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Load-carrying behaviour of web-core sandwich plates in compression

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Abstract

This paper investigates theoretically the compressive load-carrying behaviour of geometrically imperfect web-core sandwich plates. Slender plates, which first buckle globally, are considered. The study is carried out using two approaches, both solved with the finite element method. The first is the equivalent single-layer theory approach. First-order shear deformation theory is used. The second approach is a three-dimensional shell model of a sandwich plate. Plates are loaded in the web plate direction. Simply supported and clamped boundary conditions are considered with a different level of in-plane restraint on the unloaded edge. The results show that the behaviour of the sandwich plate is qualitatively equal to the isotropic plate of the same bending stiffness for deflections lower than the plate thickness. As the deflections increase, the lower in-plane stiffness of the sandwich plate results in lower post-buckling stiffness. Local buckling of face plates in the post-buckling range of the sandwich plate further reduces the structural stiffness.

Keywords: *web-core sandwich, load-carrying behaviour, global buckling, local buckling, post-buckling, geometric imperfections*

List of symbols:

a	Length of the plate in x-direction (m)
A_{ij}	Extensional (in-plane) stiffness coefficients, i,j = 1,2,3. (N/m)
b	Width of the plate in y-direction (m)
b_e	Effective width of the plate
B_{ij}	Extensional-bending stiffness coefficients, i,j = 1,2,3. (N)
d	Distance between the neutral axes of the face plates, $d = h_c + (t_t + t_b)/2$ (m)
d_b	Distance between neutral axis of the bottom face plate and the mid-plane of the sandwich
d_c	Distance between the point in the web plate and the mid-plane of the sandwich plate (m)
d_t	Distance between neutral axis of the top face plate and the mid-plane of the sandwich
D_{ij}	Bending stiffness coefficients, i,j = 1,2,3. (Nm)
D_t	Bending stiffness of top face plate (Nm)

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D_w	Bending stiffness of web plate (Nm)
D_{Qx}	Transverse shear stiffness in *x*-direction (Nm)
D_{Qy}	Transverse shear stiffness in *y*-direction (Nm)
E	Young's modulus (Pa)
G	Shear modulus (Pa)
h	Height of the sandwich plate, $h = t_t + h_c + t_b$; thickness of the isotropic plate (mm)
h_c	Height of the sandwich plate core (mm)
k_θ	Rotation stiffness of laser weld (kN)
m	Number of buckling half-waves in *x*-direction
n	Number of buckling half-waves in *y*-direction
N_0	Buckling load per unit width (N)
N_{cr}	Plate buckling load (N)
s	Spacing of the web plates (mm)
t_b	Thickness of bottom face plate (mm)
t_f	Thickness of face plates (mm)
t_t	Thickness of top face plate (mm)
t_w	Thickness of web plate (mm)
u	Displacement component in *x*-direction, edge shortening (m)
w	Deflection of the plate (m)
ν	Poisson's ratio
σ_x	Membrane stress in *x*-direction (MPa)
$\sigma_{x, max}$	Maximum membrane stress in *x*-direction in the middle of plate length (MPa)
τ_i	Shear stress in the structural member of the cross-section (MPa)

1 Introduction

Steel sandwich plates are light-weight structures which can save space and improve safety; see Naar et al. [1], Ehlers et al. [2]. Their stiffness-to-weight and strength-to-weight ratio are higher than those of stiffened plate when bending is concerned. This study concentrates on sandwich plates which consist of two face plates separated by web plates. Sandwich plates are typically slender when used e.g. in a ship or a bridge deck. Such application exposes the structure to compressive in-plane loads and thus its stability becomes a design issue.

Buckling and post-buckling of web-core sandwich structures have only been considered in a few studies. Kolsters and Zenkert [3], Kolsters and Zenkert [4], and Kolsters [5] studied the local buckling and post-buckling behaviour of face plates. Kozak [6] studied the ultimate strength of sandwich columns. Jelovica et al. [7] studied the global bifurcation buckling strength of sandwich plates. Thus, these investigations leave a gap for understanding the geometrically non-linear load-carrying behaviour of web-core sandwich plates. Web-core sandwich plates are periodic and in that respect can be considered as being similar to corrugated-core sandwich plates. The ultimate strength of corrugated board plates was studied by Hahn et al. [8] and Nordstrand [9]. Nordstrand [10] developed an analytical expression for the load-carrying behaviour of imperfect corrugated board plates and verified it with an average experimental load-deflection curve. The plates exhibited first global and then local buckling, as in the study by Hahn et al [8]. However, the investigations did not discuss the effect of boundary conditions and local buckling of the faces plates and the core.

Byklum et al. [11] and Byklum and Amdahl [12] have developed a two stage approach for the buckling and post-buckling assessment of stiffened plates. Local buckling is calculated first and the non-linear ABD-matrix is derived for global analysis. However, the investigations do not consider the influence of out-of-plane shear deformations. As shown by Romanoff et al. [13], Nordstrand [14] and Jelovica et al. [7], the shear deformations have large effect on the response of the sandwich plates with discrete, unidirectional core. In Rhodes [15] the influence of in-plane edge restraint has been discussed for isotropic plates. From there it is concluded that the in-plane restraint has large effect on the post-buckling behaviour of the plates.

The aim of this study is to investigate the load-carrying behaviour of geometrically imperfect web-core sandwich plates under in-plane compression. The influence of imperfection amplitude and possible local buckling on load-deflection and load-end shortening curves is studied. The investigation is limited to linear-elastic material behaviour and slender plates, which buckle first globally and then locally. The interest in primarily global buckling is justified by the typically large span of girders in ships or bridges. The investigation is carried out with two theoretical approaches that have different kinematic assumptions, both of which are solved with the finite element method (FEM). The first is the large deflection equivalent single-layer (ESL) theory approach, described for example in Reddy [16]. There, first-order shear deformation theory is used. The second approach is a 3-D shell model of the sandwich plate topology. Models are compared for plate buckling with analytical solution. Plates are loaded in their main load-carrying direction, i.e. parallel to the web plates. Simply supported and clamped boundary conditions are considered with a different level of in-plane restraint on the unloaded edge. A cross-section of relevant industrial applications is considered. Sandwich plate behaviour is compared with that of an isotropic plate, since the isotropic plate post-buckling theory is well established; see e.g. Jones [17], Timoshenko and Gere [18].

