Graphic Equalization Using Frequency-Warped Digital Filters

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The aim of this thesis is to design a graphic equalizer with frequency warped digital filters. The proposed design consists of a warped FIR filter for the low frequency bands and a standard FIR filter for the high frequency bands. This design is used to implement both an octave and a one-third octave equalizer in Matlab.

Low frequency equalization with FIR filters requires high filter orders. The frequency resolution of the lowest band of the graphic equalizer requires filter orders that are impractical for real life applications. With frequency warping filter orders can be lowered, so that a practical graphic equalizer can be designed. With this design common gain build-up problems, which are present in most of the IIR designs, can be avoided.

The proposed equalizer design is found to be accurate and comparable to the previous equalizer designs. Filter orders required are small enough to this design to be used in real life applications. The gain build-up problem is avoided in this design, as several equalizer bands are filtered with a single filter. The computational costs of the design are higher than the costs of the other compared designs. However, the difference can be smaller if the accuracy restrictions are lowered.

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Tämän työn tavoiteena on suunnitella graafinen ekvalisaattori taajuusvarpattujen digitaalisten suotimien avulla. Ehdotettu ekvalisaattorimalli koostuu taajuusvarpatusta ja tavallisesta FIR suotimesta. Varpattua suodinta käytetään alimpien taajuuskaistojen suodattamiseen ja tavallista FIR suodinta ylimpien kaistojen suodattamiseen. Tätä mallia käytetään sekä oktaavi- että terssikaista-ekvalisaattorien totetutamiseen Matlabilla.

Matalien taajuuksien ekvalisointi edellyttää korkeaa astelukua FIR suotimilta. Alimpien taajuuskaistojen taajuusresoluutio edellyttää astelukuja, jotka ovat epäkäytännöllisiä tosielämän sovelluksissa. Taajuusvarppauksella suotimien astelukuja voidaan pienentää, jolloin graafinen ekvalisaattori voidaan toteuttaa käytännössä. Tällä mallilla voidaan välttää IIR ekvalisaattorien yleinen ongelma, jossa ekvalisaattorien kaistojen vahvistus vaikuttaa viereisiin kaistoihin.

Ehdotettu ekvalisaattorimalli todetaan olevan tarkka ja vertailukelpoinen aikaisempien toteutuksien kanssa. Suotimien asteluvut ovat tarpeeksi pieniä, jotta tätä mallia voidaan käyttää tosielämän toteutuksissa. Kaistojen välinen vaikutus vältetään tällä mallilla, sillä useampi kaista suodatetaan yhdellä suotimella. Laskennallinen kuorma on tällä toteutuksella suurempi kuin muilla vertailluilla toteutuksilla. Eroa voidaan pienentää, jos ekvalisaattorin tarkkuusvaatimuksia lasketaan.

Avainsanat: Digitaaliset suotimet, graafiset taajuuskorjaimet, taajuuskorjaimet, taajuusvarppaus, äänenkäsittely

Preface

I want to thank my supervisor Professor Vesa Välimäki for introduction this subject to me and for the guidance and suggestions with this thesis.

Otaniemi, 15.7.2016

Jaakko Siiskonen

Contents

Symbols and abbreviations

Symbols

$A(z)$	transfer function of an all-pass filter
a_k	filter coefficients of the feedback section
B	bandwidth
b_k	filter coefficients of the feedforward section
f_c	crossover frequency
f_l	lower frequency limit of a frequency band
f_m	center frequency of a frequency band
f_max	maximum frequency of a system
f_p	turning point frequency
f_s	sampling rate
f_u	upper frequency limit of a frequency band
f_w	frequency in the warped domain
G	gain of an IIR peak/notch filter
$G(\omega)$	frequency response of a filter
$g(z)$	filter transformation
$H(z)$	transfer function of a filter
$H_\mathrm{w}(z)$	transfer function of a warped filter
N	filter order / number of the numerator coefficients
M	number of the denominator coefficients
$P(\omega)$	phase response of a filter
Q	quality factor of a filter
R	ratio of frequency bands
$x[n]$	digital input signal
$y[n]$	digital output signal
Δf	frequency resolution of a filter
λ	warping parameter
τ_g	group delay of a filter
ω_c	center frequency of a second order filter
ω_w	normalized frequency of in a warped domain

Abbreviations

CLS	constrained least-squares
FFT	fast Fourier transform
FIR	finite impulse response
IIR	infinite impulse response
ISO	International Organization for Standardization
LS	least-squares
LTI	linear time-invariant
MAC	multiply and accumulate
RM	Regalia-Mitra
WFIR	warped finite impulse response

1 Introduction

Audio listening conditions vary greatly from headphone listening to the stadium venues. Different listening conditions modify the input audio signal in ways that are impossible to compensate in the recording or in the mixing process. This variation requires great flexibility on the audio reproducing systems. Ideally an audio signal should be able to be reproduced unchanged despite the listening conditions.

The signal chain from a sound source to the listener's ear is long and complex. The amplitude, the frequency and the phase responses of the input signal are prone to the changes, when processed with non-ideal audio systems. Typical audio processing and reproducing systems usually modify the input audio signal. While these modifications can be desired, sometimes these changes are unwanted.

The changes in the input audio signal are caused by imperfections in audio devices and listening conditions. Imperfections in audio devices are caused by design and implementation compromises and physical restrictions. High quality audio devices are expensive. While it is easy to design an audio amplifier with nearly flat frequency response, it is very hard to design loudspeakers or headphones, which would achieve the same accuracy level than an amplifier [1]. For example the sizes of the loudspeakers are very limited in headphone listening. Listening conditions have also a huge impact to the audio signal. Reverberant rooms and loudspeaker placement change audio signal usually more than audio devices [2].

Audio equalizers are audio devices that can be used to correct unwanted changes in the frequency response. They can be seen as frequency-specific volume knobs [3]. With equalizers, the user can shape the frequency response of the signal to either restore or enhance the frequency content. Equalizers are implemented with audio filters, either with analog electronics or digital software.

Equalizer are found in many common audio devices. The range of the equalizers varies from simple tone controls to devices that can control the exact frequency response. The simplest equalizer, the tone control found in many audio devices, allows the user to shape the frequency response in a very simple way; more bass or more treble. Similar simple equalizers are found in common stereo amplifiers, in which there are a control for both the bass and the treble frequencies. With these controls the user can control the volumes of low and high frequencies independently.

For more control of the frequency response more complex equalizers have been developed. These equalizers can be divided to the parametric and the graphic equalizers [4]. Parametric equalizers allow a very good control of the frequency response. These equalizers have several different frequency bands, whose gain, center frequency and bandwidth can be modified. Parametric equalizers are usually found in professional audio devices such as in mixing consoles and preamplifiers.

In graphic equalizers the frequency range is divided to frequency bands. Each of these bands is fixed to a certain frequency and bandwidth and every band has own gain control. These controls are usually implemented as mini faders, whose positions plot the approximated frequency response. Because this graphical representation given to the user, graphic equalizers are easy to use. Number of the bands can be anything from 3 to 30, but usually graphic equalizers have 10 or 30 bands, which are

divided logarithmically across the frequency range.

The main difficulty of the design of the graphic equalizers is the gain interference across the frequency bands [5]. The most common designs use individual filters for each of the frequency bands, which affect the gains of the adjacent bands. The resulting equalization curve can differ greatly from the intended response. There are many available solutions to compensate this problem [6]. One solution is to use large finite impulse response (FIR) filters, which can equalize several frequency bands with one filter. However, as low frequency equalization with FIR filters requires large filter lengths, such filters can be hard to design or be too large for practical applications.

Frequency warping is a filter transformation method, which can be used to design filters with complex responses and lower filter orders. In frequency warping the response of the filter is moved to another frequencies. Finite impulse response filters have a good resolution at high frequencies and a poor resolution at low frequencies. With frequency warping the resolution at low frequencies can be increased at the expense of the decreased resolution at high frequencies. Frequency warping increases the complexity of the filter, but the resulting warped filter can still be more efficient than the unwarped filter.

The purpose of this thesis work is to design a graphic equalizer using frequency warped FIR filters. The goal is to design an equalizer, which could be used in real time applications and would have an accuracy that is comparable to the other equalizer designs. At the end of this thesis different equalizer designs are compared on the accuracy and complexity.

The structure of this thesis is organized in several sections. At first a short introduction to equalizers is presented in the first half of Section 2. In the second half the specifications for the graphic equalizers are defined. In Section 3 an introduction to the digital filters as in terms of equalizers and common equalizer designs are presented. Frequency warping and frequency warped filters are discussed in Section 4. After these theory sections the proposed design of the warped equalizer is presented. The designs of both an octave and a one-third octave equalizers are shown in Section 5. In Section 6 the proposed design is evaluated against the specification defined in Section 2 and against other designs. Finally, this thesis work is summarized in Section 7.

2 Equalizers and equalizer specifications

In this section the general concept of the audio equalizers is discussed. The main focus is on graphic equalizers, although many of these implementations have similarities to the other types of equalizers. The specifications and the requirements of an audio equalizer are also presented in this section. The structure of this section is as follows. First a brief introduction to the equalizers is presented. After that the specifications for graphic equalizers are defined.

2.1 Audio equalizers

Audio equalization is a process of adjusting the amplitudes of audio signals at particular frequencies [3]. The device that performs the equalization process is called an equalizer. The name equalization originates from the desire to obtain flat frequency response of an audio system by compensating the nonideal audio signal chain or room acoustics [7]. All audio equalizers are implemented using audio filters, either analog or digital.

The purpose of the equalization process is to shape the magnitude response of an audio signal. Equalizers can be used to restore the original frequency response that has been distorted by the audio chain. Nonideal audio devices, such as microphones, transmission lines and speakers, modify the frequency response of the input audio signal by attenuating or boosting certain frequencies. These modifications can be reversed to a certain degree by equalization. The operation of an equalizer is defined by the equalization curve, which defines the how the different frequencies are processed.

The first audio equalizers were designed in the 1920s to correct the audio transmission losses of the telephone lines. These equalizers were fixed to the audio system and did not have user controllable settings. A little later the first variable equalizer that allowed users to control the equalization curve were used in movie theaters to improve the sound reproduction. As the designs of the equalizers progressed, the applications of the equalizers started also cover the sound enhancement not just correction of the frequency response. [4]

Another common use of equalizers is to enhance or diminish certain frequencies. Certain frequencies can be modified to make a more pleasant audio listening experience, even though it would mean an altered frequency response from the original. Equalizers can be used in the audio mixing process as a creative tool with many different applications [2]. For example they can be used to separate different instruments in the sound field, so they can be heard more clearly on a recording or the tone of the different instruments can be altered to create different moods.

The equalizers before the 1980s were implemented with analog filter circuits. The era of the digital equalizers begun in the 1980s, when Yamaha introduced the first commercially available digital equalizer [6]. Yamaha DEQ7 was the first variable stand-alone equalizer model based on DSP technology. As the technology developed digital equalizers became much more versatile than the analog ones. While it is in theory possible to design an equivalent analog equalizer, in practice the costs and

the noise issues make digital designs more practical. However, this does not mean that the digital equalizers sound better than the analog ones. [2]

Equalizers can be categorized based on their properties [4]. Fixed equalizers are used to equalize conditions that do not change. Examples of these equalizers are RIAA filters used in phono recording and playback [8] and the loudness setting used in stereo amplifiers. Variable equalizers give the user controls to modify the equalization curve. These equalizers are more common than fixed ones, as they are more versatile. The main types of the variable equalizers are the parametric and the graphic equalizers [6].

2.1.1   Parametric equalizers

Parametric equalizers are variable multi-band filters, which allow the user to control the gain, the center frequency and the bandwidth of an individual frequency band. The controls for these parameters are provided to the user in the user interface of the equalizer. An example of the parametric equalizer is shown in Figure 1.

Figure 1: DBX 555 5-band fully parametric equalizer [9]. An example of a five band analog parametric equalizer. Every frequency band is fully parametric and has a control for the gain, the center frequency and the bandwidth. Image adapted from [9].

The number of the bands in parametric equalizers can vary. The usual designs have from three to five different frequency bands. In some designs the bands can be semiparametric and have some controls fixed to a certain value. For example in some simpler equalizer designs the bandwidth is set by the designer. Another usual design is to implement the lowest and the highest frequency bands as shelving filters.

Parametric equalizers are the most powerful and flexible type of an equalizer [6]. They allow a good control of the frequency response and can be used to create complicated responses with a limited set of frequency bands as shown in Figure 2.

The audio plugin shown in Figure 2 shows the response curve of the equalizer. This eases the use of the equalizer, as the effect of the parameter changes can be inspected visually. If this visual clue is not provided, as in the case of the analog

Figure 2: A screenshot of an equalizer curve created with EQ III audio plugin by Avid. A five band equalizer is enough to create complex response curves.

implementations, the efficient use of the parametric equalizer requires an experienced user. This is why the fully parametric equalizers are usually used in professional audio devices, such as in mixing consoles.

2.1.2 Graphic equalizers

The second type of the common equalizers is a graphic equalizer. Typical graphic equalizers consist of many adjacent mini-faders. Each fader can be used to boost or attenuate the gain of a certain frequency band. The center frequencies and the bandwidths of the bands are predefined and fixed. The frequency bands are usually spaced either an octave or a one-third octave apart. In the former case the number of the bands is usually 10 and in the latter case from 27 to 31. An example of a 30 band graphic equalizer is shown in Figure 3.

Figure 3: BSS FCS 966 stereo graphic equalizer [10]. An individual equalizer is provided for each of the audio channels. In this design frequency bands range from 25 Hz to 20 kHz with the gain range of ±15 dB. The gain faders provide a graphical presentation of the equalization curve. Image adapted from [10].

In graphic equalizers the frequency bands are usually divided logarithmically in the frequency axis to better respond to the human perception. At low frequencies the bands are placed densely close to each other and at high frequencies the bands are placed sparsely. As seen in Figure 3 the positions of the gain sliders can be

understood as a graphic presentation of the frequency response. This gives the user a visual clue of the operation of the equalizer. Graphic equalizers are highly common in live sound applications, where they are used to control the frequency response of the sound reproducing system and to prevent feedback [2].

2.2 Specifications of the graphic equalizer

Any audio device that is used to modify the frequency response of an audio signal can be called an equalizer. They are devices that give the user straightforward controls to shape the frequency response. In parametric equalizers the user can adjust the center frequencies, the bandwidths and the gains of the frequency bands. In the graphic equalizers the frequency range is divided to fixed bands and the user can control only the gains of the individual bands.

Because the applications and designs of the equalizers are so diverse, there are no standardized specifications for equalizers. Equalizers can modify the frequency response in any way that is suitable for the application. In the following some features are defined for the graphic equalizers. These features are used later in this thesis in the design of a graphic equalizer and to evaluate and compare the performance of the equalizers designs.

2.2.1 Frequency resolution

The human perception of sound follows the logarithmic scale in frequency [11]. The frequency resolution of the human hearing is about one-sixth of an octave and small changes in frequency at low frequencies are noticeable, but a large change is needed at the high frequencies for an audible sensation. For equalizer it is desirable to work in the logarithmic scale. Equalizers should have a higher frequency resolution at low frequencies than at high frequencies. To correspond better to the human perception, the frequency bands of the graphic equalizers are usually divided logarithmically across the frequency range.

The most common implementations are an octave and a one-third octave equalizers. The frequency range of an octave equalizer is divided to frequency bands, so that the center frequencies are an octave apart. A one-third octave equalizer follows the same principle, but the frequency bands are divided one-third of an octave apart. Usually the center frequencies of the frequency bands follow the standard by ISO (International Organization for Standardization) [12]. The center frequencies and the corresponding bandwidths of the octave and the one-third octave bands are show in Table A1 in Appendix A.

For the octave bands the center frequency doubles from a band to the next. The frequency bands follow the ratio $f_{m+1} = Rf_m$, where $R = 2$ for the octave bands and $R = \sqrt[3]{2}$ for one-third octave bands. The frequency bands are adjacent, so the upper frequency of the band is the lower frequency of the next band. The center frequency of a band is defined as a geometric mean of the lower and the upper frequencies of the frequency band

$$f_{\mathrm{m}} = \sqrt{f_{\mathrm{u}} f_{\mathrm{l}}}, \tag{1}$$

where f_{l} and f_{u} are the lower and the upper frequencies. The ratio of the bands is defined by R and the bands are adjacent, so $f_{\mathrm{u}} = R f_{\mathrm{l}}$. Thus the lower limit of the band is

$$
\begin{aligned}
f_{\mathrm{m}} &= \sqrt{R f_{\mathrm{l}}^2} = \sqrt{R} f_{\mathrm{l}} \\
f_{\mathrm{l}} &= \frac{f_{\mathrm{m}}}{\sqrt{R}}.
\end{aligned} \tag{2}
$$

For the upper frequency limit

$$
\begin{aligned}
f_{\mathrm{m}} &= \sqrt{\frac{1}{R} f_{\mathrm{u}}^2} = \frac{1}{\sqrt{R}} f_{\mathrm{u}} \\
f_{\mathrm{u}} &= \sqrt{R} f_{\mathrm{m}}.
\end{aligned} \tag{3}
$$

The calculations of the upper and the lower limit frequencies and the bandwidths of the frequency bands are summarized in Table 1. [7]

Table 1: The lower and the upper frequency limits and the bandwidths of the octave and the one-third octave frequency bands.

