The strangeness content of the nucleon

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We evaluate the matrix element of $\bar{q}q$ in hadron states on a lattice. We find substantial mixing of the connected and disconnected contributions so that the lattice result that the disconnected contribution to the nucleon is large does not imply that the $\bar{s}s$ content is large. This has implications for dark matter searches.

arXiv:hep-lat/0109028v1 25 Sep 2001

1. INTRODUCTION

An important challenge in physics and cosmology is to understand the nature of dark matter. One plausible candidate is for this dark matter to be the lightest supersymmetric particle: the neutralino. In this case the dark matter can be detected by scattering from nuclear targets, and experimental explorations are currently under way. To extract a physical flux from such experiments, one needs the appropriate cross section for scattering of a neutralino off a nucleon. This has been evaluated [1] and depends on MSSM parameters and on the QCD matrix elements of the scalar quark current: $\langle N|\bar{q}q|N\rangle$. The Higgs exchange terms are dominant and hence the scalar current enters multiplied by the relevant quark mass. For this reason, the strange quark contribution is expected to be especially important. Moreover the u and d quark contributions are related to the πN σ term and are relatively well known whereas the strange contribution is unknown phenomenologically to within a factor of 3.

The usual way to parametrise the contribution of the strange quark is by

$$y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} \qquad (1)$$

Estimates [2] using chiral perturbation theory suggest $y = 0.2(2)$.

This ratio of quark matrix elements can be calculated in principle using lattice methods (see [3] for a review). The required correlations are illus-

trated in fig. 1. The disconnected contribution D_3 (numerator of y) can either be calculated by evaluating the three-point correlator with a disconnected loop or using the lattice equivalent of the Feynman-Hellman theorem to relate it to the derivative of the nucleon mass with respect to the sea-quark hopping parameter (see ref. [4] for a discussion of this). Likewise the connected contribution C_3 can be evaluated either as a three-point correlation or as a derivative of the nucleon mass with respect to the valence-quark hopping parameter. Then, in this approach we expect $y = D_3/(C_3 + D_3)$.
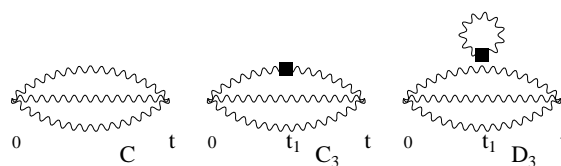

Figure 1. Connected and disconnected diagrams

Using $N_f = 2$ flavours of sea-quark, SESAM [5] obtain $y = 0.59(13)$ and, using the same method, we would obtain a similar result. Using the three-point correlator approach, it is possible to estimate y in quenched studies and results of around 0.6 were obtained [6] although the Ken-

tucky group argued [7] that renormalisation effects should reduce this to around 0.36(3) (but see criticism of this approach in ref [5]).

This large value of y obtained from lattice studies is surprising and it also has major implications for the analysis of dark matter scattering experiments. Here we discuss the status of these lattice determinations critically and we conclude that y is consistent with zero.

2. LATTICE ANALYSIS

The Feynman-Hellman theorem relates matrix elements of scalar quark currents in a nucleon to derivatives of the nucleon mass with respect to the quark mass. These identities can be derived both in the continuum and on the lattice. Here we consider Wilson-like lattice fermion formulations. The lattice equivalent of the Feynman-Hellman theorem is that [4] the following lattice observables (here we define $m^b \equiv 1/(2\kappa)$) are related:

$$\frac{\partial(aM_N)}{\partial m^b_{val}} = \lim_{t_1,(t-t_1)\to\infty} \frac{C_3(t_1,t)}{C(t)} \qquad (2)$$

$$\frac{\partial(aM_N)}{\partial m^b_{sea}} = -N_f \lim_{t_1,(t-t_1)\to\infty} \frac{D_3(t_1,t)}{C(t)} \qquad (3)$$

The input quark mass parameters can be written in terms of the physical bare quark masses, with an additive mass renormalisation (m_A).

$$m^b_{val} = m_A + am_{val} \qquad (4)$$

$$m^b_{sea} = m_A + am_{sea} \qquad (5)$$

Now m_A may be determined by varying the valence quark mass and determining the critical hopping parameter at which the pion mass (and hence am_{val}) becomes zero, or equivalently by finding the critical valence hopping parameter at which the PCAC mass becomes zero. These extrapolations to determine m_A are at fixed m^b_{sea} and hence m_A will depend on m^b_{sea}. The lattice spacing a also depends on the bare sea-quark mass parameter m^b_{sea}. These effects can be summarised by the following derivatives

$$X = \frac{dm_A}{dm^b_{sea}} \quad \text{and} \quad B = \frac{d\log a}{dm^b_{sea}} \qquad (6)$$

We are interested in the disconnected scalar matrix element which is related in the continuum to the derivative with respect to the sea quark mass. On a lattice this disconnected scalar matrix element is related to the derivative of the nucleon mass with respect to the sea-quark mass parameter at fixed valence-quark hopping parameter and fixed β by eq. 5. The key observation is then that as the sea-quark hopping parameter is varied on the lattice, both the valence quark mass and the lattice spacing also change. Thus one needs to correct for these changes to obtain the derivative of the nucleon mass at fixed valence-quark mass which is required.

