Improvements to PLSc: Remaining problems and simple solutions

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Abstract

The recent article by Dijkstra and Henseler (2015b) presents a consistent partial least squares (PLSc) estimator that corrects for measurement error attenuation and provides evidence showing that, generally, PLSc performs comparably to a wide variety of more conventional estimators for structural equation models (SEM) with latent variables. However, PLSc does not adjust for other limitations of conventional PLS, namely: (1) bias in estimates of regression coefficients due to capitalization on chance; and (2) overestimation of composite reliability due to the proportionality relation between factor loadings and indicator weights. In this article, we illustrate these problems and then propose a simple solution: the use of unit-weighted composites, rather than those constructed from PLS results, combined with errors-in-variables regression (EIV) by using reliabilities obtained from factor analysis. Our simulations show that these two improvements perform as well as or better than PLSc. We also provide examples of how our proposed estimator can be easily implemented in various proprietary and open source software packages.

Keywords: partial least squares, structural equation modeling, composite variables, latent variables, measurement error, reliability, correction for attenuation, capitalization on chance, errors-in-variables regression

1. Introduction

Partial least squares (PLS) is currently one of the most popular techniques for the estimation of structural equation models in Information Systems (IS) research (Ringle, Sarstedt, & Straub, 2012). Although appealing due to their lower computational demands relative to conventional maximum-likelihood (ML) approaches, the traditional PLS methods are inconsistent and biased estimators of latent variable models (Dijkstra, 2010). These biases occur because composite loadings overestimate factor loadings (McDonald, 1996), and approximating latent variables with composites leads to the well-known measurement error bias in estimates of structural relationships (Bollen, 1989, Chapter 5).

To address these long-standing issues, Dijkstra and Henseler (2015b) build on the classical correction for measurement error and present a new consistent PLS (PLSc) estimator. The results of their simulation study show that PLSc performs comparably to ML and a wide variety of other SEM estimators. Although these findings seem promising, earlier research (Goodhue, Lewis, & Thompson, 2012; Huang, 2013) showed that corrected PLS may not always work well. Particularly – although using a slightly different correction – Goodhue, Lewis, and Thompson (2012) demonstrated that when adjusted for measurement error, PLS path estimates may actually be positively biased. To clarify these concepts further, a consistent estimator is one that converges in probability to the true population value as the sample size approaches infinity. On the other hand, unbiasedness means that the expected value of the estimator (i.e., average after repeated samples from the same population) is the true population value, and this concept applies regardless of the sample size (Bollen, 2011). Therefore, it is possible to have a consistent but biased estimator, that is, one that tends to over or underestimate the true population

parameter in finite samples, but which still approaches the true value as the sample size increases toward infinity.

Due to several issues that we explain in this article, and illustrate via simulated datasets, PLSc can be characterized as a consistent but biased estimator. First, we demonstrate that the PLS weights are susceptible to a small-sample bias, which inflates the correlations between adjacent composites (Rönkkö, 2014; see also Goodhue, Lewis, & Thompson, 2015). Second, we show that the reliability indices used in PLSc, although consistent, only partially address the issues in the estimation of composite reliability in PLS when using sample data (Aguirre-Urreta, Marakas, & Ellis, 2013), leading to a positive bias in these estimates. Third, we build on ideas presented in various prior works (Aguirre-Urreta et al., 2013; Dijkstra, 2010; Dijkstra & Schermelleh-Engel, 2014; Lu, Kwan, Thomas, & Cedzynski, 2011) to propose further corrections to PLSc, in order to overcome the tripartite problems of measurement error, capitalization on chance, and positively biased reliability estimates. These corrections lead to errors-in-variables regression (EIV) with unit-weighted composites and reliabilities obtained from factor analysis. Although factor models can be estimated in a number of ways, we focus on the maximum likelihood confirmatory factor analysis (ML-CFA) and minres approaches because these follow logically from applying the recommendations in the existing literature to the PLSc factor loading correction. Fourth, we replicate Dijkstra and Henseler's (2015b) simulation study, comparing EIV with PLSc and maximum likelihood estimation of the full structural equation model (ML-SEM). Further, we extend the simulation study to conditions with weaker nomological networks, thus providing a direct answer to the call for future research by Dijkstra and Henseler (2015b, p. 17). We conclude our work with a discussion of our findings and

recommendations for researchers and provide examples on how to apply the proposed techniques using different statistical software.

Our research is directly relevant for readers of *MIS Quarterly* for two reasons. First, although PLS is commonly used in the IS discipline, it is largely ignored by the leading research methods journals (Rönkkö & Evermann, 2013); consequently, a large share of current methodological writings on PLS have appeared in *MIS Quarterly* (Rönkkö, 2014). In addition to providing a direct answer to a call for future research by Dijkstra and Henseler (2015b, p. 17), we contribute to the methodological discussion by explaining the reason why the PLSc results obtained by Dijkstra and Henseler (2015b) differ from the positively biased results for disattenuated PLS presented by Goodhue et al. (2012). Second, and more importantly, IS researchers rely on composite-based approximations of factor models perhaps more than practitioners in any other discipline. Therefore, it is important that they use composites that are robust, as well as estimation techniques that take measurement error into account. We thus contribute to applied practice by outlining a more robust estimation approach, which can be used immediately with either existing commercial statistical packages or freely available software.

2. Issue 1: Capitalization on chance by PLS weights

A number of recent studies have shown that traditional PLS is highly susceptible to capitalization on chance in finite samples (Goodhue et al., 2015; Goodhue, Thompson, & Lewis, 2013; Rönkkö, 2014; Rönkkö & Evermann, 2013). We explain this issue using the simple structural equation model shown in Figure 1. When applied to data produced by this model, the PLS algorithm produces two composites (weighted sums) of the indicators. The correlation between these composites, which serves as the starting point for the PLSc path estimate, depends on both the indicator weights as well as the between-block indicator correlations. In finite

samples, these correlations vary around the population values due to natural sampling variability. This creates a problem because the weights also depend on these correlations: if a correlation between indicators a_i and b_j happens to be large simply by chance, both indicators receive larger weights. Conversely, small correlations lead to decreased weights. This overweighting of highly correlated indicators at the expense of indicators with smaller correlations leads to an overall positive bias in the composite correlation (Rönkkö, 2014; Rönkkö & Ylitalo, 2010). To be sure, sampling error affects other types of correlational and multiple regression-based analyses (including ML-SEM and regression with unit-weighted summed scales), but only by increasing the variance of the estimates, and not their bias (Charles, 2005; Rönkkö, 2014; Stanley & Spence, 2014). In the case of PLS, however, sample fluctuations in the raw data-based correlations lead to both bias *and* increased variance of the estimates (Goodhue et al., 2015; Rönkkö, 2014).

---- Insert Figure 1 about here ----

We will next provide two numerical examples to demonstrate this issue. In the population model, all between-block correlations are 0.147 but, as stated above, the sample correlations, from which the PLS estimates are calculated, are never at the exact population values, and their variance depends on the sample size. When the sample size is 100, the standard error of the correlation is 0.100, which means that an observed between-block correlation falls outside the 0.147 ± 0.1 range about one third of the time. To demonstrate the effects that these small sample fluctuations have on PLS weights, we increase one between-block item correlation by 0.1 and decrease another by 0.1(that is, one standard error). These manipulations are marked with dashed lines in Figure 1. Because all other elements of the correlation matrix are exactly at their population values, the two small departures represent controlled conditions that can be used to

demonstrate the issues occasioned by the natural sampling variability that arises in any real dataset. We then used the correlation matrix thus modified to calculate the PLS Mode A weights and the disattenuated correlation between the composites² using the *matrixpls* package for R (Rönkkö, 2015) because, unlike most other PLS software implementations, the *matrixpls* package can obtain estimates directly from a correlation matrix without recourse to raw data.

The PLS Mode A algorithm converged in six iterations. Table 1 shows the weight calculation history and the PLSc estimates of both the factor loadings and the path coefficient for each set of weights. The table also shows the PLSc-specific correction factors (c's) used to rescale the indicator weights to factor loadings, and both Dijkstra and Henseler's (2015b) ρ_A coefficient as well as the true reliabilities (R's) for the composite variables. Because all indicators were equally reliable, unit weights, which are used as starting values in the PLS algorithm, produce the most reliable composites. In this scenario, the two manipulated correlations also cancel each other out exactly and all estimates using the starting weights are exactly correct. However, with the PLS indicator weighting system, the indicators with larger correlations receive higher weights and those with smaller correlations receive lower weights (Rönkkö, 2014), leading to a positive estimation error in the PLSc parameter estimates, as shown in Table 1.

---- Insert Table 1 about here ----

In the second example, we further illustrate the effect of small-sample idiosyncrasies by using a different approach. Instead of taking the population correlation matrix and manipulating it directly, we drew 10000 multivariate normal samples of 100 observations from the unaltered, correct population matrix. Next, the model in Figure 1 was estimated using PLSc under two

² In a simple univariate regression such as that used in this demonstration, the correlation is equivalent to the standardized regression coefficient.

distinct scenarios: (1) the usual case where the indicator weights are calculated and the regression coefficient estimated within the same sample; and (2) a case where the PLS weights are first computed from a calibration sample, and are then applied to an independent or "holdout" sample to calculate the PLSc regression coefficient. Figure 2 shows that these two approaches produce markedly different distributions for the estimates: When the PLS weights and the PLSc regression coefficients are estimated from the same sample, the regression coefficients are biased away from zero and have a small secondary mode on the negative side of the x-axis, as also documented in prior IS research (Goodhue, Lewis, & Thompson, 2007; Goodhue et al., 2012). Contrarily, when these weights are applied to a new sample, the effect disappears and the resulting coefficient estimates are approximately normally distributed, but negatively biased, due to bias in the reliability estimates that we discuss in the next section. In both scenarios, the weights are based on estimated indicator reliabilities, but in the first scenario those weights also capitalize on idiosyncratic variation present in the sample from which the weights were calculated. In the second scenario, this does not happen because the characteristics of the hold-out sample are not the same as in the sample that was originally used to calculate the weights. If the weights were not affected by small-sample idiosyncrasies under the first scenario, then we would expect the two approaches to estimating the regression coefficient to yield identical distributions, which is clearly not the case. Through a different approach than before, this second example also clearly demonstrates that PLS weights capitalize on the idiosyncratic features of a given sample (e.g., correlations that are large by chance only), and that these affect the accuracy of the resulting estimates. In the interest of allowing for the replication of our results, the R source code for both examples is available in Appendix D.

---- Insert Figure 2 about here -----

3. Issue 2: Biasedness of reliability estimates

An unbiased correction for measurement error attenuation in correlational and regression analyses requires unbiased reliability estimates for the variables involved (cf., Borneman, 2010; Muchinsky, 1996). In PLS analyses, these reliability estimates are typically calculated with the composite reliability (CR_{II}³) index – a measure of reliability of unweighted composites historically associated with latent variable SEM (Fornell & Larcker, 1981; Raykov, 1997; Werts, Linn, & Jöreskog, 1974). However, as pointed out by Dijkstra and Henseler (2015b), the CR_U index is inconsistent when calculated from PLS estimates because the well-known overestimation of factor loadings by composite loadings will in turn inflate CR_U. Therefore, they propose an alternative reliability coefficient, ρ_A , where the subscript A refers to the fact that Mode A estimation is used to generate the weights required for calculating the index. Although Dijkstra and Henseler do provide evidence of the consistency of ρ_A in their Figure 3, they do not provide evidence of unbiasedness – recall our earlier discussion on the difference between these two properties. Moreover, Dijkstra and Henseler address only some of the problems associated with the use of the CR_U in a PLS analysis. As recently pointed out by Aguirre-Urreta, Marakas, and Ellis (2013), there are three issues with the way composite reliability is typically estimated following a PLS analysis.

The first problem in using the CR_U with PLS analysis is that the formula for the composite reliability statistic reported in the output of PLS analyses is the one given by Fornell and Larcker (1981; see also Chin, 1998) for equally-weighted composites. In the case of PLS analyses, however, the composites involved in the estimates are not equally-weighted combinations of individual indicators, but are rather created using weights derived with the PLS

 $^{^3}$ We use the acronym CR_U for the unweighted version of composite reliability and CR_W for a weighted version that we will introduce later. The acronyms CR_W and CR_U are not conventional terms from the psychometric literature, but were used here to make a simple distinction between reliability indices for the weighted and unweighted case, respectively.

algorithm. As a result, the composite reliability formula that is commonly employed is biased. For an unbiased estimation, an alternative version that does take into account the particular weights used to construct the composite is necessary (CR_w; c.f. Aguirre-Urreta et al., 2013).

Given that the CR_W index is not as well-known as CR_U , some further elaboration is warranted here. The most general version of CR_W was developed over 70 years ago by Mosier (1943, Equation 5, p. 162; for other variants, see Rozeboom, 1989; Webb, Shavelson, & Haertel, 2006), and can be expressed as follows for the unstandardized case:

$$CR_W = 1 - \frac{\sum w_i^2 \sigma_i^2 - \sum w_i^2 \sigma_i^2 \lambda_i^2}{\sum_i^n w_i^2 \sigma_i^2 + 2\sum_{i=1}^{n-1} \sum_{j=2}^n w_i w_j \sigma_i \sigma_j \rho_{ij}} \text{ (with } i \neq j \text{),}$$

where for a given weighted composite, w_i , σ_i^2 , σ_i and λ_i^2 are, respectively, the weight, variance, standard deviation and reliability of the ith indicator (a squared standardized factor loading is the indicator reliability); w_j and σ_j are the weight and standard deviation of the jth indicator; and ρ_{ij} is the correlation between the ith and jth indicators ($i \neq j$). Therefore, the CR_W index is essentially a modified version of the CR_U that takes differential indicator weights into account, whatever their origin. Like the CR_U index, CR_W is an estimate of the ratio of true score variance in the composite to the total variance of the composite. It is evident from Equation [1] that CR_W is the more appropriate measure of composite reliability for PLS-based analyses, which rest on a differential indicator weighting scheme (cf., Aguirre-Urreta et al., 2013). Another difference between these two indices is that CR_U uses the estimated variance of a composite in the denominator, conditional on indicator error terms being exactly uncorrelated in the sample, whereas CR_W uses the variance of the actual composite. Therefore, for standardized composites, which are always the case in the PLS context, CR_W reduces to the square of the sum of products of the loadings and weights for the composite, that is:

$$CR_w = (\sum w_i \lambda_i)^2.$$
 [2]

When the indicator weights and loadings originate from a PLSc analysis, CR_W is equivalent to the ρ_A index presented by Dijkstra and Henseler (2015b).

The second concern expressed by Aguirre-Urreta et al. (2013), which was also noted by Dijkstra and Henseler (2015b), is that in traditional PLS analysis the loadings are estimated with correlations between the individual items and the composite of which they are a part, leading to biased estimation. To solve this second issue, the PLSc correction does not use the correlations between indicators and composites as loading estimates, rather, the loadings are consistenly estimated by scaling the weights as follows (Dijkstra, 2010):

$$\hat{\lambda}_i = c_i \cdot \mathbf{w}_i, \tag{3}$$

where \mathbf{w}_i is the weight vector for the *i*th block of indicators, $\hat{\lambda}_i$ is a parallel vector of estimated factor loadings, and c_i is the scaling factor.⁴ Unfortunately, consistency does not guarantee unbiasedness in finite samples, and the study by Huang (2013) demonstrates that the PLSc loading estimates are biased, mostly negatively. The same effect can be seen in our second example, where the mean PLSc corrected factor loading over 10 000 replications was 0.653, a -7% bias compared to the population value of 0.7^5 .

⁴ Our notation differs from that used by Dijkstra and Henseler (2015b), who deviate from other papers on PLSc (e.g., Dijkstra & Henseler, 2015a; Dijkstra & Schermelleh-Engel, 2014) in that they do not refer directly to the scaling factor c in the adjustment formulas. We follow the approach from earlier papers on PLSc of using c, given that it simplifies the presentation of the formulas and also directly sets the stage for our improved estimator. However, we follow Dijkstra and Henseler (2015b) in using ρ_A to denote the PLSc reliability coefficient rather than the less intuitive q index used in other PLSc studies.

⁵ Huang (2013) does not provide an explanation for the bias. Two mechanisms may lead to bias in the factor loading estimates obtained using Equation 3. First, the indicators with loadings having positive estimation error are over-weighted at the expense of other indicators, a phenomenon which is illustrated by our first example model from the previous section. It would intuitively appear that if two pairs of indicators (e.g. a_1 , b_1 and a_2 , b_2) are affected by error with the same magnitude but different directions, and the third pair (a_3 , b_3) is unaffected by the error, the weights assigned to the third pair would remain at the original values, but this is not the case: The variance of a weighted sum depends on the squares of all weights, and therefore increasing one weight while decreasing another by the same amount leads to increasing the variance of the composite. To compensate for this increase, the standardization step must downscale all weights, leading to an overall decrease in the mean standardized weight. Second, the same mechanism affects the factor loading correction factors (c's), which also depend on squares of weights and therefore further downscale the loading estimates. More methodological research addressing the small sample properties of the weights and the correction factor is clearly needed.

The final concern expressed by Aguirre-Urreta et al. (2013) is the non-independence of weights and loading estimates. In a traditional PLS estimation, loadings are obtained as correlations between the indicators and composites. Because loading estimates depend on the weights, choosing different weights (e.g. Mode A vs. Mode B) leads to different loading estimates. Because loadings are fixed population parameters that are estimated, the estimates should not depend on the way a researcher chooses to form the composites. To break the explicit dependency between loadings and weights, Aguirre-Urreta et al. (2013) recommended that loadings be obtained through alternative procedures, such as maximum likelihood estimates from a confirmatory factor analysis of each set of items separately, following Raykov (1997). This final concern, in the form presented by Aguirre-Urreta et al. (2013), is not directly applicable to PLSc because weights are always calculated by Mode A and are thus proportional to estimated loadings, and because just one weight algorithm is allowed there is no choice of weights involved.

Unfortunately, the non-independence of weights and loadings leads to an additional but slightly different problem in small samples for the case of PLSc. Assume an ideal scenario where the estimated loadings are unbiased, normally distributed, and independent from one another. In this case, the estimation error associated with a single loading estimate can be either positive or negative. A negative estimation error leads to a weight that is smaller than the "correct" weight, whereas a positive error leads to a weight that is larger. Therefore, indicators with overestimated loadings are over-weighted and those with underestimated loadings are under-weighted, leading to an overall positive bias in the composite reliability estimates (i.e., ρ_A) by inflating the effects of positive estimation error and reducing the effects of negative estimation error so that they no longer cancel out as would be expected if the weights were independent of the reliability

estimates. The effect is clearly seen in our example in Table 1 where the reliabilities are over estimated by 0.039, or by +5%.

Thus, although the composite reliabilities are estimated with an appropriate formula in PLS, the issues of biased loading estimates and non-independence of weights and loadings remain. Because the effect of the two biases – negative bias of loadings and positive bias due to proportionality of loadings and weights – are in opposing directions, their net effect on the estimated reliabilities (ρ_A) is not obvious. However, it is safe to say that these two biases would only cancel each other out exactly by chance, and that is not something on which researchers should be willing to rely. In the next section, therefore, we propose further corrections to the new PLSc estimator that address these issues and later pit our corrected estimator against PLSc to compare their performance in a study that replicates the simulation done by Dijkstra and Henseler (2015b).

4. Proposed corrections to PLSc

In order to resolve the issues presented above, we now propose some simple improvements to PLSc, beginning with the factor loading estimation. The first correction is to estimate the factor loadings using traditional factor analysis techniques instead of by rescaling the indicator weights⁶. This leads to general confirmatory factor analysis-based techniques for disattenuation (e.g., Croon, 2002; Lu et al., 2011). The proposed technique has three advantages over the per-block correction proposed by Dijkstra, namely, it: (1) breaks the explicit dependency between weights and loadings; (2) does not depend on the initial indicator weights,

⁶ This improvement can be motivated in at least two different ways. First, the scaling factor c_i should be estimated to minimize the Euclidean distance between the sample and model-implied correlation matrices for the *i*th block of indicators. However, we can get closer to the minimum by replacing the single scalar c_i , which is applied to an entire block of indicators, with a vector \mathbf{c} , such that each loading is corrected separately. Estimating the corrections for each indicator block separately essentially leads to minimum residuals or *minres* (cf., Nunnally, 1978, Chapter 11) estimates of loadings for multiple single-factor models. Second, the PLSc factor loading estimator can be considered as a special case of minres estimator, constrained to produce loadings proportional to PLS weights. All corrections can also be estimated simultaneously and, with appropriate weighting of the distance function, lead to maximum likelihood confirmatory factor analysis. See Appendix B for how these corrections can be derived.

and is therefore more general and can also be applied to other composite-based SEM estimators (e.g. PLS Mode B), which may not have the same asymptotic properties for weights as PLS Mode A; and (3) is a full-information estimator in the scenario where all loadings are estimated simultaneously, and would therefore be expected to estimate the factor loadings more efficiently, that is, the estimates should be more precise.

Although estimating the loadings with factor analysis breaks the explicit dependency between loadings and weights, this change is insufficient to make weights and loadings independent. To completely sever this connection, we introduce a second correction, which entails constructing the composites with unit weights (i.e., sums of standardized indicators, e.g., Bobko, Roth, & Buster, 2007; Cohen, 1990; Cohen, Cohen, West, & Aiken, 2003, pp. 97–98) scaled to produce standardized composites, instead of using PLS weights to construct the composites. Even though there may be some situations where this choice leads to a marginal decrease in the reliability of the composites (Henseler et al., 2014), unit-weighted composites have been shown to be robust in a broad range of scenarios (e.g., Bobko et al., 2007; Cohen, 1990; Cohen et al., 2003, pp. 97–98; Raju, Bilgic, Edwards, & Fleer, 1999). Moreover, regardless of how the indicators are weighted, the correction for attenuation should eliminate any effect that indicator weights have on the estimated composite correlation matrix, and therefore there is little if any loss in estimation accuracy. Another advantage of this second correction is that it eliminates the biasing effects of chance correlations from the results, as unit-weighted composites do not exhibit the same sensitivity for these correlations as PLS weights do (Rönkkö, 2014).

⁷ We note that starting with less reliable composites may have an effect on the estimates because less reliable composites require a higher disattenuation coefficient, and this same coefficient is also used to multiply sample imperfections such as chance fluctuations in indicator correlations. We will address the overall effect of the correction on estimation accuracy later in the paper.

Our more refined correction obviates the need for calculating the PLS weights as the initial step during estimation, but follows the remaining steps used in PLSc, that is, disattenuation of the composite correlation matrix using estimated composite reliabilities, followed by applying regression analysis to estimate the structural coefficients. This technique is equivalent to errors-in-variables (EIV) regression (Fuller & Hidiroglou, 1978; Warren, White, & Fuller, 1974), and also better known by this name⁸. Reliability can be estimated using either of the CR indices presented above, or any other model-based formulas appropriate for unit-weighted composites (Cho & Kim, 2015; Green & Yang, 2009). In the remainder of this research we will focus on two estimators of this class, namely EIV regression using unit-weighted composites with reliability estimates from maximum likelihood factor analysis (EIV_{ML-CFA}) and per-block minres factor analysis (EIV_{minres}), which is closer to PLSc, but requires at least three indicators per block⁹.

5. Replication of Dijkstra and Henseler

In the simulations that follow, we compare EIV_{ML-CFA} and EIV_{minres} with PLSc as well as simultaneous maximum likelihood estimation of the full structural equation model (ML-SEM). As explained in the previous section, our corrections to PLSc focus on unbiased reliability estimation and eliminating the small sample bias due to PLS weights from the initial composite correlations. Therefore, in addition to focusing on the bias, efficiency, and power of the estimators themselves, we also analyzed the performance of the three different reliability estimates discussed so far (Cronbach's alpha, CR_U, and CR_W) for both PLS and unit-weighted composites as well as designed the simulation to isolate the effects of capitalization on chance by

⁸ These techniques are sometimes referred to as the "bias correcting" (Croon, 2002) or "two-step" (Oberski & Satorra, 2013) method in the SEM literature.

⁹ We also studied a number of other confirmatory factor analysis estimators including GLS, WLS, DWLS, and ULS. Because these estimators are rarely used in IS research and did not show any advantage over the ML estimator in our simulations, these results are omitted from the article, but are available upon request from the first author.

the PLS weights. The true population model for the simulation is taken directly from Figure 4 in Dijkstra and Henseler's (2015b) study, and is comprised of the following set of structural equations:

$$\begin{split} &\eta_{3} = \gamma_{23}\xi_{2} + \zeta_{3} \\ &\eta_{4} = \gamma_{14}\xi_{1} + \gamma_{24}\xi_{2} + \zeta_{4} \\ &\eta_{5} = \gamma_{15}\xi_{1} + \gamma_{25}\xi_{2} + \beta_{35}\eta_{3} + \zeta_{5}, \end{split}$$
 [3]

where $\phi_{12}=0.70$, $\gamma_{23}=0.0$, $\gamma_{14}=0.0$, $\gamma_{24}=0.70$, $\gamma_{15}=0.22$, $\gamma_{25}=-0.70$, and $\beta_{35}=0.35$. All measurement equations were of the form $y=\lambda\eta+\delta$ and $x=\lambda\xi+\epsilon$, with λ 's ranging widely in value from 0.40 to 0.94. The number of indicators also varied considerably. Both ξ_1 and η_4 were only measured by two indicators each; ξ_1 and η_4 by six indicators each; and η_4 by four indicators.

To be able to analyze the performance (i.e., bias and efficiency) of reliability estimates, we constructed our datasets sequentially by first generating all latent variables and then calculating the indicators as weighted sums of their respective latent variables and random error terms. This approach is similar to past research on PLS (e.g., Goodhue et al., 2007, 2012), and was used here because it allows for straightforward calculation of true composite reliabilities, that is, the squared correlations between the latent variables and their respective composite proxies, which can then be compared to the estimated reliabilities in order to assess the performance of different reliability indices. In the rare cases where a composite correlated negatively with the latent variable, we coded the reliability as negative.

For the conventional multivariate-normal scenario, it was assumed that all exogenous latent variables (i.e., factors and errors) were joint-normally distributed, and that measurement and structural errors were mutually independent; factors were also assumed to be independent of all error vectors. In line with Dijkstra and Henseler (2015b), sample sizes were set at N = 100, N = 200, and N = 500. The non-normal data condition was constructed using the method described

by Vale and Maurelli (1983), as implemented in the R package *simsem* (version 0.5-9, Pornprasertmanit, Miller, & Schoemann, 2014). Because the data were generated sequentially rather than drawing samples from a covariance matrix as in Dijkstra and Henseler's (2015b) study, we could not directly specify the degree of excess kurtosis in the observed variables; rather, only the distributions of the exogenous latent variables could be manipulated directly, followed by generating the indicators as weighted sums of the non-normal latent variables. However, a sum of leptokurtic variables generally has smaller excess kurtosis than the raw variables. Therefore, to ensure that our observed data were sufficiently non-normal to be comparable with the datasets used by Dijkstra and Henseler, we set all exogenous latent variables to have excess kurtosis of 5.

In addition to replicating past research, we also introduced two additional experimental conditions. The first involved the use of three different scaling factors (0, 0.5, 1) for the true parameter values, in order to examine the behavior of the estimators with respect to each model structure when the population effect sizes for the relationships between the latent variables are weaker or non-existent. This is a relevant experimental manipulation because the PLSc estimator requires the latent variables to be correlated in the population in order to calibrate the indicator weights (Dijkstra & Henseler, 2015b), but this requirement may not always be fulfilled in real datasets. Moreover, this strategy allowed us to examine the null distribution of the estimates, which will help shed further light on the appropriateness of assuming a reference distribution based on normality – the current practice in PLS – to obtain *p*-values for parameter estimates (Rönkkö & Evermann, 2013).

However, one complication with applying the scaling factor of 0 to Dijkstra and Henseler's (2015b) population model is that there are two factors (ξ_1 and η_4) with only two

indicators each. As is well known, a common factor model with only two indicators is identified only if the factor is embedded in a larger system of non-zero relationships (Bollen, 1989, pp. 238–246). Therefore, setting all factors to be uncorrelated leads to empirically underidentified measurement models for both ξ_1 and η_4 (Rindskopf, 1984; see e.g., Kline, 2011, p. 147). Resolving this identification problem by imposing equality constraints on loadings was not a viable option, as such a modification would deviate too much from the original model. Therefore, to solve this issue, we followed the example by Huang (2013) and implemented the simulations also using an alternative, larger model where the indicators x_{11} , x_{12} , y_{41} , and y_{42} were each included twice in the population model such that all latent variables had at least four indicators, rendering each factor individually overidentified.

As the second additional experimental condition, we implemented a manipulation aimed at quantifying the effects of capitalization on chance across the different estimators. Rönkkö (2014) argues that the effect is mostly caused by chance correlations between the measurement errors in the indicator variables. Following his approach, we created two versions of each dataset, which we refer to here as *original* and *manipulated* data. The original data were generated using the sequential data generation approach described above, which entailed first generating the latent variable values and then generating the indicators based on these values. For each generated dataset, the corresponding *manipulated* data were constructed by orthogonalizing the error terms in the original data (but maintaining their variances), and then calculating a new set of indicators using the original latent variable values and error terms that were artificially restricted to be uncorrelated in the sample, thereby removing any chance correlations between measurement errors. Similarly to the recent study by Rönkkö (2014) and the second example presented above, when estimating the model with *manipulated* data we

always used the *original* data for calculating the indicator weights and then used the *manipulated* data for the estimation of the structural paths. This design allowed us to quantify the extent to which chance correlations inflated the estimates of the relationships between the latent variables, which are the effects of primary substantive interest. This comparison was done only for PLSc and the EIV estimators, which are the main focus of the paper, but not for ML-SEM because the estimator does not use indicator weights that could be retained from one sample to another.

For the PLSc and EIV estimators, all simulations were conducted within the R statistical programming environment (version 3.1.0, R Core Team, 2015) using the *matrixpls* package (version 0.6.0, Rönkkö, 2015). In line with the approach of Dijkstra and Henseler (2015b), the factor weighting scheme was used for inner estimation and Mode A for outer estimation. ML-SEM estimations were performed with the *lavaan* package (version 0.5-17, Rosseel, 2014). The number of Monte Carlo replications was set at 1000 for all simulation conditions.

5.1. Experiment results

We compared the composite-based PLSc, EIV_{ML-CFA}, and EIV_{minres} estimators, and ML-SEM, in terms of their ability to estimate factor loadings, composite reliabilities, and path coefficients, as well as accuracy of confidence intervals and their statistical power. These results are further disaggregated by the experimental manipulations: (1) Dijkstra and Henseler's (2015b) original model (Small Model) vs. a model with duplicated indicators for each of ξ_1 and η_4 (Large Model); (2) scaling factors (0, 0.5, 1) for effect sizes; (3) original vs. orthogonalized measurement errors; and (4) sample size. Given that the results for the normal and non-normal data conditions were virtually identical, only the results for normal data are presented here (see Appendix A for comprehensive results tables, including those for the non-normal condition).

5.1.1. Inadmissible values

In contrast to Dijkstra and Henseler (2015b), who studied only conditions where all estimators are expected to perform well, we also included conditions where the estimators can be expected to encounter difficulties. Particularly, the two-indicator measurement model is empirically underidentified in the scenario where the latent variables are uncorrelated in the population. In these scenarios, the data do not provide sufficient information to estimate the loadings, resulting in arbitrary estimates with a large number of inadmissible values for all estimators, as shown in Table 2 below. For the Small Model (where EIV_{minres} cannot be used because a factor analysis requires at least three observed variables), the number of replications with inadmissible loadings exceeded 90% for all estimation techniques at all three sample sizes when the population path coefficients were set to zero. The number of replications showing inadmissible loadings declined with increased effect and sample size for all four of the above estimators, but this effect was weaker for PLSc than for the other estimators. In the original, unscaled model, PLSc always produced more than 2.5 times as many inadmissible loadings than any of the competing techniques. These findings are also consistent with prior research showing that having only two indicators per factor leads to increased rates of inadmissible solutions (Ding, Velicer, & Harlow, 1995; Gagne & Hancock, 2006; Jackson, Voth, & Frey, 2013; Marsh, Hau, Balla, & Grayson, 1998). Highly similar patterns of inadmissible results were found under the Small Model for the composite reliability indices, which are only relevant for PLSc and EIV_{ML-CFA} because the latent variable-based ML-SEM estimation technique does not use composites.

