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Key Points:
- Double layers in radiation belt can efficiently accelerate thermal electrons
- The energization threshold is of the order of 1 keV
- Double layers are unable to provide a seed populations of 100 keV

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On the threshold energization of radiation belt electrons by double layers

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Abstract Using a Hamiltonian approach, we quantify the energization threshold of electrons interacting with radiation belts' double layers discovered by Mozer et al. (2013). We find that double layers with electric field amplitude E_0 ranging between 10 and 100 mV/m and spatial scales of the order of few Debye lengths are very efficient in energizing electrons with initial velocities $v_\| \leq v_{th}$ to 1 keV levels but are unable to energize electrons with $E \geq 100$ keV. Our results indicate that the localized electric field associated with the double layers are unlikely to generate a seed population of 100 keV necessary for a plethora of relativistic acceleration mechanisms and additional transport to higher energetic levels.

1. Introduction

Recent in situ electric field measurements by the Van Allen Probes [*Wygant et al.*, 2014] in the radiation belts have revealed the existence and ubiquitous presence of double layers (DL). Encounters with DL during 1 min burst mode intervals were both common and indicative of large cumulative potential drops [*Mozer et al.*, 2013]. With electric fields averaging 20 mV/m, and sometimes reaching as high as 100 mV/m, observed double layers have been suggested by *Mozer et al.* [2013] as possible accelerators of radiation belt electrons. More specifically, the authors suggested that DL might be capable of accelerating the thermal population with energies of the order of 25 eV to the 100 keV seed population needed to generate the relativistic particles.

It is an observational fact that planetary magnetospheres are efficient electron accelerators, with energies as high as several MeV in the Earth's belts and 50 MeV in Jupiter's belts [*Friedel et al.*, 2002]. Numerous mechanisms spanning a wide range of time scales (from minutes to days) have been theoretically inferred [*Thorne*, 2010, and references therein] and in some cases observationally confirmed [*Horne et al.*, 2005; *Reeves et al.*, 2013; *Thorne et al.*, 2013]. However, a primary difficulty in our understanding of acceleration by wave-particle interaction processes, deemed dominant in the radiation belts, stems from the fact that known mechanisms involving large-amplitude whistlers [*Cattell et al.*, 2008; *Kellogg et al.*, 2010; *Wilson et al.*, 2011] require a seed population with $E \geq 100$ keV [*Omura et al.*, 2007; *Summers and Omura*, 2007; *Osmane and Hamza*, 2012; *Artemyev et al.*, 2012; *Osmane and Hamza*, 2014]. While radial diffusion, enhanced convection and substorm injections of plasma sheet electrons can provide a seed population with $E > 100$ keV on hourly time scales [*Meredith et al.*, 2002; *Turner et al.*, 2012; *Boyd et al.*, 2014], the recently discovered DL within the radiation belts might provide a mechanism to generate a seed population on shorter kinetic scales and in association with violation of the second invariant.

We theoretically and numerically quantify the energization of electrons by writing a Hamiltonian for a test electron interacting with a DL in a dipolar magnetic field. The assumption underlying our Hamiltonian approach, also stated in the Letter by *Mozer et al.* [2013], is that the potential energy associated with the DL is cumulatively transferred to electrons. Hence, even though the Hamiltonian is not self-consistent, it is a sufficient approach to quantify the energy gains of electrons interacting with DL.

We demonstrate that while DL are very efficient to accelerate thermal electrons, the threshold energization of the order of 1 keV is well below the 100 keV levels necessary for the creation of a more energetic seed population. Test electrons with larger energies ($E \geq 100$ keV) are barely affected by DL with spatial scales of the order of 10 km and electric fields of 10–100 mV/m. Incidently, we are led to the conclusion that for DL to be responsible for the generation of a 100 keV population, a nonlinear and secondary kinetic mechanism, arising from a perturbation of the distribution function, must be at play.

