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Renormalizability of the local composite operator A_μ^2 in linear covariant gauges

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Abstract

The local composite operator A_μ^2 is analysed within the algebraic renormalization in Yang-Mills theories in linear covariant gauges. We establish that it is multiplicatively renormalizable to all orders of perturbation theory. Its anomalous dimension is computed to two-loops in the $\overline{MS}$ scheme.

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1   Introduction

The possibility that gluons might acquire a mass through a dynamical mass generation mechanism is receiving increasing attention, both from the theoretical point of view as well as from lattice simulations. Effective gluon masses have been reported in a rather large number of gauges [1]. For instance, the relevance of the local operator $A^a_\mu A^{a\mu}$ for Yang-Mills theory in the Landau gauge has been emphasized by several authors [2, 3]. That this operator has a special meaning in the Landau gauge follows by observing that, due to the transversality condition $\partial_\mu A^{a\mu} = 0$, the integrated mass dimension two operator $\int d^4x A^a_\mu A^{a\mu}$ is gauge invariant. Remarkably, the operator $A^a_\mu A^{a\mu}$ in the Landau gauge is multiplicatively renormalizable [4, 5], its anomalous dimension being given [5, 6] by a combination of the gauge beta function, $\beta(a)$, and of the anomalous dimension, $\gamma_A(a)$, of the gauge field, according to the relation

$$\gamma_{A^2}(a) \;=\; -\; \left(\frac{\beta(a)}{a} \;+\; \gamma_A(a) \right), \qquad a = \frac{g^2}{16\pi^2} \;. \tag{1}$$

Moreover, lattice simulations [3] have provided strong indications of the existence of the condensate $\left\langle A^a_\mu A^{a\mu} \right\rangle$, which is deeply related to the dynamical gluon mass. A renormalizable effective potential for this condensate in pure Yang-Mills theory has been constructed and evaluated in analytic form up to two-loop order in [4], resulting in an effective gluon mass $m_{\text{gluon}} \approx 500 MeV$. The inclusion of massless quarks has been recently worked out [7]. Another analytic study of $\left\langle A^a_\mu A^{a\mu} \right\rangle$ can be found in [8]. Also, lattice simulations [9] of the gluon propagator in the Landau gauge have reported a gluon mass $m_{\text{gluon}} \approx 600 MeV$. Concerning other gauges, an effective gluon mass has been reported in lattice simulations in the Laplacian [10] and maximal abelian [11, 12] gauges. It is worth underlining that the local operator $A^a_\mu A^{a\mu}$ of the Landau gauge can be generalized [13] to the maximal abelian gauge, which is a renormalizable gauge in the continuum [14, 15, 16]. It turns out in fact that the integrated mixed gluon-ghost operator* $\int d^4x \left(\frac{1}{2} A^\alpha_\mu A^{\alpha\mu} + \xi \overline{c}^\alpha c^\alpha \right)$ is BRST invariant on-shell [13], a property which ensures the multiplicative renormalizability to all orders of perturbation theory [17, 18] of the local operator $\left(\frac{1}{2} A^\alpha_\mu A^{\alpha\mu} + \xi \overline{c}^\alpha c^\alpha \right)$. The analytic evaluation of the effective potential for the condensate $\left\langle \frac{1}{2} A^\alpha_\mu A^{\alpha\mu} + \xi \overline{c}^\alpha c^\alpha \right\rangle$ has not yet been worked out. Nevertheless, we expect a nonvanishing value for this condensate, which would result in a dynamical gluon mass. This is supported by the fact that a renormalizable effective potential for the mixed gluon-ghost operator has been obtained [19] in the nonlinear Curci-Ferrari gauge, yielding a nonvanishing condensate $\left\langle \frac{1}{2} A^2_\mu + \xi \overline{c}c \right\rangle$, which provides a dynamical mass for the gluons. The Curci-Ferrari gauge shares a close similarity with the maximal abelian gauge. We expect thus that something similar should happen in this gauge.

A gluon condensate $\left\langle A^a_\mu A^{a\mu} \right\rangle$ has also been introduced in the Coulomb gauge [20] in order to obtain estimates for the glueball spectrum. Older works [21, 22, 23, 24] already discussed the pairing of gluons in connection with a mass generation, as a result of the instability of the perturbative Yang-Mills vacuum. Also, the dynamical mass generation for the gluons is a part of the Kugo-Ojima criterion for color confinement [25, 26]. See [27] for a review.