2 Analysis methods

Geometric non-linear analysis is carried out by increasing the compression in small steps on the initially imperfect structure. The first eigenmode is used as the shape of the initial imperfection, which was in all studied cases a global mode with a single half-wave in both longitudinal and transverse directions. The measurements in EU Sandwich project [19] suggest that this is the only imperfection for the plates of the size and the cross-section as studied here. The analyses are carried out using the Abaqus software, version 6.9. A modified Riks procedure is used in the second step; see Abaqus [20]. Shell elements with four nodes (S4) are used.

2.1 2-D model

The orthotropic sandwich plate is described through a single layer in its geometric mid-plane, where the loads and boundary conditions are also described. The equivalent stiffness properties for extension, coupling, bending and shear are described through *ABD*- and D_Q –matrices, respectively; see Appendix A. The mesh consists of 100 elements in the length and 100 elements in the width direction. The correspondence in bifurcation buckling load to analytical solution [16, 21] is excellent; see Appendix B.

This study considers two types of flexural boundary conditions: (SS) – all edges simply supported and (CC) – all edges clamped. Loaded edges are kept straight in-plane. Unloaded edges are considered with three different in-plane boundary conditions: (free) - edges free to move; (straight) - edges kept straight, and (fixed) - edges not moving in-plane. Thus, six boundary condition cases are studied. Typical deflection patterns for a plate of unit aspect ratio a/b are shown in Fig. 1.

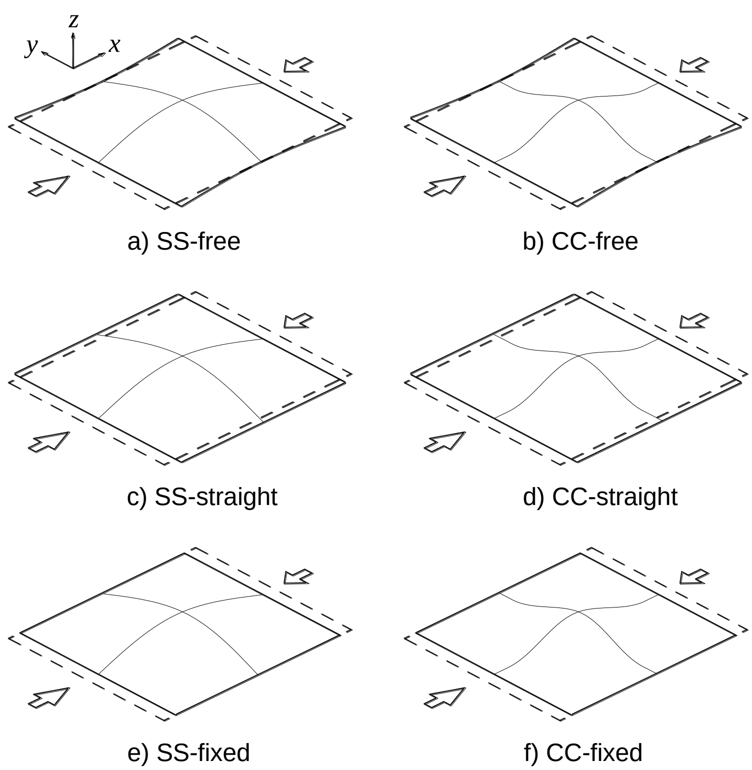


Fig. 1. *Simply supported and clamped 2-D model with unloaded edges: (a)-(b) free to move; (c)-(d) kept straight; (e)-(f) held at initial width. Dashed lines show unloaded plate. Solid lines show (exaggerated) post-buckling shape.*

2.2 3-D model

Face and web plates are modelled with shell elements to form an actual topology of the sandwich plate. Concentrated nodal forces act on the nodes in the geometric mid-plane. Six elements per web plate height are used. The face plates have six elements between the webs. Mesh density is considered sufficient since the difference in bifurcation buckling load to analytical solution [16, 21] is less than 2%; see Appendix B.

For the simply supported edges and unloaded edges free to move in-plane (SS-free), the deflection restraint is set only on the nodes at the geometric mid-plane. This allows the rotation of the plate around the mid-plane edge. Furthermore, the vertical nodes along the edge are displaced equally in the edge direction to prevent the rotation of the in-plane axis orthogonal to the edge. For example, all the nodes at a certain web plate have the same displacement in the *y*-direction, *v*; see Fig. 2. Additionally, the nodes at the geometric mid-plane at x=0 are required to have the same displacement in *x*-direction, *u*. The same is required at $x = a$.

