Aalto University
School of Business

Is "low volatility effect" that anomalous?

Finance

Master's thesis

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Abstract

PURPOSE OF STUDY
The empirical relation between expected stock returns and volatility is currently a matter of debate. The purpose of this study is to investigate whether the low volatility effect is present in the alternative data set that excludes penny stocks. I mainly focus on total volatility or the diagonal of the covariance matrix. Idiosyncratic volatility is studied to a lesser extent of testing the efficiency of the Fama French three factor portfolios.

DATA AND METHODOLOGY
My data set includes the annually reconstituted top 1000 stocks by market capitalization tracked by CRSP with at least 24 months of return history from 1983 to 2013. I form equally weighted and market capitalization weighted portfolios by ranking stocks based on 48-month total volatility and 12-month idiosyncratic volatility with respect to the Fama French three factor model into decile portfolios, from bottom (decile 1) to top (decile 10) on a monthly basis. The post-formation decile portfolio returns are controlled for the Fama French three-factor exposure and are measured by various risk and return metrics.

RESULT
I find that the equally weighted top decile portfolio sorted by total volatility statistically outperforms the equally weighted bottom decile portfolio by 1.01% (t-statistic 2.94) in monthly average return. Size and value effect cannot account for the average returns of the decile portfolios. Irrespective of whether volatility adequately captures risk, I find that the top decile is fundamentally riskier than the bottom decile by various measures. Component analysis shows that the top decile is dominated by stocks of firms in computer programing and semiconductor device related industries while the bottom decile is dominated by stocks of firms in electric service related industries. Keeping all else equal, changing from the naïve equal weighting to the market capitalization weighting distorts the positive relation between average returns and volatility. The market capitalization weighted portfolios underperform both the equally weighted counterpart portfolios and the market factor portfolio in monthly average returns. The study shows that the low volatility effect is not present in the research data set and that equal weighting scheme exposes the outperformance of the top decile portfolio. The peculiarity of stocks of firms in the dominating industries in the top decile potentially explains why volatility increases monotonically with average return but not with alpha. In addition, that idiosyncratic volatility predicts returns shows that the Fama French factor model does not adequately capture the systematic factor exposure of the decile portfolios.

Keywords: Total volatility, idiosyncratic volatility, equally weighted, market capitalization weighted

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1. Introduction

1.1. Motivation and research questions

THE RELATION BETWEEN VOLATILITY AND EXPECTED STOCK RETURNS, BOTH TOTAL AND IDIOSYNCRATIC, IS CURRENTLY UNDER DEBATE. Investigating the relation between daily stock returns and 1-month lagged idiosyncratic volatility, Ang, Hodrick, Xing, and Zhang (2006) find that stocks with high idiosyncratic volatility relative to the Fama French three (FF-3) factor model have abysmally low average returns. The quintile with the lowest idiosyncratic volatility stocks outperforms the quintile with the highest idiosyncratic volatility stocks by 1.06% (t-statistic -3.10) in monthly average return. Their data sample is consisted of all stocks listed in AMEX, NASDAQ, and the NYSE from January 1986 to December 2000. Recently, the topic has been rekindled by Blitz and van Vliet (2007), Baker, Bradley, and Wurgler (2011), and Frazzini and Pedersen (2014). Blitz and van Vliet (2007) have a comprehensive analysis of the performance persistence of low total volatility stocks from 1985 to 2006 for all constituents of FTSE World Developed Index. Frazzini et al. (2014) find that betting-against-beta factor, which is longing low beta assets and shorting high-beta assets, produces significant positive risk-adjusted return. The limits of arbitrage and bias preference for high volatility and beta assets have been the main explanations for the anomaly. Baker et al. (2011) highlight a two-fold increase in institutional holding from 1968 to 2008. The authors attribute the lack of arbitrage activities to the hesitance of institutional investors, who are under tracking error and leverage constraints, to invest in high volatility stocks. The negative relation between volatility and expected returns as a stark contrast to the prediction of the traditional asset pricing has therefore attracted significant attention in academia.

Just as interesting as the effect that is believed to be the biggest anomaly in the financial history by Baker et al. (2011) is the emerging empirical evidence of the positive relation between various risk measures and expected stock returns. Using CRSP data from 1980 to 2004, Basu and Martellini (2007) find that the quintile with the highest total volatility stocks outperforms the quintile with the lowest total volatility stocks by 0.74% (p-value 1.37%) in monthly average return. As a response, Blitz and van Vliet (2011) challenge the result of Basu and Martellini (2007) on ground of using survivor stocks in the sample. Therefore, overcoming this survivorship bias would be a solid contribution to prior research. Fu (2009) challenges the robustness of the short estimation window used by Ang et al. (2006) by arguing that idiosyncratic volatility is time varying and therefore the lagged one month idiosyncratic volatility is not a good proxy for the current month expected risk. In the robustness section, Ang et al. (2006), indeed, note that the increase in the length of the pre-defined beta estimation time window leads to the reduction in the outperformance of the bottom quintile portfolio with the lowest beta stocks against the top quintile portfolio with the highest beta stocks. The mentioned beta is the proxy for daily innovation in aggregate volatility, measured as the sensitivity of asset returns to daily changes in VIX. In response to Ang et al. (2006), Fu (2009) reports a positive relation between risk and expected return, using the EGARCH model to estimate idiosyncratic volatility rather than historical measure. Blitz and van Vliet (2011) also confirm that methodological choices can lead to less robust results for the low volatility anomaly. Most recently, Li, Sullivan, and García-Feijóo (2014) show that dropping penny stocks from their research data sample leads to the non-statistically significant return of the zero cost equally weighted portfolio that is formed by longing stocks with low FF-3 idiosyncratic volatility and shorting stocks with high FF-3 idiosyncratic volatility. Their data sample is consisted of CRSP stocks from 1963 to 2010. Without having dropping these lowly priced stocks, Li et al. (2014) would have reached the consistent conclusion with Ang et al. (2006) in favor of the low volatility effect. In a nutshell, the data sample and estimation window significantly drive differences in research results. [1]

Overall, the empirical evidence of the low volatility effect serves the considerable academic interest in light of the ongoing debate between modern finance and behavioral finance. Eugene Fama and Robert Shiller, the respective key architect of the efficient market hypothesis (EMH) and behavioral finance, along with Lars Peter Hansen, were awarded the Nobel Prize in Economics in 2013. Under EMH, arbitrage opportunities will be quickly exploited to bring asset prices back to fundamentals. Behavioral finance proponents attribute to the limits of arbitrage as the reason for the considerably long persistence of anomalies, referring to the blind spot of EMH. The limits of arbitrage could arise due to different forms of frictions. Some of these common frictions come from (i) noise traders (see, e.g., De long, Shleifer, and Summers, 1990; Barber, Odean, and Zhu, 2009) (ii) limited capital (see, e.g., Shleifer and Vishny, 1997) (iii) short selling constraints (see, e.g., Mitchell, Pulvino, and Stafford, 2002; Barberis and Thaler, 2003) (iv) slow moving capital (v) arbitrageurs' specialization (see, e.g., Shleifer and Vishny, 1997). In other words, even though prices could deviate from fundamentals, there is

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[1] In the industry, unconstrained low volatility strategy is reported to deliver -1.1% in alpha per annum with respect to the Fama-French three factor model over the most updated data from 1986 to 2013 (Robecco, 2015). Low volatility asset ranks the worst among the researched asset classes in 2013 (State Street Advisors, 2013).

not necessarily abnormal return due to different constraints that discourage arbitrageurs from capitalizing on even known mispricing.

The purpose of this paper is to investigate whether the reported low volatility effect is present in an alternative data set that excludes penny stocks. I mainly focus on total volatility which has been studied to a lesser extent than idiosyncratic volatility or aggregate volatility (see, e.g., Campbell and Hentschel, 1992; Glosten, Jagannathan, and Runkle, 1993). I will focus on 12 month idiosyncratic volatility with respect to the FF-3 model only to the extent of testing the efficiency of the FF-3 portfolios. The choice of idiosyncratic volatility with respect to the FF-3 model rather than, e.g., CAPM, is due the wider application of the FF-3 model in empirical finance. Importantly, the FF-3 model has been used by prior proponents of the low volatility effect (see, e.g., Blitz and van Vliet, 2007). Factor models generally advocate the existing systematic factors or any systematic factors orthogonal to those current ones should be priced. Many factor models, e.g., CAPM do not price idiosyncratic volatility as the variance of residuals can be eliminated by fair return risk trading. However, the empirical evidence for CAPM has been, if anything, weakened over the year (see, e.g., Gruber and Ross, 1978; Jobson and Forkie, 1982; Kandel and Stambaugh, 1987; Gibbons, Ross and Shanken, 1989; MacKinlay and Richardson, 1991). There has also been increasing evidence showing the explanatory power of the variance of the residuals in the cross section of expected returns (see, e.g., Lehmann, 1990; Goyal and Santa-Clara, 2003). Volatility and risk are not used interchangeably in my thesis, but not necessarily so in the literature review section. All the comparison, otherwise stated, refers to the comparison between the top volatility decile portfolio, decile 10 and the bottom volatility decile portfolio, decile 1. In this thesis, I answer the two main research questions as below.

Research question 1: Is there outperformance persistence of stocks with high total volatility and idiosyncratic volatility?

Following Fama and French (1992) to validate the value effect by sorting stocks based on firm size and book-to-market ratio into decile portfolios, I sort stocks into decile portfolios by volatility and control for the FF-3 factor exposure in decile portfolios. I also analyze decile portfolios' industry components that could potentially characterize the portfolio performance. By forming equally weighted portfolios sorted by volatility on a monthly basis following prior research (see, e.g., Blitz and van Vliet, 2007), I inadvertently harvest equal weighting and possibly rebalancing premium that lead to the relatively high level of the monthly average returns across the decile portfolios. The equal weighting scheme utilizes the size effect by giving more weight to small capitalization stocks in the absence of optimizing any objective function. DeMiguel, Garlappi, and Uppal (2009) report that the 1/N portfolios produce Sharpe ratios that are 50% higher on average than the mean–variance-optimized portfolios. On the other hand, portfolios weighted by market capitalization have been noted to underperform portfolios weighted by alternative weighting schemes, e.g., fundamental weighting (see, e.g., Arnott, Jason, and Philip Moore, 2005). Therefore, I also switch from the equal weighting to the market capitalization weighting to quantify the marginal weighting effect. An alternative rationality to understand the low volatility effect through weighting scheme is that market capitalization weighting is more likely to give more weight to low volatility stocks, which are more likely to be big companies. Ang et al. (2006) also report that the quintile portfolio with the highest idiosyncratic volatility stocks accounts for only a small proportion of the market value of the research data (only 1.9% on average). If market capitalization weighted portfolios underperform equally weighted portfolios, this evidence serves as an argument against the outperformance of low volatility stocks against high volatility stocks.

Research question 2: Is the portfolio with high total and idiosyncratic volatility stocks respectively riskier than the portfolio with low total and idiosyncratic volatility stocks by various measures?

A relevant question to ask is whether the portfolio with the highest volatility stocks is riskier than the portfolio with the lowest volatility stocks. If returns were to follow normal distribution, which is completely characterized by the first two orders of the distribution and the covariance matrix, volatility would be an adequate measure of risk. However, extreme ex-post events occur with a much higher probability than theoretically expected in the financial market, which has fat tail risk and whose return is well known to defy normal distribution. Fat tail risk is an ex-ante risk measure, calculated as the probability mass equal to an integral from minus infinity to a certain limit under a correct (often fat tail) distributional assumption. As a result, volatility based risk measure such as Sharpe ratio or the first two moments of the distribution may not capture all the risks in portfolios (see, e.g., Mandelbrot and Taylor, 1967; Fung and Hsieh, 1999; Francois and Bruno, 2001). Return distribution, especially skewness has also assumed the explanatory role for the preference for high volatility stocks which is more of a preference for skewness than for volatility itself (see, e.g., Baker et al., 2011; Kumar 2009; Barberis and Huang, 2008). For example, Barberis and Huang (2008) also model this preference for the lottery-like stocks with the cumulative prospect theory of Tversky and Kahneman (1992) such that investors at equilibrium overweigh the tail and overprice stocks with positive skewness, leading to negative excess returns. Just as the notion of risk varies, so does the risk measure. For example, in order for the high volatility portfolio to be riskier than other portfolios, it is expected to underperform especially in meltdown market conditions when the marginal utility of wealth is high. Even though an asset is less risky ex-ante, an ex-post huge loss of low probability could already lead to capital redemption, creating the limit of arbitrage for investment funds which are under mandate to benchmark against, e.g., major indices. Therefore, understanding the higher order of the distribution of returns and investigating the performance of the high (low) volatility portfolio, especially in relation to benchmark and in the drawdowns as the ex-post realization of the ex-ante tail risk, will therefore also shed light on the riskiness of decile portfolios.

1.2. Contribution to existing literature

My research contributes to the emerging study with empirical evidence invalidating the controversial low volatility effect, for which there is a shortage of explanations in academia (see, e.g., Baker et al., 2011). To resolve this contemporary puzzling relation between expected stock returns and volatility, I overcome prior data bias, study the weighting scheme effect, quantify portfolio components, and measure the post-formation performance of the volatility sorted deciles with benchmark-based metrics.

The choice of the data sample has a critical role in the low volatility effect. Overcoming the survivorship bias in the research of Basu et al. (2007), I have the stock universe reconstituted annually and portfolio rebalanced monthly, with both the survivor and the non-survivor stocks that have at least two year historical data. Not using index components also removes the dependence on the discretionary stock choice method of index providers as well as the chance of overestimating returns by measuring only stocks that survive in the index. The overestimation due to studying only stocks that survive in the index can be as high as 2% (Graham, B., 1949). Prior research that reports the low volatility effect in international markets focuses mainly on the stock index constituents. For example, Blitz and van Vliet (2007) have their research on the FTSE World Developed index constituents and do not report the local risk-free rates in use. As discussed briefly in Section 1.1., Li et al. (2014) report that the low volatility effect becomes non-statistically significant after dropping penny stocks from the research data. Abnormal return from the low volatility effect are mainly concentrated among penny stocks, which generally do not reflect available information as efficiently as the more actively traded ones and are more difficult to be traded in any meaningful volumes. In my data set, lowly priced stocks (less than $5) only constitute less than 0.21% of the dataset on annual average, in line with Li et al. (2014) as an independent research at the time of writing my thesis. As a robustness test, I replicate the research with the same data universe and method by Baker et al. (2011) and find results in line with the authors. However, changing the data universe and time period leads to the deviation in my research.

In terms of the historical time window and data frequency, I require that each stock has at least 24 month historical data available, following Baker et al. (2011). My goal is to estimate 48-month total volatility and 12-month idiosyncratic volatility. Blitz and van Vliet (2007) and Baker et al. (2011) estimate total volatility with 36-month and 60-month historical data respectively. As mentioned, Fu (2009) argues that lagged one month idiosyncratic volatility estimated with daily data used by Ang et al. (2006) is not a good proxy for the current month expected risk.

My methodological contribution centers on the weighting scheme and post-formation measurement. Prior published research focuses on either the value weighting scheme or the equal weighting scheme and therefore has not documented the weighting effect on volatility sorting strategy. Aiming to explain the effect from a novel angle of industry concentration, I also analyze the industry components of decile portfolios. The probability of the stocks of different industries falling into certain decile is studied through the constructed f value.

1.3. Limitations of the study

There could be a blend of various effects at the portfolio level leading to the bias of average returns estimation ostensibly attributed to volatility. On one hand, the forming and holding of portfolios might embed premium from the rebalancing on top of the equal weighting premium. As a result, the extra premiums lead to the relatively high level of the average returns of decile portfolios. On the other hand, the reconstitution drag could lead to a downward estimation of returns. By reconstituting the 1000 largest stocks by market capitalization every year, I essentially drop stocks that fail to keep up with the other stocks in terms of market capitalization. Specifically, the reconstitution is achieved by buying a stock at a high price when it reaches the 1000 largest cap threshold and selling the stock when it reduces in capitalization. The market capitalization change is very often due to the change in the share price. The reconstitution process can require buying high and selling low, creating the drag on the performance of the decile portfolio. As above mentioned, the overestimation and underestimation, if any, is not necessarily homogenous among all deciles. Low volatility stocks are usually large cap companies with less risk of dropping out of the stock universe and therefore have less reconstitution drag. My research has not comprehensively separated these various effects.

Besides, my study is subject to the limitations of the FF-3 factor model. For example, (i) Fama et al. (1992) advocate that the size factor and value factor are systematic risk factors and the FF-3 factor model is a parsimonious asset pricing model. This risk-based prediction is that stocks more exposed to the HML factor should be riskier and are expected to earn higher returns. However, Daniel and Titman (1997) sort stocks firstly by firm size and book-to-market ratio and subsequently by the HML beta. The authors find that the stock portfolio with the higher value beta is neither riskier nor has higher returns. Lakonishok et al. (1994) find little support that value stocks underperform glamor stocks in such states as low GDP growth or low market returns of the world. To be fundamentally riskier, value stocks are otherwise expected to underperform in such states when the marginal utility of wealth is high. (ii) There are also potential (non-linear) hidden risk factors other than FF-3 factors. Regardless of these setbacks, FF-3 model still exposes partial excess returns explained by factor and size factor exposure in my study. Another minor setback is that my thesis does not take the full account of the time-varying nature of beta and volatility in the industry component analysis by subsampling the data period. Instead, the occurrence probability of industries in decile portfolios is calculated across the whole time series.[2]

1.4. Main findings

For portfolios sorted by total volatility, I find that the equally weighted top volatility decile portfolio statistically outperforms the equally weighted bottom volatility decile portfolio by

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[2] For example, stocks of information technology and telecommunication service sector had relatively high beta in the global developed market during the peak of the dotcom bubble in 2000. However, as of January 2014, these sectors have among the lowest beta. The beta of the materials sector, however, was nearly twice its level in 2000 (State Street Global Advisor, 2014).

1.01% (t-statistic 2.94) in monthly average return. Monthly average returns increase monotonically with volatility and market beta in the decile portfolios. It is worth noting that the relatively high level of average returns is driven by the equal weighting premium and is not attainable with the market capitalization weighting scheme. Rebalancing premium could also have the compounding effect for portfolios in my research. The FF-3 alphas are stably flat rather than monotonically increase with volatility. Alpha varies by only 10bps between the first nine decile portfolios and significantly picks up for the highest volatility decile portfolio at 1.4% (t-statistic 4.35). Factor exposure explains excess returns the least in the most volatile portfolio. It is worth noticing that coefficients of determination, or R-squared, from the regressions of decile excess returns on FF-3 factor returns are the highest for the mid-range decile portfolios and decrease monotonically to the extreme high and low volatility deciles.

Component analysis shows that high (low) volatility decile is dominated by specific industries, while mid-range volatility deciles have the most equally distributed industry components. Decile 10 is most dominated by stocks of firms in "Services-Computer Programming, Data Processing, Etc.", "Semiconductors & Related Devices" and "Services-Prepackaged Software" industry, which all make up 16.09% of decile 10 components. Decile 1 is also highly concentrated towards stocks of firms in "Electric Services" and "Electric & Other Services Combined" industry with the combined occurrence rate of 36.48%. Decile 9 also has a similar, yet less concentrated, industry component profile to that of decile 10. As a benchmark, stocks of the industry with the highest f value across decile 2 to decile 8, makes up only about 4.43% of decile components on average. One possible explanation in relation to the R-squared is that mid-range deciles, compared with the bottom and top total volatility sorted deciles, are more diversified industry wise and therefore returns of mid-range deciles are better explained by systematic factor exposure. The peculiarity nature of the computer and semi-conductor related industry that dominates decile 10 could well assume the explanatory role for the portfolio's relatively high alpha.

The high volatility decile portfolio is more likely to underperform the market factor during meltdown periods and has more uncertainty below the benchmark threshold. Decile 10 has the biggest maximum drawdown -82.57% during the dotcom bubble and underperforms the FF-3 market factor portfolio in 13 out of the 20 months in which the factor portfolio has the lowest returns. Even one-time loss can already lead to a liquidity squeeze or a huge fund outflow that creates the limit of arbitrage. However, the portfolio of the most volatile stocks is more likely to outperform the market factor in upmarket condition. In terms of return distribution, central moment analysis and Jacque-Bera test statistic show that returns of all decile portfolios defy normal distribution. The first two orders of lower partial moment also show that top decile portfolio has higher expected shortfall below hypothetical target return and sensibly more downside variance. Decile 10 has higher excess kurtosis, indicating tail risk but also more positive skewness below the target return than decile 1. Interestingly, I also find that the low volatility portfolio has a higher Sharpe ratio in line with the proponents of the low volatility anomaly. However, the higher Sharpe ratio of the low volatility decile is partially attributable to the extraordinarily small denominator. Choosing the right benchmark e.g., the expected shortfall below VAR, is outside the scope of the thesis and is the fund specific question that eludes a definite answer.

Switching from the equal weighting scheme to the market capitalization weighting scheme shows extremely interesting results. The market capitalization weighted decile portfolios yield abysmally small average returns and have an average alpha of -0.02% per month at 99% confidence level. The market capitalization weighted decile portfolios underperform the equally weighted counterpart portfolios and the market factor portfolio. There is no monotonic increase in FF-3 alpha nor beta from decile 1 to decile 10 as previously found in the equally weighted decile portfolios. All else equal, the abysmal monthly average returns and the underperformance of market capitalization weighted decile portfolios against the equally weighted counterparts can be explained by the overweight of large market capitalization stocks which on average have lower returns than small market capitalization stocks.

For portfolios sorted by idiosyncratic volatility, I also find a positive relation between average returns and idiosyncratic volatility in line with Fu (2009). The most noteworthy difference in average return 0.81% (t-statistic 2.79) is between decile 10 and decile 1. In addition, portfolios sorted by idiosyncratic volatility bear striking similarity to portfolios sorted by total volatility by various measures, including drawdown and return distribution below the benchmark target return.

1.5. Future research

Given the findings of this paper, it would be highly interesting to study a cost-based trading strategy of longing the 10% most volatile stocks in the presence of the equal weighting, the monthly rebalancing, and the annual reconstitution of the largest 1000 stocks by average yearly market capitalization with a CRSP data sample. Furthermore, it is noted from my current research that volatility increases monotonically with average return but not with alpha. One hypothesis based on the component analysis is that the relatively high alpha of decile 10 might well be characterized by the dominating stocks of the computer programing and semiconductor device related industries. Dropping stocks of firms in these industries completely from the data set is an area for future research.

1.6. Structure of the study

The structure of this paper is as follows. Section 2 gives an overview of the literature on the relation between various proxies of risk and expected returns. In this section, I also review the weighting schemes and benchmark-based risk measurement that are relevant to my research. Section 3 presents the hypothesis. Section 4 clarifies the data and methodology on pre-formation parameter estimation and post-formation portfolio measurement. Section 5 shows the results on the performance of portfolios sorted by volatility. Section 6 explains the robustness tests. Section 7 goes through the discussion on volatility sorting strategy in the current market environment as of the first quarter of 2015. Section 8 highlights the investment implication of the key main findings which answer the research questions.