--- Insert Table 2 about here ----

With respect to factor correlations under the Small Model, inadmissible solutions were much less frequent, which is the expected result; model underidentification does not necessarily mean that all model parameters are not identified and, in these models, the factor correlations remain identified. Inadmissible correlations were produced in two scenarios. First, when the factors were uncorrelated in the population, the uncorrected correlations between PLS composites were strongly biased away from zero due to the capitalization on chance effect discussed earlier. At the same time some of the reliability estimates were very small, producing larger correction factors. The combination of these two issues leads to inadmissible correlation estimates (i.e., greater than 1.0)¹⁰. The second scenario that produced inadmissible correlations involved the original, unscaled model. In this scenario, inadmissibility of correlations is a consequence of the fact that some of the population correlations were very large, which in situations where estimates are normally distributed and have large variances, inevitably leads to some estimates that exceed the range of admissible values. The latent variable-based ML-SEM technique resulted in a slightly smaller number of inadmissible estimates in the unscaled models because the technique is generally more efficient than the traditional correction for attenuation (Charles, 2005; Kline, 2011, pp. 70–71; Muchinsky, 1996; Schmidt & Hunter, 1996; Schumacker, 2010, sec. 3.2.5).

For the Large Model, which simply duplicates each of the original indicators for ξ_1 and η_4 , the proportions of inadmissible results under PLSc closely paralleled those of the Small Model for all three types of statistics (i.e., loadings, factor correlations, and composite reliabilities). However, there was a large difference in the performance of EIV_{ML-CFA} and ML-SEM in estimating loadings and reliabilities, which results from the fact that a latent variable

¹⁰ The disattenuation correction is known to produce inadmissible correlations when the observed composite correlation is greater than the square root of the product of the two composite reliabilities (cf., Charles, 2005; Nimon, Zientek, & Henson, 2012)

with three or more indicators is identified without requiring any information from the larger model, and therefore empirical underidentification is no longer an issue. The PLSc results are largely unaffected by increasing the number of indicators because only the between-block correlations are used to estimate the proportionalities of the indicator weights, and therefore the new information that the added indicators bring to model estimation is underutilized. The pattern of results is very similar to the results by Huang (2013). That EIV_{minres} never produced any inadmissible loadings is explained by the fact that the minres implementation we used (the fa function in the psych package of R; Revelle, 2015) is constrained to always yield admissible loadings.

We will next focus on the performance of the parameter estimates. In doing so, we chose to include the inadmissible estimates in the computation of our descriptive statistics. The reason for this is two-fold. First, Dijkstra and Henseler (2015b) did not report dropping inadmissible estimates and we prefer our study to be as comparable to theirs as possible. Second, because the model contained a few very large loadings or correlations, dropping inadmissible estimates would have effectively truncated their sampling distributions, causing artificial bias in estimates that would be otherwise unbiased.

5.1.2. Factor loadings

Because the models examined here contained large numbers of factor loadings, results needed to be reported selectively. Therefore, in order to make the analysis of patterns in the results more manageable, we focus on examining six loading estimates. Of the two underidentified factors, ξ_1 and η_4 , we chose to focus on the latter because the indicator loadings have larger variances, and thus the results may be more generalizable than in the case of ξ_1 , which had two exceptionally reliable indicators. Of the remaining three identified factors, we

chose to focus on η_3 and η_5 , which are, respectively, the least and most correlated with the other factors. Therefore we expect to see large differences here between PLSc estimates – whose proportionality depends solely on the between-block correlations – and other factor loadings estimates, which are either completely unaffected (minres) or relatively less affected (ML-CFA) by the between-block correlations. With these two factors, we chose to focus on the largest and smallest (i.e., first and last) factor loadings. Table 3 shows the results for the selected loadings. (The full table of estimated loadings is presented in Appendix A). Because both estimation strategies that relied on ML estimation of the full factor model (EIV_{ML-CFA} and ML-SEM) produced nearly identical estimates, we only report results for EIV_{ML-CFA}.

--- Insert Table 3 about here ---

As shown in Table 3, the EIV_{ML-CFA} estimates were unbiased and more efficient than the PLSc estimates, across all conditions and both Models 1 and 2 for those factors that were identified (i.e., had 3 or more indicators); under the Large Model, the EIV_{minres} estimates and SDs were virtually identical to those of EIV_{ML-CFA}. EIV_{ML-CFA} faced serious difficulties with the empirically underidentified loadings in the Small Model. However, the same problem arose in PLSc where all loading estimates were biased and varied widely from sample to sample.

When discussing the three problems in estimation of composite reliabilities in PLSc (i.e., assumption of equal weights, biased estimates of loadings, and dependencies between weights and loadings; see Aguirre-Urreta et al., 2013). Except for the non-identified factors, all PLSc loading estimates are negatively biased and the magnitude of the bias depends on the strength of the correlations between the factors. The pattern of results is again very similar to the results by

Huang (2013), where the PLSc loadings were negatively biased except for doublet factors where the bias was positive¹¹.

5.1.3. Reliabilities

We analyzed the performance of several different reliability statistics for all composite-based techniques: (1) the classical Cronbach's alpha; (2) the composite reliability index (CR_U) for the unweighted case, commonly used in PLS studies; and (3) the CR_W index, designed specifically for the case of weighted composites. With the CR_U index, we estimated the variance of the composite assuming uncorrelated errors in the sample, in keeping with how the index has been used in the PLS literature; whereas with the CR_W index, we use the real composite variance so that the index is equivalent to the ρ_A used by Dijkstra and Henseler (2015b). Both CR_U and CR_W indices were calculated using the corrected loading estimates instead of using the indicator composite correlations, which Dijkstra and Henseler used for the CR_U (labeled as Jöreskog's ρ or ρ_c in their article).

Table 4 shows the estimation errors of the reliabilities calculated as differences between reliability estimates and the true reliabilities (i.e., squared correlations between the composites and the latent variable scores used to generate the data). Because the results for reliability estimates calculated with minres loading estimates were identical to the third decimal place with the reliability indices calculated with ML-CFA, only the latter are reported. We focus on the Small Model, because it is expected to show the largest discrepancies among the different reliability estimates. This approach also allowed us to compare the reliability results for the doublet factors (ξ_1 and η_4) to those with 4+ indicators. For the two doublet factors (where minres

¹¹ The positive bias in the two-indicator case may be an outcome of the PLSc correction, stemming from the fact that the scaling factor c_i reduces to the square root of the indicator correlation divided by the product of the two weights. Because the weights are scaled so that the resulting composite is standardized, this can lead to an effect where larger weights are scaled up less than smaller weights, leading to an overall positive bias in the loadings. As noted in footnote 5, more research on the small sample characteristics of the weights and the PLSc corrections is required before any definite statements about the source of the bias in the loadings can be made.

estimation was not possible), both PLSc and EIV_{ML-CFA} yielded large numbers of inadmissible values for CR_U and CR_W . For both estimators, the values for CR_U and CR_W in the doublet case improved appreciably as the scaling factor and sample size increased, and ultimately achieved less bias under EIV_{ML-CFA} than PLSc.

For the remaining composites, which corresponded to identified factors with 4 or more indicators, the EIV_{ML-CFA}-based CR_U and CR_W outperformed those obtained under PLSc by a wide margin, being unbiased and more efficient in all scenarios. As would be expected, Cronbach's alpha tended to underestimate reliability, given that it is a lower bound on reliability when the assumption of tau-equivalency (i.e., equal loadings) is violated (Green & Hershberger, 2000), as in the present case. However, this underestimation was not severe, with the bias still being substantially less than in the case of CR_U and CR_W under PLSc. Interestingly, in scenarios where the latent variables were uncorrelated, alpha overestimated reliability. This result is due to the fact that alpha is an estimate of reliability for equally weighted composites, which are more reliable than PLS composites in some scenarios (Rönkkö & Evermann, 2013). The CR_U and CR_W produce differing results for the empirically-weighted PLS composites and show an interesting pattern. Consistent with our expectations, the unweighted CR_U index is negatively biased because the effects of indicator weights are ignored. A more interesting finding – but one that is expected because weights depend on the reliability estimates – is that the CR_W indices are always positively biased.

--- Insert Table 4 about here ---

The results show that estimating the reliability of empirically-weighted composites used by PLSc is substantially more difficult than estimating the reliabilities of unit-weighted composites used by the EIV estimators. This can be intuitively explained by considering that

with PLSc, there are two sources of sampling variability, namely the variance of the weights and the variance of the loadings estimates, whereas with unit-weighted composites, only the variance of the loading estimates affects the results.

5.1.4. Path coefficients

Table 5 and Table 6 show the estimates for both the Small Model and Large Model with normally distributed data, under all modeling conditions. We start by comparing the estimates obtained with the original data to estimates obtained with manipulated data, where the correlations between the indicator error terms were artificially restricted to be zero in the sample, because the comparison reveals important differences between the estimators. As explained earlier, this manipulation was not performed for ML-SEM because the estimator does not use weighted composites. The tables demonstrate that, generally and across all conditions, the differences in the estimated path coefficients (and their SDs) across the original and manipulated data were greatest for PLSc, as compared to EIV_{ML-CFA} and EIV_{minres}. When the population value was different from zero, estimates with original data were generally further from zero than with the manipulated data. The effect can be seen most clearly for β_{35} and γ_{24} , which are correlations in the population 12 , and γ_{25} , which is the strongest predictor of η_4 . This effect is caused by PLS weights capitalizing on chance, as discussed earlier in Section 2, which is a source of bias that remains unaddressed by the PLSc correction. The differences between estimates using the manipulated and normal data with the EIV estimators are small and non-systematic, demonstrating that using unit weights instead of PLS weights in the estimator corrects for this bias. The only effect of using unit weights is a decrease in the variance of the estimates

 $^{^{12}}$ A standardized regression coefficient equals the zero-order correlation between the predictor and the crierion, if a predictor is uncorrelated with all other predictors in the model (β_{35}). The same applies if a given predictor is the only predictor with a non-zero regression coefficient (γ_{24}).

(particularly in small samples), which is consistent with earlier results using the same manipulation (Rönkkö, 2014).

Overall, the ML-SEM estimator is the most efficient across both models and both normal and non-normal data (See Appendix A for the results tables for non-normal data). For γ_{23} and γ_{14} , which were both zero in the population, none of the estimators produced mean estimates that deviated markedly from the true value; however, PLSc was less efficient than EIV_{ML-CFA} or ML-SEM. A closer inspection of the distribution diagrams revealed that this difference was largely attributed to the bimodal shape of the PLSc estimates for these paths with two peaks on both sides of zero, and only a few estimates were very close to zero (Rönkkö & Evermann, 2013). The same effect is clearly visible in the results that were obtained after scaling the latent variable paths to half or zero.

--- Insert Table 5 and Table 6 about here ---

Generally, the PLSc estimates where chance correlations between the error terms were removed were biased toward zero in nearly all scenarios. The result is expected because positively biased reliability estimates lead to undercorrecting for attenuation. However, when chance correlations were not eliminated from the data, the results were most often slightly biased away from zero. Therefore, in this case, the positive bias due to chance correlations happened to yield close to optimal recovery of the true value by compensating for the biases in reliability estimates, but of course one would not be able to rely routinely on this type of effect (Rönkkö, 2014).

5.1.5. Statistical inference

Because the sampling distribution of the PLS weights is unknown (Dijkstra, 1983), null hypothesis significance testing using a known, theoretical distribution is not possible with PLSc

(see McIntosh, Edwards, & Antonakis, 2014; Rönkkö & Evermann, 2013; Rönkkö, McIntosh, & Antonakis, 2015). Therefore, following Dijkstra and Henseler (2015b), we assess the performance of bootstrapped confidence intervals as a tool for statistical inference. Table 7 and Table 8 show the bootstrapped 95% percentile confidence intervals for PLSc, EIV_{minres}, and EIV_{ML-CFA}. Each cell in Tables 7 and 8 represents 1000 replications with 1000 bootstrap samples each, for a total of more than a million estimations each. Because of the extent of the required computational effort, we did not bootstrap the ML-SEM estimates. Given that the ML estimator has known closed-form solutions for standard errors, ML estimates are rarely bootstrapped in research practice.

Instead of focusing solely on whether zero was included in the confidence intervals, we also addressed the validity of the confidence intervals themselves by including statistics on coverage and balance of the intervals (DiCiccio & Efron, 1996): A well-constructed confidence interval should contain the population value at the stated degree of confidence and fall above and below the confidence limits in a balanced way. For the 95% intervals used here, the coverage should be close to 95% and the population value should be below or above the confidence limits for 2.5% of the time. There are no notable differences between the normal and non-normal condition or the Small Model and Large Model. The mean coverage statistics over all scenarios are .021/.962/.014¹³ for PLSc, .029/.946/.026 for EIV_{ML-CFA} and .030/.944/.026 for EIV_{minres}. The coverage values of the EIV estimator confidence intervals are closer to their nominal values and also better-balanced. These very small differences are explained by the fact that capitalization on chance by the PLS weights often created a small secondary mode in the bootstrap replications (See Figure 2), leading to skewed bootstrap distributions, which are known to cause problems for

 $^{^{13}}$ % replications with population value under lower limit of CI/% replications with population value within CI/% replications with population value over upper limit of CI. The balance values of the negative path γ_{25} are reversed. Because of rounding, the proportions do not sum to 1 exactly.

the simple percentile method (Davison & Hinkley, 1997, Chapter 5). This is an issue that can potentially be resolved by using more advanced methods for calculating the confidence intervals such as the BCa intervals which Henseler, Dijkstra, and colleagues (2014) have experimented with.

--- Insert Table 7 and Table 8 about here ---

We also assessed the false positive rates and statistical power of the confidence intervals of the three techniques by inspecting how frequently zero was not included within the confidence interval when an effect was zero or non-zero in the population, respectively. The mean false positive rates for the three techniques over both distribution conditions and models were .021 for PLSc, .055 for EIV_{ML-CFA}, and .056 for EIV_{minres}. The EIV estimators were only slightly over the nominal 5% level, whereas the confidence intervals for PLSc were clearly too wide compared to their nominal coverage when no effects existed in the population. The difference in statistical power over all conditions was .720 for PLSc, .770 for EIV_{ML-CFA}, and .778 for EIV_{minres}. Because all techniques can reliably detect the largest paths, the difference is attributed to PLSc having smaller power to detect weaker effects. These patterns in statistical power are similar to the results presented by Dijkstra and Henseler (2015b).

6. Conclusions

Ever since its introduction into mainstream IS research more than twenty years ago, PLS has become one of the most commonly employed techniques for the estimation of structural equation models in the discipline. However, the technique is not without its weaknesses. Some of these, such as the biased and inconsistent estimation of factor loadings and structural parameters unless the limiting condition of 'consistency-at-large' (an infinitely large number of indicators measuring each construct and an infinitely large sample size) applies, have been well-known

since the technique was originally developed (Wold, 1982b). Other problems, such as bias due to between-block chance correlations that are present in finite samples, have only been recently recognized (Goodhue et al., 2015; Rönkkö, 2014).

Given the increased recognition of the presence of a number of unresolved issues in the PLS approach (McIntosh et al., 2014; Rönkkö et al., 2015), researchers are left with essentially three alternatives. First, they could continue employing the technique while acknowledging that the results obtained would be biased and inaccurate, which leads to questioning their validity altogether. Second, researchers could opt to stop using the approach for the estimation of structural equation models, which would limit those to the conditions that can be examined with conventional latent variable approaches, and such a position has also been advocated in the literature (e.g., Antonakis, Bendahan, Jacquart, & Lalive, 2010). Finally, PLS could be further developed and enhanced in order to do away with, or at the very least mitigate, these issues (Dijkstra & Henseler, 2015b). A pivotal step in this direction was taken by Dijkstra and colleagues (Dijkstra & Henseler, 2015a, 2015b; Dijkstra & Schermelleh-Engel, 2014) in the form of consistent PLS.

Though certainly an important development in this literature, and one that addresses the best-known issues with PLS outlined above, the procedure proposed by Dijkstra and colleagues suffers from two important limitations. First, PLSc fails to consider bias due to capitalization on between-block chance correlations in finite samples, which result in composites that over-weight those items that, in a given sample, happened to be more strongly correlated with items in a different block, even if those correlations are solely the result of natural sampling variability (Rönkkö, 2014). Because the weights resulting from a PLS analysis are the starting point in the PLSc algorithm, any resulting biases carry over to PLSc as well. Second, the reliability estimates

employed in the PLSc procedure to correct for the effects of measurement error are biased due to biases in loading estimates and a dependency between the indicator weights and loading estimates. As a result, any adjustments made based on biased reliability statistics will also affect the resulting estimates (e.g., path coefficients).

Fortunately, both of these issues can be resolved. While developing new innovations or reusing existing, well-known results to refine our estimation techniques, we should also avoid dogmatically following past research practices. In the present research, we have shown that PLSc can be further improved by eliminating the PLS indicator weighting system from the estimator by using unit weights to form the composites (e.g., Bobko et al., 2007; Cohen, 1990; Cohen et al., 2003, pp. 97–98), and by estimating the indicator loadings directly with well-known factor analytical techniques without constraining the loadings to be proportional to the indicator weights. In doing this, we also derived an estimator that is more general than PLSc. As explained by Dijkstra and Henseler (2015b), the PLS indicator weighting system requires a strong nomological network for accurate weight calibration. What this means is that to apply PLSc or any other estimator that uses PLS weights, a researcher must have a strong theory that the constructs represented by the latent variables in the model are highly correlated. However, the need for highly correlated latent variables seems to be at odds with current PLS practice. In particular, PLS is most commonly used to test whether relationships between latent variables are non-zero (Rönkkö & Evermann, 2013), and is often recommended for exploratory research (Gefen, Rigdon, & Straub, 2011; Ringle et al., 2012), which is defined by Hair et al as the "search for latent patterns in the data in case there is no or only little prior knowledge on how the variables are related" (2014, p. 3). Moreover, the review by Goodhue et al (2015) revealed that models estimated with PLS and published in MIS Quarterly often contained composites that

were calibrated based on just one path that was also tested and therefore cannot be assumed to be non-zero. Considering that EIV techniques presented in this article do not assume highly correlated latent variables (provided that at least three indicators are available for each factor), these estimation techniques are generally more useful for the purposes and scenarios where IS researchers typically use composite-based SEM estimation.

We compared the proposed two EIV estimators with PLSc and ML-SEM by means of a simulation study, which led to several interesting findings. First, we examined the presence of inadmissible solutions for all estimators and simulation conditions, in the form of loadings greater than one, reliability estimates greater than one, or corrected correlations greater than one. Extending the work of Dijkstra and Henseler (2015b), who focused only on well-behaved scenarios, we followed their suggestion (Dijkstra & Henseler, 2015b, p. 17) and also examined the various estimators under conditions where they were not expected to perform well, such as when all latent variable pathways are null in the population. Our results show that the EIV estimators performed as well as or better than PLSc for the different models, sample sizes, and scaling factors (for effect sizes) that were studied. Second, we examined bias in the estimation of factor loadings. As was the case before with regards to inadmissible estimates, both versions of our proposed estimator performed as well as or better than PLSc for all the conditions examined, and on par with ML-SEM included in the comparison. Considering that the factor loadings from the EIV estimators are estimated directly with well-known factor analysis tools, the result is not very surprising.

Third, given that the disattenuation of structural path coefficients, which are of prime theoretical interest for researchers, is dependent on the accuracy of the reliability estimates used in the process, we compared two composite reliability statistics, CR_W and CR_U (for the weighted

and unweighted formulations, respectively; see Aguirre-Urreta et al., 2013) – calculated from the output of ML confirmatory factor analysis, per-block minres factor analysis, and PLSc – with the true reliability of each composite, which was known from the data generation process, and with Cronbach's alpha as an additional benchmark. Our results indicate that both versions of our estimator performed similarly and were superior to PLSc under all conditions. Though it is well-known that Cronbach's alpha would underestimate the true reliability under the conditions examined here (i.e., uncorrelated measurement errors in the population, non-tau-equivalent items), the underestimation observed here was not severe, and bias was nonetheless substantially lower than either the unweighted or weighted composite reliability estimates when those were calculated from PLSc outputs. These results are in agreement with a recent review demonstrating that in practice, the differences between Cronbach's alpha and CR are often small (Peterson & Kim, 2013), which means that alpha may also be a practical option in many research scenarios.

Fourth, our results demonstrated that the path coefficient estimates from the EIV approaches were nearly always less biased and substantially more efficient than PLSc, particularly for weak effects. With regard to statistical inference in particular, the bootstrapped confidence intervals from the EIV estimators also provided slightly more statistical power than PLSc, without deviating from the nominal false positive rates. Given that PLSc was able to reliably detect large paths, its slightly lower statistical power in relation to EIV is due to PLSc being the relatively underpowered approach in the case of weaker effects. Indeed, the suboptimal performance of the bootstrapped CIs under PLSc was especially notable in the case of null pathways among the latent variables, where the CIs were far too wide compared to their nominal coverage.

Given these results, we can now explain the contradictory findings by Goodhue et al (2012, pp. 996–997), who noted that some of their disattenuated PLS results were positively biased, and those from Dijkstra and Henseler (2015b), which did not show a similar effect. Goodhue and colleagues explain the bias by stating that "when a small effect size is involved [...] the reliability correction sometimes seems to over correct for PLS." (2012, p. 996), whereas Dijkstra and Henseler (2015b) attribute the discrepancies in the results to the different disattenuation approaches used in the studies. It is easy to see that neither explanation is complete because the positive bias in the estimates is already present in Figure 8a (Goodhue et al., 2012, p. 997) that shows the uncorrected estimates. This bias is due to the effect of capitalizing on chance discussed in our research, and it is a feature of the PLS weights, not any disattenuation correction. Dijkstra and Henseler (2015b) explain the differences by suggesting that Goodhue et al. used an incorrect formula and claiming that the Cronbach's alpha is a poor estimate of the reliability of PLS composites. However, considering that all paths in the model used by Goodhue et al. were correlations in the population, direct disattenuation of the path estimates using the classical formula is consistent in this scenario. The results of our research suggest that alpha works much better than given credit for, and the partial replication of the study by Goodhue et al. (2012) presented in Appendix E shows that the bias of alpha is negligible. Rather, the differences in the estimates can be attributed to positive bias in the reliability estimates used in disattenuation in PLSc. The same conditions that lead to the positive bias due to chance correlations also lead to positively biased reliability estimates and therefore attenuation is undercorrected, with these two sources of bias nearly canceling each other out in some scenarios, such as the ones studied by Dijkstra and Henseler.

The EIV estimators have a few additional advantages over PLSc that have not been discussed thus far. First, although we only addressed statistical inference with empirical confidence intervals in this study, the EIV estimators can also be used with null hypothesis significance testing. Because the estimates were approximately normal under the null hypothesis of no effects, the parametric one sample t-test can be used to calculate p-values to be used for inferential purposes. Second, the EIV estimators do not require specialized software because these estimators are available in many commonly used statistical packages such as Stata. Moreover, although we used the *minres* and ML estimators for our factor models, any other wellknown factor analysis technique can be used. Considering the availability of both more computationally effective principal axis factoring and more statistically appealing ML, the minres technique can be considered obsolete and is no longer included in many statistical packages (Bartholomew, Knott, & Moustaki, 2011, sec. 3.9). Because different factor analysis techniques generally yield highly similar estimates¹⁴, using a different factor analysis technique should not affect the final path estimates. Third, the regression part of the EIV estimator has been thoroughly studied (Fuller, 1987) and the method has known closed-form solutions for estimating standard errors¹⁵, thereby obviating the need for obtaining these with bootstrapping. This does not of course mean that bootstrapping cannot be used, but simply that EIV provides a broader array of inferential tools than are available with PLSc.

Our research provides clear evidence that the EIV estimators should be preferred over PLSc. Our results also show that the performance of EIV in terms of unbiasedness is comparable with ML-SEM, but the estimator is slightly less efficient. Nevertheless, the broader

¹⁴ We note that principal component analysis should not be confused with factor analysis. If the goal is to obtain the parameters of a latent variable model (i.e., factor loadings and intercorrelations), then factor analysis should be used (cf. Widaman, 1993).

¹⁵ The conventional procedures for calculating standard errors for errors-in-variables models may be biased in small samples because they do not take the uncertainty of the reliability estimates into consideration. Although adjustments for this issue have been proposed in the literature (Devlieger, Mayer, & Rosseel, 2015; Oberski & Satorra, 2013), as far as we know, these corrections have not been implemented in any of the commonly used software packages.

methodological literature provides conflicting advice on the usefulness of the technique. On one extreme, Moosbrugger, Schermelleh-Engel, and Klein (1997) declare that the "correction for attenuation [...] in multiple regression analysis has become obsolete with the development of structural equation models" (p. 97). However, others still see value in EIV (Antonakis et al., 2010; Oberski & Satorra, 2013) and the composite-approximation followed by disattenuation is presented as a viable estimation option even in some recent SEM textbooks as well (Schumacker, 2010, Chapter 9). These differing opinions are not new, as the various techniques based on the correction for attenuation have been surrounded with controversies since its introduction in the early 20th century (Spearman, 1904; see Charles, 2005; LeBreton, Scherer, & James, 2014; Muchinsky, 1996).

However, despite the availability of full ML-SEM, which is the most unbiased and efficient estimator of latent variable models, there are a number of reasons why the EIV techniques should still have a place in the statistical toolbox of IS researchers. First, it is clear that IS researchers have a long history of estimating factor models by composite approximations, and this is unlikely to change overnight. Therefore, introducing new and better composite-based techniques to the discipline is valuable; two critical requirements for these techniques are the ability to correct for measurement error and to produce robust composites that do not capitalize on sample idiosyncrasies. Another pragmatic reason to apply EIV instead if ML-SEM is that techniques based on composites are intuitively easier to understand and sometimes easier to apply (Oberski & Satorra, 2013), as well as more computationally feasible. Therefore, EIV may provide a simple and accurate alternative in cases where ML may be too computationally intensive to be practical, for example, with large models containing high numbers of observed indicators and latent variables. As a composite-approximation, the EIV technique also provides

straightforward diagnostics of observational residuals (e.g., Cohen et al., 2003, Chapter 4), a useful set of techniques that are more complex to implement with latent variable techniques (Bollen & Arminger, 1991); these techniques remain underutilized in the current PLS literature as well. The EIV estimators are also limited information techniques, which means that the effects of model misspecification are local and do not spread through the model, as might occur when using full information estimators (Antonakis et al., 2010). However, rather than immediately resorting to estimators that may be more robust to model misspecification, we strongly recommend that researchers first strive for correct model specification (cf., Kline, 2011, Chapter 8). Also, small sample size is not a good reason to resort to EIV over ML-SEM. The EIV techniques have been shown to perform poorly in small samples and when used with composites with poor reliabilities, and are therefore not recommended for these scenarios. For example, Zimmerman and Williams (1997) suggest a reliability cutoff of .7 and a minimum sample size of 100. On the other hand, ML-SEM has been shown to be fairly robust in small samples (e.g., Goodhue et al., 2012). Nonetheless, simulation studies on the EIV techniques demonstrate that the techniques work well, as long as both the sample sizes and composite reliabilities are reasonable (e.g., Devlieger, Mayer, & Rosseel, 2015). Thus, on balance, EIV methods should not be viewed as a routine replacement for full ML-SEM, but rather as pragmatic alternatives in certain modeling situations.

Furthermore, in order to use the EIV techniques effectively, researchers require a set of model quality statistics. Although EIV does not provide the overall model tests that ML-SEM does, there is still a large portfolio of statistics that can be used. For instance, the factor analyses used for calculating the indicator reliabilities can also be used to assess scale dimensionality, and the CR and AVE statistics used in the Fornell-Larcker (1981) testing system can be readily

calculated from the factor loadings and compared against the disattenuated correlations between the composites. The new HTMT discriminant validity statistic (Henseler, Ringle, & Sarstedt, 2015) is similarly applicable because it is calculated from raw data correlations, and therefore does not depend on any particular way of estimation. Finally, the bootstrap-based model tests suggested by Dijkstra and Henseler (2015a) should work well for evaluating model specification, although this is something to be tested in future research.

To encourage the adoption of the EIV techniques presented in this research, we have made these techniques available in the free and open source *matrixpls* (Rönkkö, 2015) package for R, which also provides an implementation of the PLSc estimator and Monte Carlo features to facilitate future comparisons (for a tutorial for using *matrixpls*, see Aguirre-Urreta & Rönkkö, 2015). The package also calculates all the model quality statistics listed in the previous paragraph. Additionally, we have included several examples of how to specify an EIV analysis with several popular statistical software packages in Appendix C.

7. References

- Aguirre-Urreta, M. I., Marakas, G. M., & Ellis, M. E. (2013). Measurement of composite reliability in research using partial least squares: Some issues and an alternative approach. *The DATA BASE for Advances in Information Systems*, *44*(4), 11–43. http://doi.org/10.1145/2544415.2544417
- Aguirre-Urreta, M. I., & Rönkkö, M. (2015). Sample Size Determination and Statistical Power

 Analysis in PLS Using R: An Annotated Tutorial. *Communications of the Association for Information Systems*, 36(1). Retrieved from http://aisel.aisnet.org/cais/vol36/iss1/3

- Antonakis, J., Bendahan, S., Jacquart, P., & Lalive, R. (2010). On making causal claims: A review and recommendations. *The Leadership Quarterly*, 21(6), 1086–1120. http://doi.org/10.1016/j.leaqua.2010.10.010
- Bartholomew, D. J., Knott, M., & Moustaki, I. (2011). *Latent Variable Models and Factor Analysis: A Unified Approach*. John Wiley & Sons.
- Bobko, P., Roth, P. L., & Buster, M. A. (2007). The usefulness of unit weights in creating composite scores: A literature review, application to content validity, and meta-analysis.

 Organizational Research Methods, 10(4), 689–709.

 http://doi.org/10.1177/1094428106294734
- Bollen, K. A. (1989). *Structural equations with latent variables*. New York, NY: John Wiley & Son Inc.
- Bollen, K. A. (2011). Evaluating effect, composite, and causal indicators in structural equation models. *MIS Quarterly*, *35*(2), 359–372.
- Bollen, K. A., & Arminger, G. (1991). Observational residuals in factor analysis and structural equation models. *Sociological Methodology*, *21*, 235–262.
- Borneman, M. J. (2010). Correction for Attenuation. In *Encyclopedia of Research Design*. 2455

 Teller Road, Thousand Oaks California 91320 United States: SAGE Publications, Inc.
- Charles, E. P. (2005). The correction for attenuation due to measurement error: Clarifying concepts and creating confidence sets. *Psychological Methods*, *10*(2), 206–226. http://doi.org/10.1037/1082-989X.10.2.206
- Chin, W. W. (1998). The partial least squares approach to structural equation modeling. In G. A. Marcoulides (Ed.), *Modern methods for business research* (pp. 295–336). Mahwah, NJ: Lawrence Erlbaum Associates Publishers.

- Cho, E., & Kim, S. (2015). Cronbach's Coefficient Alpha Well Known but Poorly Understood.

 Organizational Research Methods, 18(2), 207–230.

 http://doi.org/10.1177/1094428114555994
- Cohen, J. (1990). Things I have learned (so far). American Psychologist, 45(12), 1304–1312.
- Cohen, J., Cohen, P., West, S. G., & Aiken, L. S. (2003). *Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences*. London: Lawrence Erlbaum Associates.
- Croon, M. (2002). Using Predicted Latent Scores in General Latent Structure Models. In G. A. Marcoulides & I. Moustaki (Eds.), *Latent variable and latent structure models* (pp. 195–223).
- Davison, A. C., & Hinkley, D. V. (1997). *Bootstrap Methods and Their Application*. Cambridge University Press.
- Devlieger, I., Mayer, A., & Rosseel, Y. (2015). Hypothesis Testing Using Factor Score Regression A Comparison of Four Methods. *Educational and Psychological Measurement*, 0013164415607618. http://doi.org/10.1177/0013164415607618
- DiCiccio, T. J., & Efron, B. (1996). Bootstrap Confidence Intervals. *Statistical Science*, *11*(3), 189–212. http://doi.org/10.2307/2246110
- Dijkstra, T. K. (1983). Some comments on maximum likelihood and partial least squares methods. *Journal of Econometrics*, 22(1-2), 67–90.
- Dijkstra, T. K. (2010). Latent variables and indices: Herman Wold's basic design and partial least squares. In V. Esposito Vinzi, W. W. Chin, J. Henseler, & H. Wang (Eds.), *Handbook of partial least squares* (pp. 23–46). Berlin; New York: Spinger.

