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Demand Forecasting in a Railway Revenue Management System

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Master's thesis

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Abstract of master's thesis

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Abstract

The research in Revenue Management has tightly focused on airline markets and somewhat neglected other similar markets. The purpose of this thesis is to offer an extensive overview on RM in the railway context. The backgrounds and concepts of RM are discussed and the applicability of RM in railway markets is evaluated. The differences between railways and airlines are also explored. I am especially focused on the demand forecasting process and different methods that can be used to forecast uncertain demand for a specific train. I also discuss how demand forecasting relates to other RM components, such as capacity allocation.

Relevant RM theories and demand forecasting methods are compiled based on the existing literature. Because of the limited availability of real demand data, I use hypothetical demand data to illustrate how different forecasting methods can be applied and how the performance of each method can be evaluated. I also compile an illustrative capacity allocation example using EMSR –model.

I conclude that the applicability of RM in railway markets is evident. I find four significant differences between railways and airlines that are relevant to RM. Railways tend to have more complex networks, less price differentiation, shorter booking lead times, and less competitive markets. Illustrative demand forecasting examples indicate that the evaluation of different forecasting methods is essential, since the performance of different methods might vary substantially, depending on the available data and the time horizon of desired forecast. Capacity allocation examples suggest that it is particularly important that demand forecasts would provide the accurate predictions of total demand and demand distribution between fare classes. However, it should be taken into account that the findings of illustrative examples cannot be generalized, since the hypothetical data was used in the analysis. Thus further examination with real demand data should be required. Additionally, the issues of constrained data and network effect are omitted from the analysis.

Keywords revenue management, willingness-to-pay, price differentiation, capacity allocation, demand forecasting, railways

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1 Introduction

Railways play important role in transportation infrastructure in Europe. The market environments in European railway markets are slowly changing in consequence of deregulatory actions by the European Commission (European Commission, 1991) and increased competition by the transportation substitutes, e.g. intercity buses. Because of these changes, railway operators are forced to improve the efficiency of their operation models. A business practice called Revenue Management has significant part in these efficiency improvements.

RM has its origin in the airline industry. The extensive development of RM started in the end of 1970s after the deregulation of airline markets in United States. In consequence of the deregulation, competition increased in the industry and compelled airlines to find new procedures in order to survive in the changed market environment. (Talluri & van Ryzin, 2004) The answer to airlines' problems was RM, which purpose was to enable airlines to manage their capacity as efficiently and profitably as possible (Kimes, 1989). McGill and van Ryzin (1999) summarize that the main objective of RM is to maximize firm's revenues. In order to maximize revenues, RM aims to answer multiple demand management decisions, such as decisions related to pricing and customer segmentations, by controlling prices and inventories. (Talluri & van Ryzin, 2004) The main concepts behind these controls are related to differences in customers' willingness-to-pay (WTP). Economic concepts of *product differentiation* and *price discrimination* can be considered as the main determinants of RM. (Belobaba, 1987) By way of these concepts firms are able to separate different customer segments based on customers' WTP and then use pricing and capacity allocation to maximize revenues from these customers.

Even if RM has its roots in the airline industry and the research has also strongly focused on that specific industry, RM has its opportunities in the other industries as well. Kimes (1989) state that RM is a tool for all capacity constrained service firms and that different RM techniques are appropriate when: 1) capacity is relatively fixed, 2) customers can be segmented based on their preferences, 3) product inventory is perishable, 4) products are sold in advance, 5) demand is uncertain and fluctuating over time, and 6) marginal costs per customer are low.

But although all previous conditions would hold, how could a firm determine the most efficient capacity allocation and pricing decisions? Like all business decisions, also RM decisions should be based on accurate forecasts. Thus forecasting plays extremely important role in RM. McGill and van Ryzin (1999) emphasize that all RM decisions are related to different forecasts. Particularly important are accurate forecasts of customer demand, since they form the basis for capacity allocation and pricing decisions. In addition to that forecasting is eminently important it is evidently extremely difficult. Demands for each product change constantly due to exogenous factors but also because of firm's own actions. These changes can be uncertain, which makes it complicated to forecast future demand. (Zeni, 2001)

As the most of the researchers studying RM and RM demand forecasting have focused on the airline industry, the objective of this thesis is to investigate how RM and especially demand forecasting techniques could be applied in the railway industry. Compared to airline RM, railway RM lacks academic research and publications. Armstrong and Meissner (2010) are one of the few who consentrate solely on the railway RM. They give an extensive overview of railway RM and its models.

The motivation of why the focus of this thesis is on railway RM is related to three issues. Firstly, the impacts of RM on the airline industry have been significant. It is reported that RM applications have increased airlines' revenues by 4 to 6 % (Belobaba, 1987). Secondly, airline and railway markets appear to be quite similar in many perspectives and this begs the question of whether airline RM applications could be used in a railway context? Thirdly, there has occurred a lot of development in railway RM in the recent years. Finnish national railway operator VR has made serious changes in its pricing strategies. In 2013 VR adapted more demand based pricing system as a part of wider RM investments (VR Group, 2014). Additionally, the focus of this thesis is on demand forecasting, since it is presented to have such a significant role in RM (see e.g. McGill & van Ryzin, 1999).

In this thesis I will attempt to give answers to four research questions:

1. *What is Revenue Management?*
2. *How is RM applicable to railway markets?*
3. *How to compile accurate demand forecasts for a specific train?*
4. *How demand forecasting is related to capacity allocation?*

The first two questions are concerned in Sections 2 and 3 based on existing literature. Based on the literature I conclude that the opportunities of RM are evident in railway markets. Four significant differences between railway and airline markets relating to the applicability of RM are recognized. Railways tend to include more complex networks, less price differentiation, shorter booking lead times, and less competitive markets. Each of these factors should be taken into account in the planning and implementation of RM applications.

In illustrative examples in Section 5, I show how different forecasting methods could be used to forecast demand for a specific train. In order to show how capacity allocations are related to demand forecasting, capacity allocation for hypothetical train has also been compiled. Because both of the examples are based on the hypothetical data, any generalized conclusions cannot be drawn. However, examples indicate that the evaluation of different forecasting methods is essential, since the performance of different methods might differ substantially, depending on available data and the specifics of desired forecasts. Supporting how important is the selection of suitable method, it is presented that 10 % increase in forecast accuracy can increase revenues by even 3 % (Lee, 1990).

Examples also propose that the selection of appropriate forecasting method is impacted whether long-term or short-term forecasts are needed. Capacity allocation example suggests that two demand factors relating to demand forecasting are especially important. Firstly, demand forecasts should provide information on the distribution of total demand between fare classes. Secondly, it is essential to have accurate forecast of total demand itself.

The reminder of this paper is organized as follows. Section 2 gives an overview of RM. The basic backgrounds and concepts are covered and some relevant capacity allocation theories are presented. Section 3 deals with the relationship between RM and the railway industry. Differences between railways and airlines are discussed and a few examples of RM applications in railways are presented. In Section 4, the emphasis is on demand forecasting and different forecasting methods that have been used in RM. Section 5 contains illustrative examples of how demand forecasting methods and capacity allocation can be applied. Section 6 concludes the findings.

2 Revenue Management

All sellers of products or services face multiple essential questions related to selling and pricing. Which selling channel to use? How to set prices for those channels? How to segment customers? How to differentiate products to these segments? How to set prices for different segments? How to allocate the inventory to customer segments? How to change the price over time? All these questions relate closely to RM.

Kimes (1989) describes that RM is *"a method which can help a firm sell the right inventory unit to the right type of customer, at the right time, and for the right price"*. She says that the objective of RM is to answer these questions so that the revenue would be maximized. RM problem is then determining how much to sell at what price, to which market segment and when. Cross (1998) gives similar definition in economist's terms: *"a means of allocative efficiency that maximizes economic wealth through dynamically forecasting the self-seeking activities of each individual consumer."*

Talluri and van Ryzin (2004) present the same idea in a slightly different way. According to them, RM focuses on demand management decisions and methodologies and procedures to implement them. It consists of the methods of managing firm's interactions with the market, with an objective to maximize revenues. Additionally, Talluri and van Ryzin divide these demand management decisions further into three decisions; structural, price, and quantity decisions.

Structural decisions relate to questions like: Which selling format to use? Which segmentation and differentiation techniques apply? And what kinds of terms to include to purchases? Decisions like these are normally quite strategic and are performed infrequently. Structural decisions can also have effects on how a firm can react to price and quantity decisions. For example price advertising campaigns can limit firm's ability to adjust price decisions in short-term. (Talluri & van Ryzin, 2004)

While structural decisions are seen strategic, Talluri and van Ryzin (2004) separate two more tactical decisions areas. Price decisions are related to pricing; pricing different customer groups and pricing over time. Whereas quantity decisions try to answer questions related to the capacity controls and product rationing; capacity allocation to different customer groups and whether to accept or reject buying request of product in a particular price. When RM uses capacity allocation decisions as the main tactical tool for managing demand decisions, it is said to be *quantity-based RM* and while using prices based on these decisions, it is called *price-based RM.* These two distinct strategies of RM are actually tightly connected. The quantity-based RM balances the number of units offered available at each price, while these prices are determined by the price-based RM decisions. So both strategies affect each other and they cannot be disentangled. Additionally, both of these strategies are based on the differences in customers' WTP. (Belobaba, 1987) This dichotomy and which of the strategies is primarily used, impacts on the theories and practices of RM.

2.1 Background and concepts

2.1.1 Origin in the airline industry

Researchers are widely agreed on the origin of the business practice nowadays called RM. The first capacity allocation applications were developed in 1950's when the first overbooking applications were implemented in the airline industry. These applications allowed airlines to overbook their flight, so that the probability of full capacity usage would be maximized. (McGill, 1989) In the early 1970s, some airlines started to offer restricted number of discounted tickets. For example, BOAC (predecessor of British Airways) provided low fares to passengers that booked at least 21 days prior a flight departures. However, the number of discounted tickets was not based on passenger booking behavior but more on some fixed rule, like protecting some percentage of capacity. (McGill & van Ryzin, 1999) Wider development of modern RM applications increased after the Air Deregulation Act in 1978 in the U.S. (see e.g. Talluri & van Ryzin, 2004; Kimes, 1989; Pak & Piersma, 2002; Walczak, Boyd, & Cramer, 2012). In 1978, the U.S. Civil Aviation Board (CAB) revoked restrictions of airline fares and schedules. Primarily this meant that airlines were free to set and change their prices and operate routes without approval from the CAB. Before the act, the CAB had regulated airline ticket prices based on standardized fares and profitability targets. (Talluri & van Ryzin, 2004)

Talluri and van Ryzin (2004) give a descriptive summary of how the deregulation act changed the industry. Ticket pricing became more complex, since airlines were able to set and change prices without restraints. Because scheduling was also liberalized, major airlines started to develop hub-and-spoke[1] networks, which made possible to extend the service networks, which in turn increased the complexity of pricing even more. In order to control more complex pricing structure, many larger airlines started to invest more in the development of computerized reservation systems (CRSs) and global distribution systems (GDS)[2].

Another significant change was that the liberalization of pricing enabled airlines to compete with prices. New types of airlines, low-cost and charter airlines, entered the market. Their strategy was to decrease fares by constructing simpler point-to-point operations, lowering labor costs and cutting down services on flights. They directed their services to customer segments with more price elastic buying behaviors, such as leisure travelers, students, and other irregular travelers who would otherwise use another type of transportation.

The change in pricing and competitive environment affected the performance of traditional airlines. They needed to find new strategies to recapture lost price sensitive passengers. Talluri and van Ryzin (2004) present that one of the first solutions for this problem was presented by Robert Crandall, the vice president of marketing in American Airlines at that time, who understood that an airline faces relatively fixed costs for operating a certain flight and thus a marginal cost of selling an additional seat is near zero. Because of this fixed costs structure, an airline could in fact increase revenues by selling surplus tickets to more price sensitive passengers in discounted prices. The problem was to identify these surplus tickets and also ensure that less price sensitive passengers would not switch to buy these discounted tickets. In 1978 American Airline started to offer fixed number of discounted tickets with purchase restrictions for each flight. Purchase restrictions were designed to prevent less price sensitive passengers from buying discounted tickets. Discounted tickets had to be purchased in 30 days before the departure, those were nonrefundable and required seven days minimum stay at the destination. This is an example of one of the first modern RM applications. Talluri and van Ryzin (2004) highlight that the link between RM and the airline industry is rather unique, since there are few business practices, which origins are so strongly connected to some specific industry. (McGill & van Ryzin, 1999)

[1] A configuration of an airline's network around one or more hubs, which work as a connection points in passengers' itinerary from origin to final destination. (McGill & van Ryzin, 1999)
[2] Computer and communication system that linked the CRSs of different airlines. (McGill & van Ryzin, 1999)

2.1.2 Basic concepts

As can be seen from the previous example of airline RM application, RM is highly related to the differences in customers' WTP and other product preferences. Both quantity and price decisions are connected to the heterogeneity of customer preferences. The economic concepts behind this are price discrimination and product differentiation, which are both tightly related to RM applications. (Belobaba, 2009)

Belobaba (2009) emphasizes the importance of understanding how these terms differ. Price discrimination refers to the practice where a firm can sell the same product or service at different prices to different customers based on their individual WTP (Phlips, 1983). This is indeed extremely generalized definition of price discrimination and the studies of economics commonly separate three different degrees of price discrimination. First-degree price discrimination (also known as perfect price discrimination) refers to firm's ability to recognize the WTP of each customer and then correspond the prices for each customer separately according to their WTP (Phlips, 1983). This degree is highly theoretical, because it is basically impossible to recognize customer's individual WTP. Third-degree discrimination occurs if a firm is able to divide customers into segments according to their WTP and then set the prices for each segment separately (Phlips, 1983). This is probably the most common form of price discrimination and for instance student discounts represent this degree of discrimination (Varian, 1989). Second-degree price discrimination is connected to the product differentiation. In second-degree discrimination, a firm can identify different customer groups and differences in WTP between these groups, but it would need to induce self-selection of customers. In order to achieve this self-selection, a firm can differentiate its products so that a customer would choose the product, which corresponds his or her individual preferences and WTP. (Alderighi, 2010)

The term product differentiation indicates firm's ability to charge different prices for products that contain small differences (Botimer & Belobaba, 1999). Thus the product differentiation is actually a technique, which allows a firm to induce second-degree price discrimination. Most RM applications include characteristics of second- and third-degree price discrimination. Again an example can be seen in the airline industry. The presence of product differentiation is evident, since airlines offer differentiated tickets for different prices. An example of product differentiation (or second-degree price discrimination) is classification between economy and business classes. (Alderighi, 2010) At the same time, airlines set different prices for tickets with similar characteristics, using for example advance purchase restrictions. These price differences cannot be explained by product differentiation, because the products (seats) are totally similar and differences in prices are only based on customer's WTP for a specific product, which indicates the presence of third-degree discriminatory pricing. (Belobaba, 2009; Talluri & van Ryzin, 2004) Belobaba (2009) uses the term *differential pricing* to indicate RM practices that use both price discrimination and product differentiation principles.

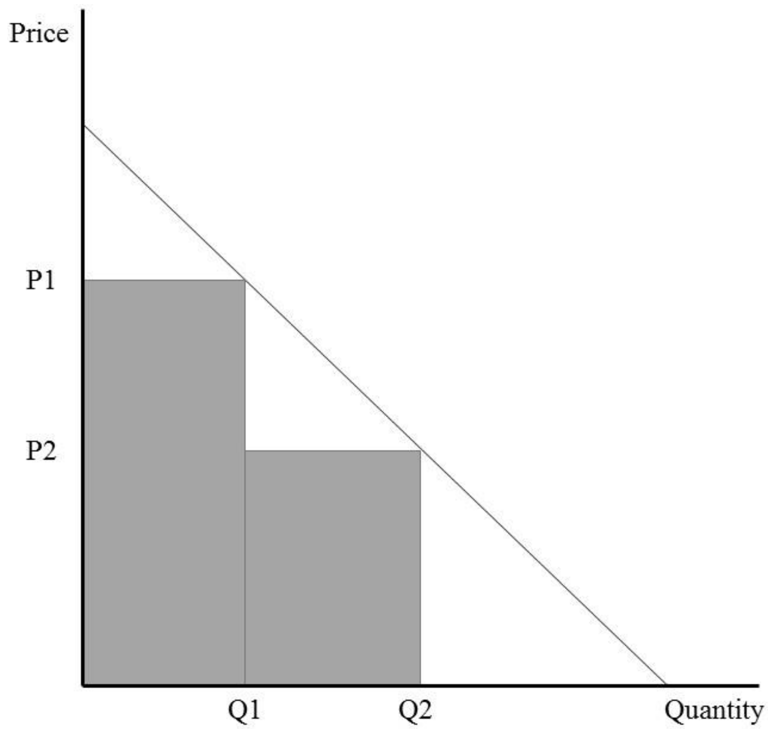

Figure 1 – Simple differential pricing model (Belobaba, 2009).

Figure 1 characterizes the basic concept of differential pricing. WTP is defined by the demand curve. Let's assume that a firm has a perfect ability to segment consumers according to their WTP. So the firm can use some price discrimination method to separate consumers into two groups and then offers different prices for these groups. Offering P1 for high WTP consumers, the firm can expect that Q1 consumers will purchase, because their WTP is equal or greater than P1. The method used to segment consumers is assumed to prevent the high WTP consumers to move purchasing P2 products and thus all high WTP consumers purchase in P1. Then offering P2 for low WTP consumers, the expected number of purchases in this lower price is Q2 – Q1, since these consumers' WTP is less than P1 but greater than P2. (Belobaba, 2009) This simple example demonstrates how the firm can increase its revenues by adopting differential pricing methods. The expected total revenue under the price differentiation is $P1 * Q1 + P2 * (Q2 - Q1)$, which is evidently larger than the expected revenue under any single price (e.g. with P1, the revenue would be P1*Q1). On the other hand, also consumers benefit from the differential pricing. Low WTP consumers purchasing P2 obviously benefit, as under single price e.g. P1, the price would be higher than their WTP and they would not consume at all. Also P1 consumers benefit if the single price without price differentiation would be higher than P1. (Belobaba, 2009)

This type of price differentiation is not feasible in all situations. There are few conditions that have to be considered. Firstly, the perishability of the product relates to the applicability of price differentiation. The economic concept called *Coase conjecture* states that if a firm sells durable good and if consumers are able to predict price changes and are patient enough, the firm loses its power to price differentiate and end up charging price that equals to the competitive price. (Coase, 1972) For example, if the product in Figure 1 is completely durable and consumers predict price changes, P1 consumers would end up to wait until the price drops to P2 and would make their purchases then. Secondly, there should be heterogeneity in consumers' preferences. If all consumers are exactly identical the price differentiation will lose its power to generate additional revenues. Third condition presumes that re-sales should be prevented. This can be achieved by prohibiting re-sales with contractual manners (an airline ticket includes passenger name) or because of natural reasons (health care services cannot be transferred to another person). The last condition relates to firms' pricing power. Under perfect competition, a firm cannot affect the prices and thus it is not capable of implementing price differentiation. (Talluri & van Ryzin, 2004)

Albeit differential pricing can be used to increase revenues, it is not enough to maximize revenues alone. Both high and low WTP consumers tend to favor same products and thus under the capacity limitations it would be preferable to sell more high priced products. Use of differential pricing methods (e.g. advance purchase restrictions) directs the low WTP consumers to buy before the high WTP consumers. So there is a risk that low WTP consumers will displace the high WTP consumers and decrease the positive revenue effect of differential pricing. (Belobaba, 2009) Belobaba (1987) points out that capacity allocation methods should be incorporated into the differential pricing in order to maximize expected revenues. A firm should set some limits to the offered numbers of products in each price class, so that revenues can be drawn also from the potential later-buying high WTP consumers. Section 2.2 presents theoretical models of how these limits are set.

Both of these concepts, differential pricing and capacity allocation, are involved in RM (Belobaba, 1987; Kimes, 1989). In the ideal environment, a firm could use these concepts to maximize total revenues by setting an optimal set of different prices and allocating the capacity in an optimal way according to the demand of each price category (Belobaba, 1987).

2.1.3 General conditions for using RM techniques

Considering the descriptions of RM and the background concepts, such like differential pricing and capacity allocation, there are several conditions for RM techniques to be appropriate. Kimes (1989) compiles the six general conditions for using RM techniques: 1) fixed capacity, 2) customer heterogeneity, 3) perishable inventory, 4) product sold in advance, 5) uncertain and fluctuating demand, and 6) low marginal costs. However, Talluri and van Ryzin (2004) suggest that these conditions are not completely restrictive, but would increase the effectiveness of RM.

1. Fixed capacity

Kimes (1989) states that the first condition relates to the capacity constraint. RM techniques are relevant only if a firm cannot (at least in the short-term) adjust its capacity to correspond current demand. Weatherford and Bodily (1992) amplify that totally fixed capacity is not necessary, but it is essential that adding additional capacity would cause high incremental costs. For example, an airline decides in advance what sort of an aircraft it will use in a particular flight. After the selection of airplane type has been done, the capacity can be considered fixed. The idea is that the airplane cannot be changed even if all seats on the flight are sold out, without significant change costs. This condition links to the sixth condition. (Kimes, 1989)

2. Customer heterogeneity

A firm implementing RM practices should be able to divide its customers into different types according to their preferences. These preferences can relate to WTP, preference for the different products or the differences in purchase behavior over time. (Talluri & van Ryzin, 2004) In the airline industry widely used manner is to segment customers using different ticket restrictions, for example advance purchase restrictions and restrictions related to cancellation and change policies. (Kimes, 1989). Time of purchase restrictions are particularly used in the airline industry and they are founded on the assumption that less price-sensitive passengers book their flights closer to the departure date (Weatherford & Bodily, 1992).

