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Acoustic Emission from Paper Fracture

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We report tensile failure experiments on paper sheets. The acoustic emission energy and the waiting times between acoustic events follow power-law distributions. This remains true while the strain rate is varied by more than 2 orders of magnitude. The energy statistics has the exponent $\beta \sim 1.25 \pm 0.10$ and the waiting times the exponent $\tau \sim 1.0 \pm 0.1$, in particular, for the energy roughly independent of the strain rate. These results do not compare well with fracture models, for (brittle) disordered media, which as such exhibit criticality. One reason may be residual stresses, neglected in most theories.

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A simple, everyday statistical physics experiment is to tear a piece of paper into two parts. During this process noise is heard [1], evidently originating from damage to the paper or the propagation of a crack. It is easy to make the fracture surfaces "rough" as told by the naked eye without any more sophisticated analysis, and the experience is that it is difficult to do the tearing so that the cut is at all close to straight. Meanwhile, one can confirm the existence of disorder in the "sample" by just looking through the sheet against any reasonable light source. The paper looks cloudy, which comes to a large degree from the local fluctuations of areal mass density. This tells that elastic and fracture properties vary locally. Such a test poses the related questions: how does a crack develop in an inhomogeneous material, and what kind of properties does the acoustic emission have?

In fracture there is evidence of scaling properties familiar from statistical physics. The roughness of cracks, as measured by, e.g., the root mean square height fluctuations vs length scale, can often be described by self-affine fractal scaling [2–4]. Another such quantity is the distribution of strength, and its average versus sample size. In particular for (quasi)-brittle materials, the average strength can follow scaling laws, if the sample size is varied, which derive from the presence of disorder in the material [5–7].

Investigating such questions combines materials science and statistical mechanics. Two fundamental problems are the following: What is the role of disorder in fracture? and how do cracks develop and interact in real materials? These join forces in the failure of a notched three-dimensional sample, visualized by the advancement of an one-dimensional crack front or line in the presence of disorder [8,9]. It has an equation of motion, in an environment with varying properties as local strength and elastic modulus. The trail left by the linelike front is the two-dimensional crack surface, which can become rough (self-affine) if the propagating linelike object develops critical fluctuations. However, it is difficult to propose such a model that would match with the experimental data. For one thing, simple models tell that the disorder and microcracks interact in crack dynamics, so

that sometimes it is not correct to consider an isolated object like the front or a crack tip. From the materials research viewpoint the dynamics is interesting since it deals with the engineering quantities of strength and toughness, or the resistance to intrinsic flaws. For instance, in fiber and two-phase ceramic composites one can control these two by varying the mixing and constituents.

We study fracture in ordinary paper, as a two-dimensional disordered material, via acoustic emission (AE) analysis. The release of acoustic energy is related in paper to irreversible deformation, microcracks, and, perhaps, to plasticity. A large-scale analogy is earthquakes. Their energies are described by a power-law probability distribution, the Gutenberg-Richter-law [10] with an *energy exponent* β. Cracks in paper may be self-affine [11,12], with a roughness exponent close to that in a scalar two-dimensional fracture model, random fuse networks (RFN's) [13,14]. Both these are near 2/3, the one for surfaces of minimal energy [8,12,14]. Paper provides an example of crack growth in the presence of disorder. The probability distributions of the released energy and the temporal statistics, describing dynamical aspects, can be compared to models for two-dimensional failure, and to other work on AE.

Here the *strain rate* $\dot{\epsilon}$ is varied in the tensile test, to study possibly time-dependent effects in the fracture process. Our first main conclusion is that in strain-controlled tests power laws are found in the statistics, with either no or at most a weak dependence on the strain rate. The power laws do not follow the predictions of theoretical models outlined below, as fuse networks or mean-field ones. It is also apparent that an eventual localization of the crack [15,16] does not change the microscopic properties of the crackling noise, as long as the crack propagation is stable. This implies that paper failure is not a "phase transition" with a diverging correlation length, but leaves open the origins of the observed scalings.

To describe the interactions in an elastic medium with randomness incorporated one starts from mean-field stress sharing. In fiber bundle models (FBM) the applied

external force $F(t)$ is shared by all the $N(t)$ fibers, and as the (random) failure threshold of the currently weakest one is reached, the stress $\sigma(t) = F(t)/N(t)$ is instantaneously distributed among the remaining fibers. As a sign of criticality there is a divergent scale, the size of a typical avalanche which is made up of all the fibers that break down due to a single, slow time scale stress increment. The total avalanche size (N) distribution follows $P(N) \sim N^{-5/2}$ [17,18]. Thus global load sharing fracture resembles a second-order phase transition.

