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Determination of Lambda in quenched and full QCD – an update*

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We present an update on our previous determination of the Lambda parameter in QCD. The main emphasis is on results for two flavours of light dynamical quarks, where we can now almost double the amount of data used, including values at smaller lattice spacings. The calculations are performed using $O(a)$ improved Wilson fermions. Little change is found to previous numerical values.

The Λ parameter is one of the fundamantal parameters of QCD, setting the scale for the running coupling constant α_s. In this contribution we shall update our previous work, [1], both for quenched ($n_f = 0$) and unquenched ($n_f = 2$) $O(a)$ improved Wilson ('clover') fermions. Specifically we are now able to use for

- quenched fermions, the force scale r_0/a up to $\beta = 6.92$, [2] (previously $\beta \le 6.4$),
- unquenched fermions, improved statistics and additional quark masses at the previous β values of 5.20, 5.25, 5.29 for r_0/a together with new results at $\beta = 5.40$ (at three quark masses).

The 'running' of the QCD coupling constant as the scale changes is controlled by the β-function

$$\frac{\partial g_{\mathcal{S}}(M)}{\partial \log M} = \beta^{\mathcal{S}}\left(g_{\mathcal{S}}(M)\right) ,$$

where, perturbatively

$$\beta^{\mathcal{S}}\left(g_{\mathcal{S}}\right) = -b_0 g_{\mathcal{S}}^3 - b_1 g_{\mathcal{S}}^5 - b_2^{\mathcal{S}} g_{\mathcal{S}}^7 - b_3^{\mathcal{S}} g_{\mathcal{S}}^9 - \ldots ,$$

renormalisation having introduced a scale M together with a scheme $\mathcal{S}$. Integrating this equation gives

$$\begin{aligned} \frac{\Lambda^{\mathcal{S}}}{M} &= \exp\left[-\frac{1}{2b_0 g_{\mathcal{S}}(M)^2}\right]\left[b_0 g_{\mathcal{S}}(M)^2\right]^{-\frac{b_1}{2b_0^2}} \times \\ & \exp\left\{-\int_0^{g_{\mathcal{S}}(M)} d\xi \left[\frac{1}{\beta^{\mathcal{S}}(\xi)} + \frac{1}{b_0\xi^3} - \frac{b_1}{b_0^2\xi}\right]\right\}, \\ &\equiv F^{\mathcal{S}}(g_{\mathcal{S}}(M)) , \end{aligned}$$

where $\Lambda^{\mathcal{S}}$, the integration constant, is the fundamental scheme dependent QCD parameter. Results are usually given in the $\overline{MS}$ scheme, with the scale M being denoted by μ. In this scheme the first four β-function coefficients are known, $b_3^{\overline{MS}}$ being found in [3]. The running coupling $\alpha_s^{\overline{MS}}(\mu) \equiv g_{\overline{MS}}(\mu)^2/4\pi$ is plotted in Fig. 1 for $n_f = 2$, by solving the previous equation (numerically) using successively more and more coefficients of the β-function. The figure shows an apparently fast convergent series (cf 3- to 4-loop), certainly in the range we are interested in, $\mu/\Lambda^{\overline{MS}} \sim 8$. A very similar result holds for $n_f = 0$ but with slightly lower curves.

*Talk given by R. Horsley at Lat04, Fermilab, USA.

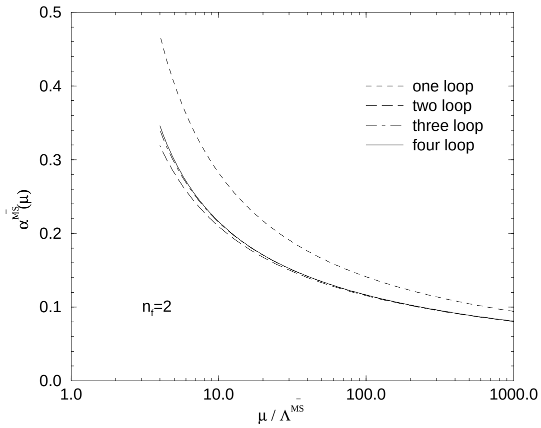

Figure 1. $\alpha_s^{\overline{MS}}(\mu)$ *versus* $\mu/\Lambda^{\overline{MS}}$ *for* $n_f = 2$.

On the lattice we also have a Λ parameter,

$$a\Lambda^{\Box} = F^{\Box}(g_{\Box}(a))\,,$$

where to help convergence of lattice perturbative expansions we use $g_{\Box}^2 \equiv g^2(a)/u_0^4$ with u_0^4 the average plaquette value. To calculate $\Lambda^{\overline{MS}}$, we shall compute $g_{\overline{MS}}$ at some appropriate scale μ^* from $g_{\Box}(a)$ and then using the r_0 scale, extrapolate

$$r_0\Lambda^{\overline{MS}} \equiv \left(\frac{r_0}{a}\right) F^{\overline{MS}}(g_{\overline{MS}}(\mu^*))a\mu^*\,,$$

to the continuum limit.