	Octave bands	1/3 octave bands
Lower frequency	$f_{\mathrm{l}} = f_{\mathrm{m}}/\sqrt{2}$	$f_{\mathrm{l}} = f_{\mathrm{m}}/\sqrt[6]{2}$
Upper frequency	$f_{\mathrm{u}} = \sqrt{2} f_{\mathrm{m}}$	$f_{\mathrm{u}} = \sqrt[6]{2} f_{\mathrm{m}}$
Bandwidth	$B = f_{\mathrm{u}} - f_{\mathrm{l}}$	$B = f_{\mathrm{u}} - f_{\mathrm{l}}$

As the bandwidths of the frequency bands depend on the center frequencies, the frequency resolution required for the equalizer is higher at low frequencies and lower at high frequencies. The bandwidth of a frequency band can be defined with the Q value as the ratio of the frequency and the bandwidth

$$Q = \frac{f}{B} = \frac{f}{f_{\mathrm{u}} - f_{\mathrm{l}}} = \frac{f}{\sqrt{R} f_{\mathrm{m}} - f_{\mathrm{m}}/\sqrt{R}}. \tag{4}$$

The Q value is calculated at the center of the band. So $f = f_{\mathrm{m}}$.

$$Q = \frac{f_{\mathrm{m}}}{\sqrt{R} f_{\mathrm{m}} - f_{\mathrm{m}}/\sqrt{R}} = \frac{1}{\sqrt{R} - 1/\sqrt{R}} = \frac{\sqrt{R}}{R - 1}. \tag{5}$$

By Equation 5 the Q values of the logarithmically divided frequency bands are constant for all of the bands. Since $R = 2$ for the octave and $R = \sqrt[3]{2}$ for the

one-third octave bands, the corresponding Q values are $Q \approx 1.41$ for the octave bands and $Q \approx 4.32$ for the one-third octave bands [13]. In this thesis the minimum resolution for the equalizer is defined as half of the frequency band to achieve the required accuracy.

2.2.2 Target frequency response

Graphic equalizers allow the user to define the target frequency response with the gain controls of the equalizer. In the ideal case the equalizer should achieve this response. However, because of the design compromises, these gain settings can be seen more as a guideline, not as the exact response of the equalizer [5]. Depending on the implementation of the equalizer, the actual response can vary.

The user controllable command gain settings can be used as a good target for the equalizer. In a graphic equalizer the user controls the gains of the different frequency bands independently and ideally, the gain of one band should not affect the gains of the adjacent bands. The gain range of a graphic equalizer is typically from ± 6 dB to ± 24 dB depending on the design.

There are several different ways to construct the target response of the equalizer [14]. The target response is precisely defined only at center of a frequency band. It is clear that the magnitudes at the center frequencies should match the command gains, but the responses at the transition bands, are not so unambiguous. Some of the methods to define the target response are shown in Figure 4.
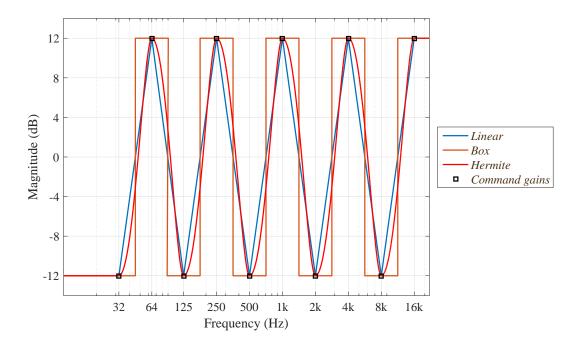

Figure 4: Different ways to define the ideal target response of an octave equalizer. Command gains are the gains defined by the user.

The linear target response is defined by simply connecting the command gains

together. It produces very sharp peaks, which are hard to achieve with equalizers. The ideal box type target response is defined by setting the gain of the whole frequency band at the command gain. This box shape target response is good for the evaluation of the transition frequencies between the bands, but it is not a good approximation for the real life equalizer. The target response used in this thesis uses the piecewise cubic Hermite interpolation [15]. It produces smooth target curves that follow the command gain points with precision without any overshoots [16].

The target response is defined to follow the command points accurately. For the real life applications it is not necessary to reach the target response precisely, as the human perception of small magnitude changes is not very sensitive. Studies have shown that smaller than ± 1 dB deviations from the target response are not noticeable to humans [11]. These result have been obtained in an A/B testing, so in the real life situations larger deviations may be acceptable. In this thesis the maximum deviation for magnitude response at the gain control points is defined to be ± 1 dB.

2.2.3 Phase response

The human perception is not as sensitive to the phase distortions as for the magnitude distortion. The phase distortion of sufficient magnitude is audible [17], but depends heavily on the frequency. The perception of the error also depends on the source material and listening conditions [18]. For example reverberant listening rooms cause bigger phase distortions than common equalizers [17].

Most equalizer designs use minimum-phase filters, as they minimize the latency caused by filters [13]. The phase response of a minimum-phase equalizer changes with parameter changes, so the phase errors and their audibility are hard to predict. An alternative to the minimum-phase filters is the linear phase filters. They do not distort the phase response, but cause a constant group delay [19]. Thus linear phase filters have a constant latency, which depends on the order of the filter.

Both of the linear and the minimum-phase equalizers cause unwanted errors to the audio signal [2]. Because the impulse response of a linear phase filter is symmetrical, pre-ringing of the signal can be heard as an unwanted by-product. Linear and minimum-phase equalizer are alternatives to each other and neither of them is a superior design [2].

2.2.4 Computational complexity and latency

Digital signal processing causes latency in the signal chain [7]. This is caused by the unit delay elements used in digital signal processing algorithms and buffering. In typically equalizer designs buffered signals are not needed, so most of the latency is caused by the unit delay elements. The latency should be small as possible, because several devices may be used in cascade, which means that the latencies will accumulate.

For the live applications the total latency of the signal chain should be less than 20 ms to be acceptable [20]. If there is more latency, the delayed signal is heard as

an echo, which can be confusing to the speakers and the musicians. For the sampling rate of 44.1 kHz the delay of 20 ms corresponds the sample length of 882 samples.

Equalizers are required to perform two separate tasks. The first task is the recalculation of the filter coefficients, when the input parameters are changed. The second task is the actual filtering. In this thesis the equalizers are primarily compared by the complexity of the filtering task. The redesign procedures are also compared, but the actual measurements are out of the scope of this thesis.

The computational complexity of a digital signal processing task means the number of multiplications and additions needed to perform the required task. Digital filters can be implemented efficiently with the multiply-accumulate (MAC) operation [21]. Most modern DSP processors can calculate the MAC operation with one instruction. In this thesis the computational complexity of the equalizer is measured both in number of multiplications and additions and in MACs.

3 Digital filters and equalizer designs

For equalizers there are a variety of different filter designs, as the use of the equalizers is so diverse. Different applications require different properties for the equalizers. In this section few of the simplest and most common equalizer designs are presented. Analog equalizer designs are out of the scope of this thesis, but their properties are briefly discussed. A comprehensive introduction to the analog equalizers is presented in an overview by Bohn in [4]. Digital equalizers are based on the digital filters [7], so a short introduction to them is presented here.

The structure of this section is as follows. First a short introduction to digital filters is presented. The properties of FIR and IIR filters are discussed. After that the different equalizer designs and implementations are discussed.

3.1 Digital filters

Digital filters are used to modify the frequency and the phase response of the digital input signal. Digital filters operate on discrete digital signals, which are discrete in both time and magnitude [21]. To convert an analog input signal to a digital signal, the input signal is sampled with even time intervals defined by a finite sampling frequency f_s. The sampling frequency determines the maximum frequency range of a system [21].

$$f_\mathrm{max} = \frac{f_\mathrm{s}}{2}. \tag{6}$$

The magnitude intervals are defined by the bit depth. Figure 5 shows the sampling of an analog input signal to a digital sampled signal.

Analog signals are continuous in time and frequency and they have in theory an infinite time and magnitude resolutions. The limiting factors for the resolution are the electronic components, which are used in the signal path. The amplitude of an analog signal is usually presented as voltages. Digital signals are presented as a sequence of numeric values. The magnitude resolution of a digital signal is limited by the bit depth used in sampling.

As an analog signal is sampled, the corresponding voltage values are quantized to the nearest numerical value, determined by the used bit depth. The error between the original and the sampled values is called the quantization noise. The time resolution of the digital signal is determined by the sampling rate. [21]

A digital filter is an algorithm implemented in hardware or software. It performs mathematical operations on a sampled discrete-time signals to reduce or enhance certain aspects of that signal [22]. Digital filter is a processor that transforms an input sequence $x[n]$ to an output sequence $y[n]$. If the filter is defined as H, the filter process can be expressed as

$$\{x_1, \ldots, x_n\} \xrightarrow{H} \{y_1, \ldots, y_n\}. \tag{7}$$

Digital filters presented here are LTI (linear time-invariant) systems [21]. LTI definition implies that the properties of the system do not depend on the input signal
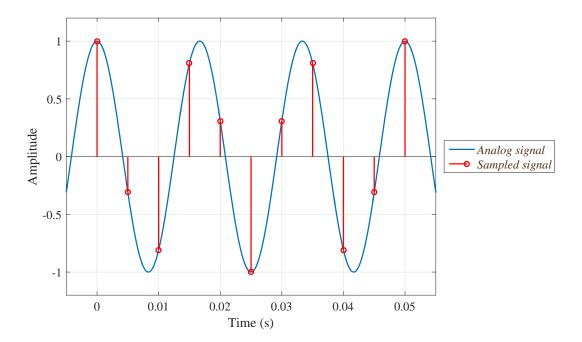
Figure 5: An example of the sampling process. A continuous analog signal is sampled with sampling rate of 0.05 s.

and do not change in time.

Digital filters are constructed with addition and multiplication elements and unit delays [21]. Addition elements sum the input signals and multiplication elements multiply the input signal with a constant value. Unit delays are signal blocks that delay the input signal samples for one sample. The order of the filter is defined as the maximum number of input samples stored in the delay line. The structure of a second order digital filter is shown in Figure 6.
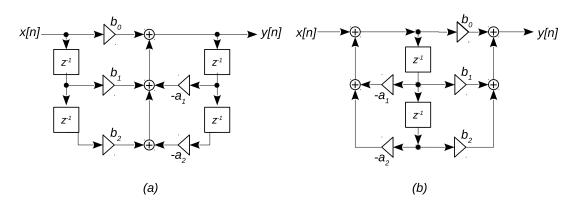

Figure 6: A second order digital filter. Two input samples are stored in the delay line. The same filter is shown with two different structures. (a) Direct form I and (b) direct form II.

The filter elements are connected so that signal loops are formed. Gains $a_1, \ldots, a_N$

control the feedback loops and gains $b_0, \ldots, b_N$ the feedforward paths. Filters can be realized in different forms. Figure 6 shows the direct form I and the direct form II realizations of the same filter. The operation of these filters is the same. The elements of the filter are just reorganized by combining the delay elements.

The functionality of a digital filter is characterized by its transfer function [21]. With the transfer function the output of a digital system can be calculated from the input signal. If the input signal is $x[n]$ and the output signal is $y[n]$, the output of the filter in Figure 6 can be presented as

$$
\begin{aligned}
y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] \\
- a_1 y[n-1] - a_2 y[n-2].
\end{aligned}
\tag{8}
$$

Equation 8 can be presented in the z-domain as

$$
\begin{aligned}
Y(z) = b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) \\
- a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z).
\end{aligned}
\tag{9}
$$

By combining the common factors $X(z)$ and $Y(z)$ the transfer function of a second order filter is obtained.

$$
\begin{aligned}
Y(z) \left[1 + a_1 z^{-1} + a_2 z^{-2} \right] = X(z) \left[b_0 + b_1 z^{-1} + b_2 z^{-2} \right] \\
\frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{N(z)}{D(z)} = H(Z).
\end{aligned}
\tag{10}
$$

$H(z)$ is the z-domain version of the transfer function. In the general case the transfer function has the form

$$
H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_N z^{-N}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_M z^{-M}}
\tag{11}
$$

The variable N is the number of the feedforward paths and M is the number of the feedback loops. N and M define the order of the filter, which is also called the length of the filter. Usually the order of the filter is defined with one variable, N or M, which is larger.

The roots of the polynomial $N(z)$ are called the zeros of the filter and the roots the $D(z)$ the poles. A graphic presentation of the transfer function is the pole-zero plot, which shows the poles and the zeros on the complex plane. An example of the plot is shown in Figure 7.

The unit circle in the pole-zero plot presents the frequency range of the system. In the case of the discrete digital signal the unit circle anticlockwise from 1 to -1 coincides with positive frequencies from zero to half of the sampling rate. The zeros in the plot can be seen as minimum points of the frequency response. The closer the zero is to the unit circle the deeper is the minimum point. The poles can be seen as
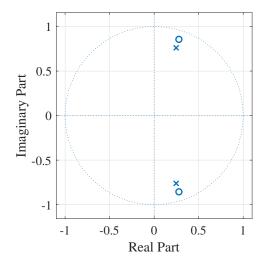
Figure 7: The pole-zero plot of a second order bandstop filter. 'o's mark the zeros and '×'s the poles.

the maximum points of the response. If a pole is outside the unit circle, the gain of the feedback loop is over one, which means that the filter is unstable.

The transfer function of a system is used to characterize the frequency and the phase responses of a system. The frequency response $G(\omega)$ defines how the system responds to different frequencies. It is defined as the magnitude of $H(\omega)$

$$G(\omega) = |H(\omega)|. \tag{12}$$

The phase response $P(w)$ is defined as the phase shift caused by the system as the phase shift of the $H(w)$

$$P(\omega) = \arg(H(\omega)). \tag{13}$$

The group delay of the system defines how much different frequency components are delayed by the system. It is defined with the phase response as

$$\tau_{\mathrm{g}}(\omega) = -\frac{d}{dw}P(\omega). \tag{14}$$

Because of the feedback loops, the impulse response, i.e. the filter's response to an impulse input, is infinite. The length of the impulse response depends on the coefficient values of $D(z)$. If a filter has no poles, i.e., $D(z)$ equals unity, the filter has a finite impulse response and its length is determined by the length of $N(z)$. These types of filters are the two basic types of digital filters.

3.1.1 Finite impulse response (FIR) filters

The impulse response of an FIR filter is finite and the same length as the filter itself. When the denominator of the transfer function equals unity, the filter has no feedback loops. In this case the filter consists only of the feedforward section. The structure

of a direct form FIR filter is presented in Figure 8. Several other possible structures for an FIR filter exists, but in this thesis the direct form structure is used.
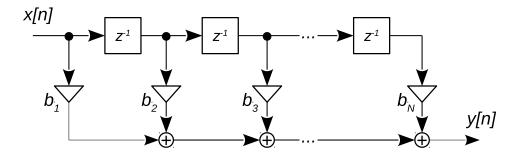

Figure 8: The structure of an Nth order FIR filter in the direct form.

The feedforward structure implies that the output of the filter does not depend on the previous output samples. The direct form FIR filters are characterized by a transfer function

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_N z^{-N} = \sum_{k=0}^{N} b_k z^{-k}, \qquad (15)$$

where N is the length of the filter i.e. the filter order. In the direct form the impulse response of an FIR filter is the same as the coefficients $b_0 \ldots b_N$.

FIR filters are always stable, because of the absence of the feedback loops. The rounding and quantization errors cannot make the filter unstable and do not compound as in the feedback loops. This makes the implementation of the FIR filters easy as the computational restrictions cannot make filters unstable. [19]

The frequency resolution of the FIR filter is determined by the filter order. As the order increases the maximum steepness of the response curves increases. For example the ideal box shape filter would require an infinite long filter. The frequency resolution of an FIR filter can be approximated as [23]

$$\Delta f = \frac{f_s}{N}, \qquad (16)$$

where f_s is the sampling frequency and N is order of the filter. The resolution of an FIR filter is constant at all frequencies and does not depend on the frequency. This is due to the fact that the unit delays delay all frequencies the same. The resolution of an FIR filter can be expressed with the Q value as [24]

$$Q = \frac{f}{\Delta f} = \frac{Nf}{f_s}. \qquad (17)$$

The resolution of an FIR filter as Q values on a logarithmic frequency scale is shown in Figure 9. With Q values it is easy to see that FIR filters have a poor resolution at low frequencies and a good resolution at high frequencies. As the order of the filter increases, the resolution improves.