3. Perishable inventory

The perishability of inventory is a main factor that distinguishes service firms from many manufacturing firms. A relevant assumption for applying RM techniques is that the inventory perishes after a specific time. This means that the output in one period cannot be used to fulfill the demand in future periods (Talluri & van Ryzin, 2004). Weatherford and Bodily (1992) emphasize that if the inventory could be stored, RM techniques would not be needed, but inventory management approaches alone would solve the problems. For example, after an airplane departs, unsold seats cannot be inventoried and thus they represent wasted inventory. By minimizing the inventory spoilage, a firm can operate more efficiently. (Kimes, 1989) This is a common characteristic for all service providers; seats for a theater, a sport events, a restaurant or any other transportation mode (Weatherford & Bodily, 1992).

4. Product sold in advance

As mentioned, RM systems often use the time of purchase restrictions to differentiate products. A reservation system allows a firm to sell inventories in finite time horizon before the actual use. This will create some security because a firm knows that some of the capacity will be used in the future. On the other hand, it creates uncertainty, because a firm has to decide whether to accept early buying requests, which often means trading between certain low prices and uncertain high prices. (Kimes, 1989)

5. Uncertain and fluctuating demand

Demand for a service or product is not necessarily constant over time. Demand can vary seasonally, weekly, daily etc. and these variations occur with some level of uncertainty. The more fluctuating and uncertain the demand is, the more difficult the demand management becomes. Different RM techniques can be used to estimate some of the demand fluctuation and uncertainty, which would help firms to allocate capacity more efficiently and increase the revenue. (Kimes, 1989).

6. Low marginal costs

This condition links to the first condition, fixed capacity. For a capacity constrained firm the marginal capacity change costs are high. However for a RM system to be efficient, the marginal cost of providing additional unit of available capacity has to be relatively low. (Kimes, 1989; Weatherford & Bodily, 1992) Again the airline industry provides a good example; when an airline chooses the type of an airplane for particular flight, this does not only fix the capacity but also the total cost of service, because the total cost does not depend on how many passengers actually are on a flight. (Talluri & van Ryzin, 2004)

In addition to these six conditions Talluri and van Ryzin (2004) add that a firm applying RM also needs different information systems to collect and store data related to demand and customer behavior. In Section 3, I evaluate how these six conditions are fulfilled in the railway industry.

2.1.2 Background of the development

In principle, the decision problems related to RM are not new at all. Far before the deregulation in the airline industry and the first simple capacity allocation methods, sellers have made decisions related to RM. Talluri and van Ryzin (2004) highlight that the demand management decisions themselves did not change after the deregulation act in 1978, but the real innovation happened in the methods of how these decisions are made.

Significant changes in the airline industry combined with advances in science and technology created a ground for establishment of new approaches for decisions making. Scientific advances in economics, statistics and operations research enabled better demand models and estimates, which further helped to compute more optimal solutions to complex decision problems. At the same time the development of information technologies provides the capabilities to automatize transactions, gather and store a growing amount of data, quickly solve algorithms and finally execute and control detailed demand management decisions. (Talluri & van Ryzin, 2004)

Talluri and van Ryzin (2004) present how the advances in science and the development of information technologies affect the decision making. One consequence is that more complex and extensive demand management, which would be impossible through manual procedures, became conceivable. Science and technology also improved significantly the quality of these demand management decisions. Talluri and van Ryzin (2004) summarize that "*models and systems are better at separating market signals from market noise, evaluating complex tradeoffs and optimizing and producing consistent decisions. The application of science and technology to demand decisions often produces an improvement in the quality of the decisions, resulting in a significant increase in revenues".* Many empirical studies manage to show that the proper use of RM applications can lead to significant increases in revenues. For example in the airline industry, numerous studies find that RM applications can increase revenues by 4 to 6 %. (see e.g. Belobaba, 1987; Smith, Leimkuhler, & Darrow, 1992)

2.2 Theoretical development of capacity allocation models

In this section I describe in more detail some of the relevant RM theories and try to give some perspective how RM has developed since the first theoretical applications in 1970's. McGill and van Ryzin (1999) list assumptions that the earliest capacity allocation models require. 1) *Sequential booking classes,* which means that all booking classes are apart and do not interlace with each other. 2) *Low-before-high price booking arrival pattern,* meaning that low priced tickets will be sold before the high priced tickets. 3) There should be *statistical independence of demand between booking classes,* i.e. any information on the actual demand on one price class cannot be derived from the actual demand on another price class. 4) *No cancellations or no-shows;* this assumption simply claims that it is not necessary to place any attention to overbookings. 5) The assumption, *no batch bookings,* justifies to look at one booking request at a time. 6) The last assumption of the earliest theories requires that only *single-leg is considered at a time.*

Additionally, demand for each fare class is assumed to be independent of the different controls set to other classes (Talluri & van Ryzin, 2004b). This indicates that the likelihood of receiving a booking request does not depend on the other substitutive products that are available at the time of the booking request. In the other words, each consumer is demanding only one type of fare class at a time.

All these assumptions create quite restrictive and unrealistic environment. However, the development of theories enabled to relax some of these assumptions so that the new models would work better in more realistic setting (McGill & van Ryzin, 1999). While more developed models are more realistic, they are also more complex. The form of the assumptions demonstrates how tightly first theories are linked to the airline industry. In this section I use the terminology relating to airlines' RM. Nevertheless these models can be applied in the other industries as well, especially in the other transportation industries.

2.2.1 Littlewood's model

Littlewood's model (Littlewood, 1972) is considered the first theoretical model of quantity-based RM (McGill & van Ryzin, 1999; Pak & Piersma, 2002; Walczak et al., 2012). The model proposes a simple solution for airline's seat inventory control problem, with a single-leg and two fare classes and it requires that all six previously stated assumptions hold (McGill & van Ryzin, 1999). Littlewood (1972) suggests that the low fare class should be closed when the expected revenue of selling the same seat at the higher fare exceeds the certain revenue of selling another low fare seat. Thus the constraint on accepting low fare class is:

$$r \geq (1 - P) \cdot R \tag{2.1}$$

$$(1 - P) \leq \frac{r}{R} \tag{2.2}$$

Where, r is the price of low fare class and R is the price of high fare class. P represents the probability that the acceptance of a low fare passenger will result in the following rejection of a high fare passenger. It is calculated from the demand distribution of high fare passengers, which is commonly assumed to follow normal distribution. Thus the low fare class should be accepted as long as $(1 - P)$ reaches the ratio of the low fare and high fare classes. (Littlewood, 1972) Although Littlewood's model is designed for airlines' capacity allocation, it would be applied also in the other industries (Talluri & van Ryzin, 2004).

2.2.2 EMSR –models

Whereas Littlewood's model concerns the seat inventory problem with a single-leg and two fare classes, Belobaba (1987) extends his model to include multiple fare classes in a nested reservation system[3] and presents the term EMSR – expected marginal seat revenue. Like Littlewood's model, also the EMSR model was at first designed for airline's capacity allocation problem. The EMSR model is a heuristic revenue maximization model that determines the booking limits for each fare classes for a future flight by using the historical demand data and average fares. Belobaba (1987) takes a probabilistic approach to capacity allocation problem. Since the demand varies stochastically over time, the future demand can be represented by a probability density function. He assumes

[3] Nested reservation system refers to booking system, in which units that are available to one particular price class are also available to all higher price classes but not the reverse. Hence the booking limit (L) defines the number of bookings accepted in that class and all lower classes and contrary protection level (C – L) can be defined for all higher classes.

that the total demand for a flight follows Gaussian (normal) distribution and defines $p_i(r_i)$ to be the probability density function for the total number of requests for fare class i. The number of seats allocated to a certain fare class is represented by s_i and so the cumulative probability that all requests for a fare class will be accepted is:

$$P_i(s_i) = P(r_i \leq s_i) \tag{2.3}$$

$$P(r_i > s_i) = 1 - P_i(s_i) = \overline{P}_i(s_i) \tag{2.4}$$

Where $\overline{P}_i(s_i)$ is the probability of receiving more than s_i request for fare class i. Figure 2 clarifies the relationship between $P_i(r_i)$ and $\overline{P}_i(s_i)$.

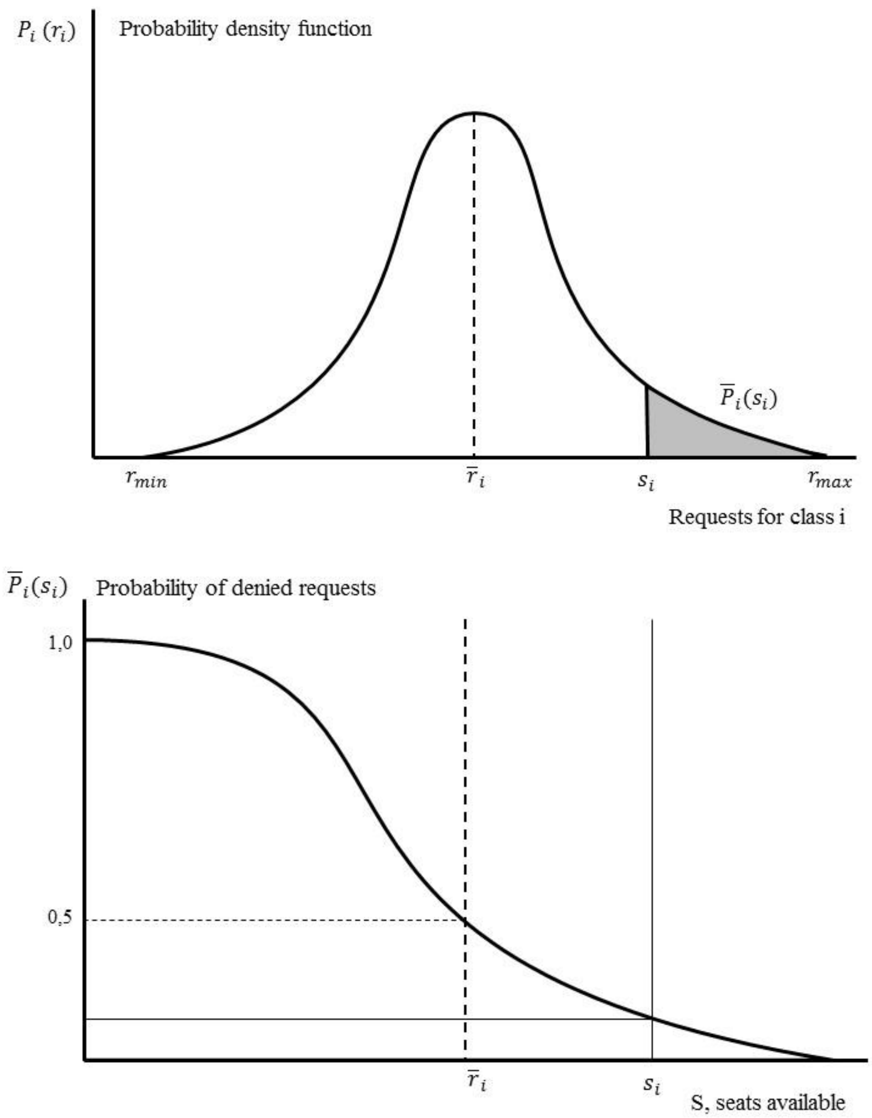

Figure 2 – Probability distribution of requests (Belobaba, 1987).

The EMSR of the s_ith seat in fare class i can be then formulated by multiplying the average historical fare level in class i, f_i, by the probability of selling at least s_i seats:

$$EMSR_i(s_i) = f_i \cdot \overline{P}_i(s_i) \quad (2.5)$$

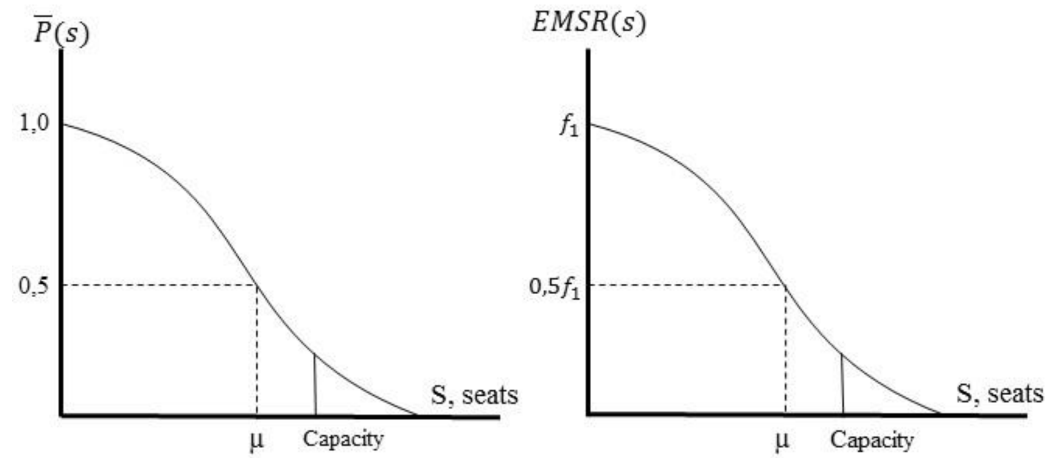

Figure 3 – Probability of selling Sth seat (left) and EMSR (right) (Belobaba, 1987).

Then in the general form of k nested fare classes offered, the optimal value of S_j^i is:

$$EMSR_i\left(s_j^i\right) = f_j \quad (2.6)$$

Where i denotes a higher fare class and j is a lower fare class. Additionally, the optimal booking limits on each fare class j can be determined by:

$$BL_j = C - \sum_{j<i} s_j^i \quad (2.7)$$

This model considers reservation system with complete nesting of fare classes under the shared inventory. (Belobaba, 1987)

Since EMSR is a heuristic model, it does not provide optimal capacity allocation unless under two fare classes (McGill & van Ryzin, 1999). However, it is shown that the losses in revenues associated with non-optimal capacity allocation of EMSR are not significant, approximately 0.5 to 1.5 percent in the worst case (Brumelle & McGill, 1993). The EMSR model is quite simple to implement and easy to understand and thus it has been used generally in RM systems.

Later Belobaba developed EMSR model to avoid pooling effect, revised model is called EMSR-b (the original model is known as EMSR-a) (Belobaba, 1992). Pooling effect, or statistical averaging effect, is produced by aggregating demand across classes. The problem arises if the fare classes are very close to each other. For example if all fare classes are the same, the EMSR-a model will set too high protection levels for each class. Actually, all fare classes under identical revenues should be aggregated. The EMSR-b model avoids this problem by aggregating demand rather than aggregating protection levels. Explicitly, the demand is aggregated and treated as one class with the weighted average revenue. (Talluri & van Ryzin, 2004)

2.2.3 Network control

After the deregulation in the airline industry, airlines started to construct hub-and-spoke networks. This change in operation networks increased the number of passenger itineraries. The capacity allocation became more complex and firms had to be able to manage capacities of numbers of connecting flights over a network (Talluri & van Ryzin, 2004). The capacity allocation problem changed from single-leg, A – B, problem to origin-destination (OD) or network problem, A – H – B (Figure 4). Thus the route A – B actually includes the combination of A – H and H – C itineraries and this affects the optimal capacity allocation. Littlewood's model and EMSR –models are designed for single-leg capacity allocation and therefore they are not suitable for network capacity control (McGill & van Ryzin, 1999).

The presence of network effect is evident in other industries as well. In the hotel industry, the problem is to control room capacity over sequential days when customers stay multiple nights. (Talluri & van Ryzin, 2004) In the railway industry, the problem is managing the capacity between OD with several legs. The rail track between OD generally includes numerous intermediate stations (Figure 4) and thus the network effect is also evident in railway markets and it cannot be neglected.

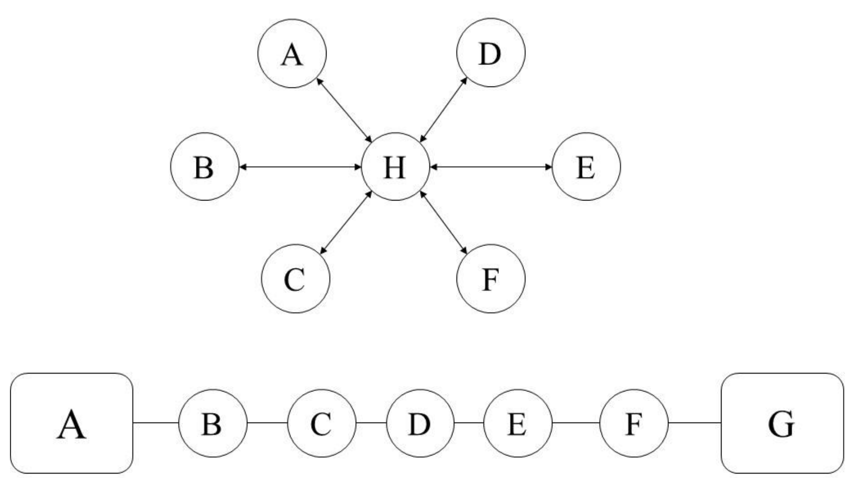

Figure 4 – Airline's hub-and-spoke network (upper) and railway network (lower).

The existence of network effect creates methodological but also implementation problems. As mentioned, the capacity allocation methods for single-leg cannot be implemented efficiently in the network context. Precise optimization in network context is actually impossible, because it creates too large and multidimensional models. However, the development of network models has led to variety of approximation methods. (van Ryzin & Vulcano, 2008) A few methods that are used to network capacity allocation are presented next: *Virtual nesting, bid-price method, and mathematical programming.*

2.2.4 Virtual Nesting

Virtual nesting method is one of the first approximation methods related to network capacity allocation and was initially developed by American Airlines (Smith et al., 1992). It is based on disintegrating the network problem into a set of single-resource problems (single-leg in the airline context) and thus it is referred to hybrid of network and single-resource controls. (Talluri & van Ryzin, 2004; van Ryzin & Vulcano, 2008)

The idea is to create single-resource controls, but while basic single-resource controls are based on fare classes, the controls of virtual nesting models are based on virtual classes. These virtual classes are constructed by connecting sets of itinerary-fare-class combinations that use a certain resource. This step is called mapping or indexing. Network value, or net network revenue benefit, is used to group these virtual classes. (Talluri & van Ryzin, 2004) To illustrate virtual nesting method, let's consider an example of OD route A –B with two legs, A – H and H – B, and three fare classes in both legs (Figure 5).

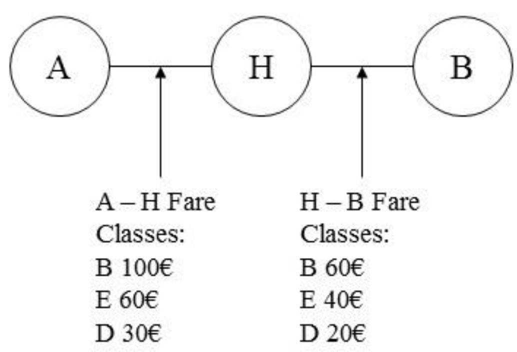

Figure 5 – Simple network with three fare classes.

In order to simplify the example, fares are assumed to represent network value and they are used to group virtual classes. Six fare classes are connected into three virtual classes based on their values (Figure 6).

Virtual Class	Value Range	Included fare classes
1	70-100 €	A – H (B)
2	40-69 €	H – B (B) A – H (E) H – B (E)
3	0-39€	A – H (D) H – B (D)

Figure 6 – Virtual classes.

These virtual classes are then used to compute booking limits for each resource using a single-resource optimization, for example EMSR –model (Williamson, 1992). An OD booking request is accepted if all the virtual classes are available for that OD itinerary and contrary request is rejected if even one virtual class is closed (van Ryzin & Vulcano, 2008).

In addition to the fact that virtual nesting model produces only approximation of optimal network capacity allocation, Talluri and van Ryzin (2004) highlight a few noteworthy drawbacks of virtual nesting model. The use of virtual classes can cause some difficulties in data collection and forecasting. If the data is in the virtual class level, it might be difficult to compute real demand levels for each resource. Virtual classes can also create practical difficulties, since interpretation of virtual classes is significantly more complex than single-resource capacity allocation problems. Despite these drawbacks, virtual nesting model manages to incorporate some of the network information in RM system and affords quite effective compromise between single-resource and full network controls.

2.2.5 Bid-price method

The first application of bid-price method has been introduced by Simpson (1989) and further examined by Williamson (1992). The difference between booking limits and bid-price controls is that the bid-price controls are based on the marginal value of each unit of inventory. Under single-leg problem, a booking request is accepted if its revenue is greater than the marginal value of that unit, i.e. the bid price. (Williamson, 1992) A simple example of single-leg and three fare classes can be used to describe the idea of bid-price system. (Talluri & van Ryzin, 2004)

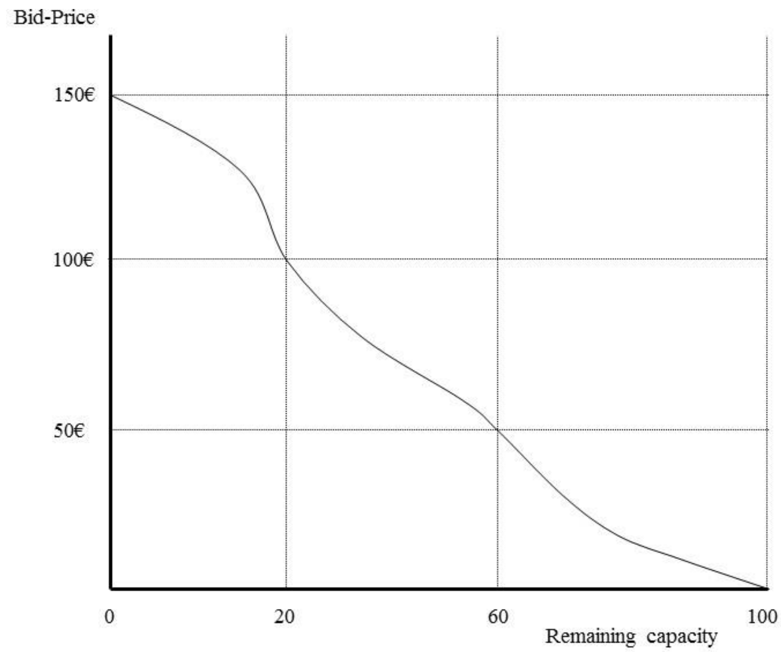

Figure 7 – Bid price as a function of remaining capacity (Talluri & van Ryzin, 2004).

