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# The spectrum of $D_s$ mesons from lattice QCD

UKQCD Collaboration

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## Abstract

The spectrum of orbitally excited  $D_s$  mesons is computed in the continuum limit of quenched lattice QCD. The results are consistent with the interpretation that the narrow resonance in the  $D_s\pi^0$  channel discovered by the BABAR Collaboration is a  $J^P = 0^+ c\bar{s}$  meson. Furthermore, within statistical errors, the  $1^+ - 1^-$  and the  $0^+ - 0^-$  mass splittings are equal, in agreement with the chiral multiplet structure predicted by heavy hadron chiral effective theory. On our coarsest lattice we present results from the first study of orbitally excited  $D_s$  mesons with two flavors of dynamical quarks, with mass slightly larger than the strange quark mass. These results are consistent with the quenched data.

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## INTRODUCTION

The recent discovery by the BABAR collaboration of a new resonance, with a mass around 2.32 GeV and a narrow width, in the  $D_s^+\pi^0$  final state [1] has provoked a great deal of interest from experimenters and theorists alike. The CLEO collaboration [2] has confirmed this resonance. Both experiments interpret this as the lowest lying of the four  $P$ -wave states, the  $^3P_0$  with  $J^P = 0^+$ . A variety of theory papers have been published [3, 4, 5, 6, 7, 8, 9, 10, 11, 12] either supporting this interpretation or presenting alternative hypotheses of “exotic” states. These alternatives are motivated by potential model results [13, 14], which suggest that the  $c\bar{s}$  scalar meson mass is around 2.48 GeV and hence above the  $DK$  threshold. One could argue the merits of a particular model, but this becomes irrelevant if the spectrum can directly determined from QCD. A mass below the  $DK$  threshold would also explain the narrowness of the state.

There have been a number of previous lattice QCD calculations of the  $L = 1$  states in the  $D_s$  meson spectrum. Hein *et al.* [15] obtained 500(80) MeV for the  $D_{s0}^* - D_s$  mass splitting (from their figure 27) using NRQCD at a fixed lattice spacing of 0.18 fm. Lewis and Woloshyn [16] obtained 530(15)(5) MeV at a fixed lattice spacing of 0.11 fm also using NRQCD. With quenched relativistic heavy quarks, Boyle [17] obtained a mass splitting of 544(20) MeV at a fixed lattice spacing of 0.07 fm. Although the calculation used the same ensemble of gauge configurations as this work, the propagators were computed using a slightly different action and different definition of the lattice quark mass. Recently Bali [10] presented results in the static limit for the heavy quark and obtained a value of 468(43)(24) MeV for the scalar-pseudoscalar mass splitting in this limit.

All the previous lattice QCD calculations of the  $D_s$  spectrum were done at fixed lattice spacing in quenched QCD. In this calculation we take the continuum limit in quenched QCD, so that lattice artifacts are under control. The lattice volumes are large enough (greater than  $1.5^3$  fm<sup>3</sup>) that finite size effects should be small. We also report results from the first unquenched calculation at fixed lattice spacing. The remaining systematic uncertainty is due to dynamical  $u$  and  $d$  quarks having unphysically large masses.

In the heavy quark limit, the spin of the heavy quark decouples from the rest of the system. The observable states can be labelled by the total angular momentum of the light quark  $j$  [18]. The  $P$ -wave states have  $L = 1$  and thus  $j = \{\frac{1}{2}, \frac{3}{2}\}$ . This combined with the

spin of the heavy quark produces two doublets. The  $j = \frac{3}{2}$  doublet contains a  $J = 2$  and a  $J = 1$  state, the  $j = \frac{1}{2}$  doublet contains a  $J = 1$  and  $J = 0$  state. The two  $J = 1$  states do not have definite charge conjugation and so can mix. On the lattice only the lightest state in this channel can be determined easily.

In the double limit of heavy quark and chiral symmetry, the two heavy light multiplets,  $\{0^-, 1^-\}$  and  $\{0^+, 1^+\}$ , are degenerate. The effect of spontaneous chiral symmetry breaking is to split these parity partners, such that the mass splittings  $1^+ - 1^-$  and  $0^+ - 0^-$  are equal [7]. This is confirmed by the CLEO Collaboration [2] who obtain splittings of 351(2) and 350(1) MeV respectively. It is interesting to explore the extent to which QCD reproduces this remarkable agreement.

