The η and η' mesons from lattice QCD.

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Abstract

Lattice QCD allows a first-principles study of QCD with the freedom to vary the number and masses of the quarks. I present results on the flavour singlet correlations (this illuminates OZI violating effects) for mesons. Concentrating on the pseudoscalar mesons, the flavour singlet mass splitting (η, η' mass splitting) appears naturally. I also present results on an investigation of decay constants for the η and η' (f_η) and discuss which quantities may be accessible in future lattice studies. The Witten-Veneziano approach can also be explored by determining the quenched topological susceptibility on a lattice.

1 Introduction

There is considerable interest in understanding hadronic decays involving η and η' in the final state. The phenomenological study of hadronic processes involving flavour singlet pseudoscalar mesons makes assumptions about their composition. Here I address the issue of the nature of the η and η' from QCD directly, making use of lattice techniques.

Lattice QCD directly provides a bridge between the underlying quark description and the non-perturbative hadrons observed in experiment. The amplitudes to create a given meson from the vacuum with a particular operator made from quark fields are measurable, an example being the determination of f_π. It also allows a quantitative study of the disconnected quark contributions that arise in the flavour singlet sector. The lattice approach provides other information such as that obtained by varying the number of quark flavours and their masses.

In the case of pseudoscalar mesons, the chiral perturbation theory approach also provides links between a quark description and the hadronic states. Indeed since lattice studies are increasingly difficult as the quark mass is reduced, the objective is to find a region of quark mass which is accessible from the lattice and for which chiral perturbation theory is reasonably convergent. As an example, lattice studies [1] can be used to estimate the size of the higher order terms in a chiral lagrangian approach.

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For the pion, lowest order chiral perturbation theory has the well known consequence that the decay constant f_π describes quantitatively both the $\mu\nu$ and $\gamma\gamma$ decays. For the flavour singlet states (η and η'), the situation is more complicated [2]. The axial anomaly now involves a gluonic component and the definition of decay constants is not straightforward. From the chiral perturbation theory description, one expects the mixing of η and η' to be most simply described in a quark model basis. In the flavour singlet sector, for pseudoscalar mesons, one then has contributions to the mass squared matrix with quark model content $(u\bar{u}+d\bar{d})/\sqrt{2}$ and $s\bar{s}$ (which are labelled as n and s respectively):

$$
\begin{pmatrix}
m_{nn}^2 + 2x_{nn} & \sqrt{2}x_{ns} \\
\sqrt{2}x_{ns} & m_{ss}^2 + x_{ss}
\end{pmatrix}
\tag{1}
$$

Here m corresponds to the mass of the flavour non-singlet eigenstate and is the contribution to the mass coming from connected fermion diagrams while x corresponds to the contribution from disconnected fermion diagrams. In the limit of no mixing (all $x = 0$, the OZI suppressed case), then the quenched QCD result is that the η is degenerate with the π meson and the η' would correspond to the $s\bar{s}$ pseudoscalar meson. This is not the case, of course, and the mixing contributions x are important.

Using as input m_{nn}, m_{ss}, m_η and $m_{\eta'}$, the three mixing parameters x cannot be fully determined. It is usual to express the resulting one parameter freedom in terms of a mixing angle, as illustrated in fig. 1, here defined by

$$
\eta = \eta_{nn}\cos\phi - \eta_{ss}\sin\phi \quad \eta' = \eta_{nn}\sin\phi + \eta_{ss}\cos\phi
\tag{2}
$$

The resulting values of the mixing parameters x are shown in fig. 2 (the input value for m_{ss} will be discussed later).

The η and η' mesons are often described in an SU(3) motivated quark basis, namely $\eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$, $\eta_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$. The mixing angle θ in this basis (see fig. 1) would be given by $\phi - 54.7^0$ in a lowest order chiral perturbation theory. In order to have $f_K \neq f_\pi$, one needs higher order terms in the chiral perturbation theory treatment and then the mixing scheme becomes more complicated [2] in this basis with more than one angle needed.