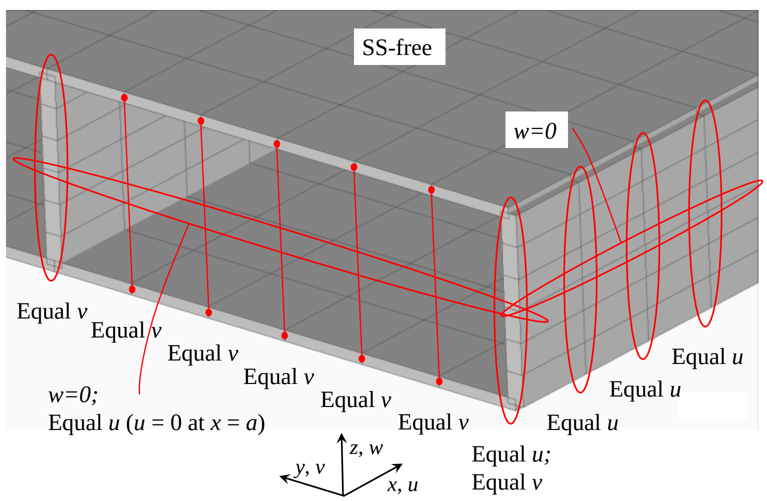


Fig. 2. *FE mesh and SS-free boundary condition for the 3-D model.*

3 Load-carrying behaviour

3.1 Description of the plates

Three plate cross-section geometries are considered: a) a standard web-core sandwich plate for marine and civil applications; b) as in Case a, but with thinner face plates, and c) an isotropic plate. The standard web-core sandwich plate has face plate and web plate thicknesses of 2.5 mm and 4 mm, respectively. The core height is 40 mm and the web plate spacing is 120 mm. A plate with 1.5-mm-thick face plates is considered as Case b. The thickness of the isotropic plate is 30.7 mm, chosen in such a way that the plate has equal bending stiffness in the loading direction, i.e. D_{11}, as Case a. The properties of the plates that are studied are given in Table 1. It can be seen that the in-plane stiffness of the web-core sandwich plate is five to seven times smaller than that of the isotropic plate. In addition, the transverse shear stiffness is $6{\cdot}10^3$ to $2{\cdot}10^4$ times smaller than that of the isotropic plate. Considered length and width of the plates are 2.76 m, and thus the aspect ratio, a/b, is equal to unity. The material behaviour is linear elastic, characterised by a Young's modulus E = 206 GPa and Poisson's ratio ν = 0.3.

Table 1. *Geometric and stiffness properties of the plates studied.*

	Sandwich plate		Isotropic plate
	t_f / $t_w \times h_c$ / s [mm]	t_f / $t_w \times h_c$ / s [mm]	h [mm]
	2.5 / 4 x 40 / 120	1.5 / 4 x 40 / 120	30.7
A_{11} [MN/m]	1 406	954	6 960
A_{22} [MN/m]	1 132	679	6 960
A_{12} [MN/m]	339	204	2 090
A_{33} [MN/m]	396	238	2 440
D_{11} [kNm]	***548***	329	***548***
D_{22} [kNm]	511	292	548
D_{12} [kNm]	153	88	164
D_{33} [kNm]	179	102	191
D_{Qx} [kNm]	$68{\cdot}10^3$	$55{\cdot}10^3$	$2\,033\cdot10^3$
D_{Qy} [kNm]	419	102	$2\,033\cdot10^3$

3.2 Influence of imperfection magnitude and stiffness coefficients

Load-deflection and load-end shortening curves for plates with different imperfection magnitudes are presented in Fig. 3 and Fig. 4. The plates are simply supported with unloaded edges free to pull-in (SS-free). The standard sandwich plate (Case a) and the isotropic plate are compared. The deflection is measured in the middle of the face plate on the convex side; local buckling does not occur there. The presented deflection includes initial imperfection. The load is normalised by the first eigenmode value. A comparison of the analytical and numerical solutions of the bifurcation buckling is given in Appendix B. Deflection is normalised by the height of the plate. End shortening with a small static load is extrapolated to the buckling load to obtain critical end shortening. The shape of the initial imperfection is taken as the shape of the first eigenmode. The behaviour of the isotropic plate is in excellent agreement with experiments [22] and textbook results [17].

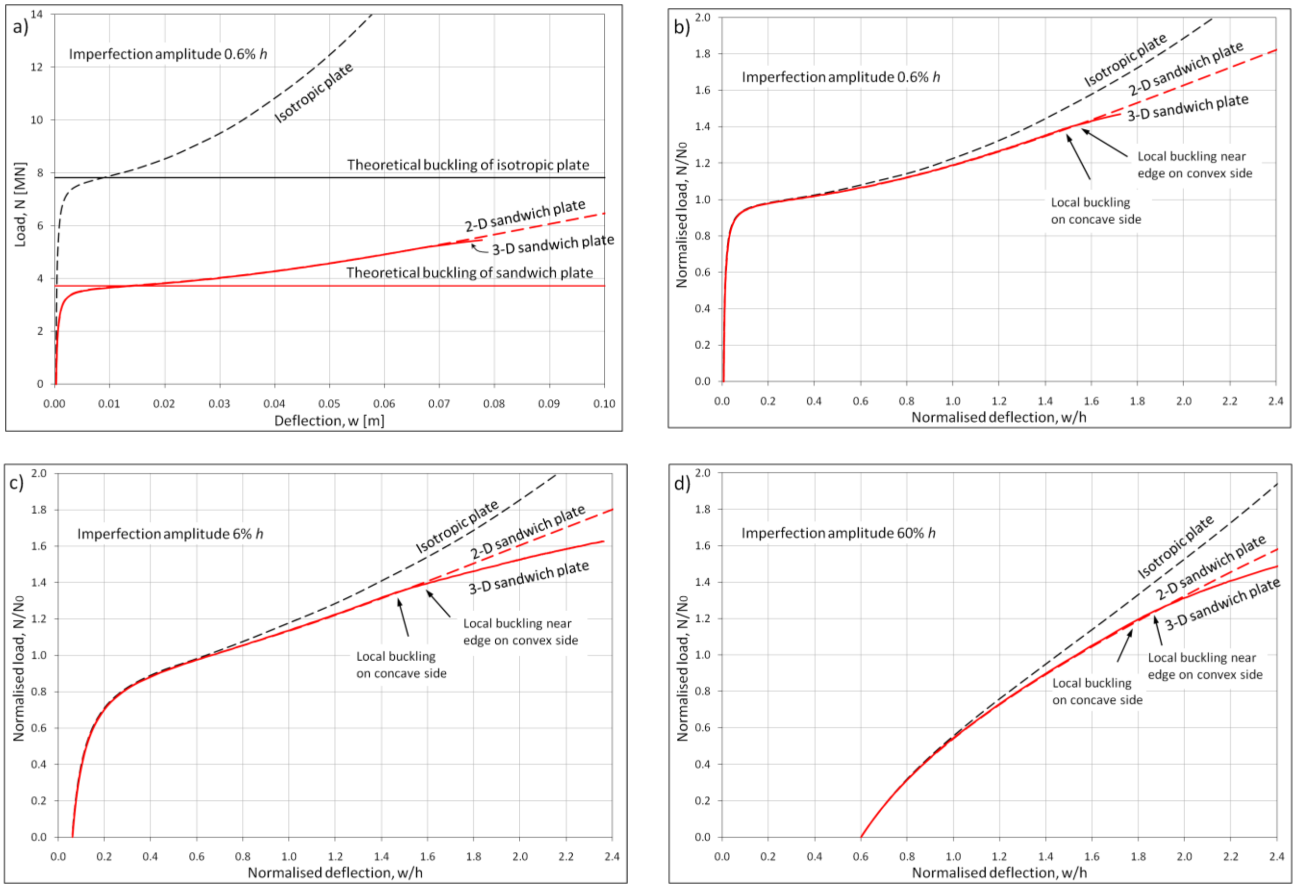
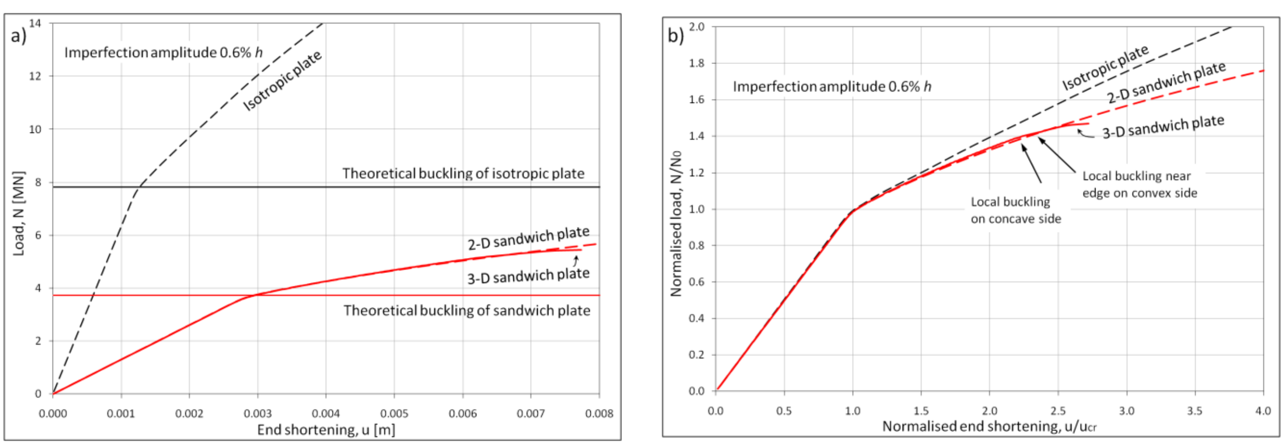


