{O}\gamma = 0$ and hence $\beta_h = 0$. Notice that in the approximation $Y = ag$ we have $h = -MY$. Thus Eq. (17) provides the generalisation of Eq. (15a) to all orders.

Just as we wanted the relation $Y = Y(g)$ to define a trajectory rather than a point in the space of couplings, thus leading to Eq. (12), we also require Eq. (17) to be RG invariant, which is true if

$$\beta_h + \left(\beta_M g + M\beta_g\right)Y' + Mg\beta_g Y'' = 0. \tag{19}$$

It is easy to verify Eq. (19), using Eqs. (3a) and (3c).

The corresponding result for the soft ϕ^2 mass term b is

$$b = -\frac{2}{3}\mu M \frac{g}{Y} Y' \tag{20}$$

and this can also be shown to be RG invariant in a similar way. If we assume that there also exists a RG trajectory for μ, of the form $\mu = \mu(g)$ then Eq. (20) can be written

$$b = -Mg\mu', \tag{21}$$

which is similar to Eq. (17), and also generalises more easily to the many-coupling case, which we shall discuss later.

We turn now to the β-function for the soft $\phi\phi^*$ mass term. As we already indicated this presents special problems. We will seek a RG invariant relation of the form

$$m^2 = \frac{1}{3}M^2 f(g), \tag{22}$$

the form of which is motivated by Eq. (15b).

For real Y we obtain from Eq. (4) and (17) that

$$\mathcal{O}\mathcal{O}^* = \frac{M^2}{4}\left[g^2\frac{\partial^2}{\partial g^2} + g\frac{\partial}{\partial g} + 2g^2 Y'\frac{\partial^2}{\partial g \partial Y} + g^2 (Y')^2 \left(\frac{\partial^2}{\partial Y^2} + \frac{1}{Y}\frac{\partial}{\partial Y}\right)\right], \tag{23}$$

and hence (using Eqs. (9), (12)) that

$$\begin{aligned}\beta_{m^2} = M^2\Big[&\tfrac{1}{2}g^2\gamma'' + \tfrac{1}{2}g\gamma' - \tfrac{1}{2}\left(g^2 Y'' + gY' - g^2\frac{(Y')^2}{Y}\right)\frac{\partial\gamma}{\partial Y} \\ &+ (g + \tilde{X})\frac{\partial\gamma}{\partial g} + f(g)Y\frac{\partial\gamma}{\partial Y}\Big],\end{aligned} \tag{24}$$

where we have written $X = M^2\tilde{X}(g)$ on the RG trajectory.

Now in order that we can obtain $\beta_{m^2} = 0$ in the finite case it is clear that the partial derivatives with respect to g and Y in the above expression will need to fit together into total derivatives with respect to g. Thus we require

$$f(g) = \left(\frac{3}{2}g + \tilde{X}\right)\frac{Y'}{Y} + \frac{1}{2}g^2\left[\frac{Y''}{Y} - \left(\frac{Y'}{Y}\right)^2\right] \tag{25}$$

whence

$$\beta_{m^2} = M^2\left[\frac{1}{2}g^2\gamma'' + \left(\frac{3}{2}g + \tilde{X}\right)\gamma'\right]. \tag{26}$$

By demanding RG invariance of Eq. (22), however, we obtain another expression for β_{m^2}:

$$\beta_{m^2} = M^2\frac{1}{3}\left[(f' - 2f/g)\,\beta_g + 2f\beta_g'\right]. \tag{27}$$

It follows that $\tilde{X}$ satisfies the equation

$$g^2\frac{d}{dg}\left(\frac{\tilde{X}\beta_g}{g^2}\right) = \tilde{X}'\beta_g - \frac{2}{g}\tilde{X}\beta_g + \tilde{X}\beta_g' = \frac{1}{2}\left(3\beta_g - 3g\beta_g' + g^2\beta_g''\right) \tag{28}$$

whence

$$X = M^2\tilde{X} = M^2\left[\frac{1}{2}g^2\frac{\beta_g'}{\beta_g} - \frac{3}{2}g + A\frac{g^2}{\beta_g}\right] \tag{29}$$

where A is an arbitrary constant. From Eq. (25) we then find

$$\begin{aligned} f(g) &= \frac{1}{2}g^2\left[\frac{Y''}{Y} - \left(\frac{Y'}{Y}\right)^2 + \frac{\beta_g'Y'}{\beta_gY}\right] + A\frac{g^2Y'}{\beta_gY} \\ &= \frac{3g^2}{2\beta_g}\gamma' + A\frac{g^2Y'}{\beta_gY}. \end{aligned} \tag{30}$$

