Hybrid meson decay from the lattice

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(Received 4 January 2002; published 18 April 2002)

We discuss the allowed decays of a hybrid meson in the heavy quark limit. We deduce that an important decay will be into a heavy quark nonhybrid state and a light quark meson, in other words, the deexcitation of an excited gluonic string by emission of a light quark-antiquark pair. We discuss the study of hadronic decays from the lattice in the heavy quark limit and apply this approach to explore the transitions from a spin-exotic hybrid to $\chi_b \eta$ and $\chi_b S$ where S is a scalar meson. We obtain a signal for the transition emitting a scalar meson and we discuss the phenomenological implications.

I. INTRODUCTION

Hybrid mesons are those with nontrivial excited gluonic components. The simplest such case is when the spin-parity is exotic, namely, not allowed in the quark model. Here we specialize to heavy quarks and so our comparisons with experiment will be for $b\bar{b}$ systems. In this context, there will be a spin-exotic ($J^{PC}=1^{-+}$) meson whose properties can be determined from lattice QCD. We review here first the information on the nature and spectrum of such excited gluonic states. We then discuss in general the allowed decay modes of such a state. In most of this discussion we focus on predictions in the heavy quark limit, so with heavy quark, spin-flip neglected.

We then review lattice methods to extract hadronic transition matrix elements. In the case of hybrid decay, we explore the creation of a light quark-antiquark state from the gluonic field of the hybrid meson. It is possible to fulfill the rather restricted conditions on a lattice and we are able to explore these transitions. We study hybrid meson transitions to $\chi_b \eta$ and $\chi_b S$ where S is a scalar meson. We obtain a signal for the transition emitting a scalar meson and we discuss the phenomenological implications.

II. HYBRID STATES ON THE LATTICE

The static quark approach gives a very straightforward way to explore hybrid quarkonia. These will be $Q\bar{Q}$ states in which the gluonic contribution is excited. The ground state of the gluonic degrees of freedom has been explored on the lattice, and, as expected, corresponds to a symmetric cigar-like distribution of color flux between the two heavy quarks at separation R. One can then construct less symmetric color distributions which would correspond to gluonic excitations. For a review see Ref. [1]. The properties of the physical states can then be obtained from these static potentials by solving the Schrödinger equation in the adiabatic approximation.

The way to organize this is to classify the gluonic fields according to the symmetries of the system. This discussion is very similar to the description of electron wave functions in diatomic molecules. The symmetries are (i) rotation around the separation axis z with representations labeled by J_z, (ii) CP with representations labeled by g and u and (iii) $C\mathcal{R}$. Here C interchanges Q and $\bar{Q}$, P is parity and $\mathcal{R}$ is a rotation of 180° about the mid-point around the y axis. The $C\mathcal{R}$ operation is only relevant to classify states with $J_z=0$. The convention is to label states of $J_z=0,1,2$ by Σ, Π, Δ respectively.

In lattice studies the rotation around the separation axis is replaced by a four-fold discrete symmetry and states are labeled by representations of the discrete group D_{4h}. The ground state configuration of the color flux is then Σ_g^+ (A_{1g} on the lattice). The exploration of the energy levels of other representations has a long history in lattice studies [2,3]. The first excited state is found to be the Π_u (E_u on a lattice)—see Fig. 1 for an illustration. This can be visualized as the symmetry of a string bowed out in the x direction minus the same deflection in the $-x$ direction (plus another component of the two-dimensional representation with the transverse direction x replaced by y), corresponding to flux states from a lattice operator which is the difference of U-shaped paths from quark to antiquark of the form $\sqcap - \sqcup$.

A summary of lattice determinations of the energy of this lowest hybrid state [1] puts it at $m(H)=10.76(7)$ GeV for b quarks, so approximately 1.3 GeV heavier than the Υ. This hybrid state in the adiabatic approximation will have lowest angular momentum $L=1$ and combining this with the heavy quark spins gives 8 degenerate J^{PC} values. Of special interest is the spin exotic state with $J^{PC}=1^{-+}$ which is expected to be the lightest spin-exotic meson. Since it is spin-exotic, it cannot mix with the non-hybrid $Q\bar{Q}$ states and is thus of considerable theoretical and experimental interest.

III. HYBRID MESON DECAYS

We shall be discussing hybrid meson decays in the heavy quark limit, so our conclusions will be more applicable to b

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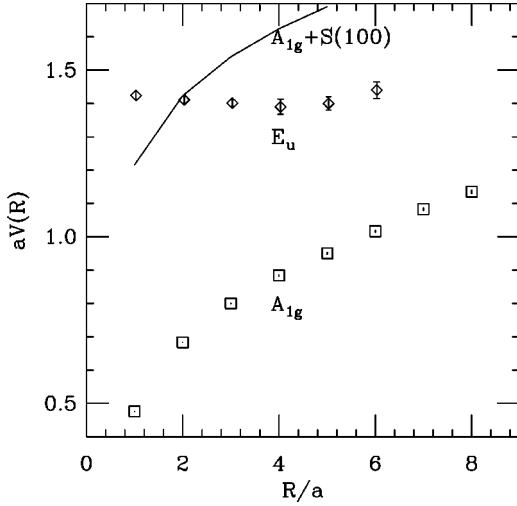
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FIG. 1. The ground state (A_{1g}) of the static potential $V(R)$ and the first gluonic excitation (E_u) from this work with $N_f=2$ flavors of sea quark (of approximately the mass of the strange quark), in lattice units with $a \approx 0.1$ fm. The energy of a scalar meson with momentum $\pi/8a$ above the ground state potential is shown by the continuous line.