Fig. 3. *Load- deflection curves for SS-free plates with imperfection magnitudes of: (a)-(b) 0.6%; (c) 6%, and (d) 60% of plate height.*



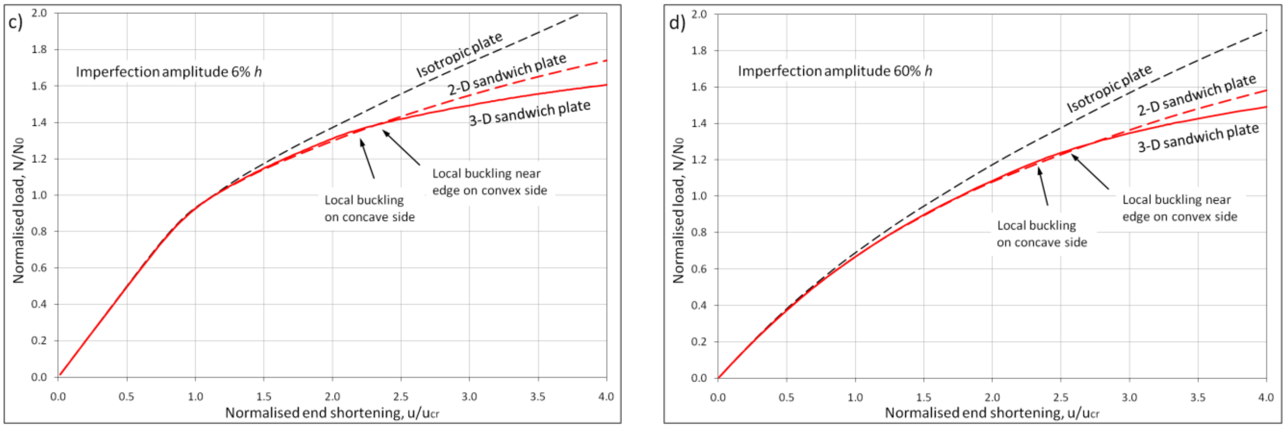


Fig. 4. *Load-end shortening curves for SS-free plates with imperfection magnitudes of: (a)-(b) 0.6%; (c) 6%, and (d) 60% of plate height.*

Fig. 3a and Fig. 4a show that the buckling load and the post-buckling stiffness are smaller for the sandwich plate in comparison to the isotropic plate. Fig. 3 and Fig. 4 furthermore show that the load-deflection and load-end shortening behaviour of the sandwich plate is qualitatively equal to the isotropic plate for deflections lower than the plate thickness. As the deflections increase, the stiffness of the sandwich plate reduces in comparison to the isotropic plate. Secondary stiffness reduction of the sandwich plate in the post-buckling range occurs due to local buckling of face plates. It can be seen from Fig. 3 and Fig. 4 that the 2-D model doesn't capture local buckling and its accuracy, perfect until that point, becomes reduced.

Fig. 5 presents the influence of stiffness coefficients on the load-deflection behaviour of the plate. By increasing the transverse shear stiffness, D_{Qy}, to the value of longitudinal shear stiffness, the buckling load is almost doubled. Small further increase in buckling load is achieved by increasing the shear and the bending stiffnsses, other than D_{11}, to that of the isotropic plate. However, these alterations do not affect the post-buckling stiffness. The post-buckling stiffness is increased by increasing extensional stiffness coefficients or A-matrix, mostly due to A_{11} since that is the loading direction.