2. Literature review

2.1. Empirical evidence for the relation between risk and expected returns

This section covers literature of the theoretical risk and return tradeoff as well as the controversial empirical evidence on the negative relation between average returns and various proxies for risk, including total volatility, idiosyncratic volatility, and beta.

2.1.1. On the non-positive relation between risk and expected returns

The evidence of risk return tradeoff in line with the Capital Asset Pricing model has been reported to be weaker over the years. Starting in the 1970s, Black, Jensen, and Scholes (1972) note that relation between risk and return was much flatter than that predicted by the CAPM. Haugen and Heins (1975) find that the relation is inverted. Later on, Reinganum (1981), Lakonishok, and Shapiro (1986) report that the positive relation between beta and average returns disappears when beta is used to explain average returns. In the same school of literature, Longstaff (1989) also find the cross sectional regression coefficient on the total variance for size-sorted portfolios carries the insignificantly negative sign using CRSP monthly stock returns from 1926 to 1985. Fama and French (1992) astoundingly state that betas are "dead".

As discussed, Ang et al. (2006) find that the quintile portfolio with the lowest idiosyncratic volatility stocks outperforms the quintile portfolio with the highest idiosyncratic volatility stocks by 1.06% per month on average returns with listed equity on AMEX, NASDAQ, and NYSE from January 1986 to December 2000. Ang at al., (2006) advocate aggregate stochastic volatility as a negative priced risk factor that is orthogonal to other factors in the cross section of stock returns. The market price of aggregate volatility is reported at -1% per annum. Ang et al. (2006) argue that under such condition, investors demand the stocks that can hedge against market volatility. These stocks usually have higher loadings or sensitivities to market volatility factors. Investors have higher demand for stocks with higher exposure to market volatility, increasing its price and thereby reducing returns. Although Ang et al. (2006) do find that the quintile with the lowest idiosyncratic volatility stocks has higher average returns than the quintile with the highest idiosyncratic volatility stocks, they fail to explain that outperformance by using their model and related arguments. Such model prediction is that higher idiosyncratic volatility stocks would have higher exposure to the market volatility risk factor and therefore would be more desirable as the hedge assets. As a result, they have higher prices or lower expected returns.

Recently, the low volatility effect has been revived with recent discussions by Blitz and van Vliet (2007), Frazzini and Pedersen (2010), and Baker, Bradley, and Wurgler (2011). Blitz and van Vliet (2007) find the performance persistence of low volatility stocks in the data from 1985 to 2006 for all the constituents of FTSE World Developed index. Frazzini and Pedersen (2010) find that going long low beta assets and going short high beta assets produces a significant risk adjusted return in 20 international equity markets, treasury bonds, corporate bonds, and futures. For the equity market, they create the self-financing portfolio to proxy for "betting against beta" factor that is found to produce significant positive risk-adjusted returns. Looking at risk from another measure, Bali, Cakici, and Whitelaw (2009) study a measure of lottery-like return distribution and find a negative relation between the risk measure and returns.

Behavioral finance explanations for the non-positive relation between risk and expected returns

Karceski (2002) explains high beta stocks are more likely to do better in the boom market and therefore are preferred. Recently, Baker et al. (2011) refer to the (i) irrational demand due to high preference for lotteries, representativeness, and overconfidence. As a result, risk loving and irrational investors overpay for high risks (ii) benchmarking by money managers leading to the hesitance to deviate from benchmarks in the short run at the expense of long term returns. Interestingly, Baker et al. (2011) observe an increase in institutional holding from 30% to 60% when the volatility anomaly is reported in the US market from 1968 to 2008. They arguably explain the anomaly not being arbitraged because institutional investors, who have leverage constraints, are refrained from increasing tracking error by investing in low risk stocks. Black (1972) also associates the leverage constraint to the flat beta and return relation.

Other explanations relate the preference for high volatility stocks more to the preference for skewness than to volatility itself. Such preference is attributed to the preference for lotteries such that buying high volatility stocks that have positive skewness enables investors to benefit from small chance of winning in the short run regardless of even larger chance of loss. Different from Baker (2011), Kumar (2009) find that individual investors have preference for the lottery like pay-off stocks which are lowly priced, have high idiosyncratic skewness, and have high idiosyncratic volatility. Barberis and Huang (2008) also model this preference for the lottery like stocks with the cumulative prospect theory of Tversky and Kahneman (1992) such that agents overweigh the tail of the distribution. More importantly, they argue that the tail distribution can be priced. At equilibrium, cumulative prospect investors who overweigh the tail highly value positive skewness. As a result, high volatility stocks with high positive skewness can be overpriced and can even earn negative excess returns.

2.1.2. On the positive relation between risk and expected returns

Traditional financial theories take a long standing stance on the risk return tradeoff. Notably, the modern portfolio theory (MPT) by Markowitz (1952) states that the higher weight in risky assets is expected to have the higher compensation. The risk adverse investor maximizes the utility by choosing the suitable optimum in relation to the risk preference. In such an efficient market, investors realize above average returns only by taking above average risk that has ex-ante positive price. Capital Asset Pricing Model, by Treynor (1961), Sharpe (1964), Lintner (1965), Mossin (1966), and Black (1972), is derived from MPT in the condition that the market portfolio is mean variance efficient. Higher beta indicates higher risk in the mean variance framework since beta is the only factor that contributes to the risk of the portfolio that contains it.

The multifactor model of Merton (1973) and Ross (1976) proposes that risk premiums are dependent on the conditional covariance between asset returns and innovations in state variables. The Intertemporal Capital Asset Pricing (I-CAPM) model corrects the static assumption of one-period utility maximization in the standard Capital Asset Pricing model. In the I-CAPM model, investors price (i) the systematic market risk (ii) the risk of the unfavorable shift in investment opportunities. The second component overcomes the CAPM in a way that investment opportunity choices are time varying. Investors are assumed to be able to identify state variables that capture these uncertainties and construct portfolios that hedge against changes in these variables. In other words, investors are concerned about risks from both market returns and changes in the forecasts of future market returns. As a result, I-CAPM is a multi-factor model that has been tested with many variables such as the changes in interest rates, gold prices, and political stability. I-CAPM advocates a positive relation between risk and return for the aggregate stock market. The Arbitrage Pricing Theory by Ross (1976) is another multi-factor model that puts no particular significance on any factor portfolio. The model is based on a linear return generating process, requiring no utility assumption beyond monotonicity and concavity or being restricted to a single period.

In general, factor models generally advocate that systematic risk factors e.g., systematic volatility should be priced in the cross section of returns. However, there is an increasing literature noting the explanatory power of idiosyncratic volatility in the cross section of expected returns. Lintner (1965) also show that idiosyncratic volatility, or the variance of residuals from the market model, significantly explains the cross section of average stock returns. However, Fama and Macbetch (1972) find the statistical pitfalls in the model of Lintner (1965). In the 90s, Lehmann (1990) report statistically significant positive coefficients on idiosyncratic volatilities in cross sectional regressions, reconfirming the above-mentioned finding of Lintner (1965). Lehman research data is monthly total returns for all the common stocks traded on NYSE as well as the equally weighted and the value weighted indices of their returns from 1926 to 1984. Idiosyncratic volatility has been found to be important in other markets. Green and Rydqvist (1997) find the idiosyncratic volatility premium on the Swedish lottery bond market. Malkiel and Xu (2002) report the portfolio with the highest idiosyncratic volatility stocks has the highest average return but did not report any significance levels for idiosyncratic volatility premium. Malkiel and Xu (2002) employ the models of Fama and Macbeth (1973) and Fama and French (1992) with the data on stocks on NYSE, AMEX, NASDAQ, and TSE from 1975 to 2000.

Goyal and Santa-Clara (2003) state that “idiosyncratic risk matters”. Focusing on the time series relation rather cross sectional and measuring idiosyncratic risk through the average of stock variance, they point out that market variance has no predictive power. The idiosyncratic volatility or the average variance of stock returns has a positive relation with the stock returns with CRSP data from 1962 to 1999. The relation is significant after the control of the macroeconomic variables that are the forecast of the stock market and can be explained from the traditional risk return compensation approach as well as the option approach. Looking at equity and debt as respective call option and put option, the value of equity goes up at the expense of debt holders as the volatility of the underlying assets increases. The correlation between average stock variance and the spread of A-rated bonds over treasuries is found to be at 0.7 from 1965 to 1999, consistent with negative impacts on bond returns but positive impacts on stock returns.

Fu (2009) use EGARCH model to estimate idiosyncratic volatility in cross sectional regressions to find its positive statistical significance as a rebuttal against Ang et al. (2006). Fu has a shark criticism of the one month lagged data window for estimating idiosyncratic volatility by Ang et al. (2006). Indeed, Ang et al. (2006) report that the longer the estimation period for pre-defined beta, the less the outperformance of the bottom beta portfolio. The quintile portfolio with the highest idiosyncratic volatility stocks lags behind the quintile portfolio with the lowest idiosyncratic volatility stocks by 0.53% (t-statistic -2.88), 0.37% (t-statistic -1.62) and 0.24% (t-statistic -1.04) with respective twelve month, three month and one month estimation period, in terms of FF-3 alpha.

The heated debate comes alive with emerging empirical evidence against the low volatility effect. As mentioned in the motivation, Basu and Martellini (2007) find that the quintile portfolio with the highest total volatility stocks outperforms the quintile portfolio with the lowest total volatility stocks by 0.74% (p-value 1.37%) with CRSP data from 1990 to 2004. As a response, Blitz and van Vliet (2011) challenge the result of Basu et al. (2007) on ground of using survivor stocks in the sample. Li et al. (2014) report that anomalous return is not found in the equally weighted portfolio betting on low idiosyncratic volatility with respect to the CAPM model. By dropping lowly priced stocks, the anomalous return disappears in a value weighted low minus high volatility portfolio. The data sample is CRSP monthly stock returns from 1963 to 2010.

2.2. Left tail risk and benchmark-based risk measures

This section presents literature on return distribution in the financial market and various benchmark-based risk measures.

2.2.1. Left tail risk

Mandelbrot (1963a, b) and Mandelbrot and Taylor (1967) are the very first to show that returns on financial markets are not normally distributed, but exhibit excess kurtosis. Most investment returns exhibit negative skewness and excess kurtosis, indicating left tail risk that can be overlooked by merely looking at volatility based measures such as Sharpe ratio. Often, fat tail risk refers to events beyond three standard deviations that happen with a probability much higher than predicted by the normal distribution due to the many asymmetries in the markets, returns, information, and investor behaviors (Focardi et al., 2003). Even two investments that have the equal mean and standard deviation and therefore the same Sharpe ratios can differ significantly in skewness and kurtosis, or tail risk (see, e.g., Keating and Shadwick 2002). Fung and Hsieh (1999) argue that the mean variance framework is simplistically suitable only for a quadratic utility function or for normally distributed returns, thus ignoring large negative returns or fat tails. With the same logic, volatility or VAR under normal distribution does not capture the fat tail risk as it neither adequately captures all the observations of empirical distribution nor informs the maximum loss. From a behavioral aspect, investors tend to have more aversion to the left tail events of the distribution than to the right hand ones, according to the Prospect Theory of Kahneman and Tversky (1979). Even when return distribution is symmetrical, volatility does not capture this distinction. Further studying distribution at smaller granularity, Longin and Solnik (2001) reject the null hypothesis of multivariate normality in positive tails and find that correlation of large negative returns does not converge to zero, but tend to increase with the threshold level.

2.2.2. Benchmark-based risk measure

Benchmarking has been one of the main explanations for the overpricing of high risk assets that dampens returns (see, e.g., Baker et al., 2011; Blitz and Vliet, 2007). Price, Price, and Nantell (1982), and Popova, Popov, Morton, and Yau (2006) introduce the higher order lower partial moments (LPMs), taking into account returns below the target threshold and the higher aversion to left tail events than to the right tail events in line with the Prospect Theory. LPM requires fewer restrictive assumptions of utility function and probability distribution than the mean variance selection rule. LPM is defined as bellows:

$$LPM_h\left(R_p\right) = \int_{-\infty}^{h}(R-r)^2 f_p(R)dR \tag{1}$$

Where h is the target level and $f_p(R)$ represents the probability density function of returns of portfolio p (see Price et al., (1982).

$$E([L-w^T R]^+)^K \tag{2}$$

Where w and R are the vectors of portfolio weights and return, L is the target return (or mean or zero), + denotes the positive values only, K is the degree of the order of the moments (K = 1, 2, 3, and 4), and E is an expectation operator. Keating et al. (2002) later develop the Omega function that integrates all the higher order moments into one single function by calculating directly from the observed distribution.

$$\Omega(L) = \frac{\int_L^b \left(1-F_{(r)}\right)dr}{\int_a^L F_{(r)}dr} \tag{3}$$

Where Ω is the Omega function, r is the one period return, L is the threshold return, (a, b) is the interval of return, and F(r) is the cumulative density function of returns. The numerator (denominator) is the probability weighted total gain (loss) with L as the threshold return. Ω is calculated as the integration of the respective function in the specified return interval. Assuming that the higher return and the less risk are the better options, the higher omega equates the better portfolio. Kazemi et al. (2003) present the mathematical derivation of this ratio of integration as the ratio of expected gain and loss, deriving the ratio of the call option price to the put option price as below:

$$\Omega(L) = \frac{\int_L^b \left(1-F_{(r)}\right)dr}{\int_a^L F_{(r)}dr} = \frac{\int_L^b (max(x-L,0)f(x)d(x)}{\int_L^b (max(L-x,0)f(x)d(x)} = \frac{E(max(x-L,0)}{E(max(L-x,0)} = \frac{e^{-rf} * E(max(x-L,0)}{e^{-rf} * E(max(L-x,0)} = \frac{C(L)}{P(L)} \tag{4}$$

Later, Kaplan .P, (2004) generalizes that Omega falls under the spectrum of Kappa ($K_{(n)}$). $K_{(n)}$ is Omega when n=1, Sortino ratio when n=2 and other measures with higher n.

$$\frac{\mu-L}{\sqrt[n]{LPM_n(L)}} \tag{5}$$

Where L is investor minimum acceptable "threshold" return and LPM is the higher order lower partial moment as above. It is proved that that Ω (L) = $K_1(L)$+1. In this thesis, I mainly use the LPM formula introduced by Price et al., (1982) and Popova et al., (2006).

2.3. Portfolio weighting scheme

2.3.1. Market capitalization weighting

The capital asset pricing model advocates that market portfolio is mean variance optimal and therefore passive investors benefit the most by investing in the market portfolio, which has the highest Sharpe ratio. A common proxy for the market portfolio is the market capitalization weighted portfolio which merits broad equity market coverage, high trading liquidity, high investment capacity, and low maintenance cost (see, e.g., Armott et al., 2005). Capitalization weighting is a passive strategy that overweighs large capitalization companies that generally have strong fundamentals. However, market capitalization weighted portfolio has been rejected as a good proxy for the market portfolio. The underlying assumptions of MPT are very much in consensus to be too simplistic. Even if relaxing the requirement for a proxy, the disadvantage of market capitalization weighting is the overweight of overvalued stocks and underweight of undervalued stocks. Hsu (2006) shows that cap weighed portfolios suffer from return drag if prices are noisy relative to the movements of company fundamentals. In other words, the market capitalization weighting can have growth bias. Armott et al. (2005) state that market capitalization scheme is a particularly volatile way to measure companies' size and its true value. They ultimately provide the mathematical derivations that cap weighted portfolios would on average produce negative alphas relative to its fair expected returns.

2.3.2. Alternative weighting

The pioneer in deviation from the market cap weighted indexation is Arnott et al. (2005) who find that the studied fundamental indexation index outperforms the market capitalization index by 1.97% per year from 1962 to 2004. They find that the market cap weighting tends to react strongly to the investor preference, e.g., the strong response to energy shift in the 1980s or technology stock in 1998-2001. On the other hand, the fundamental weighting indexing selects and weighs stable metrics of company size such as book value, revenue, and dividends. According to Arnott et al. (2005), the outperformance of fundamental indexation tends to be attributable to market mispricing, the exposure to distress factors, the superior market construction, or the mixed of all those factors. However, they remain open to further study of the true value drivers for fundamental indexation.

One interesting issue raised by Arnott et al. (2005) is whether the fundamental indexation is value biased or growth bias. The outperformance of value stocks against growth stocks in both US and international markets has generally come under consensus among academics (see, e.g., Chan et al., 1991; Fama et al., 1992). However, a much less agreed upon issue is the explanation for it. For example, the value effect has been accounted for by both modern finance and behavioral finance. Fama and French (1996) explain returns from the value effect such that higher returns are compensated for higher risk. Therefore, the value effect is still consistent with EMH. Behavioral finance proponents argue that those abnormal returns are a result of mispricing from the market. Investors tend to extrapolate too far into the past and overpay for the growth stock (see, e.g., Lakonishok, 1994; Shiller, 2003). Fama and French (1996) rebut by saying that the mean reversion of earnings growth is shorter than the distressed premium period of at least five years. As a result, extrapolation based on growth is not the valid explanation. Besides, Fama and French (1996) argue that anomalies do not pass the economic significance test that the marginal benefit of exploiting information exceeds the marginal cost. As the controversy becomes heated and fierce, Shiller (2003) argues that if value stocks are riskier, value stocks should have performed worse than growth stocks in downtime periods. However, he finds the opposite evidence and that value stocks are not more volatile than growth stocks. Against this argument, Fama (1992) argues that return spread does not correlate with GNP and that variance is not the adequate proxy for portfolio risk. Hsu et al. (2006) argue that the fundamental indexation is not merely the value indexation although it is true that the fundamental index under weighs growth stocks and tilts towards stocks with low P/E ratios and high dividend yields. They show the empirical evidence that U.S. Fundamental index 1000 outperforms Russell 1000 value and that the fundamental index outperforms the S&P 500 both in up and down markets. Value indexes are based on market capitalization and under weigh many growth companies that are growing their fundamentals equally rapidly while fundamental indexes don't under weigh growth companies that are growing in fundamentals.

Estrada J., (2006) and Perod A., (2007) also advocate the fundamental indexing over the market cap weighted by exploring different company specific metrics. The indexation proliferates in categories, including tiled factors such as size, value, and momentum factor. Among others, equal weighting scheme utilizes the size effect of giving more weight to small stocks in the absence of optimizing any objective functions (see, e.g., Clarke, de Silva, and Thorley, 2006; DeMiguel, Garlappi, and Uppal, 2009). DeMiguel et al. (2007) find that of that 1/n rule, or naïve diversification, is better than all 14 sample-based mean variance model across seven empirical data sets in terms of Sharpe ratio, certainty-equivalent return, or turnover. The results show that optimal diversification benefit is more than offset by estimation error, suggesting a long way for "optimal" portfolio choice to be realized out of sample.

3. Hypothesis

H1: There is performance persistence of stocks with high total volatility and idiosyncratic volatility.

H1a: The portfolio with high total and idiosyncratic volatility stocks respectively outperforms the portfolio with low total and idiosyncratic volatility stocks in monthly average return.

Traditional asset pricing which advocates volatility as a risk measure and a positive linear relation between risk and expected returns predicts that higher risk is compensated with higher returns. That the portfolio with low volatility stocks outperforms the portfolio with high volatility stocks therefore presents an anomaly under such framework.

H1b: The value effect and size effect cannot explain the average returns of the portfolios sorted by total volatility and idiosyncratic volatility.

In order to examine whether the low volatility effect is another representation of the value effect and size effect, I control for value and size factor exposure by the multivariate ordinary least square regression analysis with the FF-3 factor model as following:

$$E_{(ri)} = \alpha + \beta_{1i}\left(R_m - R_f\right) + \ \beta_{2i}SMB + \beta_{3i}HML \tag{6}$$

Where $E_{(ri)}$ is the expected return on stock i, SMB (Small minus Big) is the average return on the three small portfolios minus the average return on the three big portfolios, HML (High minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios, $(\mathrm{R_m} - \mathrm{R_f})$ is the excess return on the market, β_{1i} and β_{2i} are the factor loading coefficients, and α is the alpha return. If α is significantly different from zero, the returns from decile portfolios are not adequately explained by the size and value factor exposure. If α is not significantly different from zero, the size and value factor exposure explain all the excess returns.

H1c: Equally weighted decile portfolios sorted by total volatility and idiosyncratic volatility respectively outperform market capitalization weighted counterpart portfolios.

In the mean variance framework, the market portfolio is the most efficient portfolio with the highest Sharpe ratio. A common proxy for the market portfolio is the passive market capitalization portfolio. However, if the market capitalization weighted portfolio does not represent the set of all investable options or mean variance efficiency does not hold, investing in the market capitalization weighted index may not offer the best risk and return tradeoff. As a result, there could be chances to exploit the inefficiency with an alternative weighting. The equal weighting scheme that gives more weight to small market capitalization stocks could benefit from the size effect. As a result, the equally weighted portfolios could outperform the market capitalization counterpart portfolios.

H2: The portfolio with high total and idiosyncratic volatility stocks is respectively riskier than the portfolio with low total and idiosyncratic volatility stocks by various measures.

Normally distributed returns are fully characterized by the mean vector and the covariance matrix. Therefore, volatility is an adequate risk measure in the mean variance framework. However, if return distribution is not normally distributed, more volatile portfolios are not necessarily riskier. Theoretically, risk has an ex-ante positive price with a predicted distribution. However, the empirical realization of the ex-post risk return payoff can be negative at times.

4. Data and methodology

4.1. Data

My research data set is consisted of the largest 1000 listed stocks across US for the period from January 1983 to December 2013. The data is retrieved from CRSP for all stocks that are listed on AMEX, NASDAQ, and NYSE. The data set is monthly total returns that include both capital gains and dividends. North American value, size, momentum factor and monthly Treasury bill as the short term interest are downloaded from the Kenneth French library. Standard Industrial Classification (SIC) codes are downloaded from the website of Securities and Exchange

Commission. The SIC in a company's disseminated EDGAR filings indicates the company's type of business.

I start with all stocks listed in the CRSP database and require that the stock have at least 24 data point of historical data. Even large capitalization firms e.g., Berkshire Hathaway don't always have reported prices or have missing data points in the data base. Therefore, I allow for the flexibility if there is an acceptable number of missing data points by first sorting by number N of observations by PERMNO and constrain N to have at least 20 to 24 historical data points. The data set is balanced annually so that stocks whose market capitalization rise below the threshold of top 1000 stocks by the market cap will be dropped out and stocks whose market capitalization drop below the threshold are included. I calculate the market capitalization based on the "Price or Bid Ask Average" and "SharesOutstanding". In order to smooth out the fluctuation of monthly market cap, I calculate the average monthly market cap every year. The average market capitalization of these firms has also increased from $1.57bn to $20.76bn and the firm with the largest monthly average cap is Apple in 2012 at $547bn. Outliner stock price is that of Berkshire Hathaway Inc. Del with share price of $145 875 in January, 2013. I rank stocks by average market capitalization throughout the year to keep the top 1000 PERMNO with the highest market cap and retrieve holding period returns in each year. For each list of stocks corresponding to each year, I separately retrieve a set of 60 month historical data. Lowly priced stocks account for an insignificant proportion in my dataset at 0.21% on average across the year.

