Author(s): Möttönen, Mikko & Pekola, Jukka & Vartiainen, Juha J. & Brosco, Valentina & Hekking, Frank W. J.

Title: Measurement scheme of the Berry phase in superconducting circuits

Year: 2006

Version: Final published version

Please cite the original version:

Möttönen, Mikko & Pekola, Jukka & Vartiainen, Juha J. & Brosco, Valentina & Hekking, Frank W. J. 2006. Measurement scheme of the Berry phase in superconducting circuits. Physical Review B. Volume 73, Issue 21. 214523/1-10. ISSN 1098-0121 (printed). DOI: 10.1103/physrevb.73.214523.

Measurement scheme of the Berry phase in superconducting circuits

Mikko Möttönen*
Low Temperature Laboratory, Helsinki University of Technology, P.O. Box 3500, FI-02015 TKK, Finland and Laboratory of Physics, Helsinki University of Technology, P.O. Box 4100, FI-02015 TKK, Finland

Jukka P. Pekola and Juha J. Vartiainen
Low Temperature Laboratory, Helsinki University of Technology, P.O. Box 3500, FI-02015 TKK, Finland

Valentina Brosco
NEST-CNR-INFM & Dipartimento di Fisica, Universitá di Pisa, largo E. Fermi, I-56100 Pisa, Italy

Frank W. J. Hekking
LPMMC, CNRS, and Université Joseph Fourier, Bôite Postale 166, 38042 Grenoble Cedex 9, France
(Received 7 April 2006; published 26 June 2006)

We present a measurement scheme for observing the Berry phase in a flux assisted Cooper pair pump—the Cooper pair sluice. In contrast to the recent experiments, in which the sluice was employed to generate accurate current through a resistance, we consider a device in a superconducting loop. This arrangement introduces a connection between the pumped current and the Berry phase accumulated during the adiabatic pumping cycles. From the adiabaticity criterion, we derive equations for the maximum pumped current and optimize the sluice accordingly. These results apply also to the high accuracy pumping which results in a potential candidate for a metrological current standard. For measuring the pumped current, an additional Josephson junction is installed into the superconducting loop. We show in detail that the switching of this system from superconducting state into normal state as a consequence of an external current pulse through it may be employed to probe the pumped current. To our knowledge the experimental realization of our scheme would be the first observation of the Berry phase in superconducting circuits.

DOI: 10.1103/PhysRevB.73.214523 PACS number(s): 85.25.Cp, 03.65.Vf, 74.50.+r, 74.78.Na

I. INTRODUCTION

Adiabatic cyclic temporal evolution of any quantum system gives rise to geometric phases depending only on which path the system follows, not the speed of the cycle. For quantum states in degenerate subspaces, these geometric phases correspond to non-Abelian unitary operators within the subspace, also referred to as holonomies.[1,2] In holonomic quantum computation,[3] these operators are of special interest since they represent quantum gates necessary for the actual computation. In the nondegenerate case, the geometric phase arising from a closed cycle in the adiabatic temporal evolution is referred to as Berry phase[4] corresponding to a phase of the quantum state under adiabatic temporal evolution. In general, the Berry phase is not directly observable and it is observed as a phase difference between two states which have traveled a different path during the adiabatic evolution, as was proposed in Refs. 5 and 6 for the yet unrealized observation of the Berry phase superconducting quantum interference devices (SQUIDs). To date, the Berry phase has been observed,[7] for example, in systems of electrons[8] circulating about a wire carrying electric current known as the Aharonov-Bohm effect, and in systems of neutrons[9] or molecules[10] circulating about a line of electric charge known as the Aharonov-Casher effect.

The peculiarity of the Berry phase in a phase biased array of Josephson junctions is that the geometric phase accumulated in the ground state of the Hamiltonian is closely related to the charge pumped through the device. This relation was first found by Aunola *et al.*[11] and also studied in Ref. 12. Hence the yet unrealized observation of the Berry phase in superconducting circuits reduces in this structure into detection of weak currents.

We consider a flux assisted Cooper pair pump, also referred to as Cooper pair sluice,[13] consisting of a superconducting island separated by two SQUIDs which work as tunnel junctions with tunable Josephson energies. In order to achieve large and accurate pumped currents for a novel current standard, a voltage biased sluice was employed to pump current in Ref. 14. In contrast, we place the sluice into a superconducting loop, in which the device is phase biased, and hence may be employed to observe the Berry phase.

For the measurement of the pumped current, we suggest that an additional Josephson junction is installed in the loop in parallel with the sluice. The pumped current modifies the switching statistics of the whole superconducting circuit into the normal conducting state under external current pulses. This change is shown to suffice for quantitative measurements of the pumped current. Furthermore, our measurement scenario resembles that of the superconducting qubit quantronium,[15,16] and hence we expect it to be experimentally feasible.

Recently, a more sensitive measurement scheme than the one implemented in Saclay[15,16] for quantronium was implemented in Ref. 17. In this setup, the excitation and deexcitation of the additional tunnel junction introduces phase shifts to external current pulses, and hence no energy is transferred to the measured system. Whereas this method introduces improvements in the visibility and contrast of the measurement, its experimental implementation is more demanding than that

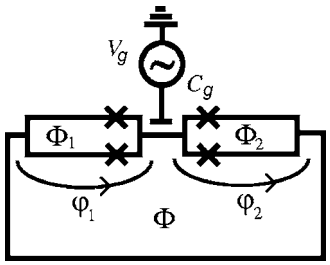

FIG. 1. Circuit for the Cooper pair sluice consisting of a superconducting island with gate capacitance C_g, and two SQUIDs with magnetic fluxes Φ_1 and Φ_2 and phase differences of the order parameter φ_1 and φ_2. The gate voltage is denoted by V_g.

of the Saclay measurement scheme.[15,16] Moreover, we will show that the Saclay measurement scheme is actually more feasible in the observation of the Berry phase than in the quantronium measurement, and hence well justified.

In the adiabatic temporal evolution of a quantum state, the parameters of the system Hamiltonian are controlled externally. The adiabaticity criterion states that the change in the parameters must be slow enough for the system to stay in the same instantaneous eigenstate of the Hamiltonian. This requirement limits the speed of pumping in the Cooper pair sluice, and hence the maximum pumped current. Thus the architecture of the sluice must be optimized in accordance with the limitations due to adiabaticity to obtain as strong a current signal as possible in the proposed measurement scheme. In Ref. 13, the errors in the pumped current due to the breaking of the adiabaticity were studied using a dynamic approach for a fixed design of the sluice. In contrast, we present both numerical and analytical results for the maximum pumped current with any error rate. The results are obtained directly from the adiabaticity theorem and the spectral analysis of the system Hamiltonian. In addition, we apply the results to optimize the architecture of the sluice for both the Berry phase measurement and for high accuracy current pumping.

The structure of this paper is the following: In Sec. II, we present the Hamiltonian and the circuit of the Cooper pair sluice. Section III is devoted to the study of the Berry phase and its connection to the pumped charge. Section IV presents the optimization methods for the design of the sluice with respect to the adiabaticity criterion. The scheme to measure the pumped current is considered in Sec. V. Section VI summarizes and concludes the results of this paper.

II. COOPER PAIR SLUICE

The Cooper pair sluice consists of a superconducting island separated by two SQUIDs. Figure 1 shows the sluice in a superconducting loop. In the case of vanishing loop inductance, the SQUIDs are equivalent to Josephson junctions with tunable Josephson energies $E_{J1}(\Phi_1)$ and $E_{J2}(\Phi_2)$, where Φ_1 and Φ_2 denote the externally controllable magnetic fluxes through the first and second SQUID, respectively. On the other hand, the Coulomb energy for one excess Cooper pair to reside on the island is $E_C=2e^2/C_\Sigma$, where C_Σ represents the total capacitance of the island. If one neglects the parasitic capacitances, the total capacitance is given by $C_\Sigma=2C_J+C_g$, where C_J is the capacitance of one of the identical SQUIDs and C_g is the gate capacitance which is used to charge the island with the gate voltage V_g. Thus the device can be considered to be a tunable Cooper pair transistor.