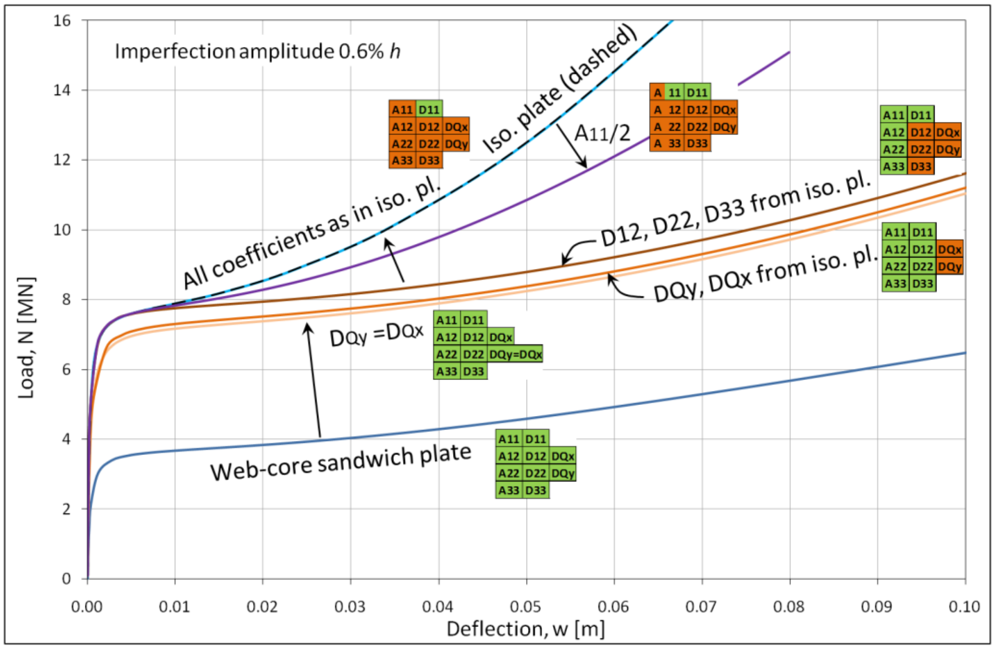


Fig. 5. *Load-deflection curves for SS-free plate with different combination of stiffness coefficients.*

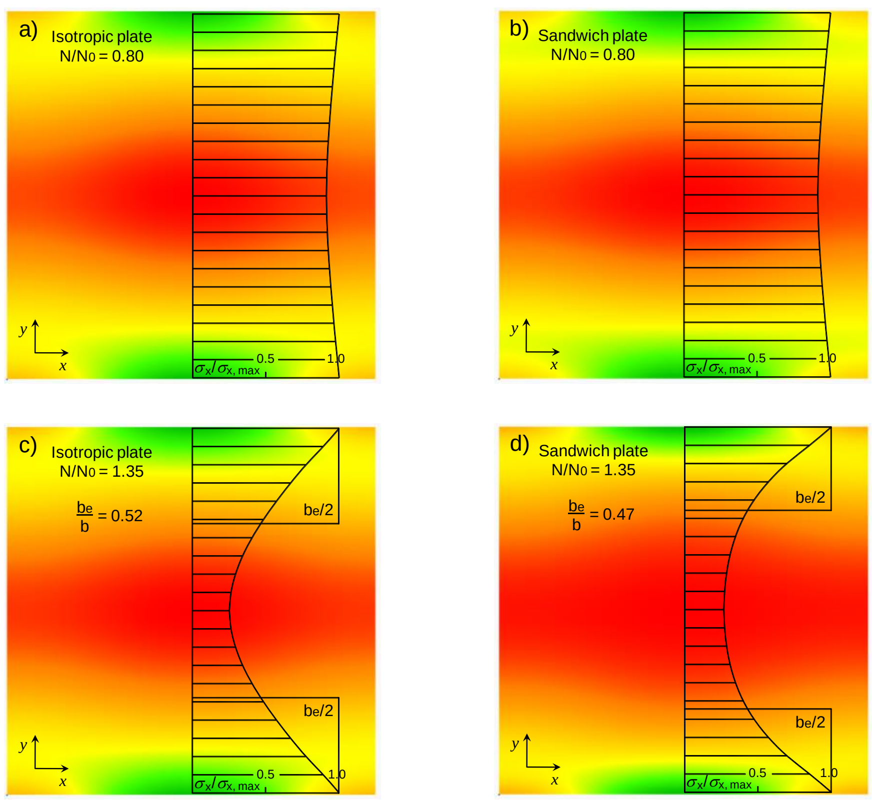


Fig. 6. *Membrane force distribution in loading direction for SS-free plates with 6% initial imperfection.*

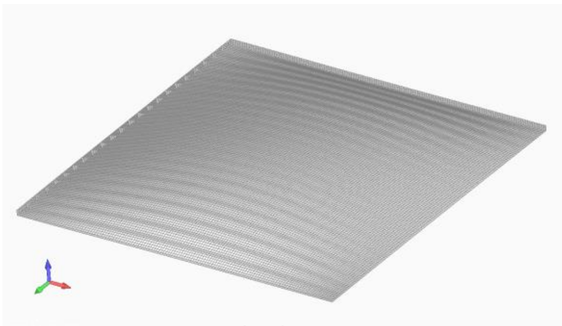
Fig. 6 presents the membrane force distribution in the loading direction for the isotropic plate and the sandwich plate Case a. It can be seen that, below the theoretical buckling load, the force distribution is equal for the sandwich plate and the isotropic plate. However, above the theoretical buckling load, the central part of the sandwich plate is less effective in carrying the membrane force. As an example, at load level 35% above the bifurcation buckling, effective width of the isotropic plate, in the middle of plate length, is 0.52 *b* while in the sandwich plate it is 0.47 *b*, which is 10% lower. Effective width is calculated as the ratio between the average membrane stress and the maximum membrane stress [23]:

$$b_e = \frac{\int_{-B/2}^{B/2} \sigma_x dy}{\sigma_{x,\max}}. \tag{1}$$

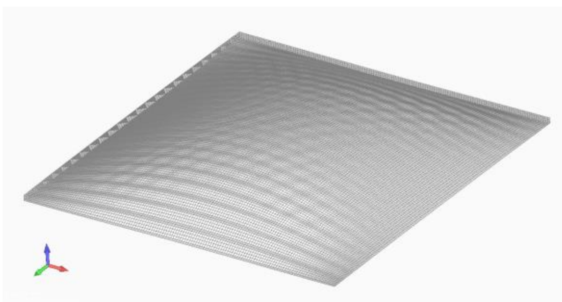
3.3 Influence of local buckling

The global deformation shape of the sandwich plate is unchanged during the initial load increase; see Fig. 7a. At a certain load, beyond the theoretical buckling load, local buckling, not initially present in the structure, occurs on the concave side of the plate and adds to the global deformation shape; see Fig. 7b. At that point, local buckling is not present on the convex side; see Fig. 7c. Further loading causes a loss of stiffness in the central part of the plate and an increase in the deflections. A further load increase is possible as a result of the high stiffness of the straight unloaded edges. The amplitude of local buckles grows, see Fig. 7d. The compressive stress at the edges increases and the face plate buckles on the convex side near the unloaded edges; see Fig. 7e. Fig. 3 and Fig. 4 show that local buckling occurs at lower loads as the plate has larger initial deformation. Local buckling leads to additional reduction of sandwich plate stiffness in the post-buckling range.