2. Hamiltonian Analysis

We start the analysis by writing the Hamiltonian for the generalized momentum $p_\parallel$ and coordinate $s_\parallel$:

$$H = \frac{p_\parallel^2}{2m_e} + \frac{m_e\omega_b^2 s_\parallel^2}{2} - \frac{a\sqrt{\pi}}{2}qE_0 \text{erf}\left(\frac{s_\parallel}{a}\right)\cos(\omega t + \theta) \tag{1}$$

for a charge q, electron rest mass m_e, bounce frequency $\omega_b = \sqrt{9\mu B_0/l^2 R_E^2}$, double layer oscillation frequency ω, and spatial scale along the background field a. The Hamiltonian describes the interaction of an electron bouncing in a magnetic mirror along a shell l and interacting with a parallel electrostatic structure of amplitude E_0. The motion is adiabatic in the first invariant, not in the second invariant, since the double layer is here assumed to only energize electrons along the parallel component of the magnetic field. If needed, the perpendicular motion can be inferred from the solution of the Hamiltonian and the conservation of the adiabatic invariant $\mu = \frac{m_e v_\perp^2}{2B_0}$. Hence, the use of the above Hamiltonian for the interaction of an electron with a double layers incurs no loss of generality.

The double layer potential is modeled in terms of the error function $\text{erf}(x) \sim \int_0^x dy \exp(-y^2)$ and consequently the electric field has a gaussian shape $E \sim \exp(-x^2)$. Choosing a different function for the electric field, e.g., $E \sim \cosh^{-2}(x)$, does not alter the conclusion of the report but complicates the theoretical analysis. For the sake of analytical tractability, we have deemed more appropriate to choose a Gaussian-shaped electric field). The Hamiltonian is composed of an integrable Hamiltonian $H_0 = \frac{p_\parallel^2}{2m_e} + \frac{m\omega_b^2 s_\parallel^2}{2}$ and a perturbed component $H_1(s_\parallel, t) = \frac{a\sqrt{\pi}}{2}qE_0\text{erf}(\frac{s_\parallel}{a})\cos(\omega t + \theta)$ mimicking the double layer electrostatic structure along the background field. The equations of motion are then easily found by computing Hamilton's equation for the parallel momentum :

$$\dot{p}_\parallel = -m_e\omega_b^2 s_\parallel - qE_0\exp\left(-s_\parallel^2/a^2\right)\cos(\omega t + \theta) \tag{2}$$

and the coordinate along the magnetic field

$$\dot{s}_\parallel = \frac{p_\parallel}{m_e}. \tag{3}$$

The equations of motion can then be numerically integrated to obtain the particle trajectories. However, before doing so, we compute an analytical estimate of the rate of energization for an electron interacting with a double layer.

2.1. Analytical Estimate of Energization by Double Layers

We first proceed by normalizing the Hamiltonian in terms of the length scale $L = v_{\text{th}}\omega_b^{-1}$, time scale $T = \omega_b^{-1}$, and thermal speed v_{th}. Consequently, the normalized variables and parameters are written as

$$P = \frac{p_\parallel}{m_e v_{\text{th}}}, \quad X = \frac{\omega_b s_\parallel}{v_{\text{th}}}, \quad \Xi = \frac{qE_0}{m_e v_{\text{th}}\omega_b}, \quad d = \frac{a\omega_b}{v_{\text{th}}}, \quad \tau = \omega_b t, \quad v = \frac{\omega}{\omega_b}, \tag{4}$$

and the normalized Hamiltonian as

$$\tilde{H} = \frac{P^2}{2} + \frac{X^2}{2} - \Xi\frac{d\sqrt{\pi}}{2}\text{erf}\left(\frac{X}{d}\right)\cos(v\tau + \theta). \tag{5}$$

The second step consists of applying a canonical transformation to action-angle variables $(X, P) \rightarrow (\alpha, J)$, resulting in the following transformed Hamiltonian:

$$K(J, \alpha, \tau) = J - \Xi\frac{d\sqrt{\pi}}{2}\text{erf}\left(\frac{\sqrt{2J}}{d}\right)\cos(v\tau + \theta), \tag{6}$$

for the action $J = H_0(P, X)$ and angle $\cos(\alpha) = X/\sqrt{2J}$. Hence, the action J is proportional to the parallel energy of the electron, and the angle α is a measure of the bouncing motion of the electron in the dipolar field as the electron is slowed down or accelerated by the double layer. Hamilton's equations for the new canonical variables yields