In Figure 7, the bid-price is presented as a function of the remaining capacity, while the total capacity being 100. When there are more than 60 units left, the bid price of an additional unit is less than 50€ and thus all three fare classes (A, B and C) are available on sale. As the remaining capacity drops below 60, but is greater than 20, the bid price increases over 50€ and thus only booking requests for classes A and B are accepted. When the remaining capacity decreases under 20, the bid price yet again increases, now over 100€, making only fare class A available on sale. Figure 8 represents the relationship between bid price and capacity allocation.

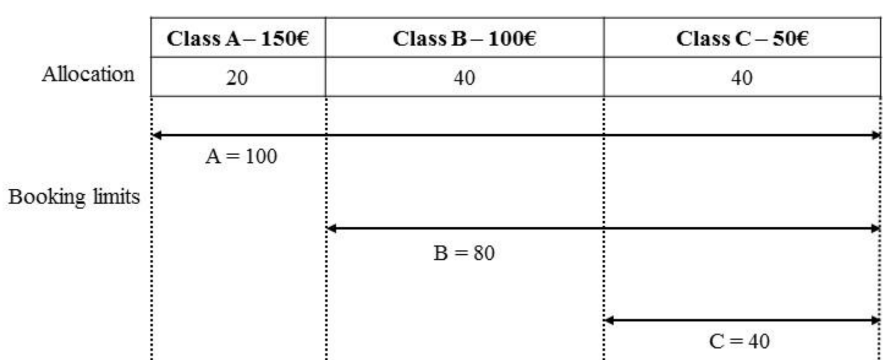

Figure 8 – Bid-price booking limits (Talluri & van Ryzin, 2004).

Although this example includes only singly-leg, bid-price method is also applicable in a network system. In that case, bid prices are set for each resource separately and a product, including one or more resources, is sold if its price exceeds the sum of the bid-prices related to its resources. (Talluri & van Ryzin, 1998)

Williamson (1992) emphasizes that bid-price control is rather simple method of capacity allocation. Only single bid price at any point of time is needed to capacity allocation, while other capacity allocation models require booking limits for each fare class. However, in order to ensure the efficiency of bid-price controls, bid prices should be updated after each sale. (Talluri & van Ryzin, 2004)

2.2.6 Mathematical programming

One of the first mathematical programming models is designed by Rothstein (1971). His model uses dynamic mathematical programming to find a solution to the overbooking problem. Later, Alstrup, Boas, Madsen and Vidal (1986) use mathematical programming to determine overbooking policies for flights with two types of passengers. Also models for network system have been developed (Glover, Glover, Lorenzo, & McMillan, 1982). Albeit these mathematical programming models give good solutions for multiple fare class allocations, they do not necessarily work well in a nested fare class system (Kimes, 1989).

Kimes (1989) describes two simple mathematical programming models: *linear programming* and *probabilistic linear programming*. The linear programming model is rather simple, its objective is to maximize revenue with two constraints: 1) the sum of the total number of units allocated to each fare class has to be less or equal to the total capacity, and 2) the number of units allocated to each fare class has to be less or equal to the expected demand for that fare class.

$$MAX \sum_{i=1}^{n} r_i x_i \tag{2.8}$$

Constraint to:

$$\sum_{i=1}^{n} x_i \leq C$$

$$x_i \leq d_i, for\ all\ i$$

$$x_i, d_i \geq 0$$

Where i is a fare class, x_i, the number of units sold in fare class i, d_i, the demand for fare class i, r_i, a given revenue from selling unit at fare i and C, the total capacity. The model assumes that the demand is deterministic and thus it might create unrealistic solutions. (Kimes, 1989)

Another basic mathematical programming approach is the probabilistic linear programming. The distinction between these two models is that the probabilistic linear programming model presents demand in a probabilistic form.

$$MAX \sum_{j=1}^{m} \sum_{i=1}^{n} p_{ij} r_{ij} x_{ij} \tag{2.9}$$

Constraint to:

$$\sum_{j=1}^{m} \sum_{i=1}^{n} x_{ij} \leq \sum_{j=1}^{m} C_j$$

In this model i is fare class and j is inventory unit. The number of allocated units x_{ij} is described:

$$x_{ij} = 0, if\ unit\ j\ is\ not\ sold\ at\ rate\ i$$

$$x_{ij} = 1, if\ unit\ j\ is\ sold\ at\ rate\ i$$

The probability of selling unit j at fare i is presented by p_{ij} and the revenue from selling unit j at fare i is r_{ij}. Again, C represents the total capacity. Like in linear programming, the problem of nested fare structure is existed also in probabilistic form. (Kimes, 1989)

2.3 Key research areas

Research on RM is mainly focused on four main areas: *forecasting*, *capacity allocation*, *overbooking*, and *pricing*. The overview of RM research by McGill and van Ryzin (1999) is used to shortly describe these four main areas.

Forecasting

Forecasting is an important part of RM, since it forms the basis for the other RM components. For example, demand and cancellation forecasts are used to determine capacity allocation, overbooking, and pricing decisions. McGill and van Ryzin (1999) underline that conducting reliable forecasts is extremely difficult. Numerous factors relate to demand forecasts; demand volatility, seasonality, sensitivity to pricing actions, and so on.

The forecasting research can be divided into three categories: *macro-level, micro-level,* and *choice modeling*. Macro-level forecasts are large-scale, aggregated forecasts of e.g. total industry level demand. Micro-level forecasting involves more disaggregated forecasts, for example forecasts of passenger demand for specific flight or train. Choice modeling, alternatively, is interested in forecasting individual's socioeconomic behavior. For example, forecasting individual's choice between transportation modes relates to choice modeling. (Lee, 1990) McGill and van Ryzin (1999) present also other factors that are related to forecasting. Firstly, it is essential to be able to derive the demand distribution. Many empirical studies (see e.g. Belobaba, 1987) use normal distribution as an approximation of demand distribution. Additionally, in dynamic models particularly, it is important to have specifically determined booking patterns. The stochastic arrival is relatively used method for constructing the distribution of total demand. Also one significant issue relating to forecasting is the constrained data. The past demand data might be biased, because the capacity restrictions may have limited the observed total demand. Therefore, forecasts can be downward biased if constrained data is used to construct forecasts. (McGill & van Ryzin, 1999)

Later in Section 4, demand forecasting methods and related issues are considered more closely in railway RM context.

Overbooking

According to McGill and van Ryzin (1999), overbooking problem is the oldest research area of RM. The first overbooking models were constructed far before the Air Deregulation Act in 1978. The focus of the first models was to control the probability of denied bookings. In the airline context, overbooking estimations depend on the predictions of how many passengers will appear for boarding at the time flight departures. Thus the overbooking research relates on the research of forecasting of passengers cancellations and no-shows. The first overbooking models were static, but later also dynamic models were developed. These models also take account of the dynamics of cancellations and reservations. (McGill & van Ryzin, 1999)

Capacity allocation

As can be seen from Section 2.2, capacity allocation models are the main determinants of efficient demand management. The research of these models evolved in the seventies, concentrating with single-resource capacity allocation problem under several conditions (see e.g. Littlewood, 1972;

Belobaba, 1987). The following researches expanded the initial theories to include capacity allocation under multiple resources, also called the network problem (see e.g. Simpson, 1989; Williamson, 1992). Since these network controls are significantly more complex, the most of these theories provide only approximations of optimal capacity allocations (Talluri & van Ryzin, 2004). Different mathematical programming and simulation approaches were also developed in the network environment. (Pak & Piersma, 2002)

Additionally, more dynamic models were developed. The earliest models determine capacity allocation controls at the start of the booking period, while dynamic models monitor the booking process over time and adjust capacity allocation controls dynamically over the booking horizon. (Pak & Piersma, 2002)

Pricing

Pricing is nowadays seen as a part of RM and it is clear why; price differentiation between customers is the main concept behind RM. Price is basically the most important factor of customer demand behavior. (McGill & van Ryzin, 1999) Since price has impact on demand, the capacity allocation decision cannot be separated from the pricing decision but these two decisions are linked together. Therefore it is essential to RM applications to have joint pricing and capacity allocations systems. (Gallego & van Ryzin, 1997)

There exists an extensive literature on different pricing models related to RM. Nevertheless, McGill and van Ryzin (1999) state that the large part of the literature concerns with monopoly pricing or price competition at industry level. For example, Borenstein and Rose (1994) study the relationship between the level of competition and price dispersion in the airline industry. Dana (1999) as for, constructs optimal pricing strategy for monopoly and oligopoly and shows that both include intra-firm price dispersion. Dana’s model is highly related to price-based RM decisions. After the overview of McGill and van Ryzin (1999), several researches have been published relating to dynamic and other pricing methods. (Bitran & Caldentey, 2003)

2.4 RM system flow

RM systems include several different components and areas of interest. So how are these components combined and implemented in organizations? Talluri and van Ryzin (2004) represent a general description of operations in RM system. RM system flow generally follows four steps: *Data collection, estimation and forecasting, optimization,* and *controls* (Figure 9).

Data collection process includes collecting and storing the relevant historical data. The data should contain information on demand, prices and other relating factors and this available data is then used in the later operations. Next step is estimation and forecasting. Collected data is used to estimate the parameters of the demand model. Forecasts are then compiled based on these parameters. Also other forecasts, such as no-show and cancellation rates, can be compiled. Optimization processes seek the optimal set of different controls (e.g. capacity allocation, prices, discounts and overbooking limits) to apply until the next optimization. These controls are then used to monitor the sales of the inventory. (Talluri & van Ryzin, 2004)

Typically RM system cycles through these steps over time. Different factors, like the volume of data, the speed of a business cycle and the type of forecasts and optimization methods used, relate to how frequently these steps are performed. For example, in the airline industry, RM system cycles through these steps repeatedly and especially demand forecasts and re-optimizations should be performed daily. (Talluri & van Ryzin, 2004)

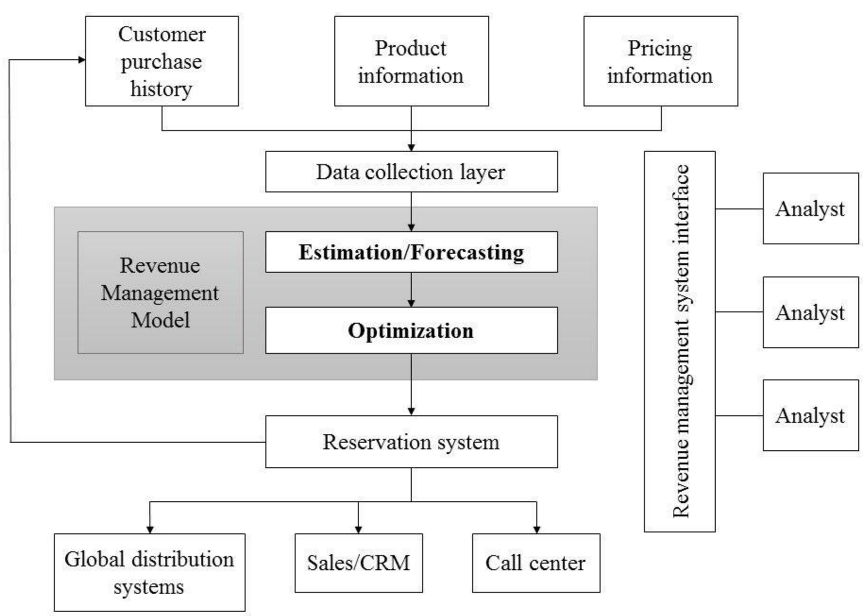

Figure 9 – RM process flow (Talluri & van Ryzin, 2004).

3 RM in the railway industry

The market environment in European railway markets has been slowly changing after the reforming directive by the European Commission in 1991 (European Commission, 1991). The directive includes three cornerstone issues: 1) it separates railway infrastructure from operations so that they are fully separated or that one operator governs both, but organizations and accounts are separated within the operator, 2) it creates independent regulatory institutions for railways, and 3) it opens the national railway markets for third parties. The objective of these issues is to increase competition and economic efficiency in European railway markets. (Friebel;Ivaldi;& Vibes, 2010) Albeit the changes in market environment have been gradual and still after nearly 25 years many European railway operators have monopoly status in national railways, the objective of improving efficiency has increased the potential of RM in railway markets.

Research regarding RM has been intensive during the last 40 years. However, researches have strongly focused on certain industries, especially on the airline industry. Also hotel and car rental industries have gathered some interest, but academic research has almost entirely neglected the railway industry. (Armstrong & Meissner, 2010) Ciancimino, Inzerillo, Lucidi and Palagi (1999) suggest that this is perhaps due to the fact that the rail transportation does not have such a significant role in the U.S., where flying is considered as the main mode of intercity transportation. Because RM started developing in the U.S. after the Air Deregulation Act, it is understandable that the main focus of the research has been on the airline industry.

Nevertheless, in Europe and Asia railways are widely used mode of transportation between cities (Ciancimino et al., 1999). Especially in Central Europe and Japan high-speed trains play crucial role in medium long intercity markets. Thus the most of the few researches considering RM in the railway industry have been published in Europe or Asia. Armstrong and Meissner (2010) compile an overview of railway RM and its models. They emphasize that the opportunities of RM in the railway industry are similar to those found in the airline industry. Ciancimino et al (1999) develop a capacity allocation model, which directly concentrates on the railway RM problem. They consider a single-fare and multi-leg capacity allocation problem. Bharill and Rangaraj (2008) study RM in the context of Indian Railways. Also the applicability of RM to high-speed railways has been studied by a couple of researches (see e.g. Wang;Lan;& Zhang, 2012 and Chuang;Chu;& Niu, 2010). Kimes (1989) gives a generalized description of RM and underline that it can be used by any capacity-constrained service firm.

In Section 2.1.3, six general conditions for using RM techniques are presented. Taking account of these conditions, it seems to be evident that RM has great opportunities in the railway industry, as Armstrong and Meissner (2010) present. The first condition, fixed capacity, seems to be appropriate assumption in railway context for two reasons. First, like airlines, also railways tend to schedule their timetables for long period and thus the capacity is fixed in short-term. Additionally, the length of platforms on each route causes capacity limitations also in the long-term, because additional cars cannot be added if the total length of the train exceeds the length of the shortest platform on the route. The second condition, customer heterogeneity, seems to be fulfilled partially in the railway industry. It is evident that railway passengers can be divided into different segments based on their WTP, but the passenger mix is shown to vary significantly between routes. The high share of business passengers is focused on particular routes, while on some routes the passenger base might include exclusively leisure passengers. Thus the applicability of RM depends on the route specifics. (Talluri & van Ryzin, 2004) The third condition, perishable inventory, is as obvious as it is in the airline industry and the other service industries. After a train has departed, unsold seats cannot be inventoried and therefore they represent a wasted inventory. The fourth condition, product sold in advance, has become more relevant in the recent years, because of the development of reservation systems but, still, this condition does not hold perfectly in the railway industry. Armstrong and Meissner (2010) remind that many railway passengers tend to buy their tickets just before the departure from the station. Finally, both fifth and sixth conditions, uncertain and fluctuating demand and low marginal costs, can be seen to hold in the railway industry. Demand can fluctuate seasonally, weekly or daily and part of these fluctuations can be uncertain. The condition of low marginal costs holds because the cost of providing a certain train does not depend on the number of passengers on that train, assuming that the condition of fixed capacity holds. All in all it can be generalized that the common conditions for RM techniques hold reasonably well in the railway industry.

3.1 Differences between airlines and railways

Though the railway and airline industries can appear pretty similar from the perspective of RM, it is crucial to also view the fundamental differences between these industries (Armstrong & Meissner, 2010). This section presents four significant differences that relates to RM and have to be considered if implementation of airline RM applications to the railway industry is planned.

More network oriented

The network problem in the railway industry is two dimensional and thus even more complex than networks in the airline industry. As Talluri and van Ryzin (2004) note, the railway network can be considered as a combination of hotels' and airlines' networks. Single OD route includes several intermediate stations and is then constructed by several legs. This OD route network corresponds to the length of stay network of hotels. While hotel revenue managers are interested in how many nights customers are staying, railway revenue managers have to consider how many legs each passenger is going to travel.

On the other hand if the whole railway network is considered instead of a single OD route, the network resembles more airline's hub-and-spoke network. The railway network contains many connection points where passengers can switch trains making it like a hub-and-spoke network. (Talluri & van Ryzin, 2004) Figure 10 illustrates two dimensionality of railway network.

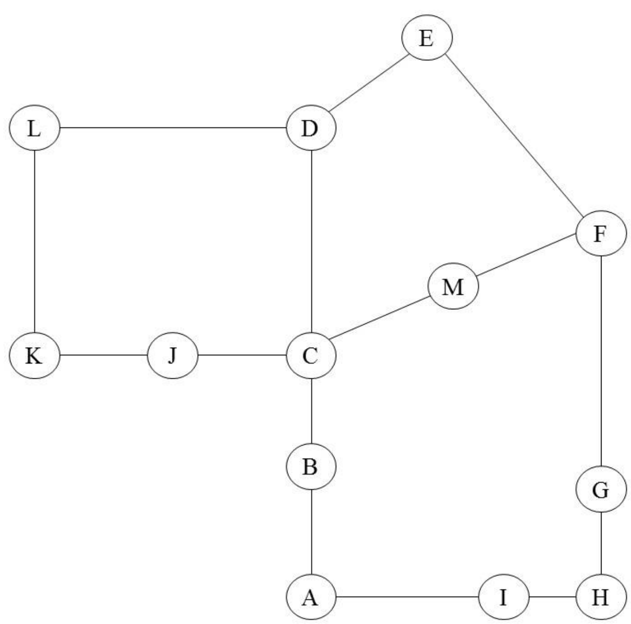

Figure 10 – Two dimensional railway network.

Less price differentiation

Another difference is the level of fare class differentiation used in RM applications. In general, railway operators tend to use less fare classes and advance purchase restrictions than airlines do. Like mentioned earlier, the passenger mix tend to vary more between railway routes than it varies between airline routes. Also the amount of business passengers can be non-existent on some railway routes. Thus many tourist-oriented railway markets rely on product differentiation (e.g. 1st class, 2nd class etc.) and identifiable customer groups (e.g. student discounts) instead of using price differentiation in a form of multiple fare classes and advance purchase restrictions. (Talluri & van Ryzin, 2004)

Shorter booking lead times

While people are used to booking and buying their airline tickets way before the departure, the convention is still slightly different in railway markets. The number of so called *walk up passengers* is significantly greater. This means that many passengers are likely to purchase their tickets from the station just before the departure. (Armstrong & Meissner, 2010) This holds especially for shorter routes and business passengers.

Less competitive markets

The fourth significant difference relates to the market environment. After the Air Deregulation Act in the U.S. and similar deregulating actions in 1990s in Europe, the airline markets have become highly competitive. On the contrary, despite similar deregulatory actions in railway industry, in many countries railways are still monopolized by the governments. Therefore they do not face direct price competition and thus they might have greater flexibility in pricing than airlines. However, also monopoly railway operators have to compete with other modes of transportation like airlines and intercity bus operators. Consequently, even monopoly railway operator’s pricing and capacity allocation decisions are affected by the prices of transportation substitutes. (Talluri & van Ryzin, 2004)

3.2 RM applications in railways

This section presents a few examples of how RM has been used by railway operators. First I describe RM system used by the French national railway operator SNCF (Société Nationale des Chemins de Fer Français). Then a description of RM scheme of Deutsche Bahn AG (DBAG) is given. Thirdly I will shortly discuss on how RM has affected on Finnish national railway operator, VR. These introductions are fairly brief and superficial because of two reasons. First, as mentioned, there is not much published research on RM in railways, which limits the available information. Another reason relates to the natural reluctance of railway operators to share details of their RM applications to the public.

3.2.1 SNCF – Socrates (France)

SNCF is a public monopoly owned by the French government. It was one of the first railway operators in the world that introduced some revisions to the traditional distance based pricing in 1970s. SNCF started to offer lower fares to trains with low utilization rates. The system was based on a tricolor year calendar with three time zones: red for peak periods, blue for average periods and white for low demand periods. The idea was to direct price sensitive passengers to use lower-priced "white trains". This year calendar was published and revised only once a year so the system was not able to dynamically adjust to changes in the markets. (Mitev, 2004) Even though the system was extremely simple, the ideology behind it was truly RM driven.

Later in 1980s SNCF expanded price differentiation by starting to use compulsory and chargeable reservations for some special routes. Prices also varied based on the travel time so that faster high-speed trains (TGV) were more expensive than slower intercity trains. (Mitev, 2004) However, the big revolution in SNCF's pricing happened in 1993 when it introduced a new computerized reservation and ticketing system, called Socrates. Socrates was created in partnership with SABRE Technology Solutions, which has earlier created RM systems for American Airlines. (Ben-Khedher, Kintanar, Queille, & Stripling, 1998) The new reservation system enabled to incorporate more parameters into the pricing model. For example type of the purchase and flexibility of the product have been included into the model and thus they also have impacts on the prices. The new system included both pricing and capacity allocation controls. The pricing model also became more dynamic, since Socrates enabled gathering new information on bookings in real time and then adjust prices and capacity allocations based on new information. (Mitev, 2004) Socrates system was divided into two parts: a long-term strategic schedule planning system, RailPlus, and a short-term tactical capacity allocation system, RailCap (Ben-Khedher et al., 1998). Figure 11 illustrates the relationship between RailCap and other RM components in Socrates system.