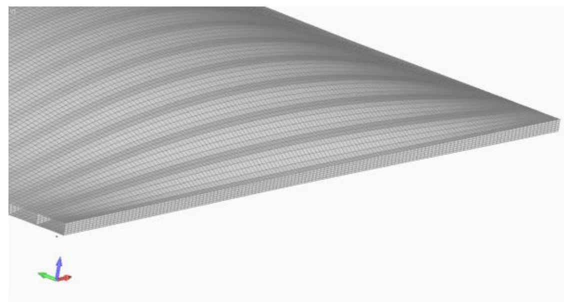
Load-deflection curves for the sandwich plate with thin faces (Case b) and 6% imperfection amplitude are presented in Fig. 8. The plate features a lower shear and in-plane stiffness coefficients than the one with 2.5-mm faces; see Table 1. They result in lower bifurcation buckling load and post-buckling stiffness, respectively; see Fig. 8a. Due to thinner face plates, local buckling occurs at lower load level; see Fig. 8b. The correspondence between 2-D and 3-D is excellent until the occurrence of local buckling.



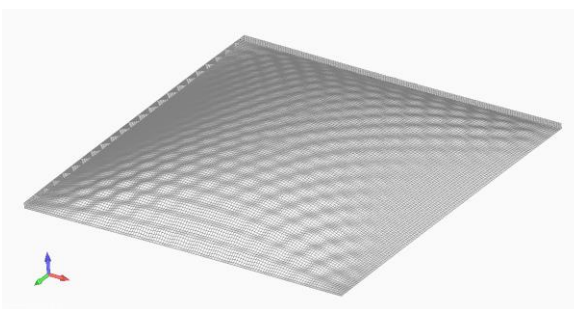
a) $N/N_0 = 1.34$



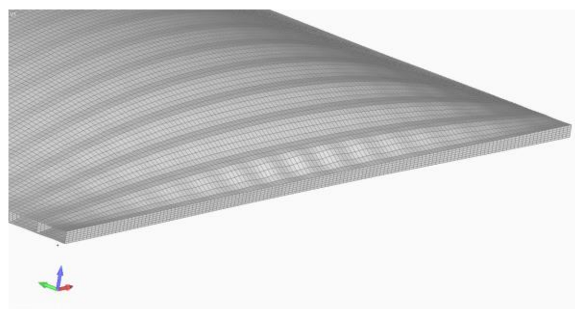
b) $N/N_0 = 1.35$



c) $N/N_0 = 1.35$



d) $N/N_0 = 1.44$



e) $N/N_0 = 1.44$

Fig. 7. *SS-free sandwich plate with 2.5 mm faces (Case a) and 6% initial deformation at different load levels.*

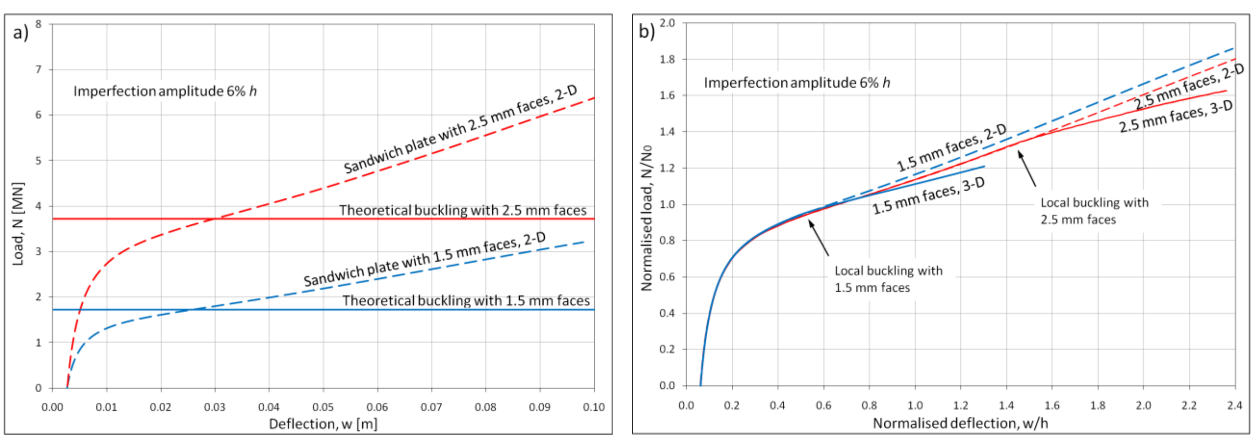


Fig. 8. *Load-deflection curves for SS-free sandwich plates with 2.5-mm- and 1.5-mm-thick face plates.*

3.4 Influence of boundary conditions

Fig. 9 presents the load-deflection curves for the standard web-core sandwich plate (Case a) and the isotropic plate with different boundary conditions. Comparing the curves for SS-free from Fig. 3 with SS-straight and SS-fixed from Fig. 9a, it is visible that more rigorous restraint of the unloaded edge results in higher post-buckling stiffness of the sandwich plate, as well as the isotropic plate; however, the stiffness of the sandwich plate is always lower than that of the isotropic plate. The same effect is present for clamped edges; see Fig. 9b, where the relative difference in stiffness between the sandwich plate and isotropic plate is even higher than for simply supported edges. Note that the clamped case with unloaded edges that are unable to move (CC-fixed) is not presented since the plate buckles locally first.