The Hamiltonian of the sluice may be written in the eigenbasis of the phase operator ϕ as[13]

$$\hat{H}_{\rm sl} = E_C(\hat{n}-n_g)^2 - E_{J1}(\Phi_1)\cos(\phi+\varphi/2) - E_{J2}(\Phi_2)\cos(\phi-\varphi/2), \tag{1}$$

where $\hat{n}=-i\partial_\phi$ is the number operator for the Cooper pairs on the island, $n_g=V_gC_g/(2e)$ is the gate charge in units of $2e$, $\varphi=\varphi_1+\varphi_2$ is the phase difference over the device, and the phase on the island is given by $\phi=(\varphi_1-\varphi_2)/2$. The phase operators ϕ and φ_i are related to the phase of the order parameter describing the many-body quantum state of the superconductor and should not be mixed with the Berry phase or the phase of the ground state of the above-mentioned Hamiltonian. Since the sluice forms a superconducting loop, the phase φ over the device is fixed by the magnetic flux Φ through the loop as

$$\varphi = \frac{2\pi\Phi}{\Phi_0}, \tag{2}$$

where $\Phi_0=\pi\hbar/e$ is the flux quantum and we have assumed the loop inductance is negligible. Thus φ may be treated as a real number.

The current operator of the kth SQUID is defined as

$$\hat{I}_k = \frac{2ie}{\hbar}[\hat{n}_k,\hat{H}] = \frac{2e}{\hbar}\frac{\partial\hat{H}}{\partial\varphi_k}, \tag{3}$$

where $\hat{n}_k=-i\partial_{\varphi_k}$ is the Cooper pair number operator of the kth SQUID. For the sluice Hamiltonian Eq. (3) reduces into

$$\hat{I}_k = \frac{2eE_{Jk}}{\hbar}\sin(\varphi_k). \tag{4}$$

Equation (3) ensures that for any states $|\xi(t)\rangle$ and $|\chi(t)\rangle$ of the system the current operator possesses the property $\partial_t\langle\xi(t)|(-2e)\hat{n}_k|\chi(t)\rangle=\langle\xi(t)|\hat{I}_k|\chi(t)\rangle$, i.e., the matrix elements of the current operator are given by the time derivative of the matrix elements of the charge operator. The critical current I_{ck} is defined as the maximum current which can flow through the kth SQUID in the superconducting state. Hence, Eq. (4) yields a relation

$$I_{ck} = \frac{2\pi E_{Jk}}{\Phi_0}. \tag{5}$$

Thus no Cooper pairs can tunnel through the SQUID in the adiabatic evolution for vanishing Josephson energy, and hence the SQUID is considered to be closed. In practice, the SQUIDs cannot be perfectly closed due to asymmetry in the tunnel junctions and finite loop inductance, which motivates us to define the minimum residual value of the Josephson energies of the SQUIDs to be $E_J^{\rm res}$. In contrast, the maximum

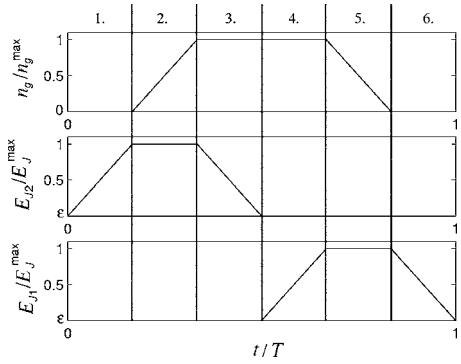

FIG. 2. Values of the externally controlled parameters $E_{J1}(\Phi_1)$, $E_{J2}(\Phi_2)$, and n_g of the Hamiltonian given by Eq. (1) during an example pumping cycle of the Cooper pair sluice. The Josephson energies E_{J1} and E_{J2}, and the gate charge n_g connected to the gate voltage $V_g=2en_g/C_g$ are expressed in terms of their maximum values $E_J^{\max}$ and $n_g^{\max}$, respectively. The period of one pumping cycle is denoted by T. The different phases in the pumping cycle are separated by vertical lines. For $\epsilon=E_J^{\mathrm{res}}/E_J^{\max}=0$, the pumping cycle is referred to as ideal, and hence either one of the SQUIDs is always perfectly closed.

Josephson energy is denoted by $E_J^{\max}$.

A typical pumping cycle of the sluice is described in Fig. 2. In the following, we assume that the SQUIDs can be perfectly closed, i.e., $E_J^{\mathrm{res}}/E_J^{\max}=0$. In the beginning of the cycle both of the SQUIDs are closed and the gate voltage is zero implying that the ground state of the Hamiltonian given by Eq. (1) is an eigenstate of charge $\hat{n}$ with eigenvalue zero. Then the second SQUID is opened by increasing Φ_2 to $\Phi_0/2$, while the first one is kept closed. The increase of the flux Φ_2 is executed adiabatically such that the system stays in its ground state which is, however, not an eigenstate of charge $\hat{n}$ for finite Φ_2. In the second step, the gate charge is brought to integer[25] $n_g^{\max}$, after which the second SQUID is closed. At this middle point of the pumping cycle, the system is again in the charge eigenstate but corresponding to eigenvalue $n_g^{\max}$. Since the first SQUID has been closed for the whole evolution, we conclude that the $n_g^{\max}$ Cooper pairs have tunneled through the second SQUID to the island. Then we direct the $n_g^{\max}$ Cooper pairs through the first SQUID by opening it, taking n_g to zero, and closing it. Thus $n_g^{\max}$ Cooper pairs have flowed through the sluice creating an effective current of $I_p=2en_g^{\max}/T$. Although the above-mentioned arguments are valid in general, we will calculate analytically the pumped current in Sec. III for $E_C \gg E_J^{\max}$ and finite E_J^{res}. In analogy with a classical pump, the gate voltage in the sluice corresponds to a piston and the SQUIDs correspond to valves.

The maximum Josephson energy $E_J^{\max}$ together with the adiabaticity criterion imposes upper bounds to the maximum pumped current, which we consider in detail in Sec. IV. In the adiabatic temporal evolution, the SQUIDs stay in the superconducting state, and hence the average current through the sluice is given by

$$\hat{I}=\frac{2e}{\hbar}\frac{\partial \hat{H}_{\mathrm{sl}}}{\partial \varphi}=\frac{e}{\hbar}\left(\frac{\partial \hat{H}_{\mathrm{sl}}}{\partial \varphi_1}+\frac{\partial \hat{H}_{\mathrm{sl}}}{\partial \varphi_2}\right)=\frac{\hat{I}_1+\hat{I}_2}{2}. \tag{6}$$

The above equation yields a rather large upper bound for the effective pumped current, i.e., the maximum critical current $I_c^{\max}=2\pi E_J^{\max}/\Phi_0$ of the SQUIDs.

III. MANIFESTATION OF THE BERRY PHASE IN COOPER PAIR PUMPS

Let us concentrate on the relation between the accumulated Berry phase and the pumped current. In the case of one-dimensional arrays of Josephson junctions, the expression of the pumped charge was first obtained in Ref. 18 and its relation to the Berry phase was found in Ref. 11. Work concerning not only the Berry phase but also non-Abelian phases and their connection to the pumped charge in rather general Josephson devices is currently in preparation.[19]

In Sec. III A, we consider a more general system than the sluice presented in Fig. 1. Here we assume in excess of cyclic and adiabatic temporal evolution only that the average current operator $\hat{I}=2e(\partial_\varphi \hat{H})/\hbar$ is well defined, i.e., that the Cooper pair pump has only one input and one output. We derive the connection between the pumped charge and the Berry phase for this rather general pump. In Sec. III B, we focus on the sluice and calculate the pumped current using a two-state approximation which is valid in the limit $E_C \gg E_J^{\max}$.