4.2. Methodology

4.2.1. Pre-formation total volatility estimation and post-formation measurement

4.2.1.1. Pre-formation parameter estimation

Trading strategy

As discussed, my goal is to use up to 48 months of historical information to form portfolios and to monitor the portfolio for the next 12 months on a rolling basis. The portfolio is formed based on the ranking of ex-post realized total volatility calculated based on 48 month historical data. I define it as the preformation period. Portfolio formation strategies are based on an estimation period of E months, an awaiting period of W months, and a holding period of H months. The E/W/H strategy is as follows. At month t, I compute total volatilities from historical monthly data over an E month period from month t−E−W to month t −W. At time t, I construct equally weighted portfolios based on these total volatilities and hold these portfolios for N months. My main analysis is based on the 1/0/1 strategy, in which I simply sort stocks into decile portfolios based on their level of total volatility computed using monthly returns over the past 48 months. I hold these equally weighted portfolios for 1 month. For simplicity, this strategy is written as 48/0/1. The decile portfolios are rebalanced each month. I also examine the robustness of my results to various choices of E, W, and H. The construction of the E/W/H portfolios for E>1 and M>1 follows Jegadeesh and Titman (1993) and Ang (2006) except that my portfolios are equally weighted. I have three variations as 36/0/1, 48/0/3, and 48/0/12. I will present those variations of holding period and estimation period in Section 6.

Ranking

Every month, I calculate the volatility of stocks return over the past four year on the rolling basis. Monthly returns have more stable informational content than weekly returns as used by Blitz and van Vliet (2007). For each month, I rank the total volatility and construct the decile portfolio following Fama (1992) methodology such that stocks are allocated in an independent sort to ten volatility group based on the ranking. I divide the ordered data into ten essentially equal-sized data subsets of 100 stocks. Stocks with the lowest 100 ranking scores are assigned to the bottom decile, decile 1 while stocks with the highest 100 scores are assigned to the top decile, decile 10. Scores are compared directly across all stocks without imposing sector or country restrictions.

Weighting scheme

For equal weighting scheme, I construct the equally weighted portfolio for each decile obtained by ranking stocks on the past four-year volatility of monthly returns on a rolling basis at the end of each month. Each stock is given weight 1/N (7) (N=100). The portfolio is form and hold from month t to t+1. For market cap weighting scheme, the cap weighted portfolio returns are calculated each month by multiplying stock returns with its corresponding weight in the decile portfolio. The market cap weight of stock i at month t is calculated by dividing its month end market cap by the month end sum of the market cap of all the stocks in the decile. The weighting calculation follows the formula $w_t = \frac{MC_{it}}{\sum_1^n MC_{it}}$ (8) where w_t is the market cap weight of stock i at month t, MC_{it} is the month end market cap of stock i at month t, and n is the number of all stocks in each decile.

4.2.1.2. Post-formation measurement

Test period portfolios are monitored in several different ways. I run the regressions of the excess returns of test period portfolios on FF-3 factor returns to estimate alpha and factor loadings. The performance of test period portfolios are measured in terms of key statistics, return distribution, and benchmark-based measure. Average return and standard deviation are measured in monthly percentage terms and apply to total, not excess, simple returns. In order to separate the low volatility effect from other effects, I use the multivariate regression method with the FF-3 market factor, the SMB (small-minus-big), and the HML (high-minus-low). By regressing the excess returns of the volatility sorted portfolios on these factor returns, I investigate how much of decile excess returns is attributed to the value and size effect. An alternative way is to use the double sorting strategy. Firstly, I can sort stocks by the size and value factor that are desirable to be controlled into deciles. Subsequently, I can sort stocks into decile portfolios by total volatility within each decile.

Component analysis

I construct the f value by counting the occurrence frequency that an industry appears in each decile across the whole time series. In the initial step, I create a list of components for each decile in certain year. As the stocks are rebalanced, the stocks can drop in and out of the decile every month by switching between different deciles. Random falls in deciles are immaterial. I trace back the industry to which each of the stock PERMNO belongs according to Standard Industrial Classification (SIC). After converting the PERMNO data into SIC data, I calculate the frequency f that certain industry appears in certain decile. F is calculated as the number of time an industry appears in a decile divided by the total number of time all industries appear in that decile.

$$f_{ij} = \frac{n_{ij}}{N_j} \tag{9}$$

where n=1 if industry i appears in decile j otherwise 0 and N is total number of time all industries appear in decile j.

Benchmark-based risk measurement

Most of the risk metrics require VBA code to compute values across the whole time series (see, e.g., appendix for the code). Lower partial moments are calculated with the formula $E([L - w^T R]^+)^K$(2). If L < R, the event is not an unwanted one and therefore is not a risk but rather desired. Therefore, I only include the positive values of E. The downside risk measure is dependent on the threshold minimum acceptable or target return L.

Drawdown is a realization of the tail risk over a period of time, as an ex-post risk measure. The maxdrawdown is calculated as the largest cumulative percentage decline in month end net asset value (NAV) due to loss, during a period in which the peak month end NAV is not equaled or exceeded by a subsequent month end NAV. $Maxdrawdown = \frac{(P-L)}{P}$ (11) where P is the peak value before the largest drop, L is the lowest value before new high established. "Months in maxdrawdown" is calculated as the lapsed time from the start of the maxdrawdown to the end of maxdrawdown. "Months to recover" is the lapsed time when the portfolio has the bottom NAV value to the time when NAV recovers the original investment.

An up percentage ratio is a measure of the number of periods that the investment outperforms the benchmark (FF-3market portfolio) when the benchmark is up, divided by the number of periods that the benchmark is up. $Up\ Percentage\ Ratio \frac{\sum_{i=1}^{N} L_i}{\sum_{i=1}^{N} LD_i}$ (12) where $\mathrm{L_i} = 1$ if $\mathrm{R_i} \geq \mathrm{RD_i}$ and $\mathrm{RD_i} \geq 0, \mathrm{LD_i} = 1$ if $\mathrm{RD_i} \geq 0$, $\mathrm{R_i}$ is the return for period i, RD_i is the benchmark return for period i, and N is the number of periods (months). Down percentage ratio is a measure of the number of periods that the investment outperforms the benchmark when the benchmark is down, divided by the number of periods that the benchmark is down. $Down\ Percentage\ Ratio \frac{\sum_{i=1}^{N} L_i}{\sum_{i=1}^{N} LD_i}$ (13) where $\mathrm{L_i} = 1$ if $\mathrm{R_i} \geq \mathrm{RD_i}$ and $\mathrm{RD_i} < 0$, $\mathrm{LD_i} = 1$ if $\mathrm{RD_i} \leq 0$, $\mathrm{R_i}$ is the return for period i, $\mathrm{RD_i}$ is the benchmark return for period i, and N is the number of periods (months).

Downside deviation is calculated as the standard deviation below the monthly average return. $Downside\ deviation = \sqrt{\frac{\sum_{r_i<c}^{n}(r_i-c)^2}{n}}$ (14) where c is the minimum acceptable return and n is the total number of returns. The sum is strictly restricted to those returns that are less than c. $Sortino\ ratio = K_2(L) \frac{\mu - L}{\sqrt[2]{LPM_2(L)}}$ (15) where μ is the month end return, L is investor minimum

acceptable return, and LPM is the second order lower partial moment. $Sharpe\ ratio = K_2(L)\frac{\overline{r_p}-r_f}{\sigma_p}$ (16) where $\overline{r_p}$ is the average portfolio return, r_f is the risk free rate, and σ_p is the portfolio standard deviation.

4.2.2. Pre-formation idiosyncratic volatility estimation and post-formation measurement

I define idiosyncratic volatility as the variance of the residual terms in the FF-3 model in (6). My goal is to estimate FF-3 parameter by running the rolling regressions of excess individual stock returns on FF factor returns for month from t-36 to month t, to obtain residual values from month t to month t+23 and subsequently to estimate idiosyncratic volatility for month t+24 to month t+35. In total, I require 60 data points. I use the same data set that was used to estimate total volatility. The same steps are repeated for 30 years for the rebalanced 1000 stocks. The total number of regression is therefore 720 000 (=24*1000*30). I rank the idiosyncratic volatility and construct the decile portfolio following Fama (1992) methodology such that stocks are allocated in an independent sort to ten volatility groups based on the ranking. I divide the ordered data into ten essentially equal-sized data subsets of 100 stocks as in the case of total volatility. Portfolio performance measurement follows similar procedures to those in the case of total volatility.

5. Results

5.1. Total volatility

5.1.1. Equally weighted decile portfolios sorted by total volatility

Table 1 reports the key statistics of deciles portfolio. I use 48/0/1 strategy as the base case of the analysis. The average returns of decile portfolios have a monotonic increase from 1.1% per month for decile 1 to 2.1% per month for decile 10. It is worth noting that this relatively high level of average return is not attainable in the market capitalization portfolios (see, e.g., Section 5.1.2.). The volatility of the returns of each decile portfolio is the lowest at 3.11% per month for the bottom volatility decile and increases monotonically to 9.54% per month for the top volatility decile, or 10.76% to 33.06% annually. Even though the largest monthly gain occurs to decile 10, it has the lowest percentage of monthly gain at 64%. Other deciles have the percentage of positive return months from 64% to 72%.

5.1.1.1. Post-formation factor loading estimation

The covariance with the market explains the most of the variation in decile portfolio returns among the factor loadings, showing that decile portfolio returns are more sensitive to innovations in the market as the state variable than to the value and size factor. Market systematic beta, ranging from 0.48 to 1.58, has a monotonic increase with return volatility and is the highest among the other factor betas with t value statistically significant with 99% confidence level, even after adjusting for robustness with the Newey-West robust standard errors. The result is consistent with traditional financial theories and evidence reported by Treynor (1961), Sharpe (1964), Lintner (1965), Mossin (1966), and Black (1972). However, changing the weighting scheme does not display such a pattern, advocating the proposition that equal weighting has a critical role in the positive beta and average return relation.

Overall, the low volatility decile tends to move positively with the value factor and negatively with the size factor. The size beta also increases monotonically with the average volatility and the market beta. However, after adjusting with Newey-West robust standard errors to remedy the heteroscedasticity and autocorrelation of residuals, only mid-range decile returns are statistically significantly sensitive to the movement of the size factor. The value beta shows the switching sign, with decile 1 having the highest positive beta coefficient 0.326 (t-statistic 5.72) and decile 10 having the lowest negative beta -0.408 (t-statistic -2.13). Among the 10 deciles, decile 1 moves the most with high P/E or value stocks while decile 10 is less so. The result is in line with the argument of Ang et al. (2006) that stocks with low (high) idiosyncratic volatility are generally large (small) market cap stocks with low (high) book-to-market ratios. I ask the question if the constituents of the decile portfolios explain the factor co-movement. One hypothesis is that the top decile is dominated by the stocks of industries that have a relatively large number of small and growth firms whereas bottom decile is dominated by the stocks of industries that have a relatively large number of big and value firms.

The most noteworthy difference in alpha is between decile 10 alpha of 1.4% (t-statistic 4.84) and decile 1 alpha of 0.3% (t-statistic 2.57). Decile 10 outperforms decile 1 by 1.1% (p-value 0.00) per month. The extreme top 10% volatility stocks still earn statistically high alpha although there is small marginal difference in alphas of the first eight volatility deciles. In other words, the ex-post realized volatility and alpha relation has been quite flat from decile 1 to decile 8. Black (1972), Jensen, and Scholes (1972), and Haugen and Heins (1975) report that the beta and return relation has been much flatter than that predicted by CAPM.[3]

By having a smaller granularity from sorting volatility into deciles rather quintiles as prior research (see, e.g., Fama et al., 1992; Baker et al., 2012), I find that alpha is significantly different mainly at the extremely high volatility decile portfolio. Bigger granularity may have obscured the relation since the marginal difference is as small as 10-140 bps. In other words, volatility premium exists only at an extreme level. The middle range decile 5, decile 6, and decile 7, are most precisely predicted by a linear model, with R-squared reduces monotonically to either the extremely low or extremely high volatility end. Such a pattern is consistent with the explanation that medium level volatility stocks are more representative of the aggregate market factors and less biased toward value or size factor due to being less concentrated towards specific industries than other deciles. As a result, it is highly interesting to identify the industry components of decile portfolios.

In summary, I use the FF-3 model to compare the relative unexplained excess returns by factor exposure across deciles, all else equal. Whether the FF-3 model effectively explains the return variation should also show in the residual terms. If the FF-3 model captures all excess returns, there is no informational content left in the residuals which follow the White noise process. Sorting the decile portfolio based on idiosyncratic volatility is therefore a potentially interesting question to study (see, e.g., Ang et al., 2006), given the R-squared values remain around 83% in the ten regressions.

5.1.1.2. Component analysis

The R-squared value from the FF-3 model is the highest for middle range decile 5 (R2=0.80), decile 6 (R2=0.82), and decile 7 (R2=0.83) and reduces slowly to the two ends, decile 1 R2=0.47 and decile 10 R2=0.64. Hodrick and Zhang (2001) argue that small and growth portfolios are typically harder to be priced by standard factor models. One hypothesis is that

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[3] Ang et al. (2006) report the underperformance of high, rather than the underperformance of low total volatility decile portfolio, of -0.97% (t-statistic -2.86) per month on average returns and -1.19% (t-statistic -5.92%) on the FF-3 alpha with daily data over the previous month.

middle range stocks are more evenly distributed with stocks from different industries while decile 10 and 1 are clustered by stocks of specific industries that harder to be priced. It is questionable whether decile portfolios gather top (bottom) volatile stocks from individual industries or gather top (bottom) volatile industries. As the portfolios are rebalanced every month, the chance that an industry falls randomly into a decile is negligible. Figure 3 shows the top 20 industries with the highest occurrence frequency in each decile portfolio. The outliner decile 1 and decile 2 are mostly dominated by stocks of firms in “Electric Services” related industry. On the other hand, stocks of “National Commercial Banks” lead in the occurrence frequency in decile 3, decile 4, and decile 5. Stocks of “Crude Petroleum & Natural Gas” dominate decile 6, decile 7, and decile 8. There is a common trend that occurrence frequency either stays stable within 5 % fluctuation or drops as the average level of volatility decreases. It is arguable that the distribution of decile components is according to the volatility of the dominating industries rather than the accumulation of most volatile stocks from a wide variety of industries. The average level of volatility of component stocks and volatility of decile portfolio returns experience the same monotonic pattern from decile 1 to decile 10. Against the concentration of industries as above, stocks of outliner pharmaceutical and oil related firms are present with relatively high f value in almost all decile portfolios.

Decile 1 is highly concentrated towards the two top industry “Electric Services” and “Electric & Other Services Combined”, which make up 36.48% of decile 1. Stocks of the top f value industry across all other nine deciles constitutes only about 4% on average. Out of 14 industries with f>1%, there are 10 industries with f >2% and five industries with f >3%. The rest has f value in (0%, 1%).

Decile 2, compared to decile 1, has more uniform distribution with the top five f value 6.68%, 4.61%, 3.97%, 3.94%, and 3.42% respectively. Still, stocks of “Electric Services” and “Electric & Other Services Combined” ranks 1st and 4th in the top five f value industries, leaving the other industries “Fire, Marine & Casualty Insurance”, “Pharmaceutical Preparations”, and “National Commercial Banks” with rank 2nd, 3rd, and 5th. Stocks of top two industries only constitute 11.29% decile 2 components (vs. 36.48% of decile 1). Out of the 22 industries with f>1%, there are 11 industries with f >2% and five industries with f >3%. The rest has f value in (0%, 1%).

Decile 3, compared with decile 2, has more uniform distribution with the top five f value 4.82%, 3.95%, 3.12%, 2.49%, and 2.37% respectively. Stocks of “National Commercial Banks” ranks 1st and stocks of the top two industries only constitute 8.76% of decile 3 components (vs.

36.48% of decile 1 and 11.29% of decile 2). Stocks of “National Commercial Banks” leads decile 3, decile 4, and decile 5. Out of 21 industries with f>1%, there are nine industries with f>2% and three industries with f >3%. The rest has f value in (0%, 1%).

Decile 4 is very similar to decile 3 in terms of industry component profile. Compared with decile 3, decile 4 has more uniform distribution with the top five f value 3.99%, 3.44%, 3.33%, 2.64%, and 2.38% respectively. Stocks of “National Commercial Banks” ranks 1st and stocks of the top two industries only constitute 7.43% of decile 4 components (vs. 36.48% of decile 1, 11.29% of decile 2, and 8.76% of decile 3). Out of 18 industries with f>1%, there are seven industries with f>2% and three industries with f >3%. The rest has f value in (0%, 1%). In terms of returns, decile 3 and decile 4 have the equal FF-3 alpha.

Decile 5, compared with decile 4, has more uniform distribution with the top five f value ranging 3.25%, 3.06 %, 2.34%, 2.21%, and 2.05 % respectively. Stocks of “National Commercial Banks” ranks 1st and stocks of the top two industries only constitute 6.34% of decile 5 components, lower than all deciles above. Out of 21 industries with f>1%, there are six industries with f >2% and 2 industries with f >3%. The rest has f value in (0%, 1%). In terms of FF-3 returns, decile 5 and decile 4 have the equal FF-3 alpha.

Decile 6’s top five f values are 3.94%, 2.19 %, 1.98%, 1.86%, and 1.8 % respectively. However, the spread of f value among the top deciles starts to pick up. Stocks of “Crude Petroleum & Natural Gas” ranks 1st and stocks of the top two industries only constitute 6.14% of decile 5 components, lower than all deciles above. Out of 19 industries with f>1%, there are two industries with f>2% and one industries with f >3%. The rest has f value in (0%, 1%). In terms of FF alpha, decile 5 and 6 are on equal terms in estimated FF-3 factors.

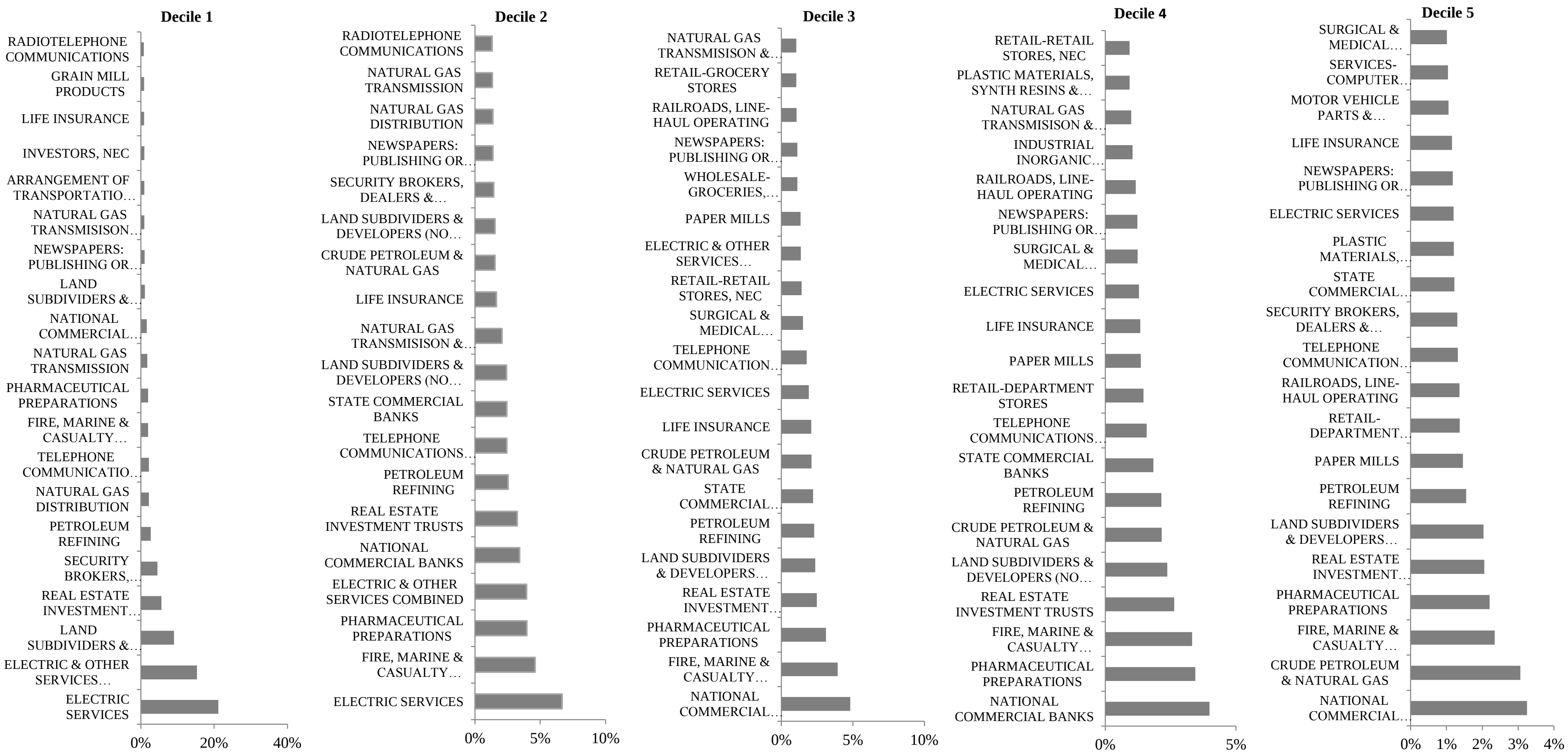
Decile 7’s top five f value are 4.16%, 2%, 1.83%, 1.66%, and 1.62 % respectively. However, the spread of f values, which drops gradually from decile 1 to decile 5, starts to increase slightly. Stocks of “Crude Petroleum & Natural Gas” ranks 1st and stocks of the top two industries only constitute 6.15% of decile 7 components. Out of 14 industries with f>1%, there are two industries with f>2% and one industry with f >3%. The rest has f value in (0%, 1%). In terms of FF-3 returns, decile 7 has higher FF-3 alpha than the above six decile portfolios by only 10bp.

Decile 8's top five f values are 4.36%, 2.35 %, 2.02%, 1.74%, and 1.72 % respectively. The spread of f values, which drops gradually from decile 1 to decile 5, starts to increase slightly since decile 7's spread pick up. Stocks of "Crude Petroleum & Natural Gas" ranks 1st and stocks of the top two industries only constitute 6.72% of decile 8. Out of 20 industries within f>1%, there are three industries with f>2% and one industry with f >3%. The rest has f value in (0%, 1%).