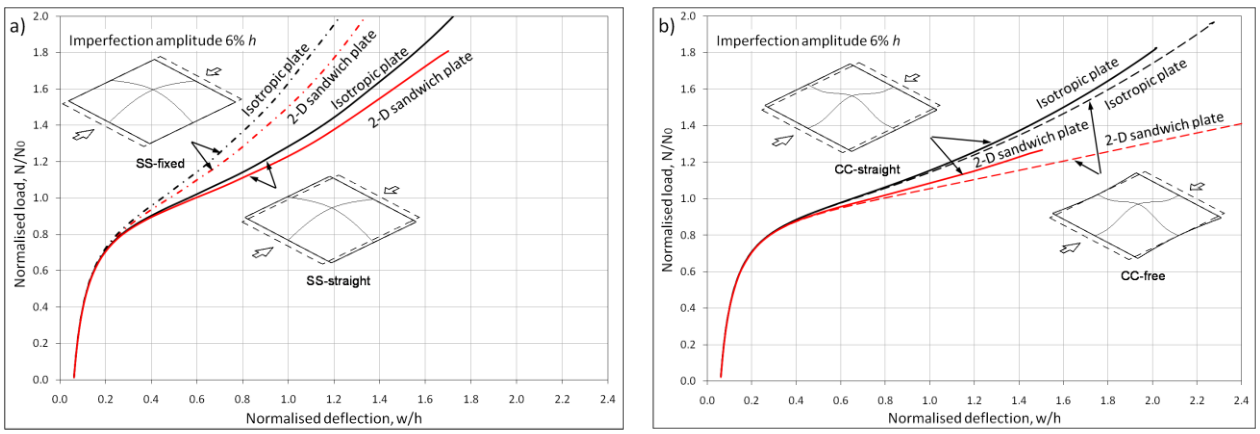


Fig. 9. *Load-deflection curves for plates with (a) simply supported and (b) clamped boundary conditions.*

4 Discussion and conclusions

This study presents the load-carrying behaviour of web-core sandwich plates in compression. Different magnitudes of initial imperfection are considered. The behaviour is compared to the isotropic plate of the same bending stiffness in the loading direction, D_{11}. It is found that the load-deflection and the load-end shortening behaviour of sandwich plate is qualitatively equal to that of isotropic plate for deflections lower than the plate thickness. As the deflections increase, the stiffness of the sandwich plate reduces in comparison to the isotropic plate. In other words, the post-buckling stiffness of web-core sandwich plate is lower than of isotropic plate with the same bending stiffness. It is shown that this is due to lower in-plane stiffness, A_{11}, of the sandwich plate, coming from its low cross-sectional area, i.e. hollow construction. Consequentially, it is less efficient in carrying the membrane forces. Also, the effective width is reduced; the sandwich plate studied here showed 10% reduction at N/N_{cr} = 1.35. Other bending and shear stiffness coefficients affect the bifurcation buckling load but not the post-buckling.

Transverse shear stiffness, D_{Qy}, is the most detrimental on the buckling load. The sensitivity of buckling load on variations in D_{Qy} was shown in Jelovica et al. [7].

The local buckling of face plates occurs during the post-buckling response of the sandwich plate as a result of high in-plane stress although it is not present in the initial imperfection. This additionally reduces the structural stiffness of the sandwich plate. First, the concave side of the plate buckles locally. The same was reported by Hahn et al. [8] and Nordstrand [10] for slender corrugated board plates. The local buckling of concave facing adds to increased stress at the edges as a result of the deflection of the middle part of the sandwich plate. Shortly after, because of increased compressive stresses, the face plate on the convex side close to the unloaded edges also buckles. The local buckling occurs at lower loads as the extent of the initial deformation is larger or as the face plates get thinner. In this study, the occurrence of local buckling is seen as a deviation of the load-deflection curves with 3-D from the 2-D method late in the post-buckling region. The later uses the displacement field described in Romanoff and Varsta [24] and cannot represent buckles between the web plates. Nonetheless, until the point of local buckling, the two methods are in perfect agreement. Kolsters [3] studied the effect of foam-filling the core on the local buckling in web-core sandwich plates. The results showed that increase in foam density increases the local buckling stress of the face plates. On the other hand, adding the foam increases the transverse shear stiffness of the sandwich plates, especially D_{Qy}. This increases the global buckling strength. However, filling the large panels with foam with adequate mechanical properties and good adhesion is not trivial in practice. Therefore, these investigations are left for the future.

This study furthermore found that in-plane boundary conditions at the unloaded edge have an important role in sandwich plate post-buckling response. The same effect is recognised for isotropic plates; see e.g. Rhodes [15]. More rigorous restraint of the unloaded edge increases the membrane forces opposite to the loading direction. In such situation the A_{22} coefficient also becomes important. A_{22} as well as A_{11} are, for the same bending stiffness, typically lower in a sandwich plate and lead to lower post-buckling stiffness.

The local buckling should be included to the ultimate strength assessment when using the equivalent single layer theory. Byklum et al. [11] have done this for the plates with single sided stiffening. As the present study shows, the local buckling modes on the face and web-plates depend on the structural dimensions and the deformation state. This means that in contrast to [11], the stiffness coefficients in *ABD*-matrix can have variation in the panel. This extension is left for the future work. Furthermore, post-buckling strength of an isotropic plate is sometimes used in the design of stiffened plates, provided that the stiffener support system is strong enough to prevent the overall panel field buckling; see Hughes and Paik [25] and DNV [26]. There, the plates are allowed to buckle elastically and it is assumed that the supporting girders have adequate strength to retain safety of the structure. This means that the effective width

[23] of the buckled plate must provide enough stiffness to the girders. However, this study indicates that the effective width is reduced; 10% reduction is observed at N/N_{cr} = 1.35. This means that design philosophy which allows elastic buckling is not directly applicable for a sandwich plates. The post-buckling strength characteristics should be studied further. Additionally, future investigations should include material non-linearity effects at least until the point of yielding as is done in [11] for a stiffened plate.