One of the advantages of FIR filters is that they can have a linear phase response [19]. Linear phase filters have a constant group delay on all frequencies, which means
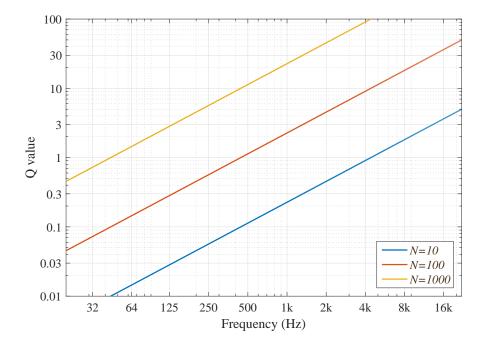
Figure 9: The resolution of an FIR filter with different filter orders shown as Q values. Higher Q means better resolution.

that all frequency components are delayed the same amount. In order to the phase to be linear the coefficient sequence must be symmetrical [19]. There are four different possibilities to the sequence to be symmetrical, as shown in Figure 10.

The impulse response of a linear phase filter is shifted by half of the length of the filter. This causes a constant delay in the filter for all frequencies [19]. This latency limits the use of the FIR filters in live audio applications, when the length of the filter increases. To reduce the latency, a linear phase filter can be converted to a minimum-phase filter [19]. Minimum-phase filters have a smaller group delay than linear phase filters. However, the group delay of the minimum-phase filter is not constant at all frequencies and the phase response is not linear.

Linear phase FIR filters have zeros either at the unit circle or at complex conjugate pairs. A minimum-phase filter can be obtained from a linear phase filter by reflecting the zeros that are outside the unit circle to the inside of the unit circle. The zeros that are located at $z = re^{jw}, (r > 1)$, r is the distance from zero point, are changed to zeros located at $z = e^{-jw}/r$. The resulting minimum-phase filter has the same magnitude response as the original linear-phase filter [19]. The phase responses and zero locations of the linear and minimum-phase filters are shown in Figure 11. Another advantage of the minimum-phase filters is that they achieve the same magnitude response as the linear-phase filters with lower filter orders.

There are several well-known methods for the design of FIR filters [19]. The most straightforward method is to calculate the inverse FFT of the target response and use the resulting impulse response as the filter coefficients. Other more optimized designs method are for example least-squares and Parks-McClellan design methods. In this
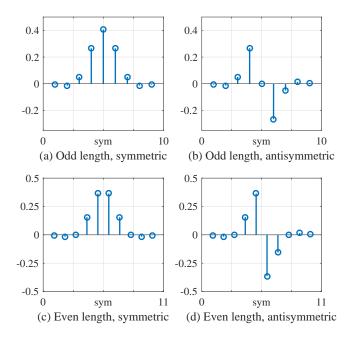
Figure 10: Four types of impulse response symmetry for linear phase filters.

thesis filters are designed with the constrained least squares (CLS) design method [25]. This method allows a multiband filter design with the specified magnitude ripples at various frequency bands, which makes it suitable for the equalizer design. The CLS design method is readily available in Matlab as the '$fircls$' command.

In general FIR filters are easy to implement. Filters that are in the direct form can be implement efficiently with one MAC (multiply–accumulate) operation. The coefficient quantization or finite-precision implementation cannot make FIR filters unstable [19].

3.1.2 Infinite impulse response (IIR) filters

IIR filters are digital filters with an infinite impulse response in theory. Unlike FIR filters, they have a feedback structure. An IIR filter may or may not have the feedforward paths. In theory, as the response of the FIR filter reaches zero at some point, the response of the IIR filter continues infinitely. The infinite response is implemented using recursive feedback loops. The structure of the basic IIR filter was shown in earlier in Figure 6 in Section 3.1. The transfer function of the basic IIR filter is

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_N z^{-N}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_N z^{-N}}. \tag{18}$$

Due to the feedback loops, IIR filters are not always stable. The stability of the filter is determined by the coefficients of the denominator $a_1 \ldots a_N$. A simple way to determine the stability is to use pole-zero diagram. If the poles are within the unit
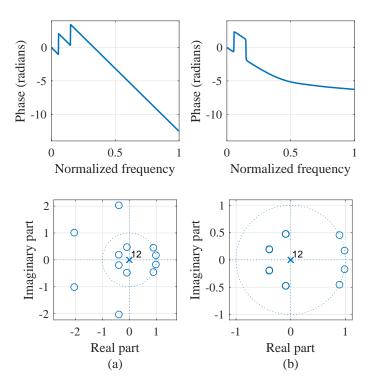
Figure 11: The phase responses and zero locations of (a) a linear phase filter and (b) a minimum-phase filter. The zeros of the minimum-phase filter that are inside the unit circle are double zeros.

circle, the filter is stable.

In general IIR filters can achieve sharper transitions between band edges than FIR filters. The reason is that IIR filters can have a pole near the edge of the pass band and a zero near the edge of the stop band [19]. This means that IIR filters require smaller filter orders as FIR filters to have similar steep magnitude responses. The frequency resolution of an IIR filter is not dependent on the frequency, as the poles and the zeros close together create sharp transition bands regardless of the filter order.

Because of the infinite impulse response, IIR filters cannot be implemented as linear phase filters. Approximations of the linear phase IIR filter exists [26], but in general IIR filters have a nonlinear phase response. If poles and zeros are closely located on the z-plane, rapid changes are produced in the phase response [19]. A larger change in magnitude causes a larger deviation in phase. As an example the frequency and the phase responses of an IIR peak filter are presented in Figure 12. Because of the nonlinear phase response, group delay of an IIR filter is not constant and varies with frequency.

The implementation of IIR filters is more demanding than FIR filters. When filter coefficients are quantized or rounded for the implementation, poles and zeros move across the z-plane. If a pole is close to the unit circle, a small change can make a filter unstable. Also because of the recursive feedback loops, the errors in the filter
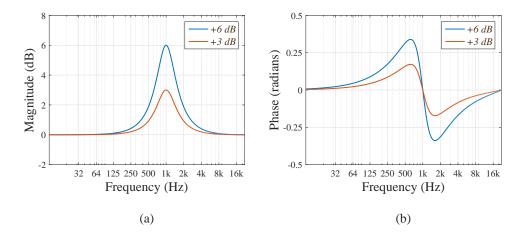
Figure 12: The (a) frequency and the (b) phase response of an IIR peak filter with the center frequency of a 1 kHz. A larger gain causes a larger phase shift.

coefficients cumulates. This causes error buildup with long IIR filters. [19]

The most common way to design an IIR filter is to use the bilinear transformation method [21]. The specifications of the filter are converted to analog filter specifications. These specifications are then used to design an analog filter with design methods, such as Butterworth, Chebyshev or elliptic filter designs. The designed analog filter is converted to digital form. Because of the feedback loop the design of the IIR filters is not as straightforward as the design of the FIR filters. FIR filters can be designed to match a certain response, but IIR filters are usually implemented as a cascade or a parallel structure of lower order filters.

3.2 Common equalizer designs

Digital equalizers can be implemented using a variety of filter structures [6]. The most common structure for the analog and the digital graphic equalizer is a bank of cascaded or parallel filters [13]. In these designs the frequency range of the equalizer is divided to frequency bands and each of them is filtered with an individual filter. In a cascade structure the output of a filter is used as an input for the next filter. In a parallel structure outputs of the filters are combined at the output of the equalizer. Because of the limited frequency range of the each filter, fairly simple filters can be used [7]. The structures of the cascade and the parallel designs are shown in Figure 13.

The center frequencies of the bands can be freely chosen, but in graphic equalizers the bands are usually divided logarithmically across the frequency range. In the cascade structures filters are peak or notch filters, which have a unity gain at the adjacent bands. The gains of the bands in the cascade structure are defined with the filter coefficients. In parallel structures each filter is a bandpass filter with a zero gain at the other bands. The external gain controls can be used and the filter coefficients do not need to be recalculated when gains are changed.

Because low order filters can be used, it is fairly simple to recalculate the filter
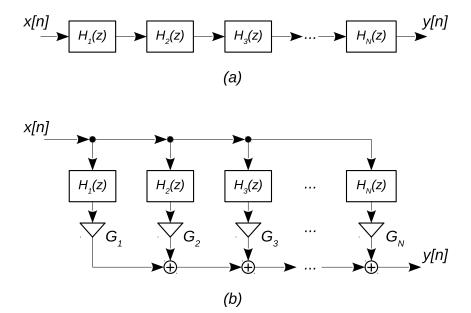
Figure 13: Two typical equalizer filter structures. In the cascade structure (a) the filters are peak/notch filters. In the parallel structure (b) filters are bandpass filters.

coefficients, when the equalizer parameters are changed. In a cascade structure only one of the filters has to be redesigned. In a parallel structure only changing one of the external gains may be needed. The accuracy of the filter bank structure depends on the complexity and the number of the filters.

Both parametric and graphic equalizers can be designed with filterbank structures. For graphic equalizers this approach is suitable, because they have predefined frequency bands. As the number of the bands is high, each band can be filtered with a low order filter.

An alternative equalizer structure is to use one higher order filter that filters the whole frequency range [27]. Depending on the target response, the filter required has to be complex, as it has to be able to filter several frequency bands. The whole filter has to be redesigned as the parameters change, which can be computationally expensive. This kind of design can be more accurate than the filter bank structures, as the filter can be designed to match a certain target response.

Digital equalizers can be implemented with both FIR and IIR filters. Designs using both of these filter types have their advantages and disadvantages and they are discussed next.

3.2.1 IIR equalizers

As stated earlier in this section, IIR filters are difficult to design. Simple IIR filters can be very efficient so IIR equalizers are usually implemented using cascade or parallel structures. In both of the cases equalizer is divided to multiple subfilters, which each ideally affects only a single frequency band. The structure of a simple three tone IIR equalizer is shown in Figure 14.
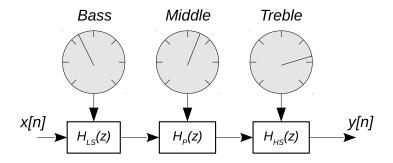
Figure 14: The structure of a simple IIR equalizer. The bass and the treble filters are shelving filters. The middle filter is a peak/notch filter.

The most common way to implement an IIR equalizer is to use second order peak/notch filters for each of the bands [3]. The first and the last bands can be shelving filters. For simple equalizers the shelving filters can be implemented as first order filters [6]. These filters are designed with simple prototype filters and bilinear transformations [6]. The transfer function of the first order shelving filter for low frequencies is

$$H_{\text{LS}}(z) = \frac{G\tan(\omega_c/2) + \sqrt{G} + (G\tan(\omega_c/2) - \sqrt{G})z^{-1}}{tan(\omega_c/2) + \sqrt{G} + (tan(\omega_c/2) - \sqrt{G})z^{-1}}, \quad (19)$$

where ω_c is the crossover frequency and G is the gain of the filter. The corresponding the first order high-frequency shelving filter is

$$H_{\text{HS}}(z) = \frac{\sqrt{G}\tan(\omega_c/2) + G + (\sqrt{G}\tan(\omega_c/2) - G)z^{-1}}{\sqrt{G}\tan(\omega_c/2) + 1 + (\sqrt{G}tan(\omega_c/2) - 1)z^{-1}}. \quad (20)$$

The transfer function of the second order peak/notch filter can be derived by applying a lowpass-to-bandpass transformation to the low frequency shelving filter [6]

$$H_{\text{PN}}(z) = \frac{\sqrt{G} + G\tan(B/2) - (2\sqrt{G}\cos(\omega_c))z^{-1} + (\sqrt{G} - G\tan(B/2))z^{-2}}{\sqrt{G} + \tan(B/2) - (2\sqrt{G}\cos(\omega_c))z^{-1} + (\sqrt{G} - \tan(B/2))z^{-2}}, \quad (21)$$

where B is the bandwidth of the peak filter and ω_c is the center frequency of the band. When $G > 1$ the filter is a peak filter and when $0 < G < 1$ the filter is a notch filter. Figure 15 shows the frequency and the phase responses of the three band equalizer presented in Figure 14.

The overlapping filters of the equalizer cause gain buildup at the transition bands. The first order shelving and the second order peak filters do not have transition bands that are steep enough to have a unity gain at the adjacent frequency bands. As the number of the bands increase, this gain buildup error increases. As seen from Figure 15, the phase response of the equalizer is highly nonlinear. The peak/notch filter in this design has a controllable center frequency, bandwidth and gain so it is suitable and often used in parametric equalizers.
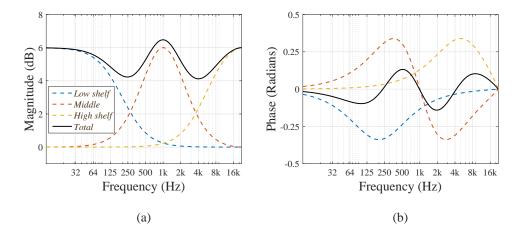
Figure 15: (a) The frequency and (b) the phase responses of the three band equalizer. A low shelving filter is used for low frequencies and a high shelving filter for high frequencies. Middle frequencies are equalized with a peak/notch filter.

Another common peaking filter design is the second order Regalia-Mitra (RM) filter [28]. In this design second order all-pass filters are used to tune the peaking filter to a certain frequency. The structure of this design is more complicated, as the output of the filter is a combination of the input signal and the output of the all-pass section. Original RM filter was used as a peaking filter, but with modifications it can be used as peak/notch filter [29].

The number of the equalizer bands can be increased by adding more peak filters to the design. A graphic equalizer can be designed with a cascade form by adding a filter for each of the frequency bands. As the center frequency of the IIR peak/notch filters can be defined, they can be divided over the frequency axis logarithmically. The frequency response of an octave equalizer with ten peaking filters is shown in Figure 16.

As seen from Figure 16, the gain buildup cause a large ripple to the response of the equalizer. The filters affect the frequencies of the adjacent frequency bands, as the second order peak filters cannot have a flat response on top of the peak and steep transition bands. This gain buildup distorts the frequency response of the equalizer and causes error compared to the target response. In Figure 16 the maximum error compared to the target response is over 4 dB.

There are many available solutions to this problem. A common solution used in the analog graphic equalizers is to use proportional Q values [13]. Q values are adjusted as the command gains are increased. This solution is easy to implement and is still a suitable solution, although it does not remove the ripple of the response curve [5]. Another solution is to compensate the gains of the individual filters [30]. The gains of the filters are allowed to differ from the command gains and are obtained by solving a system of linear equations. These two solutions do not increase the complexity of the equalizer structure, as only the filter parameters are modified. Figure 17 shows examples of these two solutions. In Figure 17a the Q values of the filters are adjusted depending on the gains of the frequency bands. This solution
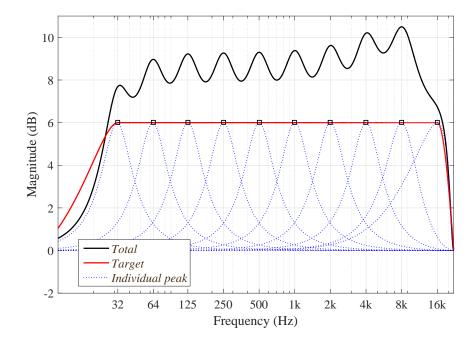
Figure 16: A graphic octave equalizer implemented with second order peak/notch filters. The command gains are set to +6 dB. The Q values of the filters are $Q = 1.3$. The last frequency band (the 16 kHz band) widens to the lower frequencies, as the peaking filter is so close to the Nyquist frequency.

reduces the gain buildup, but the magnitude ripple is still significant. As seen from Figure 17b, the gain compensation reduces both the magnitude ripple and the gain buildup. Both of these solutions are viable options, if a high accuracy of the equalizer is not needed.

There are other solutions that increase the complexity of the equalizer to achieve smoother response. One solution is to use so called opposite filters to improve the frequency response of an equalizer [31], [32]. Additional filters are inserted between the frequency bands to filter out the ripple caused by interfering frequency bands. This solution increases the complexity of the equalizer, as additional filters are needed.

The frequency band interference can also be reduced by using higher order minimum-phase filters [33]. Higher order filters have steeper transition bands and so gains of the frequency bands do not affect the gains of the adjacent bands [34]. Higher order filters increase the computational complexity of the equalizer. For example the use of the fourth order filters doubles the computational load.