- Dijkstra, T. K., & Henseler, J. (2015a). Consistent and asymptotically normal PLS estimators for linear structural equations. *Computational Statistics & Data Analysis*, 81, 10–23. http://doi.org/10.1016/j.csda.2014.07.008
- Dijkstra, T. K., & Henseler, J. (2015b). Consistent partial least squares path modeling. *MIS Quarterly*, 39(2), 297–316.
- Dijkstra, T. K., & Schermelleh-Engel, K. (2014). Consistent partial least squares for nonlinear structural equation models. *Psychometrika*, 79(4), 585–604. http://doi.org/10.1007/s11336-013-9370-0
- Ding, L., Velicer, W. F., & Harlow, L. L. (1995). Effects of estimation methods, number of indicators per factor, and improper solutions on structural equation modeling fit indices.
 Structural Equation Modeling: A Multidisciplinary Journal, 2(2), 119–143.
- Fornell, C., & Larcker, D. F. (1981). Evaluating structural equation models with unobservable variables and measurement error. *Journal of Marketing Research*, 18(1), 39–50.
- Fuller, W. A. (1987). Measurement error models. New York, NY: Wiley.
- Fuller, W. A., & Hidiroglou, M. A. (1978). Regression estimation after correcting for attenuation.

 *Journal of the American Statistical Association, 73(361), 99–104.

 http://doi.org/10.2307/2286529
- Gagne, P., & Hancock, G. R. (2006). Measurement Model Quality, Sample Size, and Solution

 Propriety in Confirmatory Factor Models. *Multivariate Behavioral Research*, *41*(1), 65–83. http://doi.org/10.1207/s15327906mbr4101_5
- Gefen, D., Rigdon, E. E., & Straub, D. W. (2011). An update and extension to SEM guidelines for administrative and social science research. *MIS Quarterly*, *35*(2), iii–xiv.

- Goodhue, D. L., Lewis, W., & Thompson, R. (2007). Statistical power in analyzing interaction effects: questioning the advantage of PLS with product indicators. *Information Systems Research*, 18(2), 211–227.
- Goodhue, D. L., Lewis, W., & Thompson, R. (2012). Does PLS have advantages for small sample size or non-normal data. *MIS Quarterly*, *36*(3), 981–1001.
- Goodhue, D. L., Lewis, W., & Thompson, R. (2015). PLS pluses and minuses in path estimation accuracy. *AMCIS 2015 Proceedings*. Retrieved from http://aisel.aisnet.org/amcis2015/ISPhil/GeneralPresentations/3
- Goodhue, D. L., Thompson, R., & Lewis, W. (2013). Why you shouldn't use PLS: Four reasons to be uneasy about using PLS in analyzing path models. In *2013 46th Hawaii International Conference on System Sciences (HICSS)* (pp. 4739–4748).

 http://doi.org/10.1109/HICSS.2013.612
- Green, S. B., & Hershberger, S. L. (2000). Correlated Errors in True Score Models and Their Effect on Coefficient Alpha. *Structural Equation Modeling: A Multidisciplinary Journal*, 7(2), 251–270. http://doi.org/10.1207/S15328007SEM0702 6
- Green, S. B., & Yang, Y. (2009). Commentary on coefficient alpha: A cautionary tale.

 Psychometrika, 74(1), 121–135. http://doi.org/http://dx.doi.org/10.1007/s11336-008-9098-4
- Hair, J. F., Hult, G. T. M., Ringle, C. M., & Sarstedt, M. (2014). *A primer on partial least squares structural equations modeling (PLS-SEM)*. Los Angeles: SAGE.
- Henseler, J., Dijkstra, T. K., Sarstedt, M., Ringle, C. M., Diamantopoulos, A., Straub, D. W., ... Calantone, R. J. (2014). Common beliefs and reality about PLS: Comments on Rönkkö

- and Evermann (2013). *Organizational Research Methods*, *17*(2), 182–209. http://doi.org/10.1177/1094428114526928
- Henseler, J., Ringle, C. M., & Sarstedt, M. (2015). A new criterion for assessing discriminant validity in variance-based structural equation modeling. *Journal of the Academy of Marketing Science*, 43(1), 115–135.
- Huang, W. (2013). *PLSe: Efficient estimators and tests for partial least squares* (Doctoral dissertation). University of California, Los Angeles.
- Jackson, D. L., Voth, J., & Frey, M. P. (2013). A Note on Sample Size and Solution Propriety for Confirmatory Factor Analytic Models. *Structural Equation Modeling: A Multidisciplinary Journal*, 20(1), 86–97. http://doi.org/10.1080/10705511.2013.742388
- Kline, R. B. (2011). *Principles and practice of structural equation modeling* (3rd ed.). New York, NY: Guilford Press.
- LeBreton, J. M., Scherer, K. T., & James, L. R. (2014). Corrections for criterion reliability in validity generalization: A false prophet in a land of suspended judgment. *Industrial and Organizational Psychology*, 7(4), 478–500.
- Lu, I. R. R., Kwan, E., Thomas, D. R., & Cedzynski, M. (2011). Two new methods for estimating structural equation models: An illustration and a comparison with two established methods. *International Journal of Research in Marketing*, *28*(3), 258–268. http://doi.org/10.1016/j.ijresmar.2011.03.006
- Marsh, H. W., Hau, K.-T., Balla, J. R., & Grayson, D. (1998). Is more ever too much? The number of indicators per factor in confirmatory factor analysis. *Multivariate Behavioral Research*, *33*(2), 181–220.

- McDonald, R. P. (1996). Path analysis with composite variables. *Multivariate Behavioral Research*, 31(2), 239–270.
- McIntosh, C. N., Edwards, J. R., & Antonakis, J. (2014). Reflections on partial least squares path modeling. *Organizational Research Methods*, *17*(2), 210–251. http://doi.org/10.1177/1094428114529165
- Moosbrugger, H., Schermelleh-Engel, K., & Klein, A. (1997). Methodological problems of estimating latent interaction effects. *Methods of Psychological Research Online*, *2*(2), 95–111.
- Mosier, C. I. (1943). On the reliability of a weighted composite. *Psychometrika*, 8(3), 161–168. http://doi.org/10.1007/BF02288700
- Muchinsky, P. M. (1996). The correction for attenuation. *Educational and Psychological Measurement*, *56*(1), 63–75. http://doi.org/10.1177/0013164496056001004
- Nimon, K., Zientek, L. R., & Henson, R. K. (2012). The assumption of a reliable instrument and other pitfalls to avoid when considering the reliability of data. *Quantitative Psychology and Measurement*, *3*(102). http://doi.org/10.3389/fpsyg.2012.00102
- Nunnally, J. (1978). *Psychometric theory*. New York: McGraw-Hill.
- Oberski, D. L., & Satorra, A. (2013). Measurement Error Models With Uncertainty About the Error Variance. *Structural Equation Modeling: A Multidisciplinary Journal*, 20(3), 409–428. http://doi.org/10.1080/10705511.2013.797820
- Peterson, R. A., & Kim, Y. (2013). On the relationship between coefficient alpha and composite reliability. *The Journal of Applied Psychology*, *98*(1), 194–198. http://doi.org/10.1037/a0030767

- Pornprasertmanit, S., Miller, P., & Schoemann, A. (2014). *simsem: SIMulated Structural Equation Modeling*. Retrieved from http://CRAN.R-project.org/package=simsem
- Raju, N. S., Bilgic, R., Edwards, J. E., & Fleer, P. F. (1999). Accuracy of population validity and cross-validity estimation: An empirical comparison of formula-based, traditional empirical, and equal weights procedures. *Applied Psychological Measurement*, 23(2), 99–115.
- Raykov, T. (1997). Estimation of Composite Reliability for Congeneric Measures. *Applied Psychological Measurement*, *21*(2), 173–184. http://doi.org/10.1177/01466216970212006
- R Core Team. (2015). R: a language and environment for statistical computing (Version 3.2.0).

 Vienna, Austria: R Foundation for Statistical Computing. Retrieved from http://www.R-project.org/
- Revelle, W. (2015). psych: Procedures for psychological, psychometric, and personality research (Version 1.5.4). Retrieved from http://cran.r-project.org/web/packages/psych/index.html
- Rindskopf, D. (1984). Structural equation models empirical identification, heywood cases, and related problems. *Sociological Methods & Research*, *13*(1), 109–119. http://doi.org/10.1177/0049124184013001004
- Ringle, C. M., Sarstedt, M., & Straub, D. W. (2012). A critical look at the use of PLS-SEM in MIS Quarterly. *MIS Quarterly*, *36*(1), iiv–8.
- Rönkkö, M. (2014). The effects of chance correlations on partial least squares path modeling.

 Organizational Research Methods, 17(2), 164–181.

 http://doi.org/10.1177/1094428114525667.
- Rönkkö, M. (2015). matrixpls: Matrix-based partial least squares estimation (Version 0.6.0). Retrieved from https://github.com/mronkko/matrixpls

- Rönkkö, M., & Evermann, J. (2013). A critical examination of common beliefs about partial least squares path modeling. *Organizational Research Methods*, *16*(3), 425–448. http://doi.org/10.1177/1094428112474693
- Rönkkö, M., McIntosh, C. N., & Antonakis, J. (2015). On the adoption of partial least squares in psychological research: Caveat emptor. *Personality and Individual Differences*, (87), 76–84. http://doi.org/10.1016/j.paid.2015.07.019
- Rönkkö, M., & Ylitalo, J. (2010). Construct Validity in Partial Least Squares Path Modeling. In *ICIS 2010 Proceedings*. Retrieved from http://aisel.aisnet.org/icis2010_submissions/155
- Rosseel, Y. (2012). lavaan: An R package for structural equation modeling. *Journal of Statistical Software*, 48(2), 1–36.
- Rozeboom, W. W. (1989). The Reliability of a Linear Composite of Nonequivalent Subtests.

 *Applied Psychological Measurement, 13(3), 277–283.

 http://doi.org/10.1177/014662168901300307
- Schmidt, F. L., & Hunter, J. E. (1996). Measurement error in psychological research: Lessons from 26 research scenarios. *Psychological Methods*, *1*(2), 199–223.
- Schumacker, R. E. (2010). *A beginner's guide to structural equation modeling* (3rd ed). New York: Routledge.
- Spearman, C. (1904). The proof and measurement of association between two things. *American Journal of Psychology*, 15(1), 72–101.
- Stanley, D. J., & Spence, J. R. (2014). Expectations for replications are yours realistic?

 *Perspectives on Psychological Science, 9(3), 305–318.

 http://doi.org/10.1177/1745691614528518

- Vale, C. D., & Maurelli, V. A. (1983). Simulating multivariate nonnormal distributions. *Psychometrika*, 48(3), 465–471.
- Warren, R. D., White, J. K., & Fuller, W. A. (1974). An errors-in-variables analysis of managerial role performance. *Journal of the American Statistical Association*, *69*(348), 886–893. http://doi.org/10.2307/2286158
- Webb, N. M., Shavelson, R. J., & Haertel, E. H. (2006). Reliability coefficients and generalizability theory. In C. R. Rao & S. Sinharay (Eds.), *Handbook of Statistics: Psychometrics* (Vol. 26, pp. 81–124). Amsterdam, The Netherlands: Elsevier Science B.V.
- Werts, C. E., Linn, R. L., & Jöreskog, K. G. (1974). Intraclass Reliability Estimates: Testing Structural Assumptions. *Educational and Psychological Measurement*, *34*(1), 25–33. http://doi.org/10.1177/001316447403400104
- Wold, H. (1982). Soft modeling The Basic Design And Some Extensions. In K. G. Jöreskog & S. Wold (Eds.), *Systems under indirect observation : causality, structure, prediction* (pp. 1–54). Amsterdam: North-Holland.
- Zimmerman, D. W., & Williams, R. H. (1997). Properties of the Spearman correction for attenuation for normal and realistic non-normal distributions. *Applied Psychological Measurement*, *21*(3), 253–270. http://doi.org/10.1177/01466216970213005

8. Tables and Figures

Table 1 Estimation history of two LV model with two chance correlations

	γ	λ_1, λ_4	λ_2, λ_5	λ_3, λ_6	$\mathbf{w}_{1}, \mathbf{w}_{4}$	$\mathbf{W}_{2}, \mathbf{W}_{5}$	W ₃ , W ₆	c_1, c_2	ρ_{A1}, ρ_{A2}	R_1, R_2
Start	.300	.700	.700	.700	.410	.410	.410	1.706	.742	.742
1	.305	.537	.851	.694	.316	.501	.409	1.698	.773	.736
2	.305	.554	.859	.672	.326	.505	.395	1.701	.774	.736
3	.304	.551	.861	.673	.324	.506	.395	1.701	.775	.736
4	.304	.551	.861	.672	.324	.506	.395	1.701	.775	.736
5	.304	.551	.861	.672	.324	.506	.395	1.701	.775	.736
6	.304	.551	.861	.672	.324	.506	.395	1.701	.775	.736

Note: The model is symmetric for the latent variables.

Table 2 Inadmissible replications over 1000 replications for normal data

					le load				iabilities			ctor corre	lations
Model	Scale	N	PLSc	EIV _{ML-CFA}	EIVminres	ML-SEM	pL.Sc	EIV _{ML-CFA}	EIVminres	pL.Sc	EIV _{ML-CFA}	EIVminres	ML-SEM
Small	0	100	944	935	N/A	941	762	741	N/A	93	0	N/A	2
Model		200	936	938	N/A	937	769	749	N/A	44	0	N/A	0
		500	957	939	N/A	936	754	746	N/A	22	0	N/A	0
	.5	100	855	673	N/A	650	537	243	N/A	11	0	N/A	0
		200	780	446	N/A	429	448	73	N/A	3	0	N/A	0
		500	612	187	N/A	184	250	2	N/A	0	0	N/A	0
	1	100	438	166	N/A	152	72	2	N/A	27	19	N/A	15
		200	268	44	N/A	45	11	0	N/A	0	0	N/A	0
		500	54	1	N/A	1	0	0	N/A	0	0	N/A	0
Large	0	100	964	43	0	40	580	0	0	144	0	0	0
Model		200	968	4	0	4	570	0	0	52	0	0	0
		500	975	0	0	0	598	0	0	28	0	0	0
	.5	100	943	42	0	42	398	0	0	24	0	0	1
		200	904	3	0	3	364	0	0	3	0	0	0
		500	817	0	0	0	143	0	0	1	0	0	0
	1	100	554	21	0	20	21	0	0	10	9	9	5
		200	311	0	0	0	2	0	0	0	0	0	0
		500	79	0	0	0	0	0	0	0	0	0	0

Note: Loadings and reliabilities are invalid if any value in a replication exceeds one in absolute value. Factor correlations are inadmissible if any disattenuated composite correlation matrix exceeds one in absolute value. Small Model: Original model, Large Model: Model with duplicated indicators for $\xi 1$ and $\eta 4$. N/A = not estimated.

Table 3 Loadings over 1000 replications for normal data

						over 1000	Гериси	iions jo				
					Model			~	Large		_	
j;				Sc		ML-CFA	PL		EIV_M		EIV _r	
Param.	Scale		Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
Ра	Sc	Z										
λ ₃₁	0	100	059	.398	003	.087	096	.413	005	.084	003	.083
N31	U	200	047	.397	003	.061	041	.410	.000	.059	.003	.059
.490												
.490	_	500	064	.387	000	.039	083	.402	000	.037	000	.037
	.5	100	065	.346	003	.089	070	.351	001	.087	.000	.086
		200	028	.302	002	.060	021	.291	001	.058	001	.058
		500	021	.212	000	.037	010	.205	.001	.037	.001	.037
	1	100	027	.241	003	.088	014	.242	000	.087	.000	.086
		200	014	.167	002	.060	016	.160	002	.058	001	.058
		500	006	.101	000	.037	009	.107	.001	.037	.001	.037
λ_{34}	0	100	354	.361	.000	.056	341	.345	.004	.055	.000	.049
		200	336	.344	.001	.037	328	.342	.001	.035	000	.034
.900		500	340	.362	001	.022	327	.346	000	.023	001	.023
	.5	100	217	.314	000	.055	220	.314	.004	.053	.001	.049
		200	122	.238	.001	.036	137	.248	000	.035	001	.035
		500	045	.144	000	.022	043	.135	000	.023	001	.023
	1	100	060	.160	000	.052	059	.158	.002	.048	.001	.049
		200	019	.099	.001	.034	018	.100	000	.033	001	.035
		500	008	.063	000	.020	006	.064	000	.022	001	.023
λ_{41}	0	100	.218	1.507	4.880	11.634	107	.397	005	.070	005	.070
7041	V	200	.188	1.104	4.668	10.970	103	.399	001	.049	001	.049
.630		500	.195	1.258	4.252	9.601	100	.402	001	.031	002	.031
.050	.5	100	.018	.245	.633	4.785	033	.227	002	.072	002	.072
	.5	200	.008	.132	.075	1.636	016	.159	.001	.048	.001	.049
		500						.101	002		002	
	1		001	.075	004	.078	006			.032		.032
	1	100	001	.089	005	.090	010	.109	002	.070	002	.071
		200	001	.064	001	.064	005	.074	002	.049	002	.050
2	0	500	001	.039	001	.038	002	.046	001	.031	002	.032
λ_{42}	0	100	073	.768	4.594	11.563	273	.366	002	.050	001	.049
0.40		200	.118	4.488	4.314	10.964	258	.346	000	.034	.000	.034
.840		500	.022	1.581	3.658	9.464	237	.356	000	.021	000	.021
	.5	100	.078	.813	1.715	7.406	047	.177	.000	.048	.001	.048
		200	.021	.228	.383	3.362	025	.122	000	.034	.001	.034
		500	.008	.103	.012	.107	002	.073	.000	.021	.000	.021
	1	100	.003	.102	.009	.113	007	.070	001	.046	001	.050
		200	.000	.064	.001	.063	005	.052	001	.031	000	.034
		500	.000	.040	.000	.039	000	.032	.000	.019	.000	.021
λ_{51}	0	100	069	.358	.001	.063	074	.356	001	.061	001	.061
		200	080	.354	000	.044	077	.360	003	.044	003	.044
.650		500	062	.351	.001	.029	073	.347	001	.028	001	.028
	.5	100	004	.198	001	.060	010	.212	003	.062	003	.062
		200	008	.154	001	.043	004	.150	.000	.042	.000	.042
		500	.000	.096	000	.028	010	.098	002	.028	002	.028
	1	100	003	.120	001	.063	007	.123	003	.065	002	.065
		200	003	.090	.001	.044	001	.086	001	.045	001	.045
		500	000	.055	.001	.027	007	.056	003	.028	003	.028
λ_{56}	0	100	196	.276	000	.026	175	.250	000	.028	001	.028
,,,,,,		200	198	.268	000	.019	188	.281	000	.019	.000	.019
.900		500	186	.265	.000	.012	186	.253	.000	.011	.000	.011
.,,,,,	.5	100	044	.134	001	.012	043	.121	000	.026	000	.026
	ر.	200	026	.090	000	.019	043	.093	.000	.020	.000	.020
	1	500	010	.060	.000	.011	010	.061	.000	.012	.000	.012
	1	100	012	.072	001	.026	009	.070	000	.026	001	.027
		200	007	.054	001	.019	008	.051	001	.019	001	.020
		500	002	.032	.000	.011	003	.032	001	.011	000	.011

Table 4 Estimation error of reliabilities with normal data for Small Model

					PLS com		n error	oj reita	dilliles wil	th normal a	veighted co			
ole			- C1				A 1	1 .	CI				A 1.	1
Variable	Scale			R _U		R _W	Alp		CI		CF		Alp	
Va	Sc	Z	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
ξ1	0	100	.153	3.037	.207	1.666	.004	.054	39.658	89.198	39.813	89.675	002	.020
		200	.067	.524	.149	.517	.005	.065	35.897	76.062	35.960	76.229	002	.013
		500	.067	.434	.175	.815	.003	.021	25.644	55.709	25.662	55.762	001	.008
	.5	100	.012	.047	.040	.136	003	.022	2.765	26.790	2.776	26.924	002	.020
		200	.007	.043	.026	.132	003	.014	.461	9.120	.461	9.134	002	.014
		500	000	.009	.006	.015	004	.009	.002	.010	.002	.010	002	.008
	1	100	001	.020	.004	.023	004	.021	.001	.021	.001	.021	002	.020
		200	002	.014	.002	.015	004	.014	.000	.014	.000	.014	002	.014
		500	002	.008	.001	.009	004	.009	.000	.008	.000	.008	002	.008
ξ2	0	100	114	.115	.054	.147	.117	.142	000	.047	002	.048	009	.048
		200	114	.105	.051	.142	.112	.135	000	.032	000	.032	007	.033
		500	127	.106	.038	.114	.116	.127	000	.021	000	.021	006	.022
	.5	100	037	.053	.020	.051	016	.050	000	.046	001	.046	009	.048
		200	028	.035	.012	.035	021	.034	000	.032	000	.032	008	.033
		500	023	.022	.005	.022	024	.022	000	.021	000	.021	006	.022
	1	100	028	.046	.004	.045	031	.047	001	.045	002	.045	009	.047
		200	023	.032	.002	.031	028	.033	001	.031	000	.031	008	.033
_	0	500	022	.021	.000	.021	027	.021	000	.021	000	.020	006	.022
η3	0	100	065	.117	.086	.205	.032	.149	000	.030	001	.030	016	.034
		200	060	.122	.089	.236	.028	.133	001	.022	001	.021	016	.024
		500	062	.114	.095	.236	.031	.130	.000	.013	000	.013	015	.015
	.5	100	062	.103	.067	.180	.007	.109	000	.031	001	.030	017	.035
		200	053	.079	.055	.134	012	.076	000	.022	000	.021	016	.025
	1	500	050	.058	.031	.060	030	.040	000	.013	000	.013	015	.015
	1	100 200	051	.072	.042	.118	026 037	.065	000	.030	000 000	.030	017	.035
		500	045 043	.028	.026	.074	037	.042	000 000	.022		.021	016 015	
η4	0	100	.563	5.925	.979	6.689	.011	.119	107.477	194.713	000 108.461	197.203	013	.015
·Ι	U	200	.673	5.700	1.113	6.048	.019	.115	96.219	169.132	96.640	170.095	020	.044
		500	.960	15.428	2.508	47.453	.022	.115	73.438	127.609	73.571	127.925	017	.026
	.5	100	.202	2.227	.420	2.785	025	.077	31.438	123.968	31.765	125.597	020	.058
		200	.072	1.422	.162	1.365	031	.056	4.942	43.370	4.965	43.621	018	.043
		500	008	.075	.044	.213	037	.032	.011	.050	.011	.050	017	.026
	1	100	020	.067	.020	.116	043	.064	.250	7.749	.251	7.770	020	.059
		200	024	.045	.005	.056	042	.047	.000	.045	.000	.045	018	.043
		500	025	.027	.001	.033	041	.028	001	.027	001	.027	016	.026
η5	0	100	041	.076	.057	.093	.021	.085	000	.016	000	.015	004	.016
		200	043	.086	.054	.099	.017	.077	000	.011	000	.010	004	.011
		500	043	.104	.054	.116	.019	.093	000	.007	000	.007	003	.007
	.5	100	017	.020	.015	.022	011	.018	000	.015	000	.015	004	.016
		200	014	.012	.009	.014	012	.012	000	.010	000	.010	004	.011
	L	500	012	.007	.003	.008	013	.007	000	.007	000	.007	003	.007
	1	100	012	.015	.001	.015	015	.016	000	.015	000	.015	004	.016
		200	012	.011	.000	.010	015	.011	000	.011	000	.010	004	.011
		500	011	.007	000	.006	014	.007	000	.007	000	.006	003	.007

Table 5 Path estimates with normal data for Small Model

			Tuble	5 Path es		viin noi	mai aaii			iei .	MIC	EM
ا بر ا			0.	PLS		1-4:1	0.	EIV _N	IL-CFA	-1-4:1	ML S	
Param.	ale			ginal	Manip		Orig		Manip		Orig	
Par	Scale	Z	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
γ_{23}	0	100	025	.521	.002	.259	004	.133	006	.135	004	.126
		200	012	.357	015	.173	005	.097	009	.098	005	.093
0		500	001	.321	002	.171	000	.061	.001	.060	001	.057
	.5	100	033	.232	007	.165	005	.133	001	.137	005	.126
		200	009	.124	005	.097	.002	.097	006	.096	.001	.092
		500	005	.067	000	.060	002	.065	001	.061	000	.060
	1	100	018	.147	.002	.130	000	.135	.006	.133	000	.122
		200	001	.094	.001	.089	.008	.095	.001	.091	.003	.085
		500	001	.061	.002	.060	.002	.063	.001	.062	.002	.056
γ_{14}	0	100	087	2.719	.001	.147	.002	.101	.002	.091	.003	.097
		200	005	.168	.007	.206	003	.069	000	.062	004	.067
0		500	.012	.438	001	.063	003	.042	001	.037	002	.041
	.5	100	.023	.629	.001	.118	004	.125	007	.120	004	.129
		200	.010	.084	.010	.080	.000	.094	.002	.090	.001	.094
		500	.002	.055	.004	.054	001	.061	.001	.059	002	.060
	1	100	022	.202	.010	.178	026	.220	023	.296	020	.212
		200	007	.136	.007	.121	007	.144	003	.133	005	.136
		500	004	.085	000	.080	004	.089	004	.084	002	.083
γ_{24}	0	100	073	2.710	.008	.228	.002	.120	.002	.110	.003	.121
_		200	001	.333	003	.226	001	.086	.002	.070	002	.084
.7		500	.008	.485	.004	.121	002	.053	.002	.046	001	.053
	.5	100	.007	.594	019	.146	016	.168	021	.160	023	.170
		200	.007	.106	022	.097	009	.115	012	.106	011	.115
		500	.002	.068	011	.061	002	.069	005	.062	002	.069
	1	100	.035	.203	017	.174	.028	.219	.023	.293	.020	.215
		200	.013	.135	011	.119	.008	.142	.003	.130	.005	.137
	0	500 100	.006	3.363	001 011	.076	.004 002	.089	.005 003	.081	.002 001	.085
γ15	0	200	117	12.085	.007	.451	002	.069	003	.066	001	.065
.22		500	016	.833	003	.102	003	.045	002	.044	000	.063
.22	.5	100	.035	1.385	003	.102	002	.121	.002	.116	002	.120
	د. ا	200	008	.145	015	.083	.000	.088	.001	.085	000	.086
		500	008	.050	013	.050	.000	.050	.002	.052	.001	.049
	1	100	.004	.194	004	.176	.002	.208	.002	.032	.010	.194
	1	200	.008	.194	004	.118	.017	.133	.032	.126	.008	.194
		500	.008	.077	003	.073	.012	.078	.005	.076	.008	.075
V	0	100	194	8.184	.004	.575	004	.129	.003	.130	001	.128
γ25		• • • •		22.527	.004	000	004	.089	.002	.087	000	.087
7		500	766 061	1.231	.003	.880	001	.057	.001	.057	001	.055
	.5	100	179	4.659	.001	.342	001	.137	005	.131	.002	.132
		200	017	.258	.013	.091	001	.094	002	.092	000	.091
		500	003	.055	.006	.054	.001	.057	002	.055	.001	.054
	1	100	023	.192	.003	.176	024	.205	037	.253	015	.192
		200	014	.123	.003	.117	014	.129	010	.125	009	.122
		500	010	.078	.002	.072	010	.080	004	.076	007	.076
β35	0	100	.268	7.348	.011	.590	005	.119	.000	.120	002	.115
рээ		200	626	18.853	.028	.887	002	.086	001	.083	002	.081
.35		500	045	.881	.003	.104	002	.052	001	.053	002	.049
	.5	100	106	4.455	009	.329	.001	.116	.006	.116	.000	.112
		200	.030	.248	001	.090	.003	.081	.004	.077	.002	.076
		500	.008	.049	001	.051	.001	.051	.001	.051	.002	.048
	1	100	.013	.112	.004	.102	.003	.108	.011	.106	.001	.092
	1	200	.008	.072	005	.072	.003	.076	001	.074	001	.067
		500	.003	.047	003	.044	.004	.048	000	.045	001	.041
<u> </u>	<u> </u>	500	.002	.04/	001	.044	.001	.070	000	.043	001	.041

Table 6 Path estimates with normal data for Large Model

				PLS		0 1 0000	estimate	EIV		ara jor .	Large M		V _{minres}		ML S	EM
am.	Scale	z	Orig		Manipu	ılated	Origi		Manip	ılated	Origi			ulated	Origi	
Param.	Sc	_														
		100	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
γ_{23}	0	100	016	.709	003	.406	009	.136	.001	.132	009	.136	.001	.131	007	.131
0		200 500	047 .009	.283	001	.358	005	.097	006	.096	005	.097	006	.096 .057	004	.092
0	.5				.002		001	.057	002		001	.057	002		002	.055
	.3	100 200	024 008	.236	003 002	.166	003 002	.134	003 004	.133	003 002	.134	003 004	.133	004 000	.127
		500	008	.064	002	.057	002	.062	.000	.060	002	.062	.000	.060	000	.059
	1	100	007	.141	.002	.129	.002	.130	.000	.129	.002	.130	.001	.129	.002	.120
	1	200	015	.097	.002	.087	.002	.097	.002	.091	.002	.097	.001	.091	.002	.087
		500	003	.060	.002	.058	.002	.061	.002	.060	.002	.061	.002	.060	000	.056
γ14	0	100	071	2.449	.008	.344	.000	.119	.003	.118	.000	.119	.004	.118	.001	.115
7 14		200	005	.279	004	.129	.000	.080	003	.081	.000	.080	003	.081	001	.079
0		500	.002	.164	018	1.219	001	.050	001	.049	001	.050	001	.049	001	.049
	.5	100	004	.153	.011	.188	006	.122	003	.125	006	.123	002	.125	005	.119
		200	.005	.081	.005	.081	001	.083	000	.085	001	.083	.000	.085	001	.082
		500	.003	.051	.002	.058	001	.053	.000	.053	001	.053	.000	.053	001	.052
	1	100	015	.183	.004	.168	019	.193	021	.198	020	.199	018	.191	014	.184
		200	008	.120	.002	.112	008	.122	010	.121	008	.124	009	.120	009	.121
		500	006	.076	001	.071	005	.078	005	.075	006	.079	005	.074	005	.075
γ_{24}	0	100	088	2.856	.006	.389	.004	.139	.001	.130	.004	.139	.001	.130	.005	.137
		200	.011	.389	001	.165	001	.093	.000	.092	001	.093	.000	.092	.001	.090
.7		500	.006	.267	012	1.243	.002	.058	000	.058	.002	.058	000	.058	.002	.056
	.5	100	.051	.157	013	.201	.009	.137	.008	.134	.009	.138	.008	.134	.008	.135
		200	.018	.091	011	.093	.000	.096	000	.094	.000	.096	000	.094	000	.093
		500	.005	.058	008	.060	001	.058	003	.058	001	.058	003	.058	002	.056
	1	100	.031	.180	007	.163	.021	.191	.022	.192	.022	.197	.020	.185	.016	.183
		200	.013	.119	004	.110	.007	.123	.011	.119	.006	.125	.010	.117	.008	.121
		500	.009	.076	.000	.069	.006	.079	.005	.073	.006	.080	.005	.072	.006	.076
γ15	0	100	.050	.708	003	.322	003	.108	004	.106	003	.108	004	.106	004	.106
22		200	.050	1.670	012	.176	003	.075	003	.074	003	.075	003	.074	003	.073
.22	.5	500	001	.150	.004	.210	002	.049	002	.048	002	.049	002	.048	002	.048
	.3	100 200	.005	.166	018 004	.260	.006	.118	.002	.116	.006	.118	.002	.116	.004	.083
		500	001	.059	004	.054	.003	.052	.003	.083	.003	.052	.003	.083	.002	.050
	1	100	.019	.188	011	.163	.026	.198	.020	.201	.027	.203	.003	.193	.016	.189
	1	200	.008	.120	002	.113	.012	.123	.012	.124	.012	.125	.017	.123	.007	.118
		500	.005	.078	002	.069	.007	.080	.005	.073	.007	.081	.005	.073	.004	.076
γ ₂₅	0	100	012	1.223	.026	.472	.002	.132	002	.128	.002	.132	002	.128	.001	.127
143		200	.227	5.259	.023	.779	.004	.088	.001	.085	.004	.089	.001	.085	.003	.087
7		500	.025	.485	000	.250	.003	.056	.001	.056	.003	.056	.001	.056	.002	.054
	.5		016	.675	.002	.308	002	.137	004	.128	002	.137	004	.128	001	.133
		200	016	.096	.008	.096	002	.096	001	.091	003	.096	001	.091	001	.093
		500	004	.063	.004	.056	.001	.057	001	.054	.001	.057	001	.054	.001	.055
	1	100	036	.197	.008	.172	031	.203	024	.206	032	.209	021	.198	022	.190
		200	014	.120	.004	.112	013	.123	012	.122	013	.126	011	.121	008	.117
		500	007	.079	.004	.070	006	.081	002	.074	007	.082	002	.074	004	.077
β35	0	100	009	1.122	.001	.309	002	.120	004	.120	002	.120	004	.119	002	.114
ľ		200	245	5.612	030	.760	004	.085	001	.082	004	.085	001	.082	002	.081
.35		500	015	.445	.005	.136	001	.053	.000	.053	001	.053	.000	.053	001	.050
	.5		.023	.698	.006	.336	.002	.118	.004	.117	.002	.118	.004	.117	.001	.110
		200	.020	.102	003	.092	.003	.079	.002	.079	.003	.079	.002	.079	.001	.074
		500	.008	.049	002	.051	.001	.051	.001	.051	.001	.051	.001	.051	.000	.048
	1	100	.016	.109	001	.106	.007	.110	.004	.107	.006	.110	.004	.106	.002	.091
		200	.007	.073	007	.070	.003	.077	003	.072	.003	.077	003	.072	.002	.064
		500	.002	.046	000	.042	.001	.047	.002	.044	.001	.047	.002	.044	000	.041

