

P-wave Radial distributions of a Heavy-light meson on a lattice

UKQCD Collaboration, A.M. Green*, J. Koponen[†] and C. Michael**

**Helsinki Institute of Physics*

P.O. Box 64, FIN-00014 University of Helsinki, Finland

[†]Department of Physical Sciences and Helsinki Institute of Physics

P.O. Box 64, FIN-00014 University of Helsinki, Finland

***Department of Mathematical Sciences, University of Liverpool, L69 3BX, UK*

Abstract. This is a follow-up to our earlier work for the charge(vector) and matter(scalar) distributions for S-wave states in a heavy-light meson, where the heavy quark is static and the light quark has a mass about that of the strange quark. The calculation is again carried out with dynamical fermions on a $16^3 \times 24$ lattice with a lattice spacing of about 0.14 fm. It is shown that several features of the S- and P-wave distributions are in qualitative agreement with what one expects from a simple one-body Dirac equation interpretation.

INTRODUCTION

Experimentalists are often able to tell us several properties of a given meson, such as its energy, width and angular momentum. However, usually they can not tell us the structure of the meson. For example, with B_s states — the topic of interest here — they can not say whether these states are $b\bar{s}$, $b\bar{s}u\bar{u}$ or BK . Unfortunately, when theoreticians try to describe B_s states, they have to decide beforehand the state structure to be used in some model, which often has sufficient freedom to fit the data with any of the possible structures. In an attempt to clarify the Experiment \longleftrightarrow Theory comparison, we suggest the use of lattice QCD. In principle, lattice QCD should give us all we need to know about B_s states. However, in practice, the results need to be corrected for the lattice spacing (a), finite lattice size (L) and quark mass (m_q) effects — but these are usually under control and with the advent of more computer resources they will decrease in importance.

The strategy followed here is to concentrate on the simplest of quark states, namely, the $Q\bar{q}$ system, where Q is an infinitely heavy quark (i.e. static) and q is a quark with about the strange quark mass. This system is sufficiently simple to enable state-of-the-art lattice calculations to generate much more “data” than can be achieved by direct experiment. This data consists of the ground state energies of S-, P- and D-wave states and also the spin average F-wave energy[1]. Not only are the ground state energies extracted, but also those of the corresponding excited states containing at least one radial node. In addition, the vector (charge) and scalar (matter) radial distributions of these states can be measured. This is an abundance of data, far beyond what has been done experimentally in the B_s meson. Given all this data the challenge is now for theorists to make models to explain it. In this quest there are two simplifications. Firstly, since Q is static, the system

is essentially reduced to a one-body problem involving only the light quark q_s . Secondly, in the lattice calculations, since the energies and radial distributions are extracted from $Q\bar{q}$ correlations propagating in Euclidean time, it is expected that the resultant states are indeed $Q\bar{q}$ states with little contamination from other possible multi-quark components. Support for this expectation (hope) is seen in Fig. 2 of Ref. [2]. There a $Q\bar{Q}$ correlation generates a linearly rising potential for interquark distances far larger than expected i.e. way beyond where $(Q\bar{q})(\bar{Q}q)$ configurations should appear through string breaking. To see this effect needs the the explicit introduction of $(Q\bar{q})(\bar{Q}q)$ correlations [3].

The "data base" for properties of $Q\bar{q}$ states measured on lattices is so far incomplete. In Ref. [4] we concentrated on the S-wave energies and radial distributions. The latter showed two distinct features. Firstly, the excited state distributions exhibited nodes as expected. Secondly, the charge distribution ($x_C(R)$) was of longer range than the matter distribution ($x_M(R)$) but with $x_C(0) \approx x_M(0)$. This is readily explained by the one-body Dirac equation, since there $x_C(R) = G(R)^2 + F(R)^2$ and $x_M(R) = G(R)^2 - F(R)^2$, where for S-waves $G(0) \gg F(0)$ with $G(R)$ and $F(R)$ becoming comparable for large R . In Ref. [1] the data base was extended to include the energies of the excited states $P_{+/-}$ and $D_{+/-}$ and also the spin averaged combinations D_{+-} and F_{+-} . Here P_- is the P -wave state with $j_q = 1/2$, since the spin of the Q does not play a dynamical role. In this note we return to measuring the charge and matter distributions as in Ref. [4], but concentrate on the P_- -state and its excitations. This state is of particular interest, since — as shown in Fig. 4 of Ref. [1] — the indications are that the predicted $B_s(0^+)$ is below the BK -threshold and so should be very narrow as was found for the $c\bar{s}$ counterpart. The outcome is seen in Figs. 1 for the ground state distributions x^{11} and the off-diagonal distributions between the ground state and the first excited state x^{12} . The latter show single nodes as