In this work we analyse the ultraviolet properties of the local composite operator $A^a_\mu A^{a\mu}$ in the linear covariant gauges, whose gauge fixing term is

$$\int d^4x \left(b^a \partial_\mu A^{a\mu} + \frac{\alpha}{2} b^a b^a + \overline{c}^a \partial^\mu D^{ab}_\mu c^b \right) \;, \tag{2}$$

---

*In the case of the maximal abelian gauge the group index α labels the off-diagonal generators T^α of $SU(N)$, with $\alpha = 1, ..., N(N-1)$. The parameter ξ is the gauge fixing parameter of the maximal abelian gauge.

where b^a stands for the Lagrange multiplier and α is the gauge parameter. Our aim is that of establishing some necessary requirements in order to study the possible condensation of this operator, which would imply the occurrence of dynamical mass generation in these gauges. Notice that, unlike the case of the Landau and maximal abelian gauges, the quantity $\int d^4x A_\mu^2$ is now not BRST invariant on-shell. However, we shall be able to prove that the local operator A_μ^2 is multiplicatively renormalizable to all orders of perturbation theory. There is a simple understanding of this property. In linear covariant gauges, due to the additional shift symmetry of the antighost, $i.e.$ $\overline{c} \to \overline{c} + \text{const.}$, the operator A_μ^2 doesn't mix with the other local dimension two composite ghost operator $\overline{c}c$, which cannot show up due to the above symmetry. We remark that the renormalizability of A_μ^2 is the first step towards the construction of a renormalizable effective potential in order to study the possible condensation of this operator and the ensuing dynamical mass generation.

The work is organized as follows. In Sect.2 we derive the Ward identities for Yang-Mills theory in linear covariant gauges in the presence of the local operator A_μ^2. These identities turn out to ensure the multiplicative renormalizability of A_μ^2. In Sect.3 the explicit two-loop calculation of the anomalous dimension of A_μ^2 is presented.

2 Algebraic proof of the renormalizability of the local operator $A_\mu^a A^{a\mu}$

We begin by recalling the expression of the pure Yang-Mills action in the linear covariant gauges

$$
\begin{aligned}
S &= S_{YM} + S_{GF+FP} \\
&= -\frac{1}{4} \int d^4x F_{\mu\nu}^a F^{a\mu\nu} + \int d^4x \left(b^a \partial_\mu A^{a\mu} + \frac{\alpha}{2} b^a b^a + \overline{c}^a \partial^\mu D_\mu^{ab} c^b \right) ,
\end{aligned}
\tag{3}
$$

where

$$
D_\mu^{ab} \equiv \partial_\mu \delta^{ab} - g f^{abc} A_\mu^c .
\tag{4}
$$

In order to study the local composite operator $A_\mu^a A^{a\mu}$, we introduce it in the action by means of a BRST doublet [28] of external sources (J, λ), namely

$$
S_J = s \int d^4x \left(\frac{1}{2} \lambda A_\mu^a A^{a\mu} + \frac{\xi}{2} \lambda J \right) = \int d^4x \left(\frac{1}{2} J A_\mu^a A^{a\mu} + \lambda A^{a\mu} \partial^\mu c^a + \frac{\xi}{2} J^2 \right) ,
\tag{5}
$$

where s denotes the BRST nilpotent operator acting as

$$
\begin{aligned}
s A_\mu^a &= -D_\mu^{ab} c^b , \\
s c^a &= \frac{1}{2} g f^{abc} c^b c^c , \\
s \overline{c}^a &= b^a , \\
s b^a &= 0 , \\
s \lambda &= J , \\
s J &= 0 .
\end{aligned}
\tag{6}
$$