A. Connection between the pumped charge and the Berry phase

In adiabatic temporal evolution, an eigenstate of the system Hamiltonian $\hat{H}$ is slowly varied with respect to a set of real parameters $\{q_k\}$, here denoted by a vector $\mathbf{q}$. The initial state of the system at $t=t_0$ must be an eigenstate $|m;\mathbf{q}\rangle$ of the Hamiltonian, for which one obtains

$$\hat{H}(\mathbf{q})|m;\mathbf{q}\rangle=\varepsilon_m(\mathbf{q})|m;\mathbf{q}\rangle, \tag{7}$$

where $\varepsilon_m(\mathbf{q})$ is the eigenenergy of the corresponding state. In this section, we assume that the system is very accurately in its nondegenerate eigenstate $|r;\mathbf{q}\rangle$. For these so-called zeroth-order assumptions, the state of the system at any time instant is described by[20]

$$|\psi_0(t)\rangle=e^{i\theta_r(t)}|r;\mathbf{q}\rangle, \tag{8}$$

where the phase $\theta_r(t)=\theta_{dr}(t)+\theta_{gr}(t)$ is a sum of the dynamic phase

$$\theta_{dr}=-\frac{1}{\hbar}\int_{t_0}^{t}\langle r;\mathbf{q}(\tau)|\hat{H}|r;\mathbf{q}(\tau)\rangle d\tau=-\frac{1}{\hbar}\int_{t_0}^{t}\varepsilon_r(\mathbf{q}(t))d\tau, \tag{9}$$

and the geometric phase

$$\theta_{gr} = i\int_{t_0}^{t} \langle r;\mathbf{q}(\tau)|\partial_\tau|r;\mathbf{q}(\tau)\rangle d\tau = \int_{\mathbf{q}(t_0)}^{\mathbf{q}(t)} \langle r;\mathbf{q}|\nabla_{\mathbf{q}}|r;\mathbf{q}\rangle \cdot d\mathbf{q}. \tag{10}$$

In cyclic temporal evolution, the dynamic phase is proportional to the period T of the cycle, but the Berry phase, defined by

$$\theta_{Br} = i\oint_{\varsigma} \langle r;\mathbf{q}|\nabla_{\mathbf{q}}|r;\mathbf{q}\rangle \cdot d\mathbf{q}, \tag{11}$$

only depends on the contour $\varsigma=\{\mathbf{q}(t)\,|\,t_0 \leqslant t < T+t_0\}$, according to which the state $|r;\mathbf{q}\rangle$ is varied in the cycle.

To obtain the connection between the Berry phase and the pumped current,[18,19] we consider the leading order corrections to the state vector $|\psi(t)\rangle$ in the speed of the pumping cycle $|\dot{\mathbf{q}}|$. Hence, we express it as

$$|\psi(t)\rangle = \sum_{k=0}^{\infty} C_k(t) e^{i\theta_k(t)} |k;\mathbf{q}\rangle, \tag{12}$$

where $C_k(t)$ are complex numbers. By differentiating Eq. (12) with respect to time and employing the Schrödinger equation we obtain

$$\dot{C}_m(t) = -\sum_{k\neq m} C_k(t) e^{i[\theta_k(t)-\theta_m(t)]} \langle m;\mathbf{q}|\partial_t|k;\mathbf{q}\rangle. \tag{13}$$

Since we control our system such that it remains almost exactly in the state $|r;\mathbf{q}\rangle$, we may neglect all other terms in Eq. (13) except the one with $k=r$. Thus we obtain for $m \neq r$ the first-order correction to the coefficients C_k as

$$C_m(t) = -\int_{t_0}^{t} \langle m;\mathbf{q}|\partial_t|r;\mathbf{q}\rangle e^{i[\theta_r(t')-\theta_m(t')]} dt'. \tag{14}$$

As shown for example in Sec. IV, the absolute value of the above coefficients C_k is proportional to $|\dot{\mathbf{q}}|$ in adiabatic and cyclic temporal evolution.

The total mean charge through the system in one pumping cycle may be written with the help of the average current operator $\hat{I}$ as

$$Q_{\text{tot}} = \int_{t_0}^{t_0+T} \langle \psi(t)|\hat{I}|\psi(t)\rangle dt. \tag{15}$$

The contribution to the total charge purely from the zeroth-order term presented in Eq. (8) is observed to originate from the usual supercurrent and is denoted by

$$Q_{sr} = \int_{t_0}^{t_0+T} \langle r;\mathbf{q}(t)|\hat{I}|r;\mathbf{q}(t)\rangle dt. \tag{16}$$

On the other hand, the pumped charge $Q_p := Q_{\text{tot}} - Q_s$ assumes the form

$$Q_{pr} = 2\,\text{Re}\int_{t_0}^{t_0+T} \left(\sum_{k\neq r} C_k(t)\langle r;\mathbf{q}(t)|\hat{I}|k;\mathbf{q}(t)\rangle e^{i[\theta_k(t)-\theta_r(t)]}\right) dt, \tag{17}$$

where we have neglected the terms containing $|C_k|^2$ which vanish in the adiabatic limit. By inserting Eq. (14) into the above equation and changing the order of the resulting integrals we obtain

$$Q_{pr} = -2\,\text{Re}\sum_{k\neq r}\int_{t_0}^{t_0+T} \left\{ \langle k;\mathbf{q}(t')|\partial_t|r;\mathbf{q}(t')\rangle e^{i[\theta_r(t')-\theta_k(t')]} \times \int_{t'}^{t_0+T} \langle r;\mathbf{q}(t)|\hat{I}|k;\mathbf{q}(t)\rangle e^{i[\theta_k(t)-\theta_r(t)]} dt \right\} dt'. \tag{18}$$

In the above integral with respect to t, the integrand may be expressed as

$$\frac{d}{dt}\{i\hbar\,\langle r;\mathbf{q}(t)|\hat{I}|k;\mathbf{q}(t)\rangle e^{i[\theta_k(t)-\theta_r(t)]}/(\varepsilon_k-\varepsilon_r)\} + \mathcal{O}(|\dot{\mathbf{q}}|). \tag{19}$$

The term $\mathcal{O}(|\dot{\mathbf{q}}|)$ in the above equation may be dropped since it becomes negligible in the adiabatic limit compared with the first term. Hence the pumped charge is obtained to be

$$Q_{pr} = 2\hbar\ \text{Im}\sum_{k\neq r}\left[\int_{t_0}^{t_0+T} \frac{\langle r;\mathbf{q}(t)|\hat{I}|k;\mathbf{q}(t)\rangle\langle k;\mathbf{q}(t)|\partial_t|r;\mathbf{q}(t)\rangle}{\varepsilon_r-\varepsilon_k} dt + \frac{C_k(t_0+T)\langle r;\mathbf{q}(t_0+T)|\hat{I}|k;\mathbf{q}(t_0+T)\rangle e^{i[\theta_k(t_0+T)-\theta_r(t_0+T)]}}{\varepsilon_r-\varepsilon_k}\right]. \tag{20}$$

Since the excitation amplitudes C_k tend to zero in the adiabatic limit, we obtain the known equation[18] for the pumped charge

$$Q_{pr} = 2\hbar\ \text{Im}\oint_{\varsigma}\sum_{k\neq r}\frac{\langle r;\mathbf{q}|\hat{I}|k;\mathbf{q}\rangle}{\varepsilon_r-\varepsilon_k}\langle k;\mathbf{q}|\nabla_{\mathbf{q}}|r;\mathbf{q}\rangle \cdot d\mathbf{q}, \tag{21}$$

which only depends on the contour ς, according to which the state $|r;\mathbf{q}\rangle$ traverses, not the speed $|\dot{\mathbf{q}}|$ of the pumping cycle. Although the above equation contains an integral over a closed contour, it is also valid for open contours. Furthermore, the matrix elements of the average current operator $\hat{I} = 2e[\partial_\varphi, \hat{H}]/\hbar$ can be expressed as

$$\langle r;\mathbf{q}|\hat{I}|k;\mathbf{q}\rangle = \frac{2e(\varepsilon_k-\varepsilon_r)}{\hbar}\langle r;\mathbf{q}|\partial_\varphi|k;\mathbf{q}\rangle, \tag{22}$$

which yields together with Eq. (21)

$$Q_{pr} = -4e\ \text{Im}\oint_{\varsigma}\langle r;\mathbf{q}|\partial_\varphi\nabla_{\mathbf{q}}|r;\mathbf{q}\rangle \cdot d\mathbf{q}. \tag{23}$$

In the above-mentioned derivation, we have employed the fact that $\langle r;\mathbf{q}|\partial_\varphi|r;\mathbf{q}\rangle\langle r;\mathbf{q}|\nabla_{\mathbf{q}}|r;\mathbf{q}\rangle$ is real and that the basis $\{|k;\mathbf{q}\rangle\}$ is normalized spanning the whole configuration space.