The parallel structure is a common design for the analog equalizers [13], as the noise introduced by the filters does not cumulate. As shown earlier in Figure 12, IIR peak filters change the phase response of the input signal. When several filters are placed in a cascade structure, these phase errors accumulate and can cause audible artifacts [7]. In the parallel structure this problem is not so prominent. Also the group delays introduced by the filters do not accumulate as in the cascade structure.
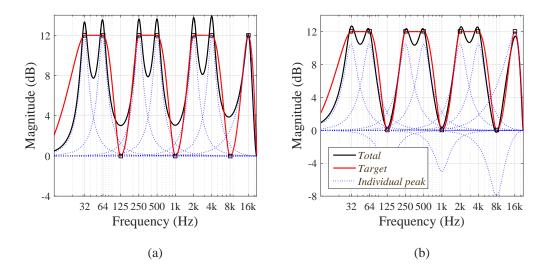
Figure 17: Two examples of IIR equalizer optimizations that require no change in filter structure. (a) Proportional-Q with Q values from $1.4 < Q < 1.8$. (b) Gain compensated equalizer with constant Q values.

The parallel structure enables equalizers to be implemented using parallel processing [16], as all the individual bands have the same input signal. Parallel equalizers have similar band interference problems as the cascade ones. Several solutions are available to solve this [16], [35].

As graphic equalizers are designed with logarithmic frequency bands, the poles and the zeros are clustered at low frequencies. The poles and the zeros of the simple equalizer presented in Figure 16 are shown in the pole-zero diagram in Figure 18.
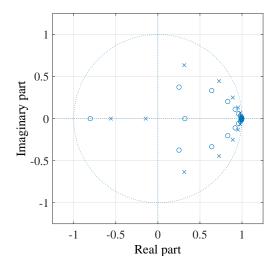

Figure 18: The pole-zero diagram of an example octave equalizer. Poles and zeros are clustered at the low frequencies.

As the number of the bands increases, this pole-zero cluster thickens. This can cause

unstability and errors in the achieved response, when the coefficients of the filters are rounded and quantized at implementation.

The main advantage of the cascade and the parallel structure of peaking filters is that they are fairly simple to implement. Simple low order filters can be used and they are efficient and easy to design. When the command gains are altered, only the filter of the band that has been changed has to be redesigned. Other filters remain unchanged. The low order of the filters means that the equalizer has a low latency and thus suitable for live applications. Additional complexity is added to the structure depending on the interfering band compensations.

3.2.2 FIR equalizers

The implementation of an equalizer using FIR filters is tempting, because FIR filters are always stable and easy to design. They can also have a linear phase response, which is sometimes a desired property. The main drawback of the FIR filters is that high order filters are required for low frequency equalization [36]. The usual structures for the FIR graphic equalizers are a single high order filter and a parallel structure [6].

As stated earlier FIR filters have a constant frequency resolution at all frequencies. This means that high order filters are required for low frequency equalization as the bandwidths of the low frequency bands are narrow. This implies that, if the low frequency bands are equalized with a high order filter, the high frequency bands can be filtered with the same filter [6]. This is why the cascade structure of FIR filters is not sensible for the equalizer design. Cascaded filters would not reduce the orders of the low frequency filters, so the total complexity of the structure would be higher than that of a single larger filter.

The advantage of the single filter design is that the equalizer can be designed to match the target response accurately. As all the bands of the equalizer are filtered using a single filter, there are no gain interferences between the bands. The accuracy of the equalizer is limited only by the order of the filter. Figure 19 shows an octave band FIR equalizer with different filter orders.

The low frequency resolution of the FIR equalizer improves as the order of the filter increases. The resolution at the high frequency bands is wasted, because the resolution is much better than needed. With the filter order of $N = 5000$ and the sampling rate of 44.1 kHz the latency of the filter is 56 ms, which is too much for live applications. For the one-third octave equalizer the required filter order would be at least from 16000 to 20000 [27]. The latency can be reduced by designing the filter as a minimum-phase filter, but in this case the linear phase response is lost.

The total computational costs of the FIR equalizer depends on the filter order. As stated earlier the required multiplications and additions are one per filter order. If the equalizer is implemented as a single larger filter, every alteration of the command gains requires the redesign of the whole filter. Depending on the order of the filter this may be a computationally expensive task.

FIR equalizers are always stable regardless of any roundings or quantizations of the filter coefficients. However, as the filter orders required are large, the zeros are
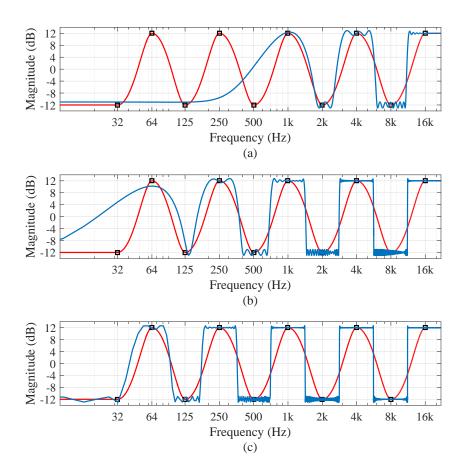
Figure 19: An octave equalizer implemented with a single high order FIR filter. (a) $N = 100$, (b) $N = 1000$ and (c) $N = 5000$. Filters are designed with CLS design algorithm. The red line presents the target frequency response and the blue line the response of the FIR equalizer.

very close to each other on the z-plane. Thus the quantization of the filter coefficients can degrade the performance of an FIR equalizer [27].

To reduce the order of the FIR equalizers, multirate processing can be used [37]. In the multirate solutions the equalizer is implemented in a parallel structure. The input signal is divided to separate frequency bands and each of the bands is filtered with an individual filter. The sampling rates of the frequency bands are lowered so that the highest frequency band uses the largest sampling rate and the lowest uses the smallest sampling rate. An example structure of a parallel multirate equalizer is shown in Figure 20.

With a lowered sampling rate lower order filters can be used [38]. For example, as the processing is done using one-fourth of the original sampling rate, the filter order can also lowered to one-fourth. After that the sampling rates of the bands are converted back to the original sampling rate.
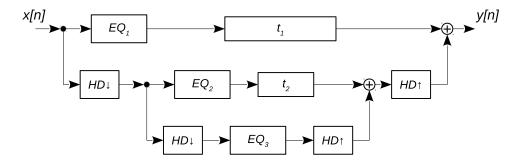
Figure 20: The structure of a three band multirate FIR equalizer [37]. Branches are in a nested form to better correspond to the logarithmic frequency scale. The delays t_1 and t_2 are used to compensate the processing delays of the other branches.

Compared to the IIR designs FIR designs are computationally more expensive. Single FIR filters are required to have large filter orders to equalize low frequencies. Parallel multirate equalizers are still more complex than second-order IIR filters, although much simpler than single filter FIR equalizers. As the filters are more complex, the change of the command gains requires significant computations to redesign the filters.

4 Frequency warping

In this section the concept and the properties of frequency warping for implementation of digital filters are discussed. Frequency warping is a method to warp a digital filter with a linear frequency response to a non-uniform frequency scale [39]. The purpose of the warping is to reduce the order of a filter. Although both FIR and with some modifications IIR filters can be warped [40], only warped FIR (WFIR) filters are discussed here.

First in this section the general concept of the frequency warping is presented. After that it is shown how frequency warping can be applied to the digital filters. Next the impact of the warping to the frequency resolution of a filter is discussed. And finally the design procedure of the warped filters is shown.

4.1 Background of the frequency warping

Digital signal processing using standard digital filters operates on the linear frequency scale. The basic building block of the digital filters, the unit delay, delays all frequencies the same amount. IIR filters can be designed with a frequency resolution that depends on the frequency, as shown in Section 3.2.1 as in the case of the peak filters. FIR filters on the other hand do not have this kind of property, as their resolution depends solely on the filter order. The linear frequency processing is not always optimal for the real life audio applications, because of the nonlinear frequency perception of human hearing [11].

The frequency resolution of the human hearing can be estimated as one-sixth of an octave. At low frequencies small changes in the frequency are noticeable, but at high frequencies, a larger change in frequency is needed for an audible change. This brings demands particularly for the FIR filters, since high order filters are needed to equalize low frequencies. For IIR filters this required high resolution causes the poles to cluster at low frequencies on the z-plane and an extra care must be taken to ensure that the filters are stable.

One solution to this problem is to use spectral transformations to warp the frequency response of a filter to another frequency scale [41]. In this transformation the structure of the filter is modified so that the magnitude characteristics are retained, but moved on the frequency axis. For now on this transformation is called frequency warping.

Frequency warping was first used to obtain a nonuniform frequency resolution for the analysis with the Fourier transform by Oppenheim et al. [42]. The idea of the frequency warped transfer function was introduced by Schüssler [39] based on the filter transformations by Constantinides [41]. The first use to warp digital filters to match the human perception was by proposed by Strube [43]. Audio equalization with warped filters was introduced by Karjalainen et al. [44].

Frequency warping of digital filters is a method to transform a filter so that its frequency response is moved to other frequencies. The goal of the warped filters is to increase the frequency resolution at low frequencies at the expense of a decreased resolution at high frequencies. Since FIR filters have a good resolution at high

frequencies, this can be used to alter FIR filters constant frequency resolution to a nonuniform resolution. The amount of how much the filter is warped is controlled by a single variable called the warping parameter λ. Frequency warping is not a filter design method, so the recalculation of the filter coefficients is not necessary, when the amount of warping is changed.

4.2   Warped filters

Frequency warping can be defined as a bilinear transformation [41]

$$z^{-1} \rightarrow g(z^{-1}) \tag{22}$$

To use the transformation in frequency warping, following properties must be met:

- $g(z^{-1})$ is a real rational function of z^{-1}

- $g(z^{-1})$ maps the unit circle to unit circle in the warped domain

- $g(z^{-1})$ maps the inside of the unit circle into inside of the unit circle in the warped domain

- $g(z^{-1})$ maps $H(\pm 1)$ to $G(\pm 1)$

The first condition ensures that the warped filter is real and rational, as only such filters can be realized. The second condition ensures that the magnitudes of the frequency response do not change. The frequencies are only moved along the unit circle. The third condition ensures that the stability of the filter is preserved when warped. The last condition makes sure that the frequency characteristics of the filter do not change, i.e., a low-pass filter stays as a low-pass.

To fulfill the conditions above the mapping $g(z^{-1})$ must be given by [41]

$$g(z^{-1}) = e^{j\theta} \prod_{i=1}^{n} \frac{z^{-1} - \lambda_i}{1 - \bar{\lambda}_i z^{-1}} \qquad |\lambda| < \pm 1, \tag{23}$$

where λ is the warping parameter and $\bar{\lambda}$ is the complex conjugate of λ. Since $g(z^{-1})$ is required to be real, the zeros λ_i must occur in complex-conjugate pairs and $e^{j\theta} = \pm 1$. This gives the spectral transformation of digital filters [41]

$$g(z^{-1}) = \pm \prod_{i=1}^{n} \frac{z^{-1} - \lambda_i}{1 - \bar{\lambda}_i z^{-1}} \frac{z^{-1} - \bar{\lambda}_i}{1 - \lambda_i z^{-1}}. \tag{24}$$

This transform maps multitude copies of $H(z)$ to $G(z)$. To ensure that the mapping is one to one, the transform is simplified to [41]

$$g(z^{-1}) = \frac{z^{-1} - \lambda}{1 - \lambda z^{-1}}. \tag{25}$$

The transform is bijective, that is, the warping can be reversed with the negative of the warping parameter

$$g^{-1}(z^{-1}) = \frac{z^{-1} + \lambda}{1 + \lambda z^{-1}}. \tag{26}$$

The filter defined by $g(z^{-1})$ is an all-pass filter $A(z)$ that preserves the magnitude response, but alters the phase response. The parameter λ is the warping parameter, which controls how the response is warped. For example, a low-pass filter remains as a low-pass filter when warped. Only the cutoff frequency changes.

The phase response of the all-pass filter determines how the frequencies are warped. This filter has a non-uniform group delay, which means that different frequencies are delayed different amounts. The group delay of the all-pass filter $A(z)$ is shown in Figure 21.
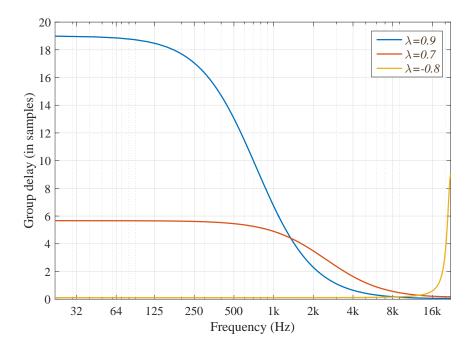

Figure 21: The group delay of the all-pass filter $A(z)$ with different warping parameters.

With a positive value of the warping parameter, the low frequency components proceed slower than the high frequency components. Thus this all-pass filter can be seen as a dispersive system and frequency warping interpreted as a frequency-dependent resampling of the signal [39]. In the z-domain the poles and the zeros are shifted depending on the warping parameter. With negative values of the parameter, zeros and poles are shifted clockwise along the unit circle and with positive values they are shifted anti-clockwise. By the definition the poles on the imaginary axis do not move. This is illustrated in Figure 22.

To warp a digital filter $H(z)$ the transformation is applied simply by replacing all of the unit delays of the filter with an all-pass filter $A(z)$ [39].

$$H_{\mathrm{w}}(z) = H(A(z)) = H\left(\frac{z^{-1} - \lambda}{1 - \lambda z^{-1}}\right) \tag{27}$$
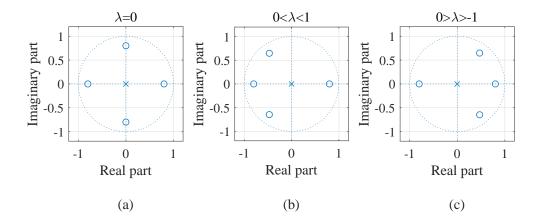
Figure 22: Warping moves the zeros of the filter on the z-plane. (a) No warping, (b) warping with a positive warping parameter and (c) warping with a negative warping parameter. The distances of the zeros from the unit circle do not change.

The original unwarped filter is called the prototype filter and the transformed filter is called the warped filter. If the prototype filter contains feedback loops, i.e., IIR filters, replacing the unit delays with all-pass filters causes delay free loops in the structure. This is why normal the IIR filters cannot be transformed directly and modifications must be made to the structure [40]. The prototype filter can be restored from the warped filter by warping with a negative of the parameter

$$H(z) = H_{\mathrm{w}}(A_r(z)) = H_{\mathrm{w}}\left(\frac{z^{-1} + \lambda}{1 + \lambda z^{-1}}\right) \tag{28}$$

For here on only warped FIR filters are considered. As stated earlier in Section 3.1.1 the standard direct form FIR filter has the transfer function

$$H(z) = \sum_{k=0}^{N} b_{\mathrm{k}} z^{-k}, \tag{29}$$

where N is the length of the filter and b_{k} are the filter coefficients. The transfer function of the warped FIR filter is obtained by replacing every unit delay of the prototype filter with an all-pass filter $A(z)$.

$$H_{\mathrm{w}}(z) = \sum_{k=0}^{N} b_{\mathrm{k}} A(z)^k = \sum_{k=0}^{N} b_{\mathrm{k}}\left(\frac{z^{-1} - \lambda}{1 - \lambda z^{-1}}\right)^k. \tag{30}$$

This transfer function can be seen as a cascade structure of the all-pass filters. The structure of a first order all-pass filter is shown in Figure 23.

When all-pass filters are placed in a cascade structure, an optimized form can be constructed by combining the delay elements of the adjoining filters [44]. By replacing the unit delays of the direct form FIR filter, the structure of the warped FIR filter is obtained, as shown in Figure 24.
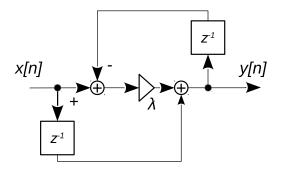
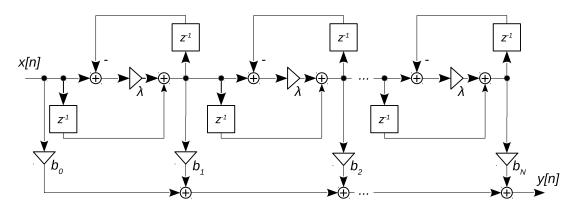Figure 23: The structure of the all-pass filter $A(z)$.

Figure 24: The structure of the warped FIR filter.

The filter coefficients $b_0 \ldots b_N$ do not change when warped. The shape of the frequency response is compressed to lower frequencies and the magnitudes of the stop and pass bands are retained. Like the magnitude response, the phase response is also compressed to lower frequencies. The linearity of the phase response is lost, but the phase distortion is still low. Figure 25 shows the frequency and the phase responses of an example FIR filter and its warped counterpart.

