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Energy Landscapes in Random Systems, Driven Interfaces, and Wetting

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We discuss the zero-temperature susceptibility of elastic manifolds with quenched randomness. It diverges with system size due to low-lying local minima. The distribution of energy gaps is deduced to be constant in the limit of vanishing gaps by comparing numerics with a probabilistic argument. The typical manifold response arises from a level-crossing phenomenon and implies that wetting in random systems begins with a discrete transition. The associated "jump field" scales as $\langle h \rangle \sim L^{-5/3}$ and $L^{-2.2}$ for $(1 + 1)$ and $(2 + 1)$ dimensional manifolds with random bond disorder.

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The physics of systems with quenched disorder is related to the energy landscape. The free energy is at low temperatures governed by zero temperature effects, which in turn are ruled by the scaling of the disorder-dependent contribution. Random magnets, as spin glasses and random field systems, flux line lattices in superconductors, and granular materials are examples of physical systems in which frustration and disorder play an important role. Disorder may dominate also in nonequilibrium conditions, like driven systems (domain walls in magnets, flux lines in superconducting materials). In that case temperature-driven dynamics (creep, aging) and the external drive change the system from one metastable state to another [1,2].

A lot of information about energy landscapes is contained in how the number of local energy minima and the typical scale of their energy differences scale with system size, L [3]. This can be interpreted in a geometric fashion in that one compares the energy difference of two states with their overlap in terms of the spin configuration (as for magnets). In spin glasses, an intense debate still goes on as to whether in the thermodynamic limit the thermodynamic state is trivial ("droplet" picture [4]) or not (as in the "replica symmetry breaking" picture [5]).

Consider now the problem of the energetics of D dimensional elastic manifolds in random media [6–9], of which the best-known case is a directed polymer (DP) in a random medium with $D = 1$, often called a "baby spin glass" [10]. For these systems the interface energy is proportional to the area, and the sample-to-sample energy fluctuations scale with the exponent θ ($\theta = 1/3$ for a DP in $d = D + 1 = 2$ embedding dimensions). The geometry is often self-affine, characterized by a roughness exponent ζ ($2/3$, when $d = 2$). In the simplest energy landscape, the valleys and excitations are separated by energy gaps proportional to l^{θ}, where l is the length scale of the perturbation.

Here, the susceptibility of elastic manifolds is studied in the presence of weak fields numerically and by scaling arguments. By investigating each sample separately, we explore the changes in the energy landscape with applied fields. These lead to discrete "jumps" in the physical configuration. As a consequence, scaling arguments of wetting in random systems do not work in the limit of weak fields if the original interface-to-wall distance is much larger than the interface roughness [11]. With preconditioned systems we obtain the detailed probability distribution of the energy differences (gaps) between local minima and the global one. We find that the average interface behavior can be explained with scaling arguments, but the susceptibility cannot, and it is directly related to the exact properties of the gap distribution. Thus, the detailed statistics of the landscape is important. This contradicts considerations for random systems that assume well-defined thermodynamic functions [12] and scaling arguments with a single parameter (L^{θ}). These findings agree with claims that the susceptibility of a DP to thermal perturbations or applied fields is anomalous [13–15]. The reason is that the response to a very weak field, say applied locally at the end point of a DP, is governed by rare samples. The disorder-averaged response differs from the typical one because the ground state can be almost degenerate with a local minimum. Likewise, numerical studies of $d = (1 + 1)$ DP susceptibility reveal aging phenomena reminiscent of real spin glasses [16,17].

The continuum Hamiltonian for a D dimensional elastic manifold [$\mathbf{x}$ is an internal coordinate and a z (scalar) displacement]

$$\mathcal{H} = \int d^D\mathbf{x}[\Gamma\{\nabla z(\mathbf{x})\}^2 + V_r(\mathbf{x}, z) + h(z)], \qquad (1)$$

with an elastic energy (Γ is the interface stiffness), and V_r is a random pinning energy [we use a random bond correlator, $\langle V_r(\mathbf{x}, z)V_r(\mathbf{x}', z')\rangle = 2\mathcal{D}\,\delta(\mathbf{x} - \mathbf{x}')\delta(z - z')$]. $h(z)$ couples the interface to an external perturbation; e.g., it describes a constant magnetic field H in Ising magnets with antiperiodic boundary conditions.