Equation (23) for the pumped charge resembles Eq. (11) except for the operator ∂_φ. This observation motivates us to study the derivative of the Berry phase with respect to the phase difference across the sluice φ which is fixed by the total magnetic flux Φ as in Eq. (2), and hence may be treated as a scalar variable. Hence, Eq. (11) yields

$$\partial_\varphi \theta_{Br} = i \oint_{\mathfrak{s}} [(\partial_\varphi |r;\mathbf{q}\rangle)^\dagger \nabla_{\mathbf{q}} |r;\mathbf{q}\rangle + \langle r;\mathbf{q}|\partial_\varphi \nabla_{\mathbf{q}}|r;\mathbf{q}\rangle \cdot d\mathbf{q}. \tag{24}$$

The first term on the right side of Eq. (24) may be written in the form

$$(\partial_\varphi |r;\mathbf{q}\rangle)^\dagger \nabla_{\mathbf{q}} |r;\mathbf{q}\rangle = \nabla_{\mathbf{q}}[(\partial_\varphi |r;\mathbf{q}\rangle)^\dagger |r;\mathbf{q}\rangle - (\nabla_{\mathbf{q}} \partial_\varphi |r;\mathbf{q}\rangle)^\dagger |r;\mathbf{q}\rangle. \tag{25}$$

The integral of the term $\nabla_{\mathbf{q}}[(\partial_\varphi |r;\mathbf{q}\rangle)^\dagger |r;\mathbf{q}\rangle]$ over a closed contour $\mathfrak{s}$ vanishes by the Stokes theorem. Thus Eq. (24) may be recast into

$$\partial_\varphi \theta_{Br} = -2\,\mathrm{Im} \oint_{\mathfrak{s}} \langle r;\mathbf{q}|\partial_\varphi \nabla_{\mathbf{q}}|r;\mathbf{q}\rangle \cdot d\mathbf{q}, \tag{26}$$

and hence the comparison of Eqs. (23) and (26) yields the relation between the Berry phase and the pumped charge

$$Q_p = 2e \partial_\varphi \theta_B. \tag{27}$$

The interpretation of Q_p is the average pumped charge through the device, since it arises from the average current operator $\hat{I}$. Furthermore, if the pump consists of a linear chain of islands separated by SQUIDs, the charge conservation and cyclic temporal evolution assures that the pumped charge through all the SQUIDs separately equal to Q_p.[18] Since the Berry phase is not directly observable in a one-dimensional subspace of any quantum system, only the derivative of the Berry phase with respect to the total phase difference appears in Eq. (27). We do not regard this property to be a limitation in measuring the Berry phase in Josephson circuits, but rather equate it with the fact that generally the Berry phase is detected as a phase difference between two orthogonal quantum states. In contrast to the Berry phase, the charge corresponding to the dynamic phase is related to the usual supercurrent through the device as

$$Q_{\mathrm{s}} = -2e \partial_\varphi \theta_d, \tag{28}$$

which is obtained by differentiating Eq. (9) with respect to φ and comparing the result with Eq. (16).

B. Berry phase in the Cooper pair sluice

Assuming $E_C \gg E_J^{\max}$ and $n_g \in [0,1]$, the state of the Cooper pair sluice can be described by two eigenstates $|0\rangle$ and $|1\rangle$ of the charge operator $\hat{n}$ corresponding to eigenvalues 0 and 1, respectively. We employ this approximation to demonstrate the results of Sec. III A for the sluice up to linear terms in

$$\delta := \frac{E_J^{\max}}{E_C}. \tag{29}$$

In the discussion of Sec. II, we found that the pumped charge is $2en_g^{\max}$ in an ideal pumping cycle shown in Fig. 2 for $E_J^{\mathrm{res}}/E_J^{\max}=0$. In contrast, we consider here SQUIDs with finite

$$\epsilon := \frac{E_J^{\mathrm{res}}}{E_J^{\max}}. \tag{30}$$

We will, however, assume that the residual Josephson energy is small, and hence study its effect only up to linear terms in ϵ.

From the expression $\hat{n} = -i\partial_\phi$ of the number operator, we observe that

$$e^{\pm i\phi}|n\rangle = |n \pm 1\rangle, \tag{31}$$

where $|n\rangle$ is an eigenstate of the number operator $\hat{n}$ with an eigenvalue n. Thus we decompose the trigonometric functions of Eq. (1) into exponential functions and obtain the approximate two-state Hamiltonian in the basis $\{|0\rangle, |1\rangle\}$ as

$$\hat{H}_{2s} = \begin{pmatrix} E_C n_g^2 & -\frac{E_{J1}}{2} e^{-i\varphi/2} - \frac{E_{J2}}{2} e^{i\varphi/2} \\ -\frac{E_{J1}}{2} e^{i\varphi/2} - \frac{E_{J2}}{2} e^{-i\varphi/2} & E_C (1-n_g)^2 \end{pmatrix}. \tag{32}$$

The ground state $|g\rangle$ and the excited state $|e\rangle$ are obtained by diagonalizing this Hamiltonian, and can be expressed as

$$|g\rangle = e^{i\gamma} a|0\rangle + b|1\rangle, \quad |e\rangle = e^{i\gamma} b|0\rangle - a|1\rangle, \tag{33}$$

where the amplitudes a and b are positive real numbers satisfying $a^2+b^2=1$ and γ is the relative phase difference of the charge states. The overall constant phase of the eigenstates is physically unobservable and can thus be chosen as in Eq. (33). The amplitudes a and b are obtained to be

$$a^2 = 1 - b^2 = \frac{1}{2}\left[1 - \frac{\eta}{\sqrt{\eta^2 + (E_{12}/E_C)^2}}\right], \tag{34}$$

where we have defined $\eta = n_g - 1/2$ and $E_{12} = \frac{1}{2}\sqrt{E_{J1}^2 + E_{J2}^2 + 2E_{J1}E_{J2}\cos\varphi}$. Moreover, the phase difference γ assumes the form

$$\gamma = \arctan\left(\frac{E_{J2} - E_{J1}}{E_{J1} + E_{J2}} \tan\frac{\varphi}{2}\right). \tag{35}$$

Our aim is to compute the pumped charge utilizing Eq. (21), and compare it with the derivative of the Berry phase with respect to the phase difference φ. In this two-state approximation, the sum in Eq. (21) consists of only one term

$$Q_{p2s} = 2\hbar\ \mathrm{Im} \oint_{\mathfrak{s}} \frac{\langle g;\mathbf{q}|\hat{I}_2|e;\mathbf{q}\rangle}{\varepsilon_g - \varepsilon_e} \langle e;\mathbf{q}|\nabla_{\mathbf{q}}|g;\mathbf{q}\rangle \cdot d\mathbf{q}, \tag{36}$$

where we have employed the fact that all the current operators $\hat{I}$, $\hat{I}_1$, and $\hat{I}_2$ result in equal pumped charges due to the charge conservation. The symbols ε_g and ε_e denote the ei-

genvalues of the Hamiltonian $\hat{H}_{2s}$ corresponding to the states $|g;\mathbf{q}\rangle$ and $|e;\mathbf{q}\rangle$, respectively. The parameters $\mathbf{q}$ are chosen to be $\mathbf{q}=(a^2,\gamma)$. With the help of Eqs. (4) and (31), the matrix element of the current operator $\hat{I}_2$ in Eq. (36) is expressed as

$$\langle g;\mathbf{q}|\hat{I}_2|e;\mathbf{q}\rangle = \frac{ie}{\hbar}E_{J2}[e^{i(\gamma-\varphi/2)}b^2 + e^{-i(\gamma-\varphi/2)}a^2]. \quad (37)$$