According to the local composite operators technique [29], the dimensionless parameter ξ is needed to account for the divergences present in the vacuum Green function $\langle A^2(x) A^2(y) \rangle$, which turn out to be proportional to J^2. As is apparent from expressions (3) and (5), the action $(S_{YM} + S_{GF+FP} + S_J)$ is BRST invariant

$$
s (S_{YM} + S_{GF+FP} + S_J) = 0 .
\tag{7}
$$

3

2.1 Ward Identities

In order to translate the BRST invariance (7) into the corresponding Slavnov-Taylor identity [28], we introduce two further external sources Ω_μ^a, L^a coupled to the non-linear BRST variations of A_μ^a and c^a

$$S_{\text{ext}} = \int d^4x \left(-\Omega^{a\mu} D_\mu^{ab} c^b + \frac{1}{2} g f^{abc} L^a c^b c^c \right) , \tag{8}$$

with

$$s\Omega_\mu^a = sL^a = 0 . \tag{9}$$

Therefore, the complete action

$$\Sigma = S_{YM} + S_{GF+FP} + S_J + S_{\text{ext}} , \tag{10}$$

obeys the following identities

- The Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) = \int d^4x \left(\frac{\delta\Sigma}{\delta A_\mu^a} \frac{\delta\Sigma}{\delta\Omega^{a\mu}} + \frac{\delta\Sigma}{\delta c^a} \frac{\delta\Sigma}{\delta L^a} + b^a \frac{\delta\Sigma}{\delta\overline{c}^a} + J \frac{\delta\Sigma}{\delta\lambda} \right) = 0 . \tag{11}$$

- The linear gauge-fixing condition

$$\frac{\delta\Sigma}{\delta b^a} = \partial_\mu A^{a\mu} + \alpha b^a . \tag{12}$$

- The antighost equation

$$\frac{\delta\Sigma}{\delta\overline{c}^a} + \partial_\mu \frac{\delta\Sigma}{\delta\Omega^{a\mu}} = 0 . \tag{13}$$

Notice also that the additional shift symmetry in the antighost present in the linear covariant gauges

$$\overline{c} \to \overline{c} + \text{ const.} \tag{14}$$

is automatically encoded in the antighost equation (13). Indeed, integrating expression (13) on space-time yields

$$\int d^4x \frac{\delta\Sigma}{\delta\overline{c}^a} = 0 , \tag{15}$$

which expresses in a functional form the shift symmetry (14). Equations (13), (15) imply that the antighost field can enter only through the combination $(\Omega^{a\mu} + \partial^\mu \overline{c}^a)$, forbidding the appearance of the counterterm $\overline{c}^a c^a$. As a consequence, the local operator $A_\mu^a A^{a\mu}$ does not mix with $\overline{c}^a c^a$ in linear α-gauges.

Let us also display, for further use, the quantum numbers of all fields and sources entering the action Σ

	A_μ	c	$\overline{c}$	b	λ	J	Ω	L
dim .	1	0	2	2	2	2	3	4
gh $-$ number	0	1	-1	0	-1	0	-1	-2

(16)

2.2 Algebraic characterization of the general local invariant counterterm

In order to characterize the most general local invariant counterterm which can be freely added to all orders of perturbation theory [28], we perturb the classical action Σ by adding an arbitrary integrated local polynomial Σ^{count} in the fields and external sources of dimension bounded by four and with zero ghost number, and we require that the perturbed action $(\Sigma + \varepsilon\Sigma^{\text{count}})$ satisfies the same Ward identities and constraints as Σ to the first order in the perturbation parameter ε, *i.e.*

$$
\begin{aligned}
\mathcal{S}(\Sigma + \varepsilon\Sigma^{\text{count}}) &= 0 + O(\varepsilon^2) \,, \\
\frac{\delta(\Sigma + \varepsilon\Sigma^{\text{count}})}{\delta b^a} &= \partial^\mu A_\mu^a + \alpha b^a + O(\varepsilon^2) \,, \\
\left(\frac{\delta}{\delta\overline{c}^a} + \partial_\mu \frac{\delta}{\delta\Omega_\mu^a} \right)(\Sigma + \varepsilon\Sigma^{\text{count}}) &= 0 + O(\varepsilon^2) \,.
\end{aligned}
\tag{17}
$$

This amounts to impose the following conditions on Σ^{count}

$$
\mathcal{B}_\Sigma \Sigma^{\text{count}} = 0 \,,
\tag{18}
$$

and

$$
\frac{\delta\Sigma^{\text{count}}}{\delta b^a} = 0 \,,
\tag{19}
$$

$$
\frac{\delta\Sigma^{\text{count}}}{\delta\overline{c}^a} + \partial_\mu \frac{\delta\Sigma^{\text{count}}}{\delta\Omega_\mu^a} = 0 \,,
\tag{20}
$$