Table 7 95% percentile confidence intervals with normal data for Small Model

abie	1	70 pei	centite	PLS		rvais w	1111 110111			111 11100
m.	le		-				_	EIV _{ML}		
Param.	Scale	Z		Coverage		Incl.		Coverage		Incl.
P	9 1		Under	Within	Over	zero	Under	Within		zero
γ_{23}	0	100	.008	.979	.013	.979	.024	.951	.025	.951
		200	.014	.972	.014	.972	.029	.938	.033	.938
0		500	.011	.977	.012	.977	.026	.945	.029	.945
	.5	100	.008	.962	.030	.962	.017	.959	.024	.959
		200	.014	.954	.032	.954	.032	.943	.025	.943
		500	.021	.936	.043	.936	.043	.925	.032	.925
	1	100	.007	.947	.046	.947	.025	.941	.034	.941
		200	.015	.958	.027	.958	.033	.945	.022	.945
		500	.023	.938	.039	.938	.040	.931	.029	.931
γ_{14}	0	100	.004	.995	.001	.995	.028	.942	.030	.942
		200	.007	.988	.005	.988	.029	.943	.028	.943
0		500	.005	.987	.008	.987	.016	.957	.027	.957
	.5	100	.011	.977	.012	.977	.019	.953	.028	.953
		200	.024	.967	.009	.967	.027	.947	.026	.947
	ļ	500	.027	.963	.010	.963	.025	.954	.021	.954
	1	100	.013	.955	.032	.955	.015	.955	.030	.955
		200	.021	.947	.032	.947	.026	.940	.034	.940
		500	.031	.941	.028	.941	.026	.952	.022	.952
γ_{24}	0	100	.013	.979	.008	.979	.033	.943	.024	.943
		200	.011	.980	.009	.980	.026	.946	.028	.946
.7	ļ	500	.009	.980	.011	.980	.025	.946	.029	.946
	.5	100	.041	.955	.004	.427	.011	.962	.027	.350
		200	.041	.951	.008	.115	.018	.950	.032	.106
		500	.031	.941	.028	.000	.023	.944	.033	.001
	1	100	.046	.944	.010	.033	.033	.955	.012	.086
		200	.043	.942	.015	.002	.030	.952	.018	.002
	_	500	.034	.946	.020	.000	.031	.945	.024	.000
γ15	0	100	.000	1.000	.000	1.000	.025	.948	.027	.948
.22		200	.001	.998	.001	.998	.016	.951	.033	.951
.22		500	.005	.985	.010	.985	.025	.942	.033	.942
	.5	100	.002	.989	.009	.951	.014	.958	.028	.849
		200	.009	.961	.030	.828	.031	.939	.030	.722
	1	500	.010	.968	.022	.486	.023	.964	.013	.430
	1	100	.017	.958	.025	.768	.034	.947	.019	.739
		200	.032	.953	.015	.489	.035	.953	.012	.515
-	^	500	.039	.938	.023	.100	.040		.018	
γ25	0	200	.002	.997	.001	.997	.029	.948	.023	.948
7		500	.004	.995	.001	.995	.031	.949	.020	.949
/	.5		.005	.990	.005	.990	.023	.949	.028	.949
	د. ا	100 200	.000	.970	.030	.618	.026	.932	.042	
			.006	.958	.036	.137	.025	.937	.038	.051
	1	100	.011	.950	.039	.000	.025	.948	.027	.000
	1	100	.011	.957	.032	.081	.015	.943	.042	.060
		200	.011	.960		.000	.008	.957	.035	.000
		500	.015	.948	.037	.000	.013	.949	.038	.000
β35	0	100	.000	.999	.001	.999	.030	.938	.032	.938
.35		200	.001	.997	.002	.997	.028	.936	.036	.936
.55	.5	100	.001	.989	.010	.989	.018	.949	.033	.949
	د. ا	100 200	.027	.973		.845	.034	.950	.016	.681
		500	.060	.938 .952	.002	.552	.034	.939	.027	.426
	1		.046		.002	.117			.028	.074
	1	200	.022	.975	.003	.295	.022	.965	.013	.208
		200	.041	.950	.009	.018	.036	.944	.020	.009
	<u> </u>	500	.029	.947	.024	.000	.025	.947	.028	.000

Table 8 95% percentile confidence intervals with normal data for Large Model

			Tuble	PLS		ne con	jiuence	<u>intervals</u> EIV _{ML} .		ormai		EIV _{mi}		
Param.	Scale	_				T., .1				Incl.				T1
are	Sc	Z		overage		Incl.		Coverage				Coverage		Incl.
1			Under	Within	Over	zero	Under	Within	Over	zero	Under	Within		zero
γ_{23}	0	100	.007	.972	.021	.972	.026	.934	.040	.934	.026	.934	.040	.934
		200	.010	.977	.013	.977	.028	.933	.039	.933	.028	.933	.039	.933
0		500	.008	.980	.012	.980	.018	.953	.029	.953	.018	.953	.029	.953
	.5	100	.007	.971	.022	.971	.028	.949	.023	.949	.028	.949	.023	.949
		200	.014	.943	.043	.943	.033	.928	.039	.928	.033	.928	.039	.928
	1	500	.009	.957	.034	.957	.025	.945	.030	.945	.025	.945	.030	.945
	1	100	.008	.950	.042	.950	.033	.944	.023	.944	.033	.944	.023	.944
		200	.012	.944	.044	.944	.029	.942	.029	.942	.029	.942	.029	.942
21	0	500 100	.013	.941	.003	.941	.028	.937	.035		.028	.940	.035	.937
γ_{14}	U	200	.004	.993	.003	.993	.030	.949	.030	.940	.030	.940	.030	.940
0		500	.007	.988	.003	.988	.029	.949	.022	.958	.029	.949	.022	.958
	.5	100	.004	.977	.009	.977	.020	.938	.023	.938	.017	.950	.023	.950
	.5	200	.022	.973	.005	.973	.024	.956	.020	.956	.024	.956	.020	.956
		500	.027	.969	.003	.969	.023	.952	.025	.952	.024	.950	.026	.950
	1	100	.013	.947	.040	.947	.013	.949	.038	.949	.015	.949	.036	.949
	1	200	.021	.942	.037	.942	.018	.944	.038	.944	.019	.943	.038	.943
		500	.017	.950	.033	.950	.016	.950	.034	.950	.016	.955	.029	.955
γ ₂₄	0	100	.011	.976	.013	.976	.039	.929	.032	.929	.039	.928	.033	.928
124		200	.007	.985	.008	.985	.023	.944	.033	.944	.023	.944	.033	.944
.7		500	.012	.979	.009	.979	.027	.947	.026	.947	.027	.947	.026	.947
	.5	100	.088	.912	.000	.384	.052	.927	.021	.283	.047	.930	.023	.281
		200	.064	.934	.002	.074	.030	.946	.024	.061	.030	.946	.024	.061
		500	.032	.957	.011	.000	.020	.954	.026	.000	.019	.954	.027	.000
	1	100	.054	.941	.005	.016	.047	.942	.011	.052	.048	.938	.014	.045
		200	.048	.937	.015	.000	.035	.949	.016	.000	.034	.951	.015	.000
		500	.039	.943	.018	.000	.033	.945	.022	.000	.028	.949	.023	.000
γ_{15}	0	100	.001	.999	.000	.999	.025	.953	.022	.953	.025	.952	.023	.952
		200	.004	.994	.002	.994	.019	.950	.031	.950	.019	.950	.031	.950
.22		500	.009	.979	.012	.979	.026	.941	.033	.941	.026	.941	.033	.941
	.5	100	.005	.992	.003	.955	.030	.955	.015	.824	.030	.955	.015	.823
		200	.028	.963	.009	.819	.040	.940	.020	.721	.040	.941	.019	.719
		500	.018	.956	.026	.482	.025	.952	.023	.406	.024	.953	.023	.409
	1	100	.031	.959	.010	.713	.041	.949	.010	.699	.049	.939	.012	.696
		200	.034	.945	.021	.469	.034	.949	.017	.494	.038	.944	.018	.503
		500	.028	.945	.027	.112	.028	.943	.029	.121	.031	.941	.028	.125
γ25	0	100	.005	.995	.000	.995	.032	.942	.026	.942	.032		.026	.942
7		200 500	.005	.993	.002	.993	.031	.949 .952	.020	.949	.031	.949 .952	.020	.949
.,	.5	100	.007	.990 .979	.003	.631	.026	.932	.022	.932	.026	.932	.022	.932
	ر.	200	.003	.950	.047	.142	.022	.937	.039	.042	.024	.935	.039	.042
		500	.011	.945	.044	.000	.025	.945	.030	.000	.025	.944	.031	.000
	1	100	.005	.951	.044	.066	.007	.932	.061	.052	.008	.935	.057	.050
	1	200	.012	.950	.038	.001	.017	.950	.033	.000	.014	.953	.033	.000
		500	.016	.940	.044	.000	.023	.939	.038	.000	.021	.940	.039	.000
β35	0	100	.000	1.000	.000	1.000	.029	.940	.031	.940	.029	.940	.031	.940
p33	Ů	200	.002	.996	.002	.996	.025	.935	.040	.935	.024	.936	.040	.936
.35		500	.000	.993	.007	.993	.029	.948	.023	.948	.029	.948	.023	.948
	.5	100	.033	.967	.000	.841	.033	.945	.022	.685	.034	.943	.023	.683
		200	.045	.953	.002	.541	.022	.944	.034	.417	.021	.947	.032	.417
		500	.047	.952	.001	.106	.031	.944	.025	.072	.031	.944	.025	.071
	1	100	.029	.967	.004	.282	.026	.952	.022	.183	.026	.951	.023	.179
	1	200	.037	.955	.008	.009	.032	.944	.024	.004	.032	.946	.022	.004
		500	.032	.937	.031	.000	.028	.940	.032	.000	.028	.942	.030	.000
		200	.032	.73/	.031	.000	.020	.740	.032	.000	.028	.542	.030	.000

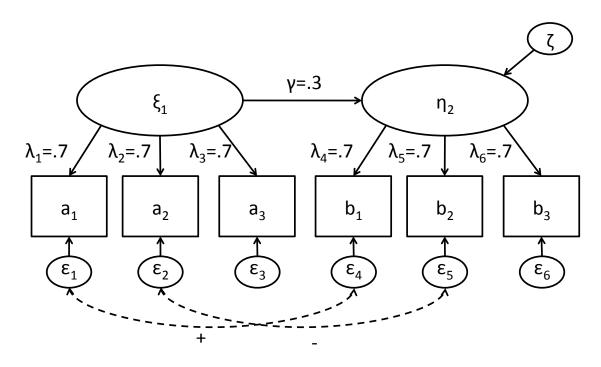


Figure 1 Example of chance correlations

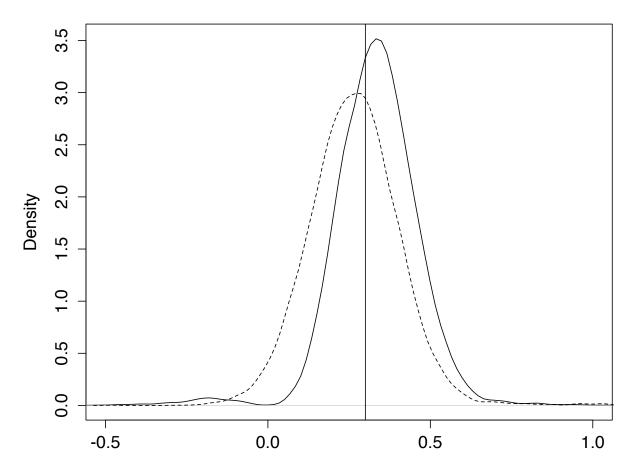


Figure 2 Normal PLSc estimates (solid line) and estimates with weights and reliabilities estimated from hold-out sample (dashed lines)

Appendix A: Full results tables

This appendix contains the full results tables for both normal and non-normal data, excluding the tables that are included in the main document.

Table 1 Inadmissible replications over 1000 replications for non-normal data

		Tubi							ations jor				
			Inac	lmissib	le load	ings	Inadmis	ssible reli	iabilities	Inadmis	ssible fa	ctor corre	lations
Model	Scale	Z	PLSc	$\mathrm{EIV}_{\mathrm{ML-CFA}}$	$\mathrm{EIV}_{\mathrm{minres}}$	ML-SEM	PLSc	EIV _{ML-CFA}	EIV_{minres}	PLSc	EIV _{ML-CFA}	EIV_{minres}	ML-SEM
Small	0	100	949	936	N/A	935	749	747	N/A	102	0	N/A	2
Model		200	938	935	N/A	945	768	748	N/A	37	0	N/A	0
		500	947	937	N/A	930	767	749	N/A	20	0	N/A	0
	.5	100	849	679	N/A	636	517	238	N/A	16	0	N/A	0
		200	780	433	N/A	416	451	79	N/A	6	0	N/A	0
		500	614	186	N/A	183	238	2	N/A	1	0	N/A	0
	1	100	438	170	N/A	162	71	3	N/A	23	18	N/A	14
		200	281	43	N/A	43	10	0	N/A	0	0	N/A	0
		500	62	1	N/A	1	0	0	N/A	0	0	N/A	0
Large	0	100	976	43	0	35	603	0	0	131	0	0	0
Model		200	971	6	0	5	592	0	0	63	0	0	0
		500	973	0	0	0	572	0	0	23	0	0	0
	.5	100	931	38	0	38	386	0	0	14	0	0	4
		200	903	2	0	1	359	0	0	1	0	0	0
		500	811	0	0	0	140	0	0	0	0	0	0
	1	100	559	18	0	18	20	0	0	7	6	9	5
		200	303	1	0	2	3	0	0	0	0	0	0
		500	83	0	0	0	0	0	0	0	0	0	0

Note: Loadings and reliabilities are invalid if any value in a replication has absolute value greater than one. Reliabilities are inadmissible if any index exceeds one in absolute value. Linear effects are inadmissible if the disattenuated composite covariance matrix is invalid. Small Model: Original model, Large Model: Model with duplicated indicators for $\xi 1$ and $\eta 4$. N/A = not estimated.

Table 2 Loadings over 1000 replications for normal data

					Model	1000 repi		<i>j</i> = 1, 1, 1, 1	Large			
m.	o		ΡΙ	Sc		-CFA	PL	Sc	ML-		min	res
Param.	Scale	_	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
Ъ	S	Ν	Dias	50	Dias	SD	Dias	SD	Dias	3D	Dias	SD
λ_{11}	0	100	.117	.776	2.649	7.864	068	.366	002	.028	003	.028
		200	.116	.695	2.717	7.685	055	.354	001	.020	001	.020
.870		500	.112	.731	2.245	6.434	062	.353	001	.012	001	.012
	.5	100	.073	.528	.164	2.259	075	.335	001	.028	002	.028
		200	.081	.706	.036	.968	045	.309	001	.020	001	.020
		500	.009	.227	000	.044	030	.192	001	.012	001	.012
	1	100	.001	.070	002	.050	008	.085	002	.027	002	.028
		200	001	.051	001	.035	001	.057	000	.019	001	.019
		500	001	.029	000	.020	003	.037	001	.012	002	.012
λ_{11b}	0	100	N/A	N/A	N/A	N/A	096	.361	002	.028	002	.028
		200	N/A	N/A	N/A	N/A	059	.351	000	.020	000	.020
.870		500	N/A	N/A	N/A	N/A	069	.356	000	.012	000	.012
	.5	100	N/A	N/A	N/A	N/A	066	.323	001	.029	001	.029
		200	N/A	N/A	N/A	N/A	036	.290	000	.020	001	.020
		500	N/A	N/A	N/A	N/A	021	.220	000	.013	001	.013
	1	100	N/A	N/A	N/A	N/A	002	.088	001	.029	002	.029
		200	N/A	N/A	N/A	N/A	002	.060	000	.019	001	.019
		500	N/A	N/A	N/A	N/A	.000	.037	000	.012	001	.013
λ_{12}	0	100	.034	.901	2.486	7.718	110	.321	000	.017	000	.017
		200	.021	.759	2.264	7.072	115	.297	001	.012	001	.012
.940		500	007	.821	1.896	5.984	095	.287	.000	.007	.000	.007
	.5	100	.022	.540	.287	2.689	079	.277	.000	.017	.000	.017
		200	.037	.799	.036	.903	076	.244	001	.012	001	.012
		500	.019	.260	.001	.048	032	.173	000	.007	.000	.008
	1	100	.001	.073	.001	.048	005	.068	000	.017	.000	.017
		200	.002	.052	.001	.033	002	.046	001	.012	000	.012
		500	.001	.029	000	.019	001	.029	000	.007	.000	.007
λ_{12b}	0	100	N/A	N/A	N/A	N/A	114	.305	001	.017	000	.017
		200	N/A	N/A	N/A	N/A	112	.288	001	.012	000	.012
.940		500	N/A	N/A	N/A	N/A	107	.303	000	.007	.000	.008
	.5	100	N/A	N/A	N/A	N/A	104	.281	.000	.017	.000	.017
		200	N/A	N/A	N/A	N/A	068	.235	001	.012	001	.012
		500	N/A	N/A	N/A	N/A	027	.172	000	.007	000	.007
	1	100	N/A	N/A	N/A	N/A	001	.066	000	.017	.000	.017
		200	N/A	N/A	N/A	N/A	003	.047	001	.011	001	.012
		500	N/A	N/A	N/A	N/A	001	.028	000	.007	000	.007
λ_{21}	0	100	144	.303	000	.107	142	.292	006	.106	005	.105
410		200	150	.293	.001	.076	153	.284	002	.076	002	.075
.410		500	153	.290	001	.047	167	.280	.001	.046	.001	.046
	.5	100	033	.195	001	.106	036	.199	005	.107	003	.109
		200	023	.146	004	.072	019	.144	004	.069	003	.070
		500	007	.093	001	.044	006	.090	002	.046	001	.047
	1	100	007	.122	000	.097	006	.121	001	.097	002	.104
		200	.001	.087	.001	.066	005	.084	003	.067	003	.073
		500	001	.054	000	.043	005	.053	003	.043	002	.046

				Small	Model				Large	Model		
ïi.	ြှ		PL			-CFA	PL	Sc	ML-		min	res
Param.	Scale	z	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
λ_{22}	0	100	159	.292	002	.110	160	.295	011	.111	010	.110
		200	160	.289	002	.075	153	.299	.002	.074	.002	.074
.410		500	159	.284	002	.046	152	.299	004	.047	004	.047
	.5	100	026	.196	003	.105	038	.193	007	.104	006	.104
		200	021	.151	005	.073	008	.140	.003	.075	.003	.077
		500	007	.092	002	.044	003	.086	001	.045	002	.047
	1	100	006	.122	004	.096	008	.119	003	.098	004	.107
		200	005	.085	003	.067	006	.082	.000	.068	.002	.075
	0	500	.001	.058	001	.043	001	.052	000	.041	001	.047
λ_{23}	0	100	190	.298	008	.100	192	.289	002	.104	001	.102
.470		200	195	.287	004	.074	181	.291	003	.071	003	.071
.470		500	200	.284	000	.046	187	.285	002	.043	002	.044
	.5	100 200	045 021	.191	008 002	.101	030 014	.182	.002 002	.100	.001 001	.103
		500	021	.088	.002	.042	009	.086	002	.042	001	.043
	1	100	013	.115	007	.042	.001	.110	.002	.090	.002	.103
	1	200	008	.081	007	.064	003	.077	002	.062	001	.069
		500	002	.052	.000	.040	000	.048	002	.039	001	.044
λ_{24}	0	100	288	.289	001	.093	271	.288	005	.094	004	.093
7624		200	265	.284	.002	.063	289	.278	003	.062	003	.061
.600		500	287	.290	001	.040	294	.285	000	.039	.000	.039
	.5	100	053	.169	002	.089	044	.161	.001	.085	.003	.090
	"	200	035	.123	002	.059	027	.121	003	.061	003	.063
		500	014	.078	001	.038	013	.078	.001	.037	.002	.039
	1	100	013	.098	002	.077	010	.100	.002	.078	000	.090
		200	009	.070	003	.053	006	.068	002	.053	002	.063
		500	004	.047	002	.035	002	.044	.000	.034	.000	.040
λ_{25}	0	100	303	.292	002	.093	286	.290	003	.095	004	.092
		200	300	.285	000	.063	291	.279	001	.064	001	.064
.630		500	296	.275	001	.039	294	.284	000	.038	000	.038
	.5	100	062	.173	.001	.086	053	.164	006	.086	007	.090
		200	025	.120	.003	.060	031	.111	.000	.057	000	.062
		500	012	.078	001	.038	010	.074	002	.037	002	.040
	1	100	010	.093	.003	.074	019	.094	008	.076	006	.092
		200	006	.069	.002	.053	004	.065	.001	.050	.001	.061
	0	500	005	.045	001	.034	005	.042	003	.032	002	.039
λ_{26}	0	100	323 305	.284	002	.094	294 301	.283	001	.094	001	.092
.650		200 500	305	.287	001 000	.060	319	.279	000 002	.060	001 002	.060
.550	.5	100	056	.162	004	.039	062	.160	002	.039	002	.038
		200	022	.116	004	.058	002	.100	004	.057	004	.060
		500	009	.077	.001	.038	011	.072	001	.037	001	.039
	1	100	014	.096	003	.073	013	.094	003	.078	001	.094
		200	008	.065	001	.051	005	.066	000	.051	001	.061
		500	003	.045	.001	.034	004	.040	001	.032	001	.039
λ_{31}	0	100	059	.398	003	.087	096	.413	005	.084	003	.083
		200	047	.397	001	.061	041	.410	.000	.059	.001	.059
.490		500	064	.387	000	.039	083	.402	000	.037	000	.037
	.5	100	065	.346	003	.089	070	.351	001	.087	.000	.086
		200	028	.302	002	.060	021	.291	001	.058	001	.058
		500	021	.212	000	.037	010	.205	.001	.037	.001	.037
	1	100	027	.241	003	.088	014	.242	000	.087	.000	.086
		200	014	.167	002	.060	016	.160	002	.058	001	.058
		500	006	.101	000	.037	009	.107	.001	.037	.001	.037

				Small	Model				Large	Model		
ım.	e		PI	Sc		-CFA	PL	Sc	ML-0		min	res
Param.	Scale	Z	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
λ_{32}	0	100	135	.406	003	.074	123	.411	006	.076	005	.074
		200	122	.407	002	.053	136	.384	002	.052	002	.052
.600		500	133	.403	002	.033	132	.398	.000	.033	.000	.033
	.5	100	096	.349	004	.072	093	.339	005	.074	003	.073
		200	076	.279	002	.051	078	.287	001	.053	001	.053
		500	038	.200	.000	.034	019	.181	000	.032	000	.032
	1	100	037	.204	004	.072	038	.215	004	.073	003	.073
		200	018	.149	001	.052	016	.150	002	.053	001	.053
		500	.000	.089	.001	.034	004	.090	000	.032	000	.032
λ_{33}	0	100	270	.374	001	.059	261	.360	003	.060	000	.056
.800		200	260	.361	.001	.041	279	.365	000	.040	.001	.040
.800		500	257	.381	.001	.026	251	.365	.000	.024	.001	.024
	.5	200	178 119	.311	001 000	.058	185 109	.308	003 001	.058	001 000	.055
		500	036	.161	.001	.025	038	.142	.001	.024	.001	.024
-	1	100	043	.174	001	.023	054	.173	002	.054	001	.055
	1	200	043	.116	000	.038	019	.110	002	.040	000	.033
		500	022	.070	.001	.024	006	.069	.001	.023	.001	.024
λ ₃₄	0	100	354	.361	.000	.056	341	.345	.004	.055	.000	.049
7034	v	200	336	.344	.001	.037	328	.342	.001	.035	000	.034
.900		500	340	.362	001	.022	327	.346	000	.023	001	.023
	.5	100	217	.314	000	.055	220	.314	.004	.053	.001	.049
		200	122	.238	.001	.036	137	.248	000	.035	001	.035
		500	045	.144	000	.022	043	.135	000	.023	001	.023
	1	100	060	.160	000	.052	059	.158	.002	.048	.001	.049
		200	019	.099	.001	.034	018	.100	000	.033	001	.035
		500	008	.063	000	.020	006	.064	000	.022	001	.023
λ_{41}	0	100	.218	1.507	4.880	11.634	107	.397	005	.070	005	.070
		200	.188	1.104	4.668	10.970	103	.399	001	.049	001	.049
.630		500	.195	1.258	4.252	9.601	100	.402	002	.031	002	.031
	.5	100	.018	.245	.633	4.785	033	.227	004	.072	004	.072
		200	.008	.132	.075	1.636	016	.159	.001	.048	.001	.049
-	1	500	001	.075	004	.078	006	.101	002	.032	002	.032
	1	100 200	001	.089	005 001	.090	010 005	.109	002 002	.070	002	.071
		500	001 001	.064	001	.064	003	.046	002	.049	002 002	.050
λ _{41b}	0	100	N/A	N/A	N/A	N/A	002	.408	001	.075	002	.074
7416	U	200	N/A	N/A	N/A	N/A	122	.402	002	.049	002	.049
.630		500	N/A	N/A	N/A	N/A	120	.395	.001	.031	.000	.031
	.5	100	N/A	N/A	N/A	N/A	034	.229	001	.071	000	.071
		200	N/A	N/A	N/A	N/A	009	.164	001	.051	001	.051
		500	N/A	N/A	N/A	N/A	011	.104	000	.030	000	.030
	1	100	N/A	N/A	N/A	N/A	009	.109	004	.070	003	.071
		200	N/A	N/A	N/A	N/A	003	.076	000	.049	000	.050
		500	N/A	N/A	N/A	N/A	003	.049	001	.030	001	.030
λ_{42}	0	100	073	.768	4.594	11.563	273	.366	002	.050	001	.049
		200	.118	4.488	4.314	10.964	258	.346	000	.034	.000	.034
.840		500	.022	1.581	3.658	9.464	237	.356	000	.021	000	.021
	.5	100	.078	.813	1.715	7.406	047	.177	.000	.048	.001	.048
		200	.021	.228	.383	3.362	025	.122	000	.034	.001	.034
		500	.008	.103	.012	.107	002	.073	.000	.021	.000	.021
	1	100	.003	.102	.009	.113	007	.070	001	.046	001	.050
		200	.000	.064	.001	.063	005	.052	001	.031	000	.034
		500	.000	.040	.000	.039	000	.032	.000	.019	.000	.021

				Small	Model				Large	Model		
am	le		PL	Sc	ML·	-CFA	PL	Sc	ML-	CFA	min	res
Param.	Scale	Z	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
λ_{42b}	0	100	N/A	N/A	N/A	N/A	247	.359	001	.048	001	.047
		200	N/A	N/A	N/A	N/A	240	.373	001	.035	001	.035
.840		500	N/A	N/A	N/A	N/A	242	.360	001	.022	000	.022
	.5	100	N/A	N/A	N/A	N/A	043	.179	000	.048	.000	.047
		200	N/A	N/A	N/A	N/A	015	.112	000	.033	.000	.034
		500	N/A	N/A	N/A	N/A	008	.074	001	.022	000	.022
	1	100	N/A	N/A	N/A	N/A	004	.074	001	.045	001	.048
		200	N/A	N/A	N/A	N/A	001	.049	001	.031	001	.034
		500	N/A	N/A	N/A	N/A	002	.032	000	.020	.000	.023
λ_{51}	0	100	069	.358	.001	.063	074	.356	001	.061	001	.061
		200	080	.354	000	.044	077	.360	003	.044	003	.044
.650		500	062	.351	.001	.029	073	.347	001	.028	001	.028
	.5	100	004	.198	001	.060	010	.212	003	.062	003	.062
		200	008	.154	001	.043	004	.150	.000	.042	.000	.042
		500	.000	.096	000	.028	010	.098	002	.028	002	.028
	1	100	003	.120	001	.063	007	.123	003	.065	002	.065
		200	003	.090	.001	.044	001	.086	001	.045	001	.045
		500	000	.055	.001	.027	007	.056	003	.028	003	.028
λ_{52}	0	100	063	.340	001	.056	065	.351	003	.063	003	.063
670		200	069	.344	002	.043	088	.347	001	.041	001	.041
.670	_	500	062	.347	000	.026	075	.338	000	.027	000	.027
	.5	100	030	.194	003	.057	038	.204	005	.059	004	.059
		200	004	.140	.001	.043	005	.143	.000	.044	000	.044
		500	007	.095	000	.027	003	.098	.000	.026	.000	.026
	1	100	010	.120	004	.061	015	.123	004	.061	004	.061
		200	002	.085	000	.042	000	.086	001	.045	001	.045
1	0	500	002	.052	001	.026	.000	.056	.000	.026	.000	.026
λ_{53}	0	200	118 123	.326	.001 001	.049	104		.002 002	.048	.002 002	.048
.750		500	123	.330	000	.033	110 114	.319	.002	.034	002	.022
.,,50	.5	100	031	.182	002	.050	022	.171	001	.046	001	.046
	.5	200	003	.122	.001	.034	022	.133	001	.035	002	.035
		500	008	.079	001	.023	002	.083	.001	.022	.001	.022
•	1	100	011	.100	001	.048	002	.099	000	.047	000	.047
	•	200	002	.072	000	.035	008	.075	002	.035	002	.035
		500	003	.046	001	.022	000	.047	.000	.021	.000	.021
λ ₅₄	0	100	112	.326	.000	.050	120	.321	.001	.049	.001	.049
-74	-	200	101	.318	000	.035	115	.323	000	.035	001	.035
.750		500	115	.323	001	.022	102	.326	000	.021	000	.021
	.5	100	022	.171	002	.050	029	.173	002	.048	002	.048
		200	019	.132	000	.034	018	.135	000	.035	000	.035
		500	005	.084	000	.023	006	.084	001	.021	001	.021
	1	100	006	.103	002	.050	010	.103	001	.047	001	.047
		200	006	.073	002	.036	007	.075	001	.037	002	.037
		500	003	.047	001	.022	002	.047	001	.021	001	.021
λ_{55}	0	100	185	.262	.000	.028	183	.265	001	.026	001	.026
		200	180	.253	001	.020	192	.278	.000	.019	.000	.019
.900		500	187	.271	000	.012	189	.264	.001	.012	.001	.012
	.5	100	044	.130	001	.028	046	.132	002	.027	003	.027
		200	026	.092	001	.019	019	.091	.000	.019	.000	.019
		500	008	.061	000	.011	008	.060	.000	.011	.000	.012
	1	100	012	.072	.000	.026	014	.073	002	.027	002	.027
		200	008	.050	001	.019	006	.050	000	.018	.000	.019
		500	002	.033	001	.012	002	.032	000	.011	000	.012

				Small	Model				Large	Model		
am	le		PL	Sc	ML-	-CFA	PL	Sc	ML-0	CFA	min	res
Param.	Scale	Z	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
λ ₅₆	0	100	196	.276	000	.026	175	.250	000	.028	001	.028
		200	198	.268	000	.019	188	.281	000	.019	.000	.019
.900		500	186	.265	.000	.012	186	.253	.000	.011	.000	.011
	.5	100	044	.134	001	.027	043	.121	000	.026	000	.026
		200	026	.090	000	.019	026	.093	.000	.019	.000	.019
		500	010	.060	.000	.011	010	.061	.000	.012	.000	.012
	1	100	012	.072	001	.026	009	.070	000	.026	001	.027
		200	007	.054	001	.019	008	.051	001	.019	001	.020
		500	002	.032	.000	.011	003	.032	001	.011	000	.011

Note: N/A = not estimated.