We write the gradient part in Eq. (36) as

$$\langle e;\mathbf{q}|\nabla_{\mathbf{q}}|g;\mathbf{q}\rangle \cdot d\mathbf{q} = iabd\gamma + \frac{d(a^2)}{2ab}, \quad (38)$$

and the energy difference as

$$\varepsilon_e - \varepsilon_g = 2E_C\sqrt{E_C^2\eta^2 + E_{12}^2}. \quad (39)$$

By a substitution of Eqs. (37)–(39) into Eq. (36) we obtain

$$Q_{p2s} = -2e\,\mathrm{Re}\oint_{\varsigma}\frac{E_{J2}}{E_{12}}[e^{i(\gamma-\varphi/2)}b^2 + e^{-i(\gamma-\varphi/2)}a^2] \times\left[i\frac{E_{12}^2 d\gamma}{4(E_C^2\eta^2+E_{12}^2)} + \frac{1}{2}d(a^2)\right]. \quad (40)$$

Since the part containing $d\gamma$ in the above integral vanishes as $\mathcal{O}(\delta^2)$, it is neglected in this approximation. With the same argument we neglect the terms containing $d(a^2)$ for the parts of the contour ς in which E_{J1} or E_{J2} is varied. Furthermore, we employ the identity $b^2=1-a^2$ to Eq. (40) and obtain

$$Q_{p2s} = -2e\int\frac{E_{J2}}{2E_{12}}\cos(\gamma-\varphi/2)d(a^2) + \mathcal{O}(\delta^2), \quad (41)$$

where the integration is over the second and fifth pumping phase, see Fig. 2. For the second pumping phase, $E_{J2}=E_J^{\max}$, and up to linear order in ϵ we have $E_{12}=E_J^{\max}(1+\epsilon\cos\varphi)/2$ and $\cos(\gamma-\varphi/2)=1$. Furthermore, a^2 varies from $1-\mathcal{O}(\delta^2)$ to $\mathcal{O}(\delta^2)$, and hence the contribution to the pumped charge from this domain is $2e(1-\epsilon\cos\varphi)$. For the fifth pumping phase, $E_{J2}=E_J^{\mathrm{res}}$, $E_{12}\approx E_J^{\max}(1+\epsilon\cos\varphi)/2$, and $\cos(\gamma-\varphi/2)\approx\cos(\varphi)$. In this part of the cycle a^2 varies from $\mathcal{O}(\delta^2)$ to $1-\mathcal{O}(\delta^2)$, and hence the contribution to the pumped charge up to linear order in ϵ is given by $-2e\epsilon\cos\varphi$. Thus the pumped charge of the sluice in the adiabatic cycle of Fig. 2 assumes the form

$$Q_{p2s} = 2e(1-2\epsilon\cos\varphi) + \mathcal{O}(\delta^2) + \mathcal{O}(\epsilon^2). \quad (42)$$

For vanishing residual Josephson energy, the above equation reduces to the general result for ideal pumping cycle $Q_p=2en_g^{\max}$, where the number of Cooper pairs pumped in one cycle $n_g^{\max}$ is unity.

To show the relation between the pumped charge and the Berry phase, we employ Eq. (11) and write

$$\theta_{B2s} = i\oint_{\varsigma}\langle g;\mathbf{q}|\nabla_{\mathbf{q}}|g;\mathbf{q}\rangle\cdot d\mathbf{q} = -\oint_{\varsigma}a^2 d\gamma, \quad (43)$$

where the vanishing integral involving the differential $d(a^2)$ is not shown. Since the parameter γ does not depend on the gate charge, the Berry phase is accumulated only when the Josephson energies are changed in the cycle of Fig. 2. Since $a^2=\mathcal{O}(\delta^2)$ for $\eta=1/2$, the only non-negligible contribution to the integral in Eq. (43) is obtained in the first and last pumping phase, for which $a^2=1+\mathcal{O}(\delta^2)$. Thus the Berry phase can be expressed as

$$\theta_{B2s} = \varphi - 2\epsilon\sin(\varphi) + \mathcal{O}(\delta^2) + \mathcal{O}(\epsilon^2). \quad (44)$$

Indeed, Eqs. (42) and (44) demonstrate that

$$Q_{P2s} = 2e\partial_{\varphi}\theta_{B2s}, \quad (45)$$

which demonstrates for the sluice the validity of the general result given by Eq. (27).

IV. ADIABATICITY CRITERION FOR THE COOPER PAIR SLUICE

In the above sections, we have assumed that the sluice is more or less perfectly in its instantaneous ground state during the temporal evolution. However, this assumption is strictly speaking valid only if the changes in the parameters $\mathbf{q}$ of the system Hamiltonian are infinitely slow. On the other hand, we wish to repeat the pumping cycle as fast as possible for a given $n_g^{\max}$ to create a detectable current. Thus a quantitative study of the pumping errors due to finite pumping rate is required. In this section, we derive upper bounds for the population of the excited states of the sluice in the spirit of adiabatic theorem. Moreover, we apply the results to the case $\delta\ll 1$ for which the two-state approximation is valid, and to the case $\delta\gg 1$ for which we employ a perturbed harmonic oscillator approach. In these two cases, we also derive the maximum pumped current for small pumping error rate.

As in Sec. III A, we study a rather general quantum system described by the Hamiltonian $\hat{H}(\mathbf{q})$, parameters $\mathbf{q}$, and the instantaneous eigenbasis $\{|m;\mathbf{q}\rangle\}$ defined in Eq. (7). We consider a representation of the state of the system given by Eq. (12). Since the aim of this section is to show how we can retain the system in its ground state, i.e., $|C_0(t)|^2\approx 1$, we may employ Eq. (14) with $r=0$ to describe the time evolution of the excitation coefficients C_k. It follows from Eq. (7) that

$$\langle m;\mathbf{q}|\partial_t|k;\mathbf{q}\rangle = \frac{\langle m;\mathbf{q}|(\partial_t\hat{H})|k;\mathbf{q}\rangle}{\varepsilon_k(t)-\varepsilon_m(t)}. \quad (46)$$

By insertion of Eq. (46) into Eq. (14) we obtain

$$C_m(t) = \int_{t_0}^{t}\frac{\langle m;\mathbf{q}|(\partial_{t'}\hat{H})|0;\mathbf{q}\rangle}{\varepsilon_m(t')-\varepsilon_0(t')}e^{i[\theta_m(t')-\theta_0(t')]}dt'. \quad (47)$$

Let us assume that we adjust the speed of the temporal evolution such that $\langle m;\mathbf{q}|(\partial_t\hat{H})|0;\mathbf{q}\rangle/[\varepsilon_m(t)-\varepsilon_0(t)]^2$ is constant in time. Furthermore, if we neglect the geometric part of the phase and assume that the energy separation is time independent, we obtain an estimate

$$|C_m(t)| \leqslant 2\hbar\frac{|\langle m;\mathbf{q}|(\partial_t\hat{H})|0;\mathbf{q}\rangle|}{[\varepsilon_m(t)-\varepsilon_0(t)]^2}. \quad (48)$$

Although the above assumptions may be considered to be crude in general, they turn out to be accurate for a sluice

Hamiltonian with at least one of the Josephson energies much larger than the Coulomb energy E_C.