quark systems than c quarks. From the adiabatic approximation, one solves the gluonic field around a static quark-antiquark at separation R to determine a potential first and then solves the Schrödinger equation in that potential. For a spin-exotic hybrid with an excited gluonic field having $J_z=1$ about the interquark axis, it follows that the quark-antiquark must have orbital angular momentum greater than or equal to one unit and we will assume the least angular momentum allowed, namely a P-wave. Hence the quark-antiquark system has $L=1$ in the hybrid and this will persist to the final state in any decay. Furthermore the spin-exotic hybrid has the heavy quark-antiquark in a spin triplet so this (and its spin projection) will also persist to the final state in any decay. The potential that binds the hybrid meson is relatively flat (see Fig. 1). Hence the hybrid meson has an extended radial wave function, for example [4,5] with R-dependence of $u_H \approx R^2 e^{-(R/0.51)^2}$ in units of fm for the inter-quark coordinate R for b quarks. This has implications for their production and decay. For instance, any vector hybrid state will only be weakly produced in e^+e^- collisions because the wave function at the origin is suppressed.

We shall later consider transitions at fixed R from the hybrid state to the P-wave χ_b state with radial wave function approximately $u_\chi \approx R^2 e^{-(R/0.33)^2}$ with R in fm. The lack of nodes in the relevant wave functions implies a spatial wave function overlap factor which is quite large (i.e., assuming the transition rate is independent of R, for the above normalized wave functions $\int u_H u_\chi dR = 0.63$).

Given the mass estimate above, the open channels for decay of a $J^{PC}=1^{-+}$ hybrid include $B\bar{B}^*$, $B^*\bar{B}^*$, $\eta_b\eta$, $\eta_b\eta'$, $\chi_b\eta$, $\chi_b S$, $\Upsilon(1s)\omega$ and $\Upsilon(1s)\phi$ where S is a scalar meson which can subsequently decay to $\pi\pi$ (note decay to $B\bar{B}$ is not allowed by C conservation). However, as dis-

cussed above, decays to quarkonia are only allowed into a χ_b state in the heavy quark limit. Thus decays to η_b or $\Upsilon(1s)$ proceed by heavy quark symmetry violations (of order $1/M_b$). We do not discuss these modes further here.

Selection rules have been proposed for hybrid decays, for example [6] that $H \not\rightarrow X+Y$ if X and Y have the same non-relativistic structure and each has $L=0$. This would rule out $B\bar{B}$, $B\bar{B}^*$ and $B^*\bar{B}^*$ and the analogous cases for charm quarks. This selection rule can be addressed directly from the static quark approach. The symmetries in this case of rotations and reflections about the separation axis have to be preserved in the strong decay. From the initial state with the gluonic field in a given symmetry representation, the $q\bar{q}$ pair must be produced in the decay in such a way that the combined symmetry of the quark pair and the final gluonic distribution matches the initial representation.

We first discuss decays of non-hybrid quarkonia to set the scene. For the ground state of the gluonic excitation (Σ_g^+, non-hybrid) we have $J_z=0$ and even CP. Thus, for this state to decay to $(Q\bar{q})(\bar{Q}q)$ with each heavy-light meson having $L=0$, the final gluonic distribution is also spatially symmetric about the separation axis (actually it is essentially two spherical blobs around each static source binding the heavy light mesons). Then any $q\bar{q}$ pair production has to respect this symmetry and have $J_z=0$ and even CP. Since each light quark has no orbital angular momentum about the separation axis, the CP condition then requires $S_{q\bar{q}}=1$, a triplet state. This conclusion for the light quark spin assignment can be tested by the ratio of $B\bar{B}, B\bar{B}^*$ and $B^*\bar{B}^*$ decays.

We now consider decays from a heavier quarkonium state to a lighter such state (with both initial and final quarkonia in the most-symmetric gluonic Σ_g^+ representation) with emission of a light quark-antiquark pair which form a flavor-singlet meson. This flavor singlet meson must be produced with CP even and $J_z=0$ again. Possible modes are a scalar meson or a vector meson with $S_z=0$ which are both allowed in a symmetric spatial state. Note that decay to a pseudo-scalar meson is not allowed since the required spatial wave function would have to be in a Σ_u^+ representation but this is not realizable for a meson with no spin.

The spin nature of the quark-antiquark pair produced in hadronic decays has been widely discussed [7]. A color-singlet light quark-antiquark pair can be produced from color flux oriented in the z-direction with vacuum quantum numbers ($CP=+1, J_z=0$, flavor singlet) either as a scalar meson ($^3P_0$ model) or as the zero-helicity component of a vector meson ($^3S_1$ model). The lattice study of flavor singlet mesons suggests that the former is a much larger [8] amplitude in general. In any specific case, however, the amplitudes can be determined explicitly, and this we undertake for hybrid decays.

For the $J^{PC}=1^{-+}$ hybrid we have a gluonic field with $J_z=1$ and odd CP. For the case of decay to a $(Q\bar{q})(\bar{Q}q)$ with each heavy-light meson having $L=0$, this would imply that the $q\bar{q}$ would have to be produced with $J_z=1$ and odd CP. This is not possible since the triplet state would have even CP while the singlet state cannot have $J_z=1$. This is

TABLE I. Hybrid decays by string de-excitation in the heavy quark limit emitting a flavor singlet light quark-antiquark meson with quantum numbers J^{PC}. This meson has a wave function relative to the heavy quark-antiquark system in the representation shown with L_z and CP as shown. This implies that it is in an orbital L-wave about the heavy quark system.