Decile 9 is the hybrid of decile 8 and decile 10 with the dominance of stocks both of technology firms (f value 13.65%) and of oil firms (f value 8.08%). Decile 9's top five f values are 4.2%, 3.55 %, 3.07%, 2.62%, and 2.35 % respectively. Stocks of "Semiconductors & Related Devices" ranks 1st and stocks of the top two industries with the highest f values constitute 7.76% of decile 9's components, slightly increasing from decile 8's and showing the pick up against the decreasing trend from decile 1 to decile 6. Out of 20 industries within f>1%, there are seven industries with f>2% and one industry with f >3%. The rest has f value in (0%, 1%).

Decile 10 is most dominantly consisted of stocks of the top three industry "Services-Computer Programming, Data Processing, Etc.", "Semiconductors & Related Devices", and "Services-Prepackaged Software", which all makes up 16.09% of decile 10 components. Still, the f spread in top volatility decile is smaller than that in decile 1. It can be reasonably argued that technology companies are more subject to informational innovations than other industries. With the highest level of average volatility of component returns, decile 10 also has the highest alpha from the FF-3 model.

In summary, the mid-range deciles are more diversified and more evenly distributed in terms of industry components compared with the other ends, the bottom or top volatility decile. R-squared has also the highest value for the mid-range deciles, consistent with the hypothesis that more diversified industry portfolios are relatively more predicable by systematic factor exposure. The top and bottom deciles are respectively dominated by stocks of semi-conductor related firms and stocks of electricity services related firms respectively. Whether other deciles are strictly characterized by certain industries are less clear.


Decile 1
RADIOTELEPHONE COMMUNICATIONS
GRAIN MILL PRODUCTS
LIFE INSURANCE
INVESTORS, NEC
ARRANGEMENT OF TRANSPORTATIO…
NATURAL GAS TRANSMISISON…
NEWSPAPERS: PUBLISHING OR…
LAND SUBDIVIDERS &…
NATIONAL COMMERCIAL…
NATURAL GAS TRANSMISSION
PHARMACEUTICAL PREPARATIONS
FIRE, MARINE & CASUALTY…
TELEPHONE COMMUNICATIO…
NATURAL GAS DISTRIBUTION
PETROLEUM REFINING
SECURITY BROKERS,…
REAL ESTATE INVESTMENT…
LAND SUBDIVIDERS &…
ELECTRIC & OTHER SERVICES…
ELECTRIC SERVICES
0%
20%
40%
Decile 2
RADIOTELEPHONE COMMUNICATIONS
NATURAL GAS TRANSMISSION
NATURAL GAS DISTRIBUTION
NEWSPAPERS: PUBLISHING OR…
SECURITY BROKERS, DEALERS &…
LAND SUBDIVIDERS & DEVELOPERS (NO…
CRUDE PETROLEUM & NATURAL GAS
LIFE INSURANCE
NATURAL GAS TRANSMISISON &…
LAND SUBDIVIDERS & DEVELOPERS (NO…
STATE COMMERCIAL BANKS
TELEPHONE COMMUNICATIONS…
PETROLEUM REFINING
REAL ESTATE INVESTMENT TRUSTS
NATIONAL COMMERCIAL BANKS
ELECTRIC & OTHER SERVICES COMBINED
PHARMACEUTICAL PREPARATIONS
FIRE, MARINE & CASUALTY…
ELECTRIC SERVICES
0%
5%
10%
Decile 3
NATURAL GAS TRANSMISISON &…
RETAIL-GROCERY STORES
RAILROADS, LINE-HAUL OPERATING
NEWSPAPERS: PUBLISHING OR…
WHOLESALE-GROCERIES,…
PAPER MILLS
ELECTRIC & OTHER SERVICES…
RETAIL-RETAIL STORES, NEC
SURGICAL & MEDICAL…
TELEPHONE COMMUNICATION…
ELECTRIC SERVICES
LIFE INSURANCE
CRUDE PETROLEUM & NATURAL GAS
STATE COMMERCIAL…
PETROLEUM REFINING
LAND SUBDIVIDERS & DEVELOPERS…
REAL ESTATE INVESTMENT…
PHARMACEUTICAL PREPARATIONS
FIRE, MARINE & CASUALTY…
NATIONAL COMMERCIAL…
0%
5%
10%
Decile 4
RETAIL-RETAIL STORES, NEC
PLASTIC MATERIALS, SYNTH RESINS &…
NATURAL GAS TRANSMISISON &…
INDUSTRIAL INORGANIC…
RAILROADS, LINE-HAUL OPERATING
NEWSPAPERS: PUBLISHING OR…
SURGICAL & MEDICAL…
ELECTRIC SERVICES
LIFE INSURANCE
PAPER MILLS
RETAIL-DEPARTMENT STORES
TELEPHONE COMMUNICATIONS…
STATE COMMERCIAL BANKS
PETROLEUM REFINING
CRUDE PETROLEUM & NATURAL GAS
LAND SUBDIVIDERS & DEVELOPERS (NO…
REAL ESTATE INVESTMENT TRUSTS
FIRE, MARINE & CASUALTY…
PHARMACEUTICAL PREPARATIONS
NATIONAL COMMERCIAL BANKS
0%
5%
Decile 5
SURGICAL & MEDICAL…
SERVICES-COMPUTER…
MOTOR VEHICLE PARTS &…
LIFE INSURANCE
NEWSPAPERS: PUBLISHING OR…
ELECTRIC SERVICES
PLASTIC MATERIALS,…
STATE COMMERCIAL…
SECURITY BROKERS, DEALERS &…
TELEPHONE COMMUNICATION…
RAILROADS, LINE-HAUL OPERATING
RETAIL-DEPARTMENT…
PAPER MILLS
PETROLEUM REFINING
LAND SUBDIVIDERS & DEVELOPERS…
REAL ESTATE INVESTMENT…
PHARMACEUTICAL PREPARATIONS
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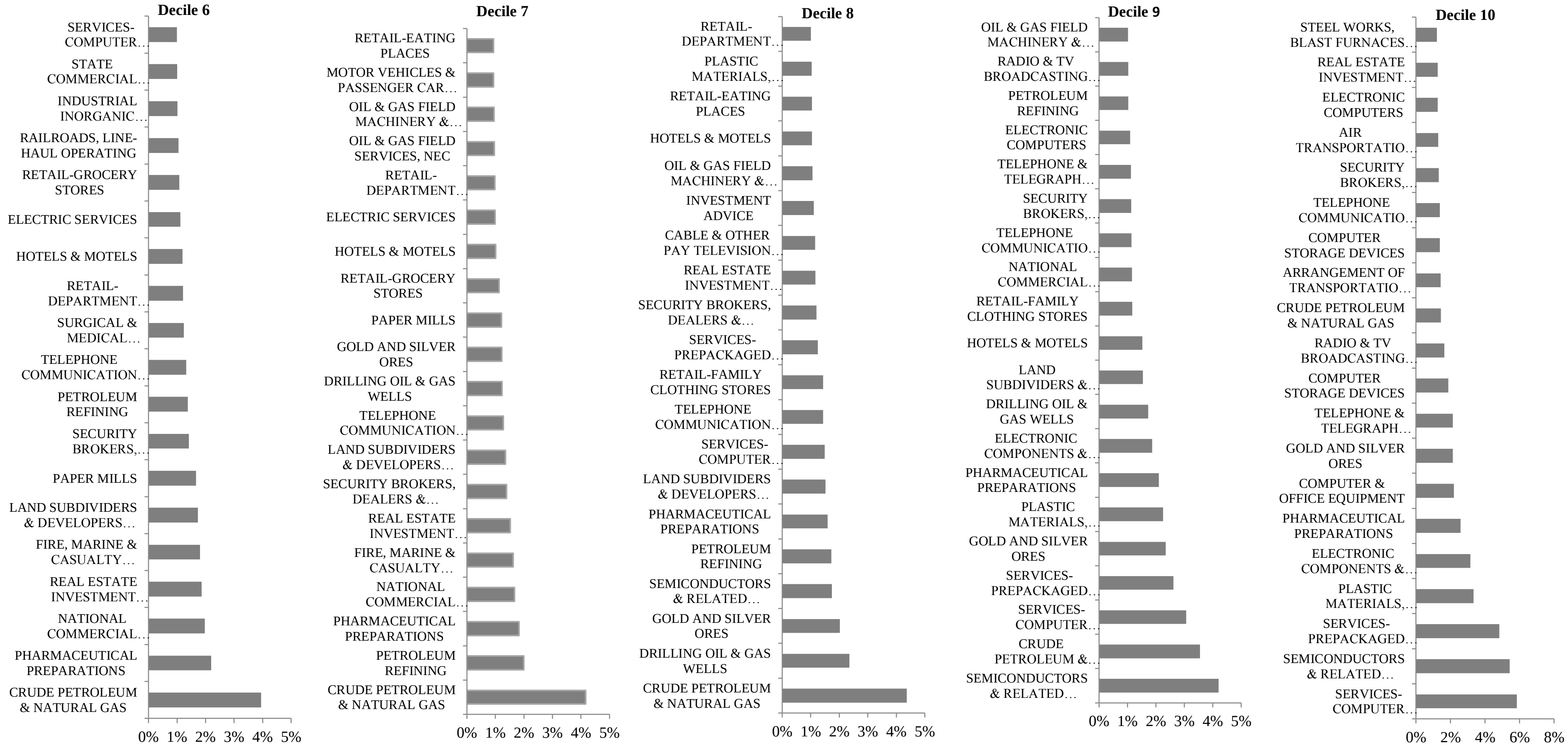

Figure 1: Top 20 industries in decile portfolios sorted by total volatility

Figure 1 presents the top 20 industries with the highest constructed f values. The f value indicates the occurrence frequency of an industry in a decile portfolio. Stock PERMNOs are traced back to Standard Industrial Classification (SIC) codes, which are downloaded from CRSP and whose industry identity corresponding to SIC codes are downloaded from the website of Securities and Exchange Commission. The SIC in a company's disseminated EDGAR filings indicates the company's type of business.

5.1.1.3. Post- formation measurement

This section presents the analysis of various metrics of the decile portfolio returns in terms of higher order central moments, higher order lower partial moments, drawdown, return volatility, and performance ratios.

An analysis of the higher order central moments shows that decile 10 has a right skewed and fat tail distribution. Positive excess kurtosis indicates the peakedness (leptokurtic) of the distribution with fatter tail or the higher probability for left tail events than that predicted by the Gaussian distribution, which has zero excess kurtosis. There is no particular monotonic pattern for the skewness among different decile portfolios. Decile 10 and decile 9 have more positive skewness while all other decile portfolios have left skewed return distribution. These negative skewed deciles have higher probability to have big loss while the mass of the distribution stays on the right.

Benchmarking and liquidity come hand in hand as managers compete with each other, refraining from high tracking errors in fear of capital withdrawal that creates the limit of arbitrage. As a result, risk management takes into account the benchmark threshold rather than the average. LPM measures the distribution below target return, which is a matter of concern for fund managers who are under benchmarking mandate to deviate only within certain range from the broad index. Even though decile 10 produces the better raw returns as well as risk adjusted returns, it has the higher expected shortfall of the investment targets. The hypothetical target returns at 10% takes into consideration risk premium, inflation, and common costs in the industry (see, e.g., Dimson, E., Marsh, P., and Staunton, M., 2009). Average monthly shortfall below the target return decreases with volatility, ranging from 6.48% to 2.62% for decile 10 to decile 1. Similar to downside deviation, second LPM measures the volatility level below the target return. The volatility below the hypothetical target returns for decile 10 is reasonably the highest among all other deciles, showing high volatility decile portfolios are more difficult to predict under the threshold. Second order LPM overcomes the setback that raw volatility penalizes the variation both above and below the mean. It is arguable that risk lies where there is more loss uncertainty. Consistent with higher average return and higher risk adjusted return, high volatility decile portfolio has the most positive skewness with third order LPM in a downward trend, ranging from 2.13 for decile 10 to only 0.1 for decile 1. 4^{th} order LPM is consistent with two side kurtosis measure, 4th order central moment that decile 10 has more peaked distribution.

Drawdowns' size, frequency, and duration provide valuable information in understanding the back tested strategy. Compared with the bottom volatility portfolio, the top volatility portfolio is the only portfolio that has the different maxdrawdown period in 2000s during the dotcom bubble. Consistently, the component analysis shows the dominance of stocks of computer programming and semiconductor related firms in the decile. Drawdown period starts in August 2000, reaches the bottom in September 2002, and recovers the initial value in June 2004. The loss magnitude has a positive relation with volatility as the corresponding loss value decreases monotonically from decile 10 to decile 1. All other decile portfolios have the maxdrawdowns in the subprime crisis, the largest financial crisis since the Great Depression of the 30s. Decile 10 takes 25 months to destroy 83% of net asset value as the biggest loss among the 10 portfolios and 21 months, the second longest time, to recover at a factor of 4.88. Decile 1, compared with decile 10, takes shorter to recover at a smaller recover rate of less than 1. On average, decile portfolios lose half of the NAV in maxdrawdown periods. Time period of loss and time period of recovering are roughly equal on average, with the average months in maxdrawdown 15.1 and the average months to recover 14.9 across 10 decile portfolios. Except for outliner decile 1 and decile 10, decile portfolios lose from 48% to 56% of its value in nine months. Decile 3, decile 4, and decile 5, which are dominated by stocks of banking sectors, take longer to destroy value and longer to recover, compared with decile 6, decile 7, and decile 8, which are dominated by stocks of oil related firms.

In this thesis, volatility is referred to as a characteristic rather than a risk measure. Among others, volatility does not justify as a risk measure on the ground of non-normality of return distribution and the oversimplification of the mean variance framework. Still, I find that volatility has the consistent informational content with other risk measures. Consistent with the drawdown period, the highest total volatility sorted decile portfolio has the highest post formation return volatility at 9.69% per month or 34% annually. The number is consistent with prior research as mentioned in key statistics. Annual downside deviation below the mean also depicts consistently decreasing pattern from 6.71% for decile 10 to 2.37% for decile 1.

Up percentage ratio and down percentage ratio respectively shows that the high volatility decile portfolio has higher chance to outperform the market in upmarket and that the low volatility decile portfolio has higher chance to outperform the market in down market. Up percentage and down percentage moves in monotonous and opposing directions as volatility increases, ranging from 0.3 to 0.73 for up percentage ratio and 0.84 to 0.33 for down percentage ratio. In addition, decile 10 actually realizes much more severe peak-to-valley drawdown than other deciles and therefore does not seem to make it a good hedge in downturns. Baker et al., (2011) propose the hypothesis that high beta stocks tend to do better in up markets and worse in down markets to explain the excessive risk taking of managers.

The low volatility portfolio has the highest Sharpe ratio both on monthly and annual basis. However, the mere analysis of Sharpe ratio is insufficient due to the setback of volatility as a risk measure. Among other reasons, volatility penalizes both the upside and downside deviation. The higher Sharpe ratio of the low volatility decile is also attributable to the extraordinarily small denominator. The spread in Sortino ratio is not large in the cross section.

Table 1: Equally weighted portfolios sorted by total volatility

Equally weighted decile portfolios are formed by sorting stocks into decile portfolios based on total volatility as the standard deviation of holding period returns over the past 48 months. For each month, I sort the top 1000 stocks by market capitalization from all publicly traded stocks tracked by CRSP with at least 24 months of return history into 10 equal deciles according to total volatility. The universe of stocks is reconstituted every year to take in stocks that increase in market capitalization and drop out stocks that decrease in market capitalization, condition on the total number of stocks unchanged at 1000. Stocks are sorted into deciles based on total volatility from the lowest (decile 1) to the highest (decile 10). The statistics in the columns labeled average return and std. dev. are measured in monthly percentage terms and apply to total, not excess, simple returns. The column "10-1" refers to the difference in the corresponding metric between decile 10 and decile 1. Pair wised t test statistic 2.94, which accounts for the statistical difference in average return between decile 10 and decile 1, is statistically significant at 1% significance level. The sample period is from January 1983 to December 2013.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
	Key statistics										
Average return	2.10	1.62	1.39	1.45	1.38	1.32	1.32	1.23	1.26	1.10	1.01
Std. dev.	9.69	7.29	5.86	5.17	4.83	4.60	4.22	3.93	3.57	3.12	6.56
Annualized std. dev.	33.56	25.27	20.29	17.92	16.73	15.92	14.62	13.60	12.38	10.83	22.73
Largest monthly gain	47.84	45.89	22.31	21.77	18.80	17.18	14.40	13.40	13.01	10.01	37.83
Largest monthly loss	-33.08	-29.36	-27.35	-25.47	-25.83	-23.95	-22.90	-19.64	-17.83	-12.46	-20.62
% of positive months	64.00	64.00	64.00	66.00	68.00	67.00	67.00	70.00	72.00	69.00	-5.00
% of negative months	36.00	36.00	36.00	34.00	32.00	33.00	33.00	30.00	28.00	31.00	5.00

Table 2: Post-formation factor loadings on the equally weighted portfolios sorted by total volatility

Equally weighted decile portfolios are formed by sorting stocks into decile portfolios based on total volatility as the standard deviation of holding period returns over the past 48 months. For each month, I sort the top 1000 stocks by market capitalization from all public traded stocks tracked by CRSP with at least 24 months of return history into 10 equal deciles according to total volatility. The universe of stocks is reconstituted every year to take in stocks that increase in market capitalization and drop out stocks that decrease in market capitalization, condition on the total number of stocks unchanged at 1000. Stocks are sorted into deciles based on total volatility from the lowest (decile 1) to the highest (decile 10). The column "10-1" refers to the difference in alpha between decile 10 and decile 1. I estimate alpha and the ex-post betas by running the three-factor regressions with FF-3 factors. The row labeled "Joint test p-value" reports a Gibbons, Ross, and Shanken (1989) test for the alphas equal to zero, and a robust joint test that the factor loadings are equal to zero. Robust Newey–West (1987) t-statistics, which are more robust than ordinary t-statistics in the presence of autocorrelation and heteroscedasticity, are reported in square brackets. The sample period is from January 1983 to December 2013.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
Mkt-RF	1.588	1.316	1.158	1.055	0.992	0.929	0.847	0.762	0.673	0.489	
	[13.35]	[16.29]	[19.80]	[23.84]	[24.03]	[23.61]	[20.76]	[18.31]	[17.20]	[12.87]	
SMB	0.206	0.163	0.141	0.179	0.17	0.17	0.122	0.052	0.003	-0.025	
	[0.72]	[0.67]	[1.05]	[2.64]	[4.20]	[2.99]	[2.13]	[0.71]	[0.03]	[-0.29]	
HML	-0.408	-0.059	0.094	0.211	0.306	0.308	0.27	0.241	0.244	0.326	
	[-2.13]	[-0.45]	[1.24]	[4.01]	[6.31]	[5.92]	[4.95]	[4.13]	[3.95]	[5.72]	
Alpha	0.014	0.007	0.004	0.005	0.004	0.004	0.004	0.004	0.004	0.003	0.011
	[4.84]	[3.53]	[2.57]	[3.29]	[2.82]	[2.89]	[3.32]	[2.80]	[3.65]	[2.57]	0.00*
Adjusted R^2	0.63	0.69	0.79	0.83	0.82	0.80	0.78	0.72	0.66	0.47	

* Joint test p value 5,731e-11

Table 3: Post-formation distribution of returns on the equally weighted portfolios sorted by total volatility

Table 3 presents various distribution measurements of decile portfolio returns. Central moments measure the distribution around the mean. 1st, 2nd, 3rd, and 4th order of central moment respectively measures the average, the variation, the skewness, and the kurtosis of the distribution. Lower partial moments measure the distribution of returns below a threshold return level. 1st, 2nd, 3rd, and 4th order of lower partial moment respectively measures the average monthly shortfall below the target return, the downside return volatility below the target return, the skewness below the target return, and the kurtosis below the target return. Jacque-Bera statistics test for the normality of the return distribution. The null hypothesis of normality of return distribution is rejected for nearly all decile portfolios at 99% confidence level. Nearly all Jacque-Bera statistics with the degree of freedom 2 are great than Chi square upper tail critical value 5.991 and 13.816 at the 1% and 5% significance level. The column "10-1" refers to the difference in the corresponding metric between decile 10 and decile 1.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
	Panel A: Higher order central moments										
Skewness	0.26	0.03	-0.70	-0.78	-0.97	-088	-0.95	-0.93	-0.78	-0.64	0.89
Kurtosis	3.27	5.10	3.29	3.63	4.62	4.27	4.60	3.83	3.22	1.52	1.75
Jacque-Bera test statistic	5.21	62.57	31.50	44.10	98.68	72.87	95.09	64.15	38.27	59.01	
	Panel B: Higher order lower partial moments										
1st order LPM	6.48	5.38	4.42	4.09	3.66	3.63	3.38	3.07	2.78	2.62	3.86
2nd order LPM	9.19	7.53	6.29	5.75	5.39	5.14	4.77	4.41	3.93	3.48	5.71
3rd order LPM	2.13	1.20	0.74	0.55	0.54	0.43	0.34	0.28	0.20	0.10	2.04
4th order LPM	2.64	-0.25	-1.37	-1.90	-1.99	-2.23	-2.41	-2.64	-2.82	-2.96	5.60

Table 4: Post-formation performance measurement of the equally weighted portfolios sorted by total volatility

Table 4 presents various performance measures of decile portfolios. The maxdrawdown is the largest cumulative percentage decline in month end net asset value (NAV), during a period in which the peak month end NAV is not equaled or exceeded by a subsequent month end NAV. Sharpe ratio measures the ratio of excess return and the standard deviation. Sortino measures the ratio of excess return and downside deviation. Up percentage ratio is a measure of the probability the portfolio outperforms the benchmark when the benchmark is up. Down percentage ratio is a measure of the probability the portfolio outperforms the benchmark when the benchmark is down. The column "10-1" refers to the difference in the corresponding metric between decile 10 and decile 1.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
	Panel A: Drawdown										
Maxdrawdown	-82.57	-56.45	-54.67	-48.04	-51.80	-49.31	-47.37	-50.18	-36.61	-38.33	-44.24
Months in maxdrawdown	25.00	9.00	9.00	9.00	9.00	16.00	21.00	21.00	16.00	16.00	9.00
Months to recover	21.00	7.00	13.00	7.00	13.00	13.00	14.00	24.00	13.00	24.00	-3.00
Peak	20000831	20080530	20080530	20080530	20080530	20071031	20070531	20070531	20071031	20071031	
Valley	20020930	20090227	20090227	20090227	20090227	20090227	20090227	20090227	20090227	20090227	
Recover	20040630	20090930	20100331	20090930	20100331	20100331	20100430	20110228	20100331	20110228	
	Panel B: Comparison to benchmark										
Sharpe ratio	0.23	0.21	0.21	0.24	0.24	0.24	0.25	0.25	0.27	0.26	-0.03
Downside deviation	6.71	5.25	4.40	3.93	3.70	3.51	3.22	3.01	2.71	2.37	4.34
Sortino ratio	0.33	0.30	0.28	0.32	0.31	0.31	0.33	0.32	0.36	0.34	-0.01
Correlation	0.78	0.83	0.89	0.90	0.89	0.87	0.86	0.83	0.79	0.61	0.17
Up percentage ratio	0.73	0.70	0.64	0.65	0.64	0.60	0.53	0.42	0.38	0.30	0.43
Down percentage ratio	0.33	0.39	0.44	0.51	0.59	0.63	0.67	0.80	0.84	0.84	-0.51

5.1.2. Market capitalization weighted decile portfolios sorted by total volatility

Tables 3 shows key statistics that the average returns and volatility of returns are abysmally small arguably due to the market capitalization weighting being biased towards large market capitalization companies. Neither is there the monotonic pattern nor significant spread between the average returns of decile portfolios. Decile portfolios have abysmally small monthly average returns compared with the equally weighted decile portfolios, ranging from 0.15% to 0.21% or 1.85% to 2.53% annually. All decile portfolios underperform the risk free rate, which has 0.34% monthly average return. On average, the market cap weighted decile portfolios have the average monthly return 0.17% while the equally weighted decile portfolios have the average monthly return 1.57%. Looking at the whole distribution, decile 9 and 10 have positive skewness while decile 1 is negatively skewed.