In the SU(3) symmetric limit, $m_{nn} = m_{ss} = m$ and $x_{nn} = x_{ns} = x_{ss} = x$, so that only one mixing parameter is relevant and the mixing matrix simplifies considerably to a diagonal form with elements m^2 (octet) and $m^2 + 3x$ (singlet). Previous lattice studies [3] have used degenerate quarks, so have explored this case and have found that the mixing parameter x is of a magnitude which can explain qualitatively the observed splitting between the η and η' mesons.

Here I describe a non-perturbative study in QCD from first principles which will be able to establish the values of the mixing parameters x, including the pattern of SU(3) breaking. This more comprehensive study would take into account the different masses of the light (u and d) quarks and the heavier s quark. Within the lattice approach, it is not at present feasible to evaluate
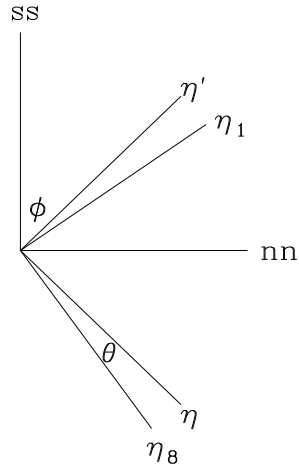
Figure 1: The mixing illustrated in the quark model basis for flavour singlet pseudoscalar mesons.

using quarks as light as the nearly massless u and d quarks and also it is more tractable to use an even number of degenerate quarks in the vacuum. As I shall show, despite these restrictions, a thorough study of the mixing between η and η' is possible.

I focus here on the results of lattice evaluations and I address four topics where lattice input permits us to construct a firm foundation for the η, η' mixing:

- From comparing pseudoscalar meson masses with valence quarks of two different masses (namely meson masses m_{11}, m_{12} and m_{22}), one can estimate the mass m_{ss} of the unmixed $\bar{s}s$ meson, given the observed m_{ns} and m_{nn} masses (ie K and π respectively).

- From measuring the mixing parameters x_{11}, x_{12} and x_{22} between initial and final flavour singlet states consisting of either quark 1 or 2 with different masses, one can establish consistently the pattern of SU(3) breaking in the mixing: obtaining a mixing angle close to 45^0 in the quark model
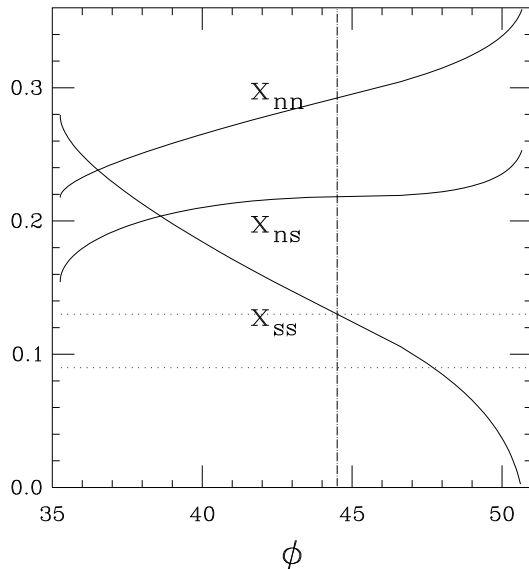
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Figure 2: The mass mixing parameters x in GeV^2 versus η, η' mixing angle ϕ in the η_{nn}, η_{ss} basis. The horizontal dotted lines give the allowed range from the lattice determination of x_{ss}. The vertical line illustrates the preferred solution.

basis.

- For $N_f = 2$ degenerate flavours of quark, one can determine the pseudoscalar decay constants for the flavour singlet and non-singlet meson. This input allows us to discuss the relation between the observed $\gamma\gamma$ decay modes of π^0, η and η' and the underlying quark content.

- The topological susceptibility can be measured in both quenched and full QCD and shows the behaviour expected for it to be related to the flavour singlet pseudoscalar mass generation.

