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Axial and tensor charge of the nucleon with dynamical fermions*

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We present preliminary results for the axial and tensor charge of the nucleon obtained from simulations with $N_f = 2$ clover fermions. A comparison with chiral perturbation theory is attempted.

Lattice QCD offers the possibility to investigate the internal structure of the nucleon without additional model assumptions. In particular, one can study how the mass, the axial charge and further properties of the nucleon change when the quark mass is varied or the nucleon is squeezed into a finite spatial box. Even if we could simulate at the physical quark mass and in a huge volume, we would like to examine the dependence on the quark mass and on the volume, because it contains important information on low-energy QCD. This information can be extracted by comparing the outcome of lattice QCD simulations with parameterisations provided by chiral effective field theories (ChEFT).

This comparison with ChEFT, if successful, leads to two important achievements: Firstly, it gives us control over the chiral extrapolation and the thermodynamic limit of the simulation results and secondly it allows us to determine phenomenologically relevant coupling constants in ChEFT.

In the following, we shall present preliminary results from simulations with $N_f = 2$ non-perturbatively improved Wilson quarks and the standard plaquette action for the gauge fields obtained by the QCDSF and UKQCD [1] collaborations. Previous quenched QCDSF results can be found in Ref. [2] and for a review see Ref. [3]. For the time being we do not yet perform an extrapolation to the continuum limit and neglect lattice artefacts.

Originally, ChEFT was invented to describe the quark mass dependence of low-energy quantities

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by means of effective hadron fields. However, for sufficiently large volumes and sufficiently small pion masses, finite size effects are dominated by pions "propagating around the world", and therefore ChEFT can also be used to calculate the volume dependence: The effective Lagrangian is independent of the volume.

Our paradigm is the analysis of the nucleon mass [4], where we could describe the pion mass dependence and the finite size effects up to surprisingly large masses by formulae derived from relativistic baryon chiral perturbation theory. We are trying to perform a similar analysis also for the axial and the tensor charge of the nucleon, and in the following we shall report on the status of these attempts. Whenever we need numbers in physical units, we shall set the scale using $r_0 = 0.5\,\mathrm{fm}$.

Compared with the case of the nucleon mass there are several additional difficulties when dealing with such matrix elements of composite operators:

- The ChEFT calculations are not as advanced as for the nucleon mass.
- In order to suppress lattice artefacts, the operators should be improved. However, the required improvement coefficients are not yet known non-perturbatively. In the following, we shall present results from the unimproved operators.
- The operators have to be renormalised. We shall use non-perturbative renormalisation factors computed by means of the Rome-Southampton method with a linear chiral extrapolation at fixed bare gauge coupling.
- The quark-line disconnected contributions to the matrix elements are hard to evaluate. Therefore we shall consider only flavour-nonsinglet quantities, where these contributions cancel.

In order to compute the axial form factor of the nucleon $g_A(q^2)$ on the lattice we use the flavour-nonsinglet axial vector current $A_\mu^{u-d} = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$ and the form factor decomposition of its proton matrix element. Using the

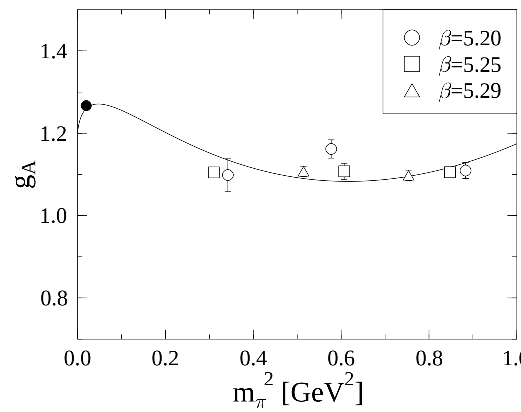

Figure 1. The axial charge g_A of the nucleon. The filled circle indicates the physical point.

so-called small scale expansion, which takes Δ degrees of freedom explicitly into account, the authors of Ref. [5] have studied the mass dependence of the nucleon axial charge $g_A \equiv g_A(q^2 = 0)$ to $O(\epsilon^3)$ and compared with quenched data.

In Fig. 1 we perform a similar comparison with our presently available dynamical data for g_A choosing some reasonable values for the parameters. The curve reproduces the weak mass dependence of the simulation results as well as the physical point, at least approximately, but the chiral logarithm dominates only for very small pion masses.