5.1.2.1. Post-formation factor loading estimation

Unlike the case of equal weighting, there is no increasing trend between volatility and market beta exposure. The market cap weighted portfolios have a bias towards the high capitalization stocks, which are in general less volatile in nature. Therefore, these portfolios are not expected to covariate with the size factor. Indeed, most deciles do not covariate with the size factor with any statistical confidence level by using robust Newey West standard error. By contrast, the value beta shows the opposite sign with decile 1 having significant and most positive beta coefficient 0.026 (t-statistic 3.77). Decile 10 has the lowest and negative beta -0.022 (t-statistic -1.75). It can be interpreted that low volatility portfolio co-moves with value firms and the high volatility portfolio with growth firms.

Decile portfolios on average have an alpha of -0.02% (t-statistic>1.96) per month for total volatility sorted decile portfolios. I run the Gibbons, Ross, and Shanken (1989) to formally test the hypothesis that a set of explanatory variables produce regression intercepts that are all equal to zero for the 10 decile portfolios. The F statistics rejects the hypothesis that market, size, and value factor exposure adequately explain the average returns on decile portfolios at the 99% confidence level. Interestingly, the alphas of decile portfolios turn negative and the positive relation between volatility and beta from the equal weighting case is distorted. Black (1972), Jensen, and Scholes (1972), Haugen and Heins (1975), and Fama and French (1992) also report that the beta and average return relation has been much flatter than that predicted by CAPM.

5.1.2.2. Post- formation measurement

The similarity between equal weighting and cap weighting is that decile 10 and decile 9 have more positively skewed return distribution. The shortfall and variation below the target return are on the same level in the cross section. Such a pattern is sensible, given the average return level does not differ significantly across deciles. The return distribution of all decile portfolios defies normality at 99% confidence level. The cap weighting has a significant effect on the low volatility deciles, with decile 1, decile 2, and decile 3 enduring the largest maximum drawdowns. However, there is no distinct pattern detected for drawdown analysis. On average, the decile portfolios lose 5.4% of NAV in the maxdrawdown periods. The time period of loss and time period of recovering are roughly equal in length, with the average months in drawdown 17.8 and the average months to recover 17.1 across 10 decile portfolios. Low volatility portfolios all underperform the risk-free rate with a negative Sharpe ratio.

In summary, replicating the similar process for equal weighting, the market cap weighted portfolio surprisingly does not show any prior monotonic increasing pattern between average returns and volatility. All the market capitalization weighted deciles underperform the FF-3 market factor and their equally weighted counterparts. However, value factor loading still provides consistent information that bottom volatility decile moves positively with the value factor while the opposite applies to the top volatility decile portfolio. Switching weighting schemes shows (i) naïve equal weighting yields better average returns than market capitalization weighting for decile portfolios (ii) a weakening argument for the outperformance of low volatility stocks.

Table 5: Market capitalization weighted portfolios sorted by total volatility

Market capitalization weighted decile portfolios are formed by sorting stocks into decile portfolios based on total volatility as the standard deviation of holding period returns over the past 48 months. For each month, I sort the top 1000 stocks by market capitalization from all publicly traded stocks tracked by CRSP with at least 24 months of return history into 10 equal deciles according to total volatility. The universe of stocks is reconstituted every year to take in stocks that increase in market capitalization and drop out stocks that decrease in market capitalization, condition on the total number of stocks unchanged at 1000. Stocks are sorted into deciles based on total volatility from the lowest (decile 1) to the highest (decile 10). The statistics in the columns labeled average return and std. dev. are measured in monthly percentage terms and apply to total, not excess, simple returns. The row "10-1" refers to the difference in the corresponding metric between decile 10 and decile 1. Pair wised t test statistic,-0.08, which accounts for statistical difference in average return between decile 10 and decile 1, is non-statistically significant at any significance level. The sample period is from January 1983 to December 2013.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
	Key statistics										
Average return	0.17	0.16	0.15	0.17	0.17	0.15	0.19	0.18	0.21	0.17	0.00
Std. dev.	0.48	0.5	0.47	0.52	0.51	0.47	0.53	0.55	0.64	0.49	-0.01
Annualised std. dev.	1.65	1.72	1.62	1.79	1.75	1.64	1.84	1.9	2.22	1.69	-0.04
Largest monthly gain	3.24	3 .71	1.77	2.28	2.35	1.7	2.06	2.68	2.83	1.98	1.26
Largest monthly loss	-1.38	-1.51	-1.66	-2.1	-2.02	-2.89	-2.18	-2.2	-2.94	-2.34	0.96
% of positive months	0.66	0.66	0.65	0.69	0.69	0.68	0.68	0.67	0.68	0.67	-0.01
% of negative months	0.34	0.34	0.35	0.31	0.31	0.32	0.32	0.33	0.32	0.33	0.01

Table 6: Post-formation factor loadings on the market capitalization weighted portfolios sorted by total volatility

Market capitalization weighted decile portfolios are formed by sorting stocks into decile portfolios based on total volatility as the standard deviation of holding period returns over the past 48 months. For each month, I sort the top 1000 stocks by market capitalization from all public traded stocks tracked by CRSP with at least 24 months of return history into 10 equal deciles according to total volatility. The universe of stocks is reconstituted every year to take in stocks that increase in market capitalization and drop out stocks that decrease in market capitalization, condition on the total number of stocks unchanged at 1000. Stocks are sorted into deciles based on total volatility from the lowest (decile 1) to the highest (decile 10). The column “10-1” refers to the difference in alphas between decile 10 and decile 1. I estimate alphas and the ex-post betas by running the three-factor regressions with FF-3 factors. The row labeled “Joint test p-value” reports a Gibbons, Ross, and Shanken (1989) test for the alphas equal to zero, and a robust joint test that the factor loadings are equal to zero. Robust Newey–West (1987) t-statistics, which are more robust than ordinary t-statistics in the presence of autocorrelation and heteroscedasticity, are reported in square brackets. Adjusted R-squared values are from OLS regressions. The sample period is from January 1983 to December 2013.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	
Mkt-RF	0.073	0.083	0.087	0.093	0.101	0.095	0.1	0.1	0.118	0.083	
	[10.16]	[11.16]	[16.62]	[16.17]	[16.67]	[17.00]	[16.63]	[16.19]	[17.51]	[18.52]	
SMB	0.007	0.004	0.002	0.006	-0.009	0.014	0.005	-0.014	-0.026	-0.014	
	[0.44]	[0.18]	[0.19]	[0.52]	[-0.95]	[2.79]	[0.50]	[-1.22]	[-2.01]	[-2.08]	
HML	-0.022	-0.014	-0.015	-0.016	0.009	0.017	0.011	0.008	0.006	0.026	
	[-1.75]	[-1.16]	[-1.73]	[-1.36]	[0.99]	[2.71]	[1.29]	[0.94]	[0.60]	[3.77]	
Alpha	-0.002	-0.002	-0.002	-0.002	-0.002	-0.003	-0.002	-0.002	-0.002	-0.002	0.00
	[-8.14]	[-9.30]	[-10.55]	[-9.65]	[-10.44]	[-11.63]	[-9.43]	[-10.10]	[-9.18]	[-12.08]	0.00*
Adjusted R^2	0.46	0.5	0.62	0.62	0.64	0.68	0.6	0.58	0.63	0.48	

* Joint test p value 0

Table 7: Post-formation distribution of returns on the market capitalization weighted decile portfolios sorted by total volatility

Table 7 presents various distribution measurements of decile portfolio returns. Central moments measure the distribution around the mean. 1st, 2nd, 3rd, and 4th order of central moment respectively measures the average, the variation, the skewness, and the kurtosis of the distribution. Lower partial moments measure the distribution of returns below a threshold return. 1st, 2nd, 3rd, and 4th order of lower partial moment respectively measures the average monthly shortfall below the target return, the downside return volatility below the target return, the skewness below the target return, and the kurtosis below the target return. Jacque-Bera statistics test for the normality of the return distribution. The null hypothesis of normality of return distribution is rejected for nearly all decile portfolios at 99% confidence level. Nearly all Jacque-Bera statistics with the degree of freedom 2 are great than Chi square upper tail critical value 5.991 and 13.816 at the 1% and 5% significance level. The column “10-1” refers to the difference in the corresponding metric between decile 10 and decile 1.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
	Panel A: Higher order central moments										
Skewness	1.32	1.09	-0.04	-0.30	0.05	-1.02	-0.23	-0.21	-0.24	-0.67	1.99
Kurtosis	7.60	8.08	1.61	3.77	2.60	5.04	2.58	3.08	3.01	4.04	3.56
Jacque-Bera test statistic	436.32	402.66	29.85	14.83	2.65	128.81	6.06	2.92	3.52	44.47	
	Panel B: Higher order lower partial moments										
1st order LPM	0.74	0.75	0.75	0.74	0.76	0.74	0.74	0.74	0.77	0.74	0.01
2nd order LPM	0.82	0.84	0.85	0.86	0.86	0.85	0.86	0.87	0.93	0.85	-0.02
3rd order LPM	1.31	1.45	0.32	0.70	0.47	0.72	0.61	0.67	1.14	0.65	0.66
4th order LPM	-1.07	-0.68	-0.51	0.47	-0.12	0.61	0.37	0.72	2.55	0.41	-1.48

Table 8: Post-formation performance measurement of the market capitalization weighted decile portfolios sorted by total volatility

Table 8 present various performance measures of decile portfolios. The maxdrawdown is the largest cumulative percentage decline in month end net asset value (NAV), during a period in which the peak month end NAV is not equaled or exceeded by a subsequent month end NAV. Sharpe ratio measures the ratio of excess return and the standard deviation. Sortino measures the ratio of excess return and downside deviation. Up percentage ratio is a measure of the probability the portfolio outperforms the benchmark when the benchmark is up. Down percentage ratio is a measure of the probability the portfolio outperforms the benchmark when the benchmark is down. The column "10-1" refers to the difference in the corresponding metric between decile 10 and decile 1.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
	Drawdown										
Maxdrawdown	-3.63	-3.43	-5.51	-5.25	-5.25	-5.21	-4.82	-7.11	-6.97	-6.79	3.16
Months in maxdrawdown	25.00	9.00	25.00	13.00	16.00	16.00	21.00	16.00	21.00	16.00	9.00
Months to recover	10.00	5.00	21.00	27.00	9.00	12.00	5.00	36.00	22.00	24.00	-14.00
Peak	20000831	20080530	20000831	20000831	20071031	20071031	20070531	20071031	20070531	20071031	
Valley	20020930	20090227	20020930	20010928	20090227	20090227	20090227	20090227	20090227	20090227	
Recover	20030731	20090731	20040630	20031231	20091130	20100226	20090731	20120229	20101231	20110228	
	Comparison to benchmark										
Sharpe ratio	-0.36	-0.36	-0.41	-0.32	-0.34	-0.39	-0.27	-0.29	-0.20	-0.35	-0.01
Downside deviation	0.30	0.32	0.33	0.37	0.36	0.36	0.38	0.40	0.46	0.36	0.36
Sortino ratio	-0.58	-0.56	-0.57	-0.45	-0.48	-0.51	-0.38	-0.40	-0.28	-0.47	-0.11
Correlation	0.73	0.77	0.86	0.84	0.86	0.88	0.82	0.78	0.79	0.70	0.03
Up percentage ratio	0.02	0.01	0.00	0.00	0.01	0.01	0.01	0.00	0.04	0.03	-0.02
Down percentage ratio	0.99	0.98	0.98	0.99	0.98	1.00	0.98	0.98	0.98	0.98	0.01

5.2. Idiosyncratic volatility

5.2.1. Equally weighted decile portfolios sorted by idiosyncratic volatility

If FF-3 factor exposure explains all excess returns, there is no information left in the residual terms that follow the White noise process. Therefore, sorting portfolios on idiosyncratic volatility provides no difference in returns. If it is not the case, FF-regression residuals may contain informational value. Table 6 shows key statistics of decile portfolios. The average return have a monotonic increase from 1.10% per month for decile 1 to 1.91% per month for decile 10. The volatility of the returns of decile portfolios is also the lowest at 3.38% per month for the bottom decile and increases monotonically to 8.85% per month for the top decile. As a reference point, Ang et al. (2006) estimate the standard deviation of monthly total returns at 3.71% per month for the quintile portfolio with the lowest total volatility stocks and 8.3% per month for the quintile portfolio with the highest total volatility stocks.

5.2.1.1. Post-formation factor loading estimation

Similar to the case of total volatility ranking, covariance with the market explains the most of the variation in decile portfolio returns. Market systematic beta coefficient, ranging from 0.68 for decile 1 to 1.49 to decile 10, shows a monotonic increase with volatility and has the highest value with t-statistic statistically significant at 99% confidence level. The mid-range decile 5, decile 6, and decile 7 have the highest R-squared value in the regressions and the highest correlation with the market movement. However, the extreme top and bottom decile portfolio have among the lowest of these parameters.

The least volatile stock portfolio moves the most with high P/B or value stocks while the most volatile stock portfolio is less sensitive to the movement of value stocks. The conclusion for the size factor exposure remains less clear after controlling for heteroscedasticity. Decile 1 has the highest positive beta coefficient 0.28 (t-statistic 6.03) and decile 10 has the lowest negative beta -0.24 (t-statistic -1.46). There is no distinct pattern as with the case of total volatility ranking. The size beta also increases with average volatility and market beta. Decile 3 to decile 10 have a positive sensitivity to the movement of small stocks, ranging from 0.98 (t-statistic 3.44) of decile 3 to 0.24 (t-statistic 2.62) of decile 10. However after controlling for heteroscedasticity and autocorrelation, the t-statistics are not statistically significant.

The most noteworthy difference in alphas lies in decile 10 alpha of 1.1% (t-statistic 4.84) and decile 1 alpha of 0.2% (t-statistic 2.89). Decile 10 outperforms decile 1 by 0.81% (t-statistic 2.79) per month. Factor exposure explains the least excess return of the most volatile portfolio. The idiosyncratic volatility and alpha relation has been quite flat from decile 1 to decile 9 with meagre difference in alpha of only 10 bps among the first nine deciles. Ang et al. (2006) report the underperformance of the quintile portfolio with the lowest total volatility stocks against the quintile portfolio with the highest total volatility stocks by 1.06% (t-statistic -3.10) per month on average returns. Noticeably, the top R-squared value is improved to up 91% from 83%, compared with that of the regressions in the total ranking method. Again, the middle range volatility deciles, namely decile 5, decile 6, and decile 7 are most precisely predicted by a linear model while R-squared values reduce monotonically to the extremely low or extremely high volatility end. Such a pattern is explainable by the argument that medium level volatility stocks are more representative of the aggregate factors or are less industry concentrated.

5.2.1.2. Post-formation measurement

Decile 10 has positive skewed return distribution while all other decile portfolios have left skewed return distribution. Excess kurtosis, on the other hand, is positive for all deciles, indicating fatter tail than predicted by the Gaussian distribution. Consistently, Jacque-Bera test rejects the null hypothesis of normality of return distribution with 99% confidence level.

Lower partial moments of all the four order follow a decreasing monotonic downward trend. The average monthly shortfall below the target return decreases with the average level of volatility, ranging from 6.09% for decile 10 to 2.74% for decile 1. Similar to the equally weighted total volatility sorted strategy, decile 10 has a higher risk of missing the investment target and is more difficult to predict. The decile also has the most positive skewness and excess kurtosis below target returns, characterized as the black swan events. Ranking by idiosyncratic volatility leads to smaller maxdrawdown by 10.37% for decile 10 but also lower average returns by 26bp and lower alpha by 20 bps, compared with ranking by total volatility. Maxdrawdown period starts in 2000 and reaches the bottom in 2002, with the cumulative return recovery to the initial value in 2004 for decile 10 and decile 9. The loss magnitude has a positive relation with idiosyncratic volatility. Sorting stocks based on idiosyncratic volatility also leads to five months faster time to recover at the expense of 10bps reduction in alphas, compared with sorting by total volatility strategy. Time to recover is critical for investors in liquidity squeeze that could lead to the limit of arbitrage. Decile 10 is the fastest to recover among the equally weighted idiosyncratic volatility sorted portfolios. Low idiosyncratic volatility decile has the worst maximum drawdown and recovers at the lowest pace with the largest loss among the deciles. The required recovery rate is the smallest at 1.7, indicating a 70% cumulative return to recover the 41% loss. On average the decile portfolios lose about half (51%) of the maxdrawdown periods. The time period of losing is longer than the time period of recovering, with average months in drawdown 18.9 and average months to recover 14.8 across 10 decile portfolios.

Compared with the total volatility ranking method, the idiosyncratic ranking method has smaller spread between the bottom and top decile of 109bps in terms of standard deviation and 20bps in terms of average return. Monthly standard deviation increases monotonically from 3.38% to 8.85% as the average return increase from 1.10% to 1.91% for monthly average return from decile 1 to decile 10. Monthly downside deviation below the mean also depicts consistently decreasing pattern in the same range with the total volatility ranking method.

Percentage ratios convey the same information for both total volatility and idiosyncratic volatility ranking method. Up percentage ratio and down percentage ratio show that the high volatility decile portfolio has the higher chance to outperform the market in upmarket while the low volatility decile portfolio has the higher chance to outperform the market in down market. Up percentage and down percentage ratios move in monotonous manner, yet opposing directions. Sharpe Ratio analysis shows that low volatility portfolio has the highest Sharpe ratio both on monthly and annual basis.

In summary, repeating sorting stocks into equally weighted portfolios based on idiosyncratic volatility with respect to the FF-3 model, I find a similar pattern of the positive relation between average returns and idiosyncratic volatility. As the FF-3 factor model does not explain all excess returns, the finding is in line with the increasing literature on the explanatory power of idiosyncratic volatility in the cross section of expected returns (see, e.g., Malkiel and Xu, 2002; Goyal and Santa-Clara, 2003).

Table 9: Equally weighted portfolios sorted by idiosyncratic volatility

I form equally weighted decile portfolios every month by regressing individual stock excess returns on factor returns using the FF-3 model with three year historical data from month t-36 to month t to obtain residual values from month the t to month t+23 and subsequently to estimate idiosyncratic volatility from month t+24 to month t+35. Stocks are sorted into deciles by ranking 12-month idiosyncratic volatility from the lowest (decile 1) to the highest (decile 10). The stock universe consists of 1000 stocks by market capitalization from all public traded stocks tracked by CRSP with at least 24 months of return history. The universe of stocks is reconstituted every year to take in stocks that increase in market capitalization and drop out stocks that decrease in market capitalization, condition on the total number of stocks unchanged at 1000. The statistics in the columns labeled average return and std. dev. are measured in monthly percentage terms and apply to total, not excess, simple returns. The row “10-1” refers to the difference in the corresponding metric between decile 10 and decile 1. Pair wised t statistic 2.79, which accounts for statistical difference in average return between decile 10 and decile 1, is statically significant at 5% significance level. The sample period is from January 1983 to December 2013.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
						Key statistics					
Average return	1.91	1.32	1.37	1.34	1.28	1.22	1.19	1.24	1.16	1.10	0.81
Std. dev.	8.85	6.73	5.75	5.09	4.89	4.59	4.31	4.11	3.70	3.38	5.47
Annualized std. dev	30.67	23.30	19.92	17.62	16.93	15.91	14.93	14.23	12.82	11.71	18.96
Largest monthly gain	46.43	32.50	25.38	20.54	17.68	14.33	14.47	14.92	11.37	10.78	35.66
Largest monthly loss	-31.19	-29.28	-25.30	-24.43	-26.18	-25.10	-22.74	-19.89	-17.15	-14.56	-16.64
% of positive months	0.65	0.61	0.64	0.64	0.67	0.66	0.66	0.68	0.68	0.69	-0.03
% of negative months	0.35	0.39	0.36	0.36	0.33	0.34	0.34	0.32	0.32	0.31	0.03

Table 10: Post-formation factor loadings on the equally weighted portfolios sorted by idiosyncratic volatility

I form equally weighted decile portfolios every month by regressing individual stock excess returns on factor returns using FF-3 model with three year historical data from month t-36 to month t to obtain residual values from month the t to month t+23 and subsequently to estimate idiosyncratic volatility from month t+24 to month t+35. Stocks are sorted into deciles by ranking 12-month idiosyncratic volatility from the lowest (decile 1) to the highest (decile 10). The stock universe consists of 1000 stocks by market capitalization from all public traded stocks tracked by CRSP with at least 24 months of return history. The universe of stocks is reconstituted every year to take in stocks that increase in market capitalization and drop out stocks that decrease in market capitalization, condition on the total number of stocks unchanged at 1000. To correspond with the Fama–French alphas, I compute the ex-post betas by running the three-factor regressions with Fama–French factors. The row labeled "Joint test p-value" reports a Gibbons, Ross, and Shanken (1989) test for the alphas equal to zero, and a robust joint test that the factor loadings are equal to zero. Robust Newey–West (1987) t-statistics, which are more robust than ordinary t-statistics in the presence of autocorrelation and heteroscedasticity, are reported in square brackets. The column "10-1" refers to the difference in alpha between decile 10 and decile 1. The sample period is from January 1983 to December 2013.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
Mkt-RF	1.494	1.313	1.186	1.091	1.05	0.989	0.922	0.863	0.778	0.683	
	[14.93]	[21.95]	[24.74]	[39.86]	[33.23]	[35.51]	[36.34]	[31.02]	[30.61]	[22.70]	
SMB	0.246	0.181	0.187	0.174	0.177	0.136	0.132	0.098	0.031	0.02	
	[0.93]	[1.01]	[1.25]	[2.86]	[2.99]	[2.92]	[2.85]	[1.55]	[0.52]	[0.27]	
HML	-0.247	-0.019	0.168	0.207	0.263	0.243	0.242	0.268	0.246	0.283	
	[-1.46]	[-0.19]	[2.13]	[4.34]	[4.73]	[5.28]	[4.73]	[4.77]	[4.93]	[6.03]	
Alpha	0.011	0.004	0.004	0.003	0.003	0.002	0.002	0.003	0.003	0.002	0.009
	[4.82]	[2.51]	[3.06]	[3.93]	[2.85]	[2.86]	[2.96]	[3.65]	[3.30]	[2.80]	0.00*
Adjusted R^2	0.65	0.8	0.85	0.91	0.91	0.9	0.89	0.85	0.83	0.76	
GRS test statistic	103.71	123.45	198.85	144.84	187.9	263.62	127.89	136.73	100.82	145.47	