2 Connected and disconnected contributions

On a lattice one can create pseudoscalar mesons by using any operator that has the required quantum numbers. The simplest choice is $\bar{\psi}\gamma_5\psi$ where ψ represents
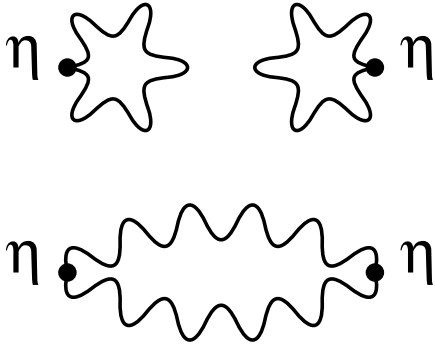
Figure 3: The disconnected (above) and connected (below) correlators for flavour singlet pseudoscalar mesons at time separation t. Here the wiggly lines represent full quark propagators.

a quark field. A suitable sum over quark flavours yields the required state (π, η, etc). By summing over spatial positions, one can create a state with zero momentum. One then studies these pseudoscalar mesons by creating them at time $t = 0$ and annihilating them at time t and using the full quark propagators evaluated on the lattice to construct the correlator $\mathcal{C}(t)$. The basic strategy is then that at large t where excited state contributions will be negligible, this correlator behaves as e^{-mt} where m is the lightest mass state with the specified quantum numbers. Hence one can determine meson masses.

A feature that occurs exclusively for flavour singlet mesons, is that there are two independent ways to link up the quarks and antiquarks between the initial and final state - as illustrated in fig. 3. Moreover, these connected and disconnected contributions can be evaluated separately on a lattice, so giving additional insight. Indeed the connected contribution $C(t)$ is just the same for flavour singlet and non-singlet mesons. Thus the disconnected contribution $D(t)$ is responsible for the difference between them. So, at large t one has for the flavour non-singlet correlator

$$\mathcal{C}(t) = C(t) \rightarrow ce^{-mt} \tag{3}$$

and for the flavour singlet correlator

$$\mathcal{C}(t) = C(t) + D(t) \rightarrow de^{-m't} \tag{4}$$

where m is the flavour non-singlet mass and m' is the flavour singlet mass.

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Note that this implies that at large t

$$\frac{D(t)}{C(t)} = \frac{d}{c}e^{-(m'-m)t} - 1 \approx \frac{d}{c} - 1 - \frac{d}{c}(m'-m)t + O[(m'-m)^2 t^2] \quad (5)$$

Thus a study of D/C gives information on the mass splitting between singlet and non-singlet. This has been explored on a lattice for mesons of all J^{PC} values that can be made by local quark bilinears [4]. This study finds that the only sizeable contributions to D come for the scalar and pseudoscalar quantum numbers. This conclusion is in agreement with the experimental observation that the meson spectrum is approximately ideal (ie as in the naive quark model) except for those two quantum numbers.

For the pseudoscalar case the behaviour of D/C is such that the flavour singlet meson is heavier than the flavour non-singlet, as indeed required by experiment. Also, for N_f degenerate flavours of quark, the flavour singlet mass is given by $m' = \sqrt{m^2 + N_f x}$, in terms of the mass matrix approach described above. This gives us access to values of x from the lattice for different values of N_f.

As well as the simple case where all quarks have the same mass and are present as sea-quarks in the vacuum, one can consider on a lattice other cases. One relevant case is with 2 flavours of sea quark (thought of as n quarks), but with mesons made out of $n\bar{s}$ or $s\bar{s}$ where the s quark is heavier and so not included in the sea. Here we have a partially quenched scenario since the s quarks are not present in the sea. In this case one can show that the disconnected diagram with nn as a source and ss as a sink can be expressed [5] as

$$\frac{D_{ns}(t)}{\sqrt{C_{nn}(t)C_{ss}(t)}} = \frac{\sqrt{2}x_{ns}t}{2\sqrt{m_{nn}m_{ss}}} + O(x^2 t^2) \quad (6)$$

which allows x_{ns} to be evaluated. Likewise x_{ss} can be evaluated.