where $\mathcal{B}_\Sigma$ is the nilpotent linearized operator

$$
\mathcal{B}_\Sigma = \int d^4x \left(\frac{\delta\Sigma}{\delta A_\mu^a}\frac{\delta}{\delta\Omega^{a\mu}} + \frac{\delta\Sigma}{\delta\Omega^{a\mu}}\frac{\delta}{\delta A_\mu^a} + \frac{\delta\Sigma}{\delta c^a}\frac{\delta}{\delta L^a} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta}{\delta c^a} + b^a\frac{\delta}{\delta\overline{c}^a} + J\frac{\delta}{\delta\lambda} \right) \,,
\tag{21}
$$

$$
\mathcal{B}_\Sigma\mathcal{B}_\Sigma = 0 \,.
\tag{22}
$$

Taking into account that (J, λ) form a BRST doublet, from the general results on the cohomology of Yang-Mills theories [30] it turns out that the external sources (J, λ) can contribute only through terms which can be expressed as pure $\mathcal{B}_\Sigma$-variations. It follows thus that the invariant local counterterm Σ^{count} can be parametrized as

$$
\Sigma^{\text{count}} = -\frac{\sigma}{4}\int d^4x\, F_{\mu\nu}^a F^{a\mu\nu} + \mathcal{B}_\Sigma\Delta^{-1} \,,
\tag{23}
$$

where σ is a free parameter and Δ^{-1} is the most general local polynomial with dimension 4 and ghost number -1, given by

$$
\begin{aligned}
\Delta^{-1} = \int d^4x \Big(&a_1\Omega_\mu^a A^{a\mu} + a_2 L^a c^a + a_3\partial_\mu\overline{c}^a A^{a\mu} + \frac{a_4}{2}g f_{abc}\overline{c}^a\overline{c}^b c^c \\
&+ a_5 b^a\overline{c}^a + a_6\frac{\lambda}{2}A^{a\mu}A_\mu^a + a_7\alpha\lambda\overline{c}^a c^a + a_8\frac{\xi}{2}\lambda J \Big) \,,
\end{aligned}
\tag{24}
$$

with $a_1, \ldots, a_8$ arbitrary parameters. From the conditions (19), (20) it follows that

$$
a_3 = a_1, \quad a_4 = a_5 = a_7 = 0 \,,
\tag{25}
$$

so that Δ^{-1} reduces to

$$\Delta^{-1} = \int d^4x \left(a_1(\Omega_\mu^a + \partial_\mu \bar{c}^a) A^{a\mu} + a_2 L^a c^a + a_6 \frac{\lambda}{2} A^{a\mu} A_\mu^a + a_8 \frac{\xi}{2} \lambda J \right) . \tag{26}$$

Notice that the vanishing of the coefficient a_7 implies the absence of the counterterm $J\bar{c}^a c^a$. As already underlined, this ensures that the operator $A^{a\mu} A_\mu^a$ does not mix with the ghost operator $\bar{c}^a c^a$. Therefore, for the final form of the invariant counterterm one obtains:

$$
\begin{aligned}
\Sigma^{\text{count}} = \int d^4x \Bigg(&-\frac{(\sigma + 4a_1)}{4} F_{\mu\nu}^a F^{a\mu\nu} + a_1 \partial_\mu A_\nu^a F^{a\mu\nu} \\
&+a_2 \Omega_\mu^a (D^\mu c)^a + a_2 \partial_\mu \bar{c}^a (D^\mu c)^a + a_1 \Omega_\mu^a (\partial^\mu c)^a - a_1 \bar{c}^a \partial^2 c^a \\
&+\frac{1}{2}(2a_1 + a_6) J A^a{}_\nu A^{a\nu} + (a_1 + a_6 - a_2)\lambda \partial_\mu c^a A^{a\mu} \\
&-\frac{a_2}{2} g f_{abc} L^a c^b c^c + \frac{a_8}{2} \xi J^2 \Bigg) .
\end{aligned}
\tag{27}
$$