Without any other assumption about the temporal evolution of the system except that it is cyclic and nearly adiabatic, Eq. (47) yields

$$C_m(NT+t_0) = C_m(T+t_0)\frac{1-\nu_m^{N+1}}{1-\nu_m}, \quad \text{for } \nu_m \neq 1, \quad (49)$$

where N is a positive integer and $\nu_m = e^{i[\theta_m(T)-\theta_0(T)]}$. Since the dynamic phase depends explicitly on the period T and the geometric phase only on the contour ς, according to which the parameters $\mathbf{q}$ vary under the cyclic temporal evolution, the period T can be chosen for example such that $\nu_m = i$ for a fixed m, which implies $|C_m(NT)| < \sqrt{2}|C_m(T)|$. Thus the probability for the sluice to be in the excited state $|\mathbf{q};m\rangle$ is bounded. The above discussion is, however, strictly valid only for the mth state, whereas ν_k can generally be very close to unity for $k \neq m$. In the extreme case $\nu_k = 1$, the amplitude of the kth state increases linearly in time, and hence the adiabaticity is lost if there is no dissipation in the system. To take the dissipation into account, we estimate from Eq. (47) that

$$\dot{C}_m(t) \leqslant \frac{|\langle m;\mathbf{q}|(\partial_t \hat{H})|0;\mathbf{q}\rangle|}{\varepsilon_m(t)-\varepsilon_0(t)} \leqslant \max_{0<t<T} \frac{|\langle m;\mathbf{q}|(\partial_t \hat{H})|0;\mathbf{q}\rangle|}{\varepsilon_m(t)-\varepsilon_0(t)} := \kappa_m. \quad (50)$$

For the excitation probability $P_m(t) = |C_m(t)|^2$, Eq. (50) and addition of relaxation to the system yields

$$\dot{P}_m \leqslant 4\kappa_m \sqrt{P_m(t)} - \Gamma_m P_m(t), \quad (51)$$

where Γ_m is the relaxation rate of the state $|\mathbf{q};m\rangle$. Equation (51) yields a bound $(4\kappa_m/\Gamma_m)^2$ for the excitation probability $P_m(t)$, and hence

$$|C_m(t)| \leqslant \frac{4\kappa_m}{\Gamma_m}. \quad (52)$$

From Eq. (52) we can conclude that if the relaxation is dominant, i.e., $\Gamma_m \geqslant 2(\varepsilon_m - \varepsilon_0)/\hbar$, bound in Eq. (48) is valid even though the assumption that the energy difference $\varepsilon_m - \varepsilon_0$ is independent of time fails. Moreover, since κ_m tends to decrease rapidly with increasing m, it may be argued that the excitation amplitudes of the high-lying states are suppressed by relaxation, and hence we have to take only the low-lying states into account if we consider an isolated quantum system as a model for the physical system.

The errors to the pumped current arise from the leakage current due to finite residual Josephson energy E_J^{res} and from the nonadiabaticity of the temporal evolution which we consider here. Since the pumped current is determined by $2en_g^{\max}/T$, the optimal way to pump is to choose the integer $n_g^{\max}$ to be as large as possible to decrease the contribution to the period T from opening and closing the SQUIDs, see Fig. 2. Thus the maximum pumped current is determined by the maximum value for $\dot{n}_g$ limited by the criteria of Eqs. (48) and (52) when one of the SQUIDs is fully open and the other one is closed. Since the energy difference $\varepsilon_e - \varepsilon_g$ depends strongly on n_g in the two state approximation valid for $\delta \ll 1$, the estimate in Eq. (48) does not hold. In contrast, we adjust $\dot{n}_g$ for all t such that

$$\frac{|\langle e;\mathbf{q}|(\partial_t \hat{H}_{\text{sl}})|g;\mathbf{q}\rangle|}{\varepsilon_e(t)-\varepsilon_g(t)} = \kappa_e, \quad (53)$$

where $\partial_t \hat{H}_{\text{sl}} = 2E_C \dot{n}_g(n_g - \hat{n})$. Substitution of Eqs. (33) and (39) into Eq. (53) results in a differential equation for the gate charge

$$\dot{n}_g = \frac{2\kappa_e}{\delta}[(n_g - 1/2)^2 + \delta^2], \quad (54)$$

which implies together with the initial condition $n_g(0) = 0$ that

$$n_g(t) = \frac{1}{2} + \delta \tan[2\kappa_e t - \arctan(\delta^{-1}/2)]. \quad (55)$$

The above equation yields the period $T = (\pi/2 - 2\delta)/\kappa_e + \mathcal{O}(\delta^3)$ and the pumped current

$$I_{p2s} = -2e\kappa_e\left(\frac{2}{\pi} + \frac{8\delta}{\pi^2}\right) + \mathcal{O}(\delta^2), \quad (56)$$

where κ_e is to be chosen such that $\kappa_e \ll \Gamma_e/4$, see Eq. (52). Although a high damping rate is desirable in light of the adiabaticity, it requires a strong coupling of the system to its environment. The strong coupling has disadvantages, since it may introduce noise to the control parameters of the Hamiltonian, and hence result in pumping errors.

Equation (56) shows that the pumped current increases with $\delta = E_J^{\max}/E_C$. This observation motivates us to work in the limit $\delta \gg 1$ for which the phase states are accurate eigenstates of the Hamiltonian in Eq. (1). Since the ground state of the system is localized at the minimum of the potential, we may approximate the Hamiltonian $\hat{H}_{\text{sl}}$ with a Hamiltonian of a perturbed harmonic oscillator as

$$\hat{H}_{\text{pho}} = -E_C \partial_{\phi'}^2 + \frac{E_{J2}}{2}(\phi')^2 - E_{J2} + \hat{H}_p, \quad (57)$$

where $\phi' = \phi - \varphi/2$, the perturbation Hamiltonian assumes the form $\hat{H}_p = -E_{J2}[\cos(\phi') - 1 + (\phi')^2/2]$, and we have assumed for simplicity that $E_{J1} = 0$. Furthermore, we have neglected the contribution from n_g since it can be taken into account in the unperturbed case by multiplying the wave function by $e^{in_g\phi'}$ which does not introduce energy shifts or changes into the matrix elements $\langle k;\mathbf{q}|(\partial_{n_g}\hat{H}_{\text{sl}})|m;\mathbf{q}\rangle$. By solving the eigenstates of the unperturbed Hamiltonian $\hat{H}_{\text{pho}} - \hat{H}_p$ and applying perturbation theory up to any order beyond the first it can be shown that

$$\frac{\varepsilon_m - \varepsilon_0}{E_C} = \sqrt{2\delta}m - 2^{-4}(2m^2 + 2m) + \mathcal{O}(\delta^{-1/2}), \quad (58)$$

and

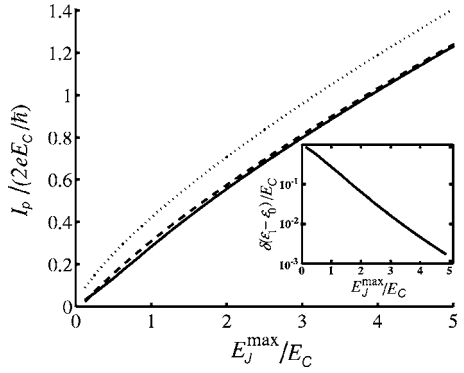

FIG. 3. Pumped current as a function of the maximum Josephson energy of the SQUIDs $E_J^{\max}$ obtained from Eq. (62) (dotted line), Eq. (61) (dashed), and full numerical solution of the system (solid line). The inset shows the maximum relative variation of the energy difference $\varepsilon_1-\varepsilon_0$ with respect to n_g as a function of the maximum Josephson energy of the SQUIDs.

$$\frac{\langle m;\mathbf{q}|(\partial_{n_g}\hat{H}_{\mathrm{sl}})|0;\mathbf{q}\rangle}{E_C} = \begin{cases} i2^{1/4}(\delta^{1/4}-2^{-7/2}\delta^{-1/4})+\mathcal{O}(\delta^{-3/4}), & \text{for } m=1 \\ \mathcal{O}(\delta^{-1/4}), & \text{for } m\neq 1. \end{cases} \tag{59}$$