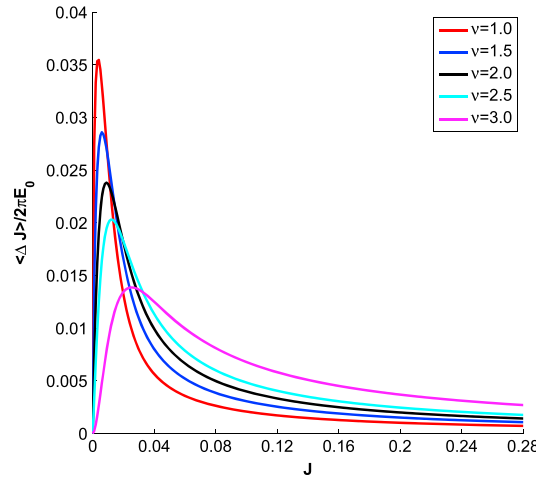$$\dot{\alpha} = 1 - \frac{\Xi}{\sqrt{J}}\cos(\alpha)\cos(v\tau + \theta)F(J, \alpha), \tag{7}$$

Figure 1. Bounce-averaged and phase-averaged $<>_{\theta_k}$ change in the action J_k normalized by the electric field amplitude E_0 for a single interaction. The double layer is much more efficient in accelerating electrons with $v_\parallel \leq v_{th}$.

and

$$\dot{J} = -2\Xi\sqrt{J}\sin(\alpha)\cos(\nu\tau + \theta)F(J,\alpha), \quad (8)$$

for the function $F(J,\alpha) = e^{\frac{-J}{d^2}(1+\cos(2\alpha))}$. This system of equations is analytically equivalent to equations (2) and (3).

In order to estimate the energization of an electron we, employ a finite-difference technique commonly used in plasma studies of linear and stochastic heating [*Smith and Kaufman*, 1978; *Karney and Bers*, 1977; *Reitzel and Morales*, 1996]. The mapping is hereafter constructed for one bounce period by substituting the solution to the unperturbed Hamiltonian $H_0 = H_0(J)$, $\alpha = \tau$, and $J = $ cst, in equations (7) and (8) and integrating for $\tau = [0,1]$. The mapping equations for $\alpha_k = \tau_k$ and J_k can therefore be written as

$$\tau_{k+1} - \tau_k = 1 - \frac{\Xi}{\sqrt{J_k}}e^{-J_k/d^2}\int_0^{2\pi}d\phi\, e^{-J/d^2\cos(2\phi)}\cos(\phi)\cos(\nu\phi + \nu\tau_k + \theta), \quad (9)$$

$$J_{k+1} - J_k = -2\Xi\sqrt{J_k}e^{-J_k/d^2}\int_0^{2\pi}d\phi\, e^{-J/d^2\cos(2\phi)}\sin(\phi)\cos(\nu\phi + \nu\tau_k + \theta). \quad (10)$$

Making use of the modified Bessel function identity for the integer index $q \geq 0$

$$I_q(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi}d\phi\,\exp[x\cos(\phi)]\cos(q\phi) \quad (11)$$
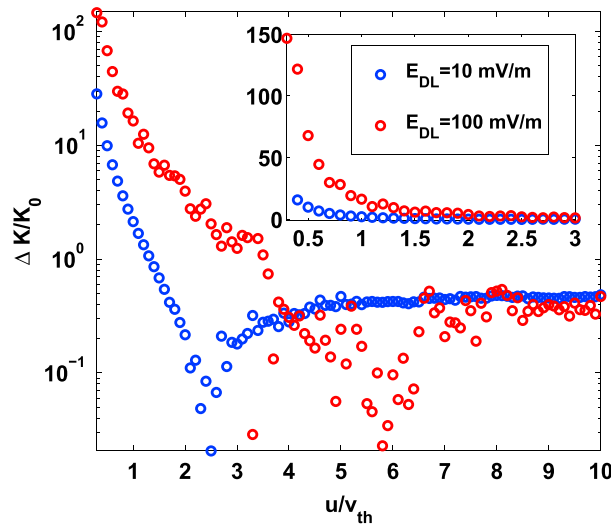

Figure 2. Numerical computation of the average gain in parallel kinetic energy as a function of initial parallel velocity. The electrons are computed after 500 interactions with double layers. The zoomed figure is plotted on a linear scale similar to that of Figure 1. Consistent with the analytical estimate, the numerical computation of the electron orbits demonstrate that double layers are much more efficient in accelerating electrons with $v_\parallel \leq v_{th} = 3000$ km/s. Electrons populating the tail of the distribution are not affected by the double layers.