The Hamiltonian (1) describes also complete wetting in a random system, where $h(z)$ equals the chemical potential difference of the wetting layer and the bulk phase [11,18–20]. For h non-negligible, the wetting-inducing external potential competes with the tendency of the interface to win pinning energy. Assuming that these balance,

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the average interface-wall separation $\langle z \rangle$ becomes $\langle z \rangle \sim h^{-\psi}$, $\psi = \frac{1}{\tau+\kappa}$, where ψ is the depinning exponent. τ measures the scaling of the elastic and pinning energy and is given by $\tau = 2(1 - \zeta)/\zeta$, and κ is the scaling exponent of the external field $h(z) \sim z^{\kappa}$ (here we use $\kappa = 1$). For random bond systems $\tau = 1$ in $d = 1 + 1$ dimensions, and $\tau \simeq 2.9$ in $d = 2 + 1$ using the known bulk roughness exponent values $2/3$ and 0.41 in $d = 2$ and 3, respectively [6,21]. In $d = 2$ numerical simulations in random Ising systems indicate, in agreement, $\psi \simeq 0.5$ [18,19].

A network flow algorithm, invented by Goldberg and Tarjan [22], is used here for the numerical procedure. It solves the minimum-cut–maximum-network-flow problem, and produces in polynomial time the exact ground state energy and interface configuration given a sample ($L \times L_z$ or $L \times L \times L_z$) with fixed quenched disorder. L_z is the z-directional system size. The algorithm is convenient when one makes systematic perturbations to the original problem ($h = 0$) [23,24]. Figure 1 illustrates the sample-to-sample behavior, as the external field $h(z)$ is switched on slowly [see Eq. (1)]. At $h = 0$ the interface is in the ground state. It has a mean wall distance $\bar{z}_0$ and a width $w \sim L^{\zeta}$ in a system of transverse size L_z. As the field is increased the interfaces move intermittently with jumps to positions $(\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_n, \ldots)$ [25]. This corresponds to a *first-order transition*. Instead of finite-size excitations the first change in the interface configuration is a macroscopic jump with zero overlap between the old and new states. The first transition point defines a jump field h_1. It assumes the role of a latent heat, and corresponds to the landscape-dependent energy to move the interface.

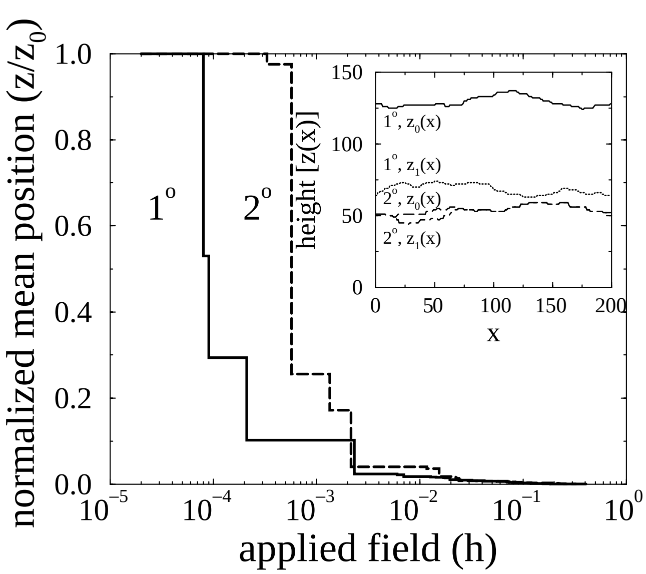

FIG. 1. Overview of two realizations of changes in mean heights $\bar{z}$ of interfaces normalized by their original (global minimum) positions $\bar{z}_0$ vs applied field h for (1 + 1) dimensional systems. Note the large jumps in both cases. $L^2 = 200^2$. $J_{ij,z} \in [0, 1]$ uniform distribution and $J_{ij,x} = 0.5$ (random bond disorder). The expected scenarios (bubble formation, jump to the lower edge of the system) before and after the first moves from global minima $z_0(x)$ to $z_1(x)$ are shown in the inset.

