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## Comment on “Pipe Network Model for Scaling of Dynamic Interfaces in Porous Media”

In a recent Letter [1], Lam and Horváth (LH) present a pipe network model of imbibition. The network was made of small pipes uniformly distributed in sizes and large pipes of a single radius. By tuning the size and relative number of large to small pipes LH were able to find power law exponents close to those measured in the experiment of one of the authors [2]. Model parameters could be chosen such that the mean velocity of the front ( $v$ ) is related to the mean height ( $H$ ) as  $v \sim H^{-\Omega}$  with  $\Omega = 1.6$ . A new exponent identity is proposed [Eq. (7) of the Letter] relating temporal and spatial correlation functions. Here we point out that LH neglect an important length scale and that their exponent identity, whose underlying assumption holds trivially for standard kinetic roughening, should not apply to imbibition.

To explain the main flaw in LH's arguments we recall the picture of spontaneous imbibition developed in Refs. [3,4]. Competition between the effective line tension and liquid transport imposes to the front a lateral length scale  $\xi_{\times}$  regardless of the value of  $\Omega$  (in experiments using paper a variety of  $\Omega$ 's different from unity and a variety of scaling behaviors have been reported [4]).  $\xi_{\times}$  is important since for any fixed height the interface width is finite and does not obey a power law relationship with system size or time. The width diverges only if the average height of the interface diverges as is the case for freely rising fronts. For example, in the model of Ref. [3] the width follows  $w \sim \xi_{\times}^{\chi}$  with a global roughness exponent  $\chi = 1.25$  and the lateral correlation length scales as  $\xi_{\times} \sim v^{-1/2} \sim t^{1/4}$ .

The existence of an intrinsic lateral correlation length becomes obvious when systems of different sizes are considered. Although such an analysis was not presented by LH a lateral correlation length can be inferred from their description of the height difference correlation function given as

$$C(l) = v^{-\kappa} g(lv^{(\theta_l + \kappa)/\alpha}), \quad (1)$$

where  $g(x) \sim x^{\alpha}$  for  $x \ll 1$ , and constant for large  $x$ . The reported values of the exponents are  $\kappa = 0.49$ ,  $\alpha = 0.61$ , and  $\theta_l = -0.25$ . Equation (1) clearly defines a length scale  $\xi_v \sim v^{-\gamma}$ , where  $\gamma \equiv (\theta_l + \kappa)/\alpha \approx 0.4$ , which then implies  $C(l) \sim \xi_v^{\chi} g(r/\xi_v)$ . Thus,  $\xi_v \sim v^{-0.4}$  is analogous to the  $\xi_{\times} \sim v^{-1/2}$  in Ref. [3]. Equation (1) also defines a global roughness exponent,  $\chi = \kappa\alpha/(\kappa + \theta_l) = 1.25$ , which implies a “superrough” interface with anomalous scaling [5], as could be verified from the structure factor  $S(k, t) \equiv \langle |h_k(t)|^2 \rangle$ .

To derive their exponent identity, LH assume that the dynamics are controlled by a single time scale which can be obtained by two means. For a moving interface, a width  $w$  is reached after a time  $t_1 \sim w^{(\Omega+1)/(\kappa\Omega)}$ . Another time scale can be obtained from the interfacial fluctuations when

the average interface height above a reservoir is kept fixed. In the steady state, LH write the time correlation function in the form

$$C(t) = v^{-\kappa} f(tv^{(\theta_t + \kappa)/\beta}) \quad (2)$$

with the scaling function  $f(x) \sim x^{\beta}$  for  $x \ll 1$  and constant at large  $x$ . This suggests a time scale  $t_2 \sim v^{-(\theta_t + \kappa)/\beta} \sim w^{(\theta_t + \kappa)/\kappa\beta}$ , where  $\theta_t = 0.38$  and  $\beta = 0.63$  are found. Assuming that  $t_1 \propto t_2$ , LH obtain  $\beta = \Omega(\theta_t + \kappa)/(\Omega + 1) \approx 0.54$  which differs from the direct numerical fit quoted above by  $\sim 14\%$ .

Our main objection to the new exponent identity is that it lacks justification since  $t_1$  and  $t_2$  describe two distinct physical time scales. It takes a time  $t_1$  for the correlation length to have value  $\xi_v(t_1)$ . This in turn controls the width as  $w \sim \xi_v^{\chi}$ . On the other hand,  $t_2$  is the relaxation time of the fluctuations when the interface is kept at a fixed height. In this case, the correlation length  $\xi_v$  is a pre-determined constant and the saturation within this zone is obtained when the spatial extent of the correlations equals  $\xi_v$ . The scenario proposed by LH cannot occur if  $t_2 \ll t_1$  and indeed their data indicate  $t_2 \sim w^{2.8}$  and  $t_1 \sim w^{3.25}$  which shows their assumption to be false, asymptotically. Notice that the exponents are close, so a reasonable range of scaling is needed.

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