Local stress-enhancements can be added to FBM's, e.g., by considering fiber chains in which the stress from microcracks, of adjacent missing fibers, is transfered to the ones neighboring the crack. A more catastrophic growth results (with an exponent much larger than $5/2$), resembling a first-order phase transition since the elastic modulus has a finite drop at σ_c/ϵ_c [17,19,20]. A finite-dimensional, but scalar, approximation is given by random fuse networks [21]. RFN's reproduce with strong enough disorder many of the features of mean-field FBM's, in spite of including the stress enhancements and shielding from other microcracks [22,23]. The dynamics of crack growth is not understood well even in this simple model, including why the cracks seem to become self-affine, with a roughness exponent close to $2/3$ [14]. The models' event statistics can be compared with experiments by considering the energies. For the RFN, in an event the lost energy is $E_i \sim \Delta G_i \epsilon_i^2$, where $\Delta G_i \sim N_{\text{broken}}$ is the change in the "elastic modulus" due to the AE event, at ϵ_i. Simulations on RFN's for strong disorder that reproduces the FBM exponent ($5/2$) for the avalanche sizes yield the exponent $\beta_{\text{RFN}} \sim 1.8$ and the FBM value is $\beta_{\text{FBM}} \sim 2$ [24].

Experiments on, mostly, three-dimensional systems [15,16,25–27] have yielded power laws for the AE energy release; the typical exponent β is $1 \cdots 1.5$. One idea is that approaching final failure resembles a phase transition: in the Lyon group's stress-controlled experiments indications were seen of a critical energy release rate [15]. This means that the AE energy would diverge like a power law in the proximity of a critical point, the sample strength σ_c. Another problem is the nature of the disorder in the material: whether quenched disorder is able to give such power-law statistics for the energy [22,25].

Normal newsprint paper samples were tested in the machine direction [28] on a mode I laboratory testing machine of type MTS 400/M. Because of the lack of constraints the samples could have out-of-plane deformations, and none of the three fracture modes (I, II, III) is excluded on the microscopic level. The deformation rates $\dot{\epsilon}$ varied between 0.1 %/min and 100 %/min. The AE system consisted of a piezoelectric transducer, a rectifying amplifier and continuous data acquisition. The time resolution of the measurements was 10 μs and the data acquisition was free of dead time. The stress was measured simultaneously to AE with a time resolution of 0.01 s. We made 20 identical repetitions for statistics.

The 100 by 100 mm samples had initial notches (size 15 mm) to achieve stable crack growth. Typical sheet thickness is about 100 μm. The fracture statistics are not affected by such a notch [24]. For these strain rates the sound velocity time scale is much faster than that implied by $1/\dot{\epsilon}$. Each individual test contributes, at most, 1000–2000 events, so we can look at only integrated probability distributions and not in detail at local averages of, e.g., the acoustic event size vs ϵ.

Figure 1 shows an example of two tests under strain control. Stress-strain curves have typically three parts: prefailure (almost) linearly elastic one, the regime close to the maximum stress where the the final crack starts to propagate or is formed, and a tail that arises due to the cohesive properties of paper which allow for stable crack propagation. The faster the strain rate the less is the role of the tail. For the smallest strain rate most of the AE originates from tail (more than 90%) while for the highest the situation is the opposite. Quantities of interest are the statistical properties before the maximum, after it, and the integrated totals, in particular, the energy distributions. The time series of events allows one to make qualitative observations of the correlations between subsequent events and to draw conclusions about the event properties as such. For this rather brittle paper grade, and the strain rates used, the elastic modulus is independent of $\dot{\epsilon}$, and by AE we are able to detect a constant fraction of the elastic energy [24].

Figure 2 shows the scaling of the energy for a fixed strain rate. The behavior is power-law-like, with several orders of magnitude of scaling. The same exponent fits all three different cases: the pre- and postmaximum stress cases, and the sum distribution. If the strain rate is varied the same conclusion holds and the exponent only fluctuates at random [24]. We have $\beta = 1.25 \pm 0.10$, in disagreement with the fiber bundle ones and with that from the fuse networks, though β_{RFN} is closer than β_{FBM}. The practical implication is, that the material can withstand
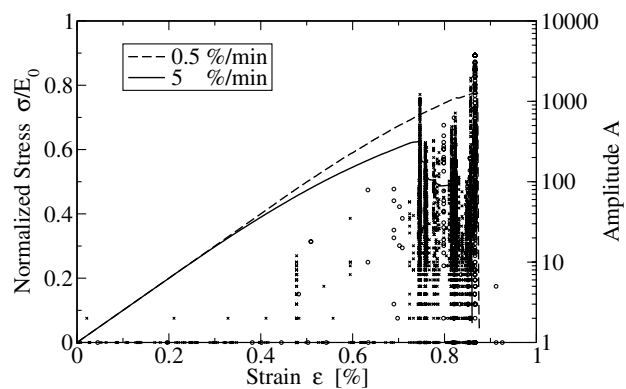