Table 3 Loadings over 1000 replications for non-normal data

				Small	Model		Ĭ		Large	Model		
am	ıle		PL	Sc	ML-	-CFA	PL	Sc	ML-	CFA	min	res
Param.	Scale	Z	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
λ ₁₁	0	100	.107	.743	2.540	7.480	061	.355	002	.028	003	.028
		200	.123	1.001	2.547	7.346	054	.352	001	.020	001	.020
.870		500	.114	.773	2.269	6.470	063	.346	001	.012	001	.012
	.5	100	.081	.554	.209	2.340	065	.327	001	.028	002	.028
		200	.115	.943	.087	1.529	043	.303	000	.020	001	.020
		500	.012	.200	001	.045	027	.197	001	.012	001	.013
	1	100	.002	.070	001	.049	008	.086	002	.027	002	.028
		200	001	.051	002	.035	001	.057	000	.019	001	.019
		500	001	.029	000	.020	003	.037	001	.012	001	.012
λ_{11b}	0	100	N/A	N/A	N/A	N/A	084	.355	002	.028	002	.028
		200	N/A	N/A	N/A	N/A	068	.350	.000	.020	000	.020
.870		500	N/A	N/A	N/A	N/A	065	.350	000	.012	000	.012
	.5	100	N/A	N/A	N/A	N/A	068	.318	001	.029	001	.029
		200	N/A	N/A	N/A	N/A	038	.294	000	.020	000	.020
		500	N/A	N/A	N/A	N/A	023	.220	000	.012	001	.013
	1	100	N/A	N/A	N/A	N/A	003	.089	001	.029	002	.029
		200	N/A	N/A	N/A	N/A	002	.059	000	.019	000	.020
		500	N/A	N/A	N/A	N/A	000	.038	000	.012	001	.013
λ_{12}	0	100	.031	.908	2.596	8.092	109	.312	000	.017	.000	.017
		200	.010	.654	2.163	6.864	120	.297	001	.012	001	.012
.940		500	.029	.966	1.843	5.877	098	.297	.000	.007	.000	.007
	.5	100	.052	.878	.209	2.246	085	.289	.000	.017	.001	.017
		200	.027	.717	.034	.856	077	.244	001	.012	001	.012
		500	.053	1.062	.002	.048	031	.173	000	.007	000	.007
	1	100	.001	.074	.001	.049	005	.069	.000	.017	.000	.017
		200	.002	.052	.001	.033	002	.046	001	.012	001	.012
		500	.000	.030	001	.020	001	.028	000	.007	.000	.007
λ_{12b}	0	100	N/A	N/A	N/A	N/A	112	.296	001	.017	001	.017
0.40		200	N/A	N/A	N/A	N/A	109	.297	001	.012	001	.012
.940		500	N/A	N/A	N/A	N/A	105	.297	.000	.007	.000	.008
	.5	100	N/A	N/A	N/A	N/A	105	.278	000	.017	.000	.017
		200	N/A	N/A	N/A	N/A	070	.239	001	.012	001	.012
		500	N/A	N/A	N/A	N/A	027	.171	000	.007	000	.007
	1	100	N/A	N/A	N/A	N/A	001	.067	000	.017	000	.017
		200	N/A	N/A	N/A	N/A	003	.047	001	.011	001	.012
		500	N/A	N/A	N/A	N/A	001	.028	000	.007	000	.007

				Small	Model				Large	Model		
am.	le le		PL			-CFA	PL	Sc	ML-		min	res
Param.	Scale	z	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
λ_{21}	0	100	148	.302	.001	.108	141	.294	007	.108	006	.107
		200	151	.291	.001	.075	149	.284	002	.076	002	.076
.410		500	150	.290	002	.048	160	.284	.001	.046	.001	.046
	.5	100	032	.193	000	.105	034	.196	005	.106	004	.109
		200	022	.143	003	.072	020	.145	004	.070	003	.071
		500	005	.094	.000	.044	006	.089	001	.046	001	.047
	1	100	008	.123	.000	.097	006	.121	001	.098	003	.104
		200	.001	.088	.001	.066	005	.084	003	.068	003	.074
		500	000	.054	.001	.043	005	.053	003	.043	002	.046
λ_{22}	0	100	160	.293	002	.109	153	.291	010	.111	009	.111
410		200	153	.291	002	.074	157	.291	.002	.075	.003	.075
.410		500	153	.282	002	.045	146	.295	004	.047	004	.047
	.5	100	025	.196	003	.105	034	.194	006	.104	006	.104
		200	022	.150	005	.073	007	.140	.003	.075	.003	.077
	1	500 100	007 007	.091	002 003	.044	003 007	.086	002 002	.045	003 003	.047
	1	200	007	.085	003	.067	007	.082	.002	.068	.003	.075
		500	.000	.057	003	.044	003	.052	000	.041	002	.047
λ_{23}	0	100	192	.037	001	.100	190	.288	002	.104	002	.103
N ₂₃	U	200	194	.286	004	.074	183	.289	002	.072	001	.071
.470		500	206	.287	000	.046	186	.288	002	.043	003	.043
, 0	.5	100	044	.189	008	.102	031	.180	.002	.100	.001	.103
		200	019	.141	002	.071	013	.131	001	.067	001	.069
		500	006	.088	.001	.042	009	.085	002	.041	002	.043
	1	100	013	.116	006	.095	.000	.110	.004	.090	.004	.103
		200	007	.082	002	.064	003	.076	002	.062	001	.069
		500	002	.053	.000	.040	000	.048	001	.039	001	.043
λ_{24}	0	100	290	.288	002	.095	268	.292	005	.095	005	.094
		200	257	.286	.002	.063	291	.284	003	.062	003	.061
.600		500	284	.290	001	.040	293	.283	000	.040	000	.039
	.5	100	051	.169	002	.089	043	.159	.001	.085	.003	.091
		200	034	.122	002	.059	026	.120	003	.061	003	.063
		500	014	.078	001	.038	014	.078	.000	.038	.002	.039
	1	100	013	.097	003	.077	010	.100	.002	.078	000	.091
		200	008	.070	002	.053	006	.069	002	.053	002	.063
		500	004	.047	002	.035	002	.044	.000	.034	.000	.040
λ_{25}	0	100	311	.293	001	.093	283	.293	003	.096	004	.093
.630		200	299	.283	.000	.063	294	.276	001	.063	001	.063
.030	.5	500 100	288 061	.267 .171	002 .001	.040	295 051	.286	000 005	.037	000 007	.037
		200	024	.171	.001	.061	031	.112	003	.057	007	.062
		500	024	.077	001	.037	032	.074	002	.037	002	.039
	1	100	012	.094	.003	.074	018	.095	002	.075	002	.091
	1	200	005	.070	.003	.053	004	.065	.001	.050	.002	.061
		500	005	.045	002	.034	004	.042	003	.032	002	.039
λ_{26}	0	100	320	.288	002	.093	298	.291	000	.092	001	.090
-20		200	303	.288	002	.060	310	.291	001	.060	001	.059
.650		500	299	.281	000	.039	323	.275	003	.038	003	.038
	.5	100	055	.161	003	.086	062	.158	003	.088	003	.090
		200	022	.117	001	.058	026	.110	001	.057	001	.060
		500	009	.077	.002	.037	011	.070	001	.036	002	.038
	1	100	014	.096	002	.073	014	.094	003	.078	000	.093
		200	008	.066	002	.051	005	.066	000	.050	001	.061
		500	003	.045	.001	.034	003	.040	001	.032	001	.038

				Small	Model				Large	Model		
am.	le		PL	Sc		-CFA	PL	Sc	ML-0		min	ires
Param.	Scale	Z	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
λ_{31}	0	100	058	.393	003	.086	088	.399	005	.084	003	.082
		200	048	.396	000	.060	044	.437	.000	.059	.001	.059
.490		500	065	.380	000	.039	081	.400	000	.037	000	.037
	.5	100	068	.352	003	.089	067	.354	000	.086	.001	.085
		200	031	.304	002	.060	019	.286	002	.059	002	.058
		500	020	.211	000	.037	008	.203	.001	.037	.001	.037
	1	100	030	.244	003	.088	012	.239	.000	.086	.001	.085
		200	015	.171	002	.060	015	.160	002	.059	002	.058
		500	005	.101	000	.037	008	.105	.001	.037	.001	.037
λ_{32}	0	100	128	.387	002	.074	114	.394	005	.075	004	.074
600		200	124	.403	002	.053	131	.391	002	.052	001	.052
.600	-	500	127	.404	002	.033	138	.389	000	.032	000	.032
	.5	100	089	.333	003	.073	090	.338	004	.074	003	.073
		200	074	.282	002	.052	076	.292	002	.053	002	.053
-	1	500 100	039 034	.200	.001 003	.033	021 039	.186	000 004	.032	000 003	.032
	1					.072				.073		
		500	017 001	.147	002 .001	.052	018 004	.090	002 000	.053	002 000	.053
λ_{33}	0	100	259	.368	001	.060	267	.373	003	.060	000	.056
N33	U	200	264	.374	.001	.041	274	.356	000	.040	.001	.040
.800		500	239	.367	.001	.026	247	.366	.000	.024	.001	.024
	.5	100	173	.304	000	.059	181	.305	004	.057	001	.055
		200	120	.250	000	.040	106	.246	001	.040	.000	.040
		500	035	.160	.001	.025	039	.149	.001	.024	.001	.024
•	1	100	044	.173	000	.056	054	.170	003	.054	001	.055
		200	022	.116	000	.038	020	.110	001	.039	.000	.040
		500	008	.069	.001	.024	005	.069	.001	.023	.001	.024
λ_{34}	0	100	342	.362	.000	.057	341	.348	.004	.055	.000	.049
		200	333	.344	.001	.037	327	.337	.000	.036	001	.034
.900		500	334	.360	000	.022	329	.348	000	.023	001	.022
	.5	100	219	.312	001	.055	213	.308	.003	.053	.001	.050
		200	123	.244	.001	.036	136	.243	001	.035	001	.035
		500	044	.141	.000	.021	045	.141	000	.023	001	.023
	1	100	059	.160	001	.052	057	.155	.002	.048	.001	.050
		200	020	.098	.001	.034	018	.102	000	.034	001	.035
1	0	500	007	.063	.000	.020	007	.064	000	.021	001	.023
λ_{41}	0	100	.225	1.542	4.875	11.864	105	.393	005 001	.070	005	.070
.630		500	.167	1.123	4.849	9.325	108			.049	002	.049
.030	.5	100	.016	.229	.934	5.645	108 035	.399	002 004	.031	002 003	.031
	.5	200	.008	.130	.096	1.741	017	.161	.001	.049	.001	.049
		500	001	.074	004	.077	006	.102	002	.032	002	.032
-	1	100	002	.090	005	.090	011	.108	002	.070	002	.071
	-	200	001	.064	001	.064	005	.075	002	.049	002	.050
		500	001	.039	001	.038	002	.047	001	.031	001	.032
λ_{41b}	0	100	N/A	N/A	N/A	N/A	107	.416	001	.074	001	.074
		200	N/A	N/A	N/A	N/A	119	.402	003	.050	003	.050
.630		500	N/A	N/A	N/A	N/A	108	.397	.000	.031	000	.031
	.5	100	N/A	N/A	N/A	N/A	033	.226	000	.071	000	.071
		200	N/A	N/A	N/A	N/A	007	.164	000	.052	001	.052
		500	N/A	N/A	N/A	N/A	011	.104	000	.030	000	.030
	1	100	N/A	N/A	N/A	N/A	008	.108	003	.070	003	.071
		200	N/A	N/A	N/A	N/A	002	.077	.000	.049	000	.050
		500	N/A	N/A	N/A	N/A	003	.049	001	.030	001	.030

				Small	Model				Large	Model		
am.	le		PL	Sc		-CFA	PL	Sc	ML-		min	res
Param.	Scale	Z	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
λ ₄₂	0	100	017	.907	4.954	12.049	269	.362	002	.050	001	.049
		200	004	1.423	4.339	11.042	253	.351	000	.034	.000	.034
.840		500	046	.890	3.404	9.181	247	.367	000	.021	000	.021
	.5	100	.052	.559	1.521	6.870	047	.176	.001	.048	.001	.048
		200	.020	.217	.338	3.191	025	.122	000	.033	.001	.034
		500	.008	.104	.013	.108	003	.073	.000	.021	.000	.021
	1	100	.004	.103	.009	.103	006	.070	001	.045	000	.050
		200	.000	.064	.001	.064	005	.052	001	.031	001	.034
		500	000	.041	000	.039	.000	.032	.000	.019	.000	.021
λ_{42b}	0	100	N/A	N/A	N/A	N/A	253	.364	001	.049	001	.048
		200	N/A	N/A	N/A	N/A	233	.365	001	.035	001	.035
.840		500	N/A	N/A	N/A	N/A	252	.357	001	.022	000	.022
	.5	100	N/A	N/A	N/A	N/A	042	.174	000	.048	000	.048
		200	N/A	N/A	N/A	N/A	016	.111	000	.033	000	.034
		500	N/A	N/A	N/A	N/A	008	.073	000	.022	.000	.022
	1	100	N/A	N/A	N/A	N/A	003	.073	001	.045	001	.049
		200	N/A	N/A	N/A	N/A	002	.049	001	.031	001	.034
		500	N/A	N/A	N/A	N/A	001	.032	000	.020	.000	.022
λ_{51}	0	100	070	.364	.001	.063	085	.371	001	.061	001	.061
		200	082	.358	000	.044	069	.355	003	.044	003	.044
.650		500	064	.356	.001	.029	065	.355	001	.028	001	.028
	.5	100	003	.195	001	.060	012	.213	003	.062	003	.062
		200	010	.157	001	.043	002	.149	.000	.043	000	.043
		500	001	.096	.000	.028	010	.096	002	.027	002	.027
	1	100	003	.119	001	.062	009	.123	003	.065	003	.065
		200	004	.091	.001	.044	001	.086	000	.045	000	.045
		500	001	.055	.001	.027	007	.055	002	.028	002	.028
λ_{52}	0	100	069	.335	001	.056	065	.358	003	.063	003	.063
		200	064	.344	002	.043	091	.347	001	.041	002	.041
.670		500	067	.338	000	.026	087	.352	000	.027	000	.027
	.5	100	028	.193	003	.057	038	.208	005	.060	004	.059
		200	004	.141	.000	.043	005	.144	000	.044	000	.044
		500	006	.095	000	.027	003	.097	.000	.026	.000	.027
	1	100	009	.119	003	.061	015	.121	004	.061	004	.061
		200	002	.085	000	.042	.000	.086	001	.044	002	.044
		500	002	.052	001	.026	.000	.055	.000	.026	.000	.026
λ_{53}	0	100	123	.327	.000	.050	109	.301	.002	.048	.001	.048
.750		200	122	.326	001	.033	112	.327	002	.034	002	.034
./30	_	500	112	.322	000	.022	116	.323	000	.022	000	.022
	.5	100	031	.181	002	.050	024	.177	001	.047	001	.047
		200	003	.122	.001	.034	018	.133	001	.035	002	.035
	1	500	007	.080	001	.023	001	.082	.000	.022	.000	.022
	1	100	011 002	.101	001	.047	001	.098	000	.047	001	.047
		200		.073	000	.035	007	.074	002	.035	002	.035
2	0	500 100	003 108	.046	001 .000	.022	000 119	.047	.000	.021	000 .001	.021
λ_{54}	U	200	108	.330	000	.049	119	.321	001	.049	001	.049
.750		500	116	.322	001	.022	098	.321	001	.034	001	.034
.,50	.5	100	020	.170	001	.050	029	.176	002	.048	002	.048
	ر.	200	020	.170	002	.035	029	.170	002	.035	002	.046
		500	020	.083	001	.023	006	.084	001	.021	001	.021
	1	100	004	.103	002	.049	010	.102	001	.047	001	.047
	1	200	006	.074	002	.036	007	.074	001	.036	001	.036
		500	002	.047	001	.022	007	.047	001	.022	002	.022
		500	002	.07/	001	.022	002	.04/	001	.022	001	.022

_:				Small	Model				Large	Model		
Param.	ale		PL	Sc	ML-	-CFA	PL	Sc	ML-0	CFA	min	res
Раг	Scale	Z	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
λ_{55}	0	100	187	.262	.000	.028	184	.267	001	.026	001	.026
		200	189	.263	001	.019	191	.266	.000	.019	.000	.019
.900		500	194	.273	000	.012	185	.262	.001	.012	.001	.012
	.5	100	046	.128	001	.028	048	.133	002	.027	002	.027
		200	025	.091	001	.019	019	.091	.000	.018	.000	.019
		500	008	.060	000	.011	008	.060	.000	.012	.001	.012
	1	100	013	.072	.000	.026	015	.074	002	.027	002	.027
		200	008	.050	001	.019	007	.051	000	.018	000	.019
		500	002	.032	001	.012	002	.032	000	.011	.000	.012
λ_{56}	0	100	195	.277	000	.026	179	.262	001	.028	001	.028
		200	202	.273	000	.019	190	.274	000	.019	000	.019
.900		500	197	.271	.001	.012	188	.253	.000	.011	.000	.011
	.5	100	044	.132	002	.027	046	.134	000	.026	001	.026
		200	026	.090	000	.019	026	.095	.000	.018	.000	.018
		500	010	.060	.000	.012	009	.061	.000	.012	.000	.012
	1	100	013	.071	001	.026	009	.070	001	.026	001	.027
		200	007	.054	001	.019	008	.052	001	.019	001	.020
		500	003	.032	.000	.011	003	.032	001	.011	000	.012

Note: N/A = not estimated.

Table 4 Estimation error of reliabilities with normal data for Large Model

		1	able 4 E					s with	normal a					
ပ					LS com	_	5				veighted		osites	
ت Variable	<u>e</u>		CR	-U	CR	W	Alp	ha	CR	-U	CR	W	Alp	ha
Var	Scale	z	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
ξ1	0	100	005	.023	.043	.059	.002	.012	000	.009	000	.009	001	.009
		200	003	.036	.047	.082	.004	.056	.000	.006	.000	.006	000	.006
		500	005	.031	.044	.062	.002	.009	.000	.004	.000	.004	000	.004
	.5	100	002	.009	.015	.019	001	.009	000	.009	000	.009	001	.009
		200	001	.006	.008	.010	001	.006	.000	.006	.000	.006	000	.006
		500	001	.004	.004	.005	002	.004	.000	.004	000	.004	000	.004
	1	100	002	.009	.002	.009	002	.009	000	.009	000	.009	001	.009
		200	001	.006	.001	.006	002	.006	.000	.006	.000	.006	000	.006
		500	001	.004	.000	.004	002	.004	.000	.004	.000	.004	000	.004
ξ2	0	100	113	.109	.052	.127	.108	.139	003	.047	005	.048	012	.049
		200	115	.104	.049	.130	.116	.137	.000	.032	000	.032	007	.033
		500	114	.099	.048	.121	.110	.129	000	.020	000	.020	007	.021
	.5	100	039	.054	.016	.051	021	.050	003	.048	004	.048	012	.049
		200	028	.036	.011	.034	021	.034	.000	.032	000	.032	007	.033
		500	023	.022	.004	.021	024	.022	000	.020	000	.020	007	.021
	1	100	030	.048	.001	.047	034	.050	004	.047	004	.047	012	.049
		200	024	.033	.002	.031	028	.034	000	.031	001	.031	007	.033
		500	022	.021	.000	.020	027	.022	001	.020	001	.020	007	.021
η3	0	100	066	.122	.088	.242	.026	.131	.000	.030	001	.030	016	.034
		200	064	.112	.076	.210	.021	.113	.000	.021	000	.021	015	.024
		500	066	.115	.077	.212	.019	.105	.000	.013	000	.013	015	.015
	.5	100	068	.107	.052	.132	003	.084	.001	.029	.000	.029	016	.033
		200	053	.080	.053	.127	014	.064	.000	.021	.000	.021	015	.024
		500	045	.050	.035	.081	030	.048	000	.013	000	.013	015	.015
	1	100	054	.077	.035	.087	024	.065	.001	.029	.000	.029	016	.033
		200	046	.048	.026	.060	038	.037	.000	.021	000	.020	015	.024
		500	041	.026	.014	.033	043	.026	000	.013	000	.013	015	.015
η4	0	100	036	.098	.097	.232	.030	.107	001	.027	001	.026	008	.028
		200	043	.098	.092	.208	.026	.092	.000	.019	.000	.019	006	.020
		500	034	.085	.098	.202	.023	.087	.000	.012	.000	.012	006	.013
	.5	100	026	.052	.041	.070	008	.044	001	.026	001	.026	008	.028
		200	021	.034	.028	.040	013	.025	.000	.018	000	.018	006	.020
		500	018	.016	.014	.023	020	.015	.000	.012	.000	.012	006	.013
	1	100	020	.029	.006	.029	024	.030	001	.026	001	.026	008	.028
		200	017	.019	.004	.020	022	.020	.000	.018	.000	.018	006	.020
		500	017	.013	.002	.013	022	.013	.000	.012	.000	.012	006	.013
η5	0	100	040	.069	.052	.081	.012	.041	000	.016	000	.015	004	.016
		200	045	.079	.051	.088	.015	.050	.000	.011	000	.011	003	.012
		500	042	.071	.056	.087	.013	.046	000	.007	000	.007	003	.007
	.5	100	016	.020	.015	.023	011	.018	000	.016	000	.016	004	.017
		200	013	.012	.008	.013	012	.012	.000	.011	.000	.011	003	.011
		500	012	.007	.003	.008	013	.007	000	.007	000	.007	003	.007
	1	100	012	.015	.001	.015	015	.016	000	.016	000	.015	004	.016
		200	011	.011	.001	.011	014	.011	.000	.011	.000	.010	003	.012
		500	011	.007	.000	.007	014	.007	000	.007	.000	.006	003	.007

Table 5 Estimation error of reliabilities with non-normal data for Small Model

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The color of th	le						<u> </u>			0.7			•		
200	iab	le le		C.			₹ _W	Alp	ha			CF		Alp	ha
The color of th	Vai	Sca	z	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Sol	ξ1		100	.125	1.482	.210	1.031	.007	.086	39.676	89.480	39.839	89.993	002	.020
S			200	.093	1.300	.165	.805	.003	.021	33.200	71.734	33.256	71.886	002	.013
The color Part P			500	.061	.311	.188	1.066	.002	.018	25.384	54.363	25.399	54.403	002	.009
S00		.5	100	.011	.041	.039	.114	003	.022	2.429	23.863	2.439	23.986	002	.020
The color The			200	.007	.060	.026	.197	003	.014	.433	8.180	.434	8.189	002	.013
\$\frac{200}{500} \ \ \cdot \cdot 0.02 \ \ \ 0.13 \ \ \ 0.002 \ \ \ 0.014 \ \ \ \ \ \ 0.09 \ \ \ \ \ \ 0.004 \ \ \ \ \ 0.000 \ \ \ \ \ \ \ 0.000 \ \ \ \			500	000	.010	.007	.016	004	.009	.002	.010	.002	.010	002	.009
\$\frac{500}{0}\$ \$.003 \$.009 \$.001 \$.009 \$004 \$.009 \$000 \$009 \$000 \$009 \$000 \$009 \$000 \$009 \$000 \$009 \$000 \$009 \$000 \$009 \$000 \$001 \$004 \$008 \$001 \$004 \$008 \$001 \$004 \$008 \$007 \$033 \$007 \$033 \$003 \$005 \$000 \$031 \$007 \$003 \$007 \$033 \$007 \$033 \$000 \$037 \$055 \$021 \$053 \$015 \$051 \$000 \$047 \$000 \$047 \$008 \$049 \$000 \$021 \$000 \$021 \$000 \$021 \$000 \$021 \$000 \$021 \$000 \$021 \$000 \$021 \$000 \$021 \$000 \$022 \$000 \$021 \$000 \$022 \$000 \$022 \$000 \$022 \$000 \$023 \$022 \$003 \$022 \$004 \$031 \$000 \$021 \$000 \$002 \$006 \$022 \$000 \$023 \$023 \$002 \$031 \$028 \$033 \$000 \$001 \$046 \$001 \$046 \$009 \$048 \$025 \$000 \$022 \$000 \$022 \$000 \$023 \$022 \$000 \$022 \$000 \$023 \$000 \$023 \$000 \$001 \$000 \$001 \$000 \$002 \$006 \$002 \$000 \$002 \$006 \$002 \$000 \$002 \$000 \$002 \$000 \$002 \$000 \$002 \$000 \$002 \$000 \$002 \$000 \$002 \$000 \$002 \$000 \$003 \$000 \$001 \$000 \$002 \$000 \$000 \$000 \$000 \$000 \$000		1	100	001	.020	.004	.022	004	.021	.001		.001	.020	002	.020
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Soo -122 .106 .042 .115 .116 .136 000 .021 000 .021 007 .022 .020 .027 .035 .021 .053 .015 .051 .000 .044 001 .047 008 .049 .000 .023 .022 .005 .022 .024 .022 000 .021 000 .032 007 .033 .022 .005 .022 .024 .022 000 .021 000 .021 006 .022 .005 .022 .024 .022 000 .021 000 .021 006 .022 .005 .022 .024 .022 000 .021 000 .021 006 .022 .024 .022 .000 .021 000 .021 000 .021 000 .024 .009 .046 .009 .046 .009 .046 .009 .046 .009 .046 .009 .046 .009 .046 .009 .046 .009 .046 .009 .046 .009 .046 .009 .046 .000 .022 .021 .000 .021 .027 .022 000 .021 000 .020 006 .022 .006 .022 .006 .022 .006 .025 .008 .018 .009 .231 .029 .150 .000 .030 001 .030 016 .033 .000 .058 .118 .096 .263 .030 .134 000 .022 000 .022 016 .025 .005 .005 .118 .096 .263 .030 .126 .000 .013 .000 .013 .015	ξ2	0	100		.111	.054			.141	.000	.048	001		008	.049
100 -0.37 0.55 0.021 0.53 -0.15 0.00 0.047 -0.01 0.047 -0.008 0.049															
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200 014 .012 .009 .014 012 .012 000 .010 000 .010 004 .011 500 012 .007 .003 .008 013 .007 000 .007 000 .007 000 .007 003 .007 1 100 012 .015 .001 .015 015 .016 000 .015 001 .015 004 .016 200 012 .010 .000 .010 015 .011 000 .010 000 .010 004 .011		.5									1			1	
500 012 .007 .003 .008 013 .007 000 .007 000 .007 003 .007 1 100 012 .015 .001 .015 015 .016 000 .015 001 .015 004 .016 200 012 .010 .000 .010 015 .011 000 .010 000 .010 004 .011															
1 100 012 .015 .001 .015 015 .016 000 .015 001 .015 004 .016 200 012 .010 .000 .010 015 .011 000 .010 000 .010 004 .011															_
200012 .010 .000 .010015 .011000 .010000 .010004 .011		1												1	
			200		.010						1				
															.007

Table 6 Estimation error of reliabilities with non-normal data for Large Model

		1 40	ne o Est				<u>bilities v</u>	viin no	n-norma					
le			an.		PLS com	•			an.		veighted			
iab	le		CR	U	CR	W	Alp	ha	CR		CR	W	Alp	ha
ت Variable	Scale	z	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
ξ1	0	100	005	.032	.045	.079	.002	.012	000	.009	000	.009	001	.009
		200	005	.033	.044	.067	.002	.010	.000	.006	.000	.006	000	.006
		500	004	.027	.042	.050	.002	.008	.000	.004	.000	.004	000	.004
	.5	100	002	.009	.015	.019	001	.009	000	.009	000	.009	001	.009
		200	001	.006	.009	.011	001	.006	.000	.006	.000	.006	000	.006
		500	001	.004	.004	.005	002	.004	.000	.004	.000	.004	000	.004
	1	100	002	.009	.002	.009	002	.009	000	.009	000	.009	001	.009
		200	001	.006	.001	.006	002	.006	.000	.006	.000	.006	000	.006
		500	001	.004	.000	.004	002	.004	.000	.004	.000	.004	000	.004
ξ2	0	100	112	.110	.052	.135	.111	.144	003	.048	004	.049	012	.049
		200	118	.106	.046	.130	.120	.142	.000	.032	000	.032	007	.033
		500	116	.100	.048	.121	.111	.129	000	.020	000	.020	007	.021
	.5	100	038	.055	.017	.052	020	.050	002	.048	003	.049	011	.050
		200	027	.036	.011	.033	021	.034	.000	.032	000	.032	007	.033
		500	023	.021	.004	.021	024	.021	001	.020	000	.020	007	.021
	1	100	030	.048	.001	.047	033	.050	003	.047	004	.047	011	.050
		200	024	.032	.001	.031	028	.034	000	.031	000	.031	007	.033
		500	022	.021	.000	.020	027	.021	001	.020	001	.020	007	.021
η3	0	100	065	.113	.090	.217	.027	.131	.000	.030	001	.030	016	.034
		200	063	.156	.087	.521	.023	.112	.000	.021	000	.021	015	.024
		500	067	.112	.071	.173	.018	.107	000	.013	000	.013	015	.015
	.5	100	065	.101	.054	.127	001	.087	.001	.030	000	.030	016	.034
		200	050	.075	.056	.140	015	.062	.001	.021	.000	.021	015	.025
		500	045	.051	.035	.080	029	.048	000	.013	000	.013	015	.015
	1	100	055	.081	.037	.095	022	.071	.000	.030	.000	.030	016	.034
		200	046	.047	.026	.057	038	.037	.001	.021	.000	.020	015	.025
		500	041	.027	.014	.033	044	.026	000	.013	000	.013	015	.015
η4	0	100	038	.093	.088	.183	.028	.098	000	.027	000	.027	007	.029
		200	044	.107	.084	.202	.025	.089	.000	.018	.000	.018	006	.019
		500	036	.091	.098	.202	.024	.088	.000	.012	.000	.012	006	.013
	.5	100	025	.052	.042	.078	009	.044	001	.027	001	.027	008	.029
		200	021	.036	.029	.040	013	.025	.000	.018	.000	.018	006	.019
		500	018	.015	.014	.022	019	.015	.000	.012	.000	.012	006	.013
	1	100	020	.029	.006	.029	024	.030	001	.027	001	.027	008	.029
		200	017	.019	.004	.020	022	.020	.000	.018	.000	.018	006	.019
		500	017	.013	.002	.013	022	.013	.000	.012	.000	.012	006	.013
η5	0	100	040	.076	.052	.082	.014	.067	000	.016	001	.015	004	.016
		200	046	.087	.051	.095	.017	.071	.000	.011	000	.011	003	.012
		500	040	.083	.058	.131	.016	.070	000	.007	000	.007	003	.007
1	.5	100	017	.020	.015	.022	011	.018	000	.016	001	.015	004	.016
1		200	013	.012	.008	.014	012	.012	.000	.011	000	.011	003	.011
		500	012	.007	.003	.008	013	.007	000	.007	000	.007	003	.007
1	1	100	012	.015	.000	.015	015	.016	000	.015	000	.015	004	.016
1		200	011	.011	.001	.011	014	.011	.000	.011	000	.010	003	.012
		500	011	.007	.000	.006	014	.007	000	.007	.000	.006	003	.007