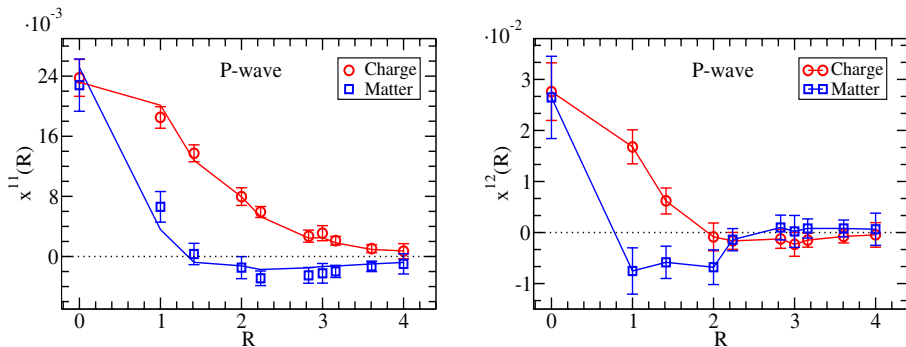


FIGURE 1. The P_- charge and matter distributions for $x^{11}(R)$ and $x^{12}(R)$.

expected from a first excited state. However, x^{11} shows two features which at first sight seem surprising:

- 1) The distributions are *finite* at $R = 0$, even though they are P -waves.
- 2) The matter distribution has a *node*, even though it involves only the ground state.

But again this is precisely what one expects from solutions of the Dirac equation, where for the P_- -state both G and F are *non-zero* at $R = 0$ and, furthermore, the "small" component F can be *larger* than G at small R . In Figs. 2 the above data, now with a factor of R^2 included, are compared with the Dirac distributions using a quark mass of

100 MeV and an interquark potential $V = -a/R + bR$, where $a = 0.6$ and $b = 1.3$ GeV/fm. These three parameters are very sensible and were not tuned to get an optimal fit.

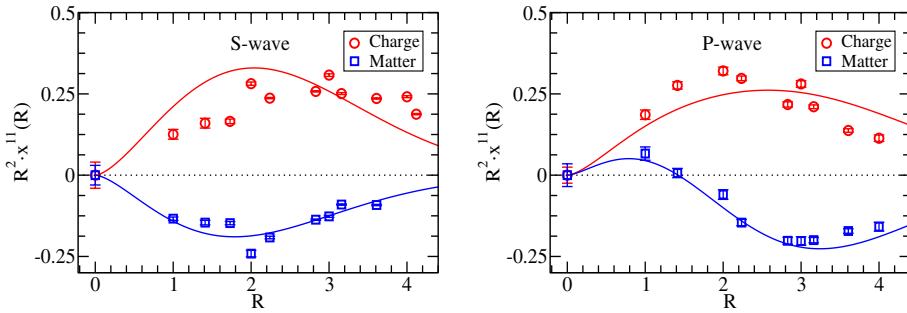


FIGURE 2. The S and P- radial distributions from Fig. 1 compared with Dirac equation fits.

CONCLUSIONS

The S- and P-wave charge and matter distributions in Figs. 2 suggest that they can be understood qualitatively in terms of the one-body Dirac equation. The challenge is now to see to what extent there is an analogous *quantitative* description of the energies and distributions of all the S-, P-, D- and F-wave states — both ground and excited, when they become available. It is possible that the strategy used in studying the NN -potential is appropriate, namely, to first concentrate on the higher partial waves and so avoid or reduce complications, such as the effect of form factors needed to regulate the one-gluon-exchange potential and also instanton-induced interactions, that enter at small values of R .

ACKNOWLEDGMENTS

The authors wish to thank the Center for Scientific Computing in Espoo, Finland and the ULGrid project of the University of Liverpool for making available ample computer resources. Two of us, AMG and JK, acknowledge support by the Academy of Finland contract 54038 and the EU grant HPRN-CT-2002-00311 Euridice. JK thanks the Finnish Cultural Foundation and the Magnus Ehrnrooth Foundation for financial support.

REFERENCES

1. Green, A., Koponen, J., McNeile, C., Michael, C., Thompson, G., and UKQCD, *Phys. Rev. D*, **69**, 094505 (2004).
2. Bolder, B. e. a., *Phys. Rev. D*, **63**, 074504 (2001).
3. Michael, C., Pennanen, P., and UKQCD, *hep-lat/000105* (2000).
4. Green, A., Koponen, J., Michael, C., Pennanen, P., and UKQCD, *Eur. Phys. J. C*, **28**, 79 (2003).