FIG. 1.   The acoustic data series (crosses and circles) and the stress-strain curves, for two strain rates. There is a difference in the post-stress maximum part of the stress-strain curve, which is more important for $\dot{\epsilon}$ 0.5 (%/min) (solid line and crosses) than 5 (%/min) (dashed line and circles).
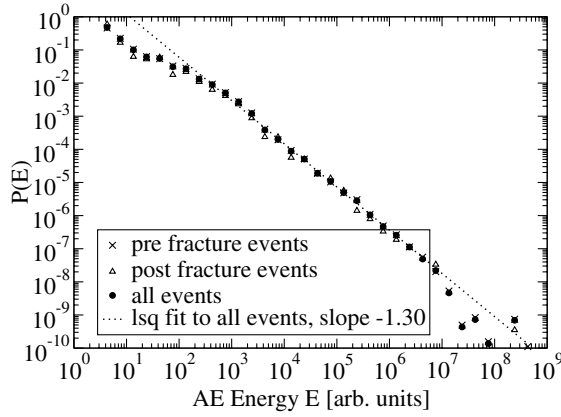
FIG. 2. The three (post/pre/total) energy distributions, $\dot{\epsilon}$ 1.0 (%/min) (for which the post/pre-contributions to AE energy are roughly the same).

FIG. 3. Plots of the event interval τ (waiting time) distributions, for three strain rates.

more damage than expected since $\beta < \beta_{\mathrm{models}}$. In paper, the postmaximum events correlate with the advancement of the final crack. The remaining ligament length/width contracts, thus the elastic modulus drops should (assuming a constant stress state) here be qualitatively related to AE event energies (see also [29]).

In an experiment with a varying strain rate $\dot{\epsilon}$ both the event durations and the waiting time between any two events may (consider Fig. 1) depend on $\dot{\epsilon}$. If the crack dynamics becomes "fast" then events take place on time scales set by the sound velocity. This establishes a time scale $t_s = \Delta x / v_s$ where Δx is the spatial separation of two events and the sound velocity $v_s \sim 2 \times 10^3$ m/s. In a sample of linear size 0.1 m this results in a maximum $t_s \sim 10^{-4}$ s. In the failure of brittle carbon foams the eventual critical crack growth is dictated by such fast events [16].

In quasistatic fracture models the dynamics of cracks is assumed such that the stress field is equilibrated infinitely fast during microfracture events, between further adiabatic increases of strain or stress. Thus the only time dependence of any temporal statistics is in the average time interval, between AE events, proportional to $1/\dot{\epsilon}$. For the FBM model there are no correlations between the waiting times and event size or durations, except for the trend that the average waiting time decreases as σ_c is approached. It is "critical" and thus the energy release rate follows a power law close to σ_c. Waiting time results do not generally exist for more complicated models [30]. One can compare to the experimental signatures of the intervals between AE events and the integrated energy release rate integral $\int d\epsilon E_{\mathrm{event}}$. As noted above there is some evidence—from stress-controlled experiments— for a critical behavior for this quantity [15].