Table 7 Path estimates with non-normal data for Small Model

			Tuble /	PL		ith non-n	ormai ai		IL-CFA	Juei	ML S	EEM
m.	e		Omic			ulatad	Orio			ulatad		
Param.	Scale	7	Orig			ulated	Orig		Manip		Orig	
P		Z	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
γ23	0	100	015	.769	016	.381	005	.133	007	.135	005	.127
0		200	015	.359	008	.176	006	.097	007	.098	006	.093
0	_	500	008	.231	001	.099	000	.061	.001	.059	001	.058
	.5	100	021	.308	010	.186	005	.132	002	.138	006	.126
		200	013	.149	006	.105	.001	.098	007	.096	000	.092
	1	500	005	.067	000	.060	001	.065	001	.061	000	.060
	1	100	020	.146	.001	.132	001	.134	.004	.134	000	.122
		200	002	.094	000	.088	.008	.094	001	.090	.003	.085
	0	500	001 017	.061	.001	.060	.002	.063	.000	.061	.002	.056
γ14	U	100		.522	.008	.241	.002	.101	.002	.090	.003	.098
0		200	.003	.221	.002	.089	003	.069	000	.062	004	.068
U	_	500	006	.087	001	.098	003	.043	002	.038	002	.041
	.5	100	.005	.118	.002	.111	004		009	.119	002	.129
		200	.009	.084	.011	.079	001	.094	.003	.090	000	.095
	1	500 100	.002	.056	.003	.054	002	.061	000 024	.059	002 023	.060
	1	200	025 009	.203	.012	.179	028 008	.144	024	.376	023	.215
		500	009	.086	.006	.078	008	.089	003	.082	007	.084
- 01	0	100	.007	.639	.005	.276	.002	.119	.003	.109	000	.118
γ ₂₄	U	200	.007	.350	.003	.143	.002	.086	.003	.071	000	.085
.7		500	012	.205	.011	.142	001	.054	.003	.047	001	.053
.,	.5	100	.026	.151	023	.143	018	.170	025	.161	024	.171
	.5	200	.020	.106	023	.097	008	.115	023	.106	010	.115
		500	.002	.068	023	.061	002	.069	004	.061	003	.069
	1	100	.037	.203	021	.174	.029	.220	.022	.376	.023	.217
		200	.014	.137	012	.119	.009	.143	.002	.130	.006	.138
		500	.006	.086	002	.075	.004	.089	.004	.079	.003	.086
γ15	0	100	017	.370	006	.214	003	.104	003	.097	001	.097
113		200	000	1.094	006	.145	006	.068	003	.066	005	.065
.22		500	003	.109	.002	.191	002	.045	002	.043	002	.043
	.5	100	.032	1.018	.011	1.009	001	.121	.001	.117	006	.119
		200	004	.163	028	.505	.001	.088	.003	.085	000	.087
		500	003	.052	005	.051	.002	.051	.003	.052	.001	.049
	1	100	.007	.193	005	.170	.017	.212	.033	.287	.009	.195
		200	.007	.126	003	.118	.011	.130	.010	.125	.008	.126
		500	.008	.077	001	.074	.010	.078	.005	.077	.007	.076
γ ₂₅	0	100	044	1.003	.019	.468	003	.130	.003	.129	001	.128
		200	019	3.279	.019	.586	000	.090	001	.088	.000	.088
7		500	005	.320	.015	.296	001	.057	.001	.057	001	.055
	.5	100	.024	1.058	.008	.313	001	.136	005	.131	.002	.131
		200	016	.162	.017	.149	001	.094	001	.091	.000	.091
		500	003	.055	.005	.055	.001	.057	001	.056	.001	.055
	1	100	023	.197	.006	.174	023	.211	037	.289	015	.194
		200	013	.122	.006	.116	012	.127	009	.123	008	.121
		500	010	.077	.002	.073	010	.079	004	.077	007	.076
β35	0	100	052	.915	009	.433	004	.119	.001	.120	002	.115
2.5		200	.053	3.051	.021	.566	002	.086	.001	.083	001	.081
.35	_	500	014	.327	.007	.249	002	.052	001	.053	002	.049
	.5	100	.020	1.636	.038	.990	.002	.116	.005	.115	.001	.112
		200	.016	.177	019	.475	.003	.081	.003	.078	.002	.076
	1	500	.010	.056	001	.050	.001	.051	.001	.051	.002	.048
	1	100	.013	.106	.004	.107	.004	.107	.011	.106	.002	.092
		200	.007	.072	004	.072	.004	.076	000	.074	001	.067
		500	.002	.047	002	.043	.001	.049	000	.044	000	.041

Table 8 Path estimates with non-normal data for Large Model

					Sc.	in estim	uies wi		<i>i-norma</i> ML-CFA	ii aaia _.	jor Lai		minres		ML	SEM
Param.	Scale	Z	Orig	ginal	Manip	oulated	Orig	inal	Manip	ulated	Orig	inal	Manip	ulated	Orig	ginal
Ь	01		Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD	Bias	SD
γ23	0	100	.092	4.365	008	.429	009	.136	001	.133	009	.136	001	.133	007	.132
123		200	023	.395	000	.263	005	.096	005	.095	005	.096	005	.095	004	.092
0		500	010	.700	001	.185	002	.057	002	.058	002	.057	002	.058	002	.055
	.5	100	025	.206	002	.148	003	.134	003	.133	003	.134	003	.132	003	.128
		200	011	.122	002	.096	002	.100	005	.094	002	.100	005	.094	000	.094
		500	007	.064	.000	.058	003	.062	000	.060	003	.062	000	.060	002	.059
	1	100	015	.140	.001	.126	.002	.130	000	.129	.002	.130	000	.128	.002	.121
		200	005	.097	.002	.086	.002	.097	.002	.090	.002	.097	.002	.090	.002	.086
	•	500	002	.060	.003	.058	.002	.061	.003	.059	.002	.061	.003	.059	.000	.056
γ_{14}	0	100	051	1.393	.022	.494	.001	.118	.004	.118	.001	.118	.004	.118	.002	.115
0		200 500	.004	.260	006 .001	.142	000 001	.081	004 001	.081	000 001	.081	004 001	.081	001 001	.079
0	.5	100	001	.176	.001	.134	001	.123	001	.124	001	.123	001	.124	001	.120
	.5	200	.005	.080	.003	.081	002	.083	002	.085	003	.083	002	.085	003	.082
		500	.003	.050	.003	.051	002	.053	.000	.053	002	.053	.000	.053	001	.052
	1	100	014	.179	.004	.168	017	.189	020	.193	018	.195	019	.190	.257	8.509
	•	200	008	.119	.003	.110	008	.123	010	.119	008	.124	008	.117	009	.121
		500	005	.076	002	.071	005	.078	006	.075	005	.079	006	.074	005	.075
γ ₂₄	0	100	.049	1.492	.025	.560	.004	.140	.001	.130	.004	.140	.001	.130	.004	.138
		200	.002	.465	003	.206	001	.093	.001	.092	001	.093	.001	.092	.001	.090
.7		500	.017	.217	.001	.136	.002	.058	000	.058	.002	.058	000	.058	.002	.056
	.5	100	.046	.190	012	.143	.007	.137	.005	.133	.007	.138	.005	.133	.007	.132
		200	.017	.091	008	.093	000	.096	.002	.095	000	.096	.002	.095	001	.093
		500	.005	.057	009	.057	001	.059	004	.057	001	.059	004	.058	002	.057
	1	100	.029	.176	007	.161	.019	.187	.021	.185	.020	.194	.020	.182	255	8.516
		200	.013	.119	004	.108	.007	.124	.010	.117	.007	.126	.009	.115	.008	.121
	•	500	.008	.077	000	.069	.005	.079	.005	.073	.005	.080	.005	.072	.005	.076
γ15	0	100	003	.759 .523	.019	.316	003 003	.108	004 004	.108	003 003	.108	004 004	.108	003 003	.106
.22		200 500	.008	.100	010 002	.133	003	.075	004	.048	003	.075	004	.074	003	.073
.22	.5	100	003	.369	.002	1.033	.002	.118	.002	.116	.002	.118	.002	.116	.004	.114
	.5	200	.001	.090	004	.083	.003	.085	.002	.084	.003	.085	.002	.084	.002	.083
		500	001	.053	002	.052	.003	.052	.002	.051	.003	.052	.002	.051	.002	.050
	1	100	.016	.183	002	.175	.023	.193	.016	.197	.024	.199	.015	.194	263	8.710
	•	200	.008	.118	.000	.116	.011	.121	.015	.126	.011	.124	.014	.125	.006	.116
		500	.006	.078	000	.068	.007	.081	.005	.073	.007	.082	.005	.073	.005	.077
γ ₂₅	0	100	.061	1.317	018	.543	.002	.131	001	.128	.001	.131	001	.128	.001	.127
,		200	018	1.554	.009	.228	.003	.089	.001	.085	.003	.089	.001	.085	.002	.087
7		500	.003	.222	002	.127	.003	.056	.001	.056	.003	.056	.001	.056	.002	.054
	.5	100	026	.535	089	3.280	001	.137	004	.127	001	.137	004	.127	000	.133
		200	015	.099	.008	.091	002	.096	001	.092	002	.096	001	.092	002	.093
		500	005	.057	.004	.056	.001	.057	000	.055	.001	.057	000	.055	.001	.055
	1	100	032	.188	.010	.173	028	.198	020	.200	029	.204	019	.198	.257	8.708
		200	013	.118	.001	.113	012	.122	014	.123	012	.125	014	.122	006	.116
		500	007	.080	.004	.070	007	.082	002	.074	007	.083	002	.073	005	.079
β35	0	100	041	1.067	.012	.470	002	.121	001	.119	002	.120	001	.118		.114
.35		200	031	1.514	001	.168	004	.086	001	.083	004	.086	001	.083	003	.082
.33	_	500	001	.199	002	.087	001	.053	.000	.053	001	.053	.000	.053	001	.049
	.5	200	.025	.654	.094	3.086	.002	.118	.004	.116	.002	.118	.004	.115	.001	.110
		200 500	.023	.096	002 002	.082	.003	.079	.002	.078	.003	.079	.002	.078	.001	.074
	1	100	.008	.105	002	.114	.001	.108	.001	.108	.007	.109	.001	.107	.002	.048
	1	200	.007	.073	003	.070	.007	.076	003	.073	.007	.077	003	.073	.002	.064
		500	.007	.046	007	.043	.003	.047	.001	.044	.003	.047	.001	.044		.041
\Box		200	.002	.0-10	.000	.043	.001	.07/	.001	.0-1-1	.001	.07/	.001	.0-1-1	.000	.071

Table~9~95%~percentile~confidence~intervals~with~non-normal~data~for~Small~Model

	Param.	Scale	z	PLSc				EIV _{ML-CFA}			
				Coverage			Incl.	Coverage			Incl.
	Раі	Sc			Within				Within		zero
	γ23	0	100	.006	.980	.014	.980	.022	.952	.026	.952
	123		200	.010	.972	.018	.972	.028	.937	.035	.937
	0		500	.011	.975	.014	.975	.023	.950	.027	.950
		.5	100	.006	.964	.030	.964	.019	.961	.020	.961
			200	.015	.955	.030	.955	.031	.937	.032	.937
			500	.023	.932	.045	.932	.038	.927	.035	.927
		1	100	.007	.947	.046	.947	.023	.945	.032	.945
			200	.013	.954	.033	.954	.026	.954	.020	.954
			500	.023	.941	.036	.941	.047	.923	.030	.923
	γ_{14}	0	100	.003	.995	.002	.995	.030	.940	.030	.940
	_		200	.009	.987	.004	.987	.025	.942	.033	.942
	0		500	.005	.985	.010	.985	.018	.950	.032	.950
		.5	100	.012	.976	.012	.976	.022	.953	.025	.953
			200	.024	.965	.011	.965	.026	.946	.028	.946
			500	.024	.966	.010	.966	.022	.954	.024	.954
		1	100	.013	.954	.033	.954	.016	.952	.032	.952
			200	.020	.944	.036	.944	.022	.946	.032	.946
		Λ	500	.028	.943	.029	.943	.028	.943	.029	.943
	γ ₂₄	0	100 200	.010	.983	.007	.983	.032	.945		.945
	.7		500	.012	.979 .978	.009	.979 .978	.026	.946 .953	.028	.953
	. /	.5	100	.043	.953	.004	.421	.023	.958	.024	.351
		.5	200	.039	.952	.004	.113	.014	.961	.028	.104
			500	.033	.945	.022	.001	.027	.946	.022	.001
		1	100	.041	.952	.007	.031	.029	.960	.011	.074
		1	200	.043	.941	.016	.002	.036	.945	.019	.002
			500	.035	.943	.022	.000	.031	.944	.025	.000
	γ ₁₅	0	100	.000	1.000	.000	1.000	.028	.946	.026	.946
	113		200	.001	.998	.001	.998	.017	.949	.034	.949
	.22		500	.006	.984	.010	.984	.024	.945	.031	.945
		.5	100	.002	.989	.009	.950	.013	.954	.033	.851
			200	.010	.954	.036	.827	.031	.941	.028	.725
			500	.006	.967	.027	.492	.021	.964	.015	.446
		1	100	.018	.957	.025	.763	.030	.949	.021	.740
			200	.036	.949	.015	.492	.041	.944	.015	.505
			500	.037	.940	.023	.096	.035	.946	.019	.114
	γ_{25}	0	100	.001	.999	.000	.999	.030	.947	.023	.947
	_		200	.006	.994	.000	.994	.029	.946	.025	
	7		500	.004	.989	.007	.989	.022	.948	.030	.948
		.5	100	.000	.976	.024	.605	.020	.944	.036	.280
			200	.006	.954	.040	.136	.030	.930	.040	.048
		_	500	.013	.947	.040	.000	.021	.948	.031	.000
		1	100	.012	.959	.029	.075	.018	.936	.046	.066
			200	.009	.957	.034	.001	.011	.952	.037	.000
		^	500	.015	.949	.036	.000	.016	.951	.033	.000
	β35	0	200	.000	1.000	.000	1.000	.030	.940	.030	.940
	.35		200	.002	.997	.001	.997	.027	.935	.038	.935
	دد.	.5	500 100	.002	.992 .972	.006	.992 .844	.015	.953 .952	.032	.953 .674
		ر.	200	.028	.972	.002	.547	.030	.932	.026	.425
			500	.048	.949	.002	.122	.028	.944	.028	.075
		1	100	.020	.977	.003	.289	.020	.964	.016	.191
		1	200	.042	.950	.008	.019	.036	.944	.020	.010
			500	.034	.946	.020	.000	.026	.944	.030	.000
ı			200	.051	., 10	.020	.000	.020	.,, 1.1	.550	.500

Table 10 95% percentile confidence intervals with non-normal data for Large Model

Fig. Fig. Fig.			uvie	10 9370	percent		jiuenc	e iniervo			тин и	aia jor .			
Table Tabl	E.	le		PLSc				EIV _{ML-CFA}				EIV _{minres}			
Table Tabl	ara	Sca	Z	(Incl.	(Coverage		Incl.	(Coverage		
Description Part	Ъ	9 1		Under	Within	Over	zero	Under	Within	Over	zero	Under	Within	Over	zero
0	γ23	0	100	.013	.962	.025	.962	.030	.933	.037	.933	.030	.933	.037	.933
0 500 .009 .979 .012 .979 .018 .958 .024 .958 .018 .958 .024 .958 .024 .958 .024 .958 .024 .033 .946 .031 .941 .030 .944 .033 .944 .033 .944 .033 .944 .033 .944 .033 .944 .033 .944 .033 .944 .033 .944 .030 .941 .030 .942 .030 .942 .030 .943 .030 .943 .030 .943 .030 .943 .030 .943 .030 .943 .030 .943 .030 .943 .030 .943 .030 .943 .030 .943 .030 .944 .030 .944 .030 .944 .030 .944 .030 .944 .030 .94	,			.012	.972	.016	.972		.931		.931		.931	.040	.931
S 100 0.06 9.73 0.21 9.73 0.31 9.46 0.23 9.46 0.31 9.46 0.23 9.46 0.30 9.40 0.30 9.40 0.30 9.40 0.30 9.40 0.30 9.41 0.30 9.41 0.30 9.41 0.30 9.41 0.30 9.41 0.30 9.41 0.30 9.41 0.30 9.41 0.30 0.31 9.46 0.41 9.46 0.29 9.41 0.30 9.41 0.30 9.41 0.30 9.41 0.30 0.30 9.41 0.30 0.41 0.30 0.41 0.30 0.41 0.30 0.41 0.30 0.41 0.30 0.41 0.30 0.41 0.30 0.41 0.30 0.41 0.30 0.31 0.44 0.46 0.41 0.29 9.46 0.25 9.46 0.29 9.46 0.23 9.43 0.33 9.44 0.34 9.45 0.34 0.45 0.4	0		500	.009	.979	.012	.979	.018		.024	.958	.018	.958	.024	.958
Page 200 0.013 9.44 0.43 9.44 0.034 9.28 0.038 9.28 0.034 9.28 0.038 9.28 0.034 0.030 9.41 0.00 0.011 9.951 0.038 9.951 0.034 9.43 0.023 9.43 0.034 9.43 0.023 9.43 0.034 9.43 0.023 9.44 0.032 9.44 0.033 9.44 0.034 9.45 0.05		.5	100	.006	.973		.973	.031		.023				.023	
S00			200		.944	.043	.944		.928	.038	.928		.928	.038	.928
The color The			500	.013	.946	.041	.946	.029	.941		.941	.029	.941		.941
The color The		1	100	.011	.951	.038	.951	.034	.943	.023	.943	.034	.943	.023	.943
The column Th			200	.013	.941		.941		.946		.946		.946	.025	.946
0 200 0.04 9.89 .007 9.89 .026 .950 .024 .950 .026 .949 .025 .949 .500 .005 .990 .005 .990 .020 .949 .031 .949 .020 .949 .031 .949 .020 .949 .031 .949 .020 .949 .031 .949 .020 .949 .031 .949 .020 .949 .031 .949 .020 .949 .031 .949 .020 .949 .031 .949 .020 .949 .031 .949 .020 .940 .038 .940 .020 .940 .038 .940 .022 .955 .023 .955 .022 .955 .023 .955 .022 .955 .023 .955 .022 .955 .023 .955 .020 .023 .939 .038 .945 .015 .948 .037 .948 .017 .949 .034 .034 .948 .037 .944 .029 .944 .029 .944 .027 .944 .029 .944 .027 .944 .029 .944 .027 .944 .029 .944 .027 .944 .029 .944 .027 .944 .029 .944 .027 .05			500	.017	.942	.041	.942	.023	.944	.033	.944	.023	.944	.033	.944
0 200 0.04 9.89 .007 9.89 .026 .950 .024 .950 .026 .949 .025 .949 .500 .005 .990 .005 .990 .020 .949 .031 .949 .020 .949 .031 .949 .020 .949 .031 .949 .020 .949 .031 .949 .020 .949 .031 .949 .020 .034 .940 .038 .940 .020 .949 .031 .949 .020 .024 .971 .005 .971 .023 .957 .020 .957 .023 .955 .022 .955 .023 .955 .022 .955 .023 .955 .020 .023 .935 .038 .945 .015 .948 .037 .948 .017 .949 .034 .949 .034 .949 .020 .023 .939 .038 .939 .022 .940 .038 .940 .022 .946 .032 .946 .035 .947 .017 .949 .034 .949 .018 .948 .037 .948 .037 .948 .037 .948 .037 .948 .037 .948 .037 .948 .037 .948 .037 .948 .037 .948 .037 .948 .037 .948 .037 .948 .037 .948 .037 .948 .038 .940 .022 .946 .032 .946 .025 .946 .029 .946 .025 .946 .028 .946 .028 .946 .028 .946 .028 .946 .946 .946 .946 .946 .946 .946 .946 .946 .946 .946 .946 .946 .946 .946	γ_{14}	0	100	.004	.991	.005	.991	.030	.937	.033	.937	.030	.937	.033	.937
1 100 105 107 100 107 100 107 100 107 100 107 100 107 100 107 100 107 100 107 100 100 101 100			200	.004	.989	.007	.989	.026	.950	.024	.950	.026	.949	.025	.949
200 0.024 9.71 0.05 9.71 0.023 9.57 0.20 9.57 0.23 9.58 0.19 9.58 500 0.028 9.66 0.006 9.66 0.022 9.55 0.23 9.55 0.023 9.55 1 100 0.17 9.945 0.38 9.945 0.15 9.48 0.37 9.48 0.17 9.949 0.34 9.949 200 0.23 9.39 0.38 9.39 0.022 9.40 0.38 9.40 0.022 9.46 0.32 9.46 500 0.18 9.47 0.35 9.47 0.17 9.49 0.34 9.49 0.18 9.48 0.34 9.48 724 0 100 0.012 9.76 0.12 9.76 0.40 9.23 0.37 9.23 0.40 9.23 0.37 9.23 200 0.007 9.86 0.007 9.86 0.025 9.46 0.029 9.46 0.025 9.46 0.029 9.44 .5 100 0.82 9.17 0.01 3.79 0.43 9.36 0.21 2.80 0.45 9.35 0.20 2.77 200 0.63 9.34 0.03 0.70 0.32 9.41 0.27 0.57 0.31 9.42 0.27 0.57 500 0.03 9.56 0.11 0.000 0.23 9.49 0.016 0.00 0.23 9.50 0.27 100 0.52 9.43 0.005 0.013 0.43 9.44 0.29 0.044 0.49 0.41 0.16 100 0.52 9.43 0.00 0.03 0.43 9.44 0.29 0.00 0.023 9.50 0.27 0.00 715 0 100 0.002 9.98 0.000 9.98 0.027 9.51 0.22 9.51 0.22 9.51 0.22 200 0.045 9.93 0.016 0.00 0.33 9.44 0.23 0.00 0.33 9.38 0.27 0.00 715 0 100 0.002 9.998 0.002 9.993 0.27 9.51 0.22 9.51 0.27 9.50 0.23 9.50 200 0.05 9.993 0.02 9.993 0.27 9.51 0.22 9.51 0.27 9.50 0.23 9.50 200 0.05 9.965 0.01 8.16 0.39 9.44 0.11 8.22 0.28 9.61 0.11 8.18 200 0.025 9.65 0.10 8.16 0.39 9.44 0.11 6.88 0.43 9.43 0.14 6.93 725 0 100 0.005 9.995 0.002 9.91 0.02	0		500	.005	.990	.005	.990	.020	.949	.031	.949	.020	.949	.031	.949
Soo .028 .966 .006 .966 .022 .955 .023 .955 .022 .955 .023 .955 .024 .945 .034 .945 .015 .948 .037 .948 .017 .949 .034 .949 .946 .020 .946 .032 .946 .032 .946 .038 .940 .022 .946 .032 .946 .032 .946 .032 .946 .032 .946 .032 .946 .032 .946 .032 .946 .032 .946 .034 .948 .948 .034 .948		.5	100	.016	.977	.007	.977	.022	.941	.037	.941	.022	.940	.038	.940
1 100 0.017 0.945 0.38 9.45 0.015 0.948 0.037 9.48 0.017 0.949 0.34 9.49 0.020 0.23 9.39 0.38 9.39 0.022 9.40 0.038 9.40 0.022 9.46 0.32 9.46 0.32 9.46 0.32 9.46 0.32 9.46 0.32 9.46 0.34 9.49 0.18 9.48 0.34 0.34 9.44 0.27 9.44 0.29 9.44 0.27 9.44 0.29 9.44 0.27 9.44 0.29 9.44 0.27 9.44 0.29 0.44 0.27 9.44 0.29 0.45 9.35 0.20 0.277 0.00 0.63 9.34 0.03 0.70 0.032 9.44 0.027 0.57 0.31 9.42 0.27 0.57 0.50 0.03 0.35 9.50 0.27 0.00 0.23 0.49 0.28 0.00 0.23 9.50 0.27 0.00 0.20 0.45 9.39 0.16 0.00 0.37 9.47 0.16 0.00 0.34 9.48 0.18 0.00 0.30 0.41 9.42 0.17 0.00 0.33 9.44 0.23 0.00 0.35 9.38 0.27 0.00 0.35 0.34 0.3			200	.024	.971	.005	.971	.023	.957	.020	.957	.023	.958	.019	.958
1 100 .017 .945 .038 .945 .015 .948 .037 .948 .017 .949 .034 .949 .900 .920 .933 .939 .032 .940 .038 .940 .022 .946 .032 .946 .946 .932 .946 .946 .946 .948 .94			<u>50</u> 0	.028		.006	.966		<u>.9</u> 55	.023	.955	.022	<u>.9</u> 55	.023	.955
\$\begin{array}{c c c c c c c c c c c c c c c c c c c		1	100		.945	.038	.945	.015		.037	.948	.017	.949	.034	.949
\$\begin{array}{c c c c c c c c c c c c c c c c c c c			200	.023	.939	.038	.939	.022	.940	.038	.940	.022	.946	.032	.946
200 .007 .986 .007 .986 .025 .946 .029 .946 .025 .946 .029 .946 .025 .946 .029 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .944 .027 .957 .057			500	.018	.947	.035	.947	.017	.949	.034	.949	.018	.948	.034	.948
100 0.007 0.986 0.007 0.986 0.025 0.946 0.029 0.946 0.025 0.946 0.029 0.946 0.025 0.008 0.008 0.008 0.009 0.009 0.009 0.001 0.009 0.001 0.009 0.001 0.009 0.001 0.009 0.001 0.009 0.001 0.009 0.001 0.009 0.001 0.009 0.001 0.009 0.001 0.009 0.001 0.009 0.001	γ_{24}	0	100	.012			.976	.040		.037		.040			
S 100 0.82 9.917 0.01 3.79 0.43 9.36 0.21 2.80 0.45 9.35 0.20 2.77			200	.007	.986	.007	.986	.025	.946	.029	.946	.025	.946	.029	.946
200 .063 .934 .003 .070 .032 .941 .027 .057 .031 .942 .027 .057 .030 .033 .956 .011 .000 .023 .949 .028 .000 .023 .950 .027 .000	.7		500	.008	.982	.010	.982	.029	.944	.027	.944	.029	.944	.027	.944
S00		.5	100	.082	.917	.001	.379	.043	.936	.021	.280	.045	.935	.020	.277
1 100 .052 .943 .005 .013 .043 .941 .016 .046 .044 .941 .015 .043 .000 .045 .939 .016 .000 .037 .947 .016 .000 .034 .948 .018 .000 .000 .041 .942 .017 .000 .033 .944 .023 .000 .035 .938 .027 .000 .000 .000 .000 .0000 .0000 .998 .000 .998 .027 .951 .022 .951 .027 .950 .023 .950 .020 .005 .993 .002 .993 .021 .949 .030 .949 .021 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .034 .			200	.063	.934	.003	.070	.032	.941	.027		.031	.942	.027	.057
200 .045 .939 .016 .000 .037 .947 .016 .000 .034 .948 .018 .000			500	.033	.956	.011	.000	.023	.949	.028	.000	.023	.950	.027	.000
Soo .041 .942 .017 .000 .033 .944 .023 .000 .035 .938 .027 .000 γ15		1	100	.052	.943	.005	.013	.043	.941	.016	.046	.044	.941	.015	.043
γ15 0 100 .002 .998 .000 .998 .027 .951 .022 .951 .027 .950 .023 .950 .22 200 .005 .993 .002 .993 .021 .949 .030 .949 .021 .949 .030 .949 .021 .949 .030 .949 .021 .949 .030 .949 .021 .949 .030 .949 .021 .949 .030 .949 .021 .949 .030 .949 .024 .939 .037 .939 .024 .939 .037 .939 .024 .939 .037 .939 .024 .939 .031 .939 .034 .031 .939 .944 .017 .709 .039 .946 .015 .710 .940 .021 .471 .046 .943 .011 .688 .043 .943 .014 .693 .949 .015 .482 .036 .946 .018 <td></td> <td></td> <td>200</td> <td>.045</td> <td>.939</td> <td>.016</td> <td>.000</td> <td>.037</td> <td>.947</td> <td>.016</td> <td>.000</td> <td>.034</td> <td>.948</td> <td>.018</td> <td>.000</td>			200	.045	.939	.016	.000	.037	.947	.016	.000	.034	.948	.018	.000
200 .005 .993 .002 .993 .021 .949 .030 .949 .021 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .030 .949 .031 .939 .037 .939 .037 .939 .037 .939 .037 .939 .037 .939 .037 .939 .037 .939 .037 .939 .037 .939 .030 .030 .949 .015 .024 .017 .709 .039 .946 .015 .710 .710 .030 .949 .015 .024 .407 .026 .951 .023 .401 .030 .949 .021 .472 .036 .949 .015 .482 .036 .946 .018 .488 .043 .943 .014 .693 .030 .030 .949 .021 .472 .036 .949 .015 .482 .036 .946 .018 .488 .030 .026 .943 .031 .106 .031 .939 .030 .119 .036 .935 .029 .126 .024 .036 .030 .946 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .946 .024 .946 .030 .946 .024 .946 .030 .946 .024 .946 .030 .946 .024 .946 .030 .946 .024 .946 .030 .946 .024 .946 .030 .946 .024 .946 .030 .946 .024 .946 .030 .946			500	.041	.942	.017	.000	.033	.944	.023	.000	.035	.938	.027	.000
Solution Solution	γ_{15}	0	100	.002	.998	.000	.998	.027	.951	.022	.951	.027	.950	.023	.950
5 100 .004 .994 .002 .961 .028 .961 .011 .822 .028 .961 .011 .818 200 .025 .965 .010 .816 .039 .944 .017 .709 .039 .946 .015 .710 500 .017 .960 .023 .475 .025 .951 .024 .407 .026 .951 .023 .401 1 100 .027 .961 .012 .711 .046 .943 .011 .688 .043 .943 .014 .693 200 .030 .949 .021 .472 .036 .949 .015 .482 .036 .946 .018 .488 500 .026 .943 .031 .106 .031 .939 .030 .119 .036 .935 .029 .126 725 0 100 .005 .995 .000 .995 .032 .938 .030 .938 .032 .938 .030 .938 200 .007 .991 .002 .991 .028 .951 .021 .951 .028 .951 .021 .951 7 500 .009 .986 .005 .986 .030 .946 .024 .946 .030 .946 .024 .946 7 500 .000 .975 .025 .626 .023 .937 .040 .276 .024 .938 .038 .274 200 .001 .947 .042 .002 .023 .948 .029 .000 .022 .950 .028 .000 1 100 .006 .957 .037 .062 .007 .938 .055 .053 .007 .941 .052 .049 200 .014 .952 .034 .001 .015 .953 .032 .000 .014 .954 .032 .000 500 .021 .939 .040 .000 .021 .941 .038 .000 .020 .940 .040 .000 635 0 100 .000 .998 .002 .998 .030 .937 .033 .937 .030 .936 .034 .936 835 0 100 .000 .998 .002 .998 .030 .937 .033 .937 .030 .936 .034 .936 200 .002 .997 .001 .997 .025 .931 .044 .931 .025 .932 .043 .932 835 0 100 .000 .998 .002 .998 .030 .937 .033 .937 .030 .936 .034 .936 835 0 100 .000 .998 .002 .998 .030 .937 .033 .937 .030 .936 .034 .936 835 0 100 .000 .998 .002 .998 .030 .937 .033 .937 .030 .936 .034 .936 836 200 .002 .997 .001 .997 .025 .931 .044 .931 .025 .932 .043 .932 836 200 .002			200	.005	.993	.002	.993	.021	.949	.030	.949	.021	.949	.030	.949
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$.22		500	.010	.979	.011	.979	.024	.939	.037	.939	.024	.939	.037	.939
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$.5	100	.004	.994	.002	.961	.028	.961	.011	.822	.028	.961	.011	.818
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			200	.025	.965	.010	.816	.039	.944	.017	.709	.039	.946	.015	.710
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			500	.017	.960	.023	.475	.025	.951	.024	.407	.026	.951	.023	.401
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		1	100	.027	.961	.012	.711	.046	.943		.688	.043		.014	.693
γ25 0 100 .005 .995 .000 .995 .032 .938 .030 .938 .032 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .938 .030 .946 .024 .946 .030 .946 .024 .946 .030 .946 .024 .946 .030 .946 .024 .946 .030 .946 .024 .946 .030 .946 .024 .946 .030 .946 .024 .946 .024 .946 .024 .946 .024 .938 .038 .274 200 .003 .959 .038 .133 .027 .933 .040 .042 .027 .931 .042 .042 500 .011 .947 .042 .002			200	.030	.949	.021	.472	.036	.949	.015	.482	.036	.946	.018	.488
7777897777777777			500	.026	.943	.031		.031	.939	.030		.036	.935	.029	.126
7	γ_{25}	0													.938
100 .000 .975 .025 .626 .023 .937 .040 .276 .024 .938 .038 .274				.007	.991	.002	.991	.028	.951	.021		.028	.951	.021	.951
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	7														.946
500 .011 .947 .042 .002 .023 .948 .029 .000 .022 .950 .028 .000 .000 .006 .957 .037 .062 .007 .938 .055 .053 .007 .941 .052 .049 .000 .014 .952 .034 .001 .015 .953 .032 .000 .014 .954 .032 .000 .000 .021 .939 .040 .000 .021 .941 .038 .000 .020 .940 .040 .000 .035		.5												.038	.274
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$.042
β35 0 .001 .952 .034 .001 .015 .953 .032 .000 .014 .954 .032 .000 β35 0 .021 .939 .040 .000 .021 .941 .038 .000 .020 .940 .040 .000 β35 0 .000 .998 .002 .998 .030 .937 .033 .937 .030 .936 .034 .936 200 .002 .997 .001 .997 .025 .931 .044 .931 .025 .932 .043 .932															.000
500 .021 .939 .040 .000 .021 .941 .038 .000 .020 .940 .040 .000 β35 0 100 .000 .998 .002 .998 .030 .937 .033 .937 .030 .936 .034 .936 200 .002 .997 .001 .997 .025 .931 .044 .931 .025 .932 .043 .932		1	100			.037	.062			.055	.053	.007			.049
β ₃₅ 0 100 .000 .998 .002 .998 .030 .937 .033 .937 .030 .936 .034 .936 200 .002 .997 .001 .997 .025 .931 .044 .931 .025 .932 .043 .932															.000
200 .002 .997 .001 .997 .025 .931 .044 .931 .025 .932 .043 .932				.021								.020			.000
	β35	0										.030			.936
1.35 1500 .002 .992 .006 .992 .030 .945 .025 .945 .030 .945 .025 .945				.002				.025		.044		.025			.932
	.35		500	.002		.006	.992	.030	.945	.025	.945	.030	.945	.025	.945
.5 100 .031 .969 .000 .834 .033 .945 .022 .678 .033 .944 .023 .671		.5													
			200	.045		.000	.536	.017	.950	.033		.017	.949	.034	.409
											.067	.032			.067
		1													.168
			200	.034			.009	.035		.019	.004	.036			.005
500 .036 .943 .021 .000 .033 .941 .026 .000 .031 .942 .027 .000			500	.036	.943	.021	.000	.033	.941	.026	.000	.031	.942	.027	.000