Because feedback loops are added to the filter structure by warping, the filter has no longer a finite impulse response. However, the warped filter retains the stability of the original filter if the warping parameter is $-1 < \lambda < 1$.

4.3 Warping parameter

The gain introduced into the filter structure is the so called warping parameter and it defines how the prototype filter is warped. The parameter is the same for every all-pass filter in the structure. Positive values of the parameter stretches the low frequencies and compresses the high frequencies. For the negative values the effect is opposite. Low frequencies are stretched and high frequencies compressed. By the definition of the transformation the original frequencies ω relate to the warped frequencies ω_{w} by

$$ e^{j\omega_{\mathrm{w}}} = \frac{e^{-j\omega} - \lambda}{1 - \lambda e^{-j\omega}}. \tag{31} $$
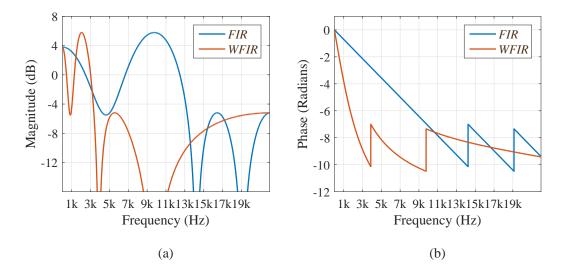
(a)

(b)

Figure 25: (a) The frequency and (b) the phase responses the FIR and WFIR filters. The prototype FIR filter is warped with the parameter $\lambda = 0.7$. The linear phase response of the prototype filter is no longer linear.

from which follows [43]

$$\omega_{\mathrm{w}}(\omega) = \arg(A(Z)) = \omega + 2\arctan\left(\frac{\lambda \sin(\omega)}{1 - \lambda \cos(\omega)}\right) \tag{32}$$

With Equation 32 the warped frequency of the sampled signal in the terms of the warping factor and the original frequency is given as [24].

$$f_{\mathrm{w}}(f, \lambda) = f + \frac{f_{\mathrm{s}}}{\pi} \arctan\left(\frac{\lambda \sin(2\pi f/f_{\mathrm{s}})}{1 - \lambda \cos(2\pi f/f_{\mathrm{s}})}\right) \tag{33}$$

The warping of frequencies for different values of the warping parameter is shown in Figure 26. The effect of the warping increases as the λ approaches one. If $\lambda = 0$ the all-pass filter simplifies to a unit delay and the warping effect disappears.

4.4 Resolution of warped filters

The main advantage of the frequency warping is that it transforms the linear frequency resolution of the prototype FIR filter to a non-linear one. The change of the resolution is determined by the warping factor. When frequencies are stretched, the resolution at those frequencies is increased and decreased when frequencies are compressed. The resolution of the warped filter is defined with the resolution of the prototype filter. The resolution of the prototype FIR filter is

$$\Delta f = \frac{f_{\mathrm{s}}}{N}, \tag{34}$$

where f_{s} is the sampling frequency and N is the filter order. The resolution of a prototype FIR filter is constant at all frequencies and depends only on the order
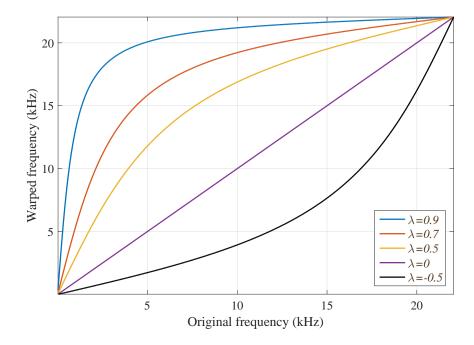
Figure 26: The mapping of the frequencies from the original frequency axis to the warped axis with different warping parameters.

of the filter. As the order increases the resolution increases. The resolution of the warped filter is obtained, when the resolution of the prototype FIR filter is multiplied with the derivate of the frequency function of the warped filter (Equation 33) [24].

$$\Delta f_{\mathrm{w}}(f, \lambda) = \Delta f \left(\frac{1 + \lambda^2 - 2\lambda \cos(2\pi f / f_{\mathrm{s}})}{1 - \lambda^2} \right) \tag{35}$$

The resolution of the warped filter depends on both the frequency and the warping parameter. If λ is positive, the frequency resolution increases at low frequencies and decreases at high frequencies. The amount of the increase depends on the λ. As the value of the warping parameter is increased, the resolution at low frequencies is increased. However, the resolution at high frequencies is decreased. This is illustrated in Figure 27, in which the resolution of the warped filter is compared to the resolution of the prototype filter.

As the warping parameter gets closer to one, the resolution at low frequencies compared to an FIR filter increases. The increase of the resolution is significant and illustrates how frequency warping is a valuable tool for the low frequency equalization. However, at the same time the resolution at high frequencies decreases. With large warping parameter values the resolution is considerably poorer than the resolution of a standard FIR filter.

As seen in Figure 27, the frequency at which the warped filter has the same resolution as the standard FIR filter, depends on the warping parameter. This frequency point is called the turning point frequency f_{p} and defined by [39]
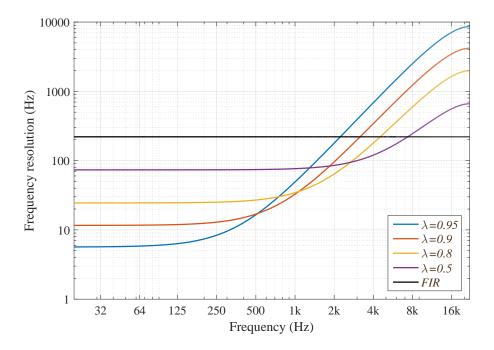
Figure 27: The resolution of a warped FIR filter. The order of the prototype FIR filter is $N = 200$. The prototype FIR filter has a constant resolution of 220 Hz.

$$f_{\mathrm{p}} = \frac{f_{\mathrm{s}}}{2\pi} \arccos(\lambda). \tag{36}$$

The resolution of a filter can illustrated in more convenient way by presenting it with the Q values. Q values are used to measure the bandwidth of the filter and it is defined by the ratio of frequency and bandwidth

$$Q = \frac{f}{\Delta f}. \tag{37}$$

The higher Q value means more shaper filter, i.e., better resolution. The Q value of a warped filter is defined in the same way as

$$Q = \frac{f}{\Delta f_{\mathrm{s}}(f, \lambda)}. \tag{38}$$

The Q value of an FIR filter increases linearly as the frequency increases. In the case of the warped filters the Q value depend also from the warping parameter. The Q values of a standard FIR filter and a warped filter are shown in Figure 28.

As the higher Q value means better resolution, it can be seen from Figure 28 how the warping parameter affects the frequency resolution. With larger λ values the maximum resolution of the filter is moved towards the low frequencies. The maximum Q value of the warped filter does not change much as the warping parameter changes. These Q values can be used to choose the optimal warping parameter for the warped filter. If the maximum resolution is wanted on the certain frequency range, Q values can be used to tune the resolution of the warped filter [24].
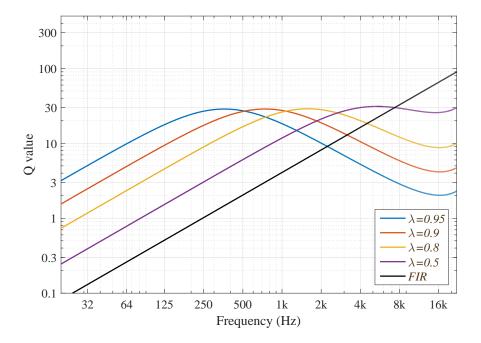
Figure 28: The *Q* values of an FIR and a warped FIR filters with different warping parameters.

4.5 Computational complexity and robustness

Because the unit delays of the prototype filter are replaced with all-pass filters, the overall structure of the warped filter is more complex than the original filter. Additional multiplications and additions are added to the structure, as shown in Figure 24. The computational complexity of the filter is increased by a factor 3 to 4 depending on the implementation architecture [39]. For every order of the prototype filter one additional multiplication and two additions are needed. However, warping can reduce the filter order by a factor about 5 and even more in some applications [39].

Frequency warping has some additional benefits. The required high resolution at low frequencies in audio applications leads to clustered poles and zeros close to each other. Clustered poles and zeros are more sensitive to the parameter accuracy and the quantization effects [21]. In the case of the warped filter the prototype filter can be designed so that zeros are spread more uniformly across the z-plane. When the filter is implemented, the filter coefficients are quantized before the warping [45]. Thus quantization errors are smaller after the filter is warped. The all-pass sections of the warped filter will remain as all-pass filters, as they have the same parameter in the numerator and the denominator of the transfer function. The warping parameter can also be freely chosen, so it is not vulnerable to the quantization [45]. However, a small error in the warping parameter can change the frequency response of the warped filter considerably.

4.6 Design of warped filters

The design of a warped filter begins by defining the target frequency response $X[z]$, which can be an artificial or a measured response. This target response is transformed to the warped frequency scale by sampling the frequency points and warping them to the warped scale using Equation 33. For every point of $X[z]$ the corresponding $X_{\mathrm{w}}[z]$ is calculated. The final target response is interpolated from $X_{\mathrm{z}}[z]$. The warping parameter used in frequency warping can be freely chosen. As there are no exact methods for choosing the correct parameter, it is usually chosen by trial and error [36]. The estimated Q value with different warping parameters as shown in Figure 28 can be used as a guideline, if it is known, where the best frequency resolution is required.

The prototype FIR filter is designed to match this warped target response. Any of the conventional FIR filter design methods can be used, as it is a standard FIR filter [39]. The order of the prototype should be as low as possible, because the warping increases the complexity of the filter. The prototype can be designed as a minimum-phase filter as warping retains this property [46].

The designed prototype filter is warped back to the original frequency scale by replacing the unit delays with all-pass filters. As the transform is bijective, the warping parameter used to warp filter back to the original frequency scale is the negative of the parameter used in the target response warping. If the required target response $X[z]$ is not met, the process is started from the beginning with a redefined warping parameter.

The selection of the order of the prototype filter and the warping parameter is a tradeoff between the demand of the equalization of the low frequencies and the high frequencies. The larger warping parameter increases resolution at low frequencies but decrease at high frequencies. This resolution loss of the high frequencies can be compensated by increasing the order of the prototype filter. However, for the every increase of the prototype filter order the complexity of the warped filter is increased by a factor of three.

5 Design of a warped FIR equalizer

In this section the design of a graphic equalizer using warped filters is presented. The equalizer designed here is a ten band octave equalizer. The same design is later extended to a 31 band one-third octave equalizer. These equalizers are implemented using Matlab with the equalizer specifications defined in Section 2.2. In these designs FIR and warped FIR filters are used. The Matlab code used to implement the octave equalizer is listed in Appendix B.

First, a comparison octave equalizer is designed using a standard FIR filter. Next the corresponding warped equalizer is presented. After that, the cascade equalizer structure, using warped FIR and standard FIR filters, is defined and finally this octave equalizer is extended to a one-third octave band equalizer.

5.1 FIR equalizer

As presented in Section 3.2.2, FIR equalizers are usually implemented as one large filter or a parallel multirate structure. The single filter design is presented here as a comparison to show the benefits of the warped filters. More efficient FIR equalizers are available, as shown in Section 3.2.2, but this simple design was chosen to show how the computational requirements can be decreased with warping

The FIR equalizer is designed as an octave equalizer with logarithmically spaced frequency bands listed in Appendix A. The frequency spacing follows the ISO standard [12], as do the most of the equalizer designs. The first frequency band is defined as a low shelf filter and the last band as a high shelf filter. FIR equalizers using a single filter are designed to match the given target frequency response, which is defined by the user with the command gains of the equalizer.

In this thesis FIR filters are designed with the constrained least-square design algorithm, which is readily available in Matlab (the 'fircls' function). This algorithm was chosen, because the limits of the magnitude ripple at different frequency bands can be defined. The algorithm tries to implement this design and fails to converge, if the desired ripples cannot be achieve with the given filter order. With the CLS design algorithm the minimum filter order that meets the equalizer specifications, can be found by trial and error. Other FIR design methods can also be used, but the control of the magnitude ripples with different gain settings is more difficult.

For an octave band equalizer, the bandwidth of the lowest band is 22 Hz. The required frequency resolution is defined to be half of the bandwidth. At the sample rate of 44.1 kHz, filter order of 4000 is needed in order to achieve the adequate resolution. This gives the frequency resolution of 11 Hz. The maximum magnitude ripple is defined to be ± 1 dB. An example of the FIR equalizer designed with CLS algorithm is shown in Figure 29.

The equalizer command gain setting used was chosen, because it demonstrates clearly how well the equalizer follows the frequency bands. When the order of the filter is decreased, the accuracy of the equalizer decreases at the low frequency bands. The high frequency bands are filtered accurately even with a lower order. As shown in Figure 29, the frequency resolution at the low frequency bands is just adequate,
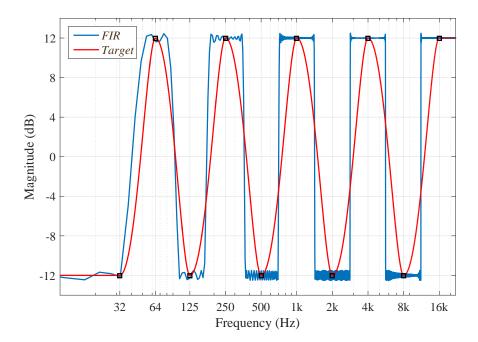
Figure 29: An octave equalizer using a single FIR filter. The order of the filter is $N = 4000$.

when the filter order is 4000. The high frequencies, bands from 1 kHz and up, are over-equalized with very narrow transition bands. The frequency resolution of 11 Hz is completely unnecessary for the frequencies over 2 kHz, when compared to the resolution of the human hearing. With the CLS algorithm the transition bands between the equalizer bands are not defined. As the filter order increases the transition bands of the filter become unnecessary narrow, which could produce fast unnatural changes in frequency response for real life signals.

Because the equalizer is implemented as one larger filter, the change of one of the command gains requires the redesign of the whole filter. This recalculation of the filter is a computationally expensive task, so this equalizer design is unsuitable for the real time parameter changes.

The filters designed with the CLS algorithm are linear phase filters. With the filter order of 4000, the group delay of the filter is constant 2000 samples for all frequencies. This equals to the latency of 45 ms for the sample rate of 44.1 kHz, which is too high for this kind of filter to be used for live applications. The latency can be decreased if the equalizer filter is converted to be a minimum-phase filter. Also the order of the filter can be decreased with the minimum-phase conversion.

With the same design a 31 band one-third octave version of the equalizer can be designed. Again standard ISO frequency bands are used. The first band of the equalizer is defined as a low shelf filter with the center frequency of 20 Hz and the last band is defined as a high shelf filter with the center frequency of 20 kHz. The center frequencies of the bands and corresponding bandwidths are listed in Appendix A. The required frequency resolution to equalize the lowest bands is about 2.3 Hz,

which equals to a filter order of 19000 with the sampling rate of 44.1 kHz. A linear phase equalizer with this size filter order would have a latency of 380 ms. The filter this size would be highly impractical for design and implementation.

5.2 Single warped filter

The main cause of the poor performance of the FIR equalizer is the constant frequency resolution of the FIR filter. The filter order must be increased for low frequency equalization and much of the processing is wasted at the excessive filtering of the high frequencies. With this in mind the use of warped filters is tempting, as it allows a non-uniform frequency resolution with FIR filters.

The implementation of the graphic equalizer using warped filters was introduced in a patent by Olivier [47]. The idea was to create an FIR equalizer in the linear frequency scale as a prototype filter and to warp it to match the logarithmic frequency scale. As the warping triples the computational costs of the FIR filter, the prototype filter should be designed to have the lowest possible order.

The normal design procedure of the warped filters begins with the warping the target frequency response to the warped domain. In the ideal case the target response would warp to the linear scale in the warped domain. This would allow the design of the prototype with a linear spacing of the frequency bands. The simplest prototype filter would be a filter that has frequency bands linearly divided across the frequency range.

For the octave equalizer the prototype filter is designed in the warped domain with ten bands with equal bandwidths on the linear scale. With ten frequency bands, the bandwidth is of a single band is 2 kHz and thus the required resolution is 1 kHz. This resolution requires the filter order of 44. This prototype filter designed with CLS algorithm is shown in Figure 30 with the same equalizer gain settings as in the previous case.

This prototype is warped by replacing every unit delay of the filter with an all-pass filter. The results of the warping using different warping parameter values are shown in Figure 31.

From these resulting filters it is obvious that none of the warping parameters can fit this linearly spaced prototype filter to the logarithmic target response. This is due to the fact that no such warping parameter can be found that would map the linear frequency scale to a logarithmic scale [47]. The mapping of the frequency scale by warping is shown in Figure 32. It is obvious that it is impossible to warp the linear frequency axis directly to the logarithmic one with first-order all-pass filters.