The two possible mechanisms are compared in the inset of Fig. 1. Either the interface adjusts itself gradually by forming "bubbles" or local excitations, or it jumps completely (compare with the main figure). The scenarios are linked to the structure of the energy landscape. If the first excitation is localized and has the transverse spatial extension Δ ($l \simeq \Delta^{1/\zeta}$) [14], the energy cost scales with $\Delta^{a/\zeta}$ and the energy win in the field scales with $h_1 \Delta^{1+(d-1)/\zeta}$. Assuming that $a = \theta$ the jump field $h_1 \sim \Delta^{\bar{\alpha}} = \Delta^{\theta/\zeta - 1 - (d-1)/\zeta}$. The exponent is negative, and thus small excitations are the more expensive ones [26]. Numerically, the fraction of jumps leading to a nonzero overlap with the ground state decreases towards zero slowly with L. Also, the scaling function of the interface jump lengths approaches a constant shape. The mean jump length ($\Delta z_1 = \bar{z}_0 - \bar{z}_1$, $\bar{z}_1 < \bar{z}_0$) scales extensively, $\Delta z_1 \sim L_z$, not with, e.g., L^{ζ}.

So, for small fields h and $L^{\zeta} \ll L_z$ the sample-to-sample fluctuations lead to a *discrete* (wetting) transition. The average behavior with $\langle z(h) \rangle$ and typical interface behavior with $\bar{z}(h)$ do not coincide, since the asymptotic $h \rightarrow 0$ limit is dominated by the near degeneracy of the ground state. In the limit $L^{\zeta} \ll L_z$, there are many independent "valleys" in the energy landscape for directed surfaces. Each of these has an energy E_n corresponding to a local minimum and their energy difference to the ground state (with E_0) is expected to scale as with two independent sets of disorder. That is $E_n - E_0 \sim L^{\theta}$. This energy difference equated with the jump energy $h_1 L^D \Delta z_1$ leads (with the choice $L_z = L$) to the scaling:

$$h_1 \sim L^{\theta - d} = L^{-\alpha}. \tag{2}$$

The jump field exponents are $\alpha = 5/3$ and $\alpha \simeq 2.18$ in $d = 2$ and $d = 3$ random bond systems, respectively. In $d = 3$ random field interfaces have $\alpha = 5/3$ ($\zeta = 2/3$ and $\theta = 2\zeta + D - 2$ [7,8]). It is assumed that $\Delta z_1 \sim L$, since the valley energies are independent, except for the bias caused by the field h. Figure 2 compares the exponent values to numerical data with only the nonoverlapping jumps being considered (without this pruning the same exponent is obtained asymptotically). For $D = 1$ α becomes 1.62 ± 0.04, close to the scaling estimate of $5/3$. The inset shows the disorder-averaged jump distance $\langle \Delta z_1 \rangle$ vs L and shows that the interface response geometry scales linearly with L (as discussed above). For $D = 2$ random bond manifolds we obtain $\alpha \simeq 2.2$, in reasonable agreement again. In the limit $\langle z_n(h) \rangle \simeq \bar{z}_n(h) \simeq w \sim L^{\zeta}$ (after n jumps of sizes $\Delta z_n = \bar{z}_{n-1} - \bar{z}_n$) the mean-field wetting theory applies, and indeed we obtain for the depinning exponent for $d = 2$ $\psi \simeq 1/2$, and for $d = (2 + 1)$ $\psi \simeq 0.26$, in rough accordance with the Lipowsky-Fisher [11] prediction. In $d = (2 + 1)$ there are deviations including a dewetting transition for weak disorder [21] and the exponent converges very slowly ($\bar{z}_0 \simeq w \sim L^{\zeta}$ at $L \simeq 10^4$ if $L_z = 50$ [27]).

If the initial interface position is random, the jump statistics are an average over the initial number of available