Appendix B – Deriving the Refined Factor Loading Corrections

In this appendix we explain how applying the refined per-indicator correction leads to well-established factor analysis techniques. The PLSc correction for factor loadings was derived from the asymptotic case, leading to a consistent estimator. In PLSc, the loadings are estimated (rescaled) as follows (Dijkstra, 2010):

$$\hat{\lambda}_i = c_i \cdot \mathbf{w}_i, \tag{B1}$$

where \mathbf{w}_i is the weight vector for the *i*th block of indicators, $\hat{\lambda}_i$ is a parallel vector of factor loading estimates, and c_i is the scaling factor. Because the indicator covariances are implied by the products of factor loadings, c_i is chosen to minimize the Euclidean distance (i.e., sum of squared discrepancies) between (Dijkstra & Henseler, 2015b, p. 12):

$$[\mathbf{S}_{ii} - diag(\mathbf{S}_{ii})]$$
 and $[(c_i \cdot \mathbf{w}_i)(c_i \cdot \mathbf{w}_i)^t - diag((c_i \cdot \mathbf{w}_i)(c_i \cdot \mathbf{w}_i)^t)],$ [B2]

where S_{ii} is the within-block sample covariance matrix of the observed indicators, the superscript t is the transposition operator, and diag is an operator that zeroes the off-diagonal elements of a matrix. The current estimator of choice for \hat{c}_i is (Dijkstra, 2014. Equation 11):

$$\hat{c}_i = \sqrt{\frac{\mathbf{w}_i^t(\mathbf{S}_{ii} - diag(\mathbf{S}_{ii}))\mathbf{w}_i}{\mathbf{w}_i^t(\mathbf{w}_i\mathbf{w}_i^t - diag(\mathbf{w}_i\mathbf{w}_i^t))\mathbf{w}_i}} .$$
 [B3]

A more refined correction can be developed by replacing the single scalars c_i with a vector \mathbf{c} , such that each loading is corrected separately. Unfortunately, there is no closed form solution for minimizing the Euclidean distance after this modification, and thus numerical optimization must be used. Another complication arises from the fact that two of the LVs in the model specified by Dijkstra and Henseler (2015b) have only two indicators, and correcting the loadings on these latent variables using information from only one indicator block is not possible because the minimization problem is not identified; it is well known that at least three indicators

are required for identification of a single factor model, that is, to obtain a unique solution for the parameter estimates (Bollen, 1989, pp. 238–246)¹. Given these two considerations, we can estimate all of the correction factors simultaneously by using the full covariance matrix **S** for all p observed indicators of the q latent variables (q < p), and therefore the distance to be minimized becomes:

[S - diag(S)] and $[bd(\mathbf{c} \circ \mathbf{w}) \widehat{\Phi} bd(\mathbf{c} \circ \mathbf{w})^t - diag(bd(\mathbf{c} \circ \mathbf{w}) \widehat{\Phi} bd(\mathbf{c} \circ \mathbf{w})^t)]$, [B4] where \mathbf{c} is a $p \times 1$ vector of estimated scaling factors, \mathbf{w} is a parallel vector of indicator weights, the symbol \circ denotes the Hadamard product of vectors (i.e., element-wise multiplication of \mathbf{c} and \mathbf{w}), $\widehat{\Phi}$ is a $q \times q$ correlation matrix with freely estimated elements, and bd is an operator that transforms a p-dimensional vector into a block-diagonal $p \times q$ matrix, where a given value on the ith column and jth row is the ith element of the input vector if the ith indicator is linked to the jth latent variable, and zero otherwise. In other words, bd converts the product vector $\mathbf{c} \circ \mathbf{w}$ into a factor loading matrix (or its transpose) with zero cross-loadings. Because minimizing the Euclidean distance is equivalent to minimizing the sum of squares, the minimization problem can be expressed conveniently as:

$$tr\Big(\Big([bd(\mathbf{c}\circ\mathbf{w})\widehat{\mathbf{\Phi}}bd(\mathbf{c}\circ\mathbf{w})^t - diag(bd(\mathbf{c}\circ\mathbf{w})\widehat{\mathbf{\Phi}}bd(\mathbf{c}\circ\mathbf{w})^t)] - \big(\mathbf{S} - diag(\mathbf{S})\big)\Big)^2\Big), \quad [B5]$$

Raising a symmetric matrix to the second power collects the sums of squares of rows on the diagonals and *tr* is the *trace* operator that sums the elements on the main diagonal of the resulting matrix, thus giving the distance to be minimized.

Numerical minimization requires starting values. A reasonable assumption is to set all elements of \mathbf{c} to 1. We can further simplify the equations to be minimized by considering that the

¹ Identification can be achieved for smaller models by constraining some of the free parameters. A typical example would be constraining the loadings of a two-indicator factor to be equal. However, such models are rare and are not encouraged, as they are unlikely to be capable of providing useful substantive information about the phenomenon under study.

products of weights and correction factors equal factor loadings, as per Equation [B1] above. The $bd(\mathbf{c} \circ \mathbf{w})$ terms in the equation can therefore be replaced with the factor loading matrix $\mathbf{\Lambda}$, which we initially populate with PLS weights. This alternative representation would be equivalent to using the PLS weights as starting values for numerical optimization, which was originally suggested by Dijkstra (1981) and later developed by Huang (2013; see also Bentler & Huang, 2014). Equation [B5] can therefore be equivalently and more compactly written in terms of loadings that would be estimated. Dijkstra (Dijkstra & Schermelleh-Engel, 2013) further suggests that the distance minimization problem can be refined by differentially weighting each element of the indicator covariance matrix. Given these considerations, the distance to be minimized can be written as:

$$tr\left(\left(\left[\left(\widehat{\boldsymbol{\Lambda}}\widehat{\boldsymbol{\Phi}}\widehat{\boldsymbol{\Lambda}}^{t}\right)-diag(\widehat{\boldsymbol{\Lambda}}\widehat{\boldsymbol{\Phi}}\widehat{\boldsymbol{\Lambda}}^{t}\right)\right]-\left(\mathbf{S}-diag(\mathbf{S})\right)\right)\mathbf{V}\right)^{2}\right),$$
[B6]

where **V** is a weight matrix that can be chosen in multiple ways. Using the inverse of **S** leads to Generalized Least Squares (GLS) estimation. Another alternative would be to use estimation weights **V** calculated based on the inverse of the model-implied correlations, updating the estimation weights **V** after each estimation round. This iteratively reweighted least squares (IRLS) approach is equivalent to maximum likelihood estimation (ML) for correctly specified models (Yuan & Chan, 2005) and is used to obtain ML estimates in, for example, the EQS software package for covariance-based SEM (Bentler, 1995, pp. 134–135).

Finally, consistent estimates of the factor correlations, which are required for consistent estimation of the paths between latent variables, can also be estimated more directly without requiring calculation of the composites. In the *Handbook of Partial Least Squares*, Dijkstra (Dijkstra, 2010, p. 38, eq. 1.33) suggests that although disattenuation could be used to produce consistent estimators of the factor correlations, a more direct way to estimate the correlations

between factors would be to minimizing the difference between the model-implied and observed between-block correlation matrix. This approach can be further extended from estimating all between-factor correlations separately to estimating the full factor correlation matrix, which is equivalent to using $\widehat{\Phi}$ from our Equation [B6]. We tested also an estimator where we calculated regressions directly from this correlation matrix without calculating any composites variables. The results were in all respect nearly identical with maximum likelihood SEM estimates. The similarity is explained by the fact that the SEM model studied by Dijkstra and Henseler (2015b, fig. 4) only imposes two additional constraints over the CFA model: the correlation between ξ_1 and η_3 equals the product of correlations between ξ_1 , ξ_2 , ξ_2 and η_3 . Similarly, the correlation between η_3 and η_4 equals the product of correlations between η_4 , ξ_2 , ξ_2 , and η_3^2 . Therefore the constraints on the model implied correlation matrix by these two estimators are very similar.

1. References

- Bentler, P. M. (1995). *EQS structural equations program manual*. Encino, CA: Multivariate Software.
- Bentler, P. M., & Huang, W. (2014). On Components, Latent Variables, PLS and Simple

 Methods: Reactions to Rigdon's Rethinking of PLS. *Long Range Planning*, 47(3), 138–

 145. doi:10.1016/j.lrp.2014.02.005
- Bollen, K. A. (1989). Structural Equations with Latent Variables. New York, NY: John Wiley & Son Inc.
- Dijkstra, T. K. (1981). Latent variables in linear stochastic models: reflections on "maximum likelihood" and "partial least squares" methods (Doctoral dissertation). Rijksuniversiteit te Groningen, Groningen.

² The correlation between η_4 and η_5 could be similarly derived, however, the error terms of these two latent variables are typically allowed to be correlated.

- Dijkstra, T. K. (2010). Latent variables and indices: Herman Wold's basic design and partial least squares. In E. V. Vinzi, W. W. Chin, J. Henseler, & H. Wang (Eds.), *Handbook of Partial Least Squares* (pp. 23–46). Berlin; New York: Spinger.
- Dijkstra, T. K. (2014). PLS' Janus Face Response to Professor Rigdon's "Rethinking Partial Least Squares Modeling: In Praise of Simple Methods." *Long Range Planning*, *47*(3), 146–153. doi:10.1016/j.lrp.2014.02.004
- Dijkstra, T. K., & Henseler, J. (2015a). Consistent and asymptotically normal PLS estimators for linear structural equations. *Computational Statistics & Data Analysis*, 81, 10–23. doi:10.1016/j.csda.2014.07.008
- Dijkstra, T. K., & Henseler, J. (2015b). Consistent Partial Least Squares Path Modeling. *MIS Quarterly*, (forthcoming).
- Dijkstra, T. K., & Schermelleh-Engel, K. (2013). Consistent Partial Least Squares for Nonlinear Structural Equation Models. *Psychometrika*, 1–20. doi:10.1007/s11336-013-9370-0
- Huang, W. (2013). *PLSe: Efficient Estimators and Tests for Partial Least Squares* (Doctoral dissertation). University of California, Los Angeles.
- Yuan, K.-H., & Chan, W. (2005). On Nonequivalence of Several Procedures of Structural Equation Modeling. *Psychometrika*, 70(4), 791–798. doi:10.1007/s11336-001-0930-9

Appendix C – Examples of Applying the EIV Estimators

In this appendix, we provide several examples of how the errors-in-variables (EIV) estimators can be implemented with commonly used statistical software. The general workflow of the analysis is to first estimate either a series of single—block factor analyses or one multi-block confirmatory factor analysis and either print out or programmatically store the standardized factor loadings. Next, the indicators are standardized and summed by block as composites, followed by standardization of those composites. A simple way to calculate the weights that can be used to calculate a standardized composite from standardized indicators is to regress each standardized composite on its standardized indicators. Equation [2] from the main document is then used to calculate estimated reliabilities for the composites.

The procedures for performing EIV regression vary between statistical packages. A popular approach that can be used with nearly any commonly used statistical package is to estimate the EIV model with a SEM function or software package. In this procedure, each variable in the regression analysis is replaced with a latent variable, which has the corresponding composite as a single indicator. The loadings are fixed at the square roots of the reliability estimates and error variances are fixed at 1-reliability estimates. Because the resulting model is just-identified, all estimators converge to the same solution and the model fits perfectly. This is a well-known model configuration and is documented in the user manuals of many commonly-used SEM software packages (e.g., SAS Institute Inc., 2010, pp. 1317–1342; SPSS Inc, 2009; StataCorp, 2013, pp. 275–278).

We will next provide self-contained example analysis files written for R, SAS, and Stata using data generated based on the indicator covariance matrix reported by Chin, Johnson, and Schwarz (2008, Appendix C).

References

- Chin, W. W., Johnson, N., & Schwarz, A. (2008). A Fast Form Approach to Measuring Technology Acceptance and Other Constructs. *MIS Quarterly*, 32(4), 687–703.
- SAS Institute Inc. (2010). SAS/STAT® 9.22 User's Guide. Cary, NC: SAS Institute Inc.
- SPSS Inc. (2009). Amos | Structural Equation Modeling. Retrieved March 3, 2015, from http://www.spss.com.hk/amos/measurement error application.htm
- StataCorp. (2013). *Stata Statistical Software: Release 13*. College Station, TX: StataCorp LP.

1 Example in R.R.

```
# This file contains example analyses using the EIV estimator in various
2
    \# configurations using the data from Appendix C of
3
    \# Chin, W. W., Johnson, N., & Schwarz, A. (2008). A Fast Form Approach to \# Measuring Technology Acceptance and Other Constructs. MIS Quarterly, 32(4),
4
5
6
    # 687 703.
9
    # Load libraries
10
    library(psych)
11
    library(matrixpls)
12
    library(lavaan)
13
    library(MASS)
14
   library(boot)
15
16
    # Set up the data
17
18
   covariances <- matrix(NA, 16, 16)
19
20
    covariances[upper.tri(covariances, diag = TRUE)] <- c(</pre>
21
      1.694, 1.903,
22
23
      1.662, 1.551, 1.875,
24
      1.675, 1.570, 1.581, 1.985,
25
      0.880, 0.798, 0.851, 0.757, 2.137,
      0.929, 0.877, 0.800, 0.765, 1.619, 1.846,
      0.938, 0.895, 0.853, 0.815, 1.541, 1.437, 1.760,
      0.920\,,\; 0.864\,,\; 0.850\,,\; 0.794\,,\; 1.199\,,\; 1.248\,,\; 1.251\,,\; 1.648\,,
28
29
      0.922, 0.864, 0.889, 0.850, 1.535, 1.428, 1.443, 1.258, 1.964,
      0.912,\ 0.873,\ 0.806,\ 0.838,\ 1.620,\ 1.543,\ 1.579,\ 1.346,\ 1.640,\ 2.032,
      1.007, 0.939, 0.925, 0.985, 0.644, 0.743, 0.679, 0.676, 0.703, 0.708, 1.436,
      1.026, 0.896, 0.893, 0.883, 0.669, 0.661, 0.666, 0.694, 0.739, 0.693, 1.086, 1.503,
33
      0.895,\ 0.864,\ 0.836,\ 0.842,\ 0.564,\ 0.587,\ 0.583,\ 0.641,\ 0.554,\ 0.538,\ 0.956,\ 0.897,\ 1.212,
      0.934, 0.945, 0.804, 0.902, 0.526, 0.615, 0.568, 0.602, 0.554, 0.574, 0.978, 0.844, 0.874, 1.311, 0.831, 0.815, 0.811, 0.856, 0.497, 0.537, 0.576, 0.559, 0.551, 0.528, 0.857, 0.781, 0.778, 0.800,
          1.106,
36
      0.734, 0.740, 0.677, 0.771, 0.505, 0.535, 0.555, 0.528, 0.531, 0.498, 0.748, 0.705, 0.731, 0.773,
           0.714, 1.015)
37
38
    covariances[lower.tri(covariances)] <- t(covariances)[lower.tri(covariances)]</pre>
    colnames(covariances) <- rownames(covariances) <- c("LU4", "LU3", "LU2", "LU1", "E0U1", "E0U2", "E0U3", "E0U4", "E0U5", "E0U6", "Use6", "Use5", "Use4", "Use3", "Use2", "Use1")
39
40
    # Create multivariate normal dataset with the exact covariance structure
41
42
   rawData <- mvrnorm(283, mu = rep(0,16), Sigma = covariances, empirical = TRUE)
43
44
45
    46
47
    # Examples using matrixpls package
48
    49
50
    # Specify the model using Lavaan syntax. (http://lavaan.ugent.be/tutorial/syntax1.html)
51
52
53
    modelSpecification <- "
54
55
    # Define the factors
    PredictedUsage = "LU1 + LU2 + LU3 + LU4
Usefulness = "Use1 + Use2 + Use3 + Use4 + Use5 + Use6
56
57
    EaseOfUse = EOU1 + EOU2 + EOU3 + EOU4 + EOU5 + EOU6
58
59
    # Regression between factors
PredictedUsage ~ Usefulness + EaseOfUse
60
61
62
63
64
    # EIV minres with matrixpls
65
66
    est1 <- matrixpls(covariances, modelSpecification,</pre>
                        # Use unit weights
67
68
                        weightFunction = weight.fixed,
69
70
                        # Use disattenuation
71
                        disattenuate = TRUE,
73
                        # Use blockwise EFA with minres to estimate the reflective part of
75
                        parametersReflective = estimator.EFALoadings)
```

```
76
     summary(est1)
 77
 78
     # EIV_ML-CFA with matrixpls
 79
 80
     \verb"est2 <- matrixpls(covariances, modelSpecification,\\
 81
 82
                        # Use unit weights
 83
                       weightFunction = weight.fixed,
 84
                       # Use disattenuation
 85
 86
                       disattenuate = TRUE,
 87
                       # Full model CFA with ML to estimate the reflective part of
 88
 89
                       # the model
                       parametersReflective = estimator.CFALoadings)
 90
 91
 92
     summary(est2)
93
 94
     # Bootstrap EIVminres
95
 96
     est1.boot <- matrixpls.boot(rawData, modelSpecification,</pre>
97
                                  # Use unit weights
98
                                  weightFunction = weight.fixed,
99
100
                                  # Use disattenuation
101
                                  disattenuate = TRUE.
102
103
                                  # Use blockwise EFA with minres to estimate the reflective part
104
                                  # of the model
105
                                  parametersReflective = estimator.EFALoadings,
106
107
                                  # 500 bootstrap replications
108
                                  R = 500)
109
110
     # Calculate percentile confidence intervals from bootstrap results
111
112
     CIs <- lapply(1:length(est1), function(i){
       CI <- boot.ci(est1.boot, type = "perc", index = i) percent[,4:5] names(CI) <- c("lower", "upper")
113
114
115
    })
116
117
     cbind(est1,do.call(rbind,CIs))
118
119
120 # Calculate two-tailed t-tests based on bootstrap SEs
121
122
     se <- apply(est1.boot$t,2,sd)
123
    t <- est1 / se
    p <- (1-pt(abs(t), 283-1))*2
124
125
     cbind(est1, se, t, p)
126
127
128
     129
     # Examples using psych and lavaan packages
130
131
     132
133
134
     # Calculate three different reliability estiamtes
135
     reliabilities.alpha <- list(LU = alpha(covariances[1:4,1:4])$total$std.alpha,
136
                                  EOU = alpha(covariances[5:10,5:10]) $total $std.alpha,
Use = alpha(covariances[11:16,11:16]) $total $std.alpha)
137
138
139
140
     loadings.minres <- list(LU = fa(covariances[1:4,1:4])$loadings,</pre>
141
                              EOU = fa(covariances[5:10,5:10]) $loadings,
Use = fa(covariances[11:16,11:16]) $loadings)
142
143
144
145
     # Calculate unit weights
146
147
     correlations <- cov2cor(covariances)</pre>
148
     weights <- list(LU = 1/sqrt(sum(correlations[1:4,1:4])),</pre>
149
                     EOU = 1/sqrt(sum(correlations[5:10,5:10])),
150
151
                     Use = 1/sqrt(sum(correlations[11:16,11:16])))
152
153
     # Use the equation from the article to calculate reliabilities
154
155
     reliabilities.minres <- mapply(function(loadings, weight){</pre>
156
       sum(weight * loadings)^2
```

```
157 }, loadings.minres, weights)
158
159
     # Specify the CFA model using Lavaan syntax.
160
161
     modelSpecification <- "
162
163
164
     # Define the factors
     PredictedUsage = "LU1 + LU2 + LU3 + LU4
Usefulness = "Use1 + Use2 + Use3 + Use4 + Use5 + Use6
165
166
      EaseOfUse = EOU1 + EOU2 + EOU3 + EOU4 + EOU5 + EOU6
167
168
     169
170
171
     loadings.CFA <- list(LU = inspect(cfa1, "est") $lambda[1:4,1],</pre>
172
                               EOU = inspect(cfa1,"est")$lambda[11:16,3],
Use = inspect(cfa1,"est")$lambda[5:10,2])
173
174
175
176
     \tt reliabilities.CFA <- mapply(function(loadings, weight)\{
177
       sum(weight * loadings)^2
     }, loadings.CFA, weights)
178
179
180
181
182
     # Calculate unit weighted composites
183
184
185
      composites <- scale(cbind(PredictedUsage = rowSums(scale(rawData)[,1:4]),</pre>
                                     Usefulness = rowSums(scale(rawData)[,11:16]),
EaseOfUse = rowSums(scale(rawData)[,5:10])))
186
187
188
189
190
     # Estimate errors in variables model using lavaan and the minres reliability
191
192
193
     modelSpecification <- paste("
PredictedUsage_star =~ ",sqrt</pre>
194
                                 ",sqrt(reliabilities.minres["LU"]),"*PredictedUsage
     Usefulness_star = ",sqrt(reliabilities.minres["Use"]),"*Usefulness
195
     EaseOfUse_star = ", sqrt(reliabilities.minres["EOU"]), "*EaseOfUse
196
     PredictedUsage ~~ ",1-reliabilities.minres["LU"],"*PredictedUsage
Usefulness ~~ ",1-reliabilities.minres["Use"],"*Usefulness
EaseOfUse ~~ ",1-reliabilities.minres["EOU"],"*EaseOfUse
198
200
201
202
     PredictedUsage_star ~ Usefulness_star + EaseOfUse_star
     ",sep="")
203
204
205
     eiv <- sem(modelSpecification,composites)</pre>
206
     summary(eiv)
```

2 Example in SAS.sas

```
1
   This file contains example analyses using the {\tt EIV} estimator in various
   configurations using the data from Appendix C of
4
   Chin, W. W., Johnson, N., & Schwarz, A. (2008). A Fast Form Approach to
6
   Measuring Technology Acceptance and Other Constructs. MIS Quarterly, 32(4),
8
   687 703.
Q
10
   */
11
12
13
   /* Set up the data */
14
   FILENAME foo URL 'http://www.ats.ucla.edu/stat/sas/macros/corr2data.sas';
15
16
   %INCLUDE foo;
17
18
   INPUT LU4 LU3 LU2 LU1 EOU1 EOU2 EOU3 EOU4 EOU5 EOU6 Use6 Use5 Use4 Use3 Use2 Use1;
19
   DATALINES;
20
   2.111 . .
   1.694 1.903 .
23 1.662 1.551 1.875 .
   1.675 1.570 1.581 1.985 . . . . . . . . . . . . .
```

```
0.938 0.895 0.853 0.815 1.541 1.437 1.760 . . . . . .
 27
    0.920 0.864 0.850 0.794 1.199 1.248 1.251 1.648 . .
    0.922 0.864 0.889 0.850 1.535 1.428 1.443 1.258 1.964 .
    0.912 0.873 0.806 0.838 1.620 1.543 1.579 1.346 1.640 2.032
    1.007 0.939 0.925 0.985 0.644 0.743 0.679 0.676 0.703 0.708 1.436
 31
    1.026 0.896 0.893 0.883 0.669 0.661 0.666 0.694 0.739 0.693 1.086 1.503 . .
    0.895 0.864 0.836 0.842 0.564 0.587 0.583 0.641 0.554 0.538 0.956 0.897 1.212 .
 33
    0.934 0.945 0.804 0.902 0.526 0.615 0.568 0.602 0.554 0.574 0.978 0.844 0.874 1.311 .
    0.831 \ 0.815 \ 0.811 \ 0.856 \ 0.497 \ 0.537 \ 0.576 \ 0.559 \ 0.551 \ 0.528 \ 0.857 \ 0.781 \ 0.778 \ 0.800 \ 1.106
 35
    0.734 \ 0.740 \ 0.677 \ 0.771 \ 0.505 \ 0.535 \ 0.555 \ 0.528 \ 0.531 \ 0.498 \ 0.748 \ 0.705 \ 0.731 \ 0.773 \ 0.714 \ 1.015
 36
 37
    RUN;
 38
 39
    /* Create multivariate normal dataset with the exact covariance structure. */
 40
 41
    %corr2data(rawData, C, 283, FULL='F', corr='F');
 42
 43
 44
    DATA rawData;
 45
             SET rawData;
             RENAME COL1=LU4 COL2=LU3 COL3=LU2 COL4=LU1 COL5=EOU1 COL6=EOU2 COL7=EOU3 COL8=EOU4 COL9=EOU5
 46
                 COL10=E0U6 COL11=Use6 COL12=Use5 COL13=Use4 COL14=Use3 COL15=Use2 COL16=Use1:
47
     RUN:
 48
 49
    /***********************************
 50
 51
 52
    Example using FACTOR and CALIS
 53
 54
    ****************
 55
 56
    /* Create unit weighted composites and standardize */
 57
 58
    PROC STANDARD DATA=rawData MEAN=0 STD=1 OUT=zrawData;
 59
     RUN;
 60
 61
     DATA zrawData;
 62
            SET zrawData;
 63
             PredictedUsage = LU4 + LU3 + LU2 + LU1;
             Usefulness = Use6 + Use5 + Use4 + Use3 + Use2 + Use1;
 64
 65
             EaseOfUse = EOU1 + EOU2 + EOU3 + EOU4 + EOU5 + EOU6;
    RUN;
     PROC STANDARD DATA=zrawData MEAN=0 STD=1 OUT=zrawData;
 68
 69
 70
     /* minres based reliability estimate for PredictedUsage */
 72
 73
 74
     PROC FACTOR data=zrawData NFACTORS=1 METHOD=ULS OUTSTAT=factorResults;
            VAR LU4 LU3 LU2 LU1;
 75
 76
 77
 78
     PROC REG DATA=zrawData OUTEST=regressionResults PLOTS=none;
 79
      MODEL PredictedUsage = LU4 LU3 LU2 LU1;
 80
     RUN;
 81
 82
     DATA _null_;
            SET regressionResults;
 83
            CALL symput("PredictedUsageWeight", LU4);
 84
 85
    RUN:
 86
    87
 88
             IF _NAME_ NE "Factor1" THEN DELETE;
Reliability = ((LU4 + LU3 + LU2 + LU1) * &PredictedUsageWeight)**2;
 89
 90
             CALL symput("PredictedUsageReliability", Reliability);
 91
 92
    RUN:
 93
    /* minres based reliability estimate for Usefulness */
 94
 95
96
    PROC FACTOR data=zrawData NFACTORS=1 METHOD=ULS OUTSTAT=factorResults;
 97
98
            VAR Use6 Use5 Use4 Use3 Use2 Use1;
    RIIN:
99
100
101
     PROC REG DATA=zrawData OUTEST=regressionResults PLOTS=none;
102
      MODEL Usefulness = Use6 Use5 Use4 Use3 Use2 Use1;
103
    RUN;
104
     DATA _null_;
105
106
            SET regressionResults;
```

```
CALL symput("UsefulnessWeight",Use6);
107
108
     RUN;
109
     DATA _null_;
110
              SET factorResults:
111
               IF _NAME_ NE "Factor1" THEN DELETE;
112
              Reliability = ((Use6 + Use5 + Use4 + Use3 + Use2 + Use1) * &UsefulnessWeight)**2; CALL symput("UsefulnessReliability", Reliability);
113
114
     RUN;
115
116
117
     /* minres based reliability estimate for EaseOfUse */
118
119
     PROC FACTOR data=zrawData NFACTORS=1 METHOD=ULS OUTSTAT=factorResults;
120
               VAR EOU1 EOU2 EOU3 EOU4 EOU5 EOU6;
121
     RUN:
122
123
     PROC REG DATA=zrawData OUTEST=regressionResults PLOTS=none;
124
125
       MODEL EaseOfUse = EOU1 EOU2 EOU3 EOU4 EOU5 EOU6;
126
     RIIN:
127
     DATA _null_;
128
               SET regressionResults;
129
               CALL symput("EaseOfUseWeight", EOU1);
130
131
     RUN;
132
     DATA _null_;
133
134
              SET factorResults;
               IF _NAME_ NE "Factor1" THEN DELETE;
Reliability = ((EOU1 + EOU2 + EOU3 + EOU4 + EOU5 + EOU6) * &EaseOfUseWeight)**2;
135
136
               CALL symput("EaseOfUseReliability", Reliability);
137
138
     RUN;
139
140
     /* ML-CFA based reliability estimates for all variables */
141
142
     PROC CALIS DATA=zrawData OUTMODEL=cfaResults;
143
              FACTOR
144
                        FPredictedUsage ---> LU4-LU1,
                        FUsefulness ---> Use6-Use1,
FEaseOfUse ---> EOU1-EOU6;
145
146
147
               PVAR
148
                        FPredictedUsage FUsefulness FEaseOfUse = 3 * 1;
     RUN;
149
150
151
     DATA cfaLoadings;
152
         SET cfaResults;
         IF _TYPE_ NE "LOADING" THEN DELETE;
153
     RUN;
154
155
     PROC MEANS DATA=cfaLoadings NWAY;
156
157
               CLASS _VAR2_;
158
               VAR _ESTIM_;
               OUTPUT OUT=loadingSums SUM=sum;
159
160
     RUN;
161
162
     DATA _null_;
              SET loadingSums;
163
              IF _VAR2_ NE "FPredictedUsage" THEN DELETE;
Reliability = (sum * &PredictedUsageWeight)**2;
164
165
              CALL symput("PredictedUsageReliabilityCFA", Reliability);
166
     RUN;
167
168
169
     DATA _null_;
170
              SET loadingSums;
               F _VAR2_ NE "FEaseOfUse" THEN DELETE;
Reliability = (sum * &EaseOfUseWeight)**2;
171
172
              CALL symput ("EaseOfUseReliabilityCFA", Reliability);
173
174
     RUN;
175
     DATA _null_;
176
              SET loadingSums;
177
              TF _VAR2_ NE "FUsefulness" THEN DELETE;
Reliability = (sum * &UsefulnessWeight)**2;
178
179
               CALL symput("UsefulnessReliabilityCFA", Reliability);
180
181
     RUN;
182
183
184
     /* Errors in variables regression */
185
186
     PROC CALIS DATA=zrawData;
187
              LINEQS
```

```
188
                     FPredictedUsage = b1 * FUsefulness+ b2 * FEaseOfUse + DFPredictedUsage,
                     PredictedUsage = %SYSEVALF(&PredictedUsageReliability**2) * FPredictedUsage +
189
                          EPredictedUsage,
190
                     Usefulness = %SYSEVALF(&UsefulnessReliability**2) * FUsefulness + EUsefulness,
                     EaseOfUse = %SYSEVALF(&EaseOfUseReliability**2) * FEaseOfUse + EEaseOfUse;
191
192
             VARIANCE
                     EPredictedUsage = %SYSEVALF(1-&PredictedUsageReliability),
193
                     EUsefulness = %SYSEVALF(1-&UsefulnessReliability),
194
                     EEaseOfUse = %SYSEVALF(1-&EaseOfUseReliability);
195
     RUN:
196
197
     /* List the user macros to see the reliability estimates */
198
     %PUT _USER_;
```

