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Internal Magnus Effects in Superfluid ^{3}He-A

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The orbital angular momentum of the coherently aligned Cooper pairs in superfluid ^{3}He-A is encountered by an object immersed in the condensate. We evaluate the associated quasiparticle-scattering asymmetry experienced by a negative ion; this leads to a measurable, purely quantum-mechanical reactive force deflecting the ion's trajectory. Possible hydrodynamic Magnus effects are also discussed.

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The discovery of superfluidity in ^{3}He was followed by evidence for other novel types of superconductivity for which the Cooper pairs may also exist in states with nontrivial internal structure. Therefore, it is important to find probes sensitive to the internal degrees of freedom of the Cooper pairs in condensates with complicated broken symmetries, such as heavy fermions and high-T_c superconductors. The superfluid ^{3}He order parameter $A_{\alpha i}$ is a complex 3×3 matrix where α and i denote the spin and orbital parts. The spin angular momentum in ^{3}He-A_1 has been probed in experiments on magnetically driven superflow by Ruel and Kojima.[1] We suggest that the orbital angular momentum of the Cooper pairs in ^{3}He-A (and ^{3}He-A_1) could also be detected.

The Magnus force is well known within hydrodynamics. This effect arises when a classical viscous fluid flows past a rotating body, and a Bernoulli pressure difference is created across the object for points where the fluid flow reinforces or counteracts its circulation; the deflecting force is perpendicular to both the direction of motion and the axis of rotation. In ^{3}He-A, the picture is reversed: the condensate itself possesses spontaneous angular momentum, which is experienced by a moving nonrotating object. Here the circulation is an inherent property of the superfluid—thus we suggest to call the ensuing phenomena *internal Magnus effects*.

In ^{3}He-A, there exist Cooper pairs with the spin projections $\uparrow\uparrow$ and $\downarrow\downarrow$; both correspond to the same eigenvalue of the orbital angular momentum, $L_z=1$.[2] A common direction, $\hat{\mathbf{l}}$, can thus be ascribed to the internal circulation; the special nature of this direction is manifested in the energy gap $\Delta(\hat{\mathbf{p}})=\Delta_A(T)\sin\theta\exp(i\phi)$, which displays nodes (pointlike vortices on the Fermi sphere) along $\hat{\mathbf{l}}$. Here θ is measured from $\hat{\mathbf{l}}$, while ϕ is the azimuthal angle; $\Delta_A(T)$ is the maximum gap and $\hat{\mathbf{p}}$ is the direction of a quasiparticle momentum on the Fermi surface. The nature of the interaction between the superfluid and the foreign body is determined by the size of the impurity with respect to the coherence length ξ over which the order parameter may vary; $\xi_0\approx 20$ nm at T_c. The various scenarios are illustrated in Fig. 1.

(i) The motion of a tiny particle with radius $R\ll\xi_0$, such as a negative ion (pressure-dependent radius $R=1$–2 nm), is determined by the dissipative quasiparti-

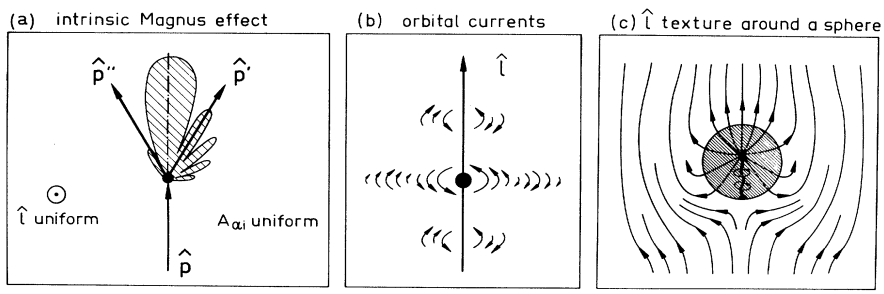

FIG. 1. Internal Magnus effects on different scales in ^{3}He-A: (a) For tiny objects ($R\ll\xi_0$), there is a quantum-mechanical asymmetry in the ^{3}He quasiparticle scattering, i.e., an *intrinsic Magnus effect*. (b) Small particles ($R<\xi_0$), only slightly perturbing the order parameter, generate orbital currents (Ref. 3) which lead to a hydrodynamical *current-induced Magnus effect*. (c) For a large object ($R>\xi_0$) (Ref. 4), the order-parameter texture is essentially redistributed by making $\hat{\mathbf{l}}$ (arrows) normal to its surface. A virtual vortex is formed (heavy curve with circular arrows), which experiences a hydrodynamical *topological Magnus force*.