Meson	J^{PC}	wave fn.	L_z	CP	L	Example
η, η'	0^{-+}	Π_g	1	+	2	Hybrid→χ_b+ η
scalar	0^{++}	Π_u	1	−	1	Hybrid→χ_b+ π+ π

then equivalent to the selection rule described above. There will presumably be small corrections to this selection rule coming from retardation effects. Decay to $(Q\bar{q})(\bar{Q}q)$ with one heavy-light meson having a non-zero orbital excitation is allowed from symmetry but is not allowed energetically with conventional mass assignments [9] for the P-wave excited B meson multiplet.

Decays of a hybrid meson to $(Q\bar{Q})(q\bar{q})$ are also possible since there is enough excitation energy to create a light quark meson. This meson must be created in a flavor singlet state and the lightest candidates are η, ω and scalar (S) channels. In a lattice context, this production is via a disconnected quark loop while the normalization of the meson will involve the connected correlator. Thus the relative strength of the disconnected correlator to the connected correlator enters and this has been studied for different meson quantum numbers on a lattice [8]. As expected from the phenomenology of meson spectra, the pseudoscalar and scalar mesons are the only two cases with relatively large disconnected contributions. For pseudoscalar mesons, the flavor singlet mixture of η and η' is mainly η' with only an amplitude of $\sin(10°)$ of η for the conventional mixing scheme (see [10]). For the scalar meson, the discrete states coming from mixing of the glueball and $q\bar{q}$ meson are relatively heavy so they may not be allowed from energy considerations, but one should also consider the $\pi\pi$ continuum with flavor singlet and scalar quantum numbers, which is experimentally known to be big (i.e., have large $\pi\pi$ phase shift) around 700 MeV.

In these decays from a hybrid meson with Π_u representation to a de-excited string with Σ_g^+ representation, the light quark meson must have a wave function with a net Π_u representation with $J_z=1$ and $CP=-1$. If this meson has an angular momentum of L about the final state heavy-heavy meson, then this implies that the orbital wave function has $L_z \leq L$ and $CP=(-1)^L$. Using this orbital angular momentum then allows the decay products from the decay of a $J^{PC}=1^{-+}$ meson to be identified. Thus in the static limit the decays allowed by symmetry for a Π_u representation hybrid to a Σ_g^+ representation state plus flavor-singlet meson are shown in Table. I. The examples shown take account of the adiabatic approximation and the nature of the $J^{PC}=1^{-+}$ hybrid wave function.

As for the case of quarkonium decays and string breaking [12,11], it is possible in principle to explore on the lattice some aspects of these hybrid meson decays. One can study matrix elements between ground states which are degenerate

in energy such as the 1^{-+} hybrid and the $\chi_b\eta$ final state where the light quark mass is adjusted so that there is equal energy in both systems. This and similar lattice studies will enable some further guidance to be given for experimental searches for hybrid mesons.

IV. DECAYS FROM THE LATTICE

Consider the generic transition $H \rightarrow A + B$ where A and B represent stable particles and H is unstable to decay. Here we assume that the two-body state has exactly all the symmetries of the state H. For simplicity we will consider H at rest and then A and B have momenta k and $-k$ respectively. Thus the two-body state, if non-interacting, has energy $E_{AB} = \sqrt{m_A + k^2} + \sqrt{m_B + k^2}$. In Euclidean time, the properties of this decay transition are very different in practice [13] from the Minkowski case, in particular the large time correlator will be dominated by the lightest two-body state which will be that with minimum momentum.

One way to explore this system in detail is to consider a finite volume. We make the usual assumption that the theory is defined independently of the boundary conditions. From the lattice viewpoint, the finite volume result can be obtained by taking the continuum limit at fixed physical volume. For a cubic spatial volume L^3 with periodic boundary conditions, the momenta are discrete ($\mathbf{k}=2\pi\mathbf{n}/L$) where $\mathbf{n} =(n_1,n_2,n_3)$ is an integer vector. The two-body states are then also discrete in energy. One expects that as L increases beyond the range of the two body interaction, the two body energy levels become close to the non-interacting case. This has been studied [14–17] and detailed formulas obtained for the energy shifts at sufficiently large L in terms of the scattering phase shift in the $A+B$ system, provided inelasticity is negligible. This allows, in principle, to measure the phase shift at various energies by varying L and $\mathbf{n}$. From the phase shift one can then deduce the properties of the decay in the large volume limit. To measure the lightest two-body state accurately [typically with $n=(1,0,0)$] is already a challenge and to obtain accurate energy determinations for excited states with higher momentum will be much harder. Moreover, in practice the energy shifts are small and so it will be extremely difficult to measure accurately the phase shifts on a lattice [18].

For some applications, it is possible to measure the transition amplitude directly. This is clearly the case in a quenched (or partially quenched) approach where the decay transition does not actually take place in the lattice version of the theory. For example, the ρ meson does not decay to $\pi + \pi$ in quenched studies. Let us describe how this can be measured in principle: Create H at $t=0$ and annihilate a two-body state with relative momenta k and $-k$ at time t. Then the contribution to the correlator from an H state with mass m_H and a two-body state with energy E_{AB} is given by

$$C_{H-AB}(t)=\sum_{t_1} h e^{-m_H t_1} x e^{-E_{AB}(t-t_1)} b \qquad (1)$$

where the summation over the intermediate t-value t_1 will be an integral in the continuum and where h and b are the am-

plitudes to make each state from the lattice operators used and x is the required transition amplitude $\langle H|AB \rangle$. Here we are assuming that the states H and AB are normalized to 1. By obtaining h and b from the $H \rightarrow H$ and $AB \rightarrow AB$ correlators, one can hope [12,19] to extract x.