In order to warp the prototype filter to a logarithmic axis, the frequency bands of the prototype filter must be shifted. From Figure 32 it can be seen that the low frequency bands of the prototype filter must be moved to the lower frequencies and the high bands to the higher frequencies. This means that the prototype filter, which has equally divided frequency bands, cannot be used.

To keep the order of the prototype filter as low as possible, a warping parameter is chosen, which requires minimal changes to the frequencies of the prototype filter. With the help of Figure 32 the warping parameter of $\lambda = 0.9$ was chosen. The warped
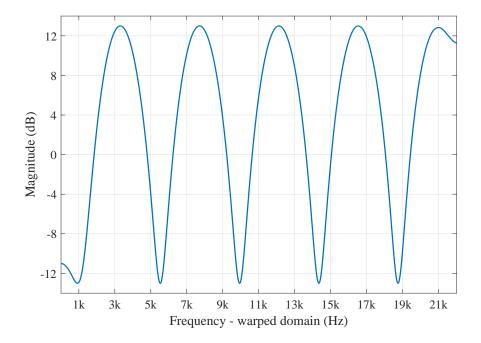
Figure 30: The frequency response of the linearly spaced prototype filter in warped domain. Filter order is $N = 44$.

frequencies for the bands of the prototype filter are calculated using Equation 33. From these calculated frequencies the minimum bandwidth is determined as 720 Hz and required resolution as 360 Hz. This resolution requires the filter order of 124 for the prototype filter. The warped equalizer and the corresponding prototype filter are shown in Figure 33.

The prototype filter has unequal frequency bands. The narrowest band are the first and the last bands, and they define the required order for the prototype filter. From Figure 33 it can be seen that the maximum resolution of the equalizer is moved from the high frequencies to the mid frequencies. This can be verified with plot of the Q value for the $\lambda = 0.9$ shown in Figure 28. However, the frequency resolution is still divided unequally across the bands. The low and the high frequency bands have considerably poorer resolution than the middle frequency bands.

As shown in Figure 33 it is possible to implement an equalizer using a single warped filter. Warping enables the equalization of the low frequency bands with considerable lower order filters than with the standard FIR filters. However, because the frequency bands of the prototype filter must be divided unequally, the increase in the required frequency resolution increases the filter order of the prototype filter. In this case the filter order 124 is required. With warping the numerical computations is increased threefold to 372. Compared to the unwarped FIR equalizer ($N = 4000$), the computational costs of the warped equalizer is considerably smaller.
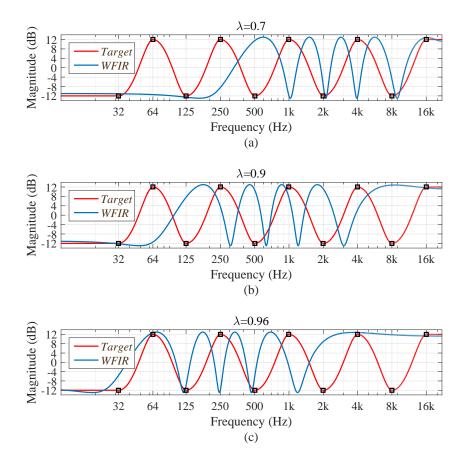
Figure 31: The warped prototype filter with different warping parameters. (a) $\lambda = 0.7$, (b) $\lambda = 0.9$ and (c) $\lambda = 0.96$. None of the parameter values map the prototype to the logarithmic axis.

5.3 Cascaded WFIR and FIR structure

As seen in the case of the single FIR filter equalizer, FIR filters have an excellent frequency resolution at high frequencies. For the high frequency equalization, the FIR filters are a viable option. When an FIR filter is warped the resolution of the high frequencies is decreased and increased on low and mid frequencies depending on the warping factor. With high values of the warping factor, the warped filters can be very efficient on the low frequency equalization.

To make use of the good qualities of both of these filters a cascade structure was presented by Ramos et al. [24]. The idea of the structure is to use a warped filter to equalize the low frequency bands of the equalizer and a standard FIR filter to equalize the high frequency bands. This structure of two cascaded filters is shown in Figure 34.

The low and the low-mid frequencies of the input signal are filtered with a warped filter $H_{\mathrm{WFIR}}(z)$ and the mid-high and the high frequencies are filtered with an FIR
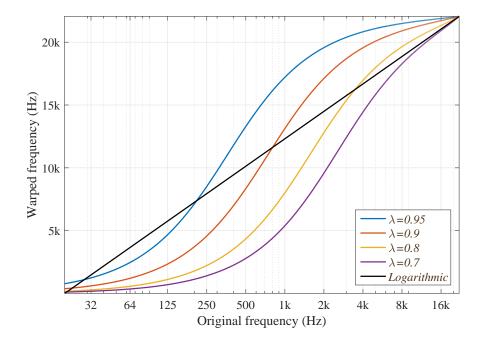
Figure 32: The mapping of the frequencies from warped domain to original frequency scale with different warping parameters.
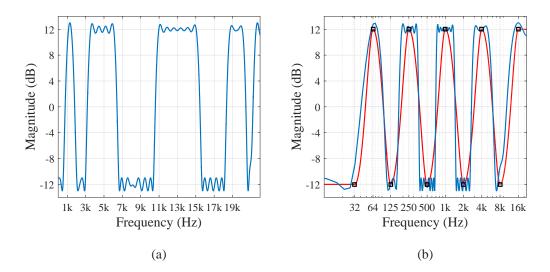

(a)

(b)

Figure 33: (a) The unevenly spaced prototype filter and (b) the warped counterpart. Warping parameter $\lambda = 0.9$.

filter $H(z)$. Filters are placed in a cascade structure in which the output of the warped filter is the input of the FIR filter. The total response of the structure is given by

$$H_{\text{eq}}(z) = H_{\text{WFIR}}(z) \cdot H_{\text{FIR}}(z). \tag{39}$$
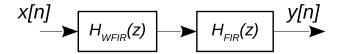
Figure 34: The structure of the cascaded equalizer.

In a cascade structure the frequency bands, which are outside the range of an individual filter, should be left intact. A parallel structure of these filters is also possible, in which case the filters would be band-pass filters. In this thesis the cascade structure is used.

In the cascade structure of two filters the frequency range of the equalizer is divided in two at the crossover frequency f_c. In the ideal case the magnitude of the warped filter would be 0 dB over the crossover frequency and the magnitude of the FIR filter 0 dB below the crossover frequency. This would allow the filters to be designed and operated independently.

The choice of the f_c determines the required orders of the filters. If the f_c is at high frequencies, a higher order warped filter is needed and vice versa if f_c is at low frequencies a higher order FIR filter is needed. The total computational cost of this cascade structure in terms of the MACs is [24]

$$N_{\text{eq}} = 3N_{\text{WFIR}} + N_{\text{FIR}}. \tag{40}$$

where the N_{WFIR} is the order of the prototype of the warped filter and N_{FIR} the order of the FIR filter. As the computational cost of the warped filter is threefold compared to the FIR filter, the order of the prototype filter should be as low as possible.

With the graphic equalizers it makes sense to choose the crossover point so that it is between the frequency bands, i.e., on the octave equalizer the warped filter equalizes the frequency bands $1 \ldots n$ and the FIR filter bands $n + 1 \ldots 10$. If the crossover point is chosen to be at the center of one of the bands, both of the filters would have to filter the same band, which would make both filters more complex. Both filters would need better resolution to filter frequencies of the additional band.

The frequency resolution of the FIR filter is easily determined with the Equation 16. For the octave equalizer, the 1 kHz band has the bandwidth of 710 Hz and for the 2 KHz band 1420 Hz. As the required frequency resolution is defined as half of the bandwidth, the order of the FIR filter should be 125 for 1 kHz band and 62 for the 2 kHz band. It is easy to see that the required order of the FIR filter doubles with every additional band.

The estimation of the filter order for the warped filter is not as straightforward, because the resolution of the warped filter depends on the frequency, the filter order and the warping parameter. The required order of the warped filter can be approximated with the required Q value. For an octave equalizer at the center of a frequency band, half of the bandwidth equals to $Q = 2.8$. Different values of the warping parameter are used to evaluate the required filter order at various frequency bands. When the Q values are plotted with different filter orders and

warping parameter values, the estimations for these values can be found. The Q values of the warped filters with different filter orders and warping parameter values are shown in Figure 35.
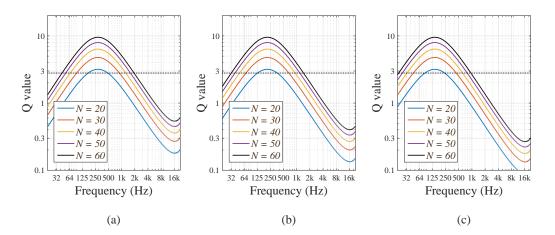

Figure 35: The Q values of the warped filter with different prototype filter orders and warping parameters. The warping parameters are (a) $\lambda = 0.96$, (b) $\lambda = 0.97$ and (c) $\lambda = 0.98$.

With the warping parameter $\lambda = 0.96$ the required Q value is reached at the 2 kHz band with a prototype filter order of 65. With $\lambda = 0.97$ the 1 kHz band is reached with an order of 45 and the warping parameter $\lambda = 0.98$ gives a filter order of 35 at the 500 Hz band. Although these are just approximations of the filter order, these numbers show that the filter order of the warped filter increases and decreases slower than the order of the FIR filter as the crossover point of the equalizer is changed. With these values the optimal crossover point can be determined. The required filter orders of the cascaded filters with the crossover points of 500 Hz, 1 kHz and 2 kHz are listed in Table 2.

Table 2: The required orders of the cascaded structure.

f_c	WFIR	FIR	Total
500 Hz	105 ($35 \cdot 3$)	125	230
1 kHz	135 ($45 \cdot 3$)	62	197
2 kHz	195 ($65 \cdot 3$)	31	226

As the complexity of the warped filter is threefold compared to the FIR filter, the most optimal crossover point of the equalizer is between the equalizer bands 5 and 6 (the 1 kHz band and the 2 kHz band). With the lower crossover point, the order of the FIR filter increases rapidly and raises the total complexity of the filter. With higher crossover point the order of the warped filter increases. This increase is not linear as the exact mapping from the logarithmic scale to the linear scale is not

possible. As the frequency range of the warped filter increases, the frequencies of the prototype filter are moved further from the linear frequency spacing thus making the prototype filter more complex.

The cascaded equalizer is implemented using the crossover point and the estimated filter orders. The filters are designed with the CLS design algorithm. The magnitude ripple is defined to ±1 dB on the equalizer bands and ±0.2 dB on the pass bands. Because the filter orders discussed above are just approximations, the effective filter orders must be found by trial and error. Both of the filters are forced to unity gain right next to the crossover point. Otherwise wide transition bands would be introduced by the CLS algorithm. The resulting filters and the cascaded equalizer are designed with the estimated filter orders. The results are shown in Figure 36.
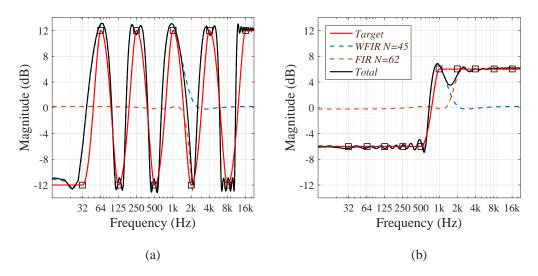

Figure 36: The frequency response of the cascaded structure. (a) Command gains in the alternating -12 dB and +12 dB pattern, (b) gains in step pattern. The crossover point of the equalizer is between 1 kHz and 2 kHz bands.

From the results it can be seen that the cascaded equalizer does not reach the target response. At low frequencies the equalizer fails to reach the target response (Fig. 36a) and the response peaks seem to be slightly off from the target. This can be corrected by tweaking the transition frequencies of the bands. The second problem of this design can be seen from the step response, where a dent in the response is present at the crossover frequency (Fig. 36b). In the ideal case of the cascaded filters, both of the filters would have a unity gain at the adjacent frequency bands. However, because the frequency resolutions of both of the filters are the lowest at the crossover frequency, the responses of the filters expand outside from their frequency bands. This expansion causes errors in the frequency response, when the responses are combined. These filters could be forced to unity with a narrow transition band, but this would require more frequency resolution and thus increase the orders of the filters. If the slopes of the filters at the crossover frequency had symmetrical shapes, the resulting responses would sum up nicely to have a smooth response. Because the gain values of the last band of the warped filter and the first band of the FIR filter

can vary, and thus have variable slopes, the smooth response is not possible without corrections.

One solution would be to use gain compensations similar to the compensations used with the cascaded structure of IIR filters. The gains of the frequency bands beside the crossover point could be modified according to the gains of those bands. Another solution would be to tweak the transition frequencies between the bands to better match the slopes of the response curves. Both of these solutions would require complex calculations in order to function properly and would complicate the design procedures of the filters. The frequencies of the equalizer bands and the gains of the prototype filters would have to be recalculated at every parameter change.

A simpler solution in presented here. Because the most of the error is caused by forcing both of the filters to unity, the outermost frequency band of one of the filters is left at its gain value at the pass band, thus making it a shelf filter. The other filter is affected by this constant gain, which must be compensated. This compensation does not increase the order of the filter much as the required frequency resolution stays the same. The FIR filter was chosen to be the shelving filter and the warped filter to be the compensating filter. As the compensating filter must be still forced to unity at the crossover point, the order of the FIR filter would have to be increased more, if it would be the compensating filter. The warped filter is compensated by subtracting this constant gain from its gain values before the calculation of the filter. With this solution the magnitude error at the crossover point is virtually removed as shown in Figure 37.

5.4 Filter optimizations

The approximations described earlier are used as a starting point for the orders of the filters and they are increased until the specifications are met. The orders must be increased as additional resolution is required to ensure that the equalizer reaches the target response at the crossover bands. Also the frequencies of the transition bands must be tweaked in order to match the target response. The frequency accuracy is inspected visually by comparing the response of the equalizer to the target response. As mentioned earlier in the equalizer specifications in Section 2.2, the main concern is the response at the command gain points of the frequency bands. Larger errors are allowed in the transition frequencies.

The filter order of $N = 52$ for the warped filters prototype and $N = 62$ for the FIR filter were found, by trial and error, to be adequate for most of the gain settings. The order of the warped filter is larger than the approximated order ($N = 45$) presented in Table 2, as additional resolution is required for the crossover compensation. Lower orders of the filters cause larger errors in the response and make the equalizer miss the ± 1 dB requirement. The response curves of the equalizer with different gain setting are shown in Figure 38.

As the frequencies of the bands at the crossover point are tweaked and the filter orders are increased, the equalizer follows the target response reasonable well. For some of the gain settings lower filter orders would be adequate. However, the filter orders are defined as constants, as the dynamic altering of the orders would make
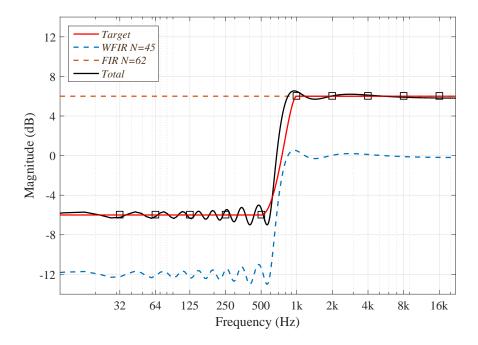
Figure 37: The cascaded equalizer with the crossover correction. The pass band of the FIR filter (<2 kHz) is left at the value of the 2 kHz band. The warped filter is compensated by subtracting the value of the 2 kHz band from the gains of the warped filter.

the equalizer unnecessary complicated. In this implementation both of the filters are redesigned at every parameter change for simplicity. The FIR filter would not be required to be redesigned, when the parameters of the warped filter are changed, as it does not depend on the shape of the warped filter. The warped filter must be redesigned only when the gain of the first frequency band of the FIR filter changes, as it defines the constant gain on the pass band of the FIR filter.