cle collisions. When an ion moves in the quantum liquid, an *intrinsic Magnus effect* arises: The asymmetric differential scattering cross section $d\sigma/d\Omega$ in Fig. 1(a) illustrates the broken reflection symmetry in the plane perpendicular to $\hat{l}$, due to the uneven quasiparticle "pressure" created by the perpetual circulation of the Cooper pairs.

(ii) For a small particle with radius $R<\xi_0$, in Fig. 1(b), introducing only a minor distortion of the order parameter, there also exists a nondissipative hydrodynamic *current-induced Magnus effect*. In this case, first discussed close to T_c by Rainer and Vuorio,[3] there results a superflow and also a countersuperflow at a distance $r\approx\xi$, such that the total net angular momentum of the liquid surrounding the sphere amounts to zero. However, a net hydrodynamic contribution to the Magnus force from the superflow, F_M, remains: $F_M\approx(m^*/\hbar)\sigma_N \times\xi_0 N(0)\Delta_A^2 v$, where m^* is the effective mass of a quasiparticle, σ_N is the transport cross section in the normal phase, $N(0)$ is the density of states on the Fermi surface, and v is the perpendicular (to $\hat{l}$) component of the drift velocity for the particle. In the Knudsen limit, the drag force experienced by an impurity, e.g., an ion, is $F_{\text{drag}} \approx(e/\mu_\perp)v\approx(\rho/m^*)p_F\sigma_N(\mu_N/\mu_\perp)v$. Here p_F is the ^{3}He Fermi momentum, ρ is the mass density, and e is an electron's charge; μ_N is the isotropic normal-phase mobility. For clarity, we only discuss motion in the plane perpendicular to $\hat{l}$; the corresponding mobility $\mu_\perp$ and the mobility component $\mu_\parallel$ along $\hat{l}$ were calculated in Ref. 5. For a small particle, the ratio between the hydrodynamic Magnus force and the drag force is, for $T\rightarrow T_c$,

$$\frac{F_M}{F_{\text{drag}}}\approx\left(\frac{\Delta_A}{k_BT_c}\right)^2\left(\frac{k_BT_c}{E_F}\right)\frac{\mu_\perp}{\mu_N}\quad(R<\xi_0)\,. \tag{1}$$

(iii) A macroscopic body with $R>\xi_0$, shown in Fig. 1(c), redistributes both the phase and the amplitude of the order parameter. The object aligns the angular momentum vector $\hat{l}$ perpendicular to its surface, which is a topological dilemma: A virtual vortex line emerges.[4] The *topological Magnus force* exerted on this vortex may be estimated by noting that its circulation is of order one quantum of circulation, h/m; hence $\mathbf{F}_M\approx\rho_s \times(h/m)(\hat{l}\times\mathbf{v})R$, where ρ_s denotes the superfluid density. We use Stokes's formula $F_{\text{drag}}=6\pi\eta Rv$, with η the ^{3}He shear viscosity,[6] to compare F_M with the drag force in the hydrodynamic regime; we find

$$\frac{F_M}{F_{\text{drag}}}\approx\frac{\rho_s}{\rho}\left(\frac{k_BT_c}{E_F}\right)^2 \xrightarrow{T\rightarrow T_c}\left(\frac{\Delta_A}{k_BT_c}\right)^2\left(\frac{k_BT_c}{E_F}\right)^2\quad(R>\xi_0)\,. \tag{2}$$

Therefore, this hydrodynamic Magnus force is negligible close to T_c. In the Knudsen regime where $\xi_0<R$ and, moreover, $R\ll L$ with L the quasiparticle mean free path[6,7] of order 1 μm, we estimate

$$\frac{F_M}{F_{\text{drag}}}\approx\frac{a}{R}\frac{\rho_s}{\rho}\frac{\mu_\perp}{\mu_N}\xrightarrow{T\rightarrow T_c}\frac{\xi_0}{R}\left(\frac{\Delta_A}{k_BT_c}\right)^2\frac{k_BT_c}{E_F} \tag{3}$$

$$(\xi_0<R\ll L)\,,$$

where a denotes an interatomic distance. Because of the divergence of the mobility, the Magnus and drag forces may become comparable for $T\approx0.3T_c$; in the A phase, this would require applied magnetic fields of order 1 T.