The complication, however, is that removal of excited state contributions is tricky. For example, if $m_H - E_{AB} > 0$ then the transition time t_1 will be preferentially near 0 (since the heavier state then propagates less far in time) and one can complete the sum over t_1 obtaining a t-dependence of Eq. (1) as $e^{-E_{AB}t}$. This same t-dependence would be obtained if the state with mass m_H were to be replaced with an excited state with an even heavier mass. Thus one cannot separate the ground state and excited state contributions even in principle. See Ref. [19] for a fuller discussion.

The way forward is that if $m_H = E_{AB}$, the ground state contributions have a t-dependence as $te^{-E_{AB}t}$ whereas any excited state contributions behave as $e^{-E_{AB}t}$ as above. So now we do have a way to isolate the required ground state contribution:

$$x = \lim_{t \to \infty} \frac{1}{t} \frac{C_{H-AB}(t)}{[C_{H-H}(t)C_{AB-AB}(t)]^{1/2}}. \qquad (2)$$

Note that this separation is only by a power of t which is less than the case for diagonal correlations where the excited state contributions are suppressed by an exponential $e^{(m'_H - m_H)t}$.

In practice the requirement of energy equality can be relaxed. Defining $\Delta = m_H - E_{AB}$, then the ground state contribution to the expression of Eq. (2) evaluates to $2x \sinh(t\Delta/2)/(t\Delta) = x[1 + (t\Delta)^2/24 + \cdots]$. So this will be equivalent to the expression with $\Delta = 0$ provided

$$(m_H - E_{AB})t \ll 5. \qquad (3)$$

So far we have described the behavior of the $C_{H-AB}(t)$ in the quenched approximation. In full QCD, there will be a mixing of these two states. Let us illustrate this for the case of interest where the energies are approximately the same (namely E). Then the energy mixing matrix has the form

$$\begin{pmatrix} E & x \\ x & E \end{pmatrix} \qquad (4)$$

which has eigenvalues $E \pm x$. An accurate measurement of these energy eigenstates would then give the transition amplitude x. If x is numerically small, it is actually possible to follow an approach similar to that described above for the quenched approximation. Namely, if x is small, one can work to a given low order in x. Then provided Eq. (3) is satisfied, to first order in x, we again find that $C_{H-AB}(t)$ will have a contribution with a t-dependence behaving as $xte^{-E_{AB}t}$ from Eq. (1) just as described above. In addition to further transitions and corrections from the mixing energy shifts which will both be of higher order in xt, one must also consider the intrinsic mixing of the initial H state with AB (and vice versa). This intrinsic mixing [i.e., not the mixing induced by the propagation from the energy matrix of Eq. (4)—see Fig.
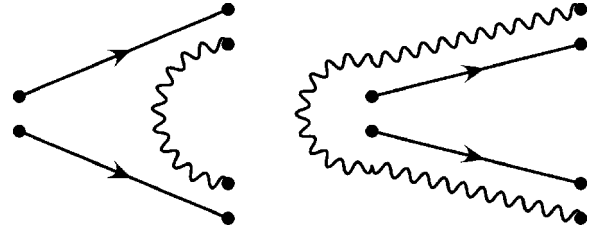

FIG. 2. Light quark pair production (wiggly lines) for the three point function $H \rightarrow A + B$ in Euclidean time (running horizontally). The straight lines represent quarks which may be heavy or light. The left hand diagram has the interpretation of a transition (our x) at an intermediate time while the right hand diagram can be thought of as some intrinsic mixing in the H state.

2 for an illustration of a typical contribution] is expected to be of order $x/\mathcal{E}$ where $\mathcal{E}$ is the energy of the quark pair and so will contribute a term like $xe^{-Et}/\mathcal{E}$. This is a contribution similar to that from excited states and so will be dominated at large t by the xte^{-Et} term we are looking for. So we need both xt to be small and t to be large. This implies that x must be small for this simplified approach.

We now discuss whether x is generically small. Provided the ranges of the interactions between A and B and between H and AB are effectively finite and smaller than the spatial extent L, then the transition probability x^2 will be proportional to $1/L^3$ and hence the transition amplitude x behaves as $1/L^{3/2}$. As L is increased, the different momentum states of $A + B$ become closer together in energy and the density of states behaves like L^3. Hence the net transition probability to states close to a given momentum will be independent of L at large L as expected. Thus we conclude that x is indeed small at large volume but that off diagonal transitions between different momentum states will become important. Thus at large volume there will be many small x's to take into account.

So a practical method will be possible if the lowest energy AB state that couples to H has a similar energy to H. This lowest energy state will have relative momentum $\mathbf{n} = 0$ for S-wave decays and $\mathbf{n} = (1,0,0)$ for P-wave decays etc. By adjusting the lattice volume and quark mass, it may be possible to arrange for approximate energy equality: this is often called an on-shell transition. From studying the correlations as above, one can then extract the transition amplitude x. One example was to explore glueball decay to two pseudoscalar mesons in the quenched approximation [20,21].