Filters designed with the CLS algorithm are linear phase. A further decrease of the orders of the filters can be achieved by converting both of the filters to minimum-phase filters. The orders of the minimum-phase filters can be reduced by removing the least significant filter coefficients. This reduction modifies the frequency response of the equalizer and causes errors in the response. The minimum-phase implementation with different order reductions are shown in Figure 39. As the order of the filters are reduced the ripples of the frequency bands increase. Otherwise the shape of the equalizer curve seems to retain its shape reasonably well. By comparing the modified response to the target response, the order of the warped filter can be reduced by 8 and the order of the FIR filter by 10. This yields the filter order $N = 44$ for the prototype of the warped filter and $N = 52$ for the FIR filter. Matlab code of the presented octave equalizer is listed in Appendix B.
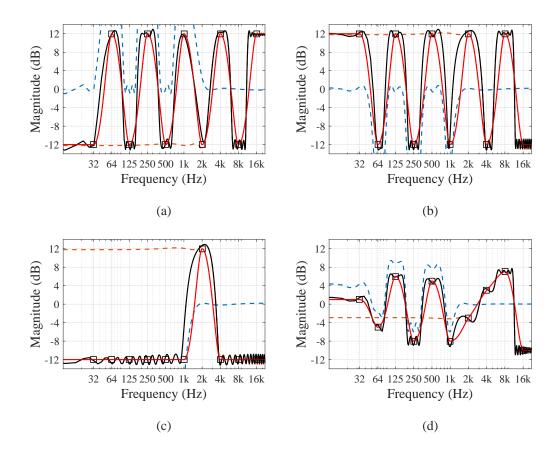
Figure 38: The cascaded equalizer with the crossover correction and tweaked frequencies. Order of the prototype of the WFIR is $N = 52$ and FIR $N = 62$. The increase of the filter orders enables the equalizer to follow the target response correctly at the crossover point.

5.5 31 band one-third octave equalizer

Another common equalizer design is an equalizer, in which frequency bands are divided logarithmically one-third octave apart. The number of the bands is usually from 27 to 31. In this thesis 31 bands are used. The frequency bands are divided logarithmically from 20 Hz to 20 kHz. The exact frequencies of the bands are listed in Appendix A. As mentioned earlier, the implementation of the 31 band equalizer using a single FIR filter requires very high filter order of $N = 19000$. A far more efficient equalizer can be designed by extending the design of the octave equalizer to a one-third octave version.

The design of the cascaded structure begins with the definition of the crossover frequency. As the number of the bands is increased, the increases to the frequency resolutions of the filters are required. The required order for the FIR filter depends on the bandwidth of the lowest band in its range. Again the required resolution of the equalizer is defined as half of the bandwidth. The required order of FIR filter for
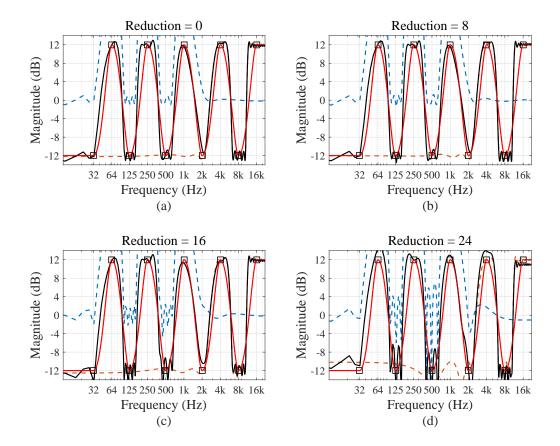
Figure 39: The effect of the minimum-phase conversion and the reduction of the filter order with different reduction amounts. The reduction by 8 produces still acceptable results.

a band can be calculated as

$$N = \frac{f_\mathrm{s}}{\frac{1}{2}(f_\mathrm{u} - f_\mathrm{l})}. \tag{41}$$

where f_u is the upper limit of the frequency band and f_l the lower limit. The minimum filter order of the FIR filter per band is shown in Figure 40.

With logarithmically spaced frequency bands, the filter order of the FIR increases logarithmically, when an additional lower frequency band is added to its range. If the crossover point is at the 1 kHz (band 18), the required filter order is 381 and if it is at the 2 kHz (band 21), the required order is 191. The required order of the warped filter is determined with the Q value charts. For the one-third octave bands $Q \approx 4.3$ and for half of the band $Q \approx 8.6$. The Q values of the warped filter with different filter orders are show in Figure 41.

From Figure 41 it can be seen that if the crossover point is set to the 2 kHz band, the prototype filter order of 250 is required. With the crossover point set to the 1 kHz and the warping parameter is $\lambda = 0.98$, the order of 200 is required. The
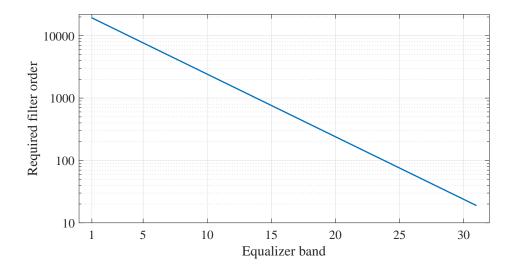
Figure 40: The required FIR filter order as a function of the equalizer bands. The required resolution is defined as half of a frequency band.
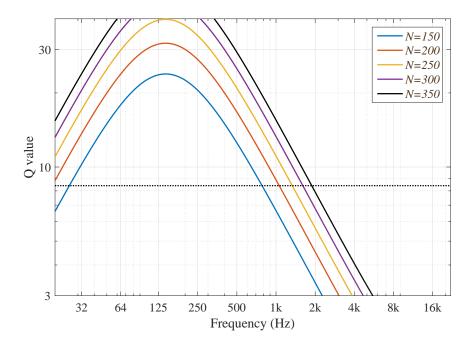
Figure 41: The Q values of a warped filter. The required resolution is $Q = 8.6$. Warping parameter is $\lambda = 0.97$.

computational costs of the cascaded equalizer structure are show in Table 3

No big difference can be seen whether the crossover point is at 1 kHz or at 2 kHz. The computational cost of the cascaded filter structure is considerably lower than with a single FIR equalizer ($N = 19000$).

From Figure 41 it can be seen that the warped filter has a much better frequency resolution at the frequencies 125 Hz - 500 Hz than at its lower and upper frequencies.

Table 3: The required orders of the cascaded structure for the one-third octave equalizer.

f_c	WFIR	FIR	Total
1 kHz	600 $(200 \cdot 3)$	381	981
2 kHz	750 $(250 \cdot 3)$	191	941

This high resolution is unnecessary for the equalization purposes. To smooth the resolution equally to all frequencies and to lower the order of the filters, the equalizer is divided further into three filters. The frequency range of the warped filter is divided into two ranges, which are filtered with an individual warped filter. All three filters are placed in a cascade structure. The structure of the three filter equalizer is shown in Figure 42.
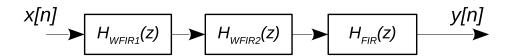

Figure 42: The structure of the three filter equalizer. $H_{\text{WFIR1}}(z)$ and $H_{\text{WFIR2}}(z)$ are warped filters and $H_{\text{FIR}}(z)$ is an FIR filter.

The filter $H_{\text{WFIR1}}(z)$ is a warped filter that filters low frequencies, the warped filter $H_{\text{WFIR2}}(z)$ filters the middle frequencies and $H_{\text{FIR}}(z)$ is a standard FIR filter for high frequencies. The order of the FIR filter is determined in the same way as in the two filter structure. Lower order warped filters are sufficient, as the warped filter is divided to two filters. In Figure 43 the Q values of two warped filters with different warping parameter are presented.

In this case the crossover point of the warped filters is between bands 11 and 12 or 200 Hz and 250 Hz bands. The second crossover point is moved further to higher frequencies between bands 22 and 23 or 2500 Hz and 3150 Hz bands. As the second crossover point is moved, the order of the FIR filter can be decreased to 120. With these approximations the total computational costs are estimated in Table 4. It can be seen that the three filter design can be more efficient than the two filter version.

Table 4: The required orders of the cascaded structure for the one-third octave equalizer.

WFIR1	WFIR2	FIR	Total
315 $(105 \cdot 3)$	315 $(105 \cdot 3)$	120	750

For the crossover point compensations, the same solution, as in the two filter structure, is used. The outermost bands of the second warped filter are defined
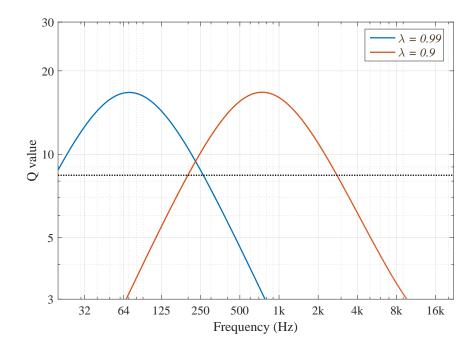
Figure 43: The Q values of two warped filters. The order of the filters is $N = 105$.

as shelving filters and the first warped filter and the FIR filter are designed to compensate. By making the middle filter the shelving filter, the outermost filters must only compensate the gain of one filter.

As in the case of the two filter structure, the filter orders must be increased from the approximations and the frequencies of the bands near the crossover points tweaked. To meet the equalizer requirements, the orders of the warped filters must be increased to 144 and the order of the FIR filter to 132. The orders of the filters can be reduced by converting filters to minimum-phase filters in the same way as in the two filter structure. The orders of the warped filter can be reduced by 35 to 109 and the order of the FIR by 25 to 107. The frequency response of the resulting three filter equalizer is shown in Figure 44.

5.6   Computational complexity

In order to minimize the computational complexity of the equalizer, the filter orders of the warped filters should be minimized, as they are computationally more expensive. However, as shown earlier the order of the high frequency FIR filter raises logarithmically, as additional low frequency bands are included in its range. This is why the crossover point must be chosen to minimize the total cost of the equalizer.

As shown in Section 4.2 the structure of a warped filter is more complex than the structure of an ordinary FIR filter. In this thesis the computational increase of the warped filter is defined as threefold. In this case the total computational costs of the cascaded equalizer structure of a warped filter and an FIR filter is
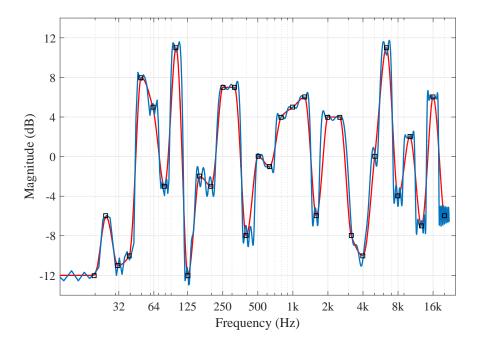
Figure 44: An example of the magnitude response of the one-third octave equalizer.

$$N_\mathrm{t} = 3N_\mathrm{WFIR} + N_\mathrm{FIR}, \tag{42}$$

where N_WFIR is the order of the prototype of the warped filter and N_FIR the order of the FIR filter. In the case of the three filter structure the total complexity is

$$N_\mathrm{t} = 3N_\mathrm{WFIR1} + 3N_\mathrm{WFIR2} + N_\mathrm{FIR}, \tag{43}$$

where N_WFIR1 and N_WFIR2 are the orders of the prototype filters of the warped filters. The minimum-phase conversion of the filters reduces the computational costs by reducing the filter orders

$$N_\mathrm{tm} = 3(N_\mathrm{WFIR1} - M_1) + 3(N_\mathrm{WFIR2} - M_2) + (N_\mathrm{FIR} - M_3), \tag{44}$$

where the $M_1 \ldots M_3$ are the number of the coefficients reduced from each filter.

6   Results

In this section the proposed design is evaluated using the previously defined specifications. The accuracy and the complexity of the design are the main evaluation factors. Later in this section the proposed design is compared to the other equalizer designs. Listening tests of the equalizers were not in the scope of this thesis.

6.1   The accuracy of the design

The specifications for a graphic equalizer were defined earlier in Section 2.2. The accuracy of the frequency response of the equalizer was defined to be ± 1 dB from the target response. As the target response can be defined in several different ways, the accuracy of the equalizer is evaluated as how well the command gains are reached. A larger error in the response is allowed at the transition frequencies between the bands. The accuracy of the octave equalizer is visualized in Figure 45.
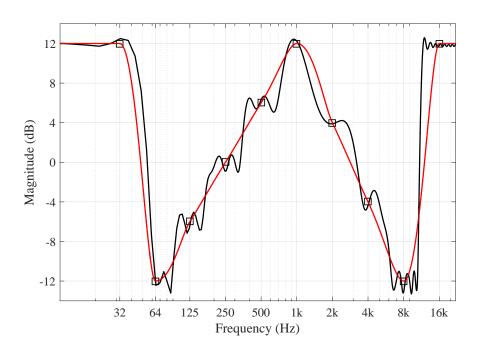
Figure 45: The accuracy of the frequency response of the octave equalizer design. The command gains are reached reasonably well.

The command gains are reached within the ± 1 dB limit. The error compared to the target response is larger at the transition frequencies. The FIR design algorithm (CLS) used in this design causes steep transition curves, as the frequency resolution increases. This is illustrated for example in Figure 45 at the frequencies between the 8 kHz and the 10 kHz bands. Also the smooth target curve is not reached well in the step-like gain settings (64 Hz to 1 kHz). This box shaped response of the equalizer is pronounced, when the orders of the filters are increased. This implies that the

increase of the filter orders do not increase the accuracy of the equalizer at all gain settings.

The equalizer was also implemented as a comparison with least-squares (LS) design algorithm [19]. The main difficulty of the equalizer design with other algorithms is that the magnitude ripples cannot be controlled. LS algorithm designs filters in the linear magnitude scale. As equalizers are designed on a logarithmic one, the linear scale ripple on the negative gain values is larger than on the positive values. This easily causes uncontrollable ripples with large negative command gains. The order of the filters must be increased, if the LS algorithm is used and similar accuracy than with the CLS algorithm is desired.

However, with LS algorithm the response curve can be smoother with less step-like behavior. Figure 46 shows the response curve of the warped equalizer designed using LS algorithm. The same gain settings and filter orders as with the CLS designed equalizer in Figure 45 are used. The LS designed equalizer has larger magnitude ripple at large negative gain values. At the 64 Hz band the magnitude error is 4 dB. With different gain setting this error can be even larger.
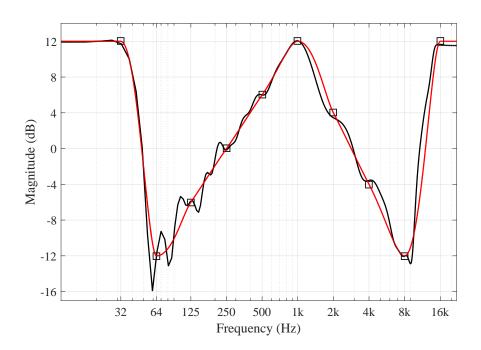
Figure 46: The equalizer designed with the LS algorithm. Large overshoots are present at the negative gain values.

The main difficulty of the cascaded IIR filter equalizers is the interference of the adjacent frequency bands. The simple second order peaking filters cause gain build-up to the equalizers response. This was illustrated in Figure 16 in Section 3.2.1. The design presented here does not have this problem, as several bands of the equalizer are filtered with one filter. This is illustrated in Figure 47, where the same values of the adjacent equalizer bands do not interfere other bands. The crossover point compensation used in this thesis removes the interference of the cascaded filters.
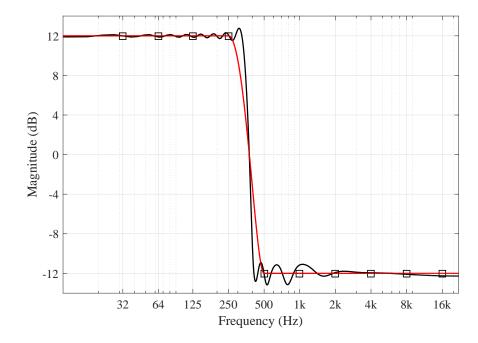
Figure 47: The frequency response of the equalizer shows that no gain buildup is present as the adjacent frequency bands have the same gain value.

The phase response of the warped equalizer structure is nonlinear, as the warping compresses the phase response according to the warping parameter. The prototype of the warped filter and the high frequency FIR filter are designed as linear phase filters. If the minimum-phase conversion is not made, the phase response of the warped filter has little phase distortion and the FIR filter is linear. As the warping parameter is constant in this design, the phase response of the equalizer does not change, when the command gains are altered. However, the all-pass filters increases the group delay at the low frequencies. When those filters are in the cascade structure, the group delay cumulates. This can cause audible phase errors at low frequencies. This nicely behaving phase response is lost, when the equalizer is converted to minimum-phase. However, the group delay at low frequencies is reduced.

The accuracy of the one-third octave version is similar to the octave version. The command gains are reached with required precision, but again the box shaped response causes errors in the transition frequencies. The accuracy of the one-third octave equalizer is visualized in Figure 48.