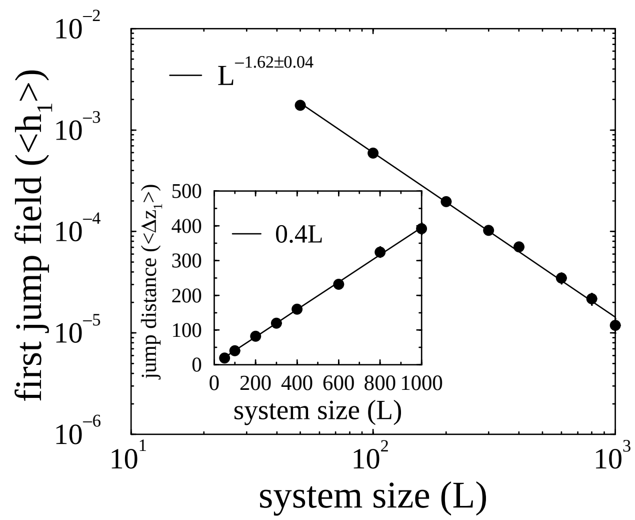

FIG. 2. Finite-size scaling of the average first jump field $\langle h_1 \rangle$ for one dimensional DP's. The line is the least squares fit to data. The scaling argument gives $\alpha = 5/3$. The inset shows the average jump distance $\langle \Delta z_1 \rangle$ at the corresponding field h_1 with a linear fit to data. $\langle\ \rangle$ is the disorder average over $N = 1000$ realizations for the system sizes $L \times L_z = L^2 = 50^2$ and 100^2, $N = 500$ for $L^2 = 200^2$–400^2, and $N = 200$ for $L^2 = 600^2$–1000^2. The disorder is of random bond type.

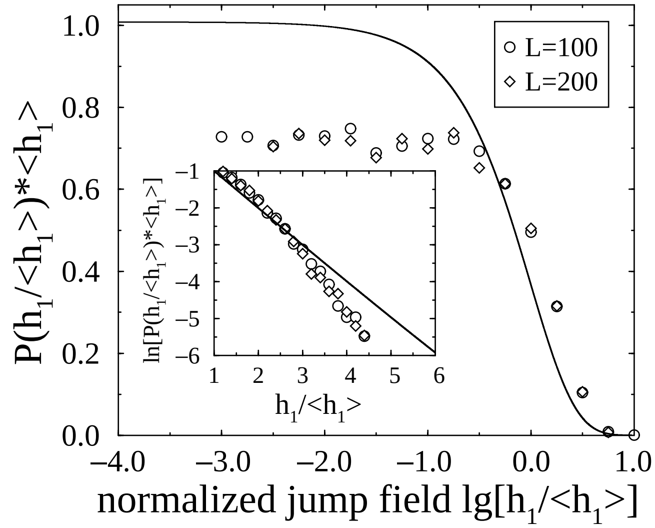

FIG. 3. The scaling function of the probability distribution $P(h_1/\langle h_1 \rangle) \times \langle h_1 \rangle$ for the first jump field values h_1 normalized by their disorder average $\langle h_1 \rangle$ in a (10-base) semilogarithmic scale for the system sizes $L \times L_z = L^2 = 100^2$ and 200^2. The inset shows the tails in the natural-log scale. The initial global minimum position $\bar{z}_0/L_z = \text{const}$ for all L. The number of realizations $N = 10^4$ for both system sizes. The line is the analytic result from Eq. (3) with a uniform distribution $\hat{P}(x)$ and $N_z = 20$.

valleys (recall that the field breaks the up-and-down symmetry, see Fig. 1). Thus, we also consider the limit in which the initial position is set to be inside a fixed-size window, $\bar{z}_0/L_z \simeq \text{const}$. We expect that the number of local valleys in the landscape, accessible with $h > 0$, has a well-defined average (in the grand-canonical sense), and that the relevant scaling parameter is L_z/L^ζ. Figure 3 shows the scaling function of the probability distribution $P(h_1)$ obtained with this initial condition. We find the form $P(h_1/\langle h_1 \rangle) = A(L) f(h_1/\langle h_1 \rangle)$, where A depends on the energy gap scale L^θ, and f is a scaling function with the limiting behaviors $f(x) \rightarrow 1$, $x \rightarrow 0$ and $f(x) \sim \exp(-ax^\beta)$, $x > 1$, $\beta \simeq 1.3$. The distribution is constant for small fields and has an almost exponential cutoff. The scaling properties imply, in particular, that the disorder-averaged susceptibility *diverges*. The change in magnetization is given by the number of interfaces that have moved times the mean distance $\langle \Delta z_1 \rangle$. Thus the divergence is not $\chi_{\text{tot}} \sim L^3$ [12]. Figure 4 shows the average jump field in the fixed height ensemble with varying L_z and constant L. We have fitted the data with $\langle h_1 \rangle \sim L_z^{-\gamma}$, and the best fit is obtained by the scaling exponent $\gamma \simeq 4/3$.