3   Lattice results

3.1   The $s\bar{s}$ pseudoscalar mass

A meson made from $s\bar{s}$ quarks is necessarily flavour singlet and so can mix with other flavour singlet operators via gluonic interactions. In order to understand the mixing between the η and η', it is important to ask a hypothetical question: what would be the mass of a pseudoscalar meson made of $s\bar{s}$ quarks but which is flavour non-singlet? This question can be addressed readily by lattice studies where the extra interactions (disconnected diagrams) associated with the flavour singlet case can be removed explicitly.

Chiral symmetry considerations lead to the expectation that the pseudoscalar meson composed of quarks of mass M_q has mass squared m^2 which behaves linearly with M_q at small quark mass. However, at large quark mass (c and b

quarks for instance), one expects the meson mass to vary approximately linearly with the quark mass. Here we are not concerned with the region of very small quark mass where chiral logs are important [2], so one can summarise this behaviour by

$$m^2 = bM_q + cM_q^2 + O(M_q^3) \tag{7}$$

For a pseudoscalar meson made of two different quarks of mass M_n and M_s, we shall assume its mass only depends on $(M_n + M_s)/2$ and not on $(M_s - M_n)/2$ as found in lattice studies [6] and in lowest order chiral perturbation theory. If eq. 7 were valid with just the linear term in the quark mass(ie $c = 0$), then one directly obtains the required mass of the pseudoscalar meson composed of s quarks, $m_{ss}^2 = 2m_{sn}^2 - m_{nn}^2$, that is $2K^2 - \pi^2$, leading to $m_{ss} = 0.687$ GeV.

This can be explored on a lattice by measuring the pseudoscalar meson mass for valence quarks in combinations 11, 22 and 12. Then, for small c/b, one has

$$\frac{c}{4b^2} = \frac{\frac{1}{2}(m_{11}^2 + m_{22}^2) - m_{12}^2}{(m_{22}^2 - m_{11}^2)^2} \tag{8}$$

One can then to use dynamical configurations with sea quarks of type 1 and consider the propagation of mesons made of either quark 1 or quark 2, where quark 2 corresponds to a heavier quark.

From such lattice studies, a statistically significant curvature from the c term is obtained. Applying this value of c to the determination of the m_{ss} mass from the π and K masses, gives [5] a relative shift upwards due to the curvature term (c) of 1.1(3)%, corresponding to a value of $m_{ss} = 0.687 + 0.008$ GeV.

3.2 Flavour-singlet mixing

The mass splitting between flavour non-singlet and singlet mesons can be measured, as discussed above, using lattice evaluation of disconnected quark propagators. This is not an easy task: the contamination from excited states is difficult to remove and the statistical errors turn out to be relatively large. Initial studies have been in the quenched approximation [3, 7, 8]. Here, although there is no flavour splitting of the masses, the mass splitting matrix element x can be evaluated. It is, however, preferable to be able to study the mass splitting directly and hence here I focus on results from full QCD simulations [9, 10, 5]. Some technical details of lattice simulations of disconnected correlators can be found in ref. [7].

Already in the first quenched studies [3], it was found that x increases as the quark mass is decreased. Moreover, it will be interesting to check to see if there is a factorisation of x as expected in some chiral perturbation theory descriptions [2], namely $x_{12}^2 = x_{11}x_{22}$. Consider now the more realistic (partially quenched) case: with heavier quarks of type 2 in a sea of two flavours of quarks of type 1.

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Setting the sea quark mass to strange in both quenched and $N_f = 2$ evaluations leads to a consistent lattice estimate [5] of x_{ss} in the range 0.09 to 0.13 GeV^2. This value is also consistent with that reported from a study of $N_f = 2$ by the CP-PACS collaboration [11] with strange quarks which gives values of $x_{ss} = 0.10$ GeV^2 and 0.14 GeV^2 (depending on using $t_{\min} = 2$, 3 in fits, respectively). These lattice values are obtained at quite coarse lattice spacings and there may be some additional systematic error arising from the extrapolation to the continuum limit. The authors [5] have, however, chosen to use a non-perturbatively improved fermion action [12] to minimise this extrapolation error.