6.2 Computational complexity of the equalizer design

The computational complexity of a digital equalizer should be as low as possible. The latency and the computational requirements increase, as the orders of the filters are increased. To function properly, the equalizer must perform two different tasks. The first task is to calculate the filter coefficients according to the command gains. These calculations must be only done when the command gains are altered. The
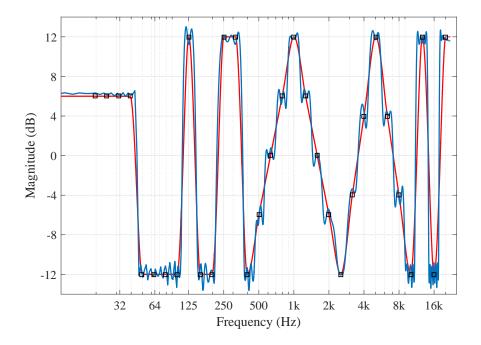
Figure 48: The accuracy of the one-third octave equalizer.

second task of an equalizer is to do the actual filtering. This task is done for the every sample of the digital input signal.

The calculation of the filter coefficients is an expensive task in this design. The filters are used to equalize several frequency bands, so the response curves of the filters are complex. The calculations of the filter coefficients in these implementations cannot be done in real time in Matlab. The implementations could be optimized more, but still the real time command gain changes would not be possible. However, graphic equalizers are usually used in conditions, where the rapid changes of the command gains are not necessary.

The actual filtering task is more essential in the terms of the real time operation. The required filter orders for the designs presented here are low enough to be used in real time applications. The total orders of the filters are listed in Table 5. The calculation cost of the warped filter is defined as threefold to the prototype filter. Based on the optimized structure of the warped filter [44], the number of the operations are two multiplications and three additions per filter order. For the FIR filter the required operations are one multiplication and one addition. The required operations per input sample are shown in Table 6.

Equalizers in this thesis are implemented in Matlab with high computational precision. In the real world applications the accuracy may be lower. Warped filters are relative insensitive to the coefficient quantization [45] and because the filter orders are relatively small, especially in the case of the octave equalizer, these designs should be easy to implement with a limited numerical accuracy.

Table 5: The required filter order of the equalizers.

	Order	Mp reduction	Mp order	Cost
WFIR	52	8	44	132 $(3 \cdot 44)$
FIR	62	10	52	52
Total				184
WFIR1	144	35	109	327 $(3 \cdot 109)$
WFIR2	144	35	109	327 $(3 \cdot 109)$
FIR	132	25	107	107
Total				761

Table 6: The required operations per input sample for the equalizers.

Operation	WFIR	FIR		Total
ADD	132 $(3 \cdot 44)$	52		184
MUL	88 $(2 \cdot 44)$	52		140
Total	220	104		324

Operation	WFIR1	WFIR2	FIR	Total
ADD	327 $(3 \cdot 109)$	327 $(3 \cdot 109)$	107	761
MUL	218 $(2 \cdot 109)$	218 $(2 \cdot 109)$	107	543
Total	545	545	214	1304

6.3 Comparison to previous designs

In this section the equalizer design presented here is compared to other designs. As stated earlier the warped design reduces the filter orders considerably, when compared to a single filter FIR design. The one-third octave equalizer using a large FIR filter is not a sensible design for the real time applications. With warped filters this equalizer is possible to implement.

Several other equalizer designs were chosen for the comparison. These equalizers present a wide range of complexity and accuracy. Equalizers used in the comparison are a cascade structure of second order Regalia-Mitra (RM) filters, a cascade structure of the fourth-order filters [34] and a high-precision parallel equalizer [16]. The frequency responses of these equalizers are from article by Rämö et al. [16]. The RM filter design is a good example of the simple cascaded filter structure. It has no interfering band compensations. The fourth-order filter equalizer is an example of a design that uses steep transition bands to reduce the gain buildup between the bands. The high-precision parallel equalizer is an example of an accurate equalizer design.

These designs compared here are the 31 band one-third octave implementations with frequency bands from 20 Hz to 20 kHz. The warped equalizer used here uses two WFIR and one FIR filters in a cascade structure. The frequency responses of the equalizers are compared in Figures 49–51.
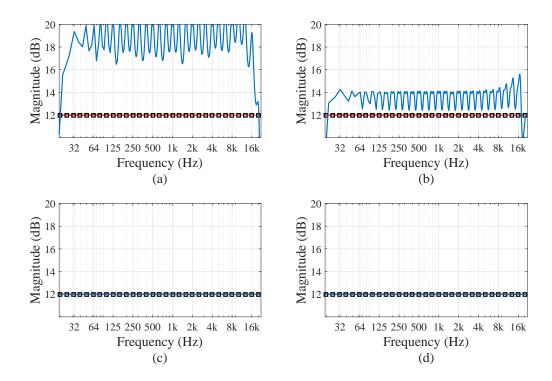

Figure 49: The frequency responses of the different equalizers when the command gains are set to +12 dB. (a) The response of the RM-equalizer, (b) the fourth-order equalizer, (c) the high-precision parallel equalizer and (d) the cascaded warped equalizer.

In Figure 49 the constant gain setting shows the disadvantage of the cascaded structure of IIR filters. The responses of the individual filters spread to the adjacent bands, which causes gain build-up. Both of the cascaded IIR equalizers have a significant error in the response. The high-precision and the warped equalizers have no error in this setting. The warped equalizer processes several frequency bands with one filter and when gains have the same value, the gain compensation used reduces the filters as unity filters. In Figure 50 the frequency resolutions of the equalizers are compared. The cascaded IIR equalizers fail to reach the command gains, as the gains of the adjacent bands reduces the peaks. The high-precision and the warped equalizers reach the command gains, although the warped equalizer has more magnitude ripple. In the last comparison (Figure 51) the equalizers ability to reach zero gain after a high peak is compared. The RM equalizer fails to reach zero gain after a peak. Other equalizers reach the target response well, although the warped equalizer has more ripple than the other two.

Another aspect of the equalizer comparison is the computational complexity. The
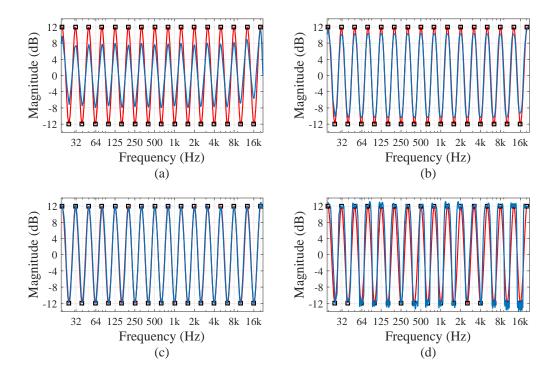
Figure 50: The frequency responses of the different equalizers when the command gains are set to alternating +12 dB and -12 dB. (a) The response of the RM-equalizer, (b) the fourth-order equalizer, (c) the high-precision parallel equalizer and (d) the cascaded warped equalizer.

computational requirements for the command gain changes could not be measured, because only the measurement data of the comparison equalizers were used. However, it is safe to say that the RM and the fourth-order equalizers are computationally lightweight considering the altering of the command gains. The high precision and the warped equalizer require complex calculations, when the gains are changed. The computational requirements for the filtering process are summarized in Table 7.

Table 7: The required operations per sample for the equalizers.

Operation	RM	4th order	High-precision	Warped
ADD	217	434	248	761
MUL	186	465	249	543
Total	403	899	497	1304

It is clear that the warped equalizer is more expensive in the computational costs than the other equalizers. However, as seen from Figures 49–51, the accuracies of the RM and the fourth-order equalizers are far from perfect. The warped equalizer is designed to be accurate within ±1 dB from the command gains. If the requirements
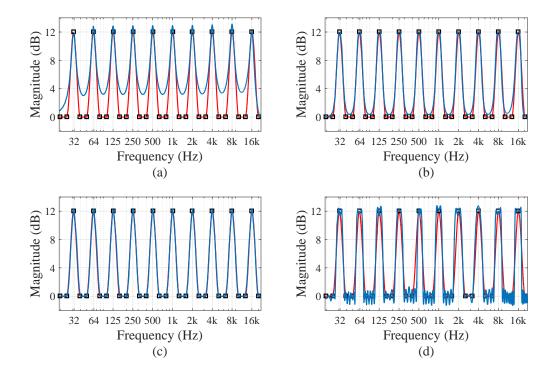
Figure 51: The frequency responses of the different equalizers when every third command gain is set to +12 dB. (a) The response of the RM-equalizer, (b) the fourth-order equalizer, (c) the high-precision parallel equalizer and (d) the cascaded warped equalizer.

of the accuracy are lowered, the warped equalizer can be implemented with lower filter orders. For example the frequency responses of the warped filter with filter orders reduced to half is shown in Figure 52. The frequency resolution is not enough for the crossover points (Figure 52a). On other frequency bands of the warped equalizer reaches command gains with the required accuracy. With simpler equalizer settings (Figure 52b), the warped equalizer can reach the target accurately.

The difference between the accuracies of the IIR and the reduced order warped FIR equalizers is that the responses of the IIR equalizers, although inaccurate, are predictable. It is easy to estimate the gain interference between the frequency bands. The accuracy of the warped FIR equalizer is not so easy to estimate, as several frequency bands are processed with one filter. The crossover points are the weak points, because the resolutions of the filters are the lowest at those frequencies. The accuracy of the equalizer at other frequencies is hard to determine, as the command gain settings can vary. Low order filters can cause larger overshoots in the frequency ripple, which can be hard to anticipate. This complicates the estimation of the minimum orders of the filters.
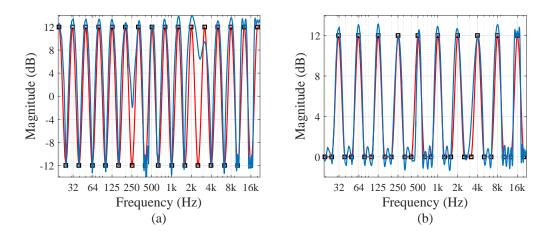
Figure 52: The frequency responses of the warped equalizer with reduced filter orders. The order of each filter is halved. (a) The command gains are set to alternating +12 dB and -12 dB. (b) Every third command gain is set to +12 dB.

7 Conclusions

The goal of this thesis work was to design a graphic equalizer with warped FIR filter that is comparable to other equalizer designs. An equalizer implemented with a single warped FIR filter was shown to be inefficient, so the cascade structure of two filters was presented.

The design of a graphic equalizer presented in this thesis uses warped and standard FIR filters. A design for both an octave and a one-third octave equalizers was presented. The equalizers use warped FIR filters for the low frequency equalization and a standard unwarped FIR filter for high frequencies. As shown in this thesis, warped filters can have a better frequency resolution at low frequencies than standard FIR filters. Thus the computational costs of the equalizer can be considerably lower than the costs of an equalizer implemented with a single FIR filter.

The key point of the equalizer design is how target frequency response is reached. The analog and the simplest digital equalizers fail to reach the target response accurately. The presented warped equalizer is designed to be accurate, but the computational costs of the implementation are higher than the costs of the conventional designs. If the design specifications for the accuracy are lowered, the computational costs of the warped equalizers can be decreased.

With the lowered accuracy requirements the warped equalizer design can be comparable to the previous designs in the terms of the computational costs. It can be implemented in the real time applications. One drawback of this design is the unpredictability of the response curve with lower filter orders, as large ripple peaks can appear in the response curve. Another drawback compared to IIR designs is the computational complexity of the filter redesign, when the parameters of the equalizer are changed. While conventional IIR designs can be operated in real time, the redesign of the warped equalizer cannot be done in real time at least in Matlab. However, more optimized redesign implementation would reduce the latency in the operation.

Several aspects of this design could be addressed for the future research. First the design presented here follows very accurate specifications. For practical applications specifications this strict can be unnecessary. As the complexity of the design can be decreased with lower requirements, the optimal design for the real life applications could use smaller filter orders.

The second aspect for future work is further optimization of the filter design procedure. This implementation uses constrained least square FIR design algorithm. The CLS algorithm produces steep transition bands, which are not optimal for the accuracy of the equalizer. Higher filter orders produce steeper transitions, so the accuracy of the design cannot be increased with larger filter orders. With least-squares design algorithm the target response can be reached more accurately at the transition bands. However, in this thesis least-squares design required larger filter orders than the CLS algorithm to have a similar accuracy. With more optimized least-squares design more accurate warped equalizer might be possible.

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A   ISO 266 frequency bands

Table A1: The center frequencies and the bandwidths of the octave and the one-third octave bands according the ISO 266 standard [12].

Octave bands	Center frequency	Bandwidth (Hz)	1/3 octave bands	Center frequency	Bandwidth (Hz)
			1	20	4.6
			2	25	5.8
1	31.5	22	3	31.5	7.3
			4	40	9.2
			5	50	11.5
2	63	44	6	63	14.6
			7	80	18.3
			8	100	22.9
3	125	89	9	125	29
			10	160	37
			11	200	46
4	250	178	12	250	58
			13	315	73
			14	400	92
5	500	355	15	500	115
			16	630	146
			17	800	183
6	1000	710	18	1000	231
			19	1250	291
			20	1600	365
7	2000	1420	21	2000	461
			22	2500	579
			23	3150	730
8	4000	2840	24	4000	919
			25	5000	1156
			26	6300	1456
9	8000	5680	27	8000	1834
			28	10000	2307
			29	12500	2910
10	16000	11360	30	16000	3650
			31	20000	4610

B Matlab code

The Matlab code used to implement the warped ten band octave equalizer.

```
% warpedeq10.m - 10 band graphic equalizer using FIR and WFIR filters
%
% Usage: y = warpedeq10(x,eqVals,Fs)
%
% x = input signal
% y = output signal
% eqVals = gain values of the equalizer bands in dB
% Fs = sampling rate (Note only 44,1 is tested properly
%

function y_out = warpedeq10(x,eqVals,Fs)
    a = 0.97;   % value of the warping parameter
    r = 1;      % ripple at the equalizer bands -r...+r
    n1 = 52;    % order of the WFIR prototype
    n2 = 62;    % order of the FIR filter
    rr1 = 8;    % coefficient reduced from WFIR by mp conversion
    rr2 = 10;   % coefficient reduced from FIR by mp conversion

    %Frequency vectors for filters
    [f1,f2] = calFreqPoints(a,Fs);

    %Gain values for the warped filter
    [amp,up,lo] = gains([eqVals(1:6)-eqVals(7) 0],r);
    %Adjusted ripple for the pass band > 2kHz
    [amp(length(amp)),up(length(up)),lo(length(lo))]=gains(0,0.2);
    %Coefficients of the prototype filter
    b = fircls(n1,f1,amp,up,lo);

    %Gain values for fir filter;
    [amp2,up2,lo2] = gains([0 eqVals(7:10)],r);
    %Adjusted ripple for the pass band < 2kHz
    [amp2(1),up2(1),lo2(1)]=gains(eqVals(7),0.2);
    %Coefficients of the fir filter
    b2= fircls(n2,f2,amp2,up2,lo2);

    %Minimum phase conversion of the filters
    [xxx,b1min] = rceps([b zeros(1,1000)]);
    [xxx,b2min] = rceps([b2 zeros(1,1000)]);
    %Reduction of the least significant filter coefficients
    b =  b1min(1:(n1+1)-rr1);
    b2 = b2min(1:(n2+1)-rr2);
```

```
    %Warping
    B=[-a 1];
    A=[1 -a];
    g(1,:) = x;
    for m = 2:length(b);
        g(m,:)=filter(B,A,g(m-1,:));
    end
    y=zeros(1,length(x));
    for m=1:length(b)
        y= b(m)*g(m,:) +y;
    end

    %Filter combination
    y_out = filter(b2,1,y);
end


% Function to calculate frequency vector of filter designs
function [fn1,fn2] = calFreqPoints(a,Fs)
    %Calculate the required frequencies to match the warped
    %prototype to a logarithmic frequency scale

    %Frequency points for warped filter

    %Desired frequencies for warped filter. Frequencies are tweaked
    %at the crossover point. Additional frequency for crossover
    %optimization
    feq = [47 89.1 178 355 768 1250];

    %Required frequencies for the prototype filter
    fn1 = feq+(Fs/pi)*atan2((a(1)*sin(2*pi*feq/Fs)),...
        (1-a(1)*cos(2*pi*feq/Fs)));

    %Frequency points for FIR filter
    feq = [1810 3020 5620 11200];
    fn2 = feq;

    %Normalized frequency vectors
    fn1 = [0 fToNorm(fn1,Fs) 1];
    fn2 = [0 fToNorm(fn2,Fs) 1];
end

% Function to normalize frequencies to 0...1
function r = fToNorm(f,Fs)
```

```
        r=2*f/Fs;
end

% Function calculate linear scale gains and upper and lower gain
% limits of 'fircls' command
function [g,gup,glo] = gains(x,inc)
    l=length(x);
    for n = 1:l
        g(n) = 1*10^(x(n)/20);
        gup(n) = 1*10^((x(n)+inc)/20);
        glo(n) = 1*10^((x(n)-inc)/20);
    end
 end
```