A careful discussion of the matching [22] between finite volume and infinite volume relies on a quantitative treatment of the interactions between the two bodies ($A + B$). In our treatment, we are neglecting this interaction, so one can obtain the matching directly from phase space considerations [13,20,21]. The key step is that we are normalizing the states H, A and B to one. Then the density of $A + B$ states in energy is given from $E(\mathbf{n}) = \sqrt{m_A^2 + k^2} + \sqrt{m_B^2 + k^2}$ with $\mathbf{k} = 2\pi\mathbf{n}/L$. In our application here, we shall treat m_A as infinite so the density of states $\rho(E) = 4\pi n^2 dn/dE = L^3 k E_B/(2\pi^2)$. Then first order perturbation theory (Fermi's Golden Rule) implies a transition rate $\Gamma = 2\pi x^2 \rho(E)$. Here we explicitly see the factor of L^3 from the density of states cancelling the implicit

factor of $L^{-3/2}$ in x. At first sight the fact that the density of states is rather sparse in a finite volume is worrying [23,24]—but the matching is done at the level of the transition amplitude so the density of states is needed in a large volume only.

In conclusion, we are able to evaluate the transition amplitude x on a lattice when $E_H \approx E_A + E_B$ for momentum k which is the minimum lattice momentum at that finite volume, provided

$$(E_H - E_A - E_B)t \ll 5, \quad xt \ll 1,$$

and

$$(E' - E)t \ll 1 \tag{5}$$

where $E' - E$ is the energy gap to an excited state. Then we use this lattice result to determine the large volume physical decay rate where k is the momentum of the decay product.

V. LATTICE TRANSITION MATRIX ELEMENT

A. Energies

Here we use 207 dynamical fermion configurations [25] with $N_f = 2$ flavors of sea-quark with masses around the strange quark mass [$\kappa = 0.1355$ with NP improved clover having $C_{SW} = 2.02$ at $\beta = 5.2$ for a $16^3 \times 32$ lattice, with lattice spacing given by $r_0/a = 5.04(4)$ which corresponds to a of around 0.1 fm].

We are interested in the heavy quark limit so we evaluate the usual static potential (A_{1g}) and the excited-gluonic potential which corresponds to the E_u representation. Results are shown in Fig. 1.

We can now evaluate the energy release at each value of R and compare it with known flavor-singlet meson masses. We are interested in the minimum momentum allowed which will be $\mathbf{n} = (1,0,0)$ for the scalar meson emission and $\mathbf{n} = (1,1,0)$ for the pseudoscalar emission. We can evaluate the energies of these states with non-zero momentum by assuming the usual energy-momentum relationship and taking the masses as determined on a lattice [25]. We also check these energy estimates from our results here. For example from the pion mass $am = 0.294(4)$ and flavor singlet mass enhancement of around 0.06 one gets $aE(110) = 0.66$ for this pseudoscalar state. For the scalar meson $am = 0.628(30)$ implying $aE(100) = 0.74(3)$ for this scalar state. These two energy values are comparable with the energy release for R values of around 0.2 fm—see Fig. 1 where this is illustrated for the scalar meson emission.

B. Transitions

We now discuss the transition matrix elements in the heavy quark limit where the quarkonia states will be accurately treated by the static approximation. We then discuss the creation of the light quark antiquark pair which forms the flavor singlet meson. In particular we discuss how to create operators for the E_u, $A_{1g} + S(0^{++})$, and $A_{1g} + \eta$ states.

Let the static quarks be separated by R in the z-direction with the midpoint at $\mathbf{r}$. Then under rotations about the z-axis we have a two-dimensional representation (like $J_z = 1$). These two states correspond to flux states from a lattice operator which is the difference of U-shaped paths from quark to antiquark of the form $\sqcap - \sqcup$ where the transverse extent can be in the x or y direction respectively.

For the ground state (A_{1g} on a lattice) we take a straight path from the static quark to antiquark. Then we need to discuss the spatial distribution of the light quark meson with respect to the static quarks. We have to ensure that the initial and final states are in the same representation of the symmetries of the heavy quark state. This can be achieved by constructing the two body state using the lattice operator

$$O(\mathbf{r}) = \sum_{\mathbf{s}} a(\mathbf{r}) M(\mathbf{s}) w(\mathbf{s}, \mathbf{r}) \tag{6}$$

where a represents the color field in the z-direction from $\mathbf{r} - \mathbf{e_z}R/2$ to $\mathbf{r} + \mathbf{e_z}R/2$ and w is the distribution function of the flavor singlet meson operator M (which will be either a pseudoscalar or scalar meson). Because of translational invariance, we can express this meson distribution function w most efficiently in momentum space:

$$w(\mathbf{s}, \mathbf{r}) = \sum_{\mathbf{k}} e^{i\mathbf{k}(\mathbf{r} - \mathbf{s})} w(\mathbf{k}). \tag{7}$$

The symmetries of $w(\mathbf{r}, \mathbf{s})$ depend on those of the meson produced. For scalar meson production (with $J^{PC} = 0^{++}$), then $w(\mathbf{s}, \mathbf{r})$ is in an E_u representation and this can be achieved by making $w(\mathbf{k})$ odd in k_x and even in k_y and k_z where x is the direction of transverse extent of the E_u state described above.

For pseudoscalar meson production (which has $CP = -1$), then $w(\mathbf{r}, \mathbf{s})$ is in an E_g representation and this can be achieved by making $w(\mathbf{k})$ odd in k_y and k_z and even in k_x. Another way to see that this is the correct symmetry configuration is by considering space inversions, since P_x, P_y and CP_z are conserved in the transition. Now consider the E_u representation which is odd under P_x and even under P_y and CP_z, while the η operator, being pseudoscalar, is odd under all three operations. The A_{1g} operator is even under all three operations, so we need to introduce a wave function w which is odd under inversions P_y and P_z (since $C = +1$ for the η) and even under P_x.