## LATTICE DETAILS

The spectrum of  $c\bar{s}$  mesons has been determined on four ensembles of gauge configurations. Three have different lattice spacings ( $a$ ) and were generated in the quenched approximation, which enables the continuum limit to be taken. The fourth ensemble was generated with two degenerate flavours of dynamical quarks, and the lattice spacing matched to that of the coarsest quenched ensemble. For all the ensembles, the Wilson gauge action and the non-perturbatively  $\mathcal{O}(a)$  improved Wilson fermion action were used. The lattice parameters are detailed in Table I. The procedures for generating the dynamical ensemble and matching to the coarsest quenched ensemble are described in [19].

Meson correlation functions were computed with several different heavy quark masses which span the charm quark mass, and several light quark masses around the strange quark mass. For the dynamical ensemble only one sea quark mass is used, for which the ratio  $m_{PS}/m_V = 0.70(1)$  when  $m_{\text{sea}} = m_{\text{valence}}$ . This corresponds to QCD with two dynamical flavours of mass slightly above the strange quark mass. The details of extracting the spectrum from lattice correlation functions, and the results for the  $S$ -wave  $D$  meson spectrum for the finer two lattice spacings can be found in [20]. To measure the mass splittings, the ratio of correlation functions at large times is fitted. We have checked that the results from computing the  $1^+ - 0^+$  mass splitting directly is the same, with the same statistical errors as obtained by combining the results obtained for the  $1^+ - 1^-$ ,  $0^+ - 0^-$  and  $1^- - 0^-$  splittings.

In quenched QCD, or in simulations with unphysical heavy sea quarks, there is an am-

biguity in the determination of the lattice spacing in physical units. For typical simulation parameters used today this is estimated to be of the order of 10%. The scale is set throughout this calculation from the static quark potential using  $r_0$  [21, 22]. The value of  $r_0/a$  is unambiguous for each ensemble, and so is a good choice for comparing results from different ensembles. However, there is no agreed experimental value for  $r_0$ . Sommer originally advocated  $r_0 = 0.5$  fm. The lattice spacing obtained from the  $K^*/K$  mass ratio (method of planes) [23] corresponds to  $r_0 \sim 0.55$  fm on the ensembles used in this work. Determinations of the lattice spacing from the kaon decay constant, the nucleon mass or the rho mass correspond to  $r_0$  values ranging from approximately 0.5 to 0.55 fm [19, 20, 24, 25, 26]. For these reasons we take the value of  $r_0$  to be 0.55 fm. The analysis has been repeated using  $r_0 = 0.5$  fm throughout, and the difference is taken as an estimate of the systematic uncertainty in the scale.

The strange quark mass is set by from the light-light pseudoscalar mass with the experimental kaon mass as input [27]. Similarly the charm quark mass is set from the heavy-light pseudoscalar mass with the experimental  $D_s$  meson mass as input.

## RESULTS

For each of the four ensembles, the mass splittings  $1^+ - 1^-$  and  $0^+ - 0^-$  are equal within statistical errors. These results and the continuum extrapolation which is linear in  $a^2$  are shown in figure 1. Furthermore, these splittings are also equal in the continuum limit, in agreement with heavy hadron chiral effective theory and experiment. At the coarsest lattice spacing the effect of introducing sea quarks with a mass close to the strange is to slightly lower the  $1^+ - 1^-$  and the  $0^+ - 0^-$  mass splittings, but this is not statistically significant.

Shown in figure 2 and Table II is the comparison of the lattice results along with the experimentally measured spectrum. The dynamical results appear to have smaller error bars and to be systematically higher than the quenched result. These effects are due to extrapolating the quenched results and are absent at fixed lattice spacing. The computed  $1^- - 0^-$  splitting is too small, a well known failing of the quenched approximation [28, 29]. Evidently the dynamical sea quark mass is too large to change this, as has been observed before in the light hadron spectrum on the same ensemble [19]. The lattice results for the  $c\bar{s}$   $0^+$  and the lightest  $1^+$  mesons are consistent, albeit within large statistical and

systematic uncertainties, with the masses of the states discovered recently by BABAR and CLEO. Although  $u$  and  $d$  sea-quark effects are not yet properly included, these lattice results provide the most reliable computation of the  $c\bar{s}$  spectrum to date. Our errors are too large to exclude exotic states based on potential models. However, there is no evidence from our lattice QCD calculations that exotics are required to explain the BABAR and CLEO discoveries.

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TABLE I: Ensemble of gauge configurations.  $\kappa_{\text{sea}} = 0$  denotes a quenched ensemble.