It remains to discuss the stability of the classical action [28], i.e. to check that Σ^{count} can be reabsorbed in the classical action Σ by means of a multiplicative renormalization of the coupling constant g, the parameters α and ξ, the fields $\{\phi = A, c, \bar{c}, b\}$ and the sources $\{\Phi = J, \lambda, L, \Omega\}$, namely

$$\Sigma(g, \xi, \alpha, \phi, \Phi) + \varepsilon \Sigma^{\text{count}} = \Sigma(g_0, \xi_0, \alpha_0, \phi_0, \Phi_0) + O(\varepsilon^2) , \tag{28}$$

with the bare fields and parameters defined as

$$
\begin{aligned}
A_{0\mu}^a &= Z_A^{1/2} A_\mu^a , & \Omega_{0\mu}^a &= Z_\Omega \Omega_\mu^a , & g_0 &= Z_g g , \\
c_0^a &= Z_c^{1/2} c^a , & L_0^a &= Z_L L^a , & \alpha_0 &= Z_\alpha \alpha , \\
\bar{c}_0^a &= Z_{\bar{c}}^{1/2} \bar{c}^a , & J_0 &= Z_J J , & \xi_0 &= Z_\xi \xi , \\
b_0^a &= Z_b^{1/2} b^a , & \lambda_0 &= Z_\lambda \lambda .
\end{aligned}
\tag{29}
$$

The parameters σ, a_1, a_2, a_6, a_8 turn out to be related to the renormalization of the gauge coupling constant g, of A_μ^a, c^a, J, λ, and ξ , according to

$$
\begin{aligned}
Z_g &= 1 - \varepsilon \frac{\sigma}{2} , \\
Z_A^{1/2} &= 1 + \varepsilon \left(\frac{\sigma}{2} + a_1 \right) , \\
Z_c^{1/2} &= 1 - \varepsilon \left(\frac{a_1 + a_2}{2} \right) , \\
Z_J &= 1 + \varepsilon (a_6 - \sigma) , \\
Z_\lambda &= 1 + \varepsilon \left(a_6 + \frac{a_1 - a_2 - \sigma}{2} \right) , \\
Z_\xi &= 1 + \varepsilon (a_8 - 2a_6 + 2\sigma) .
\end{aligned}
\tag{30}
$$

Concerning the other fields and the sources Ω_μ^a, L^a, it can be verified that they are renormalized as

$$
\begin{aligned}
Z_{\bar{c}} &= Z_c , & Z_b &= Z_A^{-1} , & Z_\Omega &= Z_c^{1/2} \\
Z_L &= Z_A^{1/2} , & Z_\alpha &= Z_A .
\end{aligned}
\tag{31}
$$

This completes the proof of the multiplicative renormalizability of the local composite operator A_μ^2 in linear covariant gauges. Finally, it is useful to observe that, from eqs.(30), one has

$$Z_\lambda = Z_J Z_c^{1/2} Z_A^{1/2}, \tag{32}$$

from which it follows that the anomalous dimension of A_μ^2 turns out to be related to that of the composite operator $A_\mu^a \partial^\mu c^a$

$$\gamma_{A\partial c} = \gamma_{A^2} + \gamma_c + \gamma_A , \tag{33}$$

where γ_c, γ_A, γ_{A^2}, and $\gamma_{A\partial c}$ are the anomalous dimensions of the Faddeev-Popov ghost c^a, of the gauge field A_μ^a, of the operator A_μ^2, and of the composite operator $A_\mu^a \partial^\mu c^a$, which are defined as

$$\gamma_c = \mu \partial_\mu \ln Z_c^{1/2} , \quad \gamma_A = \mu \partial_\mu \ln Z_A^{1/2} , \quad \gamma_{A^2} = \mu \partial_\mu \ln Z_J , \quad \gamma_{A\partial c} = \mu \partial_\mu \ln Z_\lambda , \tag{34}$$

where μ is the renormalization scale. As expected, property (33) relies on the fact that $A_\mu^a \partial^\mu c^a$ is the BRST variation of $\frac{1}{2} A_\mu^2$, i.e.

$$s \frac{A^{a\mu} A_\mu^a}{2} = -A_\mu^a \partial^\mu c^a . \tag{35}$$

Although we did not consider matter fields in the previous analysis, it can be checked that the renormalizability of A_μ^2 and the relation (33) remain unchanged if matter fields are included.