Appendix A – Stiffness properties of sandwich plate

The extensional, extensional-bending, and bending stiffness matrices respectively are [24]:

$$\left([A],[B],[D_0]\right)=\int_{-h/2}^{h/2}[E]_i\left(1,d_i,d_i z\right)dz,\ i=\mathrm{t,c,b} \tag{2}$$

and the local bending stiffness of the face plates is

$$\begin{aligned}[D]_\mathrm{t}&=\frac{t_\mathrm{t}^{\,3}}{12}[E]_\mathrm{t},\\ [D]_\mathrm{b}&=\frac{t_\mathrm{b}^{\,3}}{12}[E]_\mathrm{b},\end{aligned} \tag{3}$$

where the distance from the mid-plane of the plate is

$$\begin{aligned}d_\mathrm{t}&=\frac{h}{2}-\frac{t_\mathrm{t}}{2},\\ d_\mathrm{c}&=z,\\ d_\mathrm{b}&=-\frac{h}{2}+\frac{t_\mathrm{b}}{2}.\end{aligned} \tag{4}$$

The elasticity matrix [*E*] of the face plates is

$$[E]_i=\frac{1}{1-\nu_i^2}\begin{bmatrix}E_i & \nu_i E_i & 0\\ \nu_i E_i & E_i & 0\\ 0 & 0 & G_i\left(1-\nu_i^2\right)\end{bmatrix},\quad i=\mathrm{t,b}, \tag{5}$$

while the core has the elasticity matrix

$$[E]_\mathrm{c}=\frac{E_\mathrm{w}t_\mathrm{w}}{s}\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}. \tag{6}$$

The shear stiffness in transverse direction is given as [13]:

$$D_{Qy} = \frac{12D_{\mathrm{w}}}{s^2\left(k_Q\left(\frac{D_{\mathrm{w}}}{D_{\mathrm{t}}} + 6\frac{d}{s}\right) + 12\frac{D_{\mathrm{w}}}{k_\theta \cdot s} - 2\frac{d}{s}\right)} \tag{7}$$

where k_Q is defined as

$$k_Q = \frac{1 + 12\frac{D_{\mathrm{t}}}{s}\left(\frac{1}{k_\theta^{\mathrm{t}}} - \frac{1}{k_\theta^{\mathrm{b}}}\right) + 6\frac{D_{\mathrm{t}}}{D_{\mathrm{w}}}\frac{d}{s}}{1 + 12\frac{D_{\mathrm{t}}}{D_{\mathrm{w}}}\frac{d}{s} + \frac{D_{\mathrm{t}}}{D_{\mathrm{b}}}} \tag{8}$$

and laser weld rotation stiffness, k_θ, is equal to infinity in this study; see Romanoff et al. [13] and Jelovica et al. [7].

The shear stiffness in the longitudinal direction is

$$D_{Q\mathrm{x}} = k_{11}^2\left(G_{\mathrm{t}}t_{\mathrm{t}} + G_{\mathrm{b}}t_{\mathrm{b}} + \frac{t_{\mathrm{w}}}{s}G_{\mathrm{w}}h_{\mathrm{c}}\right), \tag{9}$$

where

$$k_{11} = \sqrt{\frac{1}{A\left(\sum_i \int \left(\frac{\tau_i}{Q_{Q\mathrm{x}}s}\right)^2 t_i \mathrm{d}s_i\right)}}, \quad i = \mathrm{t,c,b}, \tag{10}$$

Appendix B – Bifurcation buckling solution

Symmetric web-core sandwich plate is a special type of orthotropic plate where stiffness coefficients A_{13}, A_{23}, D_{13}, D_{23} and B_{ij} are equal to zero. The exact buckling load N_0 per unit width of a simply supported plate that follows the first-order shear deformation theory is given by Reddy [16] and Robinson [21]. The expression is presented in closed form:

$$N_0 = \frac{c_{33} + \left(\frac{\alpha^2}{D_{Qy}} + \frac{\beta^2}{D_{Qx}}\right)\cdot c_1}{\alpha^2 \cdot \left(1 + \frac{c_1}{D_{Qx}\cdot D_{Qy}} + \frac{c_2}{D_{Qx}} + \frac{c_3}{D_{Qy}}\right)}. \tag{11}$$

The coefficients are:

$\alpha = m \cdot \pi / a$;

$\beta = n \cdot \pi / b$;

$$c_{33} = D_{11} \cdot \alpha^4 + 2\left(D_{12} + 2D_{33}\right)\alpha^2\beta^2 + D_{22}\beta^4;$$

$$c_1 = c_2 \cdot c_3 - \left(c_4\right)^2;$$

$$c_2 = D_{11} \cdot \alpha^2 + D_{33} \cdot \beta^2;$$

$$c_3 = D_{33} \cdot \alpha^2 + D_{22} \cdot \beta^2;$$

$$c_4 = \left(D_{12} + D_{33}\right) \cdot \alpha \cdot \beta.$$

Setting the shear stiffness to infinity, number of buckling half-waves in *y*-direction to one, and minimizing Eq. (11) with respect to *m*, gives the expression for isotropic steel plate buckling:

$$N_0 = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2 \frac{\pi^2 \cdot D}{b^2}, \tag{12}$$

which is used in typical rules for ship structural design [26], additionally simplified for high aspect ratios, *a*/*b*, to

$$N_0 = 4\frac{\pi^2 \cdot D}{b^2}. \tag{13}$$

Bifurcation buckling load for the plates with SS-free boundary condition is presented in Table B.1. It shows the buckling load obtained with analytical solution, 2-D and 3-D analysis method. The agreement between these three approaches is quite satisfactory.

Table B.1. *Comparison of bifurcation buckling load with analytical solution, 2-D and 3-D analysis method for SS-free boundary condition. The percentage in brackets is the difference from analytical solution.*

	Bifurcation buckling load, N_{cr} [MN]		
	Sandwich plate		Isotropic plate
	2.5 mm faces	1.5 mm faces	
Eq. (10) or Eq. (12)	3.72	1.72	7.83
2-D	3.72	1.72	7.83
3-D	3.78 (1.6%)	1.75 (1.7%)	-

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