In practice we evaluate the difference of two Wilson loops corresponding to creating the E_u state at $\mathbf{r}, t$ with transverse extent in the x and $-x$ directions and annihilating the A_{1g} state at $\mathbf{r}, t + T$. Let us call this observable $\text{Æ}(\mathbf{r})$ and its spatial Fourier transform $\text{Æ}(\mathbf{q})$. The disconnected fermion loop (from operator $\eta = \bar{q} \gamma_5 q$ for pseudoscalar mesons or $S = \bar{q}q$ for scalar mesons) is evaluated at each spatial point $\mathbf{s}$ at time $t + T$ and its Fourier transform is $M(\mathbf{p})$ corresponding to $\eta(\mathbf{p})$ or $S(\mathbf{p})$. Then the required correlation is given after summing over $\mathbf{r}$ as

$$\sum_{\mathbf{k}} w(\mathbf{k}) \text{Æ}(-\mathbf{k}) M(\mathbf{k}). \tag{8}$$

Here $\cancel{E}$ represents a Wilson loop which has a zero expectation on its own since it has an E_u state at one end and an A_{1g} state at the other end in time, and M represents a fermionic disconnected loop with non-zero momentum which also has a zero expectation value on its own. Here M is evaluated by stochastic methods [19]. The product of these two operators is constructed to have a non-zero expectation value and that is the target of this investigation. We actually used fuzzed sources for the spatial ends of $\cancel{E}$ [2 or 13 iterations of $U \rightarrow \mathcal{P}(c U_{\text{Straight}} + \Sigma U_{\text{Staples}})$ with $c=2.5$ for the A_{1g} end but only the higher iteration level for the E_u end] and different sizes (1 or 2 lattice spacings) for the transverse extent of the E_u end while we use fuzzed and local fermionic operators for the light-quark meson.

We have here described the correlation in terms of a specific orientation of the static quark separation and of the transverse extent of the E_u state. On a lattice we sum over all cubic rotations, translations and reflections to increase statistics.

1. Pseudoscalar decays

For pseudoscalar decays, since $w(\mathbf{k})$ is odd in both k_y and in k_z, these momenta must be non-zero. The simplest assumption which corresponds to the lightest allowed state is used in this exploratory study; namely, that $n_x=0, n_y=\pm 1$ and $n_z=\pm 1$ where the lattice momentum $\mathbf{k}=2\mathbf{n}\pi/L$. In terms of these components of the momentum, the required correlation is

$$2\text{Re}[\cancel{E}(0,1,1)\eta(0,-1,-1)-\cancel{E}(0,1,-1)\eta(0,-1,1)] \quad (9)$$

which we evaluate as (here cc means cos transform in y and z, etc.)

$$4(-\cancel{E}_{cc}\eta_{ss}-\cancel{E}_{ss}\eta_{cc}+\cancel{E}_{sc}\eta_{cs}+\cancel{E}_{cs}\eta_{sc}). \quad (10)$$

Note that $\eta(\mathbf{s})$ is real for Wilson-like fermion formalisms, so we take the real part of the stochastic estimate, while $\cancel{E}(\mathbf{r})$ is complex since the Wilson loop in SU(3) has an orientation, but η has even charge conjugation so we need to take the real part here also.

For this minimum momentum, we have energy equality at $R=2a$.

Following Eq. (2), we normalize states to 1 and evaluate the transition matrix element $x=\langle H|A\eta\rangle$ from the ratio at each t value

$$xt=\frac{C_{H-A\eta}(t)}{[C_{A-A}(t)C_{H-H}(t)C_{\eta-\eta}(t)]^{1/2}} \quad (11)$$

where we have neglected interactions in the $A\eta$ two body state so have used for its correlator the direct product of the A and η propagation. This amounts to neglecting the correlation between the $R\times t$ Wilson loop giving $C_{A-A}(t)$ and the η correlator involving light quarks, so that

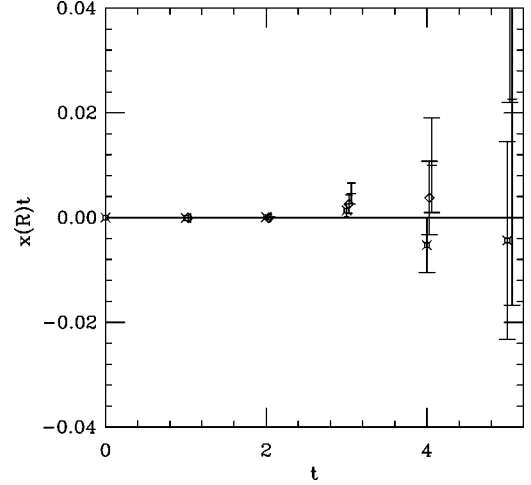$$C_{A\eta-A\eta}(t)=C_{A-A}(t)C_{\eta-\eta}(t). \quad (12)$$

FIG. 3. The transition matrix element xt for $H \rightarrow AP$ with momentum $\mathbf{n}=(1,1,0)$ versus t. Here $R=0.1$ to 0.3 fm is represented by symbols: fancy square, diamond, $+$.

Here the $C_{\eta-\eta}(t)$ contribution includes the connected and disconnected contributions to the η propagation.