3 Calculation of the two-loop anomalous dimension of A_μ^2

We now turn to the computation of the anomalous dimension of A_μ^2 in an arbitrary linear gauge. The method exploits the lack of mixing in the linear covariant gauges between A_μ^2 and the other dimension two Lorentz scalar zero ghost number operator $\bar{c}^a c^a$, which we have already noted. For instance, in the Curci-Ferrari gauge although both operators mix there is a combination, $\mathcal{O} = \frac{1}{2} A_\mu^2 + \alpha \bar{c}^a c^a$, which remains multiplicatively renormalizable. Prior to the proof of [6] that the anomalous dimension of $\mathcal{O}$ was related to the β-function and the gluon anomalous dimension, $\gamma_{\mathcal{O}}(a)$ was explicitly computed at three loops in $\overline{\text{MS}}$ in [5]. That method involved substituting the operator in a ghost two-point function with a non-zero momentum flowing through the operator itself and one external ghost momentum nullified. This configuration allowed for the application of the MINCER algorithm, [31], written in the symbolic manipulation language FORM, [32, 33]. A ghost two-point function was chosen to avoid the appearance of spurious infrared infinities which would arise for this momentum configuration if the external legs were gluons. To determine $\gamma_{A^2}(a)$ in the linear gauges we are forced into the same approach as [5] due to the infrared issue with gluon external legs. Hence, we have renormalized the momentum space Green's function $\langle c^a(p) \frac{1}{2}[A_\mu^2](-p)\bar{c}^b(0)\rangle$ where p is the external momentum. Clearly, this has no tree term and therefore to deduce $\gamma_{A^2}(a)$ at n-loops requires renormalizing the Green's function at $(n+1)$-loops as the one loop term corresponds to the tree term of $\langle A_\mu^a(p) \frac{1}{2}[A_\mu^2](-p)A_\nu^b(0)\rangle$. This is evident, for example, by drawing one and two loop diagrams for the various Green's functions based on the interactions of the Yang-Mills action, eq.(3). Since the MINCER algorithm currently only extracts the simple poles in ϵ in dimensional regularization to three loops, where $d = 4 - 2\epsilon$, this means we have only computed $\gamma_{A^2}(a)$ to two loops. Though this will be sufficient to deduce the effective potential of A_μ^2 to one loop. The Feynman diagrams for our Green's function are generated with QGRAF, [34], and converted into FORM input notation, [5]. At one loop there is one diagram which plays the role of the tree diagram and at two loops there are 15 diagrams.

The bulk of the calculation, however, is in the evaluation of the 314 three loop graphs. Since there is no operator mixing we can apply the rescaling technique of [35] for automatic multiloop computations to find the renormalization constant Z_{A^2}. From this we deduce

$$
\begin{aligned}
\gamma_{A^2}(a) &= \left[(35 + 3\alpha)\, C_A - 16 T_F N_f\right] \frac{a}{6} \\
&+ \left[\left(449 + 33\alpha + 18\alpha^2\right) C_A^2 - 280 C_A T_F N_f - 192 C_F T_F N_f\right] \frac{a^2}{24} \ + \ O(a^3)
\end{aligned} \tag{36}
$$

in the $\overline{\text{MS}}$ scheme where N_f is the number of quarks and the colour group Casimirs are defined by $\text{Tr}\left(T^a T^b\right) = T_F \delta^{ab}$, $T^a T^a = C_F I$ and $f^{acd} f^{bcd} = C_A \delta^{ab}$. In deriving (36) from the corresponding renormalization constant we have verified that the double pole in ϵ is correctly reproduced for all α. Moreover, (36) reduces to the Landau gauge expression of [36].

4   Conclusion

We have investigated the renormalizability of the dimension two operator A_μ^2 in arbitrary covariant linear gauges in Yang-Mills theories, due to the possibility that it might condense and develop a non-zero vacuum expectation value. This would generalize to these gauges previous results obtained in the Landau gauge [2, 3, 4, 7]. One feature of our analysis is that, unlike the Curci-Ferrari gauges [5, 18], the operator A_μ^2 does not mix with the other dimension two local composite operator $\bar{c}^a c^a$. This is a general feature of the linear covariant α-gauges, present also in the Landau gauge [6], $\alpha = 0$, which is a consequence of the additional shift symmetry in the antighost (14). Importantly the operator A_μ^2 can thus be treated in isolation as it does not require any ghost dependent operator.

Finally, we underline that the multiplicative renormalizability of A_μ^2 and the explicit knowledge of its anomalous dimension for all α, eq.(36), are central ingredients towards the construction of a renormalizable effective potential for studying the possible condensation of this operator and the related dynamical mass generation, as was carried out in the Landau [4, 7] and Curci-Ferrari [19] gauges.

Acknowledgments

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