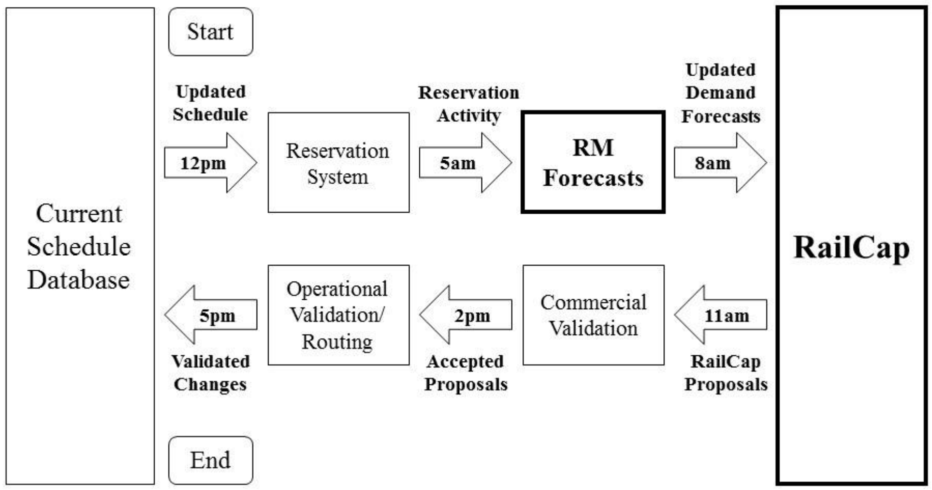

Figure 11 – The components of Socrates RM system. (Ben-Khedher et al., 1998).

The first steps after the introductions of Socrates system were catastrophic. So many issues went wrong: failed bookings, impossible reservations on some trains, wrong train connections, and inaccurate traffic information led to long queues of disappointed passengers in all main stations in France. Also sales staff resisted the new system because it was considered too complicated and they felt that the training had been inadequate. (Mitev, 1996) However, despite of the initial difficulties, the Socrates system is estimated to provide incremental revenues of 17 million euros per year and it has also shown to substantially decrease operating costs (Ben-Khedher et al., 1998).

3.2.2 DBAG – PEP (Germany)

In December 2002 Deutsche Bahn AG (DBAG) introduced a new fare system, called PEP, for its long-distance passenger routes. The PEP system can be considered as an example of unsuccessful implementation of RM system. After implementation Germans protested strongly against new fares, which lead to such a considerable drop in economic performance that DBAG had to rectify the system only 10 months after the introduction. (Link, 2004)

The PEP fare system was a traditional RM system with goals of maximizing profits and enhances capacity usage. It included four main pricing elements: a base fare, early booking discounts, BahnCard discounts and discounts for accompanying persons. In contrast to the traditional distance based fare structure, new base fare was based on the demand and was also differentiated by train type and class. Early booking discounts had been divided into three levels; a 40 % discount for booking one week in advance, a 25 % discount for booking three days in advance, and a 10 % discount if the booking was done one day before the departure. BahnCard was chargeable member card, which entitled to get a 25 % discount on all tickets. Discounts for accompanying persons meant that small groups were allowed to get a 50% discount on the base fare. Additionally to these pricing changes, new system included high cancellation and rebooking penalties for early bookers. (Link, 2004)

But as mentioned, the PEP system was not success. Link (2004) presents a list of matters that might have gone wrong in the PEP system. First of all, PEP's advanced booking discounts were not enough to shift passengers from peak trains to low-demand trains, but also specialized peak-load pricing should have been implemented. Secondly, Link states that even if PEP managed to segment passengers based on their WTP, the system overlooked the issue that passengers might make a general long-term decisions between modes of transportation instead of only considering trip specific decisions. Also complexity of railway network created highly complicated calculations of different optimal ticket restrictions. DBAG has assumed that there are 22 million possible route combinations in German railway network. Additionally, DBAG had problems with handling complex fare structures and transparency and passenger friendliness of the system was questioned. Lastly, Link emphasizes that in a way PEP reduced flexibility of rail transportation, which can be considered a major advantage of railways compared to airlines.

The PEP fare system is an excellent example of why differences between railways and airlines have to be inspected carefully when implementations of airline RM applications are considered in railway industry.

3.2.3 VR Group (Finland)

VR Group is a limited company, which is completely owned by the Finnish government. The group comprises three main business areas: passenger traffic (VR), logistics (VR Transpoint) and infrastructure (VR Tracks). VR is responsible for operating passenger rail transportation in Finland. It operates approximately 300 long distance trains and 900 commuter trains daily. (VR Group, 2015)

Like the other European railway operators also VR has felt pressure to respond to changing market and demand environments. In autumn 2011 VR implemented new ticket selling system, which had been designed to enable demand based pricing in the future (VR Group, 2012). In the following year, VR continued to revise its pricing systems. The objective of the reform was to revise both pricing structure and selling channels. In February 2012 first discount campaigns were launched. Campaigns included different types of discounts, for example “two for one” and “even prices”. VR presents in the annual report that new pricing systems increased the sales of railway tickets in 2012. (VR Group, 2013) The change of pricing systems did not happen without problems. Serious technical and implementation problems led to dissatisfied passengers, but the firm reports that those problems were fixed during the spring 2012 (VR Group, 2013).

Even greater changes in pricing structure occurred in 2013, when VR started to use demand based pricing. Also mobile store was opened, which increased the share of self-service channels in ticket sales. (VR Group, 2014) In 2014, the competitive environment tightened in a consequence of new “low-cost operators” in the long-distance bus markets. VR responded by increasing price campaigns targeted to the price sensitive passengers, for example, students got additional 50 % discount on already discounted fares. Also investigations of new pricing reform were started in 2014. It is stated in the annual report that VR partly failed to prepare to these changes in competition. Because of the competition and more price-conscious consumers, it is emphasized that the pricing system should be more transparent and customer-oriented. That’s why VR will continue to develop its pricing so that the differences between different fare classes would be more explicit. Like in the previous years, also in 2014 the share of self-service channels continued to grow. In the end of 2014 the share of self-service channels in ticket sales was 61.3 %, which was the highest share in VR’s history. (VR Group, 2015)

Because of the limited availability of information, it is not possible to draw more precise conclusions on VR's RM systems. However, the latest annual reports indicate that VR has made remarkable investments in the development of RM systems in the last five years and that this investment has paid off. Figure 12 concludes the development of RM and pricing in VR since 2011.

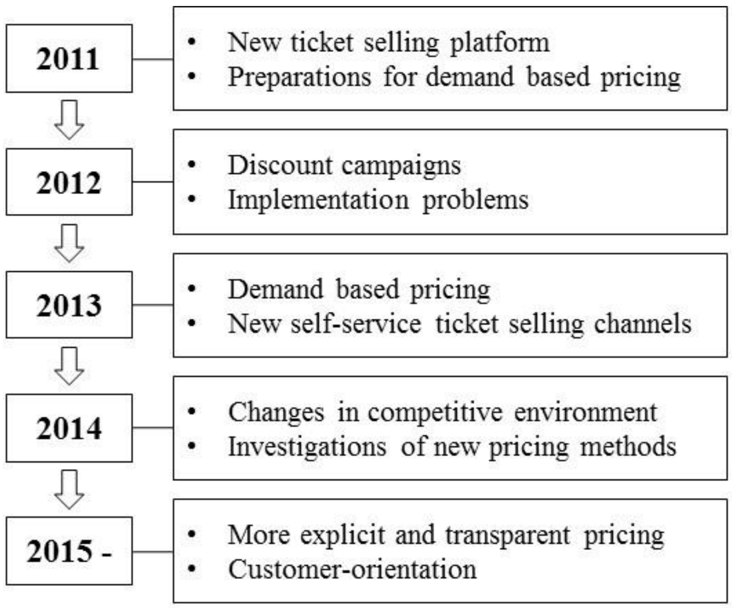

Figure 12 – Development of VR's RM and pricing.

4 Demand forecasting

This section describes the process of demand forecasting in a RM system. Some relevant methods of demand forecasting are presented. Additionally, important issues relating demand forecasting are considered. The emphasize is on disaggregated micro-level forecasting, because the purpose of this thesis is to study how to construct demand forecasts for each fare class on a specific train, which can be then used to determine efficient capacity allocation and pricing decisions.

Lee (1990) presents that the goal of airline's micro-level demand forecasting process is to provide reliable and accurate predictions of future demand on each date, flight and fare class for capacity allocation and pricing optimizations. Thus, because of the industry similarities, the demand forecasting in the railway industry can be described correspondingly: the future demand for each train departure and each fare class has to be forecasted in order to produce effective capacity allocation and pricing optimizations.

The purpose of this sort of forecasting is to provide some quantitative predictions of the future uncertain demand. Therefore, three general conditions for applying quantitative forecasting are required: 1) information about the past must be available, 2) this information can be presented in numerical form, and 3) this information of the past must represent the future in some level. The third condition is known as the *assumption of continuity* and it forms the basis for all quantitative forecasting methods. (Makridakis, Wheelwright, & Hyndman, 1998)

As McGill and van Ryzin (1999) present, forecasting passenger demand includes numbers of complicating factors, which make it extremely difficult to accurately forecast upcoming demand. Zeni (2001) lists some significant factors contributing to this difficulty.

Demand fluctuation and peaks

Demand for a certain travel route varies over time. This variation of demand includes variation between seasons, day of week, time of day, and also because of the special events. Seasonal variation features variation in demand due to different time of year. For example, there is greater demand for air and rail transportation to Lapland during the winter holiday season. Day of week variation arises because business travelers are more likely to travel on weekdays. Business travelers also prefer early morning and afternoon departures, which generates variation on time of day demand.

Special events, such as rock concerts, might create significant temporary increases in demand for a specific route.

Sensitivity to pricing actions

Because of the price elasticity of passenger demand, price changes result in shifts in demand. While it can be generalized that a price increase will lower the demand, the level of demand change due to price change differ between different passenger types.

Demand dependencies between fare classes

A passenger who meets the restrictions of low fare ticket, can be forced to buy high fare ticket because there is no available low fare tickets due to the booking limit of that class. On the contrary, a passenger who books the low fare ticket might also have been willing to buy higher fare ticket, if the low fare ticket had not been available.

Additionally, several other factors, such as *booking cancellations, no-shows, group bookings* etc., have their own effects on demand forecasts. Thus establishing accurate demand forecasts is certainly complicated. (Zeni, 2001)

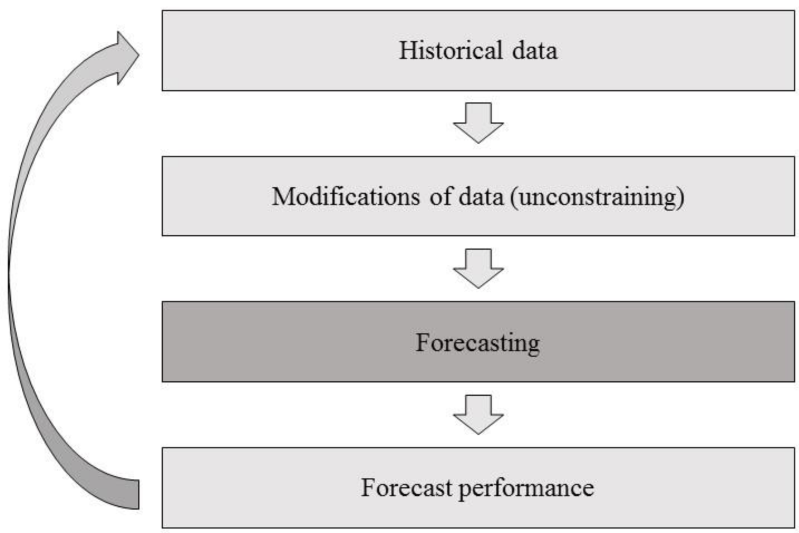

Figure 13 – Forecasting process.

The forecasting process can be divided into four steps (Figure 13). The first step consists of the available historical data. Next, the data has to be modified by unconstraining censored observations. Then the actual forecasts can be constructed applying suitable forecasting model. Finally, the forecast performance has to be evaluated, so that the capacity allocation and pricing decisions would be based on accurate and realistic forecasts. After the capacity allocations have been determined and new information on demand has been attained, the process is repeated over again. Thus forecasting process, similar to RM process, is cyclical process that cycles through these four steps over time.

4.1 Level of forecasting

As presented, in this thesis my focus is on the disaggregated demand forecasting. This is a common practice in RM applications and Weatherford and Kimes (2003) state that the majority of the modern RM systems rely on disaggregated forecasts. Additionally, these disaggregated forecasts are seen to perform better than general aggregated forecasts, which are then divided into the disaggregated level (Weatherford & Kimes, 2003). However, also the other levels of forecasting are somewhat relevant in the context of RM and thus the next two sub-sections give a short presentation of macro-level and passenger choice forecasting. After that the following sections consider solely the forecasting process related to the micro-level demand forecasting.

Macro-level (aggregated level)

The broadest level of forecasting is macro-level or aggregated forecasting. In demand forecasting these forecasts construct predictions of total industry level demand. In the transportation industry, the groundbreaking studies of macro-level forecasting are presented by Kanafani (1983) and Taneja (1978) (Lee, 1990). Taneja (1978) provides an extensive study of airline traffic forecasting. His emphasize is on forecasting total airline traffic and future national traffic growth. Kanafani (1983) as for gives overview of transportation demand theories.

Macro-level forecasts have been also compiled in the railway industry. For example, Rao (1978) constructs a forecasting system for railway freight services. The model predicts the impacts of macro-economic activity and intermodal competition on rail freight demand. Wardman (2006) compiles a model for forecasting aggregated passenger railway demand. He tries to explain the growth of railway demand in the 1990s in the United Kingdom and comes to the conclusion that the most significant demand stimulator is the GDP growth.

Passenger choice models

The most disaggregated type of demand forecasting is related to passenger behavior. Passenger choice models are interested in forecasting demand based on passengers' choices between available alternatives (competitors' products and different transportation modes). (Lee, 1990) Ben-Akiva and Lerman (1985) present a general description of a discrete choice modeling in transportation industry. Their general choice process is based on four steps. At first, individual decision-maker has to be defined. It is relevant to be able to explain the variation of preferences between different decision-makers and thus characteristics like income, age, gender, education, etc. have to be included in the model. The next step is to separate different alternatives that are available for the decision-maker. When analyzing individual's decision making, it is relevant to not only know what has been chosen, but also the alternatives that could have been chosen. These alternatives include all alternatives that can be considered as substitutes. For example, a decision maker planning to travel from A to B has to choose whether to use car, bus, train or airplane. These transportation means additionally include numerous options. Decision maker must also decide which company to use if competitive alternatives are available and also which time to depart on what day. All these available alternatives are characterized by different attributes, which represent the costs and benefits of each alternative to the decision maker. (Ben-Akiva & Lerman, 1985) In this example the airplane would probably be the fastest alternative but also most likely the most expensive. Moreover some consumers would prefer travelling in early morning while some other would prefer late night departure. Decision maker compares the costs and benefits of different attributes of available alternatives and determines which alternative would maximize her or his utility. After these three steps are determined, a decision rule can be applied. The decision rule describes how the decision maker makes the choice between alternatives. (Ben-Akiva & Lerman, 1985) Thus the discrete choice model predicts a decision maker's choice between different alternatives based on the utility of each alternative (Rasouli & Timmermans, 2012).

Talluri and van Ryzin (2004b) are one of the first who develop a methodology to analyze passenger choices in a single-leg RM problem. They create a general choice model that determines the probability of purchase of each fare class as a function of all available fare classes. Thus the assumption of independency between fare classes is not needed, making the model more realistic. The initial model does not take account of substitutive products, but Talluri and van Ryzin (2004b) emphasize that their approach could be revised so that the analyze of passenger choices between substitutes would be possible. Then the information regarding both firm's own products and competitors' products should be included into choice set model.

4.2 Historical data

Data is the heart of a forecasting system. Therefore, the first step of generating any sort of forecasts is to identify what sort of data is available and from which sources. The most used data to construct demand forecasts is historical sales or booking data. This data covers information on previously occurred sales and bookings. Often historical booking data is relatively easily available and thus it creates efficient data collection, estimation and forecasts processes. (Talluri & van Ryzin, 2004) The available historical booking data can be divided into two types: *complete* and *partial* data. The complete booking data includes information on bookings that have been realized. (Lee, 1990) For example in the railway context, this type of data includes information on the bookings of a train that has already departed. Alternatively, the partial booking data is comprised of booking information on trains that have not yet departed. Thus partial booking curve is constructed by the demand data available at the current time, t days before departure. (Lee, 1990) Talluri and van Ryzin (2004) note that partial booking data can be for instance used to forecast the short-term increments of demand for each booking day.

Besides historical booking data, also other data sources are often available for a firm. Lee (1990) mentions that for example additional information about seasonality can offer a tool for a firm to account the seasonal variation in the bookings. Seasonality can be addressed by the seasonal index, which describes the variation of the demand of a particular day from the average day. RM databases also gather information on the booking process itself. Information on past prices, overbookings, bid prices, promotions and when each fare classes was closed, are commonly recorded. Different industry level factors, such as the availability of competitors' bookings and prices, can also give additional information to forecasting. (Talluri & van Ryzin, 2004)

4.3 Constrained data

RM applications use different capacity allocation methods to optimize revenues of the inventory. These methods are based on controlling the available capacity of each fare class. For making these capacity allocations firms have to construct forecasts of the future demand. The problem of *constrained data* arises, since these forecasts are generally based on the historical booking data, which might be constrained because of the previous capacity controls (Figure 14). Weatherford and Pölt (2002) explain that booking requests can be rejected because of the booking limits of each fare class and thus the historical booking data might not reflect the real demand for each class. After a certain booking class is closed, the additional information of demand for this class is lost. In statistics, this problem is called constrained or censored data (Guo, Xiao, & Li, 2012).

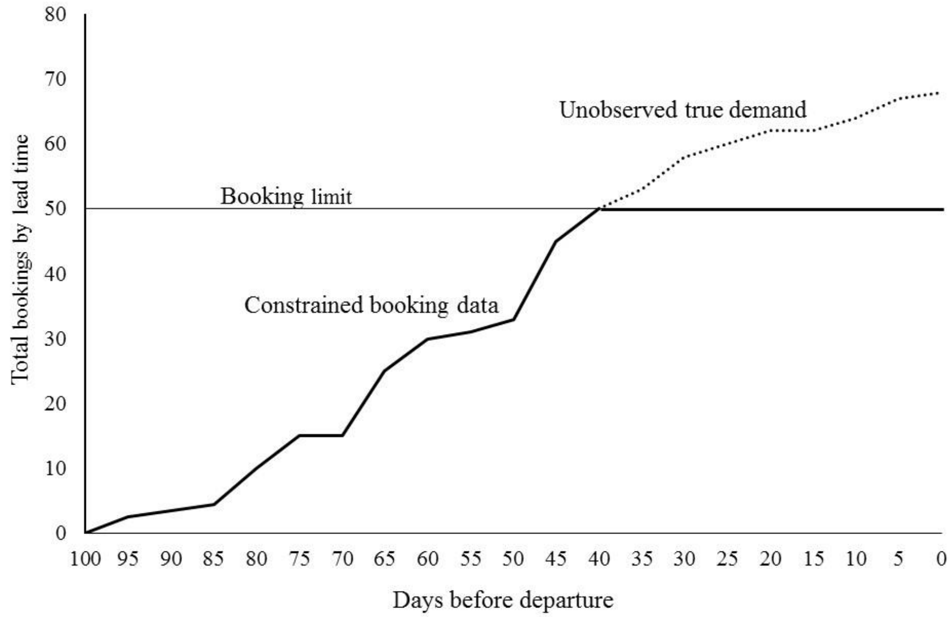

Figure 14 – Constrained booking data caused by the booking limit (Guo et al., 2012).

If a demand forecast is generated using constrained data, it is probable that the forecast would give underestimated predictions, because it does not take account of the demand information after the booking class is closed (Guo et al., 2012). According to Weatherford and Belobaba (2002), underestimating demand can result in significant losses in revenue. They estimate that underestimating demand by 12.5 – 25 % can lead to a loss of revenue from 1 – 3 %. Additionally, because of the relationship between forecasts and capacity controls, the constrained data might create a spiral-down effect on the total revenues. This spiral-down effect means that if underestimated forecasts have been used repeatedly to construct the capacity controls, revenues will systematically decrease over time. (Cooper, Homem-de-Mello, & Kleywegt, 2006)

To prevent this problem, the constrained data can be extrapolated to match the true demand distribution. This process is termed *demand unconstraining*. (Weatherford & Pölt, 2002; Guo et al., 2012) Demand unconstraining has been seen to have significant impacts on demand forecasting accuracy and consequently on revenues. Weatherford and Plöt (2002) report that the demand unconstraining process is related to 2 – 12 % increase in revenues in the airline context.

Several mathematical methods for demand unconstraining have been proposed and various studies have evaluated the performance of these methods (see e.g. Weatherford & Pölt, 2002). A widely used mathematical method is *Expected Maximization (EM),* which is especially used in quantity-based RM (Talluri & van Ryzin, 2004). It was first applied to unconstraining censored demand by Salch (1997).

Expected Maximization (EM)

The EM model is an iterative model including two steps. In the first step, the invisible demand observations are replaced with the estimated values of the true demand. This estimation is done using a maximum likelihood estimation. The second step is to re-estimate the original probability distribution of demand. These steps are then repeated until convergence. (Weatherford & Pölt, 2002) Salch (1997) assumes that the data is normally distributed $N(\mu, \sigma^2)$ and the capacity control, C, determines the bound of unconstrained and constrained observations. Then the best estimate of the unconstrained value U is determined by $E(X|X > C)$. Figure 15 illustrates the expectations of constrained demand. Zeni (2001) gives an extensive description of the EM model in his doctoral thesis.