* Joint test p value 4.619e-14

Table 11: Post-formation distribution of returns on the equally weighted portfolios sorted by idiosyncratic volatility

Table 11 presents various distribution measurements of decile portfolio returns. Central moments measure the distribution around the mean. 1^{st}, 2^{nd}, 3rd, and 4^{th} order of central moment respectively measures the average, the variation, the skewness, and the kurtosis of the distribution. Lower partial moments measure the distribution of returns below a threshold return. 1^{st}, 2^{nd},3^{rd}, and 4^{th} order of lower partial moment respectively measures the average monthly shortfall below the target return, the downside return volatility below target return, the skewness below target return, and the kurtosis below target return. Jacque-Bera statistics test for the normality of the return distribution. The null hypothesis of normality of return distribution is rejected for nearly all decile portfolios at 99% confidence level. Nearly all Jacque-Bera statistics with the degree of freedom 2 are great than Chi square upper tail critical value 5.991 and 13.816 at the 1% and 5% significance level. The column "10-1" refers to the difference in the corresponding metric between decile 10 and decile 1.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
	Panel A: Higher order central moments										
Skewness	0.28	-0.41	-0.54	-0.72	-0.98	-0.98	-0.90	-0.75	-0.77	-0.66	0.94
Kurtosis	3.65	3.11	3.03	3.42	4.48	4.44	3.94	3.25	3.00	2.15	1.50
Jacque-Bera test statistic	11.46	11.39	17.92	35.02	93.15	92.20	63.68	35.94	36.30	38.37	
	Panel B: Higher order lower partial moments										
1st order LPM	6.09	5.03	4.39	3.83	3.88	3.66	3.30	3.34	2.92	2.74	3.35
2nd order LPM	8.59	7.12	6.15	5.46	5.55	5.20	4.75	4.63	4.08	3.73	4.87
3rd order LPM	2.14	1.25	0.80	0.59	0.66	0.54	0.42	0.34	0.24	0.16	1.98
4th order LPM	2.72	-0.09	-1.29	-1.76	-1.58	-1.88	-2.27	-2.47	-2.70	-2.85	5.57

Table 12: Post-formation performance measurement of the equally weighted portfolios sorted by idiosyncratic volatility

Table 12 presents various performance measures of decile portfolios. The maxdrawdown is the largest cumulative percentage decline in month end net asset value (NAV), during a period in which the peak month end NAV is not equaled or exceeded by a subsequent month end NAV. Sharpe ratio measures the ratio of excess return and the standard deviation. Sortino measures the ratio of excess return and downside deviation. Up percentage ratio is a measure of the probability the portfolio outperforms the benchmark when the benchmark is up. Down percentage ratio is a measure of the probability the portfolio outperforms the benchmark when the benchmark is down. The column "10-1" refers to the difference in the corresponding metric between decile 10 and decile 1.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
	Drawdown										
Maxdrawdown	-72.20	-61.22	-53.01	-50.29	-50.54	-50.24	-44.95	-44.75	-41.31	-41.18	-31.01
Months in maxdrawdown	25.00	30.00	9.00	9.00	16.00	21.00	21.00	21.00	21.00	16.00	9.00
Months to recover	11.00	16.00	13.00	10.00	13.00	19.00	13.00	14.00	19.00	20.00	-9.00
Peak	20000831	20000331	20080530	20080530	20071031	20070531	20070531	20070531	20070531	20071031	
Valley	20020930	20020930	20090227	20090227	20090227	20090227	20090227	20090227	20090227	20090227	
Recover	20030829	20040130	20100331	20091231	20100331	20100930	20100331	20100430	20100930	20101029	
	Comparison to benchmark										
Sharpe ratio	0.22	0.18	0.21	0.22	0.22	0.21	0.22	0.24	0.24	0.24	-0.02
Downside deviation	6.11	4.98	4.29	3.82	3.76	3.52	3.29	3.11	2.81	2.56	3.56
Sortino ratio	0.32	0.24	0.28	0.30	0.28	0.28	0.29	0.32	0.32	0.32	-0.00
Correlation	0.79	0.89	0.92	0.95	0.94	0.94	0.93	0.90	0.89	0.84	-0.05
Up percentage ratio	0.73	0.69	0.66	0.65	0.63	0.63	0.53	0.50	0.38	0.35	0.38
Down percentage ratio	0.40	0.39	0.43	0.57	0.55	0.64	0.70	0.77	0.81	0.84	-0.44

5.2.2. Market capitalization weighted decile portfolios sorted by idiosyncratic volatility

The market cap weighted portfolios sorted by idiosyncratic volatility deliver abysmally small average returns from 0.13% to 0.19% per month, underperforming the risk free rate return and the FF-3 market factor portfolio return. Neither is there a definite pattern detected nor significant spread between volatility and average returns.

5.2.2.1. Post-formation factor loading estimation

The market beta does not increase monotonically with volatility. Decile 2, decile 3, and decile 4 have the highest correlation coefficients with the market and the highest R-squared values. Size beta does not depict any distinct pattern to be explainable with conventional arguments as in the case of the equal weighting method. Market capitalization portfolios have a bias towards large capitalization stocks and are logically expected to not covariate with the size factor. Indeed, decile 10 logically moves positively with the size factor while decile 1 moves negatively. As average returns from decile portfolios do not statistically differ, alphas stay on the same level at -0.02% on average at 99% confidence level.

5.2.2.2. Post-formation measurement

There is no particular monotonic pattern for the skewness among different decile portfolios. All decile portfolio return distribution has positive kurtosis and skewed distribution, defying normality. Shortfall and variation below the target returns are on the same level in the cross section. The top decile 10 and decile 9 have more positive skewness than other decile portfolios. On average, the decile portfolios lose 5.13% of NAV in peak to valley drawdown periods. The time period of loss and time period of recovery is roughly equal, with average months in drawdown 14.2 and average months to recover 13.2 months across 10 decile portfolios. The Sharpe ratio and Sortino ratio stay negative, indicating no compensation for both downside variations.

Table 13: Market capitalization weighted portfolios sorted by idiosyncratic volatility

I form market capitalization decile portfolios every month by regressing individual stock excess returns on factor returns using the FF-3 model with three year historical data from month t-36 to month t to obtain residual values from month t to month t+23 and subsequently to estimate idiosyncratic volatility for month t+24 to month t+35. Stocks are sorted into deciles by ranking 12-month idiosyncratic volatility from the lowest (decile 1) to the highest (decile 10). The stock universe consists of 1000 stocks by market capitalization from all public traded stocks tracked by CRSP with at least 24 months of return history. The universe of stocks is reconstituted every year to take in stocks that increase in market capitalization and drop out stocks that decrease in market capitalization, condition on the total number of stocks unchanged at 1000. The statistics in the columns labeled average returns and std. dev. are measured in monthly percentage terms and apply to total, not excess, simple returns. The row "10-1" refers to the difference in the corresponding metric between decile 10 and decile 1. The statistics in the columns labeled average return and std. dev. are measured in monthly percentage terms and apply to total, not excess, simple returns. Pair wised t-statistic -0.82, which accounts for statistical difference in average returns between decile 10 and decile 1, is not statistically significant. The sample period is from January 1983 to December 2013.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
	Key statistics										
Average return	0.16	0.13	0.14	0.17	0.16	0.16	0.16	0.17	0.19	0.19	-0.02
Std. dev.	0.43	0.44	0.45	0.46	0.49	0.50	0.50	0.56	0.53	0.58	-0.14
Annualized returns	1.97	1.58	1.69	2.02	1.89	1.97	1.94	2.07	2.34	2.24	-0.27
Annualized std. dev.	1.50	1.53	1.57	1.60	1.71	1.72	1.75	1.94	1.85	1.99	-0.49
Largest monthly gain	2.82	2.51	1.99	1.96	1.50	2.14	1.75	1.99	1.82	2.79	0.03
Largest monthly loss	-1.43	-1.36	-1.85	-1.79	-2.44	-2.17	-2.96	-2.28	-2.44	-3.04	1.60
% of positive months	0.67	0.63	0.65	0.66	0.67	0.67	0.68	0.66	0.69	0.67	-0.01
% of negative months	0.33	0.37	0.35	0.34	0.33	0.33	0.32	0.34	0.31	0.33	0.01

Table 14: Post-formation factor loadings on the market capitalization weighted portfolios sorted by idiosyncratic volatility

I form market capitalization decile portfolios every month by regressing individual stock excess returns on Fama factor returns using the FF-3 model with three year historical data from month t-36 to month t to obtain residual values for the (t, t+23) period and subsequently to estimate idiosyncratic volatility for month t+24 to month t+35. Stocks are sorted into deciles by ranking 12-month idiosyncratic volatility from the lowest (decile 1) to the highest (decile 10). The stock universe consists of 1000 stocks by market capitalization from all public traded stocks tracked by CRSP with at least 24 months of return history. The universe of stocks is reconstituted every year to take in stocks that increase in market capitalization and drop out stocks that decrease in market capitalization, condition on the total number of stocks unchanged at 1000. The column "10-1" refers to the difference in the corresponding metric between decile 10 and decile 1. To correspond with the Fama–French alphas, I compute the ex-post betas by running the three-factor regression with Fama–French factors. The row labeled "Joint test p-value" reports a Gibbons, Ross, and Shanken (1989) test for the alphas equal to zero, and a robust joint test that the factor loadings are equal to zero. Robust Newey–West (1987) t-statistics, which are more robust than ordinary t-statistics in the presence of autocorrelation and heteroscedasticity, are reported in square brackets. The sample period is from January 1983 to December 2013.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
Mkt-RF	0.071	0.081	0.089	0.094	0.099	0.104	0.106	0.117	0.111	0.111	
	[10.46]	[14.58]	[19.08]	[18.66]	[28.56]	[25.91]	[21.59]	[25.27]	[21.45]	[14.51]	
SMB	0.006	0.006	0.010	-0.001	0.007	-0.001	0.001	-0.008	-0.013	-0.023	
	[0.42]	[0.49]	[0.92]	[-0.06]	[1.42]	[-0.13]	[0.15]	[-0.88]	[-1.98]	[-1.92]	
HML	-0.014	-0.008	0.003	0.006	-0.006	0.003	0.012	0.016	0.004	-0.001	
	[-1.24]	[-0.89]	[0.38]	[0.68]	[-1.17]	[0.41]	[1.67]	[2.25]	[0.57]	[-0.09]	
Alpha	-0.002	-0.003	-0.003	-0.002	-0.002	-0.002	-0.003	-0.003	-0.002	-0.002	0.00
	[-8.76]	[-11.02]	[-11.68]	[-10.31]	[-16.11]	[-11.67]	[-12.23]	[-11.91]	[-10.11]	[-10.47]	0.00*
OLS adjusted R^2	0.4592	0.5675	0.6393	0.6569	0.7189	0.7316	0.7400	0.7378	0.7397	0.6835	

* Joint test p value 0.00

Table 15: Post-formation distribution of returns of the market capitalization weighted portfolios sorted by idiosyncratic volatility

Table 15 presents various distribution measurements of decile portfolio returns. Central moments measure the distribution around the mean. 1st, 2nd, 3rd, and 4th order of central moment respectively measures the average, variation, skewness, and kurtosis of the distribution. Lower partial moment measures the distribution of returns below a threshold return. 1st, 2nd, 3rd, and 4th order of lower partial moment respectively measures the average monthly shortfall from the target return, the downside return volatility below the target return, the skewness below the target return, and the kurtosis below the target return. Jacque-Bera statistics test for the normality of the return distribution. The null hypothesis of normality of return distribution is rejected for nearly all decile portfolios at 99% confidence level. Jacque-Bera statistic with the degree of freedom 2 is great than Chi square upper tail critical value 5.991 and 13.816 at 1% and 5% significance level. The column "10-1" refers to the difference in the corresponding metric between decile 10 and decile 1. The column "10-1" refers to the difference in the corresponding metric between decile 10 and decile 1.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
	Panel A: Higher order central moments										
Skewness	0.98	0.61	-0.29	0.09	-0.95	-0.30	-0.78	-0.45	-0.62	-0.27	1.25
Kurtosis	6.33	3.83	2.81	2.59	4.30	2.98	4.26	2.17	2.22	4.19	2.14
Jacque-Bera test statistic	231.75	24.53	5.74	3.11	81.53	5.59	62.01	23.49	33.43	26.22	
	Panel B: Higher order lower partial moments										
1st order LPM	0.72	0.77	0.76	0.73	0.75	0.74	0.74	0.76	0.74	0.78	-0.06
2nd order LPM	0.80	0.84	0.84	0.82	0.86	0.85	0.86	0.90	0.87	0.90	-0.10
3rd order LPM	1.31	1.54	0.42	0.33	0.83	0.58	0.83	0.84	0.78	0.99	0.33
4th order LPM	-1.03	-0.62	-0.01	-0.43	1.32	0.50	1.35	1.70	1.22	2.28	-3.31

Table 16: Post-formation performance measurement of the market capitalization weighted portfolios sorted by idiosyncratic volatility

Table 16 presents various performance measures of decile portfolios. The maxdrawdown is the largest cumulative percentage decline in month end net asset value (NAV), during a period in which the peak month end NAV is not equaled or exceeded by a subsequent month end NAV. Sharpe ratio measures the ratio of excess return and the standard deviation. Sortino measures the ratio of excess return and downside deviation. Up percentage ratio is a measure of the probability the portfolio outperforms the benchmark when the benchmark is up. Down percentage ratio is a measure of the probability the portfolio outperforms the benchmark when the benchmark is down. The column "10-1" refers to the difference in the corresponding metric between decile 10 and decile 1.

	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	10-1
						Drawdown					
Maxdrawdown	-3.20	-4.82	-5.33	-5.14	-3.47	-5.95	-6.09	-7.12	-4.68	-5.45	2.25
Months in maxdrawdown	7.00	9.00	16.00	9.00	16.00	16.00	21.00	16.00	16.00	16.00	-9.00
Months to recover	26.00	9.00	19.00	7.00	5.00	13.00	19.00	13.00	7.00	14.00	12.00
Peak	20000831	20080530	20071031	20080530	20071031	20071031	20070531	20071031	20071031	20071031	
Valley	20010330	20090227	20090227	20090227	20090227	20090227	20090227	20090227	20090227	20090227	
Recover	20030530	20091130	20100930	20090930	20090731	20100331	20100930	20100331	20090930	20100430	
						Comparison to benchmark					
Sharpe ratio	-0.41	-0.47	-0.44	-0.37	-0.37	-0.36	-0.35	-0.30	-0.27	-0.27	-0.14
Downside deviation	0.28	0.30	0.33	0.32	0.38	0.36	0.38	0.41	0.40	0.41	-0.13
Sortino ratio	-0.63	-0.70	-0.61	-0.53	-0.49	-0.49	-0.47	-0.40	-0.36	-0.37	-0.25
Correlation	0.75	0.83	0.88	0.89	0.91	0.92	0.92	0.90	0.91	0.84	-0.09
Up percentage ratio	0.01	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.03	-0.02
Down percentage ratio	0.99	0.99	0.98	0.99	0.99	0.99	0.98	1.00	0.98		

6. Robustness Tests

6.1. Robustness to portfolio formation and test window

I investigate the result robustness to the estimation window of total volatility and the holding period. For example, there is the probability that high total volatility stocks may underreact and therefore force returns to be high in the first month. As a result, holding decile portfolios for a longer period can expose the negative relation between volatility and average returns. However, with the variation of the holding and formation period, the outperformance of the decile portfolio with the highest total volatility stocks still holds at 99% confidence level for strategy 36/0/1, 48/0/3, and 48/0/12.

Table 17: Robustness of FF-3 alphas to the portfolio formation and test window

Portfolio formation strategies are based on an estimation period of E months, an awaiting period of W months, and a holding period of H months. The E/W/H strategy is as follows. At month t, I compute total volatility from historical monthly data over an E month period from month t −E−W to month t −W. At time t, I construct equally weighted portfolios based on total volatility and hold these portfolios for N months. My main analysis is based on the 48/0/1 strategy, in which I simply sort stocks into decile portfolios based on their level of total volatility computed using monthly returns over the past 48 months, and I hold these equally weighted portfolios for 1 month. Robust Newey–West (1987) t-statistics, which are more robust than ordinary t-statistics in the presence of autocorrelation and heteroscedasticity, are reported in square brackets.

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	10-1
					Key Statistics						
Alpha	0.003	0.004	0.004	0.004	0.004	0.004	0.005	0.004	0.007	0.014	0.011
	[2.57]	[3.65]	[2.80]	[3.32]	[2.89]	[2.82]	[3.29]	[2.57]	[3.53]	[4.84]	
					Control for Size and Value Effect						
Past 3 year 36/0/1	0.003	0.004	0.004	0.004	0.003	0.004	0.004	0.005	0.007	0.013	0.010
	[2.68]	[3.80]	[3.04]	[3.62]	[2.83]	[2.87]	[3.08]	[2.94]	[3.64]	[4.61]	
3-month holding 48/0/3	0.003	0.004	0.004	0.004	0.004	0.004	0.005	0.005	0.007	0.014	0.011
	[2.71]	[3.53]	[3.14]	[3.27]	[3.07]	[2.72]	[3.37]	[2.97]	[3.39]	[4.85]	
12-month holding 36/0/12	0.003	0.004	0.004	0.003	0.004	0.004	0.004	0.005	0.008	0.013	0.010
	[2.59]	[3.82]	[3.43]	[2.89]	[2.96]	[2.71]	[2.87]	[3.19]	[3.84]	[4.80]	

6.2. *Robustness of alphas*

I use the F statistics of Gibbon, Ross, and Shanken (1989) to formally test the hypothesis that a set of explanatory variables produces regression intercepts for the 10 decile portfolios that are all equal to zero. The test effectively examines the efficiency of the factor portfolios. The F statistics reject the hypothesis that market, size, and value factor suffice to explain the average returns of decile portfolios at 99% confidence level. It can be concluded that the FF-3 factor exposure cannot explain all excess stock returns. The Gibbons statistics, with df_1=372, df_2=358, are bigger than the F critical value at 1.27 and p-value of is less than 0.01% from its F distribution. As a robust check for the weighting experiments, both the custom automated weighting model and the manual weighting model yield the same results conditioning on the weight in use. For example, I replace weight 1/n with the market cap weight in each model and get the same result regardless of the model used.

6.3. *Subsample analysis*

I also investigate if the volatility effect is driven by a single year or a particular crisis period in the following section. However, the outperformance of the decile portfolio with the highest total volatility stocks against the decile portfolio with the lowest total volatility stocks does not seem to be driven by any single year or crisis periods. For example, I drop the data of each year and run the FF-3 regressions without finding significant changes in alphas. The decile portfolio with the highest total volatility stocks not only has larger maxdrawdown but also underperforms the market factor in 13 out of 20 months when the FF-3 market factor portfolio has the largest monthly loss, or when the marginal utility of wealth is high.

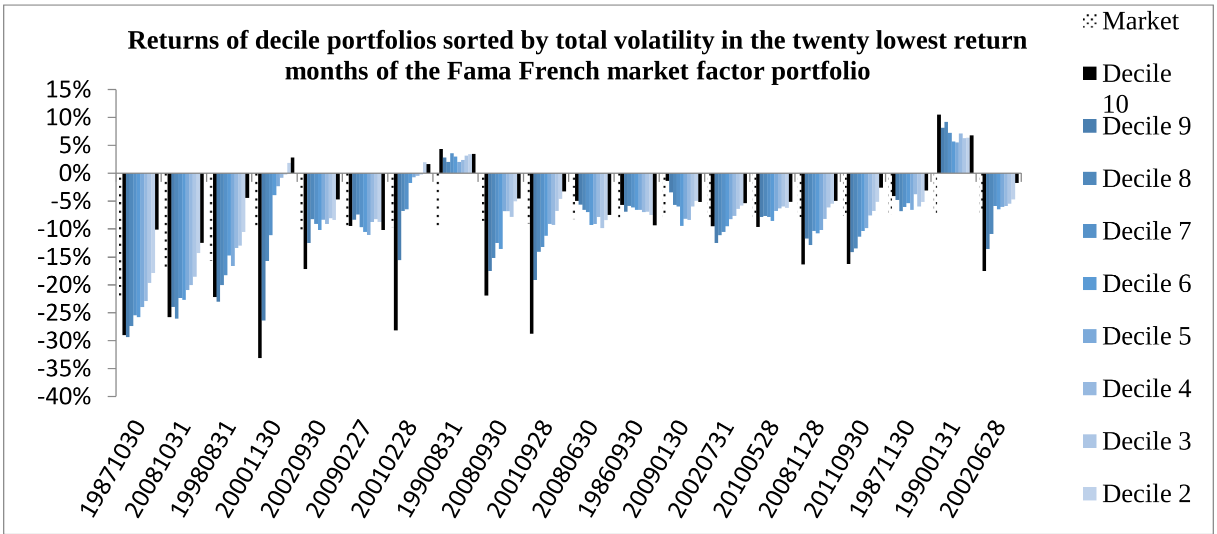

Figure 2: Returns of decile portfolios sorted by total volatility in the twenty lowest return months of the Fama French market factor portfolio

Figure 2 shows decile portfolio returns in the 20 months in which the Fama French market portfolio has the lowest returns. I rank market portfolio returns according to month end returns and benchmark the decile portfolio returns in these corresponding months of market meltdowns. The leftmost white doted bar is the market portfolio. The subsequent black doted bar is the decile portfolio with the highest total volatility stocks. The rightmost black bar is the decile portfolio with the lowest total volatility stocks. Decile 10 underperforms the market factor portfolio in 13 out of 20 worst return months, when the marginal utility of wealth is high.

6.4. Stationarity test

For stationary series, shocks to the system will gradually die away. For non-stationary series, the effect of shocks during time t will not have smaller effect in time t+1, t+2 and so on. If variables in regressions model are not stationary, the standard assumptions of asymptotic analysis are not valid. For example, the usual t statistics will not follow the t distribution and the F statistics will not follow an F distribution. I run the unit root test by Dickey Fuller (1981) to test the null hypothesis of a unit root (non-stationarity) against the alternative hypothesis of stationarity. The Dickey-Fuller statistics are much bigger than the critical value in absolute terms, or more negative. As a result, the null hypothesis of a unit root, or non-stationary, is convincingly rejected for all decile return time series.