In fact the eigenstates of the unperturbed Hamiltonian are also the eigenstates of the perturbed Hamiltonian up to $\mathcal{O}(\delta^{-1/2})$, which justifies our assumption that n_g may be neglected also in the perturbed case. We showed in the above discussion that the assumptions leading to the bound in Eq. (48) are valid here. Substitution of Eqs. (58) and (59) into Eq. (48) for $m=1$ yields

$$|C_1(t)| \leqslant \frac{\hbar \dot{n}_g}{E_C}[2^{1/4}\delta^{-3/4}+3\times 2^{-13/4}\delta^{-5/4}+\mathcal{O}(\delta^{-7/4})] := \lambda. \tag{60}$$

Hence, the pumped current in this limit $E_J^{\max} \gg E_C$ is obtained from

$$I_{\mathrm{pho}} = e\dot{n}_g = \frac{eE_C\lambda}{\hbar}[2^{-1/4}\delta^{3/4}-3\times 2^{-15/4}\delta^{1/4}+\mathcal{O}(\delta^{-1/4})], \tag{61}$$

where we choose $\lambda \ll 1$ to assure that the system is accurately in its ground state. The leading contribution to the current may be expressed as

$$I_{\mathrm{pho}} \approx \frac{2^{-1/4}e\lambda E_{J2}^{3/4}E_C^{1/4}}{\hbar}, \tag{62}$$

where the Josephson energy $E_{J2}=E_J^{\max}$. To justify the accuracy of above approximations, we show in Fig. 3 the estimates for the pumped current obtained from Eqs. (62) and (61) compared with a numerical solution, for which the spectrum of the Hamiltonian $\hat{H}_{\mathrm{sl}}$ was calculated, and the gate charge n_g was ramped up employing Eq. (48) and the condition $|C_m(t)|<\lambda$ for all m. On the other hand, the inset in Fig. 3 shows that dependence of the energy difference $\varepsilon_1-\varepsilon_0$ on n_g, and hence time, weakens with increasing δ, which renders Eq. (48) to be valid. Thus we conclude that Eq. (61) yields an accurate analytic expression for the pumped current in a Cooper pair sluice for $E_J^{\max} \gtrsim E_C$.

The calculation of the detailed dependence of the pumping errors δI_p on the parameter λ is beyond the scope of this paper. Instead, we will employ an approximation

$$\frac{\delta I_p}{I_p} \approx \lambda^2. \tag{63}$$

Thus we only give order of magnitude estimates for the maximum pumped current with a given error rate.

Figure 3 shows that to maximize the pumped current, we should ultimately maximize $E_J^{\max}$. Apart from choosing the material of the superconductor such that its energy gap is as large as possible and using very transparent tunnel junctions, the only way to increase the maximum Josephson energy of the SQUIDs is to increase the surface area of the tunnel junctions. However, the junction capacitance increases with the surface area, and hence the Coulomb energy E_C decreases. On the other hand, the Coulomb energy should be large compared with the temperature for the finite temperature effects to be minimal. Thus the attainable base temperature limits $E_J^{\max}$. For standard experimental parameters, i.e., relatively transparent aluminum junctions, $E_C/k_B=1$ K, and for $\lambda=0.01$ we obtain currents of the order of a nanoampere. The pumping errors are not critical for observation of the Berry phase, and hence the parameter λ can be chosen to be, e.g., 0.1, which implies lower than 1% population in the first excited state and a pumped current of the order of 10 nA. On the other hand, if the sluice is used to define the current standard, relative pumping errors should be less than 10^{-7} implying $\lambda<3.1\times10^{-4}$. In this case, the standard parameters still yield I_{pho} to be of the order of 10 pA. For superconducting niobium, the gap in the quasiparticle energy spectrum is an order of magnitude higher than for aluminum leading to about six times larger pumped currents than for aluminum devices.

V. MEASUREMENT SCHEME FOR THE PUMPED CURRENT

To observe the pumped current of a phase biased Cooper pair sluice, we install a third tunnel junction and a dc current source with current I_{dc} in parallel with the sluice, as shown in Fig. 4. The applied current I_{dc} is assumed to be smaller than the critical current of the system, and hence there is no voltage over the third junction. In addition, the capacitance of the third junction is assumed to be large enough such that its charging energy is negligible. Thus the third junction is essentially in such an eigenstate of its phase operator φ_3 that the free energy of the system is minimized. We write the free energy of the circuit shown in Fig. 4 as

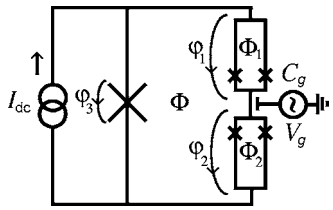

FIG. 4. Circuit diagram for the measurement of the pumped current through the Cooper pair sluice. The third tunnel junction and a dc current source with current I_{dc} is installed in parallel with the sluice shown in Fig. 1. The phase difference of the third junction is denoted by φ_3.

$$\mathcal{G} = \hat{H}_{sl} - E_{J3}\cos\varphi_3 - \frac{\hbar I_{dc}}{2e}\varphi_3, \tag{64}$$

where the second and the third terms correspond to the Josephson energy of the third junction and to the work done by the current source, respectively. Since the system is in its ground state in the adiabatic temporal evolution, the free energy must achieve its minimum at φ_3, leading to

$$\partial_{\varphi_3}\mathcal{G} = E_{J3}\sin\varphi_3 + \frac{\hbar}{2e}(I_{sl} - I_{dc}) = 0, \tag{65}$$

where $I_{sl}=(E_{J1}\sin\varphi_1+E_{J2}\sin\varphi_2)/2$ denotes the total average supercurrent through the sluice, and we have used the relation $2\pi\Phi/\Phi_0=\varphi_1+\varphi_2-\varphi_3$. In Eq. (65), we did not, however, include the effective current $I_p=2en_g^{max}/T$ through the third junction arising from the pumping. This current has to be subtracted from the applied dc current and thus we obtain a relation

$$\tilde{I} := I_{dc} - I_p = \frac{2eE_{J3}}{\hbar}\sin\varphi_3 + I_{sl}. \tag{66}$$

Equation (66) shows that the pumping of Cooper pairs alters the current through the system, which can be measured as a shift in the critical current of the device.

In the case of ideal SQUIDs, for which $\epsilon=E_J^{res}/E_J^{max}=0$, it follows from Hamiltonian of Eq. (57) that the pumped current is independent of the total phase difference φ. Thus we obtain from Eq. (66)

$$\tilde{I} = \frac{2eE_{J3}}{\hbar}\sin\varphi_3, \tag{67}$$

which is equivalent to the case where the sluice is replaced by an ideal current source. Since the pumping shifts the effective current $\tilde{I}$ through the third junction by I_p, the pumping can be observed by switching measurements, i.e., by applying dc current pulses of duration τ and different magnitudes I_{dc} while monitoring the voltage across the device. If the device switches to the normal conducting state, a nonzero voltage is observed.

We estimate the switching probability in the zero temperature limit as

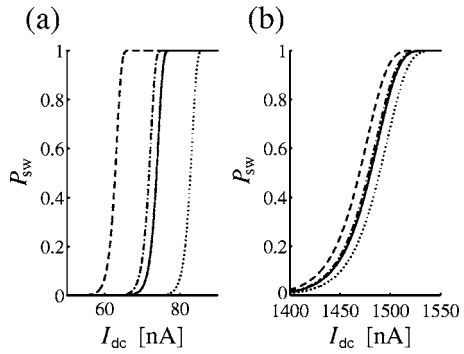

FIG. 5. The probability of the system of Fig. 4 to switch to the normal conducting state as a function of current I_{dc} with 10 nA backward pumping (dashed line), 1 nA backward pumping (dash-dotted line), 1 nA forward pumping (solid line), and 10 nA forward pumping (dotted line). The parameters of the third junction are chosen to be (a) $I_{c3}=0.1$ μA and $C_{J3}=200$ fF, and (b) $I_{c3}=2$ μA and $C_{J3}=15$ fF. The duration of the dc current pulse is $\tau=100$ μs.