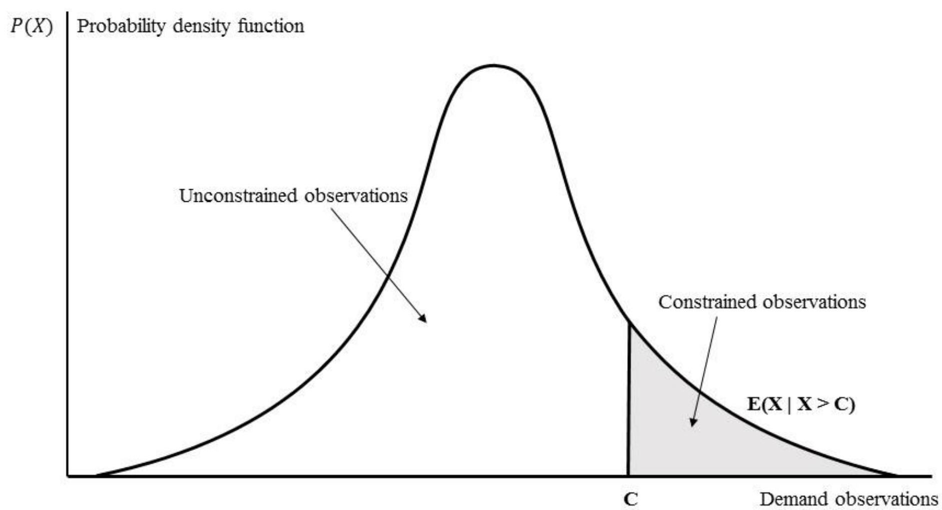

Figure 15 – Expectation of constrained demand (Zeni, 2001).

4.4 Forecasting methods

This section describes some of the commonly used micro-level demand forecasting models in passenger transportation. Like the research of RM in general, also the research of demand forecasting in transportation industry is strongly focused on the air travel demand (see e.g. Sa, 1987; Lee, 1990; Weatherford & Belobaba, 2002). However, later researches have also been conducted solely in the railway industry, for example Tsai, Lee and Wei (2005) and Sharif Azadeh (2013) in his doctoral thesis, study demand forecasting in railway markets.

In a railway RM system, the objective of demand forecasting is to construct predictions of future total demand or bookings for each departure time, given the partial and complete historical demand data. This data can be illustrated by an imaginary booking matrix (Table 1).

Week	DB0	DB7	DB14	DB21	DB60	DB90	DB180
-4	100	90	70	55	25	10	0
-3	97	85	65	50	23	9	0
-2	94	80	60	45	21	8	0
-1	91	77	57	42	19	7	0
0	88	73	54	39	17	6	0
1	-	70	50	35	15	5	0
2	-	-	45	30	13	4	0
3	-	-	-	25	11	3	0

Table 1 – An illustrative booking matrix (Zeni, 2001).

Table 1 describes the booking history of a route with a single fare class over an eight-week period. The negative week numbers present past departures, while the positive week numbers are future departures. Week 0 refers to the last departure. The DB columns present the current bookings at certain days before the departure. Hence for example, the three weeks ago departed train had 65 bookings on hand 14 days before the departure.

DP 0 indicates the final number of bookings at the departure time. The booking data of future trains (week numbers 1, 2, and 3) is partial, because of the missing values of the booking matrix.

Forecasting methods can be divided into two types depending on which data in the booking matrix they use to perform forecasts. *Historical models* use the booking data of previous departures to forecast future bookings (Lee, 1990). These are time series models and the objective of these models is to recognize the patterns in the historical data and extrapolate that pattern into the future demand. For example in Table 1, a historical model would use booking data in column DB 0 to forecast future total bookings of Week 1. Alternatively, *advanced models* determine demand forecasts using both historical and advanced booking data, the rows of the booking matrix (Lee, 1990). In Table 1, an advanced model would use historical booking data and advanced booking data in row Week 1 to forecasts total bookings of Week 1 train at the time of departure.

The most of the models used in transportation demand forecasting do not attempt to detect any causal factors that have impacts on the demand. The demand system is treated as a black box from which forecasting models attempt to recognize the patterns for future forecasts (Makridakis et al., 1998). Two reasons for using this type of forecasting methods are presented by Makridakis et al. (1998). First, as previously noted, the demand is impacted by several different factors. This multi-dimensionality creates such a complex system that it might be impossible to measure and separate the impacts of different factors. Secondly, the main objective of short-term demand forecasting is to know what the demand will be, rather than knowing which factors affect the demand. The long-term forecasting is more interested in identifying the underlying factors.

Based on the studies of Zeni (2001) and Weatherford, Gentry and Wilamowski (2002), there are four forecasting model groups that are particularly used in transportation demand forecasting. These groups are *moving averages, exponential smoothing, pickup,* and *regression* models. These models are considered as traditional forecasting models and the extensive descriptions can be found from any forecasting handbook (see e.g. Makridakis et al., 1998). Moving average and exponential smoothing models are pure historical time series models, whereas regression and pickup models can be considered as advanced models.

Booking histories can also be presented in graphical forms, which are called booking profiles (Figure 16). Each curve in booking profile reflects the advance of bookings of each departure. The booking curves tend to increase as the departure date approach, but they might also decrease because of the booking cancellations. (Zeni, 2001)

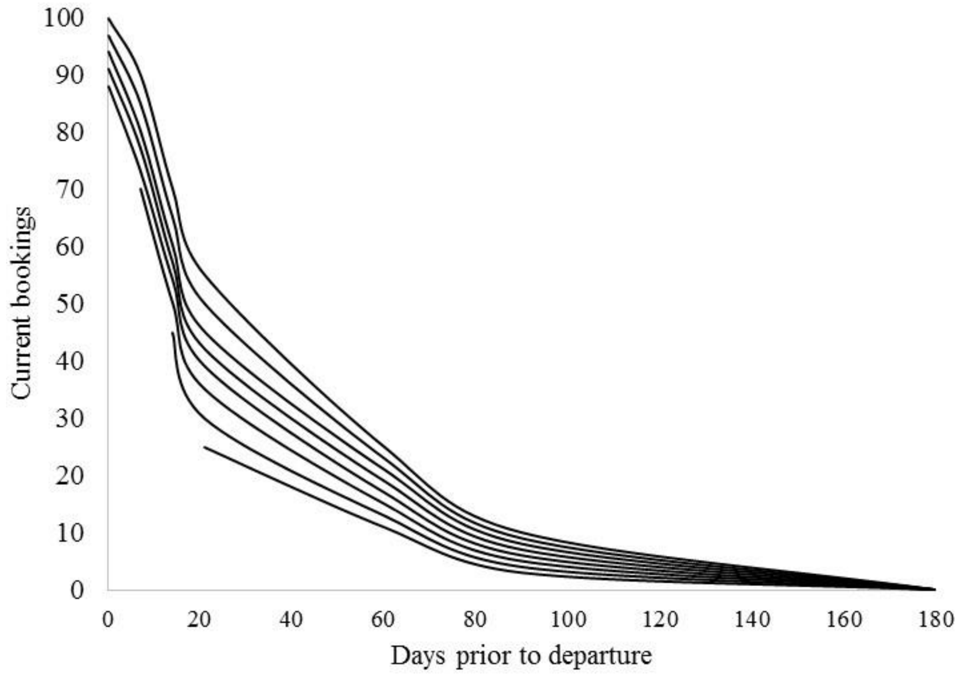

Figure 16 – Booking profiles (Zeni, 2001).

Figure 17 defines the notations used in the following descriptions of different methods.

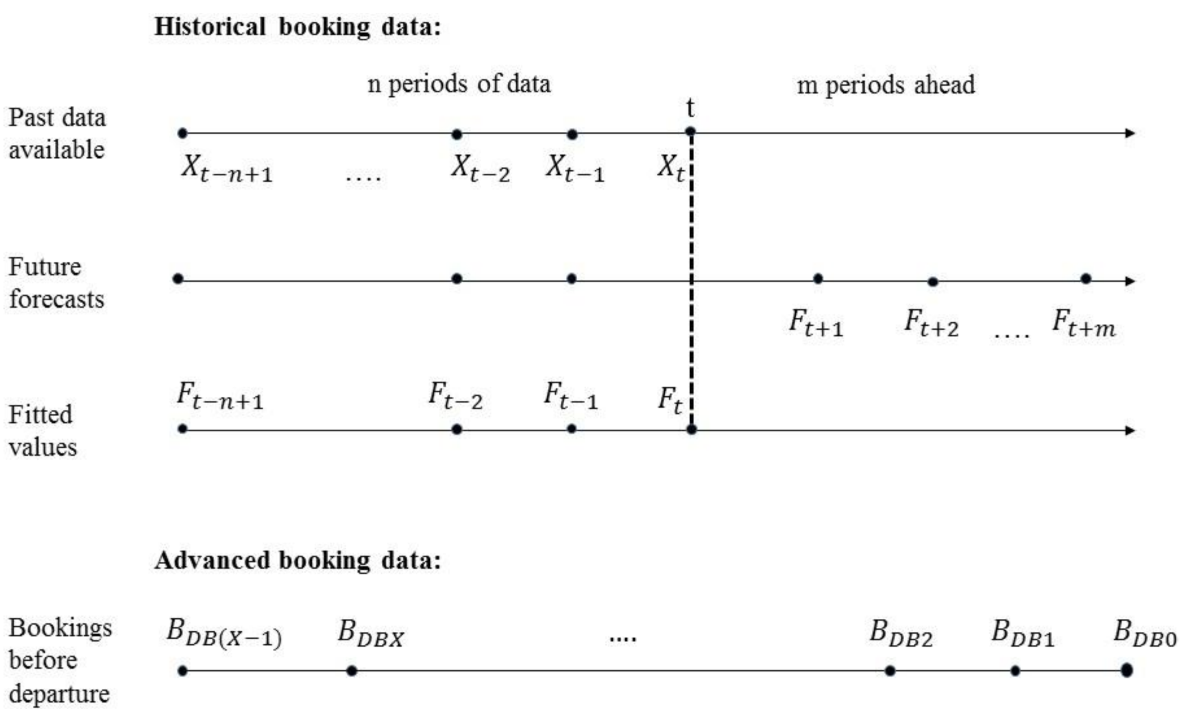

Figure 17 – Forecasting notations (remodeled from Makridakis et al., 1998).

4.4.1 Moving averages

Weatherford et al. (2002) use two different moving average models in their comparative analysis: *simple* and *weighted moving averages.* Both models are based on the assumption that the demands of recent departures are good predictors for the demand of future departure.

Simple moving average:

$$\mathrm{F}_{t+1} = \frac{1}{n}\sum_{t}^{t-n+1} X_t \qquad (4.1)$$

The term F_{t+1} is the forecasted demand for the next period. Values of X_t represent the observed values of demand, up to a given number of n historical periods. Determining the appropriate number of n is the key decision in using moving average model. (Weatherford et al., 2002) Makridakis et al. (1998) emphasize that the large number of n increases the likelihood that the model will eliminate the randomness from the forecast. On the other hand, the model will also waste information regarding the seasonality and trends, when using large number of n.

Weighted moving average:

The weighted moving average model emphasizes more recent demand over more distant demand. The weights are set so that they equal to one.

$$F_{t+1} = W_1 X_t + W_2 X_{t-1} + W_3 X_{t-2} \dots W_N X_{t-N+1} \qquad (4.2)$$

The model is similar than the simple moving average model, but instead of averaging by the numbers of n, the averaging is done by weighting each historical demand. The sum of these weights equals one and generally more weight is put on more recent observations. The advantage of using weighted average is that it results in smoother trend-cycle than the simple moving average model. (Weatherford et al., 2002)

4.4.2 Exponential smoothing

Exponential smoothing model is similar to weighted moving average model in the sense that it puts less weight on more distant demand observations. Only a small amount of data is required to compile forecasts using exponential smoothing model, since it is based only on the most recent demand observation and forecast. It is also simple to implement and thus it has its advantages over moving average methods. (Zeni, 2001; Weatherford et al., 2002) Three different exponential smoothing models are described next.

Simple exponential smoothing:

$$F_{t+1} = \alpha X_t + (1 - \alpha)F_t \quad (4.3)$$

The forecast F_{t+1} is compiled by weighting the most recent demand observation, X_t, with a smoothing constant, α, and the most recent forecast, F_t, with (1 – α). The smoothing constant is set between zero and one. (Makridakis et al., 1998)

Exponential smoothing with trend: Holt's method

Holt first presented exponential smoothing with trend component in 1957. (Holt, 2004) The model is an extended version of simple exponential smoothing method:

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1} \quad (4.4)$$

$$F_{t+1} = \alpha X_t + (1 - \alpha)F_t + T_t \quad (4.5)$$

Equation 4.4 defines the trend component. The value of β represents the smoothing constant of the trend. Then Equation 4.5 provides a complete formula for exponential smoothing with trend. (Makridakis et al., 1998)

Exponential smoothing with trend and seasonality: Holt-Winter's method

Winters (1960) extends further Holt's model to capture seasonality. The model is similar to simple exponential smoothing and Holt's models, but it also includes seasonal component:

$$S_t = \gamma(X_t - F_t) + (1 - \gamma)S_{t-s} \quad (4.6)$$

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1} \quad (4.7)$$

$$F_{t+1} = \alpha X_t + (1 - \alpha)F_t + T_t + S_t \quad (4.8)$$

Equation 4.6 defines the seasonal component, the lowercase s represents the length of seasonality and γ is the smoothing constant of seasonality. Like in the Holt's model, Equation 4.7 provides a complete formula for the forecast. (Makridakis et al., 1998)

The choice of smoothing constant α (and also β and γ in the extended models) has an effect on the responsiveness of the model. If small value of alpha is used, the model will respond slowly to changes in the observed demand. This will result in relatively stable forecast. Alternatively, if large value of alpha is used the forecast will be more responsive to changes in observed demand. (Zeni, 2001) Mean squared error (MSE) is commonly used to determine smoothing constant. Smoothing constant is simply set so that it minimizes the MSE of the forecast. (Makridakis et al., 1998)

4.4.3 Pickup models

Pickup models are advanced forecasting models. The main idea of pickup models is to produce a forecast of incremental demand between time periods and then aggregate these increments to compile a forecast of total demand at the departure (Talluri & van Ryzin, 2004). Therefore forecasted demand is a function of the current bookings and the amount of incremental bookings between the current time and the departure. These incremental bookings are illustrated by the amount of bookings picked up between the current time and the departure. (Zeni, 2001)

Zeni (2001) presents two basic pickup models: *additive* and *multiplicative pickups.* Both of these models can be performed using *classical pickup* or *advance pickup.* The difference between these model specifics is how they use the historical data. The classical pickup uses only complete historical booking data to compile incremental bookings between periods, while the advanced pickup uses all available data, including partial booking data. The formulations of pickup models are illustrated here using the classical versions.

Additive pickup model:

$$\overline{PU}_{DB(X,0)} = \bar{B}_{DB0} - \bar{B}_{DBX} \tag{4.9}$$

Equation 4.10 presents the average pickup between day X before departure and departure ($\overline{PU}_{DB(X,0)}$). The average bookings at departure ($\bar{B}_{DB0}$) and X days before departure ($\bar{B}_{DBX}$) are compiled from the historical data of previous departures. For example moving average method can be used to calculate these averages. The forecast of bookings at departure can be then presented by combining the bookings at X days before departure and the average pickup between day X and departure:

$$B_{DB0} = B_{DBX} + \overline{PU}_{DB(X,0)} \tag{4.10}$$

This model is called additive pickup, since it adds the average incremental bookings to current bookings at certain time. In addition to this, also multiplicative pickup models have been developed. They are similar to additional models in the sense that they use information on incremental bookings to forecast future bookings, but instead of adding the average pickups they multiplies current bookings by the average pickup ratio. (Zeni, 2001)

Multiplicative pickup model:

The average pickup ratio is determined by:

$$\overline{PUR}_{DB(x,0)} = \frac{\bar{B}_{DB0}}{\bar{B}_{DBX}} \tag{4.11}$$

A forecast of bookings at departure can be the compiled from equation:

$$B_{DB0} = B_{DBX} * \overline{PUR}_{DB(X,0)} \tag{4.12}$$

4.4.4 Regression

In RM, regressions can be used to predict the future demand of a certain departure, without making any conclusions about the underlying factors that might affect the demand. Regression forecasting in RM assumes that there exists some relationship between the final bookings and the bookings of a certain time before the departure. (Zeni, 2001) Both historical and advanced booking data are used to compile regression forecasts. Zeni (2001) presents a simple regression using both historical and advanced data. The dependent variable is the total demand at the time of departure and the demand at a specific time before departure has been used as an explanatory variable. For example using demand at day X as an independent variable, the linear regression model is:

$$B_{DB0} = \beta_0 + \beta_1 B_{DBX} \tag{4.13}$$

Where, B_{DB0} is the number of total bookings at the departure, B_{DBX} is bookings at X days before the departure. β_0 and β_1 are regression parameters that are estimated from the historical data using some estimation technique, for example ordinary least square (OLS). (Zeni, 2001) The demand forecast of bookings at the day of departure is then compiled inserting the number of bookings at X days before the departure in the regression equation. In this phase advanced booking data can be used to obtain more accurate forecasts.

Zeni (2001) emphasizes that this regression model differs from the general econometric models of demand, since it does not include any economic causal variables, such as price. The forecast of total demand is only dependent on the previous bookings and thus demand forecasts should be defined to all fare classes separately.

While previous linear regression model assumes linear relationship between dependent variable and independent variable, also other regression models could be applied. Weatherford et al. (2002) state that demand forecast can also be presented by quadratic or cubic regressions.

4.5 Forecast performance

As noted, demand forecasting plays a crucial role in RM. In order to produce efficient capacity allocation and pricing decisions, these decisions should be based on demand forecasts. However, it does not provide any additional value to base these decisions on inaccurate forecasts – quite the contrary, resting inaccurate forecasts might lead to even more inefficient capacity allocation and pricing decisions. Talluri and van Ryzin (2004) underline that the analysis of the forecast performance is as important as the forecasting itself. It is suggested that improvement of forecast accuracy by 10 % can increase revenue by 0.5 – 3.0 % (Lee, 1990).

The term forecast performance or accuracy refers to how well the forecasting model is able to extrapolate the available data to the future (Makridakis et al., 1998). The forecast performance is measured using forecast errors. The idea of forecast error is simple:

$$e_t = X_t - F_t \tag{4.14}$$

The forecast error, e_t, is the difference between the observed demand, X_t, and the forecasted demand, F_t, at time t. This is an example of one-step forecast error, since it takes only account of one period, t. (Makridakis et al., 1998)

Besides that forecast errors can be used to evaluate forecast performance, errors can be useful also for other reasons. The variance of demand can be estimated using forecast errors, which would enable to estimate the demand distribution. Errors can also be used to identify and exclude outliers from the data. Moreover, errors can give the signals of unusual events or instability in the demand. (Talluri & van Ryzin, 2004)

Since the forecasting system generally includes observations and forecasts for more than one time period, it is necessary to compile a measure of total forecast error. There are several commonly used methods to measure the total forecast error, which all assume that there is available information on forecasts and observed demands for n periods. (Makridakis et al., 1998; Talluri & van Ryzin, 2004)

Mean error (ME):

$$ME = \frac{1}{n}\sum_{t=0}^{n} e_t \qquad (4.15)$$

The mean error is the average of the errors of each period n. It represents an estimate of the forecast bias. When the forecasting system is unbiased, the mean error will converge to zero once N increases. (Talluri & van Ryzin, 2004) Each e_t can be calculated using Equation 4.14. The problem of the mean error method is that positive and negative errors tend to offset each other. The mean error represents only over- or under-forecasting of the model and thus it does not provide a good illustration of the size of the errors. (Makridakis et al., 1998)

Mean squared error (MSE):

$$MSE = \frac{1}{n}\sum_{t=0}^{n} e_t^2 \qquad (4.16)$$

The mean squared error can be used to overcome the problem of the mean error method. The model makes all errors positive by squaring them, making it possible to interpret the size of total error. Consequently it gives a better illustration of the forecast accuracy. (Makridakis et al., 1998)

Mean percentage error (MPE):

$$PE_t = \frac{X_t - F_t}{X_t} * 100 \qquad (4.17)$$

$$MPE = \frac{1}{n}\sum_{t=0}^{n} PE_t \qquad (4.18)$$

In order to compile the mean percentage error, the percentage error, PE_t, must be calculated first (Equation 4.17). The mean percentage error is then formulated by averaging the percentage errors. (Makridakis et al., 1998)

Mean absolute percentage error (MAPE):

The mean percentage error suffers the same problem as the mean error method and thus the mean absolute percentage error can be more efficient to describe the forecast accuracy.

$$MAPE = \frac{1}{n}\sum_{t=0}^{n} |PE_t| \qquad (4.19)$$

The mean absolute percentage error is constructed by averaging the absolute values of percentage errors. (Makridakis et al., 1998)

4.6 Network forecasting

Because of the prevailing network effect in the railway industry, leg-level demand forecasts may not be sufficient to set efficient capacity allocations. The network capacity allocation models require that also demand forecasts are compiled in the network level (Walczak et al., 2012). As mentioned in Section 2.2.3 the network effect makes the capacity allocation problem extremely complex and thus also the forecasting process becomes more challenging.

Walczak et al. (2012) present how the network effect creates challenges for demand forecasting. Perhaps the most significant complication is that the number of necessary forecasts increases substantially. This complication can be illustrated by a simple example. Let's consider a railway network, comprising seven stations (Figure 18).

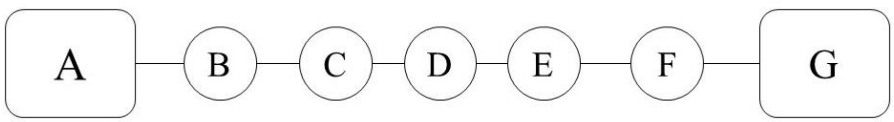

Figure 18 – A simple railway network.

If a train travels from A to G without making any intermediate stops, the itinerary A – G can be considered as a single-leg and a demand forecast only for leg A – G would be needed to allocate capacity efficiently. But if the train stops at all five intermediate stations, the itinerary A – G should be considered as a network, which consists of six separated legs (A – B, B – C … F – G). These legs then form 21 different OD pairs (A – B, A – C, A – D … E – G, F – G). Thus in order to make efficient capacity allocation for this train, 21 different demand forecasts should be performed. Additionally, if we take into consideration multiple fare classes, the number of forecasts needed increases even more. Also the second dimension of railway network (presented in Figure 10) would increase the number of necessary forecasts.