The most profound Magnus-type phenomenon is, however, the purely quantum-mechanical effect depicted in Fig. 1(a). To observe the maximum deflection, one must move ions perpendicular to $\hat{l}$ in an external electric field $\mathcal{E}$. This situation allows for a precise calculation, with no adjustable parameters; it directly measures the microscopic quasiparticle-scattering anomaly from tiny objects.

The nodes in the energy gap lead to the principal anisotropy—$\mu_\parallel$ vs $\mu_\perp$—first measured by Roach, Ketterson, and Roach,[8] for the mobility of negative ions in ^{3}He-A.[9] Here we discuss an *additional anisotropy* in the plane perpendicular to $\hat{l}$, due to the nonzero angular momentum of the condensate fraction, manifested in the $\exp(i\phi)$ factor of the energy gap. The accurate computation of the matrix $T(\hat{\mathbf{p}}',\hat{\mathbf{p}},E)$ for a quasiparticle scattering off the ion requires a careful, separate treatment of the initial and final directions of the quasiparticle momentum, $\hat{\mathbf{p}}(\theta,\phi)$ and $\hat{\mathbf{p}}'(\theta',\phi')$, respectively. In the plane perpendicular to $\hat{l}\parallel\hat{\mathbf{z}}$, only the relative projection $(\phi'-\phi)$ is relevant. The ϕ term in the A-phase energy gap, however, breaks the reflection symmetry, making the directions $+(\phi'-\phi)$ and $-(\phi'-\phi)$ disparate.

We solve the Lippman-Schwinger equation $T=V+VGT$ exactly as in Refs. 5 and 10 to find the scattering T matrix. Here V is the scattering potential for a hard sphere[5] and G is the superfluid propagator. T is expressed as a sum of partial waves, whose number is determined by the impact parameter p_FR. At A-phase pressures of $20,\ldots,29$ bars, $p_FR=8.85,\ldots,8.45$, and the inclusion of thirteen partial waves serves as an exact quantum-mechanical solution. Because of the symmetry of the A-phase energy gap, it is convenient to express T in terms of states of definite m, i.e., as projections of the partial waves on $\hat{l}$. Averaging over the four scattering channels $\pm\hat{\mathbf{p}}\rightarrow\pm\hat{\mathbf{p}}'$, we obtain the effective squared T-matrix element $\langle|t_{\hat{\mathbf{p}}'\hat{\mathbf{p}}}|^2\rangle$. Multiplying this by the momentum transfers $\Delta p_i\Delta p_j$, with $i=x,y,z$, we find that, in addition to the principal anisotropy, there also emerges an anomalous skew-scattering contribution to the momentum-transfer cross section in the plane perpendicular to $\hat{l}$. This term was not considered before.[5] All the nonvanishing cross sections σ_{ij} at quasiparticle energy E thus are the following:

$$\sigma_{\parallel}(E) = \int d\Omega \int d\Omega' (\cos\theta' - \cos\theta)^2 d\sigma/d\Omega \,,$$
$$\sigma_{\perp}(E) = \int d\Omega \int d\Omega' [\sin^2\theta + \sin^2\theta' - 2\sin\theta\sin\theta'\cos(\phi'-\phi)] d\sigma/d\Omega \,, \tag{4}$$
$$\sigma_{xy}(E) = \int d\Omega \int d\Omega' [-\sin\theta\sin\theta'\sin(\phi'-\phi) + \sin^2\theta\sin(\phi'-\phi)\cos(\phi'-\phi)] d\sigma/d\Omega \,,$$

where

$$d\sigma/d\Omega = \tfrac{3}{2}(1/2\pi)^2 (\partial E/\partial\hat{\mathbf{p}}) \langle |t_{\hat{\mathbf{p}}'\hat{\mathbf{p}}}|^2 \rangle (\partial E/\partial\hat{\mathbf{p}}') \,,$$

$\sigma_{\parallel} = \sigma_{zz}$, and $\sigma_{\perp} = \frac{1}{2}(\sigma_{xx} + \sigma_{yy})$ is an average over the indistinguishable directions $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$; σ_{xz} and σ_{yz} equal zero. The *dissipative* terms $\sigma_{\parallel}$ and $\sigma_{\perp}$ are discussed in Ref. 4. We shall now consider the consequences of the novel *reactive* component σ_{xy}.