we can solve the integral for indices $q_\pm = \frac{\nu}{2} \pm \frac{1}{2}$ approximated by positive integer values. The resulting bounce-averaged equation yields the following mapping set for the phase θ_k:

$$\theta_{k+1} - \theta_k = 2\pi\nu + \Xi\nu\pi\cos(\theta_k)e^{-\frac{J_k}{d^2}}\left[I_{q_-}\left(\frac{J_k}{d^2}\right) + I_{q_+}\left(\frac{J_k}{d^2}\right)\right] \quad (12)$$

and action J_k

$$J_{k+1} - J_k = -2\Xi\pi\sin(\theta_k)\sqrt{J_k}e^{-\frac{J_k}{d^2}}\left[I_{q_-}\left(\frac{J_k}{d^2}\right) - I_{q_+}\left(\frac{J_k}{d^2}\right)\right] \quad (13)$$

Whereas the two mapped equations of the dynamical system are provided for completeness, only the mapping for J_k is required for an estimation of the bounce-averaged energization of an electron. Figure 1 shows the phase-averaged change of the action for given values of $\nu = \frac{\omega}{\omega_b}$. Since the double layers reported by *Mozer et al.* [2013]
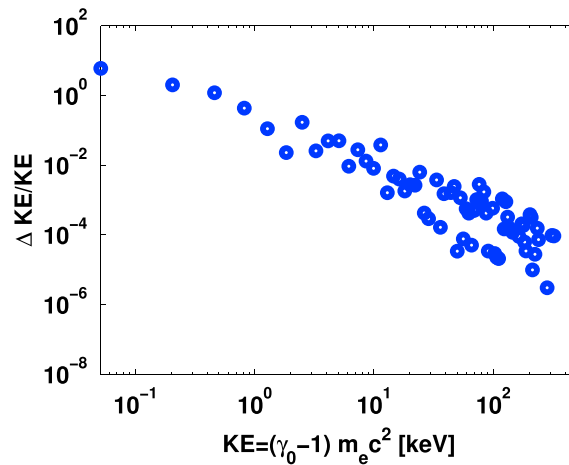
Figure 3. Average gain in kinetic energy as a function of initial kinetic energy computed for relativistic equations and for kinetic energies up to 500 keV. The change in kinetic energy $\frac{\Delta K}{K}$ for electrons of 1 keV and below is of the order of 1–100, whereas electrons with $K_0 > 10$ keV experience a gain of less than 1%. Since relativistic effects in the equation of motion becomes apparent for $K_0 \sim 100$ keV, the Hamiltonian analysis for nonrelativistic energies appears qualitatively valid for relativistic energies.

are quasi-stationary, we set $\nu \leq 1$. We also note from Figure 1 that double layers primarily impact core electrons if ν is increased to larger integer values. The estimate of the phase-averaged change in action, $\sqrt{<\Delta J^2>}/2\pi E_0$ $\sim \exp(-\frac{J_k}{d^2})[I_{q_-}(\frac{J_k}{d^2}) - I_{q_+}(\frac{J_k}{d^2})]$ plotted in Figure 1, indicates that electrons with parallel energies much greater than thermal speed will be neither accelerated nor decelerated by double layers; i.e., for parallel momentums $p_\| \gg mv_{th}$ the change in action converges to zero. We can now numerically integrate the equation of motion to test the above analytical estimate.

2.2. Numerical Estimation of Double Layers Energization

Using equations (2) and (3), we numerically computed the change in parallel kinetic energy for an electron interacting with a double layer electric field. The result is plotted in Figure 2 for double layers' electric fields $E_0 = 10$ mV/m (blue) and $E_0 = 100$ mV/m (red). For thermal velocities $v_{th} \sim 3000$ km/s, the energization for E_0 in the range (10, 100) mV/m is of the order of $\Delta v \sim (\sqrt{10}, 10)$. We also notice from (2) that particles with $v_\| \gg v_{th}$ are not energized by the DL. Energetic electrons with parallel energy $mv^2 \gg q\phi_{DL}$ are unperturbed by the spatially narrow field of the DL. The analytical estimate of previous section is therefore confirmed in that only orbits with $v_\| \leq v_{th}$ are efficiently accelerated along the background field. Hence, the double layers affect the core distribution function of electrons in the radiation belts, rather than the tail, and are very efficient in providing a seed population of the order of 1 keV. For several interactions with large electric fields ($E_0 \geq 100$ mV/m) electrons can reach 10 keV, but even such threshold is difficult to reach for double layers properties described by *Mozer et al.* [2013].