As shown in Fig. 3, we obtain no signal for the ratio of Eq. (11) which curtails our investigation. We can obtain limits, however. For instance at $R=2a$ and at $t=3$, a value $xt =0.0026\pm0.0017$ is obtained. This can be turned into a limit on this transition rate of $\Gamma<1.5$ MeV. Note that this value is for quarks of strange mass at $R=0.2$ fm so with no account of wave function effects, for $N_f=2$ with no account of η, η' mixing, with no account of excited state contamination and without any continuum limit. Because of the lack of any signal, we are unable to pursue these corrections and extrapolations. Perhaps the most useful conclusion is that this transition appears weak, maybe because it is a D wave and so involves cancellations between different spatial components of the η wave function.

2. Scalar decays

For scalar decays, since $w(\mathbf{k})$ is odd in k_x, this momentum must be non-zero. The simplest assumption which corresponds to the lightest allowed state is used in this exploratory study; namely, that $n_x=\pm 1, n_y=0$ and $n_z=0$. For this minimum momentum, we have energy equality in the transition at $R=2a$—see Fig. 1. In terms of these components of the momentum, the required correlation is

$$\text{Im}[\cancel{E}(1,0,0)S(-1,0,0)-\cancel{E}(-1,0,0)S(1,0,0)] \quad (13)$$

which we evaluate as (here c means cos transform in x, etc.)

$$2\text{ Re}(-\cancel{E}_c S_s+\cancel{E}_s S_c). \quad (14)$$

We evaluate the ratio at each t value

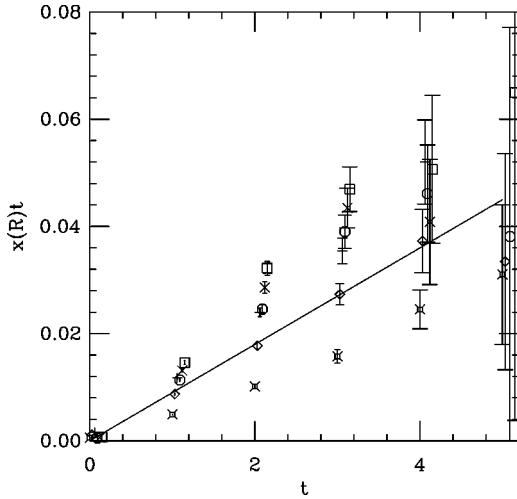$$xt=\frac{C_{H-AS}(t)}{[C_{A-A}(t)C_{H-H}(t)C_{S-S}(t)]^{1/2}} \quad (15)$$

FIG. 4. The transition matrix element xt for $H \to AS$ with momentum $\mathbf{n} = (1,0,0)$ versus t. Here $R = 0.1$ to 0.6 fm is represented by symbols: fancy square, diamond, $+$, octagon, $\times$, square. The line represents a linear fit to the $R = 0.2$ fm case.

where the scalar propagation in $C_{S-S}(t)$ again involves both connected and disconnected contributions. In this case we do obtain a signal and the values of x extracted are shown in Fig. 4. Moreover we do see good evidence for a linear dependence on t as needed to ensure excited state contributions are removed. This linear dependence sets in from very small t-values which may be explained if the off-diagonal transition matrix elements (i.e., the corresponding x-values for the transition from excited state to ground state) are small compared to the diagonal case. From the slope we can extract x obtaining $ax = 0.009(1)$ at $R = 2a$. This is indeed a small value and our assumptions about using the three point function analysis are thus fully justified. It would indeed be very difficult to detect directly the shift of ± 0.01 in the aE-values of Fig. 1 arising from this mixing where the two levels cross at $R \approx 0.2$ fm.

Using this x-value and an energy release of $aE = 0.73$, we obtain a transition rate of $\Gamma = 0.061(14)$ GeV. Note that this result is at a fixed R-value (0.2 fm), for strange quarks and with no continuum limit.

Although we are only able to evaluate x at one R-value in principle (where we have energy equality) in this study, we can explore other R values where the energy equality is only approximate. Indeed since the energy difference increases to only $a\Delta \approx 0.35$ at $R = 6/a$, we find that the criterion of approximate energy equality [Eq. (3)] is met for the range of t-values considered here. Moreover, we do find for R-values from a to $6a$ that the correlator ratio of Eq. (15) is consistent with a linear fit in t over the range of t from a to $5a$. Thus we can still estimate the x-values by a linear fit in t, with the understanding that excited state effects may be less completely removed. We find an increase of x with R (see Fig. 4) with some sign of a saturation at large R [namely fit values of $ax = 0.005(1)$, $0.009(1)$, $0.012(1)$, $0.013(2)$, $0.015(2)$, $0.017(2)$ at $R = 0.1, \ldots 0.6$ fm, respectively]. Since the transition to the scalar meson with momentum k is a P-wave, one

would expect the transition amplitude to have a factor of k. However, we are working at a fixed volume, so the minimum momentum k is fixed as R varies. Thus although the energy release varies with R, we have a fixed momentum k and hence this consideration should not affect the dependence of x on R.

One way to interpret the R-dependence of x is by noting that the scalar meson wavefunction has a node at the center of the E_u state in the transverse direction (since w is odd in relative transverse spatial coordinate) and so it is sensitive to the transverse width of the excited gluonic flux in the E_u state. This increases [26] with longitudinal extent R and then starts to saturate, just as we find.