6.5. Diagnostic tests for ordinary least squares

Classical linear regression models are based on five assumptions i.e. zero average value of error terms, constant variance of error terms, zero covariance between error terms over time, zero covariance between error term and explanatory variables, and normal distribution of error terms. The violation of assumptions without remedies could lead to any combination of the following problems: biased beta estimation, biased standard error (e.g., due to heteroscedasticity) and inappropriate distribution for the test statistics (e.g., due to non-stationarity). The mean value of the residuals is zero provided that there are constant terms in the regressions (see, e.g., Brooks, C. 2014). I run diagnostic tests for heteroscedasticity, autocorrelation, model specification, and normal distribution of residuals. Remedy will be implemented when applicable. The number of observations in each regression is 372. All results are reported in the appendix.

Heteroscedasticity

If the error is heteroscedastic, ordinary least squares (OLS) estimator still give unbiased and consistent coefficient estimates but no longer have minimum variance among the class of unbiased estimators. White (1980) test for heteroscedasticity in the error distribution by regressing the squared residuals on all distinct regressors, cross-products, and squares of regressors. The null hypothesis of homoscedasticity is tested against the alternative hypothesis of unrestricted heteroscedasticity. The test statistic, a Lagrange multiplier measure, follows Chi-squared (p) distribution under the null hypothesis of homoscedasticity. It is a special case of the Breusch-Pagan test for heteroscedasticity, where the assumption of normally distributed errors has been relaxed and an auxiliary variable list is specified. Chi square statistics are much higher than Chi square critical value with 9 degree of freedom at 10%, 5%, 2.5%, 1%, and 0.5% significance level 14.68, 16.92, 19.02, 21.67, and 23.59 respectively, rejecting the null hypothesis of homoscedasticity. The cause of heteroscedasticity can be traced back to the data points of the crisis periods, e.g., October 1987 that cause the variation in the residuals.

Autocorrelation of residual terms

Ignoring autocorrelation when it is present has the similar effect to that from ignoring heteroscedasticity such that the coefficient estimates obtained from OLS are still unbiased but inefficient. I run Breusch-Godfrey tests for autocorrelation up to the 12^{th} order under the null hypothesis of no correlation. The number of lags is set to 12, given the monthly data frequency. Test statistics follow Chi square distribution with 12 degrees of freedom. Autocorrelation is present in all decile portfolios.

Normality of residual terms

It is common that financial time series to have non-normally distributed residual terms. As the number of observations is large enough, the law of large numbers predicts that the sample mean will converge to the population mean. The definite solution to the non-normality of residuals in linear regressions remains unobvious. The robustness tests for the residuals show that all residuals for regressions defy normal distribution.

6.6. Correction for the heteroscedasticity and autocorrelation of residuals

The remedy would be to use heteroscedasticity and autocorrelation is to use robust standard error by Newey-West rather than the ordinary standard error in the regressions, following Ang et al. (2006). The OLS parameter estimates remain unchanged. OLS assumes that errors are both independent and identically distributed. Robust standard errors relax either or both of those assumptions. When heteroscedasticity is present, robust standard errors tend to be more trustworthy. The error structure is assumed to be heteroscedastic and possibly autocorrelated up to some lags in the Newey-West test. The first step is to run the OLS regression to identify the residuals. The second step is to estimate the correlation between the residuals with its lag based on the correlogram of the residuals up to its 12^{th} lag given the normal fiscal cycle in financial data. The third step is to identify the number of lags to include in the Newey West regression. The number of lags for Newey-West is chosen on a trial basis until the correlation between the residuals and its lags is insignificant as shown when Prob > Q is bigger than 5%. Robust Newey–West (1987) t-statistics are reported in the square brackets of the regressions table 2, table 5, table 8, and table 11 instead of ordinary t statistics. In most cases, autocorrelation at lag 2 has already become insignificant.

6.7. Multicollinearity

Independent variables do not have the problem of multicollinearity. The correlation structure between the variables is quite flat with the correlation between the market factor and the size factor 0.19, the market factor and the value factor at -0.27, and the size factor and the value factor at -0.32.

6.8. Model misspecification

RESET test statistics advocate that linearity is not a perfect model for predicting returns. As stated in the limitation section, the research is subject to the limitations of the fitted model and the current development of financial theories. Still, there is some variations across deciles in the fitness of functional form, with the null hypothesis of linearity fitness. The result is presented in the appendix.

6.9. Normality of decile returns

Jacque-Bera (JB) statistics test for the normality of the return distribution. I run two-sided tests, allowing for the skewness and kurtosis to be both positive and negative. The null hypothesis of normality is rejected if Jacque-Bera statistic is great than Chi square upper tail critical value with the degree of freedom 2, at 5.99 and 13.81 with 99% and 55% confidence level respectively. The null hypothesis of normality is rejected for nearly all decile portfolios at 99% confidence.

7. Discussion

The performance of commercialized low volatility index might well be a short regime switching phenomena rather than a long standing anomaly. For example, stocks with low volatility have been reported to outperform stocks with high volatility in very low interest rate market conditions possibly due to the intervention of central banks during the last decade. The low interest rate is more likely to be linked to low economic activities (see, e.g., French, Schwert, and Stambaugh, 1987; Campbell and Hentschel, 1992). In downturn market conditions, low volatility stocks that are more likely to be defensive high dividend yield stocks tend to pare better due to their low co-movement with the market. These companies are on average more highly leveraged and therefore benefit more from the lower cost of debt. In addition, defensive stocks take advantage from the flight to income in these downturn periods. By contrast, high volatility stocks are more sensitive to negative economic environment due to their bigger need for capital, considering their nature as growth firms with higher co-movement with the market. Highly volatile stocks are therefore more likely to be discounted at higher rate and therefore have lower returns. As rates get lifted up along with the improvement in economic conditions, the bottom decile dominated by stocks of electricity service related firms are likely to find it more challenging. These firms have more difficulty to raise prices to adapt to the high cost of debt due to being more strictly regulated. On the contrary, high volatility stocks can yield better returns thanks to the benefits from the growth with the market. In light of the current returning volatility and the rising rate potential in 1Q2015, low volatility stocks such as those of utility firms can also be expected to trail due to stressed valuation and high volatility stocks such as those of technology firms start to gain momentum due to strong fundamentals. My research shows that these high volatility stocks tend to outperform the market in upswings. Baker et al. (2011) find that the spread between high volatility and low volatility stock returns is time-varying especially with the boom market, e.g., in 2000, or with the change in the Treasury yield[4].

Another possible reason for the low volatility effect reported from prior research concerns the rebalancing scheme. It can be argued that low volatility stocks have less performance cost of a "buy high sell low". For example, low volatility stocks have less risk of dropping out of reconstituted stock universe due to their smaller fluctuations in prices, given their tendency to have larger market capitalization by nature. As a result, the low volatility anomaly is achieved because of the reconstitution principle. In such case, the previous findings of the low volatility effect from prior research could be subject to the discretionary reconstitution, e.g., of the providers of low volatility indexes, which has the compounding effect on the return of the portfolio with the low volatility stocks. The existence of the reconstitution drag has not been reported in details in prior research.

8. Conclusion

I form equally weighted and market capitalization weighted portfolios by ranking stocks by total volatility and idiosyncratic volatility with respect to the FF-3 model into decile portfolios, from the lowest decile (decile 1) to the highest decile (decile 10) on a monthly basis. For preformation parameter estimation, total volatility is estimated based on 48-month holding period returns and idiosyncratic volatility is estimated based on 12-month residuals from rolling FF-3 regressions. Portfolios are formed and hold for one month as the base case. For post-formation, portfolios are measured on various benchmark-based risk and return metrics. Excess returns from decile portfolios are regressed against corresponding FF-3 factor returns to control for the size and value factor exposure.

I find statistically significant evidence that low volatility effect does not exist in equally weighted portfolios. The equally weighted decile portfolio with the highest total volatility stocks outperforms the equally weighted decile portfolio with the lowest total volatility stocks by 1.01% (t-statistic 2.94) in monthly average return. After controlling for the size and the value factor exposure, the alphas are statistically significant. Basu et al. (2007) report that the quintile portfolio with the highest total volatility stocks outperforms the quintile portfolio with the

[4] During 1990-2014, the correlation between change in yield and return of high minus low volatility return strategy has spiked to 32% over the three year from 2012 to 2014 (State Street Advisors, 2014).

lowest total volatility stocks by 0.74% (p-value 1.37%) with CRSP stocks from 1980 to 2004. The result is also consistent with the findings of Fu (2009) and most recently with those of Li et al. (2014), with even stronger statistical significance. As discussed, Li et al. (2014) report that the outperformance of low volatility stocks disappears after dropping the lowly priced stocks from the data set. Consistently, penny stocks also constitute a very insignificant portion in my research data. The realized ex-post total volatility increases monotonically with the market beta and the portfolio average return but not with the FF-3 alpha. Keeping in mind that the portfolios of the most volatile stocks are not necessarily the riskiest, I still find the consistency among various risk measures. The portfolio with the highest total volatility stocks has the biggest ex-post maxdrawdown with fat tail return distribution and is more likely to underperform the market factor portfolio in meltdowns when the marginal utility of wealth is high. However, high volatility portfolio is more likely to outperform the market factor in market upswings. Baker et al., (2011) also propose that high beta stocks tend to do better in upswings and worse in downswings. They argue that money managers, who refrain from underperforming competitors especially in the favorable market conditions, have the unjustified preference for high beta stocks in good market condition.

Interestingly, the coefficient of determination in the regressions using the FF-3 model increases with the industry diversification of decile portfolios. The component analysis shows that the decile portfolio with the highest total volatility stocks is dominated by those of companies in semiconductor device related industry while the decile portfolio with the lowest total volatility stocks is dominated by those of companies in electrics service related industry across the time series. The component analysis is also consistent with the drawdown analysis that the portfolio of the most volatile decile is the only decile that has the maxdrawdown in the dotcom bubble. The rest of the deciles have the maxdrawdowns during the 2008 crisis. Hodrick and Zhang (2001) argue that small and growth portfolios, which are characterized by the top volatility decile, are typically harder to be priced by standard factor models. The peculiarity of the computer and semiconductor related companies potentially explains why realized ex-post total volatility increases monotonically with average returns but not with FF-3 alpha. The component analysis opens a new door for future research to drop the stocks of the computer and semiconductor related firms from the research data to answer the question whether the idiosyncratic characteristics of the dominating firms, rather than solely volatility, are attributable to the relatively high alpha of the top volatility decile.

By switching from equal weighting to cap weighting, I find neither the negative nor positive relation between volatility and average returns of the decile portfolios. At the least, my research shows that naïve equally weighted decile portfolios have higher monthly average returns than market capitalization weighted counterpart portfolios. The result is consistent with prior research related to weighting scheme and the inefficiency of the market capitalization weighted portfolios (see, e.g., Arnott et al., 2005; Estrada J., 2006). Equal weighting scheme has also been noted to tilt to the size factor and therefore utilizes the size effect in decile portfolios. Switching weighting scheme, all else equal, helps to remove the equal weighting premium that leads to the relatively high level of the average returns of the equally weighted portfolios. In addition, the performance of portfolios formed by ranking volatility proves to be contingent on the portfolio weighting. The equal weighting scheme gives more weight to small market capitalization stocks, which are in general more volatile than large market capitalization stocks. At the best, the decile portfolio of the most volatile stocks proves to generate the highest and statistically significant alpha return in the presence of equal weighting premium and rebalancing premium. The results with robust test statistics hold against the variation in the monthly volatility estimation and holding period window.

I also extend the research to sort stocks of the same data set into decile portfolios by idiosyncratic volatility to the extent of testing the efficiency of the FF-3 portfolios. The equally weighted decile portfolio with the highest idiosyncratic volatility stocks outperforms the equally weighted decile portfolio with the lowest idiosyncratic volatility stocks by 0.81% (t-statistic 2.79). That idiosyncratic volatility has informational content shows that the FF-3 factor model does not capture all the excess returns. In line with Lehmann (1990), and Goyal and Santa-Clara (2003), idiosyncratic volatility proves to have the prediction power in the cross section.

The paper has investment implication in volatility investing strategy in terms of premium identification, portfolio weighting scheme, and industry concentration. The weighting scheme effect and the industry dominance effect are the two most influential factors that arguably explain the positively monotonic pattern between total volatility and expected returns and the significant increase in the FF-3 alpha only of the decile portfolio with the highest total volatility stocks. With regard to the decile portfolio with the lowest total volatility stocks, the generic market capitalization weighted portfolio with no industry control or tilted factor does not create value for investors but rather the opposite.

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Appendix

Appendix 1: Realized ex-post average total volatility across decile portfolios

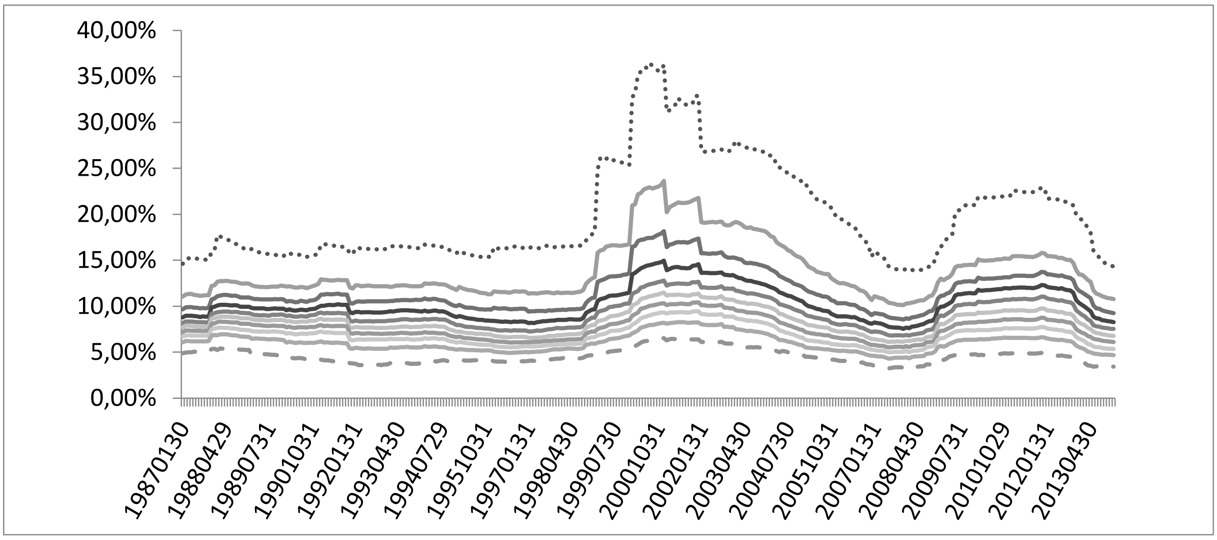
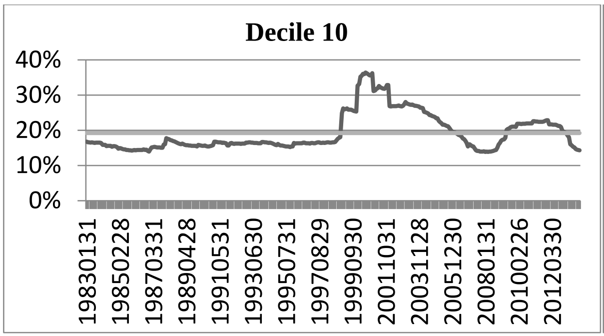
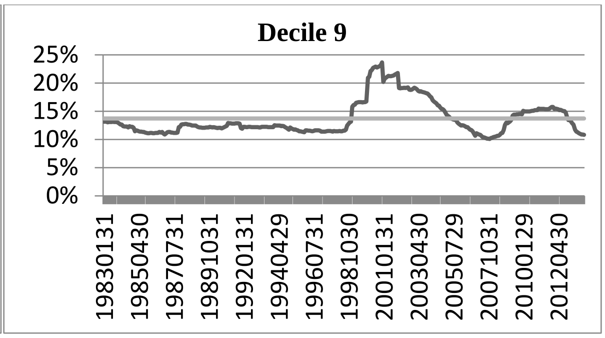


Figure 3: Realized ex-post total volatility across decile portfolios sorted by total volatility

Equally weighted decile portfolios are formed every month from January 1983 to December 2013 by sorting the NYSE, AMEX, and NASDAQ stocks based on total volatility over the past 48 months. Stocks are sorted into deciles based on total volatility from the lowest (decile 1) to the highest (decile 10). Table 1 shows the arithmetic average level of ex-post realized volatility across 100 stocks of 10 decile portfolios. The dot line (the highest) is the level of ex-post realized volatility for decile 10 and the dash line (the lowest) is the level of ex-post realized volatility for decile 1. The other lines correspond to the average level of ex-post realized volatility in the order from high (decile 9) to low (decile 2).

The average level of realized volatility across the decile portfolios has been relatively flat after the October 1987 crash, until the August-September 1998, which is marked by the Russian crisis. Volatility peaks in the dotcom bubble period when decile 10 has the highest volatility innovation at 36.47% in July 2000. Top decile portfolios experience jumps in volatility, e.g., the dotcom bubble while the bottom decile portfolio has stably low volatility level. There is a jump in the average volatility in the year 2000 and 2001 with the realized volatility above 30% for the top volatility decile portfolio. The first half of 2014 before the released favorable economic data on US economy, practitioners also consider a possibility of returning economic crisis judging from the record low level of volatility of 2014, which calls into mind the 2008 crisis's low level of both VIX and realized volatility. In December 2013, the realized volatility was the pre-crisis 2008 level at 14.33% in for decile 10 and 3.47% for decile 1. Before the crisis, the level is 14.09% and 3.66% respectively.


Decile 10
40%
30%
20%
10%
0%
19830131
19850228
19870331
19890428
19910531
19930630
19950731
19970829
19990930
20011031
20031128
20051230
20080131
20100226
20120330


Decile 9
25%
20%
15%
10%
5%
0%
19830131
19850430
19870731
19891031
19920131
19940429
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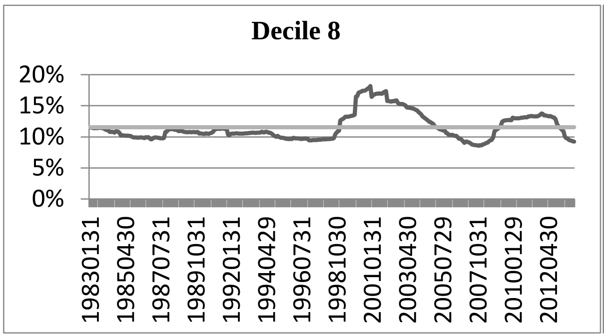
20071031
20100129
20120430


Decile 8
20%
15%
10%
5%
0%
19830131
19850430
19870731
19891031
19920131
19940429
19960731
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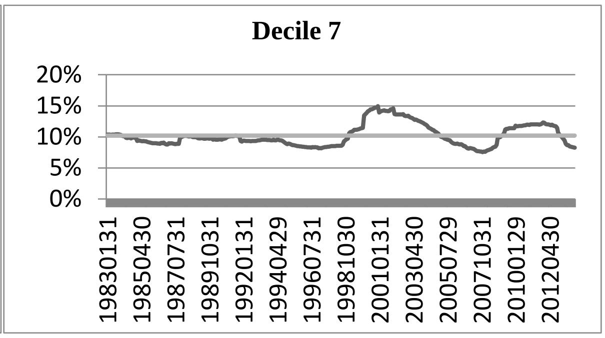
20071031
20100129
20120430


Decile 7
20%
15%
10%
5%
0%
19830131
19850430
19870731
19891031
19920131
19940429
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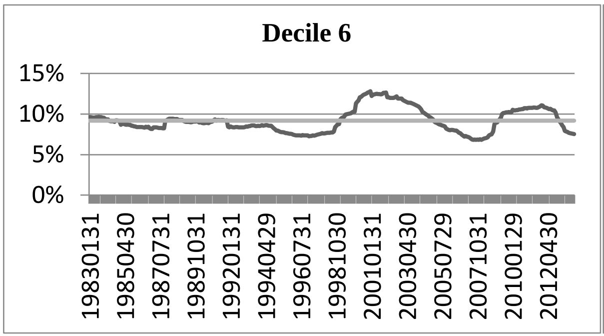
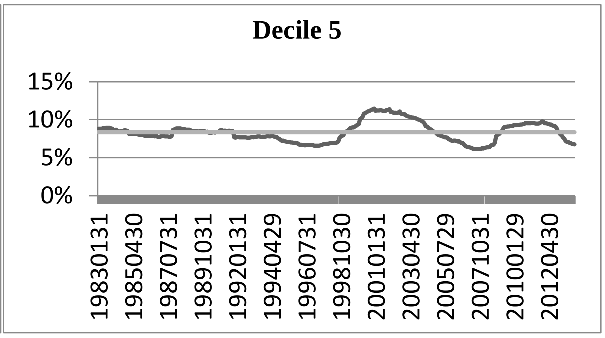
20071031
20100129
20120430


Decile 6
15%
10%
5%
0%
19830131
19850430
19870731
19891031
19920131
19940429
19960731
19981030
20010131
20030430
20050729
20071031
20100129
20120430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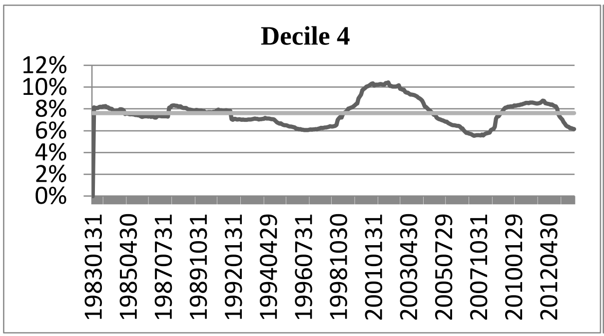
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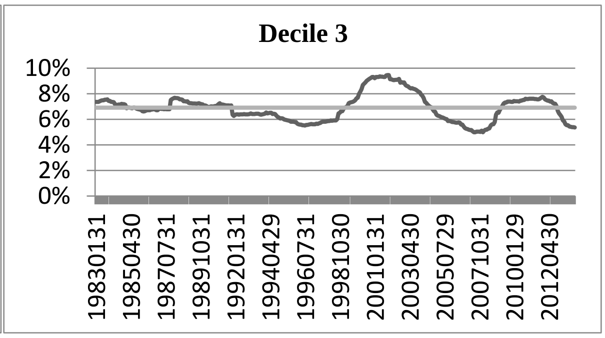
Decile 3
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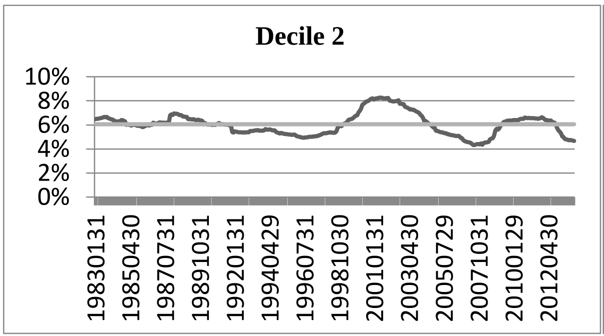
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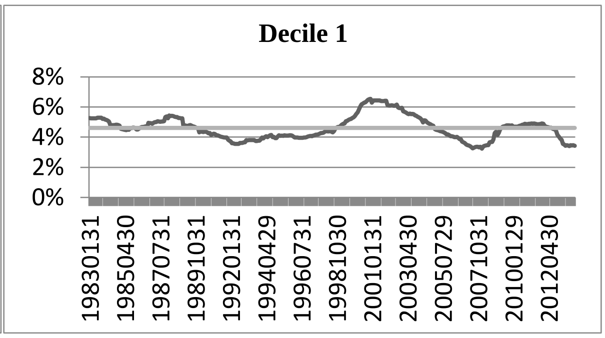
20120430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20120430

Figure 4: Long term average of total volatility across decile portfolios

Equally weighted decile portfolios are formed every month from January 1983 to December 2013 by sorting the NYSE, AMEX, and NASDAQ stocks based on total volatility over the past 48 months. Stocks are sorted into deciles based on total volatility from the lowest (decile 1) to the highest (decile 10). Figure 4 shows the average level of ex-post realized volatility across 100 stocks of ten decile portfolio over year t as curved line and its long term 30 year average as the straight line.

