Masses of singlet and non-singlet  $0^{++}$  particles.

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We compute the mass of the singlet  $0^{++}$  state using both  $\overline{\psi}\psi$  and Wilson loop operators from a  $N_f = 2$  lattice QCD calculation.

# 1. INTRODUCTION

The  $0^{++}$  singlet state is an excitation of the vacuum. The complicated non-perturbative nature of the vacuum is one of the reasons that QCD is hard to solve at low energies. The precise computation of scalar meson masses via a first principles full QCD calculation would demonstrate that low energy QCD could be tamed.

The work of many groups[[1,2,3](#page-2-0)] has shown that thelightest  $0^{++}$  state is  $1611(30)(160)$  MeV [[4\]](#page-2-0) in pure SU(3) Yang-Mills theory. The priority now is to find evidence for glueballs in nature. This almost certainly requires an understanding of the mixing of glueballs with quark anti-quark states. Experimentally, there are more singlet  $0^{++}$  states in the region 1200 to 2000 MeV than can be organised into SU(3) nonets[[5\]](#page-2-0). An understanding of the non-singlet  $0^{++}$  states is also required to show there are additional degrees of freedom beyond  $\overline{\psi}\psi$  states.

In full QCD all operators with  $J^{PC} = 0^{++}$ quantum numbers can mix. The singlet  $\overline{\psi}\psi$  operator also has  $0^{++}$  quantum numbers, hence these operators will mix with Wilson loop  $0^{++}$ operators used for glueballs in quenched QCD. The mixing of Wilson loop and  $\overline{\psi}\psi$  states has been studied in quenched QCD by Lee and Weingarten [\[6](#page-2-0)]. They claimed that in the continuum limit the mixing between Wilson loop and  $\overline{\psi}\psi$  $0^{++}$  states is small. Constructive criticism of the Lee-Weingarten calculation is in [\[4](#page-2-0),[7\]](#page-2-0).

In this work we report results for the masses of singlet and non-singlet  $0^{++}$  states. We use unquenched gauge configurations[[8\]](#page-2-0) with a smaller lattice spacing  $(a \sim 0.1 \text{ fm})$  than our previous study[[7\]](#page-2-0)  $(a \sim 0.13 \text{ fm}).$ 

### 2. METHOD

To extract masses we use factorising (or variational) fits of M exponentials to correlators of operators  $\{O_i\},\$ 

$$
\langle O_i^{\dagger}(t)O_j(0)\rangle = \sum_{n=0}^{M} c_i^n c_j^n e^{-E_n t}.
$$
 (1)

Using additional operators helps to stabilise the multi-exponential fit. We include  $0^{++}$  operators made out of a quark and anti-quark and Wilson loops in the same fit. This basis of operators should couple well to the glueball mixing.

The Wick contraction of the singlet  $\psi\psi$  operators requires the calculation of fermion loops[[7\]](#page-2-0).

$$
\langle \overline{\psi}(t, \underline{x}) \psi(t, \underline{x}) \overline{\psi}(0, \underline{0}) \psi(0, \underline{0}) \rangle \tag{2}
$$

The correlator in equation 2 can be computed using  $Z_2$  noise techniques. We used 100 noise sources and the double source techniques described in [\[7](#page-2-0)]. In the fermion sector we use fuzzed [\[8](#page-2-0)] and local operators as basis states. Two types of smeared Wilson loop operators [\[9](#page-2-0)] were included. We use  $M = 1$  and 2 in this calculation.

The non-perturbatively improved clover action was used to generate the configurations, with  $c_{SW} = 2.02, \ \beta = 5.2, \ \text{and volume of } 16^3 32 \ [8].$  $c_{SW} = 2.02, \ \beta = 5.2, \ \text{and volume of } 16^3 32 \ [8].$  $c_{SW} = 2.02, \ \beta = 5.2, \ \text{and volume of } 16^3 32 \ [8].$  $c_{SW} = 2.02, \ \beta = 5.2, \ \text{and volume of } 16^3 32 \ [8].$  $c_{SW} = 2.02, \ \beta = 5.2, \ \text{and volume of } 16^3 32 \ [8].$ The configurations with sea  $\kappa = 0.135$  and  $\kappa =$ 0.1355 were used. We use  $f_0$   $(a_0)$  to label the singlet (non-singlet)  $0^{++}$  states.