Although the Lippman-Schwinger equation is separable in m, a detailed calculation[10] of σ_{xy} shows important couplings of partial waves $m-2$ through $m+2$ in the cross section; hence, the intrinsic Magnus effect is a result of intricate interferences between many resonant partial waves; pure s-wave scattering does not produce the reactive force. The computed σ_{xy} is illustrated in Fig. 2(a) for $p = 29$ bars as a function of the ^{3}He quasiparticle energy. The cross section peaks for the resonant states near $E \approx \Delta_A$ and disappears for $E/\Delta_A \to \infty$ where the excitations approach normal quasiparticles, and also for $E \to 0$. The scattering resonances for $E < \Delta_A$ are due to bound states created below the gap edge in the vicinity of the ion. Their width originates from the variation of the energy gap as a function of $\hat{\mathbf{p}}$; it facilitates large momentum transfers for quasiparticles whose energy matches any of these levels; therefore, resonances appear in the cross section.

Each resonant partial wave produces a separate maximum: Eight peaks are formed as one should expect for $p_F R \approx 8$. This can be understood to originate from the multiple scattering of the ^{3}He quasiparticles. An interesting analogous phenomenon, the Fano effect, appears when there are discrete energy levels in the presence of an equienergy continuum, see Fig. 2(b): A particle in a quasibound energy level may relax into the continuum, thus resulting in resonant interferences for certain energies. We interpret the fine structure and the changes in sign of σ_{xy} at the lowest energies as an indication of Fano-type resonances.

The mobility tensor $\mu(T)$ is obtained from the cross sections:

$$e(\mu^{-1})_{ij} = n_3 p_F \int_{-\infty}^{\infty} dE \left(-\frac{\partial f}{\partial E}\right) \sigma_{ij}(E) \,, \tag{5}$$

where n_3 is the ^{3}He number density and f is the Fermi function. The drift velocity of an ion, $\mathbf{v} = \mu\boldsymbol{\mathcal{E}}$, now reads