But while the number of forecasts increases in the network context, the forecasted demands for each OD pairs decreases (Walczak et al., 2012). The problem is that the demand for a certain OD can be so small that it would be impossible to draw accurate demand forecast for that itinerary. Walczak et al. (2012) propose that this problem can be addressed by forecasting only the most significant ODs directly while forecasting demands for less demanded ODs at more aggregate level.

Third challenge relates to demand unconstraining. In a network system, the demand unconstraining process is far more complicated than in a single-leg system because the demand can be constrained by the booking limits on one or more of the legs that are overlapped. Hence all leg-level constraints over the network at any given time must be tracked, so that the network unconstraining can be performed. (Walczak et al., 2012)

5 Illustrative examples

Following sections give illustrative examples of demand forecasting and capacity allocation and their relationship in RM system. Examples consider singe-leg OD forecasting and capacity allocation problems and do not take account of network effects. Because of limited availability of real demand data, I use hypothetical booking histories to represent how different forecasting methods can be applied. I also show how the performance of different methods can be evaluated. Additionally, I show how demand forecasts would impact on capacity allocation.

The main objective is to show how different forecasting methods are applied. I perform demand forecasts using four types of methods: *moving average, exponential smoothing, regression,* and *pickup methods*. These methods differ in whether they use only historical booking data or also available information on advanced bookings. The performance of each method is then evaluated using basic measures of forecast errors: *mean error, mean squared error,* and *mean absolute percentage error.* Since forecasts are based on the hypothetical booking data, these evaluations do not provide any conclusions on the performance of these methods for real demand data. They are only presented in order to illustrate how the performance of different methods should be evaluated with real data.

In addition to demand forecasts, I show how capacity allocation can be conducted using EMSR – model. The objective of this example is to illustrate the link between demand forecasting and capacity allocation. The example pictures how demand environment effects on capacity allocation.

5.1 Data

The data that is used in demand forecasting examples is based on hypothetical booking history matrix presented by Wickham (1995) (Table 2). The booking matrix represents unconstrained booking profiles over 18 weeks period for single booking class for single-leg OD itinerary. Rows in the matrix represent booking profiles for each departure and columns show the number of bookings on hand at particular day before the departures.

Week	DB0	DB7	DB14	DB21	DB28	DB35	DB42	DB49	DB56
1	25	22	10	5	3	3	2	0	0
2	30	21	17	15	12	7	3	1	0
3	25	23	14	9	8	5	5	2	1
4	40	34	30	16	11	6	3	0	0
5	35	29	20	13	12	8	3	1	0
6	39	33	30	21	14	6	4	2	1
7	45	28	22	18	10	5	3	0	0
8	50	42	18	11	10	7	4	2	1
9	33	29	21	15	9	8	6	6	2
10	46	40	29	22	11	7	3	2	0
11	49	37	25	17	10	9	8	5	2
12	25	22	10	5	3	3	2	0	0
13	30	21	15	17	12	7	3	1	0
14	23	22	14	9	8	5	5	2	1
15	40	34	30	16	11	6	3	0	0
16	35	29	20	13	12	8	3	1	0
17	39	33	30	21	14	6	4	2	1
18	45	28	22	18	10	5	3	0	0

Table 2 – Hypothetical booking history matrix (Wickham, 1995).

In order to determine the performance of different methods, the forecasted bookings should be compared to the actual booking values. Therefore the historical booking data is divided into historical and future departures by setting some artificial present week. Week 10 is chosen to be a present week and thus it is expressed as Week 0 in Table 3, which represents the division of the original booking matrix.

All negative week numbers are considered past departures and booking data above the Week 0 thick line represents historical booking data. The positive week numbers denotes the future departures so that e.g. Week 5 represents a train, which departs in five weeks. The shaded cells under the Week 0 line picture advanced bookings of trains that are not yet departed. The bookings in the unshaded cells are bookings that are not yet recorded because they are considered to take place in the future. They are used to evaluate the forecast performance of different methods. Thus all shaded cells together represent available data that can be used forecasting at present time (Week 0). The procedure is similar to the method that Wickham (1995) uses in his thesis.

Week	DB0	DB7	DB14	DB21	DB28	DB35	DB42	DB49	DB56
-9	25	22	10	5	3	3	2	0	0
-8	30	21	17	15	12	7	3	1	0
-7	25	23	14	9	8	5	5	2	1
-6	40	34	30	16	11	6	3	0	0
-5	35	29	20	13	12	8	3	1	0
-4	39	33	30	21	14	6	4	2	1
-3	45	28	22	18	10	5	3	0	0
-2	50	42	18	11	10	7	4	2	1
-1	33	29	21	15	9	8	6	6	2
0	**46**	**40**	**29**	**22**	**11**	**7**	**3**	**2**	**0**
1	49	37	25	17	10	9	8	5	2
2	25	22	10	5	3	3	2	0	0
3	30	21	15	17	12	7	3	1	0
4	23	22	14	9	8	5	5	2	1
5	40	34	30	16	11	6	3	0	0
6	35	29	20	13	12	8	3	1	0
7	39	33	30	21	14	6	4	2	1
8	45	28	22	18	10	5	3	0	0

Table 3 – Division of booking matrix.

Grey lines in Figure 19 illustrate historical booking profiles and the black line is the average of historical profiles. The descriptive statistics of historical bookings are given in Table 4.

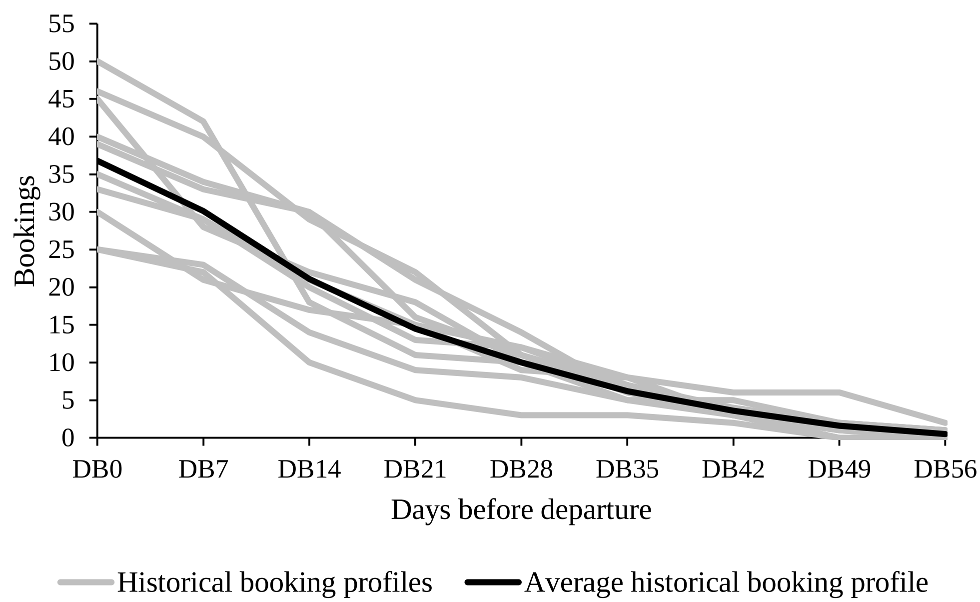

Figure 19 – Historical booking profiles.

Statistic	*DB0*	*DB7*	*DB14*	*DB21*	*DB28*	*DB35*	*DB42*	*DB49*	*DB56*
Mean	37	30	21	15	10	6	4	2	1
Std. Error	2,76	2,28	2,17	1,66	0,94	0,49	0,37	0,56	0,22
Median	37	29	21	15	11	7	3	2	0
Std. Dev.	8,72	7,22	6,85	5,25	2,98	1,55	1,17	1,78	0,71
Variance	75,96	52,10	46,99	27,61	8,89	2,40	1,38	3,16	0,50
Min.	25	21	10	5	3	3	2	0	0
Max.	50	42	30	22	14	8	6	6	2

Table 4 – Descriptive statistics of historical bookings.

5.2 Forecasting

Using the data in Table 3, demand forecasts are compiled in two different scenarios using selected methods. Table 5 presents selected methods and necessary model specifics.

	Method	Model specifics
Historical methods	Moving average	Last 10 observations are used: n = 10
	Exponential smoothing (α=0,5)	Smoothing parameter = 0,5 Average of last 10 observations is used as a last forecast (Ft)
	Exponential smoothing (α=0,2)	Smoothing parameter = 0,2 Average of last 10 observations is used as a last forecast (Ft)
Advanced methods	Additive pickup	Classical additive pickup
	Multiplicative pickup	Classical multiplicative pickup
	Regression (DB21)	Dependent variable: Bookings at DB0 Independent variable: Bookings at DB21

Table 5 – Selected forecasting methods and specifics.

5.2.1 Scenario 1

In the first scenario, the objective is to compile forecasts of final bookings for the next eight weeks. The forecast of final bookings for each week is constructed at present time (Week 0) using all available booking data at that time. Selected methods are divided into two groups, historical and advanced methods, depending on which booking data they use in forecasting (Table 5). Table 6 illustrates the difference between methods using Week 3 as an example. Grey shaded areas represent booking data used in forecasting for both groups.

Historical methods				
Week	**DB0**	**DB7**	**DB14**	**DB21**
-9	25	22	10	5
-8	30	21	17	15
-7	25	23	14	9
-6	40	34	30	16
-5	35	29	20	13
-4	39	33	30	21
-3	45	28	22	18
-2	50	42	18	11
-1	33	29	21	15
0	46	40	29	22
1		37	25	17
2			10	5
3	**F**			17

Advanced methods				
Week	**DB0**	**DB7**	**DB14**	**DB21**
-9	25	22	10	5
-8	30	21	17	15
-7	25	23	14	9
-6	40	34	30	16
-5	35	29	20	13
-4	39	33	30	21
-3	45	28	22	18
-2	50	42	18	11
-1	33	29	21	15
0	46	40	29	22
1		37	25	17
2			10	5
3	**F**			17

Table 6 – Scenario 1: Data used in forecasts of Week 3 final bookings.

As can be seen, advanced methods use the number of bookings at DB21 in current week to forecast DB0 bookings. Table 7 gives the specifics of the forecasts of Week 3 bookings.

Forecasts of Week 3 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	37	30	-7	46	23 %
Exponential smoothing (α=0,5)	41	30	-11	130	38 %
Exponential smoothing (α=0,2)	39	30	-9	75	29 %
Additive pickup	39	30	-9	86	31 %
Multiplicative pickup	43	30	-13	173	44 %
Regression (DB21)	39	30	-9	86	31 %

Table 7 – Scenario 1: Forecasts and errors for Week 3.

Similar forecasts are compiled for each week separately (Appendix 1). The performance of different methods can be evaluated using methods presented in Section 4.5.1. Both MSE and MAPE indicate that additive pickup method works best in this hypothetical data (Table 8). Also regression (DB21) performs well, but it has to be taken account that it can be compiled only for first three weeks, since the number of bookings on hand at DB21 is needed to compile regression forecast. Both exponential smoothing methods perform weakly. It is not surprising because exponential smoothing is designed to be used with long time series and moving forecast process. Multiplicative pickup model gets the largest MSE, 149, which is result of extremely bad forecast in Week 8. If forecast of Week 8 is omitted, the MSE of multiplicative pickup decreases significantly and the performance becomes nearly as good as additive pickup model.

Forecasts performance over 8 weeks: Ranked by MSE			
Method	**ME**	**MSE**	**MAPE**
Additive pickup	0	43	16 %
Regression (DB21)*	-1	62	20 %
Moving average (n=10)	-1	76	24 %
Exponential smoothing (α=0,2)	-3	84	25 %
Exponential smoothing (α=0,5)	-6	107	29 %
Multiplicative pickup	-4	149	27 %
* Forecasts only for first three weeks			

Table 8 – Scenario 1: Forecast performance.

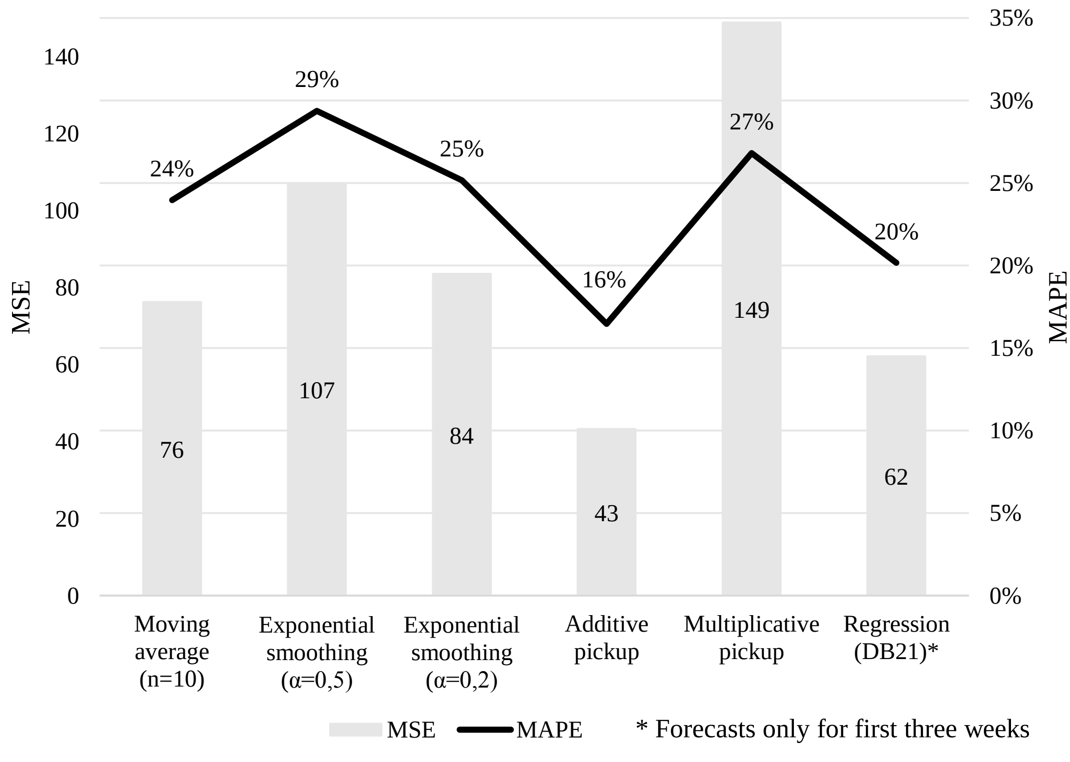

Figure 20 – Scenario 1: Forecast performance.

5.2.2 Scenario 2

While in Scenario 1 the forecasts of final bookings for the next eight weeks are compiled at static "present time" (Week 0), in Scenario 2 the forecasts of the same eight weeks are compiled but in moving "present time". Thus the objective is to produce forecast of final bookings for next period. For example the forecast of Week 3 final bookings is compiled at Week 2 using all available data at that time. This type of modification gives better illustration on how methods might work dynamically in long-term. Same six methods than in Scenario 1 are used to compile the forecasts of final bookings. Again Week 3 is used as an example to illustrate how historical and advanced methods differ (Table 9).

Historical methods				
Week	**DB0**	**DB7**	**DB14**	**DB21**
-9	25	22	10	5
-8	30	21	17	15
-7	25	23	14	9
-6	40	34	30	16
-5	35	29	20	13
-4	39	33	30	21
-3	45	28	22	18
-2	50	42	18	11
-1	33	29	21	15
0	46	40	29	22
1	49	37	25	17
2	25	22	10	5
3	**F**	21	15	17

Advanced methods				
Week	**DB0**	**DB7**	**DB14**	**DB21**
-9	25	22	10	5
-8	30	21	17	15
-7	25	23	14	9
-6	40	34	30	16
-5	35	29	20	13
-4	39	33	30	21
-3	45	28	22	18
-2	50	42	18	11
-1	33	29	21	15
0	46	40	29	22
1	49	37	25	17
2	25	22	10	5
3	**F**	21	15	17

Table 9 – Scenario 2: Data used in forecasts of Week 3 final bookings.

The forecasts of Week 3 final bookings and corresponding errors are presented in Table 10.

Forecasts of Week 3 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	39	30	-9	76	29 %
Exponential smoothing (α=0,5)	35	30	-5	26	17 %
Exponential smoothing (α=0,2)	38	30	-8	57	25 %
Additive pickup	28	30	2	4	7 %
Multiplicative pickup	26	30	4	19	15 %
Regression (DB21)	39	30	-9	84	31 %

Table 10 – Scenario 2: Forecasts and errors for Week 3.

The forecasts and errors for other seven weeks are presented in Appendix 2. The performance of historical and advanced methods are clearly different in Scenario 2 using the hypothetical data (Table 11). Advanced methods appear to perform significantly better than methods that rely only on historical data, when compiling the forecasts of the final bookings for the next period.

Forecast performance over 8 weeks: Ranked by MSE			
Method	**ME**	**MSE**	**MAPE**
Multiplicative pickup	1	25	11 %
Additive pickup	0	28	12 %
Regression (DB21)	-1	35	14 %
Exponential smoothing (0,2)	-1	91	27 %
Moving average (n=10)	-2	96	27 %
Exponential smoothing (0,5)	0	103	27 %

Table 11 – Scenario 2: Forecast performance.

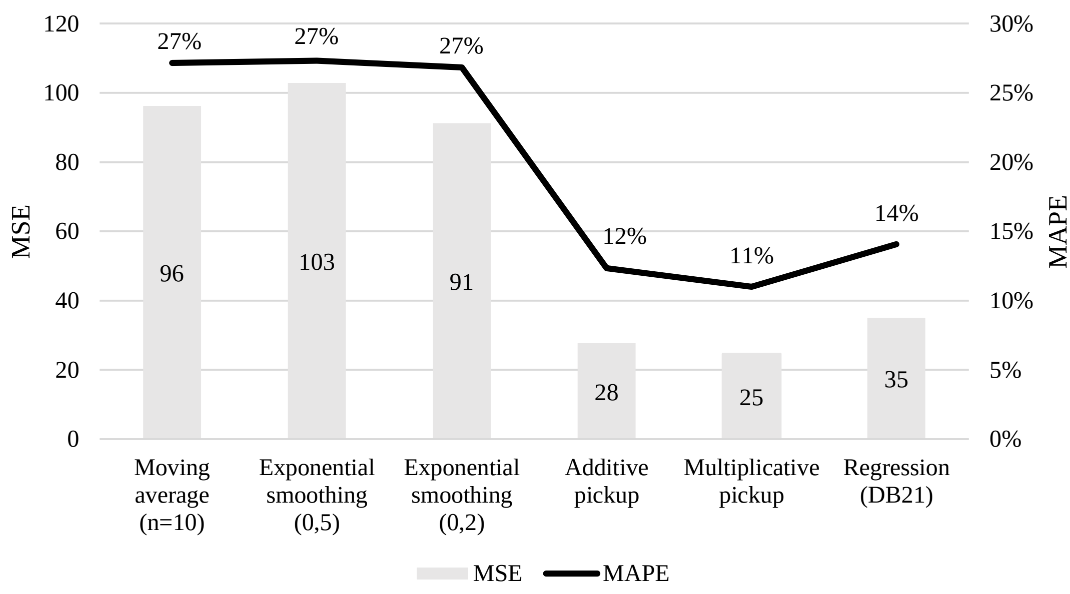

Figure 21 – Scenario 2: Forecast performance.

5.3 Capacity allocation

In this section, the effects of demand forecasts on capacity allocation have been illustrated using EMSR –model. Single-leg capacity allocation with four fare classes is compiled under four different demand cases. Four nested fare classes with average fares are Business (100€), Economy (60€), Discount (30€) and Super discount (20€). The average fares of each class are assumed to be stable in all cases. This assumption indicates that price changes cannot be used to stimulate demand. Additionally, the total capacity is fixed to 150 seats. Capacity is allocated to these fare classes assuming that the general assumptions presented in Section 2.2 hold. Demand for each fare class is also supposed to be independent between fare classes. This denotes that each passenger is demanding only one fare class. Although this is extremely theoretical assumption, it is necessary in order to present capacity allocation in such a simplistic form. Four demand cases are described next.

Case 1:

In the first case, forecasted aggregate demand of all fare classes equals the total capacity of imaginary train. Demand forecasts also suggest that the demands for higher fare classes (B and E) will be lower than for lower fare classes (D and S). (Table 12)

Case 1	Business (B)	Economy (E)	Discount (D)	Super discount (S)
Fare	100 €	60 €	30 €	20 €
Mean	15	30	50	55
Std. Dev.	5	10	17	19
Forecasted total demand: 150				

Table 12 – Case 1 details.

Case 2:

In the second case, forecasted aggregate demand equals the total capacity, but now the forecasted total demand is evenly distributed across the fare classes. (Table 13).

Case 2	Business (B)	Economy (E)	Discount (D)	Super discount (S)
Fare	100 €	60 €	30 €	20 €
Mean	37,5	37,5	37,5	37,5
Std. Dev.	5	5	5	5
Forecasted total demand: 150				

Table 13 – Case 2 details.

Case 3:

The third case represents the situation where forecasted aggregate demand exceeds the total capacity. Similarly to the first case, the forecasted demands for lower fare classes are higher than for higher fare classes. (Table 14)

Case 3	**Business (B)**	**Economy (E)**	**Discount (D)**	**Super discount (S)**
Fare	100 €	60 €	30 €	20 €
Mean	30	60	100	150
Std. Dev.	5	15	17	20
Forecasted total demand: 340				

Table 14 – Case 3 details.

Case 4:

The last case pictures the demand environment where the total demand is lower than the total capacity. Like in cases 1 and 3, the forecasted demands for lower fare classes are higher than for higher fare classes. (Table 15)

Case 4	**Business (B)**	**Economy (E)**	**Discount (D)**	**Super discount (S)**
Fare	100 €	60 €	30 €	20 €
Mean	8	30	43	55
Std. Dev.	3	9	12	17
Forecasted total demand: 136				

Table 15 – Case 4 details.