3 Example in Stata.do

```
// This file contains example analyses using the EIV estimator in various
        // configurations using the data from Appendix C of
 3
 4
        // Chin, W. W., Johnson, N., & Schwarz, A. (2008). A Fast Form Approach to
        // Measuring Technology Acceptance and Other Constructs. MIS Quarterly, 32(4),
 6
        // 687 703.
 9
10
       // Estout is used to extract unit weights from regression. Install if not already installed.
        capture which estout
        if _rc==111 ssc install estout
       // Set up the data
16
       clear
18
       matrix input C = (2.111, ///
       1.694, 1.903, ///
1.662, 1.551, 1.875, ///
       1.675, 1.570, 1.581, 1.985, ///
        0.880, 0.798, 0.851, 0.757, 2.137, ///
        0.929, 0.877, 0.800, 0.765, 1.619, 1.846, ///
       0.938, 0.895, 0.853, 0.815, 1.541, 1.437, 1.760, ///
0.920, 0.864, 0.850, 0.794, 1.199, 1.248, 1.251, 1.648, ///
        0.922, 0.864, 0.889, 0.850, 1.535, 1.428, 1.443, 1.258, 1.964, ///
       0.912, 0.873, 0.806, 0.838, 1.620, 1.543, 1.579, 1.346, 1.640, 2.032, ///
1.007, 0.939, 0.925, 0.985, 0.644, 0.743, 0.679, 0.676, 0.703, 0.708, 1.436, ///
1.026, 0.896, 0.893, 0.883, 0.669, 0.661, 0.666, 0.694, 0.739, 0.693, 1.086, 1.503, ///
        0.831,\; 0.815,\; 0.811,\; 0.856,\; 0.497,\; 0.537,\; 0.576,\; 0.559,\; 0.551,\; 0.528,\; 0.857,\; 0.781,\; 0.778,\; 0.800,\; 1.106,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.811,\; 0.81
33
        0.734, 0.740, 0.677, 0.771, 0.505, 0.535, 0.555, 0.528, 0.531, 0.498, 0.748, 0.705, 0.731, 0.773, 0.714,
                  1.015)
34
        // Create multivariate normal dataset with the exact covariance structure
35
36
        corr2data LU4 LU3 LU2 LU1 EOU1 EOU2 EOU3 EOU4 EOU5 EOU6 Use6 Use5 Use4 Use3 Use2 Use1, n(283) cov(C)
37
                  cstorage(lower)
38
        39
40
        \ensuremath{//} Examples using sem, factor, alpha and eivreg
41
42
        43
44
45
46
        // \ {\tt Calculate} \ {\tt three} \ {\tt different} \ {\tt reliability} \ {\tt estimates}
47
48
        // Alphas
49
50
51
        alpha LU4-LU1, std gen(PredictedUsage)
52
        scalar alpha_PredictedUsage = r(alpha)
53
54
        alpha Use6-Use1, std gen(Usefulness)
        scalar alpha_Usefulness = r(alpha)
55
56
57
        alpha EOU1-EOU6, std gen(EaseOfUse)
58
        scalar alpha_EaseOfUse = r(alpha)
        // Create standardized the composites
```

```
61
     egen zPredictedUsage = std(PredictedUsage)
 62
     egen zUsefulness = std(Usefulness)
 63
     egen zEaseOfUse = std(EaseOfUse)
 64
 65
 66
     // Exploratory factor analysis based reliability estimates
 67
68
    // Stata's factor command does not do minres factor analysis, but we can use maximum likelihood instead
 69
     // (minres is possible with the sem command because minres is equivalent to ULS)
 70
     factor LU4-LU1, factors(1) ml
matrix define loadings = e(L)
 71
 72
 73
 74
     qui regress PredictedUsage LU4-LU1
 75
     estadd beta
     matrix define weights_PredictedUsage = e(beta)
 76
     matrix define r = weights_PredictedUsage[1,1..4] * loadings
 77
 78
     scalar CRefa_PredictedUsage = r[1,1]^2
 79
 80
 81 factor Use6-Use1, factors(1) ml
 82
     matrix define loadings = e(L)
 83
 84
     qui regress Usefulness Use6-Use1
 85
     estadd beta
 86
     matrix define weights_Usefulness = e(beta)
 87
     matrix define r = weights_Usefulness[1,1..6] * loadings
 88
     scalar CRefa_Usefulness = r[1,1]^2
 89
 90
 91 factor EOU1-EOU6, factors(1) ml
 92 matrix define loadings = e(L)
 93
 94 qui regress EaseOfUse EOU1-EOU6
 95
     estadd beta
 96
   matrix define weights_EaseOfUse = e(beta)
     matrix define r = weights_EaseOfUse[1,1..6] * loadings
 98
     scalar CRefa_EaseOfUse = r[1,1]^2
99
100
     // Errors in variables regression using reliabilities based on EFA
101 // Note that eivreg does not adjust for error in the dependent variable.
     // To get results that are fully standardized with respect to the error
102
   // free variables, the dependent variable needs to be adjusted before
     // estimation so that the reliable part has unit variances
104
105
106
     gen sPredictedUsage = zPredictedUsage / sqrt(CRefa_PredictedUsage)
107
108
     eivreg sPredictedUsage zUsefulness zEaseOfUse, reliab(zUsefulness '=CRefa_Usefulness' zEaseOfUse '=
         CRefa_EaseOfUse')
109
110
111
112
     //The following example requires Stata 12 or later
113
114
115
     // Confirmatory factor analysis based reliability estimates
116
117
     sem (PredictedUsage -> LU4-LU1) (Usefulness -> Use6-Use1) (EaseOfUse -> EOU1-EOU6), nocapslatent latent(
118
         PredictedUsage Usefulness EaseOfUse)
119
120
    // Get the reliabilities from the factor loading matrix
     estat framework, standardized
121
     matrix define loadings = r(Gamma)
122
123
     // Calculate reliabilities
124
125
126 matrix define r = weights_PredictedUsage[1,1..4] * loadings[1..4,1]
     scalar CRcfa_PredictedUsage = r[1,1]^2
127
128
     matrix define r = weights_Usefulness[1,1..6] * loadings[5..10,2]
129
130
     scalar CRcfa_Usefulness = r[1,1]^2
131
     matrix define r = weights_EaseOfUse[1,1..6] * loadings[11..16,3]
132
133
     scalar CRcfa_EaseOfUse = r[1,1]^2
134
135
     // Errors in variables model in the SEM framework
136
     sem (PredictedUsage_star <- Usefulness_star EaseOfUse_star) ///</pre>
137
138
             (PredictedUsage_star -> zPredictedUsage) ///
139
             (Usefulness_star -> zUsefulness) ///
```

```
140 (EaseOfUse_star -> zEaseOfUse), ///
141 reliability(zPredictedUsage '=CRcfa_PredictedUsage' ///
142 zUsefulness '=CRcfa_Usefulness' zEaseOfUse '=CRcfa_EaseOfUse') ///
143 standardized
144
145 // List all reliability estimates
146 scalar list
```

Appendix D - Full R code for the simulations

This appendix contais the full R code for the simulations used in the paper. The simulation consists of two files. Parameters.R defines the simulation paremeterization and simulations.R implements the simulation.

The files to implement bootstrapping were very similar to these files and are available from the first author by request.

The source code for the two small examples presented in the theory section of the paper is available in the file example. R after the main analysis files.

1 parameters.R

```
# Model 1 and Model 2 in Lavaan syntax
MODELS <- c("
f1 = .87*i11 + .94*i12
f2 = ~ .40*i21 + .41*i22 + .47*i23 + .60*i24 + .63*i25 + .65*i26
f3 = ~ .49*i31 + .60*i32 + .80*i33 + .90*i34
f4 = ~ .63*i41 + .84*i42
f5 = ~ .65*i51 + .67*i52 + .75*i53 + .75*i54 + .90*i55 + .90*i56
f3 ~ 0*f2
f3 ~~ 1*f3
f4 ~ 0*f1 + .7*f2
f4 ~~ .51*f4
f5 ~ .22*f1 + -.7*f2 + .35*f3
f5 ~~ 0.5547*f5
f1 ~~ .7*f2
f1 ~~ 1*f1
f2 ~~ 1*f2
i11~~(1-.87^2)*i11
i12~~(1-.94^2)*i12
i21~~(1-.40^2)*i21
i22~~(1-.41^2)*i22
i23~~(1-.47^2)*i23
i24~~(1-.60^2)*i24
i25~~(1-.63^2)*i25
i26~~(1-.65^2)*i26
i31~~(1-.49^2)*i31
i32~~(1-.60^2)*i32
i33~~(1-.80^2)*i33
i34~~(1-.90^2)*i34
i41~~(1-.63^2)*i41
```

```
i42~~(1-.84^2)*i42
i51~~(1-.65^2)*i51
i52~~(1-.67^2)*i52
i53~~(1-.75^2)*i53
i54~~(1-.75^2)*i54
i55~~(1-.90^2)*i55
i56~~(1-.90^2)*i56",
"f1 =~ .87*i11a + .94*i12a +.87*i11b + .94*i12b
f2 = ~ .40*i21 + .41*i22 + .47*i23 + .60*i24 + .63*i25 + .65*i26
f3 = .49*i31 + .60*i32 + .80*i33 + .90*i34
f4 =~ .63*i41a + .84*i42a + .63*i41b + .84*i42b
f5 = ".65*i51 + .67*i52 + .75*i53 + .75*i54 + .90*i55 + .90*i56
f3 ~ 0*f2
f3 ~~ 1*f3
f4 ~ 0*f1 + .7*f2
f4 ~~ .51*f4
f5 ~ .22*f1 + -.7*f2 + .35*f3
f5 ~~ .1235*f5
f1 ~~ .7*f2
f1 ~~ 1*f1
f2 ~~ 1*f2
i11a~~(1-.87^2)*i11a
i12a~~(1-.94^2)*i12a
i11b~~(1-.87^2)*i11b
i12b~~(1-.94^2)*i12b
i21~~(1-.40^2)*i21
i22~~(1-.41^2)*i22
i23~~(1-.47^2)*i23
i24~~(1-.60^2)*i24
i25~~(1-.63^2)*i25
i26~~(1-.65^2)*i26
i31~~(1-.49^2)*i31
i32~~(1-.60^2)*i32
i33~~(1-.80^2)*i33
i34~~(1-.90^2)*i34
i41a~~(1-.63^2)*i41a
i42a~~(1-.84^2)*i42a
i41b~~(1-.63^2)*i41b
i42b~~(1-.84^2)*i42b
i51~~(1-.65^2)*i51
i52~~(1-.67^2)*i52
i53~~(1-.75^2)*i53
i54~~(1-.75^2)*i54
i55~~(1-.90^2)*i55
i56~~(1-.90^2)*i56")
designMatrix <- expand.grid(sample = c(100, 200, 500),</pre>
                            scalingFactor = c(1, 0.5, 0),
                            excessKurtosis = c(5,0),
                            estimator = c("PLSc","DRminres","DRm1","SEMm1",
```

```
"CFAml"),
model = 1:2)
```

2 simulations.R

```
library(matrixpls)
library(psych)
library(simsem)
library(lavaan)
library(parallel)
library(MASS)
library(Matrix)
library(heplots)
MULTICORE <- FALSE
DEBUG <- FALSE
SAVERESULTS <- TRUE
REPLICATIONS <- 1000
# 1 = normal, 2 = orthogonalized errors
ESTIMATESETS <- 1:2
source("parameters.R")
# Read the condition number from the command line if given. Otherwise run all
# conditions.
args <- commandArgs(trailingOnly = TRUE)</pre>
if(length(args) == 0){
 designNumbers <- 1:nrow(designMatrix)</pre>
} else {
 designNumbers <- (as.numeric(args[1]))</pre>
# Loop over designs
for(designNumber in designNumbers){
 set.seed(12345)
 design <- designMatrix[designNumber, ]</pre>
 print(paste("Starting design number", designNumber))
 print(design)
 # Prepare data generation template
```

```
MODEL <- MODELS[design$model]</pre>
parTable <- lavaanify(MODEL)</pre>
loadingsParTable <- parTable[parTable$op == "=~",]</pre>
observedVariableNames <- unique(loadingsParTable$rhs)</pre>
# Derive a SimSem model from lavaan parameter table by fitting a model to
# a diagonal matrix
cvMat <- diag(length(observedVariableNames))</pre>
colnames(cvMat) <- rownames(cvMat) <- observedVariableNames</pre>
fit <- lavaan::lavaan(MODEL, sample.cov = cvMat, sample.nobs = 100)</pre>
dataTemplate <- simsem::model.lavaan(fit)</pre>
# Rescale the data template
BE <- rawDraw(dataTemplate@dgen[[1]]$BE)$param * design$scalingFactor
PS <- rawDraw(dataTemplate@dgen[[1]]$PS)$param
i <- diag(nrow(PS)) == 0</pre>
PS[i] <- PS[i] * design$scalingFactor
diag(PS) <- 1 - diag(BE %*% PS %*% t(BE))</pre>
dataTemplate@dgen[[1]]$PS@free <- apply(PS, 2, as.character)</pre>
dataTemplate@dgen[[1]]$BE@free <- apply(BE, 2, as.character)</pre>
Lambda <- rawDraw(dataTemplate@dgen[[1]]$LY)$param</pre>
dist <- bindDist("norm", kurtosis = design$excessKurtosis)</pre>
# Generate the datasets
print("Generating datasets")
dataSets <- sim(REPLICATIONS, dataTemplate, design$sample, dataOnly = TRUE,</pre>
               factDist = dist, errorDist = dist,
                sequential = TRUE, saveLatentVar = TRUE,
               multicore = MULTICORE)
# Estimation
estimator <- design$estimator
# SEM and regressions with factor correlation matrix after CFA
if(substr(estimator,1,3)=="CFA" || substr(estimator,1,3)=="SEM"){
  if(substr(estimator,1,3)=="SEM"){
    estimatedModel <- (gsub("f[0-9] ~~ f[0-9]","",gsub("[.0-9]+\\*","",gsub("\ni.*","",MODEL))))
    lavaanfun <- "sem"
  else{
    {\tt estimatedModel} <- \ {\tt gsub}("f[0-9] \ ^ \ .*$","", gsub("[.0-9]+\*","", gsub("\ni.*","", MODEL))))
    lavaanfun <- "cfa"
```

```
inner <- matrixpls:::parseModelToNativeFormat(MODEL)$inner</pre>
exog <- which(rowSums(inner) == 0)</pre>
# Do the actual simulations
sim.res <- sim(model = estimatedModel, rawData = dataSets, lavaanfun = lavaanfun,</pre>
                outfundata = function(fit,data){
                   var.lv <- diag(inspect(fit, "cov.lv"))</pre>
                   var.ov <- unlist(lapply(data,function(x){</pre>
                    sum((x-mean(x))^2)/length(x)
                   }))
                   sd.lv <- sqrt(var.lv)</pre>
                   sd.ov <- sqrt(var.ov)</pre>
                   C <- cov2cor(inspect(fit,"cov.lv"))</pre>
                  S \leftarrow matrix(0,0,0,dimnames = list(c(),c()))
                   # Regressions from LV correlation matrix. We do this for ML
                   # estimation as well because it simplifies the code when
                   \mbox{\it \#} we do not need to treat regression from CFA and ML
                   # as separate special cases
                   regressions <- matrixpls:::estimatesMatrixToVector(</pre>
                     estimator.regression(S, inner, W = NULL, C = C),inner, "~")
                   # Standardize loadings
                   lambda <- inspect(fit, "coef")$lambda * ((1/sd.ov) %o% sd.lv)</pre>
                   loadings <- lambda[lambda != 0]</pre>
                   names(loadings) <-</pre>
                     paste(colnames(lambda)[col(lambda)[lambda != 0]],
                           rownames(lambda)[row(lambda)[lambda != 0]],sep="")
                   exogC <- C[exog,exog]</pre>
                   correlations <- exogC[lower.tri(exogC)]</pre>
                   names(correlations) <-</pre>
                     paste(colnames(exogC)[col(exogC)[lower.tri(exogC)]],
                           rownames(exogC)[row(exogC)[lower.tri(exogC)]],sep="")
                   # Check the admissibility of the correlation matrix
                   inadmissible <- ifelse(any(eigen(C)$values < 0),1,0)</pre>
                   {\it \# Return \ the \ standardized \ estimates \ from \ this \ replication}
                  c(inadmissible,regressions,loadings,correlations)
                },
                multicore = MULTICORE)
```

```
# Add the estimated regressions
  sim.res@coef <- as.data.frame(do.call(rbind,lapply(sim.res@extraOut, function(x){</pre>
    if(is.null(x)) return(NA)
   x[-1]
  })))
  # Save paths. (Loadings are saved later)
  estimates <- sim.res@coef[grep("~f",names(sim.res@coef))]</pre>
  results <- cbind(inaadmissible = unlist(lapply(sim.res@extraOut, function(x){
    if(is.null(x)) return(1)
   x[1]
  })), estimates)
  print(paste("Saving file: Paths", designNumber, estimator, 1, ".Rdata", sep="_"))
  print(results)
  print(apply(results,2,mean))
  if(SAVERESULTS) save(results, file = paste("Paths",designNumber,estimator,1,".Rdata", sep="_"))
# Composite based estimators, PLSc and EIVxx
else{
 print(paste("Estimating. Estimator:", estimator))
  # Loadings - 1: PLSc loadings, 2: CFA loadings, 3: minres loadings
  estimatorIndex <- (estimator == "PLSc") + 1 * grepl("EIV", estimator) + 1</pre>
 if(estimator == "EIVminres") estimatorIndex <- 3</pre>
  # Weights - 1: PLS weights, 2: unit weights
  modeIndex <- (estimator == "PLSc") + 1</pre>
  sim.res <- matrixpls.sim(model = MODEL, rawData = dataSets,</pre>
                            # PLSc estimation with 2 stage least squares where applicable
                            disattenuate = (estimator == "PLSc" ||
                                              grepl("EIV",estimator)),
                            # Use the factor method
                            innerEstimator = inner.factor,
                            # How to calculate loadings
                            parametersReflective = switch(estimatorIndex,
                                                           estimator.PLScLoadings,
                                                           estimator.CFALoadings,
                                                           estimator.EFALoadings),
                            # PLS Mode A or unit weights
                            weightFunction = switch(modeIndex,
                                                    weight.fixed,
                                                     weight.pls),
```

```
# We are not interested in bootstrap SEs or fit indices
                          boot.R = FALSE, fitIndices = NULL,
                          # Use parallel processing
                          multicore = MULTICORE,
                           stopOnError = DEBUG
)
print(paste(sum(sim.res@converged != 0), "non-convergent results"))
# True reliabities and a few reliability indices for all composites
results <- do.call(ifelse(MULTICORE, "mclapply", "lapply"), list(sim.res@extraOut, function(pls.res){
  if(is.null(pls.res)) return(NA)
  R <- attr(pls.res,"R")</pre>
  names(R) <- paste("R",names(R))</pre>
  W <- attr(pls.res,"W")</pre>
  1 <- loadings(pls.res)</pre>
  Q <- colSums(1 * t(W))^2
  names(Q) <- paste("Q",names(Q))</pre>
  CR <- CR(pls.res)
  names(CR) <- paste("CR",names(CR))</pre>
  S <- attr(pls.res,"S")</pre>
  a <- apply(W,1,function(x){alpha(S[x!=0,x!=0])$total[1]})</pre>
  temp <- paste("Alpha", names(a))</pre>
  a <- unlist(a)
  names(a) <- temp</pre>
  c(trueR = R,Q,CR,a)
}))
results <- do.call(rbind, results)</pre>
results <- cbind(inadmissible = apply(results[,c(6:10)],1,function(x){any(abs(x)>1)}), results)
print(paste("Saving file: QR",designNumber,estimator,".Rdata", sep="_"))
print(results)
if(SAVERESULTS) save(results, file = paste("QR",designNumber,estimator,".Rdata", sep="_"))
# Two sets of manipulated estimates (1: normal, 2:orthogonalized errors)
for(estimateSet in ESTIMATESETS){
```

```
results <- do.call(ifelse(MULTICORE, "mcmapply", "mapply"), list(function(pls.res, data){
  if(is.null(pls.res)) return(NA)
  # Possibly disattenuated composite correlations
  C <- attr(pls.res,"C")</pre>
  # Choose exogenous correlations from C
  exog = apply(attr(pls.res, "model")$inner == 0,1,all)
  # Return two sets of regression estimates and R2s
  # 1) The original estimates
  # 2) Estimates with orthogonalized errors
  # The first estimate set is with the original C.
  if(estimateSet != 1){
    d <- attr(data, "latentVar")</pre>
    Eta <- as.matrix(attr(data, "latentVar")[,grepl("^f[0-9]+",names(d))])</pre>
    Epsilon <- attr(data, "latentVar")[,grepl("^res_i[0-9]+",names(d))]</pre>
    # Orthogonalize the errors
    EpsilonOrth <- gsorth(Epsilon)</pre>
    colnames(EpsilonOrth) <- rownames(EpsilonOrth) <- NULL</pre>
    data <- Eta %*% t(Lambda) + EpsilonOrth
    S <- cor(data)
    # Weight matrix
    W <- attr(pls.res,"W")
    # Composite correlation matrix with orthogonalized error data
    C <- cov2cor(W %*% S %*% t(W))</pre>
    # Estimated composite reliabilities
    Q <- attr(pls.res,"Q")</pre>
    if(! is.null(Q)){
      # Determination of consistent estimates for the correlation between the
      \# latent variables, see (15) and (16) of Dijkstra, April 7, 2011.
      C <- C / sqrt(Q) %*% t(sqrt(Q))</pre>
      diag(C) <- 1
  \# Check the admissibility of the correlation matrix
  inadmissible <- ifelse(any(eigen(C)$values < 0),1,0)</pre>
  if(estimateSet != 1){
   model <- attr(pls.res, "model")</pre>
```

```
a <- tryCatch(
          matrixpls:::regressionsWithCovarianceMatrixAndModelPattern(
            C,model$inner) [model$inner == 1],
                      error=function(cond) {
                        print(cond)
                        print(pls.res)
                        # Choose a return value in case of error
                        return(NA)
        \# Substitute the new estimates and new C
        pls.res[1:sum(model$inner == 1)] <- a</pre>
        beta <- model$inner
        beta[beta == 1] <- a
        attr(pls.res,"beta") <- beta</pre>
        attr(pls.res,"C") <- C</pre>
      estimates <- pls.res[grep("~f",names(pls.res))]
      R2 <- R2(pls.res)
      R2 \leftarrow R2[R2 != 0]
      names(R2) <- paste("R2",names(R2))</pre>
      # Exogenous correlations
      exogC = C[exog,exog, drop=FALSE]
      indices <- which(lower.tri(exogC))</pre>
      n <- paste(colnames(exogC)[col(exogC)[indices]],"~~",</pre>
                rownames(exogC)[row(exogC)[indices]],sep="")
      exogC <- exogC[indices]</pre>
      names(exogC) <- n</pre>
      c(inadmissible,estimates,exogC,R2)
    }, sim.res@extraOut, dataSets, SIMPLIFY = FALSE))
    results <- do.call("rbind", results)</pre>
    print(paste("Saving file: Paths",designNumber,estimator,estimateSet,
                ".Rdata", sep="_"))
    # print(results)
    print(apply(results,2,mean))
    print(apply(results,2,sd))
    if(SAVERESULTS) save(results, file =
                           paste("Paths",designNumber,estimator,estimateSet,
                                 ".Rdata", sep="_"))
  } # End of looping manipulations
} # End of saving composite specific results
```

3 examples.R

```
library(matrixpls)
library(parallel)
library(MASS)
library(simsem)
SAMPLE <- 100
MODEL <- "\nA = x1 + x2 + x3\nB = x4 + x5 + x6\nB ^{\sim} A\n"
Lambda <- (diag(2) * 0.7)[rep(1:2, each = 3),]
Psi \leftarrow matrix(c(1, 0.3, 0.3, 1), 2, 2)
Sigma <- Lambda %*% Psi %*% t(Lambda)
diag(Sigma) <- 1</pre>
rownames(Sigma) <- colnames(Sigma) <- paste("x", 1:6, sep = "")</pre>
se <- sqrt((1 - Sigma[6, 1]^2)/(SAMPLE - 2))</pre>
# Adjust one correlation by +1 SD and other by -1 SD
SigmaAdj <- Sigma
SigmaAdj[4, 1] <- SigmaAdj[1, 4] <- SigmaAdj[4, 1] - se
SigmaAdj[5, 2] <- SigmaAdj[2, 5] <- SigmaAdj[5, 2] + se
converged = FALSE
iter <- 0
res <- NULL
while (!converged) {
plsc <- matrixpls(SigmaAdj, MODEL, iter = iter, parametersReflective = estimator.PLScLoadings,
```

```
disattenuate = TRUE)
    res <- rbind(res, c(plsc, attr(plsc, "c"), attr(plsc, "Q"), diag(attr(plsc,</pre>
        "W") %*% Lambda)^2))
    iter <- iter + 1
    converged <- attr(plsc, "converged")</pre>
print(res)
dataSets <- mclapply(1:10000, function(x) {</pre>
    mvrnorm(100, rep(0, 6), Sigma)
sim.res <- matrixpls.sim(model = MODEL, rawData = dataSets, multicore = TRUE,</pre>
   parametersReflective = estimator.PLScLoadings, disattenuate = TRUE, boot.R = FALSE,
    fitIndices = NULL)
{\it \# Estimate \ taking \ weights \ from \ a \ different \ sample}
estimates <- mcmapply(function(matrixpls.res, data) {</pre>
   if (is.null(matrixpls.res))
       return(NA)
    W <- attr(matrixpls.res, "W")</pre>
    Q <- attr(matrixpls.res, "Q")
    S <- cor(data)
    C <- cov2cor(W %*% S %*% t(W))
    C[1, 2]/(sqrt(Q[1]) * sqrt(Q[2]))
}, sim.res@extraOut, dataSets[c(10000, 1:9999)])
plot(density(sim.res@coef[, 1], na.rm = TRUE), xlim = c(-0.5, 1))
lines(density(unlist(estimates), na.rm = TRUE), lty = 2)
```

1. Appendix E – Partial Replication of Goodhue et al 2012

In this appendix we provide a partial replication of the study by Goodhue, Lewis, and Thompson (2012) to assess the performance of Cronbach's alpha in the conditions that they used. Goodhue et al. implemented disattenuation by correcting the regression estimates with Cronbach's alphas calculated using the population values of indicator reliabilities. Because the exogenous latent variables are uncorrelated in their model, the regression coefficients converge to bivariate correlations, and so applying the disattenuation formula directly to regression estimates produces a consistent estimator. However, as also noted by Dijkstra and Henseler (2015), Cronbach's alpha is an inconsistent estimator for PLS Mode A composites. The population reliabilities for the PLS Mode A composites for the Goodhue et al. model are all .856, whereas Cronbach's alpha from the population correlation matrix is .84, leading to bias of -.016. However, this provides an incomplete picture of how Cronbach's alpha performs as a reliability estimate for PLS composites in the small sample scenarios studied by Goodhue and colleagues. To examine this issue further, we replicated their Study 1 using 1000 Monte Carlo replications for each of the sample sizes used in their Figure 8. The R code for our replication is available in the end of this appendix. Comparing the true reliabilities of the PLS composites and Cronbach's alphas calculated for each sample produced a rather different set of results than what asymptotic comparison would suggest: Instead of being negatively biased, Cronbach's alpha showed a small positive bias of .012, .007, and .004 averaged over all composites for the three sample sizes of 90, 150, and 200 respectively. The direction of bias is positive because the PLS weights can vary widely in small samples and therefore it is possible that a single set of weights may not be close to the optimal weights, leading to decreased composite reliability (Rönkkö & Evermann, 2013).

2. References

- Dijkstra, T. K., & Henseler, J. (2015). Consistent partial least squares path modeling. *MIS Quarterly*, 39(2), 297–316.
- Goodhue, D. L., Lewis, W., & Thompson, R. (2012). Does PLS have advantages for small sample size or non-normal data. *MIS Quarterly*, *36*(3), 981–1001.
- Rönkkö, M., & Evermann, J. (2013). A critical examination of common beliefs about partial least squares path modeling. *Organizational Research Methods*, *16*(3), 425–448. http://doi.org/10.1177/1094428112474693

```
library(matrixpls)
library(simsem)
library(psych)
MODEL <- "
Ksi1 = .7*x1 + .8*x2 + .9*x3
x1 ~~ .51*x1
x2 ~~ .36*x2
x3 ~~ .19*x3
Ksi1 ~~ 1*Ksi1
Ksi2 = ~.7*x4 + .8*x5 + .9*x6
x4 ~ .51*x4
x5 ~ .36*x5
x6 ~ .19*x6
Ksi2 ~~ 1*Ksi2
Ksi3 = ^{\sim} .7*x7 + .8*x8 + .9*x9
x7 ~~ .51*x7
x7 .51*x7
x8 ~~ .36*x8
x9 ~~ .19*x9
Ksi3 ~~ 1*Ksi3
Ksi4 = ~.7*x10 + .8*x11 + .9*x12
x10 ~ .51*x10
x11 ~ .36*x11
x12 ~ .19*x12
Ksi4 ~ 1*Ksi4
Eta1 = ^{\sim} .7*y1 + .8*y2 + .9*y3
y1 ~~ .51*y1
y2 ~~ .36*y2
y3 ~~ .19*y3
Eta1 ~ .48*Ksi1 + .314*Ksi2 + .114*Ksi3
Eta1 ~ .658008*Eta1
for(N in c(90, 150, 200)){
  simres <- matrixpls.sim(1000, MODEL, N, boot.R = FALSE, multicore = TRUE,</pre>
                                 sequential = TRUE,
                                  fitIndices = function(matrixpls.res){
                                   R <- attr(matrixpls.res, "R")</pre>
                                    S <- attr(matrixpls.res, "S")</pre>
                                    for(i in 1:5){
                                      a <- alpha(S[i*3-0:2,i*3-0:2])
                                      R[i] <- R[i] - a$total[[1]]</pre>
                                    R
                                  })
   f <- inspect(simres, "fit")</pre>
   print(paste("N:",N,"mean bias:",print(mean(as.matrix(f)))))
```