They are unable to determine directly the mixing strengths x for lighter quarks than strange. However, the x values do show a decrease with increasing quark mass and also approximate factorisation. So they assume that the value of x continues to increase as the quark mass is decreased below strange in a similar way to the decrease seen from twice strange (type 2) to strange (type 1) where approximate factorisation is found.

Consider now the consequence of this lattice determination of the mixing, using input masses $m_{nn} = 0.137$ GeV, $m_{ss} = 0.695$ GeV (as discussed above) and with x values in line with the results above, namely $x_{ss} \approx 0.12$ GeV^2, $x_{ns}^2 \approx x_{nn} x_{ss}$ and, though with big errors from the extrapolation, $x_{nn}/x_{ss} \approx 2$. Fig. 2 shows the x values needed to reproduce the known η and η' masses for each mixing angle ϕ. The lattice determination of x_{ss} is shown by the dotted horizontal band. Keeping close to this band while satisfying the other lattice constraints is possible for the mixing illustrated by the vertical line. This has $x_{nn} = 0.292$, $x_{ns} = 0.218$, $x_{ss} = 0.13$ GeV^2 which gives a description of the observed η and η' masses while being consistent with the QCD inspired evidence about the mixing strengths. This assignment corresponds to a mixing angle ϕ in the η_{nn}, η_{ss} basis of 44.5^0. Note that this is almost maximal which implies that the quark content (apart from the relative sign) of the η and η' meson is the same. The corresponding mixing angle in the η_8, η_1 basis (modulo comments above) is a value of θ of -10.2^0.

3.3 Flavour-singlet decay constants

One contribution to the decays of π^0, η and η' to $\gamma\gamma$ is via the quark triangle diagram. The quark model gives a decay proportional to Q_i^2 for the contribution from a quark of charge Q_i. Thus for the π^0 meson and the flavour-singlet nn and ss mesons, the quark charge contributions to the decay amplitudes would be in the ratio $1 : 5/3 : \sqrt{2}/3$. The experimental [13] reduced decay amplitudes for π^0, η, and η' are in the ratio $1.0 : 1.00(10) : 1.27(7)$. This information can be used to analyse the quark content of the pseudoscalar mesons subject to a quantitative understanding of the decay mechanisms. The traditional approach assumes that the decay constants for the decays of the three mesons are the same and then the relative decay amplitudes give information on the quark content.

This suggests a mixing angle of $\theta \approx -20^0$ is preferred [2, 13].

For the flavour non-singlet mesons we have

$$\langle 0|A^\mu|\pi\rangle = f_\pi q^\mu \tag{9}$$

where the axial current is the local quark bilinear. From evaluating this expression on a lattice, the pion decay constant can be determined, after taking account of the differences between lattice and continuum regularisation schemes. Moreover f_K can be evaluated too and one finds that f increases with increasing quark mass, just as known from experiment. This increase is evidence for a contribution from beyond the lowest order in chiral perturbation theory.

For the η and η', however, the flavour singlet combination will have gluonic contributions from the anomaly since

$$\partial^\mu A^\mu = 2m_q P + 2N_f Q \tag{10}$$

for N_f quarks of mass m_q, where P is the local bilinear representing the pseudoscalar current and Q is known as the topological charge density and is given in terms of the gluonic field by $Q = g^2 \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}/(32\pi^2)$. This identity underlines the inevitability of gluonic contributions to the flavour singlet meson interactions. Thus for the flavour singlet (here labelled η') defining a decay constant by

$$\langle 0|A^\mu|\eta'\rangle = f_{\eta'} q^\mu \tag{11}$$

is less satisfactory since $f_{\eta'}$ will be scale dependent because of the gluonic contributions from the axial anomaly: namely there will be non-zero amplitudes

$$\langle 0|Q|\eta'\rangle \tag{12}$$

I now address the issue of determining these decay constants directly from QCD using lattice methods. The study uses 2 flavours of degenerate quark and one defines the decay constants by eq. 9, 11. For the isospin 1 state (π - like), this is on a firm footing because of the axial ward identity hence f_π will be scale invariant. For the flavour singlet pseudoscalar meson, here called η', the decay constant defined as in eq. 11 will not be scale invariant because of gluonic contributions arising from the anomaly [2] as discussed above. In an exploratory lattice study [5] the decay constants are obtained with lattice regularisation and one can compare the singlet and non-singlet values.