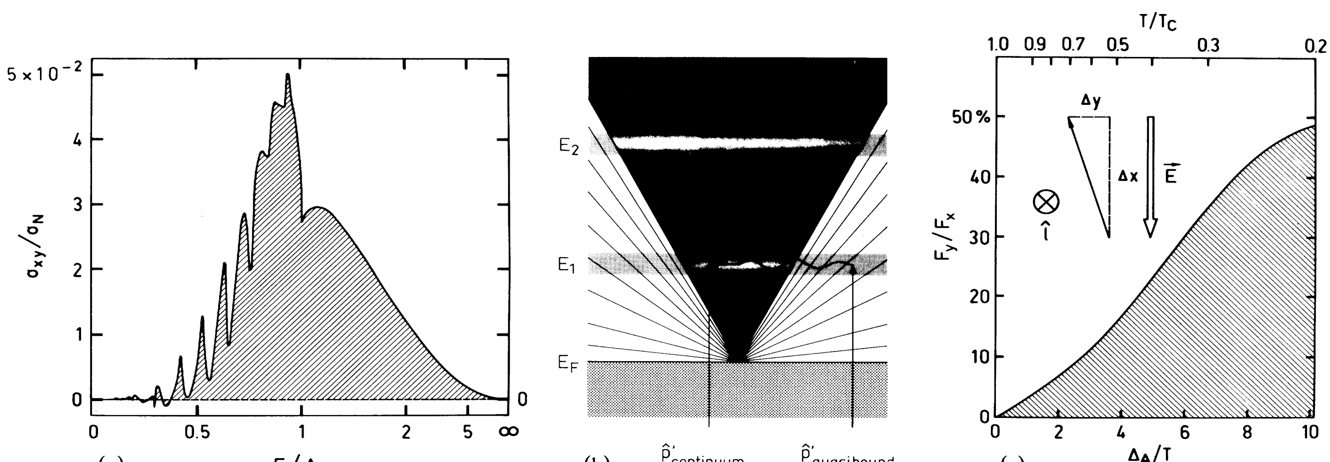

FIG. 2. (a) The computed energy dependence of the ^{3}He-quasiparticle–negative-ion skew-scattering cross section generating the intrinsic Magnus effect in the plane normal to $\hat{l}$. Scale is linear for $E \leq \Delta_A$; for $E > \Delta_A$ it varies as E^{-1}. This σ_{xy} is a measure of the reactive transfer of Cooper-pair orbital angular momentum to the moving ion; it arises from the asymmetry in Fig. 1(a). (b) Closeup near the node of the gap for the low-energy levels which undergo Fano-type interferences. (c) The magnitude of the intrinsic Magnus effect in the plane $\perp\hat{l}$ for negative ions in ^{3}He-A. An ion executes transverse motion Δy, like in the Hall effect, in addition to that (Δx) parallel with the applied electric field, $\boldsymbol{\mathcal{E}}$. The calculated force ratio, F_y/F_x, is shown as a function of Δ_A/T (lower scale) and T/T_c (upper scale, weak coupling).

as

$$\mathbf{v}=[\mu_\parallel(\hat{l}\cdot\hat{\mathscr{E}})\hat{l}+\mu_\perp\hat{l}\times(\hat{l}\times\hat{\mathscr{E}})+\mu_{xy}(\hat{l}\times\hat{\mathscr{E}})]\mathscr{E}\,. \qquad (6)$$

The calculated ratio of the transverse force F_y ($\equiv F_M$) to the component ($\equiv F_{\rm drag}$) driving the ion along $\mathscr{E}$ is seen in Fig. 2(c) for a region with uniform $\hat{l}$. Note that F_y/F_x is *linear* in Δ_A/k_BT for $T\to T_c$. Although σ_{xy} in Fig. 2(a) is rather small, of order $10^{-2}\sigma_N$, for motion perpendicular to $\hat{l}$, one needs to compare σ_{xy} not with σ_N but with $\sigma_\perp$. It is important that Eq. (5) weighs the energy-dependent cross sections with the derivative of the Fermi function. Hence, for $T\to 0$, the magnitude of the internal Magnus effect is determined by the ratio of the slopes of $\sigma_{xy}(E)$ and $\sigma_\perp(E)$. We find that, roughly, $\sigma_{xy}\approx 0.5\sigma_\perp$ for $E\to 0$ [see Fig. 2(a) and Ref. 5]. This explains why for $T\to 0$ the effect grows as large as 50%.

We may fix the $\hat{l}$ texture by applying an external magnetic field $\mathbf{B}$, which serves to orientate the spin anisotropy axis[2] $\hat{\mathbf{d}}\perp\mathbf{B}$; dipole interaction aligns $\hat{l}$ and $\hat{\mathbf{d}}$. In an experimental situation, the walls also affect $\hat{l}$ in their immediate vicinity; the resulting bulk $\hat{l}$ texture, however, lies principally in the plane perpendicular to the magnetic field.[9] It follows that the diameter of a cloud of ions moving the distance Δx, where $\hat{l}$ is oriented randomly in the plane perpendicular to the electric field, has widened by $2\Delta y$ on reaching the collector. A detectable 5° deflection would be expected already for $T=0.7T_c$, while for $T=0.5T_c$ an expansion of as much as 26° is predicted. It is of interest to extend the ion-mobility techniques to find this manifestation of the internal Cooper-pair angular momentum in superfluid ^{3}He-A.

While the hydrodynamical Magnus forces measure the asymptotic perturbation, caused by an object in the superfluid, the new quantum-mechanical effect probes the orbital helicity of the condensate at the very site of the impurity. Close to T_c, this "core" contribution leads to a Magnus force *linear* in Δ_A/k_BT_c, seen in Fig. 2(c), thus considerably exceeding the *quadratic* dependence on Δ_A/k_BT_c of the hydrodynamical contribution to the Magnus effect, cf. Eqs. (1)–(3).

In conclusion, we note that superfluid ^{3}He provides a versatile model for exotic pairing states in other condensed-matter systems. The intrinsic Magnus effect seems important for the focusing of ions by continuous vortices in ^{3}He-A; it is relevant for transport in heavy fermions;[11] in thin films it should also produce a spontaneous *internal quantized Hall effect* (even in the absence of an external magnetic field). Any unconventional (high-T_c) superconductor with Cooper pairing in $L\neq 0$ states and having boojums with noncompensated topological charges[12] on the Fermi sphere should exhibit phenomena akin to the quantum-mechanical asymmetry. Similar effects are generated for ^{3}He-B in an applied magnetic field through gap distortion and in the A-phase-like core of the ^{3}He-B vortices.