What about the finite size effects for g_A? There are indications that they could be quite large (for a quenched study see, e.g., Ref. [6]), and they might not yet be negligible for the results in Fig. 1. We have performed simulations on three different volumes at the same bare gauge coupling and quark mass, leading to $m_\pi = 0.717\,\mathrm{GeV}$ on the largest lattice, and found a clear volume dependence. First calculations within ChEFT have appeared [7]. To $O(p^3)$ in heavy baryon chiral perturbation theory (without explicit Δs) one finds

$$g_A(L) - g_A(\infty) = \frac{(g_A^0)^3 m_\pi^2}{6\pi^2 f_\pi^2} \sum_{\vec{n}}{}' K_0(L|\vec{n}|m_\pi) - \frac{g_A^0 m_\pi^2}{4\pi^2 f_\pi^2} \left(1 + \tfrac{2}{3}(g_A^0)^2\right) \sum_{\vec{n}}{}' \frac{K_1(L|\vec{n}|m_\pi)}{L|\vec{n}|m_\pi} \,,$$

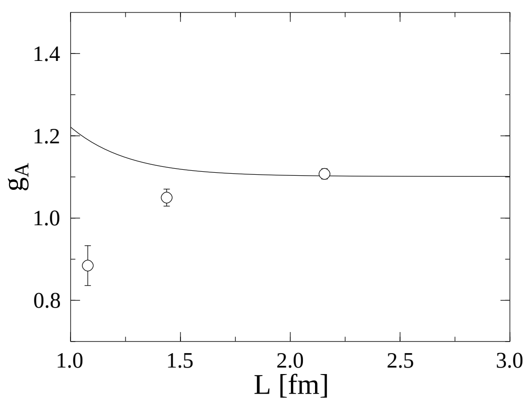

Figure 2. The volume dependence of g_A for $m_\pi = 0.717\,\mathrm{GeV}$ compared with leading order chiral perturbation theory.

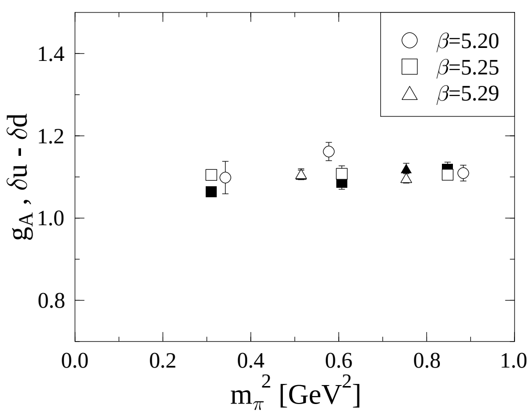

Figure 3. The tensor charge $\delta u - \delta d$ of the proton in the $\overline{\mathrm{MS}}$ scheme at the renormalisation scale $\mu = 2\,\mathrm{GeV}$ (filled symbols) together with the g_A data (open symbols).

where g_A^0 is the value of g_A in the chiral limit and f_π is the pion decay constant. In Fig. 2 we plot our data together with the above expression for the finite size effect. Not even the sign is reproduced correctly. Of course, one may blame the large pion mass in connection with the low order of chiral perturbation theory for this discrepancy, and further investigations are clearly needed.

The tensor charge of the nucleon is the lowest moment of the transversity distribution h_1. For flavour q it is given by the proton matrix element

$$\langle p, s|\bar{q}\mathrm{i}\sigma_{\mu\nu}\gamma_5 q|p, s\rangle = \frac{2}{m_N}(s_\mu p_\nu - s_\nu p_\mu)\delta q$$

with $s^2 = -m_N^2$. Again, we restrict ourselves to the flavour nonsinglet combination $\delta u - \delta d$ and plot it in Fig. 3 together with g_A data, because in the non-relativistic limit one expects that both quantities agree. Hence this comparison gives us some information on how relativistic the quarks in our simulations are and the data seem to indicate that the quarks are not very relativistic.

Concluding we may say that the interpretation of our results is only at its beginning. Once we have analysed our configurations completely, we hope to gain more insight into the structure of the nucleon. But further improvements are certainly desirable, in particular smaller quark masses and lattice spacings in the Monte Carlo simulations as well as more extensive calculations in ChEFT.

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REFERENCES

1. C.R. Allton et al., Phys. Rev. D65 (2002) 054502.
2. S. Capitani et al., Nucl. Phys. Proc. Suppl. 79 (1999) 548.
3. M. Göckeler et al., Nucl. Phys. Proc. Suppl. 119 (2003) 32.
4. A. Ali Khan et al., Nucl. Phys. B689 (2004) 175.
5. T.R. Hemmert, M. Procura and W. Weise, Phys. Rev. D68 (2003) 075009.
6. S. Sasaki, K. Orginos, S. Ohta and T. Blum, Phys. Rev. D68 (2003) 054509.

7. S.R. Beane and M.J. Savage, hep-ph/0404131.