$$P_{sw} = 1 - e^{-\tau\Gamma_{MQT}}. \tag{68}$$

The transition rate for the macroscopic quantum tunneling in a weakly dissipative environment is given by

$$\Gamma_{MQT} = 12\sqrt{6\pi}\frac{\omega_p}{2\pi}\sqrt{\Delta U/(\hbar\omega_p)}\,e^{-36\Delta U/(5\hbar\omega_p)}, \tag{69}$$

where $\Delta U \approx \frac{2}{3}E_{J3}[2(1-\tilde{I}/I_{c3})]^{3/2}$ and $\omega_p \approx \sqrt{8E_{J3}E_{C3}[2(1-\tilde{I}/I_{c3})]^{1/2}}/\hbar$ (Ref. 21) denote the barrier height and the plasma frequency of the 3rd junction, respectively. Figure 5 shows the switching probability given by Eq. (68) as a function of the dc current I_{dc} with forward and backward pumping for pumped currents of 10 and 1 nA which were found to be experimentally feasible in Sec. IV. In Fig. 5(a), we have assumed that $I_{c3}=0.1$ μA and $C_{J3}=200$ fF. Since the sluice is also assumed to be optimized for high Josephson energy, the additional assumption of the parameters of the third junction implies that for example in standard shadow evaporation combined with electron beam lithography, three-angle evaporation is to be employed to obtain a different transparency of the third junction as compared with the junctions used in the sluice. An alternative to the three-angle evaporation is to install an additional capacitor in parallel with the third junction, as realized in Refs. 15 and 16. Although these techniques in the fabrication of the sample are very feasible, we have plotted in Fig. 5(b) the switching probability without them, i.e., for $I_{c3}=2$ μA and $C_{J3}=15$ fF. To detect the pumping for the parameters of Fig. 5(b), the relative visibility in the variation of the applied dc current should be of the order of 0.1% for $I_p=1$ nA and 1% for $I_p=10$ nA. We note that in our current experimental setup for other switching measurements than the proposed one, we have detected smaller than 0.1% shifts in the current histograms similar to the ones shown in Fig. 5. Thus an accurate measurement of the pumped current for the parameters of Fig. 5(a) should be achievable.

The correction to the shift of the critical current due to finite residual Josephson energy E_J^{res} is of the order of $\max(I_{c1},I_{c2})\epsilon$ which is small compared with the pumped current I_p for typical experimental parameters. Thus we conclude that the proposed measurement scheme provides a convenient experimental arrangement for observation of Berry phase in superconducting circuits.

Furthermore, we note that Fig. 4 strongly resembles the circuit used in the Saclay experiments on the artificial two-state atom quantronium.[15,16] The sluice in Fig. 4 corresponds to the quantronium and the other part of the circuit is introduced for measuring the supercurrent through it. Since the supercurrent depends on the state of the quantronium, the switching of the superconducting system into normal state can be employed to observe the state of the quantronium. Thus the supercurrent cannot be neglected as we did in the derivation of Eq. (67), and the exact calculation of the switching becomes more difficult than in our case. For further details, see Refs. 22 and 23.

VI. CONCLUSIONS

In this paper, we analyzed a phase biased Cooper pair sluice, for which the pumped charge is closely related to the Berry phase accumulated in one pumping cycle. We showed that the adiabaticity requirement of the sluice does not restrict the pumped current to be undetectably small if the sluice is optimized according to the results derived. In fact, pumped currents of tens of nanoamperes might be achievable. Moreover, a lower bound for the maximum pumped current allowed by the adiabaticity criterion for relative pumping error of 10^{-7} and standard experimental parameters turned out to be of the order of 10 pA. In Ref. 13, a pumped current of 100 pA was obtained with the error rate 10^{-7} using a dynamical approach, which suggests that the lower bound for the maximum pumped current obtained in this paper can be increased by a more detailed derivation. Thus the sluice is a potential candidate for a metrological current standard.[24]

We presented a detailed scenario for observing the pumped current, and showed that the visibility of the current in the proposed measurement is expected to be high. In future work, we aim to experimentally observe the Berry phase employing the scheme given in this paper. Although we have neglected sources of error, e.g., decoherence in our analysis, we expect that they are not dominant in the experimental realization.[26] In fact, our goal is only to combine two already realized techniques: the Cooper pair pumping[14] and the switching measurement with a large Josephson junction as an ammeter. Thus the scheme for observing the Berry phase in superconducting circuits presented here is considered to be very promising.

ACKNOWLEDGMENTS

We thank the Academy of Finland for financial support. M.M. acknowledges the Finnish Cultural Foundation, Vilho, Yrjö, and Kalle Väisälä Foundation, and F.W.J.H. acknowledges support from Institut Universitaire de France. We would like to express our appreciation to J. Peltonen for helpful comments.

*Electronic address: mikko.mottonen@tkk.fi

[1] B. Simon, Phys. Rev. Lett. **51**, 2167 (1983).
[2] F. Wilczek and A. Zee, Phys. Rev. Lett. **52**, 2111 (1984).
[3] P. Zanardi and M. Rasetti, Phys. Lett. A **264**, 94 (1999).
[4] M. V. Berry, Proc. R. Soc. London, Ser. A **392**, 45 (1984).
[5] G. Falci, R. Fazio, G. M. Palma, J. Siewert, and V. Vedral, Nature (London) **407**, 355 (2000).
[6] Z. H. Peng, M. J. Zhang, and D. N. Zheng, Phys. Rev. B **73**, 020502(R) (2006).
[7] J. Anandan, J. Christian, and K. Wanelik, Am. J. Phys. **65**, 180 (1997), and references therein.
[8] A. Tonomura, N. Osakabe, T. Matsuda, T. Kawasaki, J. Endo, S. Yano, and H. Yamada, Phys. Rev. Lett. **56**, 792 (1986).
[9] A. Cimmino, G. I. Opat, A. G. Klein, H. Kaiser, S. A. Werner, M. Arif, and R. Clothier, Phys. Rev. Lett. **63**, 380 (1989).
[10] K. Sangster, E. A. Hinds, S. M. Barnett, and E. Riis, Phys. Rev. Lett. **71**, 3641 (1993).
[11] M. Aunola and J. J. Toppari, Phys. Rev. B **68**, 020502(R) (2003).
[12] A. O. Niskanen, Ph.D. thesis, Helsinki University of Technology, 2004.
[13] A. O. Niskanen, J. P. Pekola, and H. Seppä, Phys. Rev. Lett. **91**, 177003 (2003).
[14] A. O. Niskanen, J. M. Kivioja, H. Seppä, and J. P. Pekola, Phys. Rev. B **71**, 012513 (2005).
[15] A. Cottet, D. Vion, A. Aassime, P. Joyez, D. Esteve, and M. H. Devoret, Physica C **367**, 197 (2002).
[16] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296**, 886 (2002).
[17] I. Siddiqi, R. Vijay, F. Pierre, C. M. Wilson, L. Frunzio, M. Metcalfe, C. Rigetti, R. J. Schoelkopf, M. H. Devoret, D. Vion, and D. Esteve, Phys. Rev. Lett. **94**, 027005 (2005).
[18] J. P. Pekola, J. J. Toppari, M. Aunola, M. T. Savolainen, and D. V. Averin, Phys. Rev. B **60**, R9931 (1999).
[19] V. Brosco *et al.* (2006).
[20] B. H. Bransden and C. J. Joachain, *Introduction to Quantum Mechanics* (Addison Wesley Longman, Essex, 1989).
[21] U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1999).
[22] J. Ankerhold and H. Grabert, Phys. Rev. Lett. **91**, 016803 (2003).
[23] G. Ithier, E. Collin, P. Joyez, D. Vion, D. Esteve, J. Ankerhold, and H. Grabert, Phys. Rev. Lett. **94**, 057004 (2005).
[24] R. S. Whitney, Y. Makhlin, A. Shnirman, and Y. Gefen, Phys. Rev. Lett. **94**, 070407 (2005).
[25] Although we assume $n_g^{\max}$ to be an integer, measurements of the pumped current (Ref. 14) as a function of continuous $n_g^{\max}$ have resulted in a step-like behavior which suggests that the pumped current is insensitive to errors in the gate charge.
[26] For a study of the effect of decoherence on the Berry phase, see Ref. 24, and references therein.