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[1]R. Ruel and H. Kojima, Phys. Rev. Lett. **54**, 2238 (1985).

[2]A. J. Leggett, Rev. Mod. Phys. **47**, 331 (1975).

[3]D. Rainer and M. Vuorio, J. Phys. C **10**, 3093 (1977).

[4]G. E. Volovik and V. P. Mineev, Zh. Eksp. Teor. Fiz. **73**, 767 (1977) [Sov. Phys. JETP **46**, 401 (1977)].

[5]M. M. Salomaa, C. J. Pethick, and G. Baym, J. Low Temp. Phys. **40**, 297 (1980); R. H. Salmelin and M. M. Salomaa, J. Phys. C **20**, L681 (1987).

[6]D. Einzel and P. Wölfle, J. Low Temp. Phys. **32**, 19 (1978).

[7]N. A. Greaves and J. Leggett, J. Phys. C **16**, 4383 (1983).

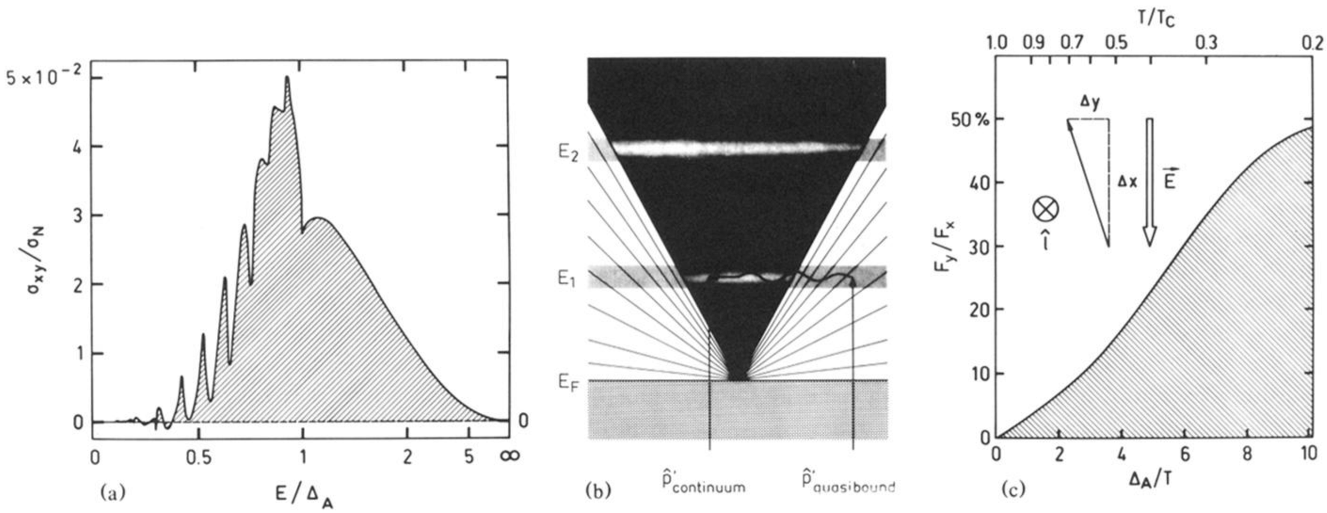
[8]P. D. Roach, J. B. Ketterson, and P. R. Roach, Phys. Rev. Lett. **39**, 626 (1977).

[9]A. I. Ahonen, J. Kokko, M. A. Paalanen, R. C. Richardson, W. Schoepe, and Y. Takano, J. Low Temp. Phys. **30**, 205 (1978).

[10]R. H. Salmelin and M. M. Salomaa (to be published).

[11]B. Arfi, H. Bahlouli, C. J. Pethick, and D. Pines, Phys. Rev. Lett. **60**, 2206 (1988); B. Arfi, H. Bahlouli, and C. J. Pethick, Phys. Rev. B **39**, 8959 (1989).

[12]G. E. Volovik and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **88**, 1412 (1985) [Sov. Phys. JETP **61**, 843 (1985)].



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