Consider now the energy landscape for small h. It has $k = 1, \ldots, N_z$ associated minima ($N_z \sim L_z/L^\zeta$) with the energies E_k picked out of an associated energy gap probability distribution $\hat{P}(\Delta E_k)$, where $\Delta E_k = E_k - E_0$ and E_0 is the ground state energy. When $h > 0$, all the local minima attain an energy of $E_k + h\Delta z_k$ with respect to the reference state with $\bar{z}_0$ and E_0. Now we make the assumption, analogous to the random energy model [28], that all the gap energies ΔE_k are independent random variables. We can now simply compute the probability for the original ground state being stable for any h (i.e., no jump has taken place) by the joint probability P_0 that all the $E_k + h\Delta z_k$'s are still higher than the original one with the given h. $-\partial P_0/\partial h$ gives then the probability that this *level crossing* occurs at exactly h. By computing

$$\frac{\partial P_0}{\partial h} = -e^{-\int_1^{N_z}\int_0^{kh/N_z}\hat{P}(x)\,dx\,dk} \times \int_1^{N_z} \frac{\hat{P}(kh/N_z)}{1 - \int_0^{kh/N_z}\hat{P}(x)\,dx}\,dk\,, \qquad (3)$$

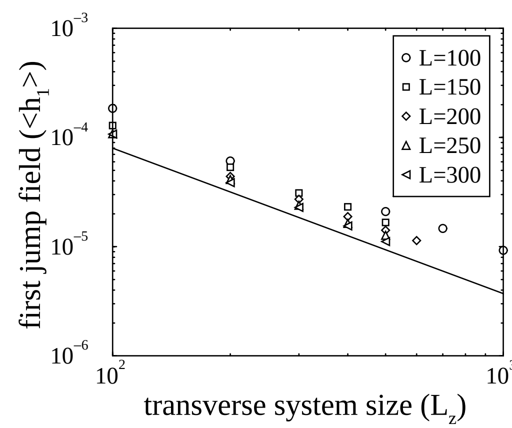

FIG. 4. The disorder average of the first jump field $\langle h_1 \rangle$ as a function of transverse system size L_z for the system sizes $L = 100$, 150, 200, 250, and 300, each with $\bar{z}_0/L_z = \text{const}$. The number of realizations ranges from $N = 500$ for $L = 300$, $L_z = 500$ to $N = 2600$ for $L = 200$, $L_z = 600$. The line $L_z^{-\gamma}, \gamma = 4/3$ is a guide to the eye.

one can show that the only $\hat{P}$ that reproduces the numerical $P(h_1)$ is a *constant* one, whereas all other functional forms of $\hat{P}$ fail (see Fig. 3). This $\hat{P}$ is in fact exactly the marginal one needed for the susceptibility per spin $\chi = \lim_{h\to 0}\langle \partial \bar{z}/\partial h\rangle$ to diverge in the thermodynamic limit. In particular, for a distribution $P(h_1)$ that vanishes in the zero field limit, the susceptibility would stay finite. Using the obtained form for the probability distribution gives $\chi \sim L^{\theta}(\frac{L_z}{L^{\zeta}})^{\gamma}$, where $\gamma \simeq 1 - \zeta$ relates to the density of valleys. This slightly disagrees with the above result ($\gamma \simeq 4/3$) since with $L = \text{const}\, \chi \sim L_z^{\gamma}$, $\gamma = 1$. In the isotropic limit $L \propto L_z$ the extensive susceptibility simply reads $\chi_{\text{tot}} = L^d \chi \sim L^{d+1+\theta-\zeta} \sim L^{2D+\zeta}$. In conclusion, χ (or χ_{tot}) is determined by the exact low-energy properties of $\hat{P}$, or by the rare events in the low ΔE tail.

In summary, we have studied the coupling between the energy landscape structure and the response of interfaces, related, for instance, to complete wetting. A disorder averaging that reflects correctly the level-crossing character of the problem reveals that the wetting starts with a discrete transition. Thus the randomness of the energy landscape drives a second-order transition to a first-order one. The jump is associated with an effective specific heat, which can be understood in terms of scaling arguments. The susceptibility is governed by the infrequent cases with low-lying local minima, which allows us to derive a constant energy gap probability distribution. The results should be relevant for other problems such as flux line lattices in superconducting materials with quenched randomness [1]. It will also be of interest to see if the energetics and the geometrical character of the response can be coupled with arguments concerning the energy barriers in each specific configuration [29]. This would allow one to understand the dynamics in the creep regime, when the interface moves between metastable states.

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