These decay constants can be thought of as giving the quark wave function at the origin of the pseudoscalar meson. In principle it would be possible to explore also the local gluonic contributions to the flavour-singlet mesons (eq. 12) but it will be difficult to relate lattice regulated results to the continuum and this has not been attempted.

Since the mass splitting between singlet and non-singlet is not reproduced directly in quenched QCD, it is essential to use lattice studies that do include sea quark effects in this study of decay matrix elements. Results have been

obtained from lattices with $N_f = 2$ flavours of degenerate sea quark. In this case there is no need for mixing angles to describe the spectrum (which will have one isoscalar and one isovector neutral particle). As a first crosscheck, one finds that lattice studies correctly give a flavour non-singlet decay constant that increases with quark mass in reasonable agreement with experiment [13] assuming a steady increase from $f_{nn} = 131$ MeV and $f_{ns} = 160$ MeV to f_{ss}.

For the flavour singlet case, the determinations of f have relatively large statistical errors and the systematic error from changing the type of fit is also comparable. For the case with $N_f = 2$ degenerate quarks, the comparison of the flavour singlet and non-singlet shows that the singlet decay constants appear to be somewhat larger, though the errors are too big to substantiate this.

Combining the mass dependence one finds in the flavour non-singlet sector with the near equality of singlet and non-singlet decay constants, we can deduce properties of the physical case with three light quarks. Thus, in terms of the traditional treatment [13], we would expect $f_\eta/f_\pi > 1$ and $f_{\eta'}/f_\pi > 1$. One way to minimise the effects of mixing is to consider $X = (a_\eta^2 + a_{\eta'}^2)/a_\pi^2$ where a refers to the reduced decay amplitude. Using the conventional formulae for the decay amplitudes would then give a value of $X = 3r^2$ (where r is a suitably weighted average of f_η/f_π and $f_{\eta'}/f_\pi$ which are both greater than 1). Thus the traditional treatment gives $X > 3$ which is significantly larger than the experimental value [13] of 2.64(24). Thus it appears unlikely that the traditional treatment (with the decay to $\gamma\gamma$ being given by the analogue of the formula for pions) is correct for any mixing angle.

I conclude that there is no support for the traditional assumption that the singlet decays are given by a similar expression to the non-singlet. As has been pointed out by many authors [2], this is plausible for at least two reasons: (i) the η and η' mesons are heavier and therefore less likely to dominate the axial current or, equivalently, higher order corrections to chiral perturbation theory will be more important (ii) the flavour-singlet axial anomaly has a gluonic component which will give additional contributions to any hadronic process.

3.4 Where does the η' mass come from?

The lattice studies show that the gluonic contributions that build up the disconnected correlators are such as to reproduce the experimental η and η' masses. This is of course as it should be - QCD is expected to be accurate for these phenomena - and it is the lattice methodology that is being tested.

One can now ask why these OZI-rule violating gluonic interactions are substantial for pseudoscalar mesons but small for most other cases (scalar mesons excluded).

The culprit must be gluonic contributions with the required quantum numbers. One possibility is a glueball, but the pseudoscalar glueball is known [14] to be heavier than 2 GeV and would contribute weakly and with the wrong sign (making the flavour singlet $q\bar{q}$ state lighter than the non-singlet). This glueball

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option is indeed appropriate for a discussion of the OZI violating effects for scalar mesons.

For the pseudoscalar case, the presence of vacuum fluctuations with those quantum numbers is the candidate. These are commonly called topological charge density fluctuations and they are necessary since the anomaly implies that $\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$ is coupled to currents with pseudoscalar quantum numbers. It must be emphasised that these contributions do not need to have anything to do with isolated instantons.