$(\beta, \kappa_{\text{sea}})$	Volume	$a^{-1}$ GeV $r_0 = 0.55\text{fm}$	number of configurations
(6.2, 0)	$24^3 \times 48$	2.64	216
(6.0, 0)	$16^3 \times 48$	1.92	302
(5.93, 0)	$16^3 \times 32$	1.70	278
(5.2, 0.1350)	$16^3 \times 32$	1.70	395

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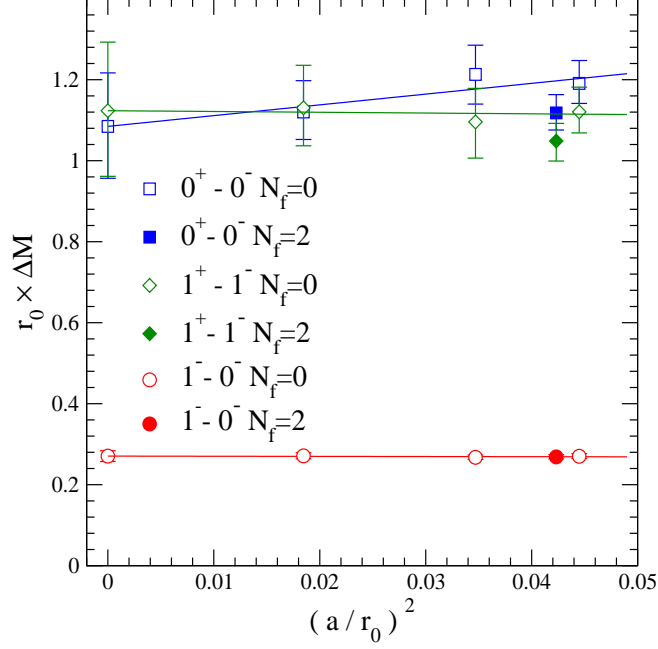


FIG. 1: Continuum extrapolation of quenched results (open symbols) and  $N_f = 2$  results in dimensionless units at fixed lattice spacing (solid symbols, offset for clarity) for mass splittings in the  $D_s$  system.

TABLE II: Comparison of lattice results with experiment. Mass splittings from the  $D_s^+(1969)$  Mass in MeV. CL denotes continuum limit,  $r_0 = 0.55$  fm is used unless otherwise noted.

$J^P$	experiment	$N_f = 0$ CL	$N_f = 0$ CL	$N_f = 2$	$N_f = 0$
		$r_0 = 0.5$ fm		$a^{-1} \sim 1.7$ GeV	$a^{-1} \sim 1.7$ GeV
$1^-$	143	121(6)	97(6)	96(2)	97(2)
$0^+$	351(1)	435(57)	389(47)	401(16)	427(20)
$1^+$ $j = 3/2$	576	—	—	—	—
$2^+$	604	—	—	—	—
$1^+$ $j = 1/2$	494(2)	572(72)	500(62)	472(20)	499(22)

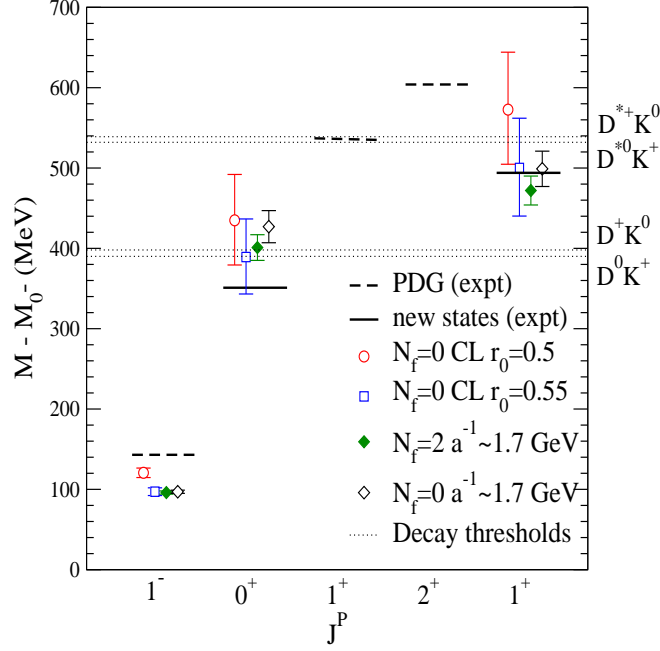


FIG. 2: Comparison of experimental results with the lattice determinations from the quenched continuum limit (open symbols) and the  $N_f = 2$  data at fixed lattice spacing (solid symbols) with  $r_0 = 0.55$  fm. For comparison we also show the quenched result at the same lattice spacing (open diamonds). Zero on the vertical scale is set by the  $D_s^+(1969)$  mass. Also plotted are the experimental  $DK$  and  $D^*K$  thresholds.