Basic EMSR –model is used to construct capacity allocations for all four cases. Protection levels between fare classes are determined respectively and these distinct protection levels are then summed to obtain the joint protection levels for classes B, E and D. Booking limits for each fare class are then compiled subtracting the joint protection level of higher fare class from the total capacity. For example in Case 1, the booking limit for class D can be expressed:

$$Booking\ limit_D = Total\ Capacity - \ Joint\ protection\ level_D^i \tag{5.1}$$

$$Booking\ limit_D = 150 - (14 + 34) = 102 \tag{5.2}$$

More detailed description of EMSR –allocations are presented in Appendix 3.

Figure 22 pictures capacity protection levels for Cases 1 and 2 and Figure 23 shows respective booking limits. These figures illustrate well how capacity allocations depend on how total demand is distributed between fare classes. When demands for lower fare classes are significantly higher than demands for higher fare classes, more capacity should be allocated to lower fares.

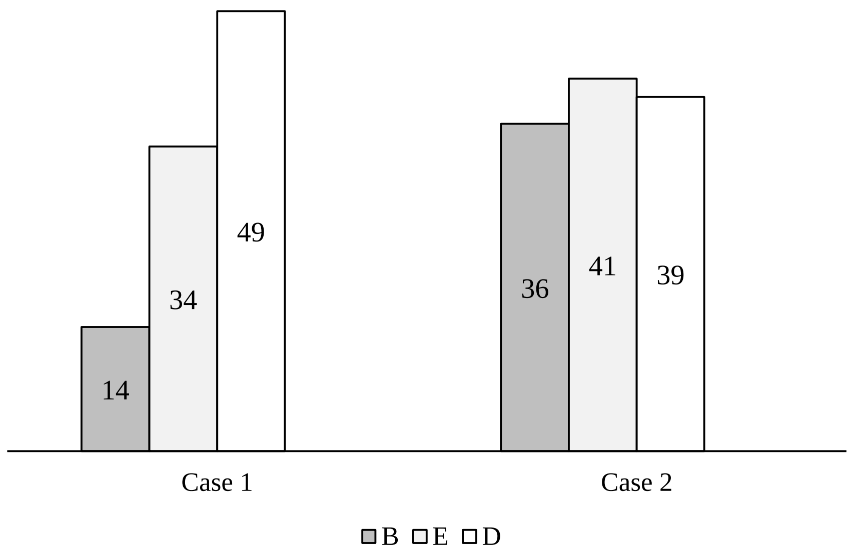

Figure 22 – Protected capacity from lower fare classes (Case 1 & 2).

For example, booking limits for classes D and S are considerable larger in Case 1 than Case 2 (Figure 23). For example, in Case 1 the booking limit of class D is 103 and the corresponding booking limit in Case 2 is only 73.

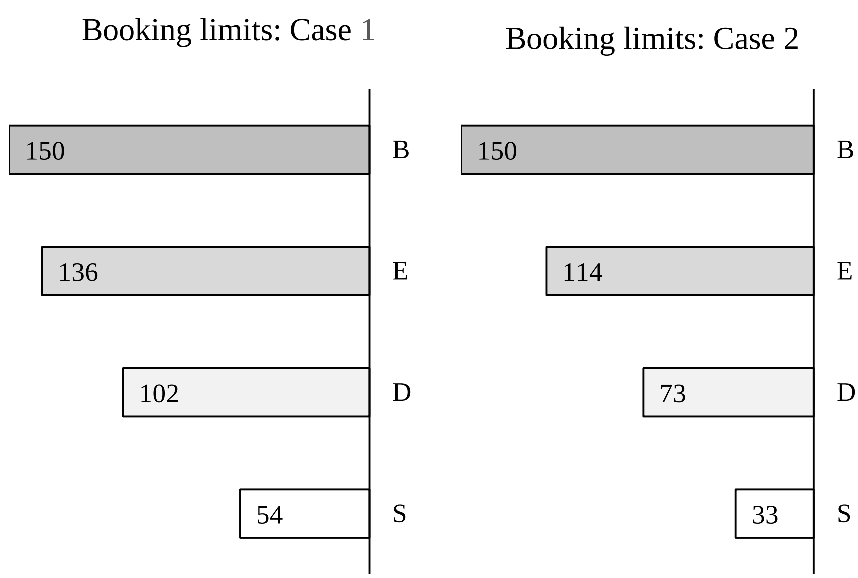

Figure 23 – Booking limits (Case 1 & 2).

Capacity protection levels and booking limits in Cases 3 and 4 reflect another considerable issue of demand forecasts. While the forecasted total demand is quite evenly distributed across fare classes in Cases 3 and 4, the forecasted total demands are significantly different. Case 3 can be seen as a peak period. The total demand is expected to be higher than the total capacity. Because of this excess demand, the expected revenues are maximized if more capacity is allocated to higher fare classes. In Case 3 nearly one thirds of the capacity (93 seats) is allocated to classes B and E (Figure 24). Actually, the expected demands for higher classes in Case 3 are so large that capacity allocation model suggests that no capacity should be allocated to the lowest fare class S. (Figure 25) On the contrary, Case 4 can be considered as a low demand state, since the forecasted total demand is less than the total capacity. In order to maximize expected revenues and load factor, capacity allocation model suggests that less capacity should be protected to higher fare classes. Thus in Case 4 only 45 seats are protected to higher classes B and E (Figure 24).

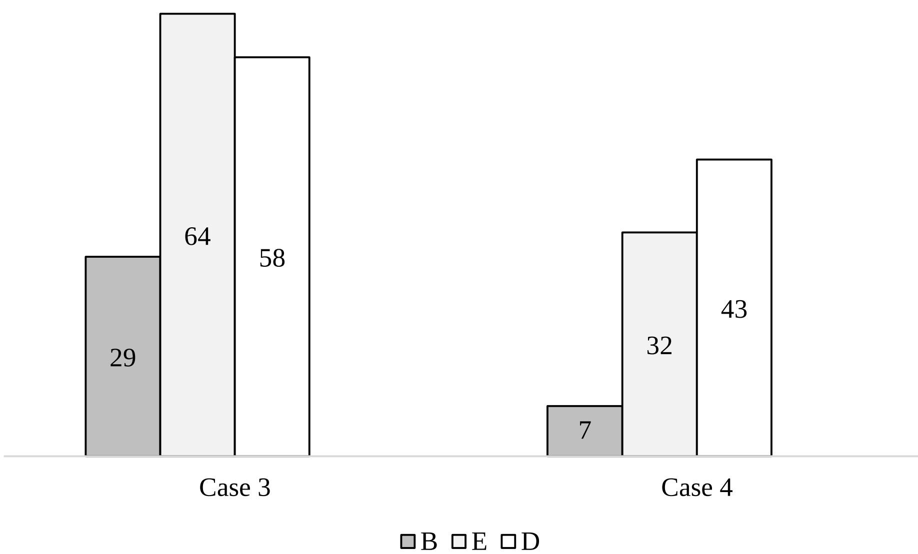

Figure 24 – Protected capacity from lower fare classes (Case 3 & 4).

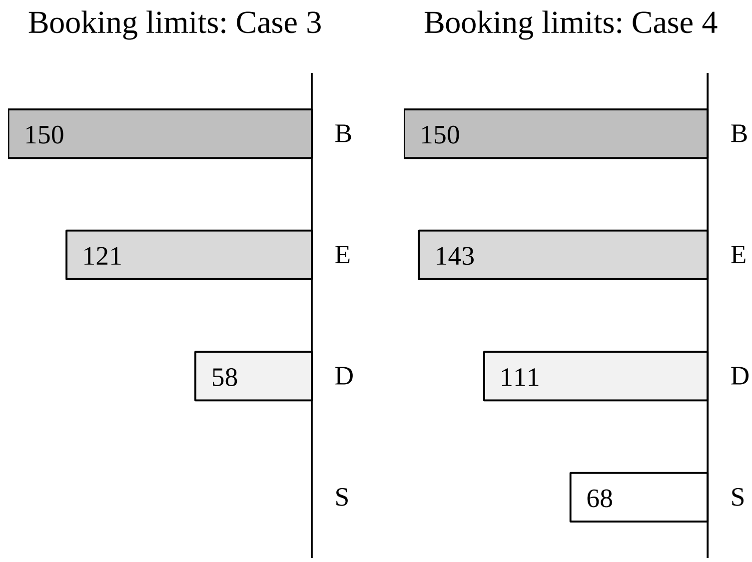

Figure 25 – Booking limits (Case 3 & 4).

5.4 Conclusions of examples

None of previous results can be interpreted as generalized results, since those are totally based on the hypothetical booking data. Nevertheless, these examples reflect some significant issues related to demand forecasting in RM.

Firstly, the time horizon of necessary forecasts has definite effect on method selection. Some methods might be suitable for long-term forecasting while some others are more suitable for forecasting in shorter-term. Thus the evaluation of different possible methods is essential in the selection of the most suitable method for each particular forecasting process. It is especially important to evaluate, could forecast performance be improved by utilizing advanced forecasting method when advanced booking data is available. It is evident that in my hypothetical data, advanced methods provide explicitly better results than methods that only rely on historical booking data.

Second implication is that demand forecasts should be performed continuously. New corrective forecasts should be compiled dynamically when new information on demand becomes available. This is supported by the demand forecast examples, since the booking forecasts appear to be more precise in Scenario 2 compared to the forecasts in Scenario 1. Also this implication relates to the use of advanced booking information.

Thirdly, the EMSR – capacity allocation examples demonstrate two reasons why demand forecasting plays such a significant role in the RM systems. In order to compile effective capacity allocations, demand forecasts should be able to give precise predictions of the total demand and the distribution of total demand across fare classes. Cases 1 and 2 illustrate how the distribution of total demand affects capacity allocation. When the forecasted total demand equals the total capacity, the number of allocated seats for a specific fare class corresponds to the relative shares of total demand of that particular class. Cases 3 and 4 account for the impacts of the forecasted total demand. In order to optimize capacity allocation under excess demand the capacity allocation model allocates more capacity to higher fares. On the contrary, in scarce demand case more capacity is allocated to lower fare classes so that higher load factor and thus higher revenues can be attained.

6 Conclusions

More than 30 years RM has gathered lots of interest among researchers in different subjects. RM has been studied from the perspective of economics, transportation research and operation research. The fundamental economic concepts, willingness-to-pay, price discrimination and product differentiation, form the ground for RM and RM research for other subjects. The majority of studies are focused on the airline industry, most likely because of the importance of the industry in the U.S. Demand forecasting has been one of the main research areas in RM because of its tight relations to other RM components; capacity allocation, overbooking and pricing.

The purpose of this thesis was to investigate demand forecasting in the context of railway RM. At first I answered two questions, what is RM and how is it applicable to railway industry, based on the existing literature. Then demand forecasting issues were covered in the context of railway RM. Finally, I performed hypothetical demand forecasting and capacity allocation examples and illustrated the specifics of demand forecasting and its impacts on capacity allocations.

Even though RM is generally linked to airlines, I find that actually it is applicable to many service industries. Like Kimes (1989) presents, RM is basically a tool for all capacity constrained service firms to manage their inventories. Studies are widely agreed on the six general conditions that are necessary to effective applicability of RM. A firm employing RM has to have relatively fixed capacity in short-term and it must be able to somehow segment its customers according to their preferences. The products are also considered perishable and the purchases of the products occur before the actual consumption. Additionally, demand can be seen changing substantially over time and these changes happen with some level of uncertainty. Finally, because the capacity is considered fixed in short-term, the marginal cost of additional consumer is low and the costs of capacity changes are high.

Taking account of these conditions it seems evident that RM is applicable in railway markets. This is in line with the study of Armstrong and Meissner (2010) and with the fact that many railway operators have used different RM techniques several decades. Railways and airlines appear to function rather similar market environments, but also a few significant differences exist. Thus it is important to evaluate the differences between these industries and also consider how these differences might effect on RM. I identified four main differences. Firstly, the network effect is more dominant in railway markets because of the two dimensional network structure. This creates highly complex forecasting and capacity allocation processes and thus efficient RM applications are even more difficult to construct than in airline markets. Secondly, railway operators have historically used more product differentiation techniques and less weight has been put on price differentiation methods. This is a result of differences in passenger mixes between routes. Third difference relates to the conventions of ticket purchasing between airlines and railways. Even if the development of information technology and self-service selling platforms in the railway industry has increased the number of advanced ticket purchases, the number of passengers who buy tickets just before the departure is still significant in railway markets. This is probably the most distinctive consideration between these two industries in RM context, since the advanced purchase restrictions have traditionally been one of the main passenger segmentation method used by the airlines. The last issue relates to the differences in competitive environments. In many countries railway operators are monopolized by the governments and despite regulatory changes, especially in Europe, movement towards more competitive markets has been slow. One significant reason for this slow change of market environment is extremely high entry costs in the railway industry. These concentrated markets indicate that railway operators might have higher pricing power than airlines have in furiously competitive airline markets. This suggests that railways might actually have more potential to utilize RM techniques than airlines, since price differentiation is more powerful under monopoly markets.

As RM techniques are shown to be effective in the railway industry, also the importance of demand forecasting is apparent. Like demand forecasting in general, also railway demand forecasting faces some considerable difficulties. One significant problem relates to the constrained demand data. The methods of how this problem can be resolved are out of my topic, but it is extremely important to understand how constrained demand data might have effects on demand forecasts. I focused on the differences between forecasting methods and presented some methods of how the performance of these methods can be evaluated. The main difference between methods is how they use available demand data in forecasting process. The illustrative examples indicate that the time horizon of forecasts relates to the performance of different methods. Advanced methods seem to perform relatively better than historical models when short-term forecasts are compiled dynamically to the next period. However, it has to be emphasized that any generalizations cannot be drawn as hypothetical booking data was used in forecasting instead of real booking information.

Capacity allocation examples highlight other important issues. The effects of the forecasted total demand and demand distribution between fare classes on capacity allocation, and thus revenue optimization, are notable. Even if only EMSR –model was used to allocate capacity in the illustrative example, also other capacity allocation models, such as probabilistic linear programming, require information on the total demand and demand distribution between fare classes. Examples demonstrate how significantly capacity allocations can differ between demand states. Thus it is extremely important that demand forecasting process manages to identify fluctuations in demand over time. Especially important is to identify abnormal demand states, such as outstandingly high and low demands.

In this thesis, an extensive overview of RM applications and demand forecasting in a railway RM system was compiled. However, the results involve some serious drawbacks that should be taken into account. The most notable drawback relates to the empirical demonstration. As the data used in the illustrative examples is completely hypothetical, any general conclusions could not be produced. This thesis only manages to point out some significant issues that should be considered in demand forecasting process. Further empirical analysis with real demand data should be needed to give more precise conclusions on which forecasting method should be applied in which circumstances. Additionally, the issue of constrained demand data could not be covered sufficiently and therefore more research on demand unconstraining methods with real railway demand data should be needed. Third significant drawback relates to existing network effects in railway markets. In this thesis capacity allocation and demand forecasting processes were mainly discussed under single-leg environment. Because the network effect is evidently relevant in railway markets, even more outstanding than in airline markets, it should be definitely incorporated into further study.

The last consideration relates to the level of forecasting. The existing literature on RM demand forecasting, as well this thesis, is mainly interested in micro-level demand forecasting. Since railways compete with other transportation modes, it would be beneficial to incorporate passenger choice models to RM demand forecasting. This is especially relevant issue at the moment in Finland because the deregulation of the bus industry has increased substitutive competition significantly and thus some competitive counteractions should be done in the railway industry.

References

Alderighi, M. (2010). Fare dispersions in airline markets: A quantitative assessment of theoretical explanations. *Journal of Air Transport Management 16*, 144-150.

Alstrup, J., Boas, S., Madsen, O. B., & Vidal, R. V. (1986). Booking policy for flights with two types of passengers. *European Journal of Operations Research, vol. 27*, 274-288.

Armstrong, A., & Meissner, J. (2010). Railway revenue management: overview and models. *Working paper*.

Belobaba, P. P. (1987). *Air Travel Demand And Airline Seat Inventory Management.* PhD Thesis, Flight Transportation Laboratory, MIT.

Belobaba, P. P. (1992). Optimal vs. heuristic methods for nested seat allocation. *Presentation at ORSA/TIMS Joint National Meeting.*

Belobaba, P. P. (2009). Fundamentals of Pricing and Revenue Management. In P. Belobaba, A. Odoni, & C. Barnhard, *The Global Airline Industry*. John Wiley & Sons, Ltd.

Ben-Akiva, M. E., & Lerman, S. R. (1985). *Discrete choice analysis: theory and application to travel demand.* Vol. 9, MIT press.

Ben-Khedher, N., Kintanar, J., Queille, C., & Stripling, W. (1998). Schedule Optimization at SNCF: From Conception to Day of Departure. *Interfaces 28(1)*, 6-23.

Bharill, R., & Rangaraj, N. (2008). Revenue management in railway operations: A study of the Rajdhani Express, Indian Railways. *Transportation Research Part A 42*, 1195-1207.

Bitran, G., & Caldentey, R. (2003). An Overview of Pricing Models for Revenue Management. *Manufacturing & Service Operations Management 5(3)*, 203-229.

Borenstein, S., & Rose, N. L. (1994). Competition and Price Dispersion in the U.S. Airline Industry. *Journal of Political Economy, Vol. 102, No. 4*, 653-683.

Botimer, T. C., & Belobaba, P. P. (1999). Airline pricing and fare product differentiation: A new theoretical framework. *The Journal of the Operation Research Society, Vol. 50, No. 11, Yield Management*, 1085-1097.

Brumelle, S. L., & McGill, J. I. (1993). Airline Seat Allocation with Multiple Nested Fare Classes. *Operation Research, Vol. 41, No. 1*, 127-137.

Chuang, H. M., Chu, C. P., & Niu, W. F. (2010). A study on revenue management of Taiwan High Speed Railway. *Industrial Engineering and Engineering Management (IEEM)*, (pp. 999-1003).

Ciancimino, A., Inzerillo, G., Lucidi, S., & Palagi, L. (1999). A mathematical programming approach for the solution of the railway yield management problem. *Transportation science 33 (2)*, 168-181.

Coase, R. H. (1972). Durability and Monopoly. *Journal of Law and Economics, Vol. 15, No. 1*, 143-149.

Cooper, W. L., Homem-de-Mello, T., & Kleywegt, A. J. (2006). Models of the Spiral-Down Effect in Revenue Management. *Operations Research, Vol. 54, No. 5*, 968-987.

Cross, R. G. (1998). *Revenue Management: hard-core tactics for market domination.* Broadway Books.

Dana, J. D. (1999). Equilibrium Price Dispersion under Demand Uncertainty: The Roles of Costly Capacity and Market Structure. *The RAND Journal of Economics, Vol. 30, No. 4*, 632-660.

European Commission. (1991). *Directive 91/440*. Brussels: Official Journal of the European Communities, No L 237/25.

Friebel, G., Ivaldi, M., & Vibes, C. (2010). Railway (De)regulation: A European Efficiency Comparison. *Economica, 77(305)*, 77-91.

Gallego, G., & van Ryzin, G. (1997). A Multiproduct Dynamic Pricing Problem and Its Applications to Network Yield Management. *Operations Research, Vol. 45, No. 1*, 24-41.

Glover, F., Glover, R., Lorenzo, J., & McMillan, C. (1982). The passenger-mix problem in the scheduled airlines. *Interface 12.3*, 72-80.

Guo, P., Xiao, B., & Li, J. (2012). Unconstraining Methods in Revenue Management Systems: Research Overview and Prospects. *Hindawi Publishing Corporation: Advances in Operations Research*.

Holt, C. C. (2004). Forecasting seasonals and trends by exponentially weighted moving averages. *International Journal of Forecasting 20*, 5-10.

Kanafani, A. (1983). *Transportation demand analysis.* McGraw-Hill, Incorporated.

Kimes, S. E. (1989). Yield Management: A Tool for Capacity-Constrained Service Firm. *Journal of Operations Management Vol. 8, No. 4*, 348-363.

Lee, A. O. (1990). *Airline Reservations Forecasting: Probabilistic and Statistical Models of the Booking Process.* PhD Thesis, Flight Transportation Laboratory, MIT.

Link, H. (2004). Pep - a yield management scheme for rail passenger fares in Germany. *Japan Railways & Transport Review 38(3)*, 50-55.

Littlewood, K. (1972). *Forecasting and Control of Passenger Bookings.* Terrigal, Israel: Agifors Symposium Proc.

Makridakis, S., Wheelwright, S. C., & Hyndman, R. J. (1998). *Forecasting methods and applications - 3rd ed.* John Wiley & Sons, Inc.

McGill, J. I. (1989). *Optimization and Estimation Problems in Airline Yield Management.* PhD Thesis, Faculty of Commerce and Business Administrations, University of British Columbia.

McGill, J. I., & van Ryzin, G. J. (1999). Revenue Management: Research Overview and Prospects. *Transportation Science 33 (2)*, 233-256.

Mitev, N. (1996). More than a failure? The computerized reservation systems at French Railways. *Information Technology & People, Vol. 9*, 8-19.

Mitev, N. (2004). Trains, planes and computers: from high-speed trains to computerized reservation systems at French railways. *Journal of transport history, 25 (2)*, 101-123.

Pak, K., & Piersma, N. (2002). *Airline Revenue Management: An Overview of OR Techniques 1982-2001.* ERIM Report Series Research in Management.

Phlips, L. (1983). *The Economics of Price Discrimination.* Cambridge: the Press Syndicate of the University of Cambridge the Pitt Building.

Rao, P. S. (1978). Forecasting the demand for railway freight services. *Journal of Transport Economics and Policy*, 7-26.

Rasouli, S., & Timmermans, H. (2012). Uncertainty in travel demand forecasting models: literature review and research agenda. *Transportation Letters 4.1*, 55-73.

Rothstein, M. (1971). An airline overbooking model. *Transportation Science 5.2*, 180-192.