This relationship between fluctuations in topological charge density and the singlet mass generation can be made semi-quantitative via the Witten-Veneziano formula:

$$m_{\eta'}^2 = \frac{2N_f}{f_\pi^2}\chi \tag{13}$$

where $\chi = \int d^4x < Q(x)Q(0) >$ is the topological susceptibility in the pure gauge (Yang-Mills or quenched) theory (here η' is to be interpreted as the flavour singlet component in the chiral limit). The derivation of this relationship (see ref [16] for a recent discussion) involves studying correlations of topological charge density assuming that $u = N_f/N_c$ can be varied. Then the $u = 0$ limit (where a massless contribution is present in the flavour-singlet channel in the quenched chiral limit) can be related to full QCD at $u \neq 0$ where there is no such massless flavour-singlet particle, hence the topological susceptibility is zero in the chiral limit. The relationship follows from assuming that the lightest flavour singlet pseudoscalar meson dominates the behaviour of topological charge correlations in this region of small u and near the chiral limit.

Lattice techniques can be used to evaluate the quenched topological susceptibility and the value obtained is consistent [7, 8] with this relationship. Evidence is also mounting [17, 19, 18] that χ does vanish in full QCD as the sea quark mass goes to zero, as implied in the above discussion.

On a lattice one can investigate whether these topological charge density fluctuations arise from instantons or not, the latter being the expectation at large N_c. The Witten-Veneziano formula is a large N_c result and does seem to be qualitatively satisfied, which is some evidence to suggest that the physical case with $N_c = 3$ may be qualitatively similar to the large N_c world. The nature of the topological charge density on the lattice can be revealed by low lying eigenmodes of the fermion operator, see ref. [20], and this suggests that some elements of an instanton description are indeed present. Varying N_c in this study does suggest that at larger N_c (e.g. 4) the instanton features are less evident, as expected. This characterisation of the relationship between the topological charge density observed and classical tunnelling (instantons) is still not finally resolved.

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4 Conclusion

Flavour singlet mesons can, in principle, be easily studied on a lattice. In practice, lattice studies have been hampered by two constraints. One is that the disconnected quark diagrams needed for a study of singlet mesons are intrinsically noisy. Much larger data sets (tens of thousands of gauge configurations) will be needed to increase precision. Another constraint is that one is unable to work with sea quarks substantially lighter than strange. Also it has not yet been possible to attempt a continuum limit extrapolation of the lattice results, although a lattice formalism that should improve this extrapolation was used.

A lattice study of the disconnected diagrams that cause flavour-singlet mesons to differ from non-singlet mesons shows that these OZI-rule violating effects are only large for pseudoscalar and scalar mesons. Here we discuss the structure of the mixing in the singlet pseudoscalar mesons.

From the careful non-perturbative study of mass formulae for flavour non-singlet pseudoscalar mesons made of different quarks, one deduces that the ss state lies at 695 MeV. One then determines the pattern of mixing for the flavour singlet sector, obtaining $x_{ss} \approx 0.12$ GeV^2, $x_{nn}/x_{ss} \approx 2$ and $x_{ns}^2 \approx x_{nn}x_{ss}$. These conditions are indeed consistent and point to a mixing close to maximal ($\phi = 45 \pm 2^0$) in the nn, ss basis (this corresponds to a conventional (η_8, η_1) mixing θ of -10 ± 2^0).

It is now possible to explore the decay constants which give the coupling to a local quark antiquark pair for singlet pseudoscalar mesons from the lattice. The results show similar decay constants for singlet and non-singlet states of the same mass but with quite large errors. This, combined with experimental data, suggests that the traditional description of $\gamma\gamma$ decays is inadequate. There will indeed be gluonic contributions to singlet meson decays and these have not yet been explored in detail from the lattice.

Lattice studies confirm the qualitative relationship between the pseudoscalar singlet mass and the topological charge density fluctuations.

Since lattice techniques give a reasonable description of the flavour singlet pseudoscalar mesons, it will be feasible to use such techniques to study hadronic decays involving η and η'.

5 Acknowledgements

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