Equating lattice and continuum expressions

$$[F^{\overline{MS}}(g_{\overline{MS}}(\mu))]^{-1} = a\mu\frac{\Lambda^{\Box}}{\Lambda^{\overline{MS}}}[F^{\Box}(g_{\Box}(a))]^{-1}\,,$$

and expanding as

$$\frac{1}{g_{\overline{MS}}^2(\mu)} = \frac{1}{g_{\Box}^2(a)} + [2b_0\ln a\mu - t_1^{\Box}] + [2b_1\ln a\mu - t_2^{\Box}]g_{\Box}^2(a)\ldots\,,$$

gives $t_1^{\Box} = 2b_0\ln\Lambda^{\overline{MS}}/\Lambda^{\Box}$ and $b_2^{\Box} = b_2^{\overline{MS}} + b_1t_1^{\Box} - b_0t_2^{\Box}$. For (hopefully) good convergence of this series we choose the scale so that the $O(1)$ term vanishes, $a\mu^* = \exp(t_1^{\Box}/2b_0)$.

For $t_1^{\Box}$ the general expression is known for n_f, c_{sw} and linear terms in $n_f am_q$, while for $t_2^{\Box}$ the $n_f am_q$ dependence is not known, [1] and references therein. We can estimate the scales as $\mu^* = 2.63/a$, $n_f = 0$ and $\mu^* \sim 1.4/a$ for $n_f = 2$. $t_3^{\Box}$ (the $g_{\overline{MS}}(\mu)^4$, $\ln a\mu$ independent term) is not known. So equivalently $b_3^{\Box}$ is not known. However a Padé estimate gives $b_3^{\mathcal{S}} \approx (b_2^{\mathcal{S}})^2/b_1$, and is small and in reasonable agreement with the known coefficient in the $\overline{MS}$ scheme, [1]. Assuming this also holds for $b_3^{\Box}$ gives little change to the results presented here. For complete $O(a)$ cancellation, [4], we need $\tilde{g}^2 = g^2(1 + b_g am_q)$ where perturbatively $b_g = 0.01200n_fg^2 + O(g^4)$, which with $c_{sw} = 1 + O(g^2)$ then gives no mass dependence in $t_1^{\Box}$. This indicates little quark mass dependence in the fit formulae (indeed there is more in the numerical data). Finally to further improve the convergence of the series, we tadpole improve the $t_i^{\Box}$ coefficients $c_{sw}^{TI} = c_{sw}u_0^3$ (for $t_1^{\Box} + t_2^{\Box}g_{\Box}^2$) further reducing the size of the n_f term in $t_2^{\Box}$.

In Fig. 2 we show the quenched ($n_f = 0$) results. The data lies on a straight line (as a function of $(a/r_0)^2$) at least over $a^{-1} \sim 2 - 6.5\,\mathrm{GeV}$ or $\mu \sim 5 - 17\,\mathrm{GeV}$, using the value for r_0 of $r_0 = 0.5\,\mathrm{fm}$. This gives a result of $r_0\Lambda^{\overline{MS}} = 0.613(2)(25)$ or $\Lambda^{\overline{MS}}(0) = 242(1)(10)\,\mathrm{MeV}$ where the first error is statistical and to estimate the systematic uncertainty, the second error takes a g^4 coeff. $= 25\% \times g^2$ coeff. (which is very much greater than when using the Padé $b_3^{\Box}$ estimate).

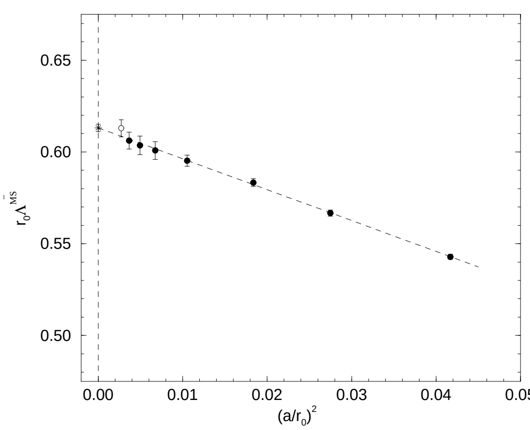

Figure 2. $r_0\Lambda^{\overline{MS}}$ *versus* $(a/r_0)^2$ *for* $n_f = 0$*, together with a linear extrapolation to* $a = 0$*. The last point has not been included in the fit.*