2.3. Extension to Include Relativistic Effects

The Hamiltonian analysis and the numerical integration of the previous sections were conducted for nonrelativistic electrons. Even though 1–10 keV electrons have kinetic energies 2 orders of magnitude smaller than their rest mass, it is appropriate to apply the same analysis for relativistic equations to test the validity of the nonrelativistic results. Using the normalized variables of equation (4) and rewriting the relativistic momentum $P = \frac{m\gamma v_\|}{mv_{th}}$, we therefore integrate the following equations of motion :

$$\dot{P} = \frac{X}{\gamma} - \Xi[X, \tau] \tag{14}$$

$$\dot{X} = \frac{P}{\gamma} \tag{15}$$

$$\dot{\gamma} = n^2 \frac{P}{\gamma} \Xi[X, \tau] \tag{16}$$

in which the constant $n = \frac{v_{th}}{c} \ll 1$. The result of the integration is shown in Figure 3 for electrons with kinetic energies ranging between a few eV and 500 keV. We notice that the change in kinetic energy $\frac{\Delta K}{K}$ for electrons of 1 keV is of the order of 0.1 keV; that is, gains are of the order of less than 1%. For 100 keV electrons and above, the energy gain is of the order of 0.01%, corresponding to a gain 0.1 keV. While the nonrelativistic Hamiltonian analysis is a simplification, it describes correctly the energization process and relativistic effects do not modify its conclusions.

3. Discussion and Conclusion

Using a Hamiltonian approach, we have theoretically quantified and numerically verified that double layers with electric field amplitudes of the order of 20–100 mV/m and length scales of the order of a few Debye lengths can energize thermal electrons from 25 eV up to energy levels of 1 keV. In order to reach energies of the order of 100 keV, electrons would need to interact with electric field of the order of 350 mV/m. Consequently, double layers should not be able to directly provide for a seed populations of 100 keV necessary for a plethora of other acceleration mechanisms in the radiation belts [*Omura et al.*, 2007; *Summers and Omura*, 2007; *Osmane and Hamza*, 2012; *Artemyev et al.*, 2012; *Osmane and Hamza*, 2014]. However, since double layers are nonlinear structures akin to BGK modes [*Bernstein et al.*, 1957] and since they primarily accelerate small velocity electrons and significantly perturb the core of the thermal distribution, we cannot completely rule out that double layers, through a reconfiguration of the distribution function, and a secondary mechanism, can generate a seed population of 100 keV electrons.

As for the necessity for a self-consistent treatment of electrons-DL interactions, our analysis indicates that we cannot use quasi-linear methods to quantify the effect of DL on the distribution functions [*Diamond et al.*, 2010; *Morales and Lee*, 1974]. In quasi-linear theory, the wave turbulence is generated by the core of the distribution functions, while the wave-particle interaction phenomena of Landau and cyclotron resonance are only significant for the tail of the distributions. The inverse is true for the problem of electrons-DL interactions in the radiation belts, i.e., the origin of the turbulence (double layers) is to be found in the tail [*Mozer et al.*, 2013], whereas the core of the distribution function is severely affected and the tail is, at least to first order, barely affected. (See *Morales and Lee* [1974] for a laboratory plasma example in which a quasi-linear approach is used to describe the effect of localized electric field structure on the velocity distribution function. Unlike the particular case treated hereafter, the DL studied by *Morales and Lee* [1974] only affect ions in the tails and a quasi-linear approach is therefore validated.)

As far as the role of DL in the acceleration of electrons from thermal energies to ultrarelativistic levels is concerned, we note that other wave-particle mechanisms [*Thorne*, 2010] can provide the last energization step between keV levels and a few to several 100 of keV levels. Recent Van Allen Probes measurements are also indicating that a three-step mechanism involving double layers, small-amplitude whistler turbulence, and finally large-amplitude waves could combine to transport thermal electrons to relativistic levels [*Agapitov et al.*, 2014]. However, such questions cannot be fully answered without a clearer understanding of the mechanisms generating and sustaining DL with electron acoustic properties in the radiation belts plasma. Future work will be focusing on the generation of the double layers and the relationship with large-amplitude oblique whistlers.

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