C. Phenomenology

In the extreme heavy quark limit, the heavy quarks are static and one can define a transition rate for each separation R. Also there will be a well defined energy release for each value of R: for example at $R = 2a = 0.2$ fm we find $E_{E_u} - E_{A_{1g}} = 0.73(1)/a = 1.4$ GeV. The energy of the scalar meson with the required momentum we take as $aE(100) = 0.74(3)$ and we also check that this energy is consistent with the value we find directly from fitting our scalar correlators with this momentum [namely $aE(100) \approx 0.7$]. Thus indeed we are close to on-shell as required. In the real world, the quarks are bound and there is a distribution of R-values as given by the wave functions. For b quarks, as discussed previously, the static potentials allow us to estimate the quark wave functions. The hybrid wave function is effectively P-wave and actually has a quite large overlap with the χ_b wave function (the overlap peaks at around 0.4 fm and the wave-function overlap integrated over all R gives a factor of 0.63 in the transition rate assuming x is independent of R). The energy release from the hybrid meson [1] at 10.76(7) GeV to the χ_b state at 9.893 GeV will be 0.87 GeV which is similar to the value at fixed R with $R \approx 0.4$ fm. Thus there is a mismatch in the energy release we study on the lattice (1.4 GeV) and that in experiment (0.9 GeV). Note that this issue could be very important: the decay rate will be proportional to k^3, so a small change of energy release will have a big effect on k and an even bigger effect on the rate. Put more bluntly: there will be no decay to a scalar meson heavier than 870 MeV in practice, but our estimates for the scalar meson mass are indeed heavier than this. Thus we will need some method to treat the virtual (below threshold) production of a scalar meson which subsequently decays to two pions. We discuss this in the context of the quark mass dependence.

We are evaluating the transition matrix element for quarks of mass near that of strange quarks. At this quark mass on our lattice the scalar meson (mass $ma = 0.63$) is already unstable to decay to two pions (mass $ma = 0.29$ each) in an S-wave. Thus we should consider, in principle, a further layer of sophistication: the sequential decay of the scalar meson to two pions. This decay will become even more significant as the quark mass is reduced further towards the physical case. Moreover it will allow scalar $\pi + \pi$ states to be produced even if the available energy is less than the mass of a scalar meson as discussed above. Thus we do not at-

tempt a naive extrapolation to light quarks of realistic mass. A study of the three body final state will be needed to resolve this issue more completely.

We could also study, in principle, the transition for higher momentum, for example S having momentum $(1,1,0)2\pi/L$ with energy $aE(110)=0.85(4)$, but then energy equality between initial and final states would be at even smaller R-values. Also this higher energy state would be coupled to the lighter $aE(100)$ state we have explored above and this would make the extraction from the lattice more prone to systematic errors.

Another possible avenue would be to vary the lattice spatial size L. This is not feasible with our current dynamical data set but is of interest for the future.

Without making a detailed study of the sequential decay of the scalar meson to two pions, our estimates of the decay rate will be qualitative. We find a rate of 61(14) MeV for the unphysical case of a transition at $R=0.2$ fm with strange quarks in the scalar meson. At the more realistic value (where the wave function overlap peaks) of $R=0.4$ fm, we have a larger transition amplitude $ax=0.013(2)$ but the energy release is insufficient for decay to an on-shell scalar meson. Including the wave function overlap factor of 0.63, we conclude that the decay rate to a scalar channel will be less than 80 MeV (here the upper limit is from assuming that the scalar meson is produced off-shell with momentum $k=2\pi/L$ at R≈0.4 fm).

Flux-tube models have been used to estimate hybrid decay widths [27]. For $b\bar{b}$ hybrid mesons, they only consider decays to BB and B^*B and these decay rates are found to be very small (less than 1 MeV).

VI. CONCLUSIONS

We have presented arguments that, in the heavy quark limit, the decay of a $J^{PC}=1^{-+}$ spin-exotic hybrid meson will be primarily through flavor singlet light quark-antiquark emission.

We have explored in lattice QCD with $N_f=2$ flavors of a sea quark (with mass near that of the strange quark) the transition between an excited gluonic state with heavy quarks at separation R and a ground state gluonic system with a flavor-singlet light quark-antiquark pair emitted. We find a very weak transition amplitude for emission of a pseudoscalar meson but a much larger rate for a scalar meson.

On a lattice it is only possible to extract a limited set of information: namely when there is an on-shell transition in the finite volume used. Our raw lattice results are a transition width of less than 1 MeV for the pseudoscalar case and 61(14) MeV for the scalar case. These results are for transitions at fixed $R\approx0.2$ fm in the heavy quark limit and for light quarks that are of strange mass.

For b quarks the relevant transitions will be $H\rightarrow\chi_b\eta$ and $H\rightarrow\chi_b S$. We argue that wave function effects will suppress these decay rates rather little (a factor of 0.6) while the choice of a more appropriate R-value (of 0.4 fm) will increase the rates. This yields an off-shell decay rate to a scalar meson of around 80 MeV which can be regarded as an upper limit. The main uncertainty comes from the sensitive dependence (like k^3) of the rate on the energy release and the complications caused by the subsequent decay of the scalar meson to two pions. More work needs to be done to build phenomenological models of scalar meson production and decay and, eventually, to explore the transition to two pions directly on the lattice.

Despite this, we consider that first principles QCD evaluation of these hadronic transitions is a very valuable component of a phenomenological study of hybrid decays. Our results are consistent with the expectation that these spin-exotic hybrid meson states are relatively narrow and hence will be detectable experimentally. We have not evaluated decay processes that are not allowed in the heavy quark limit (such as retardation effects or heavy quark spin-flip) and it would be valuable to investigate them to ensure that they are indeed negligible compared to the string deexcitation decay that we find to be important.

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