Volatility has also been known to be mean reverting, both for ex-post volatility and implied volatility (see figure 4). In other words, shocks are expected to fade away to the normal long term average. There is small difference in the long term average between consecutive deciles, except for decile 9 and decile 10. Average level of realized volatility is purely lagged arithmetic average. Volatility is not additive and therefore averaging volatility is not a measure of portfolio volatility, where an estimation of covariance matrix is necessary. It is worth noting that each data point in figure 3 and figure 4 is the lagged four year historical data return variation.

Appendix 2: VBA code

```
Function drawdown(NAV As Range) As Variant
    Dim dblValue As Double
    Dim dblMaxValue As Double
    Dim dblMinValue As Double
    Dim rngTemp As Range

    dblValue = 100
    dblMaxValue = 100
    dblMinValue = dblMaxValue
    For Each rngTemp In NAV
         '=B1*(1+A2)
        dblValue = dblValue * (1 + rngTemp.Value)
        If dblValue > dblMaxValue Then dblMaxValue = dblValue
         '=(B2-MAX($B$1:B2))/MAX($B$1:B2)
        If ((dblValue - dblMaxValue) / dblMaxValue) < dblMinValue Then _
        dblMinValue = ((dblValue - dblMaxValue) / dblMaxValue)
    Next
    drawdown = dblMinValue

End Function

Function MAXDRAWDOWNSTART(aReturnVector)

nDigits = 8

n = WorksheetFunction.Max(aReturnVector.Columns.Count,
aReturnVector.Rows.Count)

ReDim aDrawdownVector(1 To n)
ReDim aCumReturn(1 To n)

aCumReturn(1) = aReturnVector.Cells(1).Value
aMax = aCumReturn(1)
aDrawdownVector(1) = aMax - aCumReturn(1)

    For i = 2 To n
        aCumReturn(i) = aCumReturn(i - 1) + aReturnVector.Cells(i).Value
        aMax = WorksheetFunction.Max(aCumReturn(i), aMax)
        aDrawdownVector(i) = aMax - aCumReturn(i)
    Next i

    i = WorksheetFunction.Match(WorksheetFunction.Max(aDrawdownVector),
aDrawdownVector, 0)
    Do
    i = i - 1
    Loop Until
WorksheetFunction.Round(aCumReturn(WorksheetFunction.Match(WorksheetFunctio
n.Max(aDrawdownVector), aDrawdownVector, 0)) +
WorksheetFunction.Max(aDrawdownVector), nDigits) =
WorksheetFunction.Round(aCumReturn(i), nDigits)
    MAXDRAWDOWNSTART = i

End Function

Function MAXDRAWDOWNEND(aReturnVector)

n = WorksheetFunction.Max(aReturnVector.Columns.Count,
aReturnVector.Rows.Count)
```

```
ReDim aDrawdownVector(1 To n)
ReDim aCumReturn(1 To n)

aCumReturn(1) = aReturnVector.Cells(1).Value
aMax = aCumReturn(1)
aDrawdownVector(1) = aMax - aCumReturn(1)

    For i = 2 To n
        aCumReturn(i) = aCumReturn(i - 1) + aReturnVector.Cells(i).Value
        aMax = WorksheetFunction.Max(aCumReturn(i), aMax)
        aDrawdownVector(i) = aMax - aCumReturn(i)
    Next i

    MAXDRAWDOWNEND =
WorksheetFunction.Match(WorksheetFunction.Max(aDrawdownVector),
aDrawdownVector, 0)

End Function

Function MAXDRAWDOWNRECOVERY(aReturnVector)

nDigits = 8

n = WorksheetFunction.Max(aReturnVector.Columns.Count,
aReturnVector.Rows.Count)

ReDim aDrawdownVector(1 To n)
ReDim aCumReturn(1 To n)

aCumReturn(1) = aReturnVector.Cells(1).Value
aMax = aCumReturn(1)
aDrawdownVector(1) = aMax - aCumReturn(1)

    For i = 2 To n
        aCumReturn(i) = aCumReturn(i - 1) + aReturnVector.Cells(i).Value
        aMax = WorksheetFunction.Max(aCumReturn(i), aMax)
        aDrawdownVector(i) = aMax - aCumReturn(i)
    Next i

    i = WorksheetFunction.Match(WorksheetFunction.Max(aDrawdownVector),
aDrawdownVector, 0) - 1
    Do
    i = i + 1
    Loop Until
WorksheetFunction.Round(aCumReturn(WorksheetFunction.Match(WorksheetFunctio
n.Max(aDrawdownVector), aDrawdownVector, 0)) +
WorksheetFunction.Max(aDrawdownVector), nDigits) <=
WorksheetFunction.Round(aCumReturn(i), nDigits) Or i = n

    If
WorksheetFunction.Round(aCumReturn(WorksheetFunction.Match(WorksheetFunctio
n.Max(aDrawdownVector), aDrawdownVector, 0)) +
WorksheetFunction.Max(aDrawdownVector), nDigits) >
WorksheetFunction.Round(aCumReturn(i), nDigits) Then
        MAXDRAWDOWNRECOVERY = CVErr(xlErrNA)
        Else
        MAXDRAWDOWNRECOVERY = i
    End If

End Function
```

```
Function DOWNDEV(ReturnVector, ThresholdReturn)

n = WorksheetFunction.Max(ReturnVector.Columns.Count,
ReturnVector.Rows.Count)

ReDim SquaredDeviation(1 To n)

    For i = 1 To n
        If ReturnVector(i) < ThresholdReturn Then
            SquaredDeviation(i) = (ReturnVector(i) - ThresholdReturn) ^ 2
            Else
            SquaredDeviation(i) = 0
            End If
    Next i

    DOWNDEV = WorksheetFunction.Average(SquaredDeviation) ^ (0.5)

End Function

Function captureratio(updown As String, rng1 As Range, rng2 As Range) As
Double
    Dim l As Long, interim As Double, product1 As Double, product2 As
Double

    If rng1.Count <> rng2.Count Then upcaptureratio = xlErrValue
    product1 = 1
    For l = 1 To rng1.Count

        If updown = "up" Then
            interim = 1 + IIf(rng2.Cells(l) >= 0, rng1.Cells(l), 0)
        Else
            interim = 1 + IIf(rng2.Cells(l) < 0, rng1.Cells(l), 0)
        End If

        product1 = product1 * interim
    Next
    product1 = product1 - 1

    product2 = 1
    For l = 1 To rng2.Count

        If updown = "up" Then
            interim = 1 + IIf(rng2.Cells(l) >= 0, rng2.Cells(l), 0)
        Else
            interim = 1 + IIf(rng2.Cells(l) < 0, rng2.Cells(l), 0)
        End If

        product2 = product2 * interim
    Next
    product2 = product2 - 1

    captureratio = product1 / product2
End Function

Function percentageratio(updown As String, rng1 As Range, rng2 As Range) As
Double
    Dim l As Long, interim As Double, product1 As Double, product2 As
Double

    If rng1.Count <> rng2.Count Then upcaptureratio = xlErrValue
    product1 = 0
```

```
    For i = 1 To rng1.Count

        If updown = "up" Then
            interim = IIf(rng1.Cells(i) >= rng2.Cells(i), IIf(rng2.Cells(i)
>= 0, 1, 0), 0)
        Else
            interim = IIf(rng1.Cells(i) >= rng2.Cells(i), IIf(rng2.Cells(i)
< 0, 1, 0), 0)
        End If

        product1 = interim + product1
    Next
    product1 = product1

    product2 = 0
    For i = 1 To rng2.Count

        If updown = "up" Then
            interim = IIf(rng2.Cells(i) >= 0, 1, 0)
        Else
            interim = IIf(rng2.Cells(i) < 0, 1, 0)
        End If

        product2 = interim + product2
    Next
    product2 = product2

    percentageratio = product1 / product2
End Function

Function numberratio(updown As String, rng1 As Range, rng2 As Range) As
Double
    Dim l As Long, interim As Double, product1 As Double, product2 As
Double

    If rng1.Count <> rng2.Count Then upcaptureratio = xlErrValue
    product1 = 0
    For i = 1 To rng1.Count

        If updown = "up" Then
            interim = IIf(rng1.Cells(i) >= 0, IIf(rng2.Cells(i) >= 0, 1,
0), 0)
        Else
            interim = IIf(rng1.Cells(i) < 0, IIf(rng2.Cells(i) < 0, 1, 0),
0)
        End If

        product1 = interim + product1
    Next
    product1 = product1

    product2 = 0
    For i = 1 To rng2.Count

        If updown = "up" Then
            interim = IIf(rng2.Cells(i) >= 0, 1, 0)
        Else
            interim = IIf(rng2.Cells(i) < 0, 1, 0)
        End If

        product2 = interim + product2
```

```
    Next
    product2 = product2

    numberratio = product1 / product2
End Function

Sub Macro1()
'
' Macro1 Macro
'
    Application.Run "Drawdown.xls!Macro1"
    ActiveCell.Offset(40, 7).Range("A1").Select
    ActiveCell.FormulaR1C1 = ""
    ActiveCell.Offset(33, 0).Range("A1").Select
    ActiveSheet.ChartObjects("Chart 10").Activate
    ActiveCell.Offset(-25, -5).Range("A1").Select
    ActiveWindow.SmallScroll Down:=-21
End Sub

Private Sub Worksheet_SelectionChange(ByVal Target As Range)
  If Target.Count > 1 Then Exit Sub
   For i = 2 To 4
   If Target.Column = i Then
       Target.Columns.ColumnWidth = 30
   Else
       Columns(i).ColumnWidth = 10
   End If
   Next i
End Sub
```

Appendix 3: Robustness tests for regressions of excess returns of the decile portfolios sorted by volatility on FF-3 factor returns

Table 18: Robustness tests for regressions of excess returns of the equally weighted portfolios sorted by total volatility on FF-3 factor returns

Table 18 presents test statistics and the key interpretation of the following tests: Dickey Fuller unit root test for stationarity, diagnostic tests for ordinary least square key assumption violation, Gibbons, Ross, and Shanken (GRS) test for portfolio efficiency, RESET test for model misspecifications. For the stationarity test, the unit root test by Dickey Fuller (1981) is run to test the null hypothesis of a unit root (non-stationarity) against the alternative hypothesis of stationarity. The null hypothesis is rejected if Dickey Fuller statistics are much bigger than the critical value in absolute value. For the OLS diagnostic tests, the White test is run to test the null hypothesis of homoscedasticity against the alternative hypothesis of unrestricted heteroscedasticity. The test statistic, which follows Chi-squared (p) distribution, rejects the null hypothesis if Chi square statistic is higher than Chi square critical value with 9 degrees of freedom. The Breusch-Godfrey test is run to test the null hypothesis of no autocorrelation. The null hypothesis is rejected if the test statistic is bigger than Chi square critical value. The Jacque-Bera test is run to test for the normality of the return distribution. The null hypothesis of normality is rejected if JB statistic, which follows the Chi square distribution with the degree of freedom 2, is great than the critical value. The RESET test is run to test for the null hypothesis of linearity of returns, which is rejected if the test statistic is higher than the critical value. The Gibbons, Ross, and Shanken test is run to test for the alphas equal to zero, and is a robust joint test that the factor loadings are equal to zero as the null hypothesis. The null hypothesis is rejected if test statistics is higher than the critical value.

Test	Test statistics	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	1 % CV	Result
Unit root	Dickey-Fuller	-18.06	-17.57	-17.41	-17.22	-17.06	-17.60	-17.26	-17.61	-17.91	-17.35	-3.45	Stationary
Diagnostic tests for Ordinary Least Squares													
Heteroscedasticity	Chi2(9)	259.29	240.65	129.77	18.62	19.81	87.93	110.88	173.14	187.15	199.43	21.67	Heteroscedasticity
	Prob > chi2	0.00	0.00	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.00		
Autocorrelation	Breusch-Godfrey	19.21	32.05	37.89	28.52	36.07	23.54	42.41	34.12	33.83	13.50	26.21	Autocorrelation except D1,D5,D10
Normality of residuals	Skewness	-0.81	-0.89	-0.93	-0.99	-1.02	-1.03	-1.01	-0.98	-0.94	-0.87		
	Kurtosis	5.54	5.77	5.93	6.15	6.29	6.34	6.26	6.09	5.98	6.06		
	Jacque-Bera	140.68	168.04	186.69	214.56	232.28	238.69	227.97	207.54	192.43	192.06	5.99	Non-normal
Gibbons, Ross, and Shanken	GRS	7.44											
	p-value	5.73E-11										1.27	Significant alpha
RESET	F(3, 365)	0.98	0.22	1.23	4.08	6.11	4.43	5.48	2.76	2.71	9.46		Non-linear; except D9,D10
	Prob > F	0.40	0.88	0.29	0.00	0.00	0.00	0.00	0.04	0.04	0.00		

Table 19: Robustness tests for regressions of excess returns of the market capitalization weighted portfolios sorted by total volatility on FF-3 factor returns

Table 19 presents test statistics and the key interpretation of the following tests: Dickey Fuller unit root test for stationarity, diagnostic tests for ordinary least square key assumption violation, Gibbons, Ross, and Shanken (GRS) test for portfolio efficiency, RESET test for model misspecifications. For the stationarity test, the unit root test by Dickey Fuller (1981) is run to test the null hypothesis of a unit root (non-stationarity) against the alternative hypothesis of stationarity. The null hypothesis is rejected if Dickey Fuller statistics are much bigger than the critical value in absolute value. For the OLS diagnostic tests, the White test is run to test the null hypothesis of homocedasticity against the alternative hypothesis of unrestricted heteroscedasticity. The test statistic, which follows Chi-squared (p) distribution, rejects the null hypothesis if Chi square statistic is higher than Chi square critical value with 9 degrees of freedom. The Breusch-Godfrey test is run to test the null hypothesis of no autocorrelation. The null hypothesis is rejected if the test statistic is bigger than Chi square critical value. The Jacque-Bera test is run to test for the normality of the return distribution. The null hypothesis of normality is rejected if JB statistic, which follows the Chi square distribution with the degree of freedom 2, is great than the critical value. The RESET test is run to test for the null hypothesis of linearity of returns, which is rejected if the test statistics is higher than the critical value. The Gibbons, Ross, and Shanken test is run to test for the alphas equal to zero, and is a robust joint test that the factor loadings are equal to zero as the null hypothesis. The null hypothesis is rejected if test statistics is higher than the critical value.

Test	Test Statistics	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	1 % CV	Result
Unit root	Dickey-Fuller	-18.06	-17.57	-17.41	-17.22	-17.06	-17.60	-17.26	-17.61	-17.91	-17.35	-3.45	Stationary
Diagnostic tests for ordinary least squares													
Heteroscedasticity	Chi2(9)	233.50	245.19	74.63	98.59	74.21	33.18	108.88	67.10	81.79	140.27		Autocorrelation
	Prob > chi2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	21.67	Heteroscedasticity
Autocorrelation	Breusch-Godfrey	126.38	124.22	144.95	109.69	138.06	139.40	106.95	79.33	47.49	71.19	26.21	Autocorrelation
Normality of residuals	Skewness	-0.80	-0.82	-0.81	-0.83	-0.83	-0.97	-0.91	-0.79	-0.72	-0.78		
	Kurtosis	5.47	5.54	5.09	5.56	5.5	6.09	5.84	5.37	5.15	5.49		
	Jacque-Bera	134.24	141.69	108.38	144.29	139.59	206.33	176.36	125.76	103.79	133.82	5.99	Non-normal
Gibbons, Ross,	GRS						36.21						
and Shanken	p-value						0.00					1.27	Significant alpha
RESET	F(3, 365)	2.32	1.36	1.95	0.10	5.29	8.83	4.83	0.76	2.07	4.78		
	Prob > F	0.07	0.25	0.12	0.96	0.00	0.00	0.00	0.51	0.10	0.00		Non-linear

Table 20: Robustness tests for regressions of excess returns of the equally weighted portfolios sorted by idiosyncratic volatility on FF-3 factor returns

Table 20 presents test statistics and the key interpretation of the following tests: Dickey Fuller unit root test for stationarity, diagnostic tests for ordinary least square key assumption violation, Gibbons, Ross, and Shanken (GRS) test for portfolio efficiency, RESET test for model misspecifications. For the stationarity test, the unit root test by Dickey Fuller (1981) is run to test the null hypothesis of a unit root (non-stationarity) against the alternative hypothesis of stationarity. The null hypothesis is rejected if Dickey Fuller statistics are much bigger than the critical value in absolute value. For the OLS diagnostic tests, the White test is run to test the null hypothesis of homocedasticity against the alternative hypothesis of unrestricted heteroscedasticity. The test statistic, which follows Chi-squared (p) distribution, rejects the null hypothesis if Chi square statistic is higher than Chi square critical value with 9 degrees of freedom. The Breusch-Godfrey test is run to test the null hypothesis of no autocorrelation. The null hypothesis is rejected if the test statistic is bigger than Chi square critical value. The Jacque-Bera test is run to test for the normality of the return distribution. The null hypothesis of normality is rejected if JB statistic, which follows the Chi square distribution with the degree of freedom 2, is great than the critical value. The RESET test is run to test for the null hypothesis of linearity of returns, which is rejected if the test statistics is higher than the critical value. The Gibbons, Ross, and Shanken test is run to test for the alphas equal to zero, and is a robust joint test that the factor loadings are equal to zero as the null hypothesis. The null hypothesis is rejected if test statistics is higher than the critical value.

Test	Test statistics	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	1 % CV	Result
Unit root	Dickey-Fuller	-18.73	-17.16	-16.66	-17.17	-16.61	-17.08	-17.45	-17.31	-17.52	-17.50	-3.45	Stationary
Diagnostics tests for ordinary least squares													
Heteroscedasticity	Chi2(9)	296.60	255.00	232.76	113.72	123.75	141.13	281.20	321.92	289.90	267.45	21.67	Heteroscedasticity
	Prob > chi2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
Autocorrelation	Breusch-Godfrey	31.43	21.92	35.39	21.52	46.57	22.93	27.39	23.12	17.10	16.64	26.21	Autocorellation
Normality of residuals	Skewness	-0.85	-0.90	-0.96	-0.98	-1.00	-0.99	-1.00	-1.00	-0.96	-0.97		
	Kurtosis	5.66	5.82	6.06	6.12	6.22	6.16	6.19	6.16	6.03	6.09		
	Jacque-Bera	154.47	173.48	202.28	210.43	222.71	215.54	219.73	216.78	199.44	206.33	5.99	Non-normal
Gibbons, Ross,	GRS					9.31							
and Shanken	p-value					4.62E-14						1.27	Significant alpha
RESET	F(3, 365)	1.05	0.69	0.71	5.74	4.63	4.58	3.31	1.40	0.98	0.34		Non-linear; except
	Prob > F	0.37	0.55	0.54	0.00	0.00	0.00	0.02	0.24	0.40	0.79		D9, D10

Table 21: Robustness tests for regressions of excess returns of the market capitalization weighted portfolios sorted by idiosyncratic volatility on FF-3 factor returns

Table 21 presents test statistics and the key interpretation of the following tests: Dickey Fuller unit root test for stationarity, diagnostic tests for ordinary least square key assumption violation, Gibbons, Ross, and Shanken (GRS) test for portfolio efficiency, RESET test for model misspecifications. For the stationarity test, the unit root test by Dickey Fuller (1981) is run to test the null hypothesis of a unit root (non-stationarity) against the alternative hypothesis of stationarity. The null hypothesis is rejected if Dickey Fuller statistics are much bigger than the critical value in absolute value. For the OLS diagnostic tests, the White test is run to test the null hypothesis of homocedasticity against the alternative hypothesis of unrestricted heteroscedasticity. The test statistic, which follows Chi-squared (p) distribution, rejects the null hypothesis if Chi square statistic is higher than Chi square critical value with 9 degrees of freedom. The Breusch-Godfrey test is run to test the null hypothesis of no autocorrelation. The null hypothesis is rejected if the test statistic is bigger than Chi square critical value. The Jacque-Bera test is run to test for the normality of the return distribution. The null hypothesis of normality is rejected if JB statistic, which follows the Chi square distribution with the degree of freedom 2, is great than the critical value. The RESET test is run to test for the null hypothesis of linearity of returns, which is rejected if the test statistics is higher than the critical value. The Gibbons, Ross, and Shanken test is run to test for the alphas equal to zero, and is a robust joint test that the factor loadings are equal to zero as the null hypothesis. The null hypothesis is rejected if test statistics is higher than the critical value.

Test	Test statistics	D10	D9	D8	D7	D6	D5	D4	D3	D2	D1	1 % CV	Result
Unit root	Dickey-Fuller	-19.37	-18.33	-17.13	-17.57	-19.62	-17.53	-18.29	-19.21	-19.36	-18.91	-3.45	Stationary
Diagnostics tests for ordinary least squares													
Heteroscedasticity	Chi2(9)	274.86	196.76	149.28	108.66	63.24	40.73	86.91	182.12	32.87	156.47	21.67	Heteroscedasticity
	Prob > chi2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
Autocorrelation	Breusch-Godfrey	152.40	156.59	155.76	194.72	173.73	188.51	168.70	145.67	143.94	74.60	26.21	Autocorellation
Normality of residuals	Skewness	-0.83	-0.85	-0.91	-0.87	-0.87	-0.86	-0.89	-0.85	-0.80	0.72		
	Kurtosis	5.57	5.65	5.85	5.67	5.68	5.64	5.75	5.59	5.39	5.14		
	Jacque-Bera	145.09	153.64	177.24	157.43	158.26	153.88	166.33	148.77	128.22	103.12	5.99	Non-normal
Gibbons, Ross, and Shanken	GRS					50.01							
	p-value					0.00						1.27	Significant alpha
RESET	F(3, 365)	0.82	2.45	2.34	3.68	0.99	3.59	5.92	2.10	1.49	3.53		Non-linear;
	Prob > F	0.48	0.06	0.07	0.01	0.39	0.01	0.00	0.10	0.21	0.01		except D10