For unquenched ($n_f = 2$) fermions, due to the sea quark, the fit ansatz is not so simple as we must consider both chiral and continuum extrapolations. We take for finite a, $a\Lambda^{\overline{MS}}|_{m_q\neq 0,a\neq 0} = a\Lambda^{\overline{MS}}|_{m_q=0,a\neq 0} + Dam_q + \ldots$ or

$r_0\Lambda^{\overline{MS}}|_{m_q\neq 0, a\neq 0} = r_0\Lambda^{\overline{MS}}|_{m_q=0,a\neq 0} + Dr_0 m_q + \ldots$. After chiral extrapolation we would thus expect $r_0\Lambda^{\overline{MS}}|_{m_q=0,a\neq 0} = r_0\Lambda^{\overline{MS}}|_{m_q=0,a=0} + B(a/\rho)^2 + \ldots$ with $\rho \equiv r_0|_{m_q=0}$. Together with $(a/r_0)^2 = (a/\rho)^2 + Eam_q + \ldots$ this gives our fit ansatz as

$$r_0\Lambda^{\overline{MS}} = A + B(a/r_0)^2 + Cam_q + Dr_0 m_q \, .$$

So by subtracting out the B and C terms from $r_0\Lambda^{\overline{MS}}$ we can consider the chiral extrapolation and similarly by subtracting out the C and D terms we may consider the continuum extrapolation[1]. In Fig. 3 we show the results. a^{-1} ranges

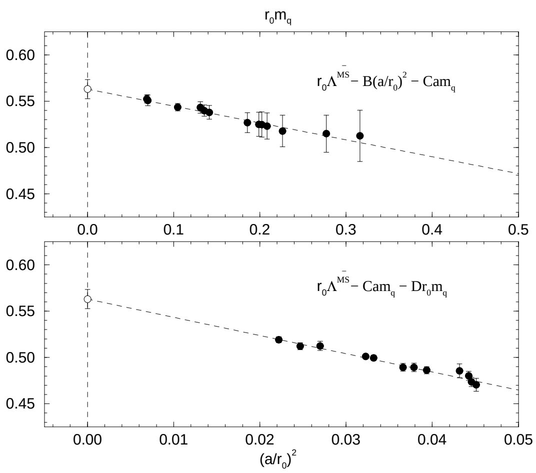

Figure 3. *$r_0\Lambda^{\overline{MS}}$ versus $r_0 m_q$ (upper picture) and versus $(a/r_0)^2$ (lower picture) for $n_f = 2$, together with appropriate extrapolations (am_q from [5]).*

at least over $a^{-1} \sim 2 - 3\,\mathrm{GeV}$ or $\mu \sim 3 - 4\,\mathrm{GeV}$ This gives a result of $r_0\Lambda^{\overline{MS}} = 0.563(10)(70)$ or $\Lambda^{\overline{MS}}(2) = 222(4)(28)\,\mathrm{MeV}$ where again the first error is statistical and the second error is obtained by taking a g^4 coeff. $= 25\% \times g^2$ coeff. which again is much larger than the error found when using a Padé $b_3^{\square}$ estimate, setting $c_{sw} = 1 + O(g^2)$ or including an additional $(am_q)^2$ fit term. Note that this result is consistent with that obtained in [6].

Finally in Fig. 4 we present results for different n_f. Our result lies somewhat low in comparison with phenomenological results. Alternatively using the matching procedure as in [1] we find for $n_f = 5$, $\alpha_s^{\overline{MS}}(m_Z) = 0.1084(6)(38)$.

[1] An alternative procedure is first to extrapolate both r_0/a and u_0^4 to the chiral limit, evalute $r_0\Lambda^{\overline{MS}}$ and then extrapolate to the continuum limit; this gives similar results, [5].

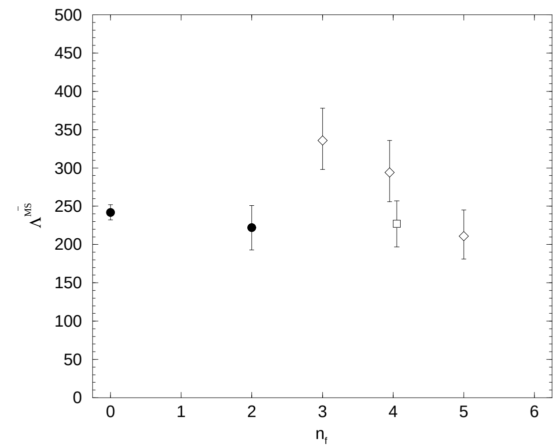

Figure 4. *$\Lambda^{\overline{MS}}(n_f)$ versus n_f The open diamonds are from [7], using $\alpha_s^{\overline{MS}}(M_Z) = 0.1183(27)$ ($n_f = 5$) to match to $n_f = 4$ and $n_f = 3$, while the open square is from [8]. The filled circles are the results reported here.*

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