We can now compare our result for X with the existing perturbative calculation [5] of β_{m^2}. Now we know that at the one loop level, β_{m^2} satisfies Eq. (9) with $X = 0$; we expect the leading contribution to X to be $O(g^3)$. It follows that we must take $A = 0$ above. Using Eq. (14) and the two–loop result for β_g [11] we obtain

$$\beta_g = Qg^3(16\pi^2)^{-1} - \frac{2}{3}Q^2g^5(16\pi^2)^{-2} + \cdots \tag{31}$$

and hence

$$X = -\frac{2}{3}QM^2g^3(16\pi^2)^{-1} + \cdots \tag{32}$$

while from Eq. (30) we obtain

$$f(g) = 1 - \frac{1}{16\pi^2}\frac{2}{3}g^2Q + \cdots. \tag{33}$$

Observe that this result for f is consistent with Eq. (15b); moreover, it is easy to show that our result for X is consistent with Eq. (10).

Let us now consider the special case of a finite theory, already considered in [10]. If we define h by Eq. (17), then it follows immediately from Eqs. (3), (5) and (18) that if $\beta_g = \gamma = 0$ then $\beta_h = \beta_M = 0$ to all orders. We also have $\beta_b = 0$; notice that this result in fact is true even if we do not impose Eq. (20); which is why Eq. (20) does not appear in [10]. Finally, from Eq. (26), we see that in the finite case we have $\beta_{m^2} = 0$ to all orders as long as X is well-defined in that case. This is not quite a trivial requirement, as can be seen from Eq. (29). But in the finite case we have

$$\beta_g(g, Y(g)) = Qg^3 F(g)(16\pi^2)^{-1} \tag{34}$$

where $F(g) = 1 - \frac{2}{3}Qg^2(16\pi^2)^{-1} + \cdots$, whence it follows that X is finite when $\beta_g = 0$. There is, however, no reason to think that X is zero to all orders, in the finite case, although the leading contribution vanishes, as can be seen from Eq. (32).

Thus we have shown that supersymmetric theories including soft terms admit RG invariant trajectories for both the Yukawa couplings and the soft terms. As a consequence the formula for β_{m^2} requires a term not predicted by a naive application of the spurion formalism. We have determined the associated RG function to all orders on the aforesaid trajectory:

$$X = M^2 \left[\frac{1}{2}g^2 \frac{\beta_g'}{\beta_g} - \frac{3}{2}g\right]. \tag{35}$$

Our results for the soft terms are:

$$h = -Mg\frac{dY}{dg} \tag{36a}$$

$$b = -Mg\frac{d\mu}{dg} \tag{36b}$$

$$m^2 = \frac{g^2}{2\beta_g} M^2 \frac{d\gamma}{dg}. \tag{36c}$$

Let us now discuss the case of a general superpotential, Eq. (1). In Eq. (35) we have merely to replace M^2 by MM^*, while in place of Eq. (36) we have:

$$h^{ijk} = -Mg\frac{dY^{ijk}}{dg} \tag{37a}$$

$$b^{ij} = -Mg\frac{d\mu^{ij}}{dg} \tag{37b}$$

$$(m^2)^i{}_j = \frac{g^2}{2\beta_g} MM^* \frac{d\gamma^i{}_j}{dg}. \tag{37c}$$

In deriving Eq. (37) we have used the generalisation of Eq. (12),

$$\beta_Y^{ijk} = Y^{l(ij}\gamma^{k)}{}_l = \frac{dY^{ijk}}{dg}\beta_g. \tag{38}$$

We found it necessary to assume that

$$Y^{ijk}\frac{\partial\gamma^l{}_m}{\partial Y^{ijk}} = Y^{*ijk}\frac{\partial\gamma^l{}_m}{\partial Y^{*ijk}}, \quad \text{(no sum on } i,j,k) \tag{39}$$

and a similar equation for β_g, and also that γ is diagonal. Using these assumptions and Eq. (38), it is easy to show that

$$Y'^{ijk}\frac{\partial\gamma^l{}_m}{\partial Y^{ijk}} = Y^{*\prime ijk}\frac{\partial\gamma^l{}_m}{\partial Y^{*ijk}}, \quad \text{(no sum on } i,j,k), \tag{40}$$

and once again a similar equation for β_g, which is necessary, for example, to establish Eq. (18) in the general case.

The fact that in the general case the soft terms preserve finiteness in finite supersymmetric theories also follows from Eq. (37). Here we are broadly in agreement with [10], but we believe our analysis places the results on a firmer footing, being associated with a specific and well-defined subtraction procedure.

An interesting question left unanswered is the form of X away from the RG invariant trajectory $Y = Y(g)$. We hope to return to this, and the phenomenological consequences of our general result Eq. (37), elsewhere.

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