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#### 3. RESULTS FOR THE  $a_0$

The interpretation of the experimental spectrum of the  $a_0$  hadrons is confused by the  $a_0(980)$ , that many people believe is a  $\overline{K}K$  molecule [\[5](#page-2-0)]. In quenched QCD the  $a_0$  channel is complicated by a severe quenched chiral artifact[[10\]](#page-2-0), caused by the  $\pi - \eta'$  state being treated incorrectly in quenched QCD.

One prediction[[10\]](#page-2-0) of the analysis of Bardeen et al. is that the mass of the  $a_0$  should not reduce smoothly with quark mass, but actually start to increase. This behavior was observed by Lee and Weingarten[[6\]](#page-2-0), who saw the mass of the  $a_0$  rise with quark mass below that of strange for small volumes (less than 1.8 fm). Other groups have seen similar behavior [\[11](#page-2-0),[12\]](#page-2-0).

The  $a_0$  channel was fitted using a variational basis of local and fuzzed operators. We do a partially quenched analysis, where the sea quark mass is fixed and the valence quark mass is varied. The partially quenched theory also has pathologies that tend to be less severe than for the quenched theory. In figure 1, we plot the mass of the  $\rho$  and  $a_0$  as a function of pseudoscalar mass squared, using  $r_0$  [\[13\]](#page-2-0) to make a dimensional ratio, in both quenched ( $\beta = 5.93$ ) and unquenched QCD. Figure 1 does show the quenched pathology predicted by[[10\]](#page-2-0). The results in figure 1 convince us that it is preferable to study the  $a_0$  mass in unquenched lattice calculations. If we chirally extrapolate the  $a_0$  mass for the unquenched data in figure 1 to  $\kappa_{critical}$  we get 1.0(2) GeV. More work is required to control the systematic errors on the mass of the  $a_0$ , so that lattice QCD calculations can distinguish between the mass of the  $a_0(980)$  and  $a_0(1450)$ .

# 4. RESULTS FOR THE  $f_0$

The results from unquenched QCD lattice calculations, with light quark masses and fine lattice spacings, should automatically include the physics of glueball  $\overline{\psi}\psi$  mixing. However, the results from  $n_f = 2$  lattice calculations from the HEMCGC[[14\]](#page-2-0) and SESAM[[15\]](#page-2-0) collaborations did not show a large deviation of the masses of  $0^{++}$  glueballs between quenched and unquenched



Figure 1. Mass dependence of the  $a_0$  and  $\rho$  in quenched and partially quenched QCD.

QCD.

In figure [2](#page-2-0) we show a compendium of lattice results for the mass of the  $f_0$  state in quenched and  $n_f = 2$  QCD [\[7](#page-2-0)]. All the calculations used the Wilson gauge action.

Hart and Teper measured the glueball correlators, using only Wilson loop interpolating opera-tors, on this data set [\[9](#page-2-0)]. They found that the  $0^{++}$ mass was significantly smaller in  $n_f = 2$  QCD, by a factor of  $(\sim 0.84 \pm 0.03)$ , over quenched QCD at comparable lattice spacing. In figure [2](#page-2-0) we plot the masses from the calculation by Hart and Teper (diamonds) with the masses obtained in this analysis (bursts). The inclusion of the  $\overline{\psi}\psi$  operators with the Wilson loop operators has produced a further suppression of the mass of the  $f_0$  at the lattice spacing we use. The combined use of both Wilson loop and  $\overline{\psi}\psi$  operators is clearly a superior technique for extracting masses than just using the Wilson loop operators on their own, hence the result for the  $f_0$  mass from this analysis is now <span id="page-2-0"></span>the preferred result.

Figure 2 shows the suppression of the mass of the  $f_0$  between  $n_f = 2$  and quenched QCD is less at this new lattice spacing than our previous result at  $a \sim 0.13 fm$  [7]. To make physical predictions about the spectrum of the  $f_0$  hadrons requires the continuum limit to be taken for the  $n_f = 2$  results. In quenched QCD it was found necessary to use a lattice spacing of 0.05 fm [1], this size of lattice spacing will also be required for full QCD calculations that use the Wilson gauge action. This will be computationally expensive [16] with clover fermions.

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Figure 2.  $f_0$  mass in units of  $r_0$  as a function of lattice spacing. The crosses are from SESAM [15]. The octagons are from UKQCD's [7] first  $n_f$  = 2 data set. The diamonds are the results from Hart and Teper [9] and the bursts are from the analysis described in this paper. The squares are the results from quenched calculations (see [7] for references).