For the valence dependence, the required derivative is directly given by the lattice observable

$$\frac{\partial M_N}{\partial m_{val}} = \frac{\partial(aM_N)}{\partial m^b_{val}} \qquad (7)$$

For the sea-quark derivative, however, there will be several other factors involved as discussed above:

$$\frac{\partial M_N}{\partial m_{sea}}(1 - am_{sea}B - X) =$$
$$\frac{\partial(aM_N)}{\partial m^b_{sea}} + (am_{val}B + X)\frac{\partial(aM_N)}{\partial m^b_{val}} - M_N B \quad (8)$$

This shows that the lattice connected and disconnected contributions are mixed when related to the more physical derivatives at fixed bare quark mass and fixed scale a. There will also be additional perturbative matching contributions to take into account to have a precise link between lattice observables and the continuum expressions., but we do not discuss these further here.

We evaluate the above expressions using UKQCD data [8,9]. We mainly use data from $\beta = 5.2$ on $16^3 32$ lattices with $N_f = 2$ flavours of sea quark with $\kappa = 0.1355$ or 0.1350 and using a NP-clover formalism with $C_{SW} = 2.0171$. These hopping parameter values correspond to quark masses around the strange quark (since the π/ρ ratio is 0.58 and 0.70 respectively). From the r_0 values [9], we obtain $B = 4.4(8)$ while from extrapolating the PCAC masses to obtain the critical hopping parameters at these two sea quark

masses we obtain $X = -0.66(4)$. This implies that there will be substantial mixing of the lattice disconnected and connected contributions in evaluating the derivative with respect to the sea-quark mass.

Setting $B = X = 0$ in eq. 8 gives the naive lattice ratio of $y = 0.53(12)$ while including the full mixing gives $y = -0.28(33)$.

Since we are using finite differences to evaluate the derivatives in eq. 8, we may instead use the value of the nucleon mass in physical units for the four cases ($\kappa_{sea} = 0.1355$, 0.1350; $\kappa_{val} = 0.1355$, 0.1350) and evaluate the quark mass in each case from the bare quark mass (eqs. 4,5). We prefer to use the bare quark mass here because of the lattice Feynman-Hellman theorem, but it would be possible to use the lattice PCAC quark mass, extrapolate to the continuum and then use the Feynman-Hellman relation in the continuum. The four combinations of sea and valence quark mass will then not lie at the corners of a rectangle, but at corners of some quadrilateral. We can then use the nucleon masses at these four values to evaluate the required derivatives with respect to the sea quark mass at constant valence mass and vice versa. The result from this approach is $y = -0.30(34)$ which is consistent with the value quoted above.

Note that y is expected to be positive, so the negative value obtained is just a reflection of the large statistical error. We have also attempted to measure the disconnected lattice correlator using a three point function approach. The result was compatible with using derivatives of the nucleon mass but the statistical errors were larger.

3. DISCUSSION

To obtain a reliable lattice determination of y, one should use $N_f = 2$ flavours of light sea quark plus a heavier (strange mass) sea quark. Then the required disconnected diagram can be obtained either as a derivative of the nucleon mass with respect to the strange sea quark mass or by evaluating the D_3 diagram with strange quarks in the disconnected loop. In practice one will need to extrapolate the sea and valence masses to to the physical u and d masses. This extrapolation is known to be non-trivial for the extraction of the πN sigma term [10]. A continuum limit of the lattice observables should also be taken as well as building in perturbative matching.

Instead we use $N_f = 2$ flavours of sea-quark which should be a good approximation for the sea. Extracting the disconnected diagram as a derivative is only possible if the quarks considered in the disconnected loop are the sea-quarks. It is possible to go beyond this by explicitly evaluating D_3 with different mass quarks in the disconnected loop and this is in progress. We are also exploring ways to reduce the large statistical errors we find.

Our main conclusion is that the current lattice data are unable to give a determination with any precision of y, but that the naive lattice ratio of $y \approx 0.6$ is not appropriate and the lattice result is indeed compatible with $y = 0$. That y is compatible with zero implies that there is no evidence for any dependence of the nucleon mass on the sea quark mass and this conclusion was also reached in an earlier lattice study [4] of the sea quark dependence of the meson (pseudoscalar and vector) masses.

REFERENCES

1. A. Bottino et al., Astropart. Phys. 13 (2000) 215, hep-ph/9909228,
2. B. Borasoy and U.G. Meissner, Phys. Lett. B365 (1996) 285, hep-ph/9508354,
3. S. Gusken, Flavor singlet phenomena in lattice qcd, hep-lat/9906034.
4. UKQCD, M. Foster and C. Michael, Phys. Rev. D59 (1999) 074503, hep-lat/9810021,
5. SESAM, S. Gusken et al., Phys. Rev. D59 (1999) 054504, hep-lat/9809066,
6. M. Fukugita et al., Phys. Rev. D51 (1995) 5319, hep-lat/9408002,
7. S.J. Dong, J.F. Lagae and K.F. Liu, Phys. Rev. D54 (1996) 5496, hep-ph/9602259,
8. UKQCD, C.R. Allton et al., Phys. Rev. D60 (1999) 034507, hep-lat/9808016,
9. UKQCD, C.R. Allton et al., hep-lat/0107021.
10. D.B. Leinweber, A.W. Thomas and S.V. Wright, Phys. Lett. B482 (2000) 109, hep-lat/0001007,