Figure 3 demonstrates waiting time τ distributions for different strain rates. There is a clear power law, whose exponent (τ) remains roughly the same for all the strain rates [31]. Importantly, this is true for the postevents regardless of the origin (before/after σ_c) of the majority of the AE energy. For the preevents there might be some
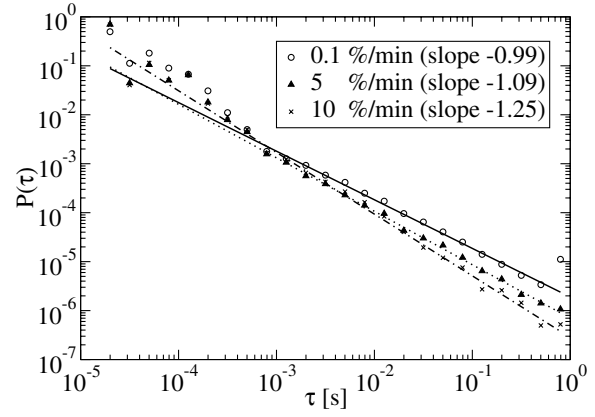
evidence of the exponent increasing with $\dot{\epsilon}$. In the time series of events, those with long durations are separated, on the average, by shorter intervals from the neighboring events before/after. Figure 4 depicts the waiting times prior to an event with two different data analysis methods for distinguishing between possibly overlapping events [24]. The interval separations are similar for both the post/pre-phases, implying that the microscopic failure dynamics does not differ, and that σ_c or ϵ_c cannot be inferred from the event characteristics. It may be so that events which are relatively long are precluded by longer waiting times. The durations of the events δt and the sizes are roughly power-law related as $\langle E \rangle \sim (\delta t)^3$.

The integral $\int d\epsilon E_{\mathrm{event}}$ demonstrates a rapid exponential growth above a typical strain of $\epsilon \sim 0.5\%$ (Fig. 5). This originates from increasing event sizes, not from an increasing density vs strain. For $\epsilon > 0.6\%$ the samples start to fail. In the regime where the exponential growth takes place the samples develop plastic, irreversible strain ϵ_{pl}, with a roughly exponential dependence on strain [24]. This does not imply that the AE measures plastic deformation work, mostly, since the rate of increase of ϵ_{pl} is much less than that of the energy integral. The
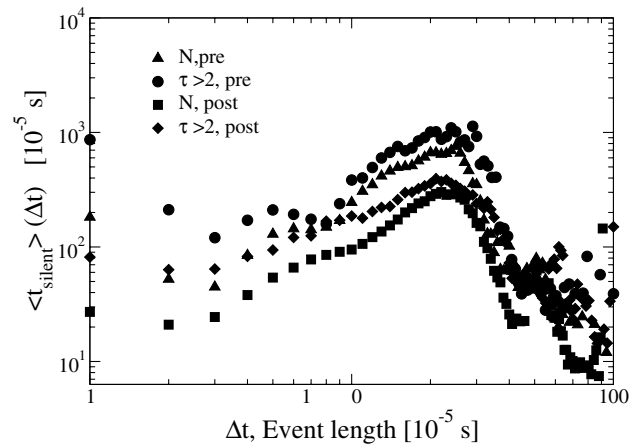

FIG. 4. The waiting time as a function of event length Δt, $\dot{\epsilon}$ 1.0 (%/min).
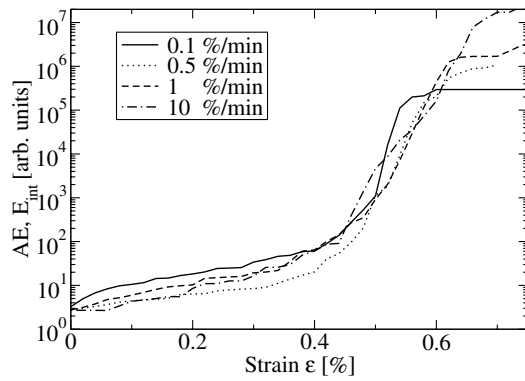
FIG. 5.   Increase of the integrated AE energy vs strain ϵ.

exponential strain dependence of the AE implies a typical length scale (as should be true for the development of plasticity), which in turn should be related to crack localization.

Concluding, failure of ordinary paper shows several features associated with critical phenomena. Our take on the experiment is that it show that (i) there is no clear sign of a "critical point," or a phase transition, in spite of the fact that the material is close to linearly elastic. In particular, while the event intervals and the event energies follow power-law-like statistics, not all the quantities do so. Also, (ii) the temporal behavior hints of complicated time-dependent phenomena not directly related to the fast relaxation of stress. A possible candidate is the viscoelastic nature of the wood fibers in paper, but note that the macroscopic stress-strain behavior remains almost linearly elastic while AE events already occur. We suspect the gradual release of internal stresses plays a role, perhaps due to frictional pullout of fibers from the network. Finally, (iii) the power laws as obtained are off those predicted by simple fracture models. It remains to be seen whether these models can be tailored closer to such tensile experiments [32]. One suggestion is that the dynamics of energy release during the events follows a different course from the model rules, in particular, finite-rate dynamics allows for stress overshoots [33,34]. The relation of the acoustic emission to why cracks get rough may be indirect. The latter could relate to the development of prefailure plastic deformation.

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