Sa, J. (1987). *Reservation forecasting in airline yield management.* Master's Thesis, Flight Transportation Laboratory, MIT.

Salch, J. (1997). Unconstraining passenger demand using the EM algorithm. *Proceedings of the INFORMS Conference.*

Sharif Azadeh, S. (2013). *Demand Forecasting in Revenue Management Systems.* PhD Thesis, École Polytechnique de Montréal.

Simpson, R. W. (1989). *Using Network Flow Techniques to Find Shadow Prices for Market and Seat Inventory Control.* Flight Transportation Laboratory, MIT.

Smith, B. C., Leimkuhler, J. F., & Darrow, R. M. (1992). Yield Management at American Airlines. *Interfaces*, (pp. 8-31).

Talluri , K., & van Ryzin, G. (2004b). Revenue Management Under a General Discrete Choice Model of Consumer Behavior. *Management Science 50(1)*, 15-33.

Talluri, K., & van Ryzin, G. (1998). An Analysis of Bid-Price Controls for Network Revenue Management. *Management Science, Vol 44., part 1*, 1577-1593.

Talluri, K., & van Ryzin, G. (2004). *The Theory and Practice of Revenue Management.* Kluwer Academic Publishers.

Taneja, N. K. (1978). *Airline traffic forecasting: A regression analysis approach.* Lexington Books.

Tsai, T. H., Lee, C. K., & Wei, C. H. (2005). Design of dynamic neural networks to forecast short-term railway passenger demand. *Journal of the Eastern Asia Society for Transportation Studies, Vol 6.*, 1651-1666.

Walczak, D., Boyd, A. E., & Cramer, R. (2012). Chapter 3 - Revenue Management. In C. Barnhart, & B. Smith, *Quantitative Problem Solving Methods in the Airline Industry - A Modeling Methodology Handbook.* Springer Science+Business Media, LLC.

van Ryzin, G., & Vulcano, G. (2008). Simulation-Based Optimization of Virtual Nesting Controls for Network Revenue Management. *Operations Research 56(4)*, 865-880.

Wang, Y., Lan, B., & Zhang, L. (2012). A Revenue Management Model for High-Speed Railway. *Proceedings of the 1st International Workshop on High-Speed and Intercity Railways* (pp. 95-103). Springer Berlin Heidelberg.

Wardman, M. (2006). Demand for rail travel and the effects of external factors. *Transportation Research Part E 42*, 129-148.

Varian, H. R. (1989). Price discrimination. In R. Schmalensee, & R. D. Willing, *Handbook of Industrial Organization, Volume I* (pp. 597-654). Elsevier Science Publisher B.V.

Weatherford, L. R., & Belobaba, P. P. (2002). Revenue Impacts of Fare Input and Demand Forecast Accuracy in Airline Management. *The Journal of the Operational Research Society, Vol. 53, No. 8*, 811-821.

Weatherford, L. R., & Bodily, S. E. (1992). A Taxonomy and Research Overview of Perishable-Asset Revenue Management: Yield Management, Overbooking, and Pricing. *Operations Research, Vol. 40, No. 5*, 831-844.

Weatherford, L. R., & Kimes, S. E. (2003). A comparison of forecasting methods for hotel revenue management. *International Journal of Forecasting, 19*, 401-415.

Weatherford, L. R., & Pölt, S. (2002). Better unconstraining of airline demand data in revenue management systems for improved forecast accuracy and greater revenues. *Journal of Revenue and Pricing Management, Vol. 1, No. 3,* 234-254.

Weatherford, L. R., Gentry, T. W., & Wilamowski, B. (2002). Neural network forecasting for airlines: A comparative analysis. *Journal of Revenue and Pricing Management, Vol. 1, No. 4,* 319-331.

Wickham, R. R. (1995). *Evaluation of Forecasting Techniques for Short-Term Demand of Air Transportation.* Master's Thesis, Flight Transportation Laboratory, MIT.

Williamson, E. L. (1992). *Airline Network Seat Inventory Control: Methodologies and Revenue Impacts.* PhD Thesis, Flight Transportation Laboratory, MIT.

Winters, P. R. (1960). Forecasting Sales by Exponentially Weighted Moving Averages. *Management Science, Vol. 6, No. 3,* 324-342.

VR Group. (2012). *Annual report 2011.* Helsinki: VR Group.

VR Group. (2013). *Annual report 2012.* Helsinki: VR Group.

VR Group. (2014). *Annual report 2013.* Helsinki: VR Group.

VR Group. (2015). *Annual report 2014.* Helsinki: VR Group.

Zeni, R. H. (2001). *Improved forecast accuracy in revenue management by unconstraining demand estimates from censored data.* PhD Thesis, The State University of New Jersey.

Appendices

Appendix 1: Forecasting Scenario 1

Formulations used in forecasts of moving average, exponential smoothing and pickup models are presented in Section 4.4.

Regression (DB21)

Simple ANOVA – regression using dependent variable Bookings at the departure and independent variable Bookings at 21 days before departure is estimated using regression tool in Excel. Booking data available at present time (Week 0) is used in estimation:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0,5944
R Square	0,3533
Adjusted R Square	0,2725
Standard Error	7,4335
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	241,5493	241,5493	4,3714	0,0699
Residual	8	442,0507	55,2563		
Total	9	683,6			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	22,5042	7,2303	3,1125	0,0144	5,8312	39,1772
Bookings DB21	0,9859	0,4716	2,0908	0,0699	-0,1015	2,0733

Estimated coefficients are β_0 = 22,5042 and β_1 = 0,9859. Thus the model used in demand forecasting is $Bookings_{DB0} = 22{,}5042 + 0{,}9859 * Bookings_{DB21}$.

According to P-values both coefficients are statistically significant at 10 % significance level. However low R^2 indicates that the estimated model fits quite badly to my hypothetical dataset. But because the purpose of these forecast examples is only to provide illustrations of each method, I will not consider the reasons of this problem more closely.

Forecasts and errors for each week in Scenario 1 using selected methods:

Forecasts of Week 1 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs % - Error**
Moving average (n=10)	37	49	12	149	25 %
Exponential smoothing (α=0,5)	41	49	8	58	16 %
Exponential smoothing (α=0,2)	39	49	10	107	21 %
Additive pickup	44	49	5	28	11 %
Multiplicative pickup	45	49	4	14	8 %
Regression (DB21)	39	49	10	95	20 %

Forecasts of Week 2 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs % - Error**
Moving average (n=10)	37	25	-12	139	47 %
Exponential smoothing (α=0,5)	41	25	-16	269	66 %
Exponential smoothing (α=0,2)	39	25	-14	186	55 %
Additive pickup	26	25	-1	0	3 %
Multiplicative pickup	17	25	8	57	30 %
Regression (DB21)	27	25	-2	6	10 %

Forecasts of Week 3 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	37	30	-7	46	23 %
Exponential smoothing (α=0,5)	41	30	-11	130	38 %
Exponential smoothing (α=0,2)	39	30	-9	75	29 %
Additive pickup	39	30	-9	86	31 %
Multiplicative pickup	43	30	-13	173	44 %
Regression (DB21)	39	30	-9	86	31 %

Forecasts of Week 4 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	37	23	-14	190	60 %
Exponential smoothing (α=0,5)	41	23	-18	339	80 %
Exponential smoothing (α=0,2)	39	23	-16	245	68 %
Additive pickup	35	23	-12	139	51 %
Multiplicative pickup	29	23	-6	41	28 %
Regression (DB21)	N/A	23	N/A	N/A	N/A

Forecasts of Week 5 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	37	40	3	10	8 %
Exponential smoothing (α=0,5)	41	40	-1	2	4 %
Exponential smoothing (α=0,2)	39	40	1	2	3 %
Additive pickup	37	40	3	12	9 %
Multiplicative pickup	36	40	4	19	11 %
Regression (DB21)	N/A	40	N/A	N/A	N/A

Forecasts of Week 6 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	37	35	-2	3	5 %
Exponential smoothing (α=0,5)	41	35	-6	41	18 %
Exponential smoothing (α=0,2)	39	35	-4	13	10 %
Additive pickup	36	35	-1	1	3 %
Multiplicative pickup	31	35	4	19	12 %
Regression (DB21)	N/A	35	N/A	N/A	N/A

Forecasts of Week 7 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	37	39	2	5	6 %
Exponential smoothing (α=0,5)	41	39	-2	6	6 %
Exponential smoothing (α=0,2)	39	39	0	0	1 %
Additive pickup	37	39	2	3	5 %
Multiplicative pickup	46	39	-7	49	18 %
Regression (DB21)	N/A	39	N/A	N/A	N/A

Forecasts of Week 8 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	37	45	8	67	18 %
Exponential smoothing (α=0,5)	41	45	4	13	8 %
Exponential smoothing (α=0,2)	39	45	6	40	14 %
Additive pickup	36	45	9	76	19 %
Multiplicative pickup	74	45	-29	818	64 %
Regression (DB21)	N/A	45	N/A	N/A	N/A

Forecasts and errors using selected methods in Scenario 1: Weeks 1 - 8

Moving average (n=10)					
Week	**Forecast**	**Actual**	**Error**	**Sq.Error**	**Abs. % - Error**
1	37	49	-12	149	25 %
2	37	25	12	139	47 %
3	37	30	7	46	23 %
4	37	23	14	190	60 %
5	37	40	-3	10	8 %
6	37	35	2	3	5 %
7	37	39	-2	5	6 %
8	37	45	-8	67	18 %
Mean	**37**	**36**	**1**	**76**	**24 %**

Additive pickup					
Week	**Forecast**	**Actual**	**Error**	**Sq.Error**	**Abs. % - Error**
1	44	49	-5	28	11 %
2	26	25	1	0	3 %
3	39	30	9	86	31 %
4	35	23	12	139	51 %
5	37	40	-3	12	9 %
6	36	35	1	1	3 %
7	37	39	-2	3	5 %
8	36	45	-9	76	19 %
Mean	**36**	**36**	**0**	**43**	**16 %**

Exponential smoothing (α=0,5)					
Week	**Forecast**	**Actual**	**Error**	**Sq.Error**	**Abs. % - Error**
1	41	49	-8	58	16 %
2	41	25	16	269	66 %
3	41	30	11	130	38 %
4	41	23	18	339	80 %
5	41	40	1	2	4 %
6	41	35	6	41	18 %
7	41	39	2	6	6 %
8	41	45	-4	13	8 %
Mean	**41**	**36**	**6**	**107**	**29 %**

Multiplicative pickup					
Week	**Forecast**	**Actual**	**Error**	**Sq.Error**	**Abs. % - Error**
1	45	49	-4	14	8 %
2	17	25	-8	57	30 %
3	43	30	13	173	44 %
4	29	23	6	41	28 %
5	36	40	-4	19	11 %
6	31	35	-4	19	12 %
7	46	39	7	49	18 %
8	74	45	29	818	64 %
Mean	**40**	**36**	**4**	**149**	**27 %**

Exponential smoothing (α=0,2)					
Week	**Forecast**	**Actual**	**Error**	**Sq.Error**	**Abs. % - Error**
1	39	49	-10	107	21 %
2	39	25	14	186	55 %
3	39	30	9	75	29 %
4	39	23	16	245	68 %
5	39	40	-1	2	3 %
6	39	35	4	13	10 %
7	39	39	0	0	1 %
8	39	45	-6	40	14 %
Mean	**39**	**36**	**3**	**84**	**25 %**

Regression (DB21)					
Week	**Forecast**	**Actual**	**Error**	**Sq.Error**	**Abs. % - Error**
1	39	49	-10	95	20 %
2	27	25	2	6	10 %
3	39	30	9	86	31 %
4	N/A	23	N/A	N/A	N/A
5	N/A	40	N/A	N/A	N/A
6	N/A	35	N/A	N/A	N/A
7	N/A	39	N/A	N/A	N/A
8	N/A	45	N/A	N/A	N/A
Mean	**35**	**36**	**1**	**62**	**20 %**

Appendix 2: Forecasting Scenario 2

Formulations used in forecasts of moving average, exponential smoothing and pickup models are presented in Section 4.4.

Regression (DB21)

Simple ANOVA – regression using dependent variable Bookings at the departure and independent variable Bookings at 21 days before departure is estimated using Excel. Data over 18 weeks is used to estimate the model parameters:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0,6593
R Square	0,4347
Adjusted R Square	0,3994
Standard Error	6,7512
Observations	18

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	560,7469	560,7469	12,3029	0,0029
Residual	16	729,2531	45,5783		
Total	17	1290			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	19,9735	4,9281	4,0529	0,0009	9,5263	30,4207
Bookings DB21	1,1283	0,3217	3,5076	0,0029	0,4464	1,8102

Estimated coefficients are β_0 = 19,9735 and β_1 = 1,1283. Thus the model used in demand forecasting is $Bookings_{DB0} = 19{,}9735 + 1{,}1283 * Bookings_{DB21}$.

According to P-values both coefficients are statistically significant at 5 % significance level. However low R^2 indicates that the estimated model fits quite badly to my hypothetical dataset. But like in Scenario 1 I will not consider the reasons of this problem more closely, because the purpose of this example is to provide illustrations of how each method is compiled.

Forecasts and errors for each week in Scenario 2 using selected methods:

Forecasts of Week 1 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	37	49	12	149	25 %
Exponential smoothing (α=0,5)	41	49	8	58	16 %
Exponential smoothing (α=0,2)	39	49	10	107	21 %
Additive pickup	44	49	5	28	11 %
Multiplicative pickup	45	49	4	14	8 %
Regression (DB21)	39	49	10	97	20 %

Forecasts of Week 2 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	39	25	-14	202	57 %
Exponential smoothing (α=0,5)	45	25	-20	408	81 %
Exponential smoothing (α=0,2)	41	25	-16	247	63 %
Additive pickup	30	25	-5	21	18 %
Multiplicative pickup	27	25	-2	5	9 %
Regression (DB21)	26	25	-1	0	2 %

Forecasts of Week 3 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	39	30	-9	76	29 %
Exponential smoothing (α=0,5)	35	30	-5	26	17 %
Exponential smoothing (α=0,2)	38	30	-8	57	25 %
Additive pickup	28	30	2	4	7 %
Multiplicative pickup	26	30	4	19	15 %
Regression (DB21)	39	30	-9	84	31 %

Forecasts of Week 4 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	39	23	-16	262	70 %
Exponential smoothing (α=0,5)	33	23	-10	91	42 %
Exponential smoothing (α=0,2)	36	23	-13	170	57 %
Additive pickup	30	23	-7	45	29 %
Multiplicative pickup	27	23	-4	19	19 %
Regression (DB21)	30	23	-7	51	31 %

Forecasts of Week 5 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	38	40	3	6	6 %
Exponential smoothing (α=0,5)	28	40	12	149	31 %
Exponential smoothing (α=0,2)	33	40	7	43	16 %
Additive pickup	41	40	-1	1	3 %
Multiplicative pickup	42	40	-2	4	5 %
Regression (DB21)	38	40	2	4	5 %

Forecasts of Week 6 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	38	35	-3	9	9 %
Exponential smoothing (α=0,5)	34	35	1	1	3 %
Exponential smoothing (α=0,2)	35	35	0	0	1 %
Additive pickup	36	35	-1	1	3 %
Multiplicative pickup	36	35	-1	1	2 %
Regression (DB21)	35	35	0	0	1 %

Forecasts of Week 7 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	38	39	1	2	4 %
Exponential smoothing (α=0,5)	34	39	5	21	12 %
Exponential smoothing (α=0,2)	35	39	4	18	11 %
Additive pickup	40	39	-1	1	3 %
Multiplicative pickup	41	39	-2	3	5 %
Regression (DB21)	44	39	-5	22	12 %

Forecasts of Week 8 bookings at DB0					
Method	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
Moving average (n=10)	37	45	8	64	18 %
Exponential smoothing (α=0,5)	37	45	8	69	18 %
Exponential smoothing (α=0,2)	36	45	9	88	21 %
Additive pickup	34	45	11	119	24 %
Multiplicative pickup	34	45	11	132	25 %
Regression (DB21)	40	45	5	22	10 %

Forecasts and errors using selected methods in Scenario 2: Weeks 1 – 8

Moving average (n=10)					
Week	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
1	37	49	12	149	25 %
2	39	25	-14	202	57 %
3	39	30	-9	76	29 %
4	39	23	-16	262	70 %
5	38	40	3	6	6 %
6	38	35	-3	9	9 %
7	38	39	1	2	4 %
8	37	45	8	64	18 %
Mean	**38**	**36**	**-2,25**	**96**	**27 %**

Additive pickup					
Week	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
1	44	49	5	28	11 %
2	30	25	-5	21	18 %
3	28	30	2	4	7 %
4	30	23	-7	45	29 %
5	41	40	-1	1	3 %
6	36	35	-1	1	3 %
7	40	39	-1	1	3 %
8	34	45	11	119	24 %
Mean	**35**	**36**	**0,41**	**28**	**12 %**

Exp. Smooth. (0,5)					
Week	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
1	41	49	8	58	16 %
2	45	25	-20	408	81 %
3	35	30	-5	26	17 %
4	33	23	-10	91	42 %
5	28	40	12	149	31 %
6	34	35	1	1	3 %
7	34	39	5	21	12 %
8	37	45	8	69	18 %
Mean	**36**	**36**	**-0,13**	**103**	**27 %**

Multip. pickup					
Week	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
1	45	49	4	14	8 %
2	27	25	-2	5	9 %
3	26	30	4	19	15 %
4	27	23	-4	19	19 %
5	42	40	-2	4	5 %
6	36	35	-1	1	2 %
7	41	39	-2	3	5 %
8	34	45	11	132	25 %
Mean	**35**	**36**	**1,03**	**25**	**11 %**

Exp. Smooth. (0,2)					
Week	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
1	39	49	10	107	21 %
2	41	25	-16	247	63 %
3	38	30	-8	57	25 %
4	36	23	-13	170	57 %
5	33	40	7	43	16 %
6	35	35	0	0	1 %
7	35	39	4	18	11 %
8	36	45	9	88	21 %
Mean	**36**	**36**	**-0,70**	**91**	**27 %**

Reg. (DB21)					
Week	**Forecast**	**Actual**	**Error**	**Sq. Error**	**Abs. % - Error**
1	39	49	10	97	20 %
2	26	25	-1	0	2 %
3	39	30	-9	84	31 %
4	30	23	-7	51	31 %
5	38	40	2	4	5 %
6	35	35	0	0	1 %
7	44	39	-5	22	12 %
8	40	45	5	22	10 %
Mean	**36**	**36**	**-0,58**	**35**	**14 %**

Appendix 3: EMSR – capacity allocation

Class (i)	Fare(i)	Forecasted		Allocated seats (S,i)	Allocated seats (D,i)	Allocated seats (E,B)
		Mean	Std. Dev.			
B	f_B	μ_B	σ_B	$EMSR_B(s_S^B) = f_E$	$EMSR_B(s_D^B) = f_E$	$EMSR_E(s_E^B) = f_E$
E	f_E	μ_E	σ_E	$EMSR_E(s_S^E) = f_D$	$EMSR_E(s_D^E) = f_D$	
D	f_D	μ_D	σ_D	$EMSR_D(s_S^D) = f_S$		
S	f_S	μ_S	σ_S			
		Allocated seats (s):		$\sum s_S^i$	$\sum s_D^i$	s_E^B

An example of determination of allocated seats: s_S^D

The number of seats allocated to fare class D from fare class S are presented by $\boldsymbol{s_S^D}$**.** The number of allocated seats comes from the equation $\boldsymbol{EMSR_D(s_S^D) = f_S}$**.**

This means that the number of allocated seats is the number of seats that equals the previous equation.

Because EMSR –model assumes that demand for each fare class is normally distributed, EMSR can be presented by $\boldsymbol{EMSR_D(s_S^D) = f_D \cdot \overline{P}_D(s_D)}$**.**

Thus the problem is to determine the value of $\boldsymbol{s_S^D}$ for which: $\boldsymbol{f_S = f_D \cdot \overline{P}_D(s_S^D)}$

Following table will describe how the number of allocated seats can be found:

Fare S	Fare D	Mean	Std. Dev.
f_S	f_D	μ_D	σ_D
s	CDF	1 - CDF	EMSR
1,2,3,…,s	$P_D(s_S^D)$	$\overline{P_D}(s_S^D)$	$f_D \cdot \bar{P}_D(s_S^D)$

CDF is the cumulative density function of selling s seats. In EMSR –model normal distribution N (μ, σ) is used to create CDF.

As all allocated seats between each fare classes have been calculated, the sum of allocated seats for each class is determined by $\sum s_S^i$, $\sum s_D^i$, s_E^B. These represent protection levels of fare classes D, E and B.

Then each booking limit can be determined by:

Booking limit = Total capacity - Allocated seats (s)				
Class:	**B**	**E**	**D**	**S**
Booking limits:	C	$C - s_E^B$	$C - \sum s_D^i$	$C - \sum s_S^i$

Calculated protection levels and bookings limits for each case:

Case 1: Capacity allocation			Total capacity:	150
Class	Fare	Joint protection	**Booking limits**	Protection level
B	100 €	14	**150**	14
E	60 €	48	**136**	34
D	30 €	96	**102**	49
S	20 €	-	**54**	-
Case 2: Capacity allocation			Total capacity:	150
Class	Fare	Joint protection	**Booking limits**	Protection level
B	100 €	36	**150**	36
E	60 €	78	**114**	41
D	30 €	117	**73**	39
S	20 €	-	**33**	-
Case 3: Capacity allocation			Total capacity:	150
Class	Fare	Joint protection	**Booking limits**	Protection level
B	100 €	29	**150**	29
E	60 €	93	**121**	64
D	30 €	194	**58**	58
S	20 €	-	**0**	-
Case 4: Capacity allocation			Total capacity:	150
Class	Fare	Joint protection	**Booking limits**	Protection level
B	100 €	7	**150**	7
E	60 €	40	**143**	32
D	30 €	82	**111**	43
S	20 €	-	**68**	-